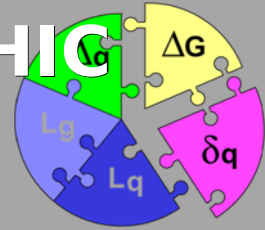


Workshop on opportunities for DY at RHIC

BNL May 10 - 13



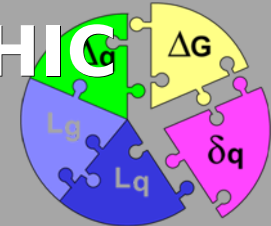
# Sivers function from SIDIS, PP



Jefferson Lab

Workshop on opportunities for DY at RHIC

BNL May 10 - 13



Zhongbo Kang  
RIKEN BNL



**BROOKHAVEN**  
NATIONAL LABORATORY

# Sivers function from SIDIS, PP

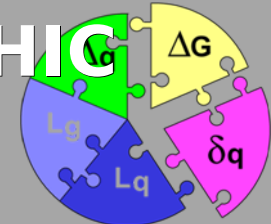
Alexei Prokudin  
Jefferson Laboratory



**Jefferson Lab**

Workshop on opportunities for DY at RHIC

BNL May 10 - 13



Zhongbo Kang  
RIKEN BNL



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**and DY**

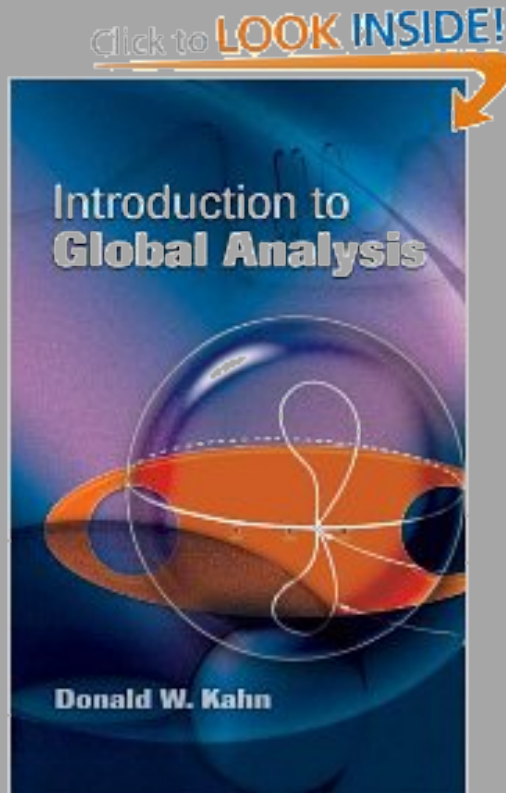
**Sivers function  
from SIDIS, PP**

Alexei Prokudin  
Jefferson Laboratory



**Jefferson Lab**

# Global analysis



# Ingredients: Factorization theorems

## • Related: Factorization Theorems:

- Semi-Inclusive deep inelastic scattering. ✓
- Drell-Yan. ✓
- $e^+/e^-$  annihilation. ✓
- ~~$p + p \rightarrow h_1 + h_2 + X$  !!~~

## • Related: Factorization Theorems:

- Semi-Inclusive deep inelastic scattering. ✓
- Drell-Yan. ✓
- $e^+/e^-$  annihilation. ✓
- $p + p \rightarrow h_1 + h_2 + X$  ✓

## • **TMD** factorization

$$\Lambda_{\text{QCD}}^2 < P_{h\perp}^2 \ll Q^2$$

Sensitive to parton transverse motion.

Talks of John Collins, Piet Mulders, Ted Rogers, Jian-Wei Qiu, George Sterman

## • **Collinear** factorization

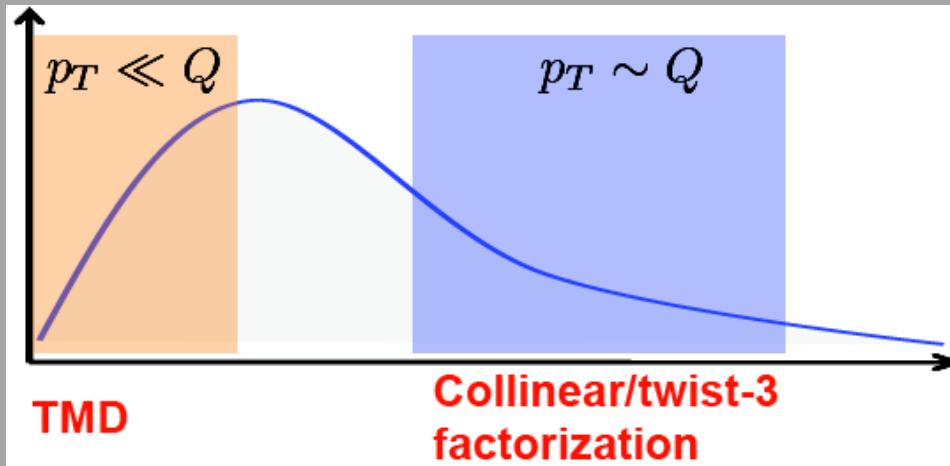
$$\Lambda_{\text{QCD}}^2 \ll P_{h\perp}^2, Q^2$$

Sensitive to multy parton correlations.

Talks of John Collins, Piet Mulders, Ted Rogers, Jian-Wei Qiu, George Sterman

# TMD and Collinear factorizations

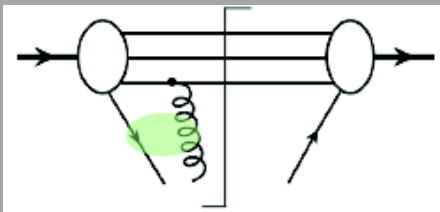
**Both** factorizations are consistent in the overlap region



Talks of John Collins, Piet Mulders, Ted Rogers, Jian-Wei Qiu, Alessandro Bacchetta

**Relation** of multiparton correlations and moments of TMDs

$$\int d^2 p_T \frac{p_T^2}{M} f_{1T}^\perp(x, p_T^2) + \text{UVCT}(\mu^2) = \mathbf{T}_F(x, x, \mu^2) \quad f_{1T}^{\perp(1)} \equiv \int d^2 p_T \frac{p_T^2}{2M^2} f_{1T}^\perp(x, p_T^2)$$

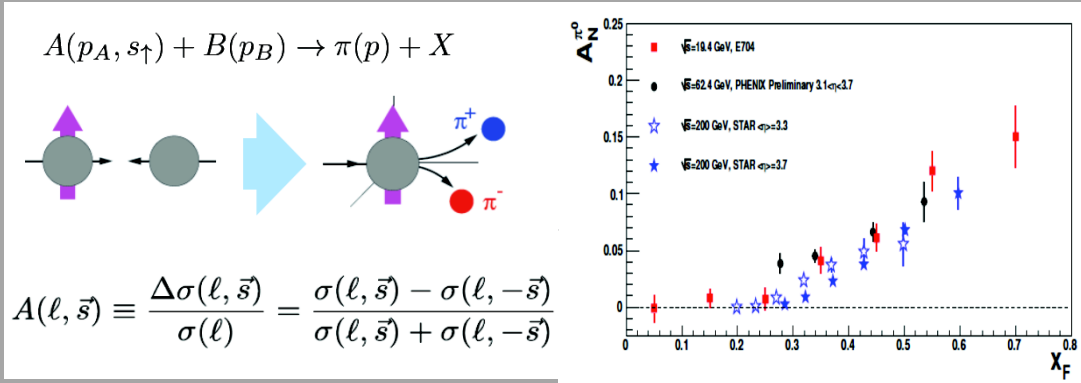


Sivers function is related to TF, but counterterm matters!

Talks of John Collins, Piet Mulders, Ted Rogers, Jian-Wei Qiu

# Data analysis

## Proton Proton



Only **one scale**  $P_T$

Collinear analysis:

Kouvaris, Qiu,

Vogelsang, Yuan (2006)

Kanazava, Koike (2010)

TMD analysis:

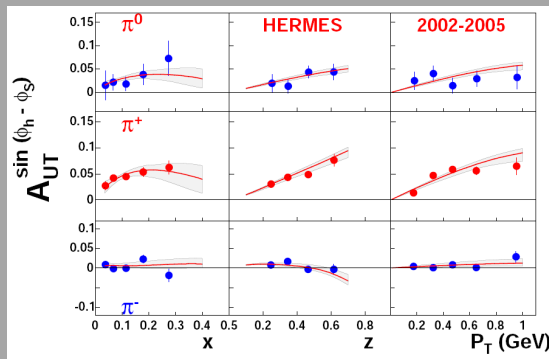
Anselmino et al (2006)

$$A_{UT} = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

$$d\sigma^\uparrow - d\sigma^\downarrow \propto \underbrace{f_{1T}^\perp \otimes D_1 \sin(\phi_h - \phi_S)}_{\text{Sivers effect}}$$

Sivers effect

## SIDIS



Two scales  $P_T, Q$

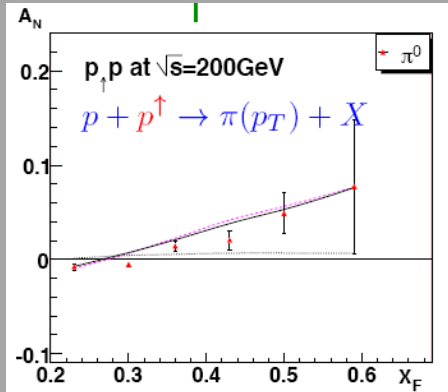
$$\Lambda_{\text{QCD}}^2 < P_{h\perp}^2 \ll Q^2$$

TMD analysis: Anselmino et al (2008);  
Collins et al (2007); Vogelsang, Yuan (2006)

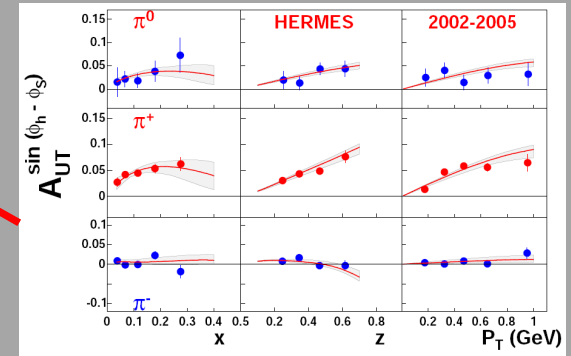
# Comparison of results

Talk of Zhongbo Kang

$$g_s T_F(x, x) = -2M f_{1T}^{\perp(1)}(x)$$



Collinear analysis: Kouvaris, Qiu, Vogelsang, Yuan (2006)



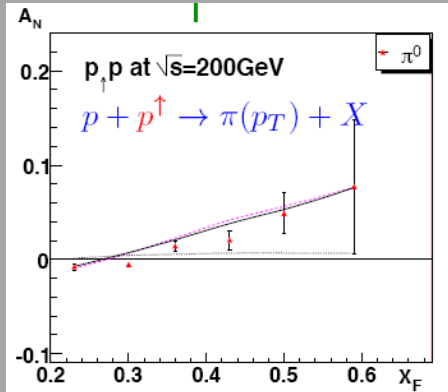
TMD analysis:  
 Anselmino et al (2008)



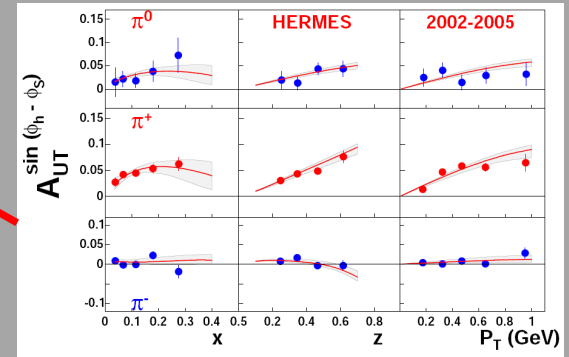
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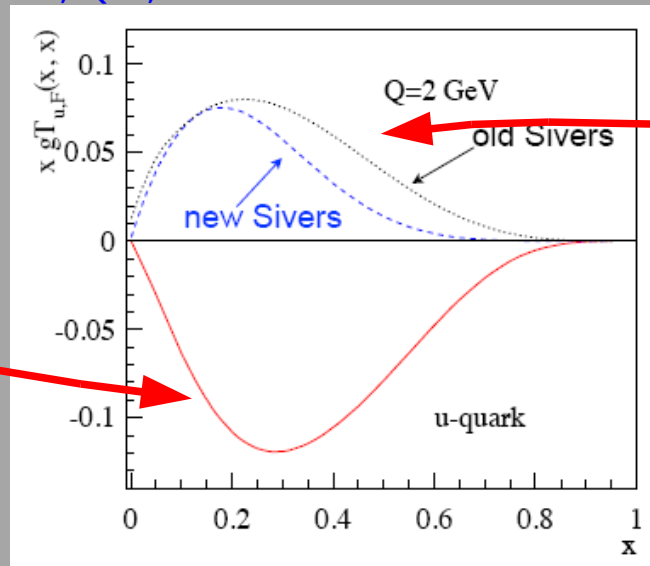


Collinear analysis: Kouvaris, Qiu, Vogelsang, Yuan (2006)



TMD analysis: Anselmino et al (2008)

Compare



Alexei Prokudin - Sivers function from SIDIS and PP

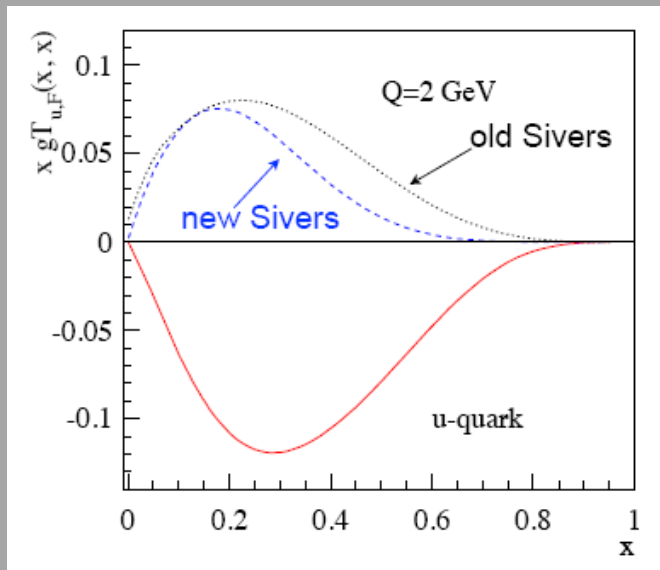
# Comparison of results

Talk of Zhongbo Kang

$$g_s T_F(x, x) = -2M f_{1T}^{\perp(1)}(x)$$

Kang, Qiu, Vogelsang, Yuan arXiv:1103.1591

- Magnitudes are similar
- Sign is **opposite**



# Comparison of results

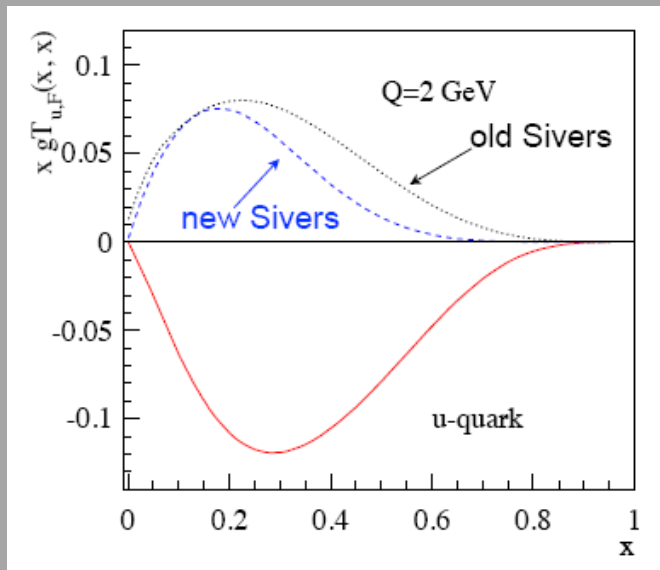
Talk of Zhongbo Kang

$$g_s T_F(x, x) = -2M f_{1T}^{\perp(1)}(x)$$

Kang, Qiu, Vogelsang, Yuan arXiv:1103.1591

- Magnitudes are similar
- Sign is **opposite**

**It is a puzzle!**



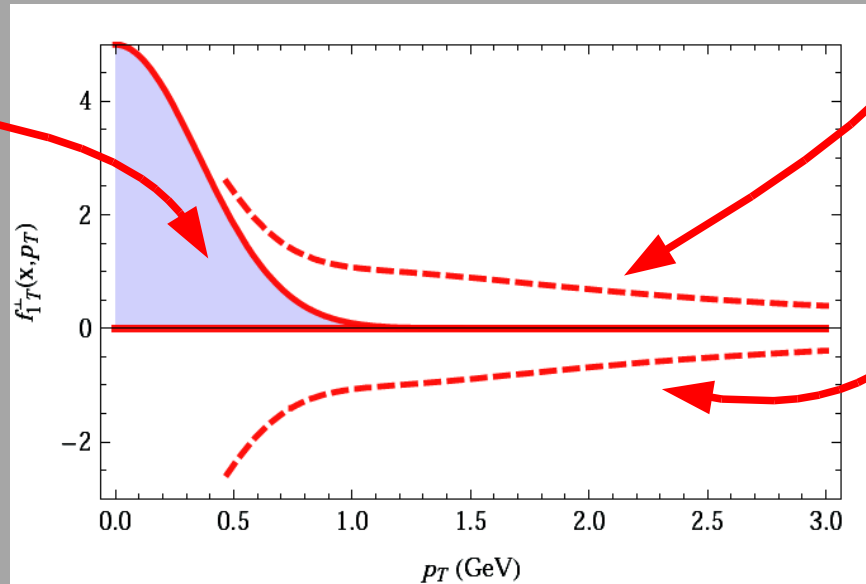
# Possible explanations

Sivers function can have nodes in  $\mathbf{p}_T$ .

Talk of Zhongbo Kang

SIDIS  $\mathbf{T}_F > \mathbf{0}$

Perturbative tail  $\propto \frac{M^2}{p_T^4} \alpha_s$



Bacchetta,  
Boer, Deihl,  
Mulders 2008

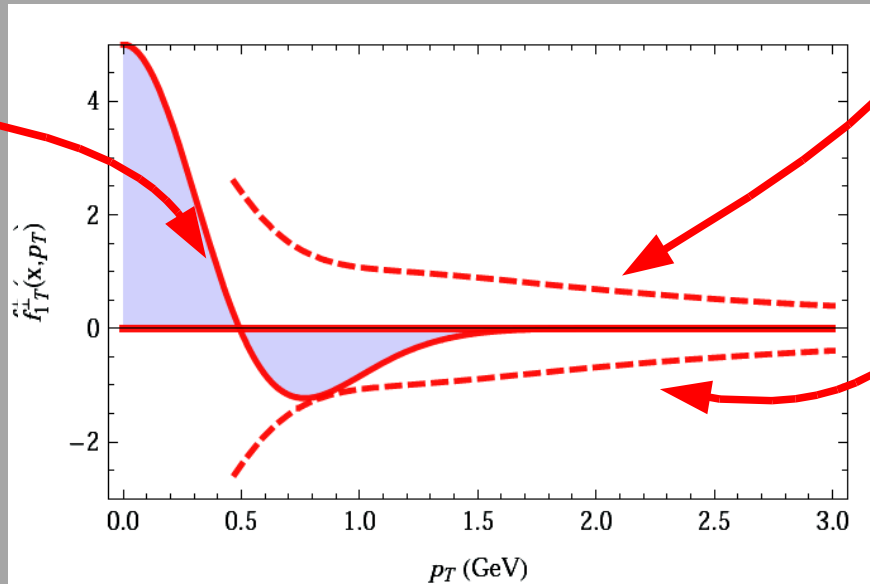
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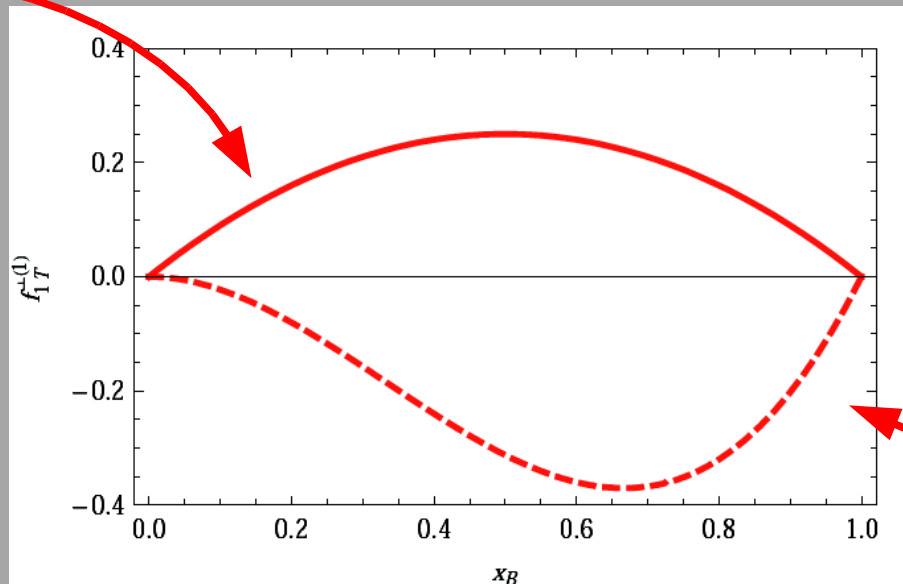
# Possible explanations

Sivers function can have nodes in  $x$ .

Boer (2011)

Bacchetta et al, model calculation (2010)

SIDIS



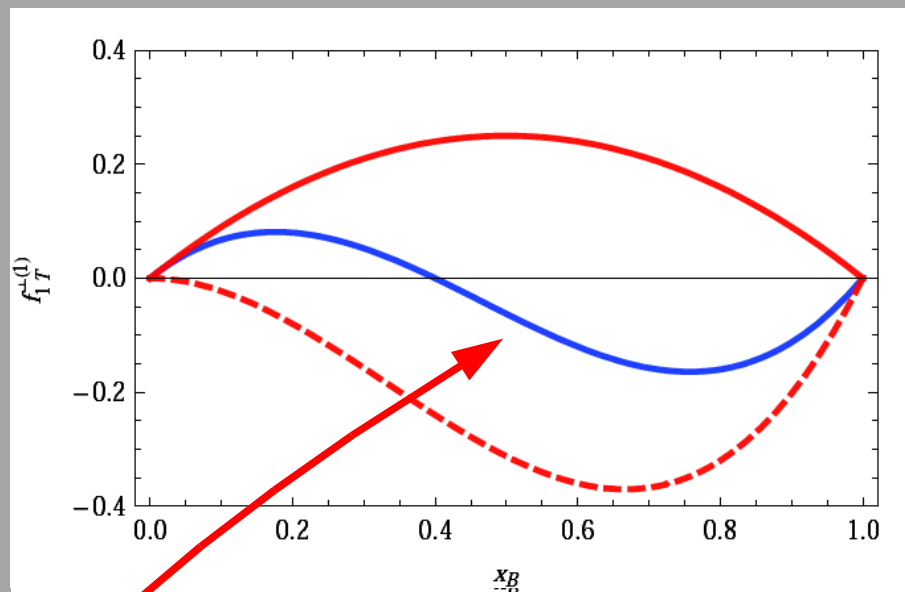
PP

# Possible explanations

Sivers function can have nodes in  $x$ .

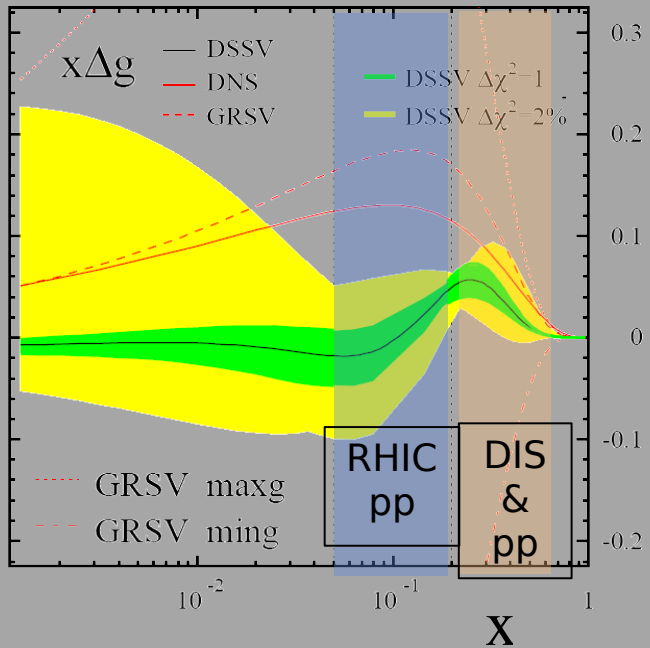
Boer (2011)

Bacchetta et al, model calculation (2010)



If PP and SIDIS probe different regions of  $x$

# Are nodes so strange?

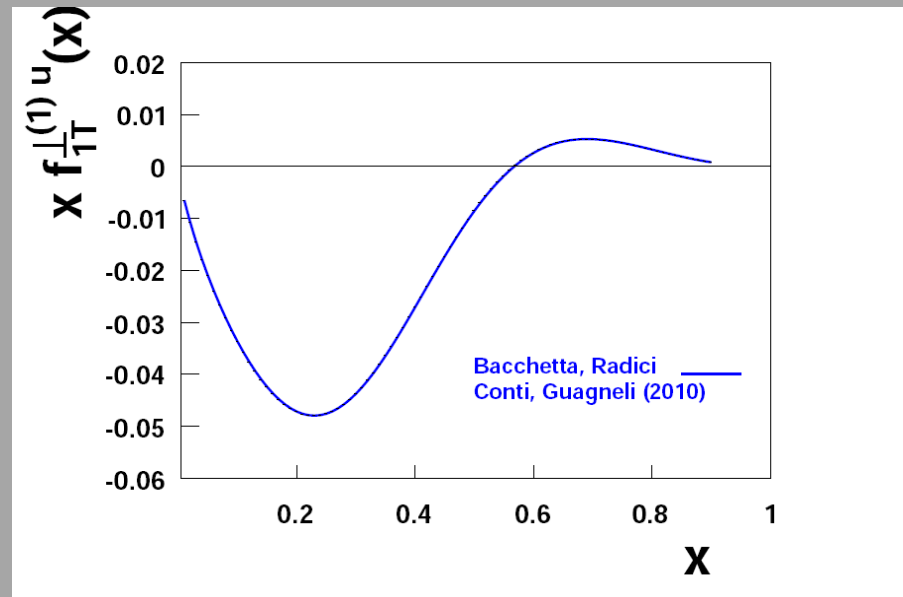


Node in  $\Delta g(\mathbf{x})$  from DSSV global fit [De Florian, Sassot, Stratmann, Vogelsang](#)

$$\Delta f \propto f(\mathbf{S}) - f(-\mathbf{S})$$

[Talk of Jian-wei Qiu](#)

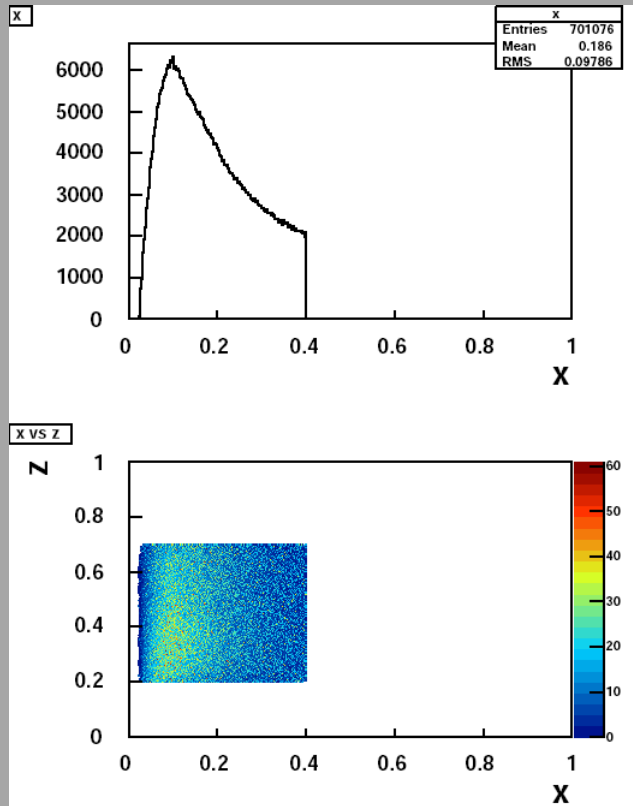
Node in Siverson function  
[Bacchetta, Radici, Conti, Guagneli \(2010\)](#)





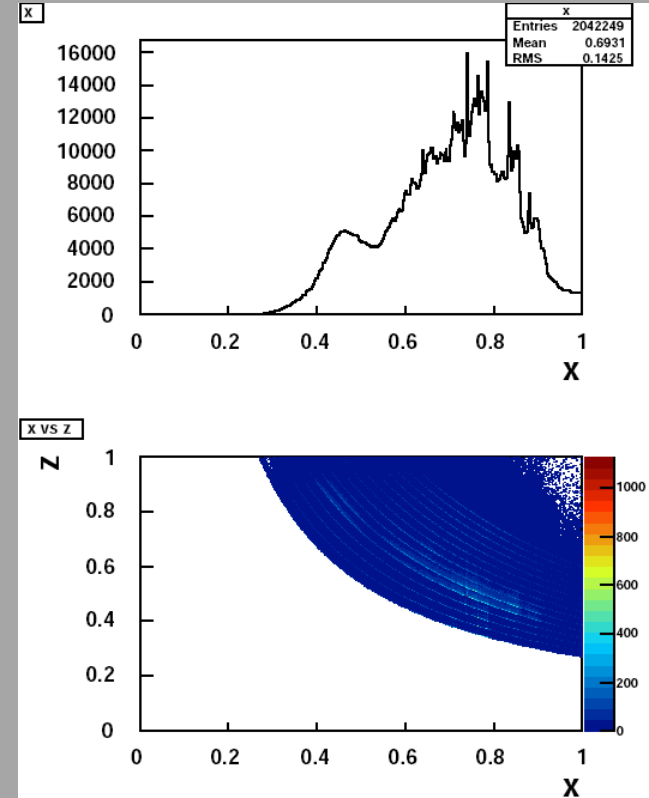
# SIDIS vs PP kinematics

## SIDIS HERMES



$x < 0.4$

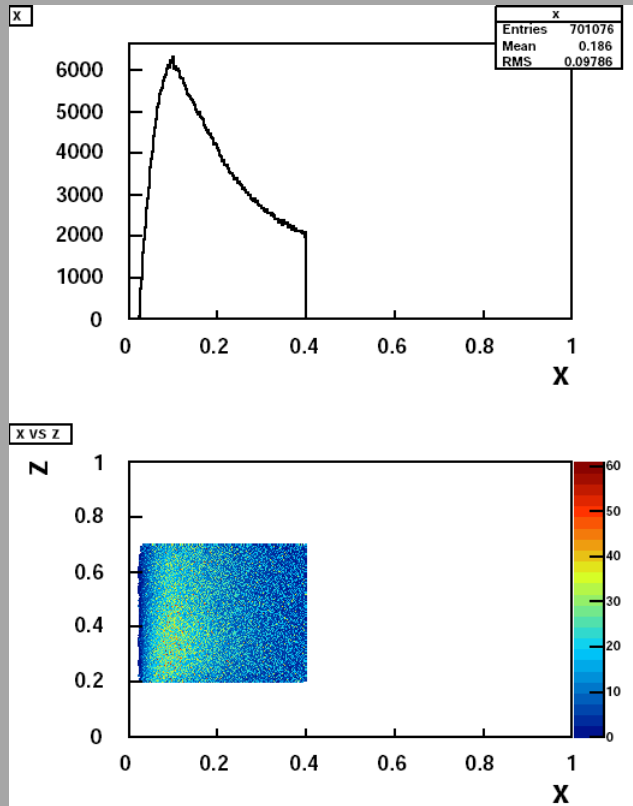
## PP STAR



$x > 0.4$

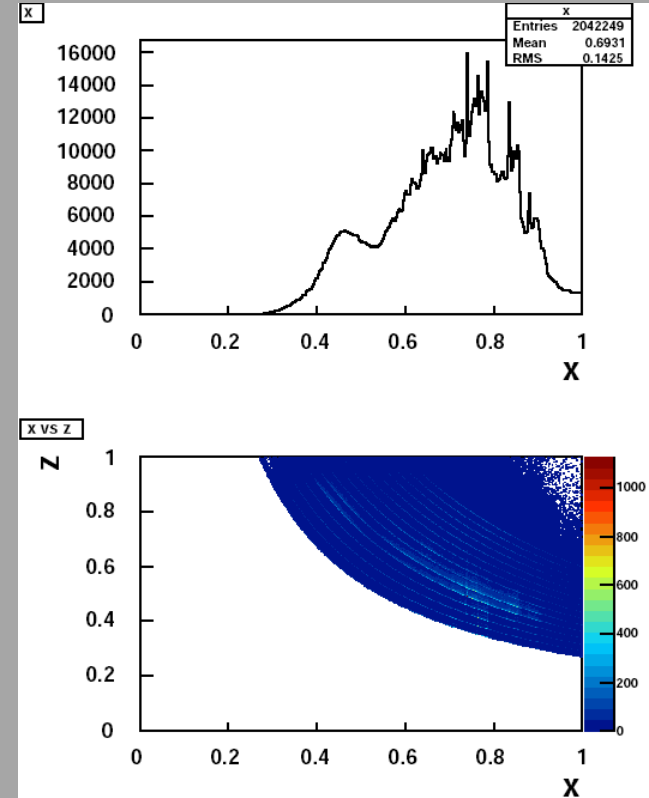
# SIDIS vs PP kinematics

## SIDIS HERMES



$$x < 0.4$$

## PP STAR



$$x > 0.4$$

SIDIS and PP probe different regions in  $x$  !

# We are looking at the same function



... from different perspectives!

# Parametrization

$$\mathbf{f}_{1\mathbf{T}}^{\perp\mathbf{q}} \propto \mathbf{x}^{\alpha_{\mathbf{q}}} (\mathbf{1} - \mathbf{x})^{\beta_{\mathbf{q}}} (\mathbf{1} - \eta_{\mathbf{q}}\mathbf{x})$$

as in [De Florian, Sassot, Stratmann, Vogelsang \(2009\)](#)

$\mathbf{1} - \eta_{\mathbf{q}}\mathbf{x}$  has a node if  $\eta_{\mathbf{q}} > 0$

**SIDIS: HERMES, COMPASS data**  $\pi^{\pm}, \mathbf{K}^{\pm}$

$$\mathbf{A}_{\mathbf{UT}}^{\sin(\Phi_{\mathbf{h}} - \Phi_{\mathbf{S}})} \sim \mathbf{f}_{1\mathbf{T}}^{\perp} \otimes \sigma \otimes \mathbf{D}_1$$

**PP: STAR data**  $\pi^0$

$$\mathbf{A}_{\mathbf{N}} \sim \mathbf{T}_{\mathbf{F}} \otimes \sigma \otimes \mathbf{D}_1$$

using [PDF GRV98](#) and [FF DSSV](#)

# Parametrization

$$\mathbf{f}_{1T}^{\perp q} \propto \mathbf{x}^{\alpha_q} (\mathbf{1} - \mathbf{x})^{\beta_q} (\mathbf{1} - \eta_q \mathbf{x})$$

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**TMD**

**PP: STAR data**  $\pi^0$

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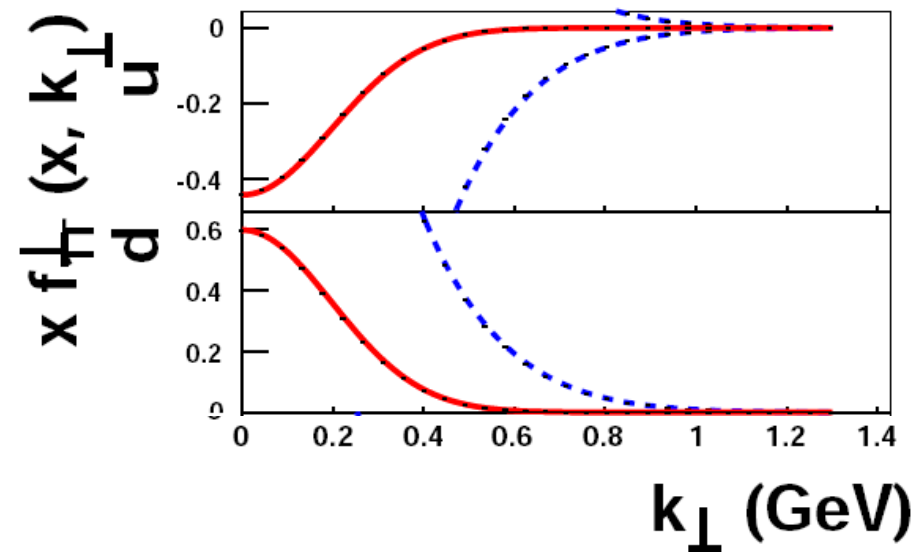
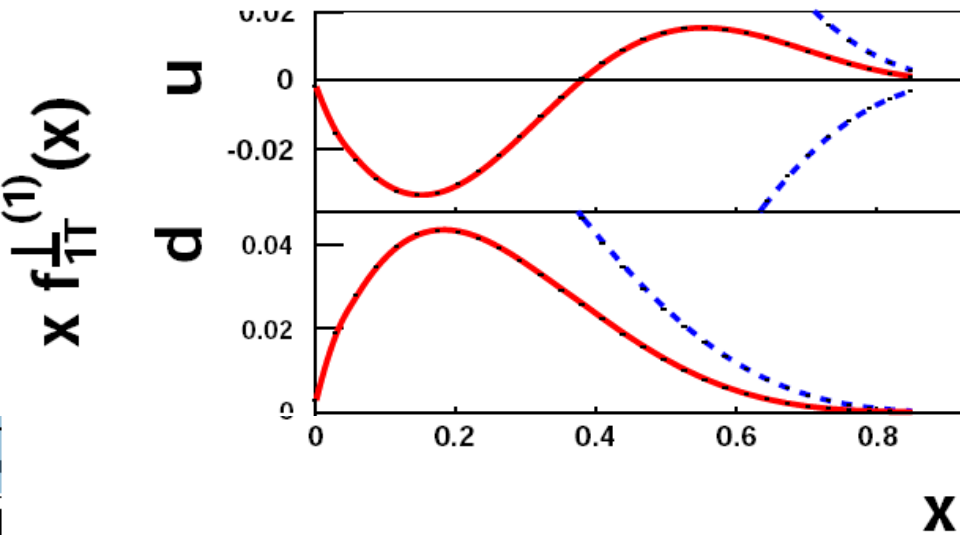
**PP: STAR data**  $\pi^0$

$$\mathbf{A}_N \sim \mathbf{T}_F \otimes \sigma \otimes \mathbf{D}_1$$

**Twist-3**

using [PDF GRV98](#) and [FF DSSV](#)

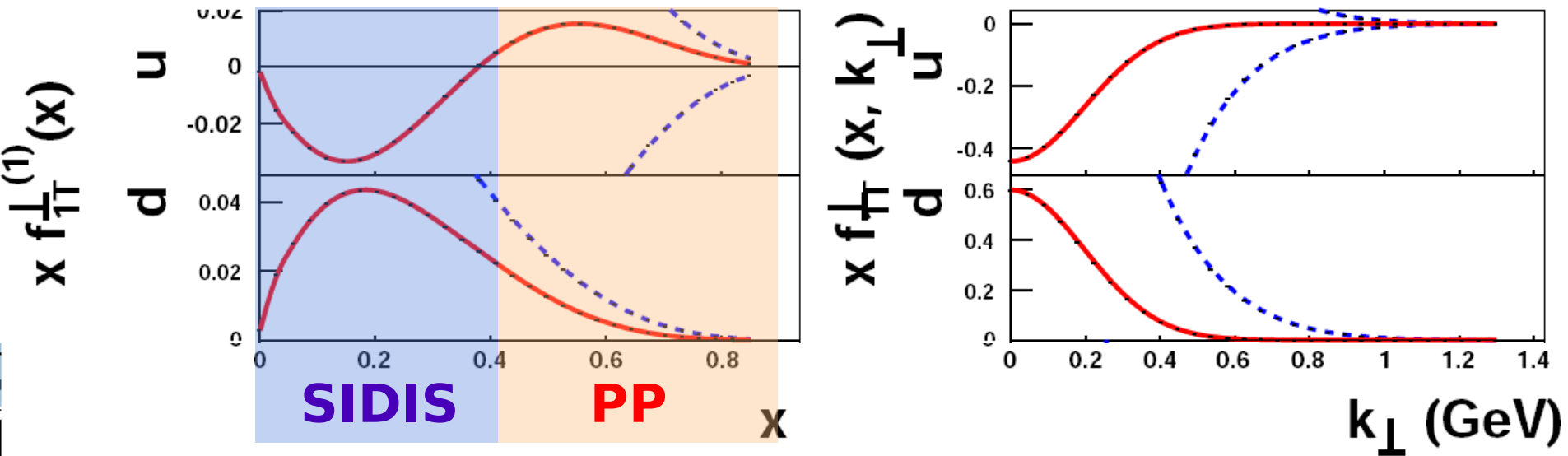
# Results: Sivers function



**Sivers function has a node!**

$$x_{\text{node}} \sim 0.4$$

# Results: Sivers function

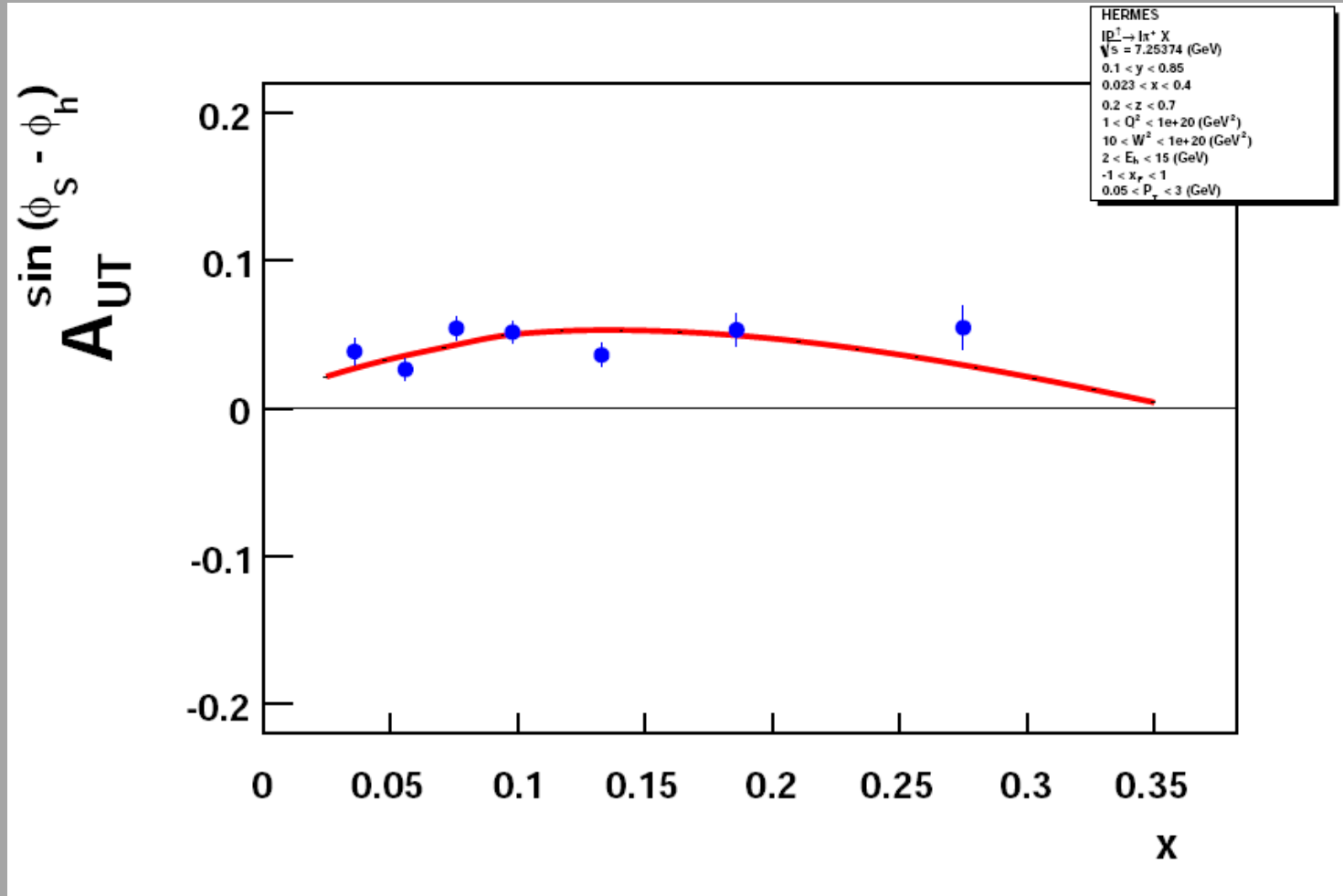


**Sivers function has a node!**

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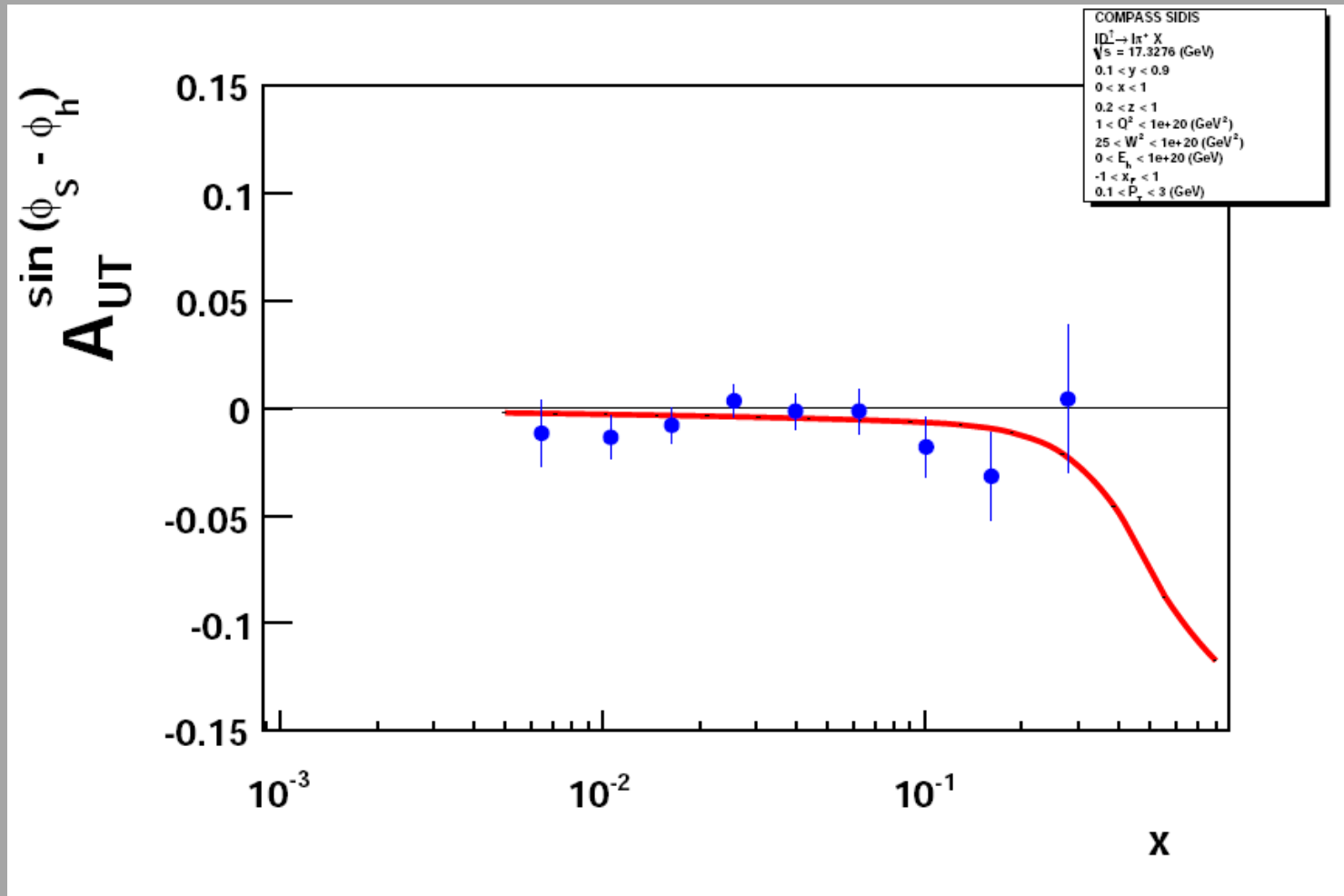


# Results: SIDIS



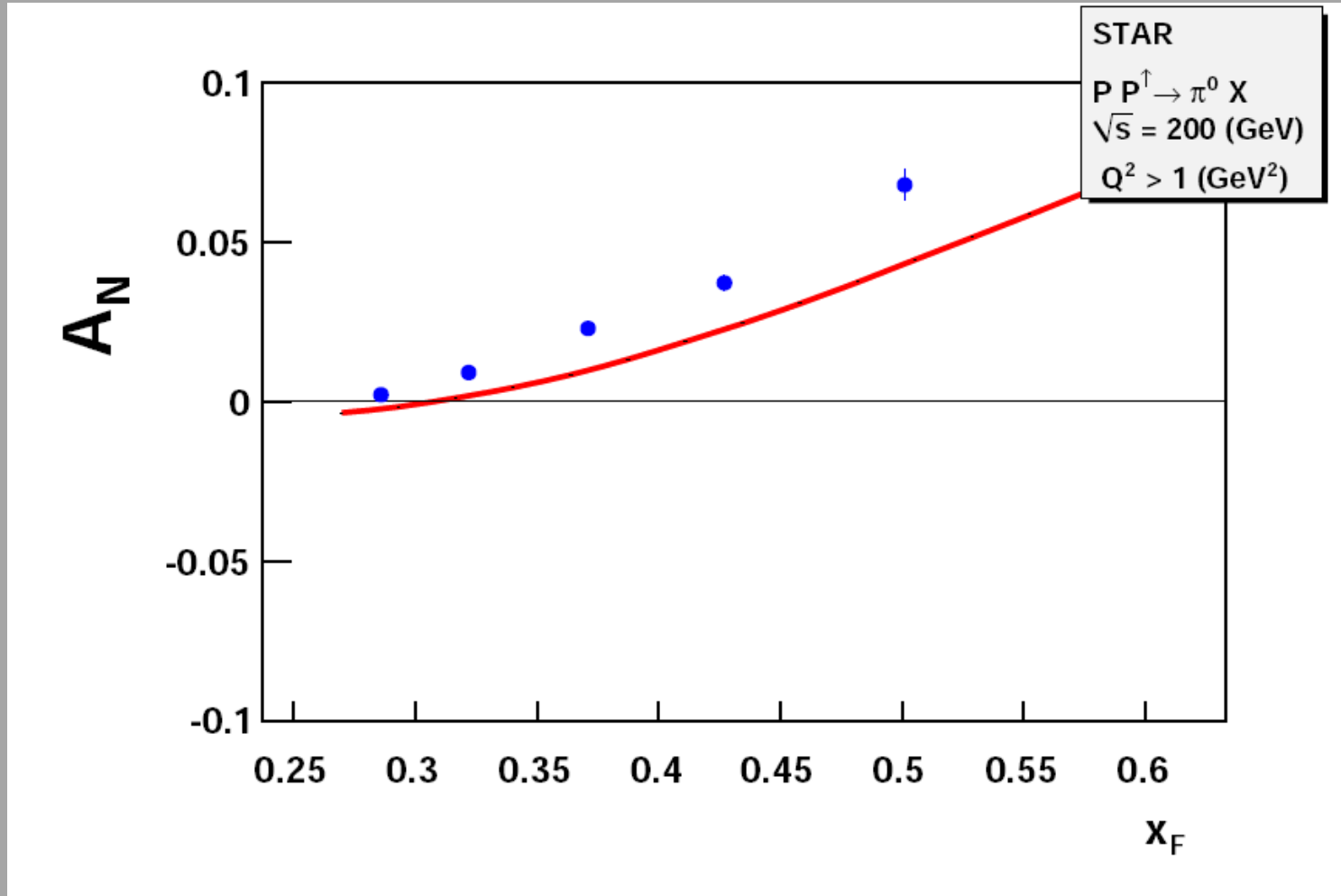
**HERMES data, good description**

# Results: SIDIS



**COMPASS data, good description**

# Results: PP



**STAR data  $\pi^0$ ,  $y = 3.7$ , reasonable description  
... but something is missing!**

# What is missing?

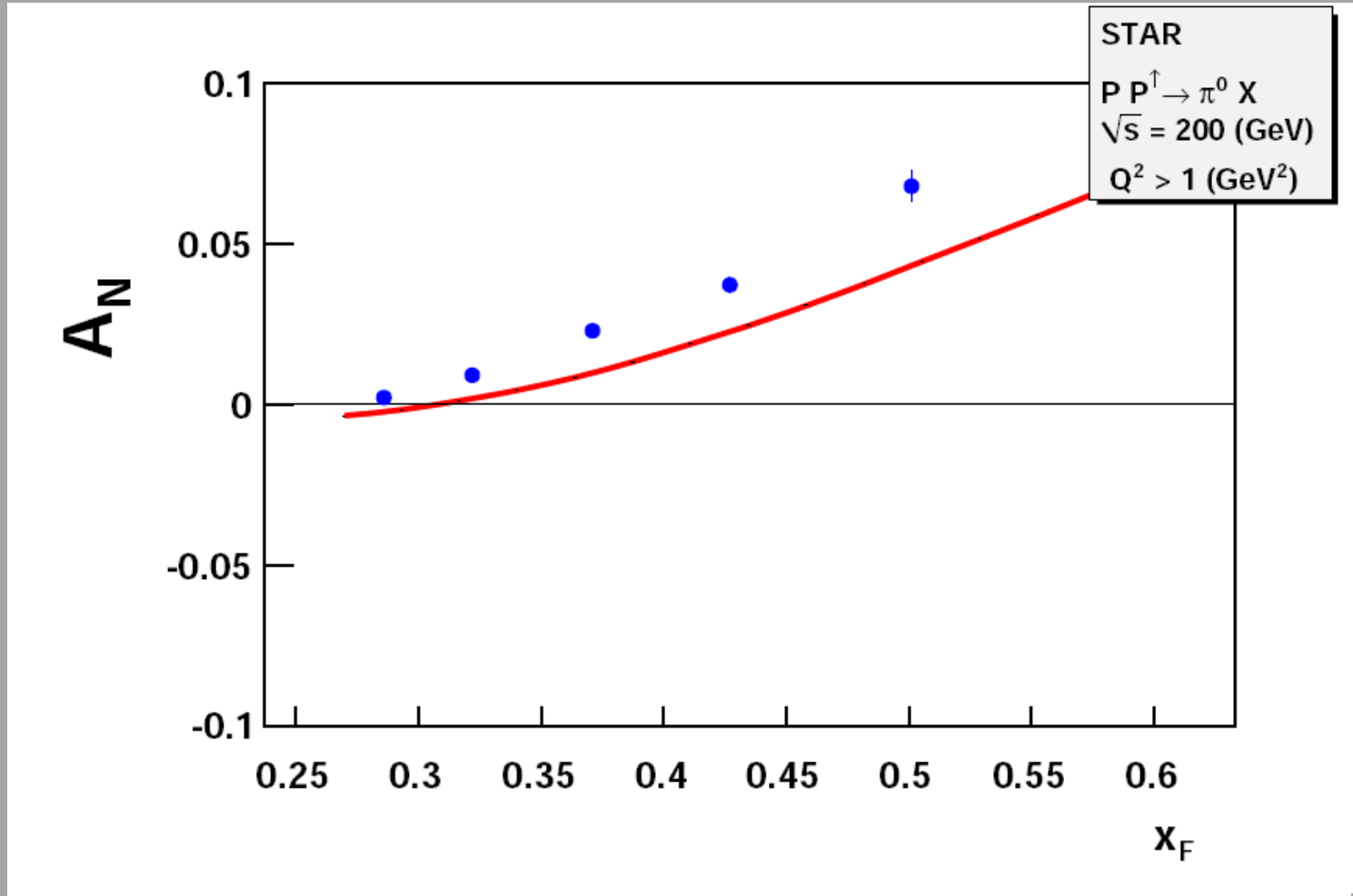
Twist-3 formalism:

$$\mathbf{A}_N \sim \mathbf{T}_F \otimes \sigma \otimes \mathbf{D}_1 + \mathbf{h}_1 \otimes \sigma \otimes \mathbf{H}_F + \dots$$

We considered only Sivers effect, SGP. Other parts should be added: sea-quarks, SFP contribution, Collins effect etc.

For **Global analysis** one should combine SIDIS, PP and  $e^+e^-$  data

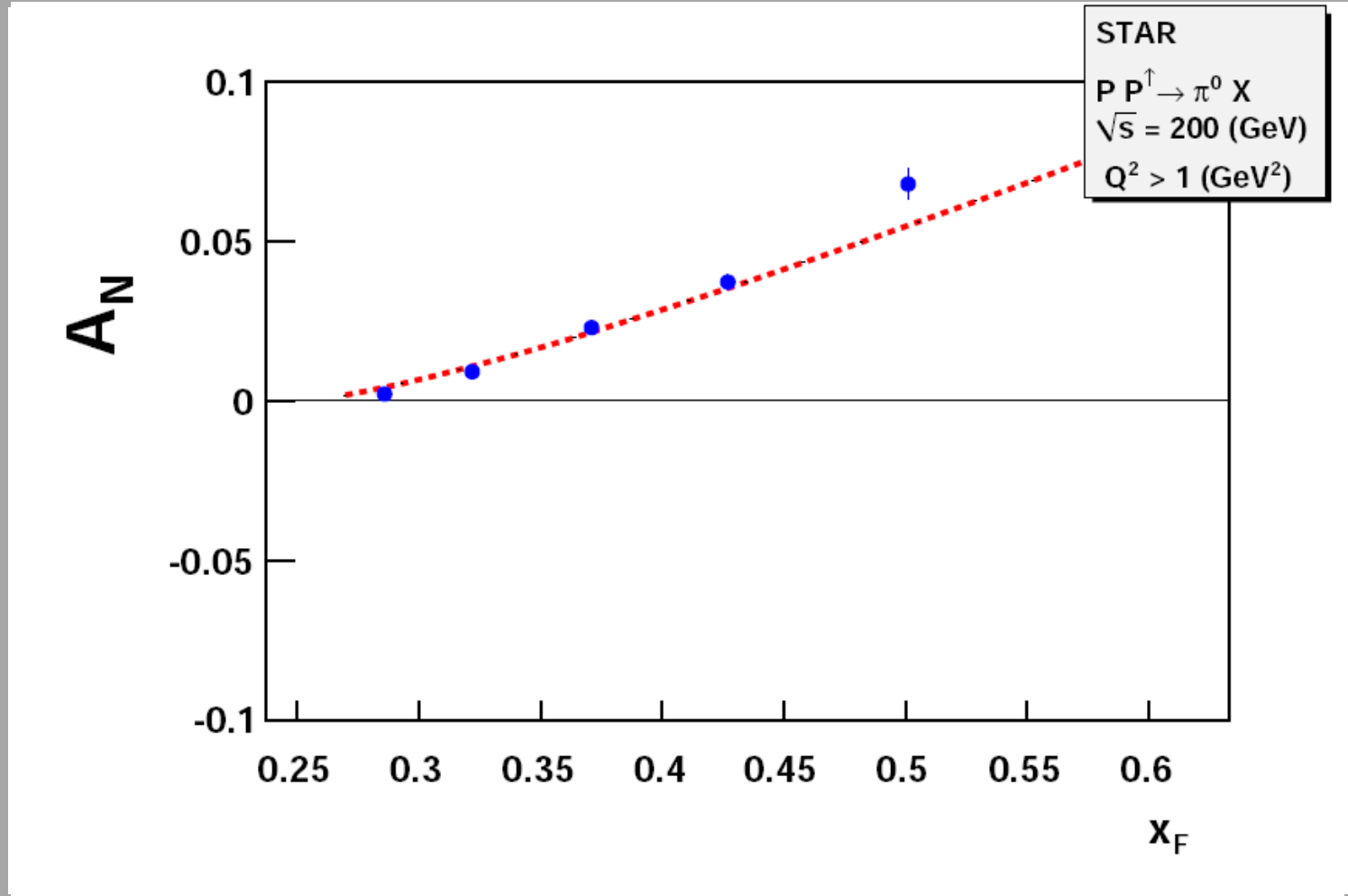
# PP scale dependence



**STAR data**

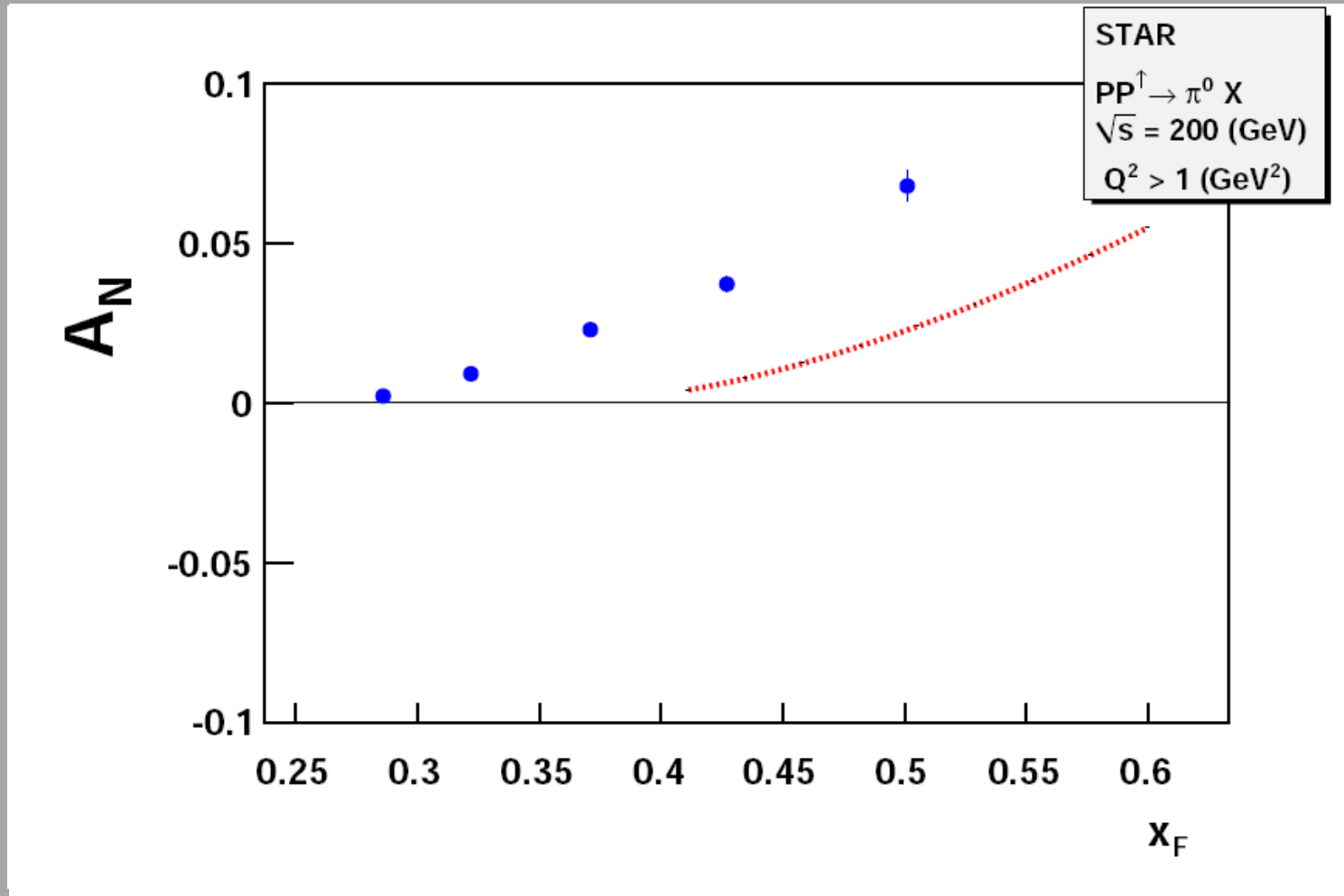
$$Q = P_T$$

# PP scale dependence



**STAR data**  $Q = 2P_T$

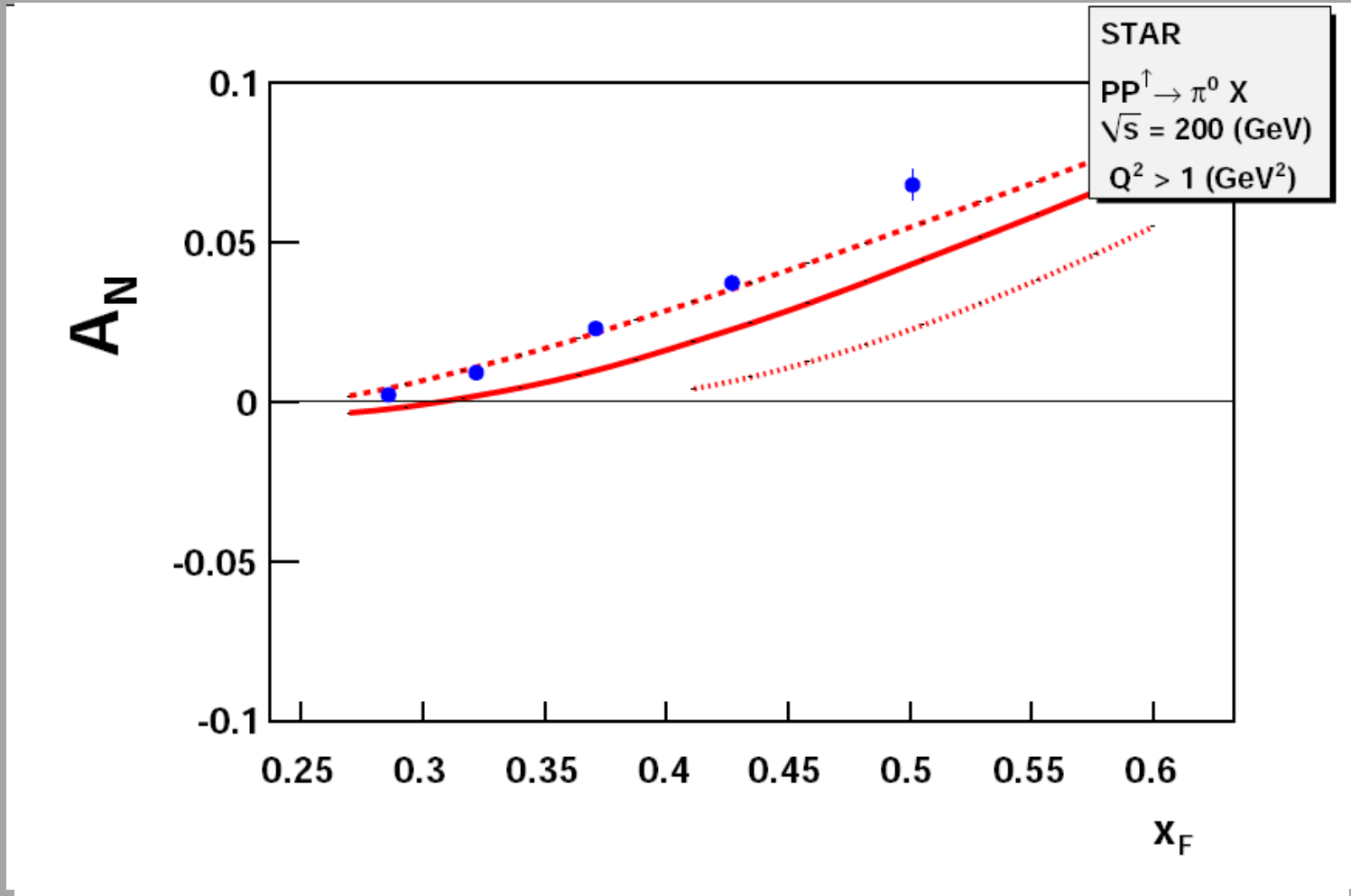
# PP scale dependence



**STAR data**

$$Q = P_T/2$$

# PP scale dependence



**STAR data**  $Q = P_T/2 \dots 2P_T$

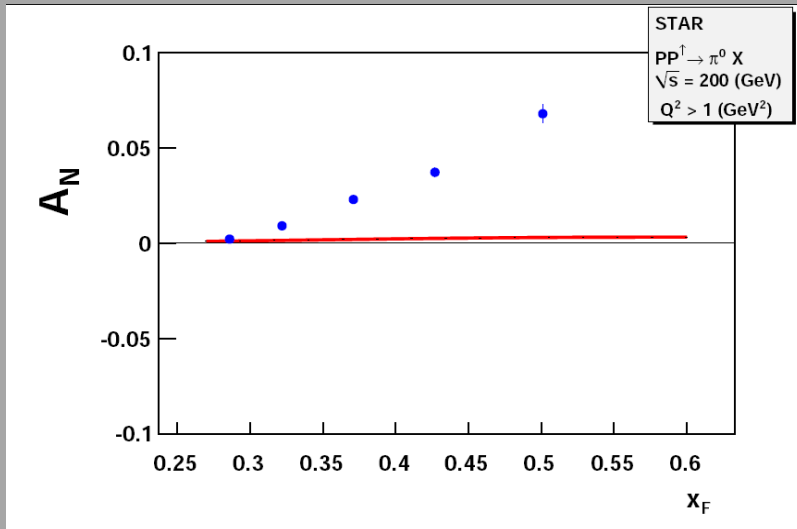


# Twist-3 Collins effect

$$\mathbf{A}_N \sim \mathbf{h}_1 \otimes \sigma \otimes \mathbf{H}_F$$

Where twist-3 function is related to Collins FF

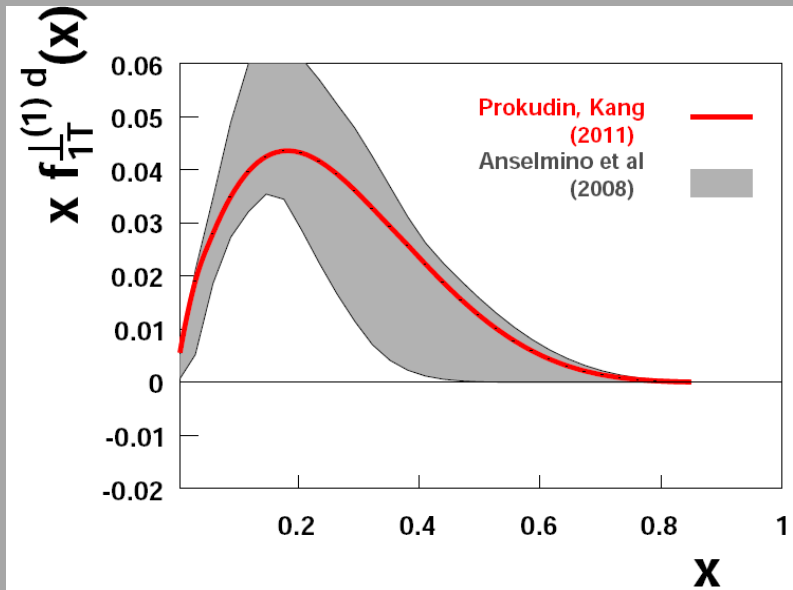
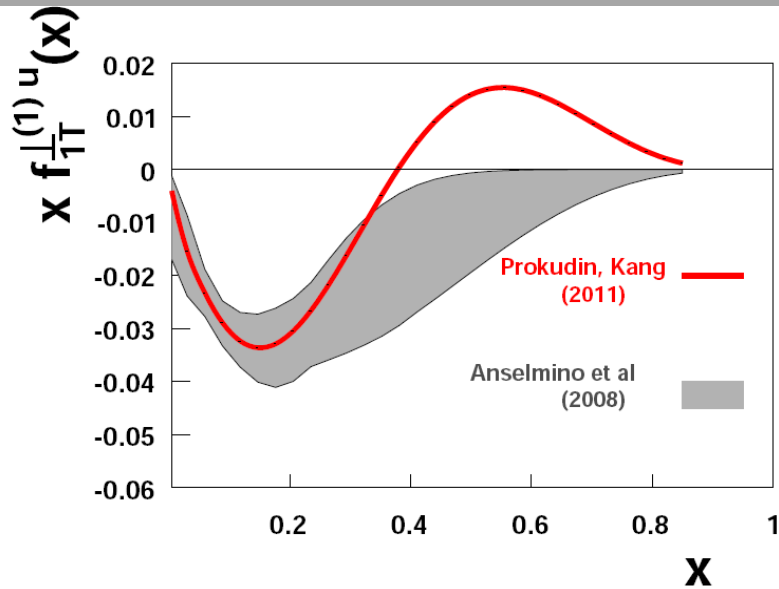
Kang, Yuan, Zhou (2009)  $\mathbf{H}_F = -\frac{zM_h}{2} \mathbf{H}_1^\perp(1)$



STAR data,  
Collins function from  
Anselmino et al (2008)

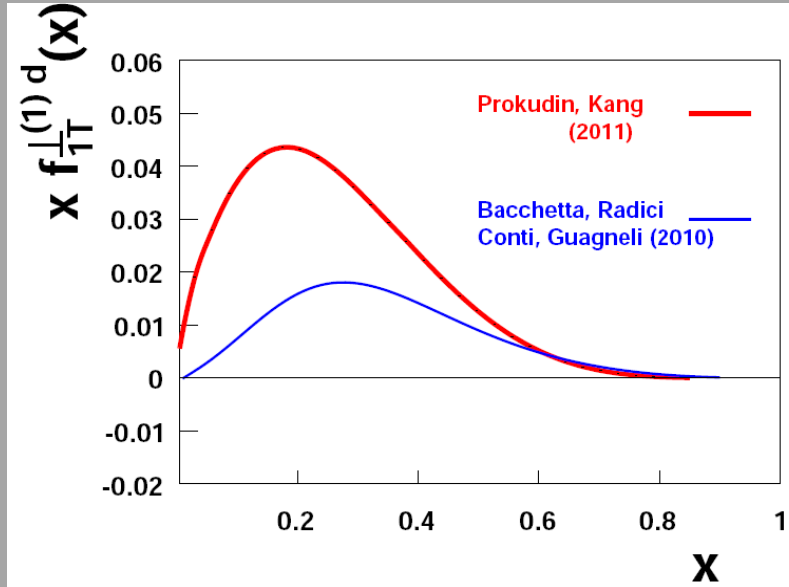
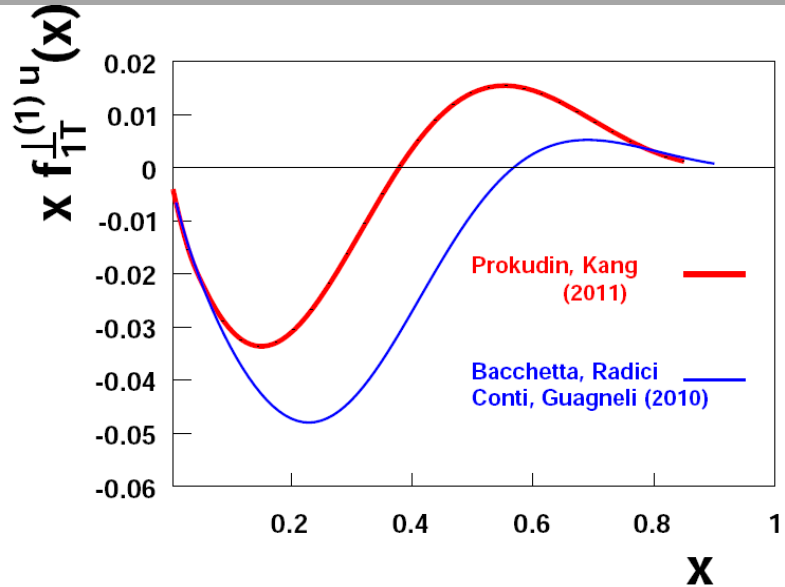
**Collins effect is suppressed, but  
z > 0.5 region is not yet explored**

# Comparison with Anselmino et al



**Comparison with Sivers function extracted from SIDIS only**

# Comparison with models



**This result compared with quark -diquark model calculation**

# Drell Yan

$$A_N = \frac{\sum_q f_{1T}^{\perp q}(\mathbf{x}_1, \mathbf{p}_T) \otimes f_1^{\bar{q}}(\mathbf{x}_1, \mathbf{p}_T) \sigma_{q\bar{q}}}{\sum_q f_1^q(\mathbf{x}_1, \mathbf{p}_T) \otimes f_1^{\bar{q}}(\mathbf{x}_1, \mathbf{p}_T) \sigma_{q\bar{q}}}$$

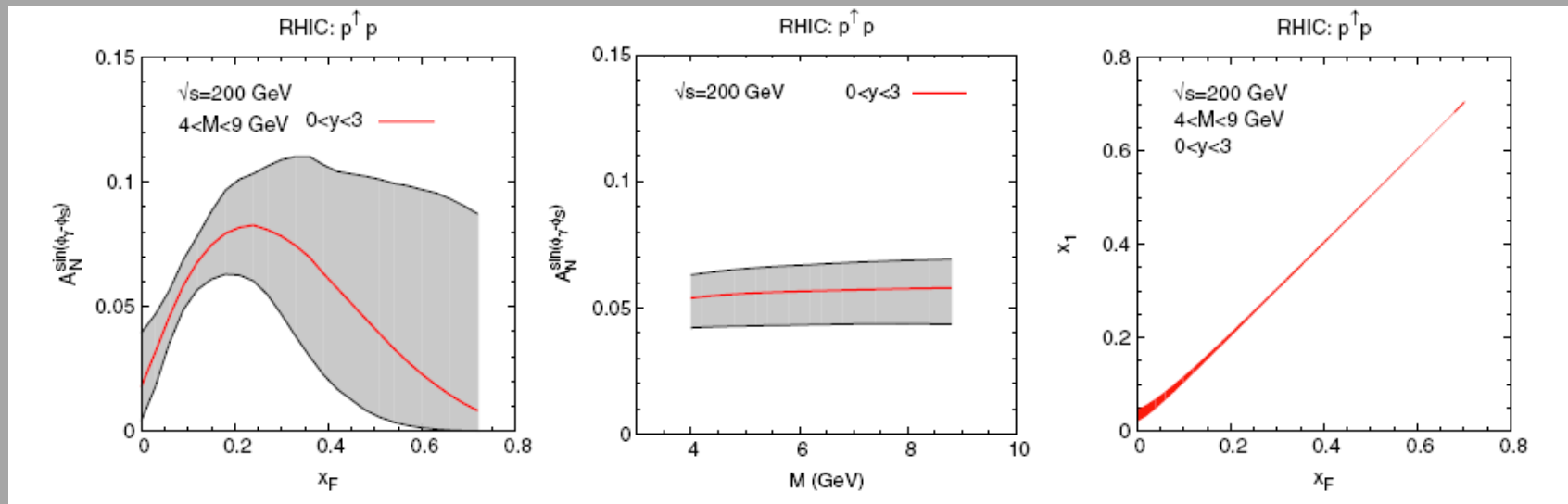
**Analysis at LO in hadronic cm frame**

Anselmino et al (2009)

$$\mathbf{x}_1 = \frac{\mathbf{x}_F + \sqrt{\mathbf{x}_F^2 + 4M^2/s}}{2} \approx \mathbf{x}_F$$

**In DY we probe Siverson function at  $\mathbf{x}_F$**

Anselmino et al (2009)



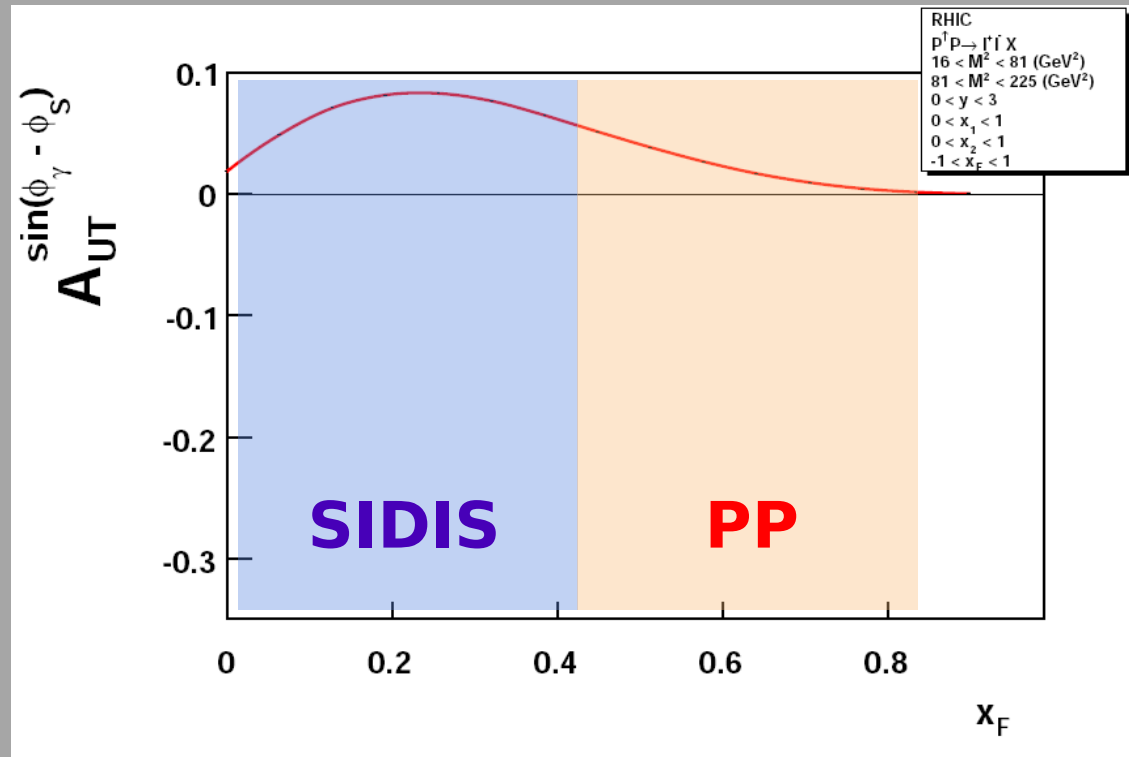
# Drell Yan

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**Analysis at LO in hadronic  
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Kang, AP (2011)

**Anselmino et al 2009**



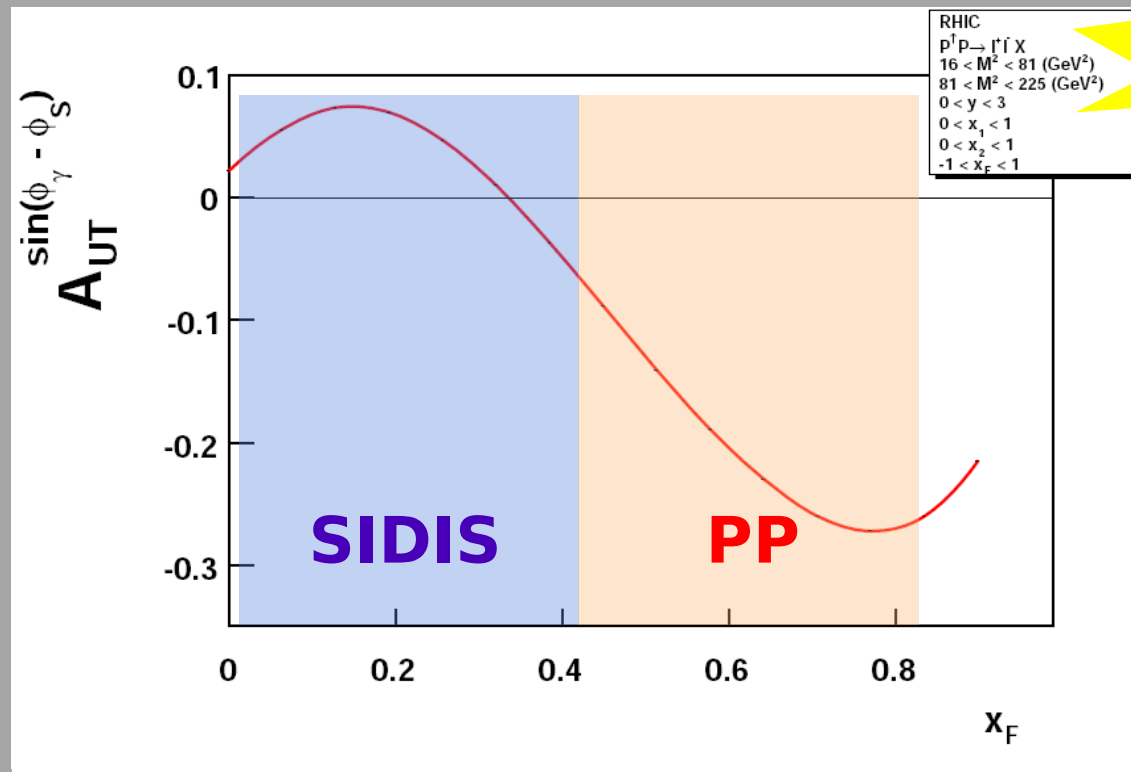
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**Analysis at LO in hadronic  
cm frame**

Kang, AP (2011)

**AP, Kang 2011**



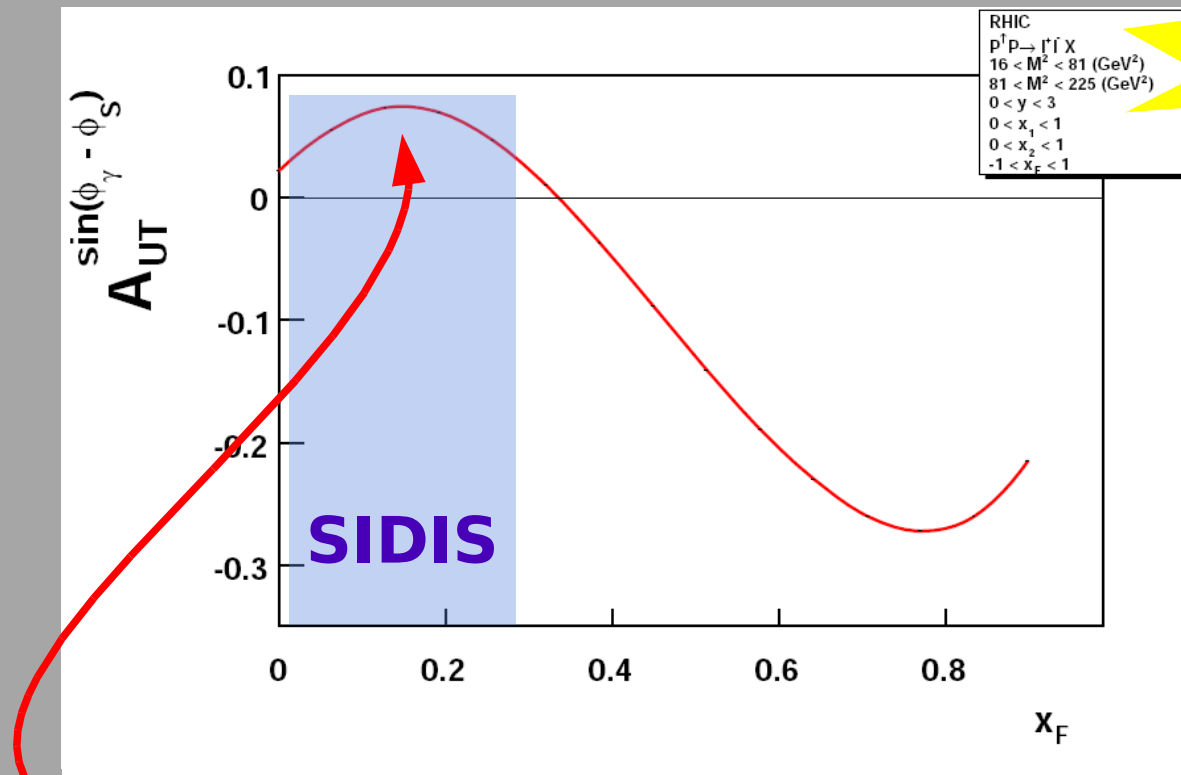
# Drell Yan

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**Analysis at LO in hadronic cm frame**

Kang, AP (2011)

**AP, Kang 2011**



**To measure in order to check**

$$- f_{1T}^{\perp} |_{DY} = f_{1T}^{\perp} |_{SIDIS}$$

Alexei Prokudin - Sivers function from SIDIS and PP

# Future of TMD&Twist-3 phenomenology

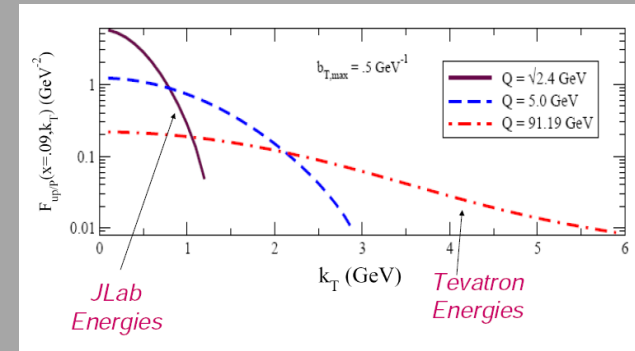
**Global analysis of SIDIS, PP and  $e^+e^-$  data using TMD and twist-3 formalisms.**

Kang, AP (2011), others ...

**TMD phenomenology:**

**NLO accuracy**

Collins (2011), Aybat, Rogers (2011), ...



**Twist-3 phenomenology:**

**NLO accuracy of hard functions**


Vogelsang, Yuan (2009), ...

$$A_N \propto \Delta\sigma(Q, S_{\perp}) \propto T_f^{(3)}(x, x) \otimes \hat{H}_f \otimes \dots$$

**Beyond LO!**



# CONCLUSIONS

- Sivers function may have a node 
- SIDIS,  $e^+e^-$  and PP data can be used in global analysis
- $x_F \sim 0 \dots 0.2$  is “safe” for DY measurement

# CONCLUSIONS

- Sivers function may have a node
- SIDIS,  $e^+e^-$  and PP data can be used in global analysis
- Working together



... we can resolve puzzles!