Unitary Coupled-Channel Model for Heavy Meson Decays into Three Mesons

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Experiment

↓

Interesting physics

Experiment

e.g., γN, πN ➔ $\pi \pi \pi N$

E852@BNL, GlueX@JLab

Interesting physics

Excited (exotic) meson properties

(J^{PC}, mass, width, ...)

Experiment

e.g., $B \rightarrow D\,K \rightarrow (\pi\pi\,K)\,K$

Belle, BABAR, LHCb

Interesting physics

CKM CP-violating phase

Experiment

✔ Raw data analysis (signal events selected)

➔ Data (cross section, polarization observables…)

✔ Data analysis

e.g.,

☛ Partial wave decomposition

☛ Analytic continuation to unphysical region

☛ Removing final state interaction effects

Reliable theoretical analysis tool is essential !

Interesting physics

Analysis of three-meson productions

e.g,

$\pi N \rightarrow \pi\pi\pi N$ ➔ **Excited (exotic) meson properties**

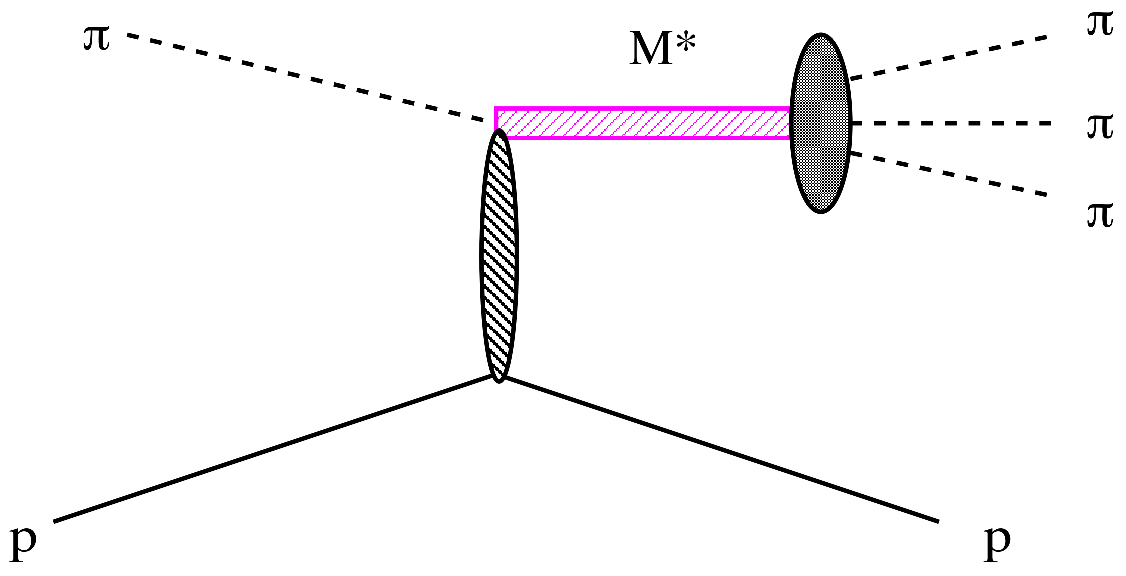
$B \rightarrow D K$

$\rightarrow (\pi\pi K) K$ ➔ **CKM CP-violating phase**

Analysis tool ?

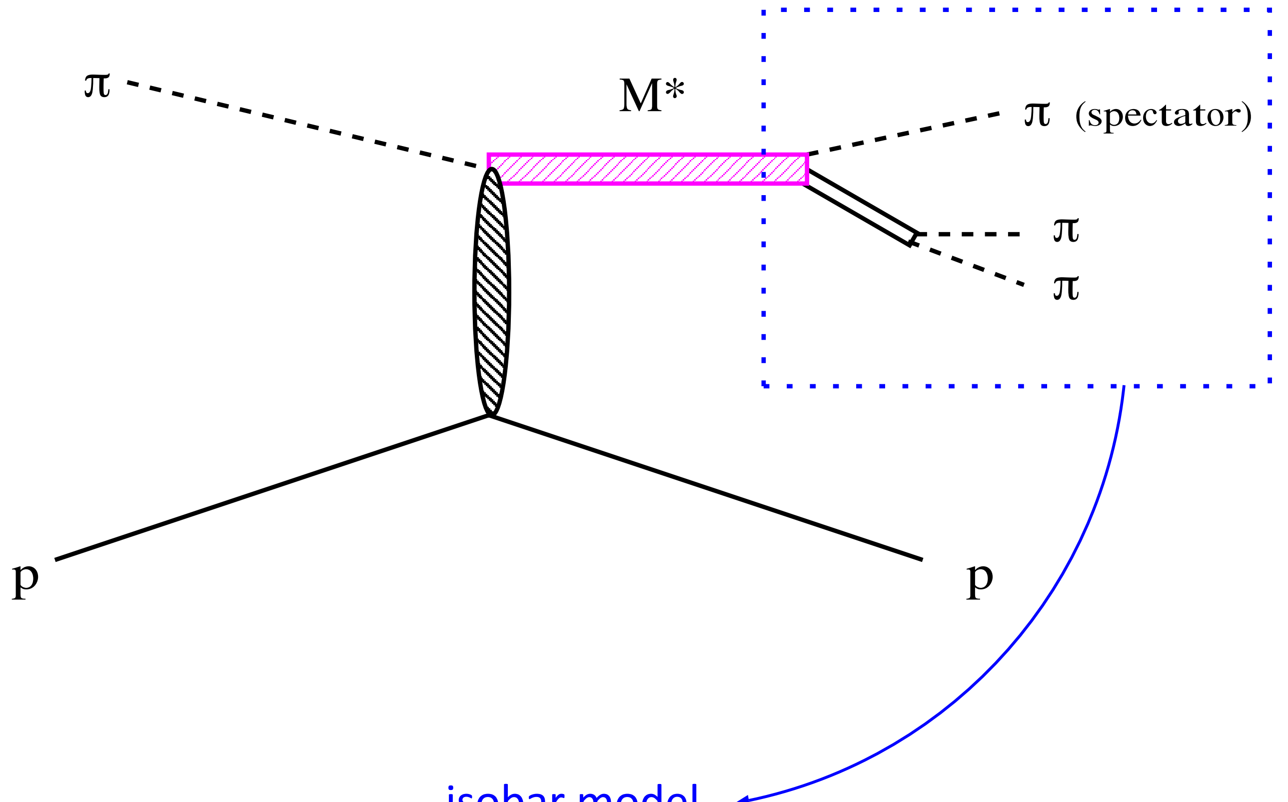
Analysis of three-meson productions

e.g., E852 (BNL) $\pi\, p \rightarrow \pi\pi\pi\, p$ Chung et al., PRD 65, 072001 (2001)



Analysis of three-meson productions

e.g., E852 (BNL) $\pi\, p \rightarrow \pi\pi\pi\, p$ Chung et al., PRD 65, 072001 (2001)

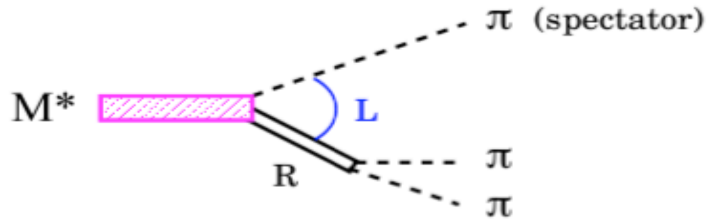


isobar model

* ππ subsystem forms a resonance
* 3rd π is a spectator

Isobar model for PWA

E852 (BNL) , Chung et al., PRD **65**, 072001 (2001)



$* \; L = 0, \; 1, \; 2$

$*$ For $R = f_0(980), \; \rho(770), \; f_2(1270), \; \rho_3(1690)$

$\Longrightarrow$ Breit-Wigner form $\quad A_R = \dfrac{F_{R\to\pi\pi}}{m_R^2 - m_{\pi\pi}^2 - i\, m_R\, \Gamma_R(m_{\pi\pi})}$

$*$ For $R = \sigma$

$\Longrightarrow$ K-matrix model [e.g., Au, Morgan, Pennington, PRD **35**, 1633 (1987)]

$* \; A_{M^*\to\pi\pi\pi} = \sum_R a_R\, e^{i\phi_R}\, A_R +$ (background)

W-dependence of partial wave amplitude is fitted with Breit-Wigner

➔ mass & width of *M**

Question

Unitarity ?

Coupled-channels ?

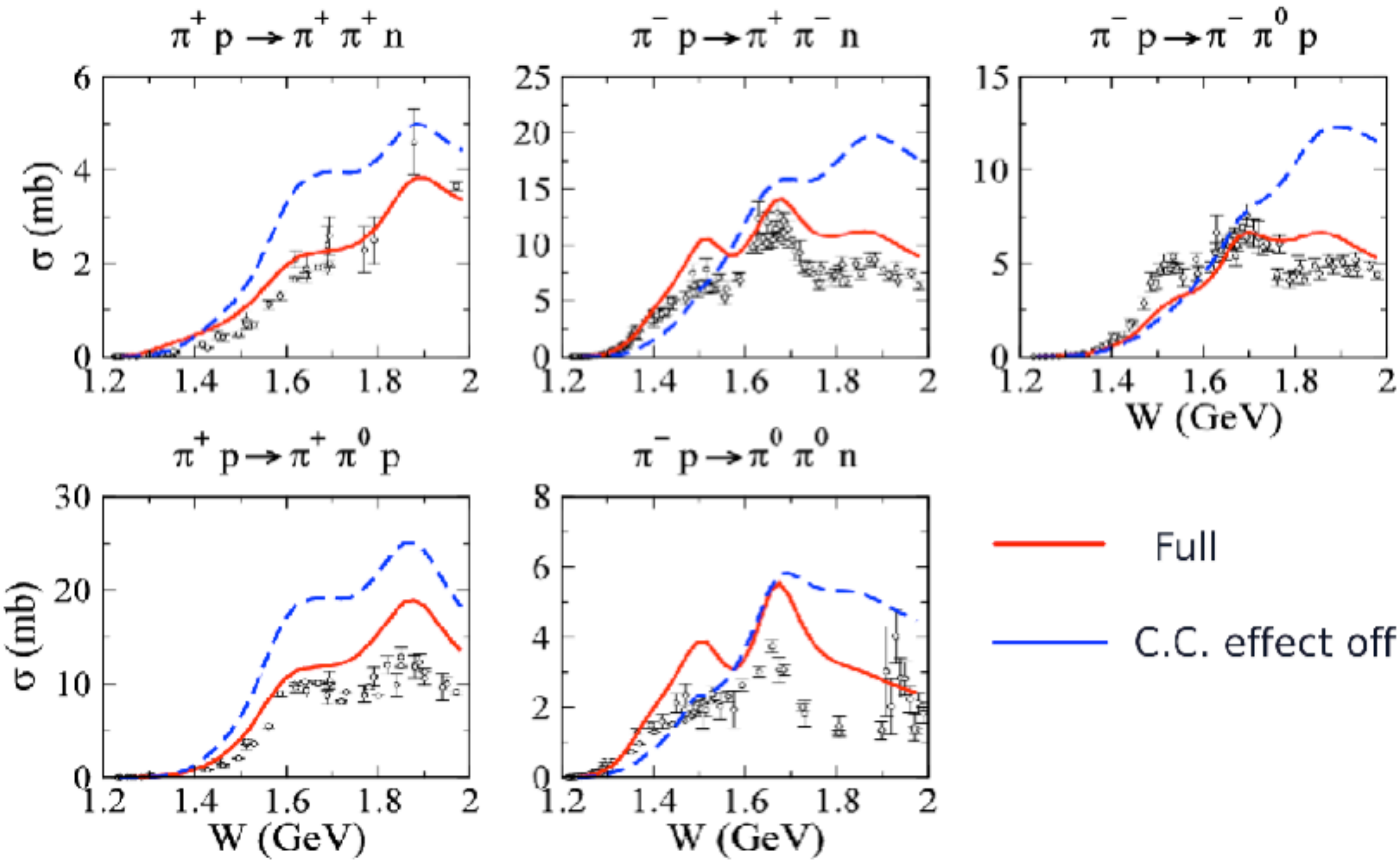
Unitarity requires coupled-channels

$$T_{ab} - T^*_{ab} \propto \sum_c T_{ac}\, T^*_{bc}$$

$$a, b, c = \pi\pi\pi\,,\ \pi K\bar{K},\ f_0\,(980)\,\pi\,,\ \rho\,(770)\,\pi\,, \ldots$$

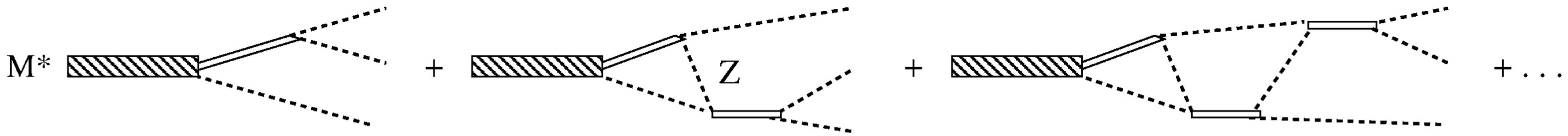
Coupled-channel effect

e.g., $\pi N \to \pi\pi N$ $[\pi N, \eta N, \pi\Delta, \rho N, \sigma N$ coupled-channels]



Kamano et al., PRC79 025206 (2009)

3-body unitarity requires … Z-diagrams



Question to be addressed

How 3-body unitarity makes a difference in extracting hadron properties from data ?

Method

1. Construct a unitary and an isobar models
2. Fit them to the same Dalitz plot
3. Extract and compare M^* properties from them
   (pole position, coupling strength to decay channels)

Coupled-Channels Model

Matsuyama, Sato, Lee, Phys. Rept. **439**, 193 (2007)

Kamano, Nakamura, Sato, Lee, PRD **84** 114019 (2011)

$M^* \to \pi R \to \pi\pi\pi$

Channels $R: \ f_0(600), f_0(980)\ , \rho(760), f_2(1270), ..$

R : resonance in $\pi\pi$ scattering amplitude (not Breit-Wigner form)

(I) Develop $\pi\pi$ model

(II) Develop πR interaction

(III) Solve πR scattering equation

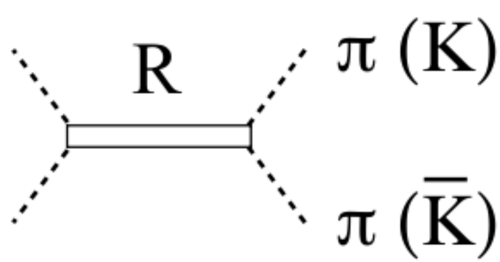
Simple $\pi\pi$ model

Coupled-channel scattering equation for $\pi\pi$ partial wave (L, I)

$$t^{LI}_{i,j}(p',p;W) = V^{LI}_{i,j} + \sum_k \int_0^\infty q^2 dq\, V^{LI}_{i,k}(p',q;W) \frac{1}{W - E_k(q) + i\epsilon} t^{LI}_{k,j}(q,p;W)$$

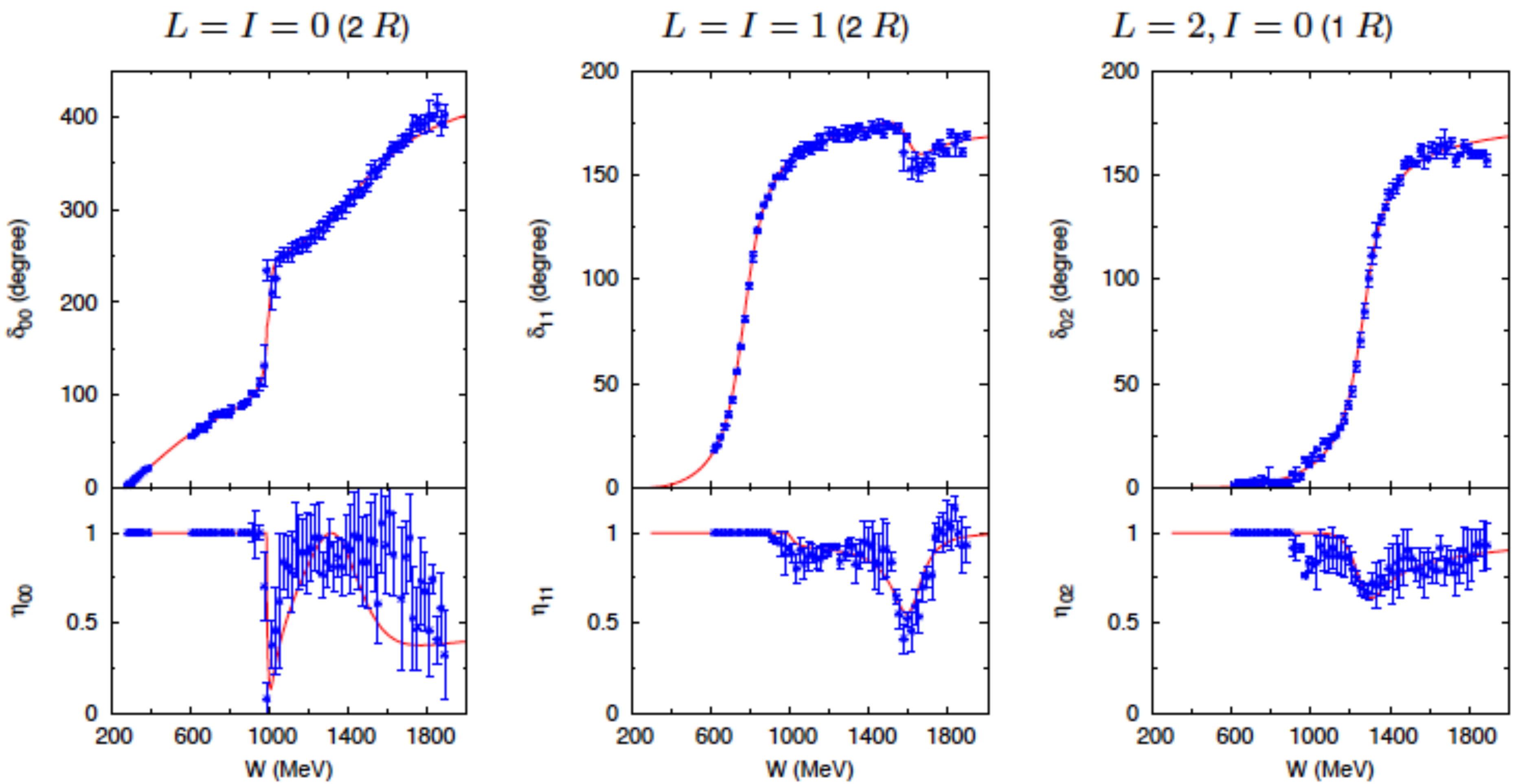
$$E_{\pi\pi}(q) = 2\sqrt{m_\pi^2 + q^2} \qquad\qquad (i,j,k = \pi\pi, K\bar{K})$$

$$V^{LI}_{i,j}(p',p;W) = \sum_R f^{LI}_{R,i}(p') \frac{1}{W - m_R} f^{LI}_{R,j}(p)$$



$$f^{LI}_{R,i}(p) = \frac{g_{R,i}}{\sqrt{m_\pi}} \frac{1}{(1 + (c_{R,i}p)^2)} \left(\frac{p}{m_\pi}\right)^L$$

Phase and inelasticity of $\pi\pi$ amplitude



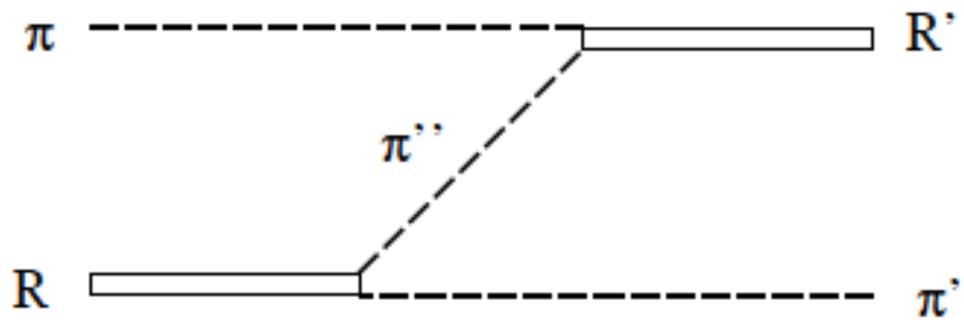
[Data: Gayer et al. (1974); Hyams et al. (1973); Batley et al. (2008)]

Pole positions in $\pi\pi$ amplitude

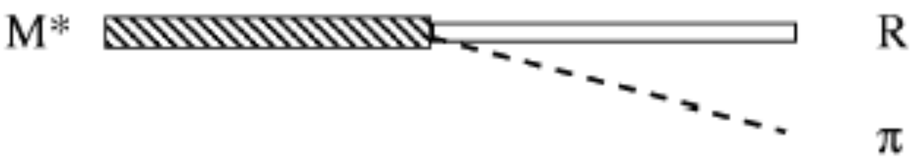
	Re[M_R] (MeV)		$-$Im[M_R] (MeV)	
	Ours	PDG	Ours	PDG
f_0 (600)	430	$400-1200$	270	$250-500$
f_0 (980)	1000	980 ± 10	9	$20-50$
f_0 (1370)	1350	$1200-1500$	170	$150-250$
ρ (760)	770	775.5 ± 0.3	81	74.5 ± 0.4
ρ (1700)	1610	$1550-1780$	120	$80-300$
f_2 (1270)	1250	1275 ± 1.2	100	92.5 ± 1.3

Quasi two-particle (πR) interaction

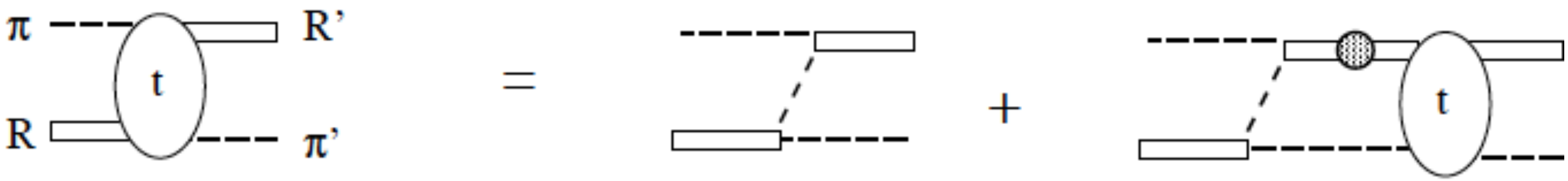
3 π Z-graph



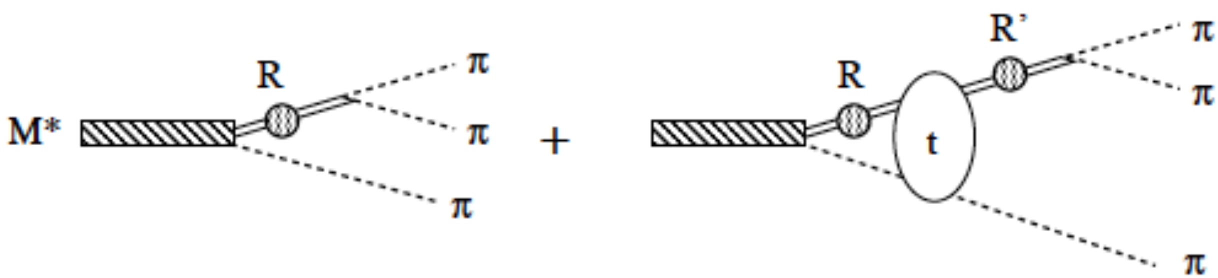
M^* graph



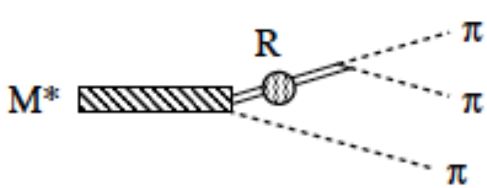
Quasi two-particle (πR) scattering equation



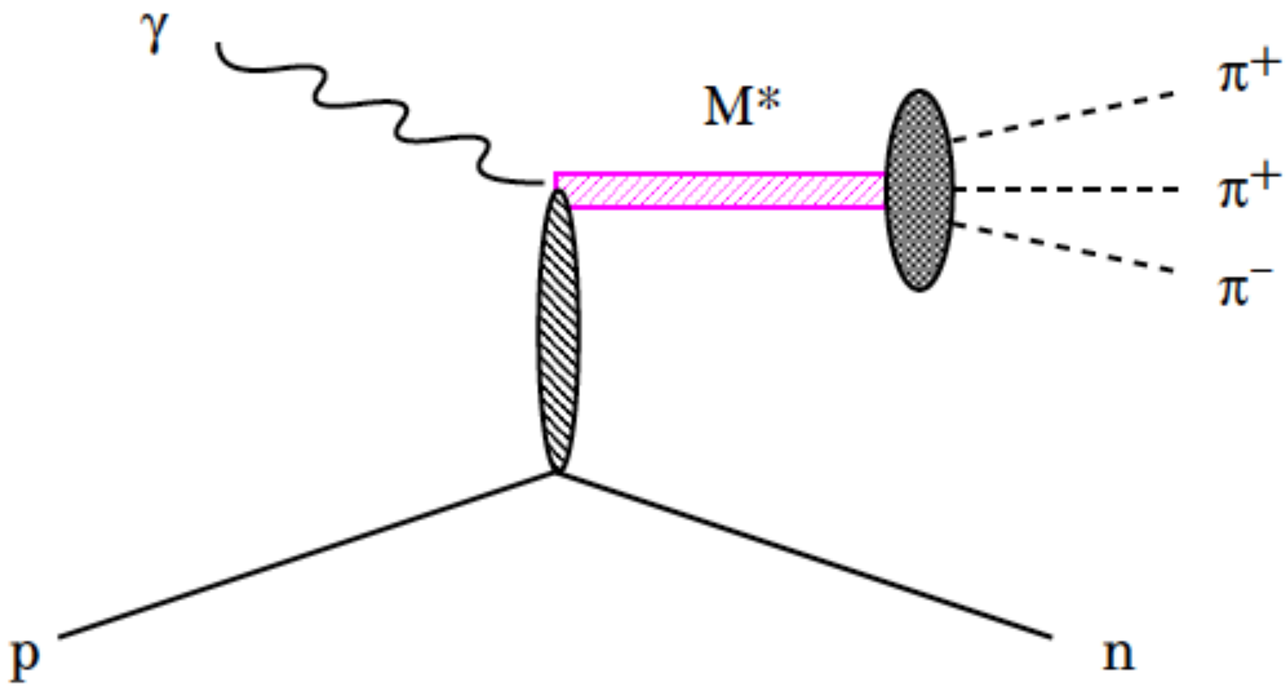
M^* decay amplitude (unitary model)



M^* decay amplitude (isobar model)



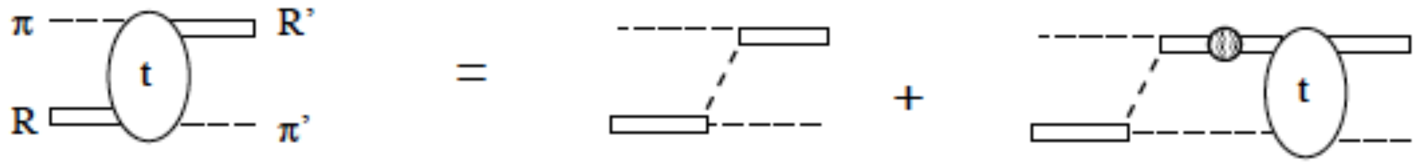
Case Study : $\gamma p \to M^* n \to \pi^+\pi^+\pi^- n$ (CLAS 6, GlueX)



Nakamura, Kamano, Sato, Lee, in preparation

How 3-body unitary makes a difference in extracting M^* properties from data ?

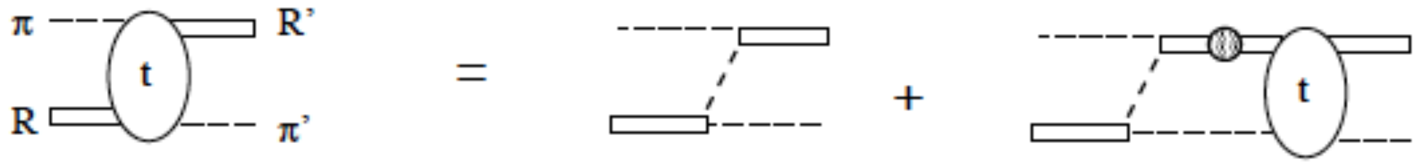
M^* propagator (unitary model)



$$G^{-1}(W) = W - M^0_{M^*} - \Sigma_{M^*}(W)$$

3-body unitarity requires consistency with M^* decay amplitude

M^* propagator (unitary model)



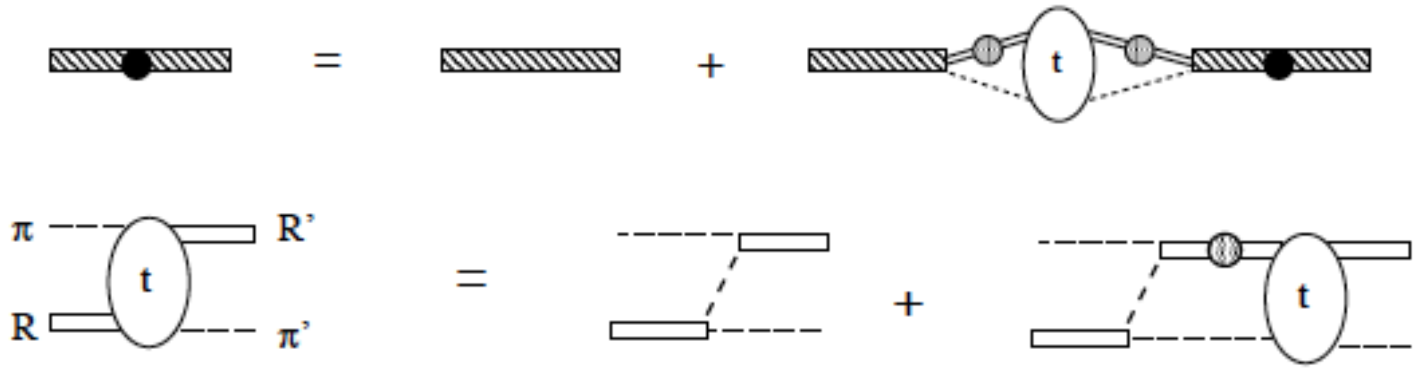
$$G^{-1}(W) = W - M^0_{M^*} - \Sigma_{M^*}(W)$$

Pole position : $M_R \Rightarrow G^{-1}(M_R) = 0$

Solved with analytic continuation to the unphysical Riemann sheet

Suzuki, Sato, Lee, PRC **79**, 025205 (2009); ibid, **82**, 045206 (2010)

M^* propagator (unitary model)



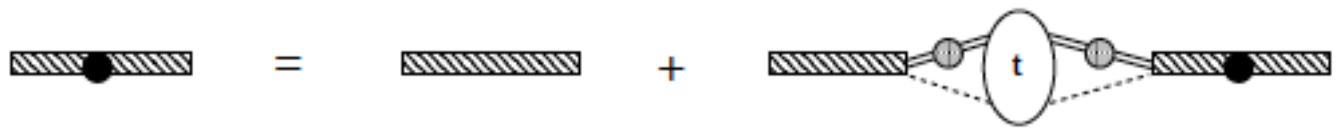
$$G^{-1}(W) = W - M^0_{M^*} - \Sigma_{M^*}(W)$$

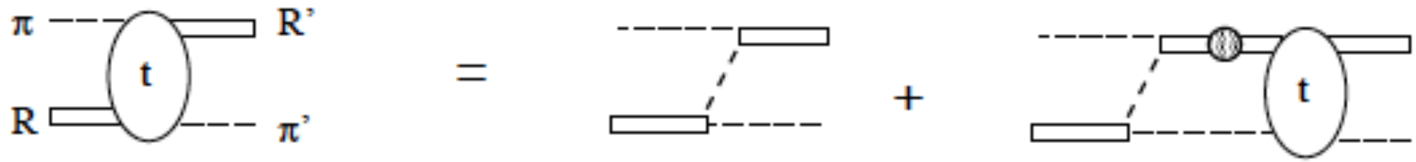
M^* propagator (isobar model)

Breit-Wigner : $G^{-1}(W) = W - M_{M^*} - i\,\frac{\Gamma(W)}{2}$

but any phenomenological parametrization should be fine ...

M^* propagator (unitary model)





$$G^{-1}(W) = W - M^0_{M^*} - \Sigma_{M^*}(W)$$

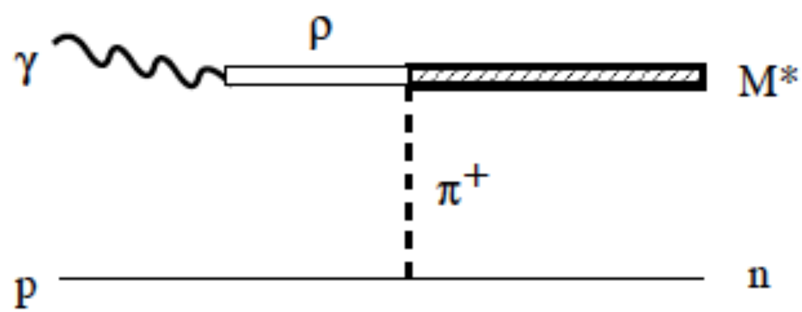
M^* propagator (isobar model)

This work :
$$G^{-1}(W) = W - M^0_{M^*} - \Sigma_{M^*}(W) - \Delta M^0_{M^*}$$

$\uparrow$ complex constant

Pole position : $M_R(\text{isobar}) \sim M_R(\text{unitary}) + \Delta M^0_{M^*}$

Production amplitude



Simple assumptions :

- t-channel π-exchange
- vector-dominance of $\gamma\pi M^*$ coupling

Not realistic but good enough

interested in effect of 3-body unitarity implemented in M^* propagation and decay

Kinematics

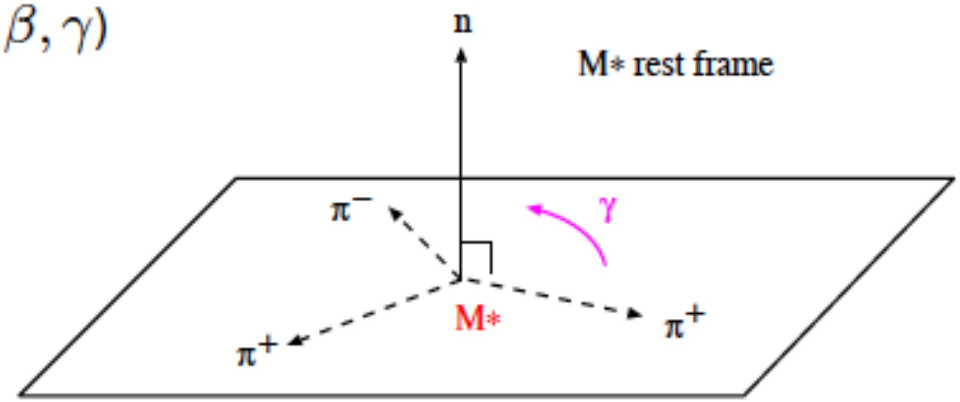
[cf. CLAS 6, PRL **102**, 102002 (2009)]

$*$ E_γ = 5 GeV

$*$ t = $-$ 0.4 GeV^2

$*$ 0.8 GeV $\leq W \leq$ 2 GeV ; W : 3 π invariant mass

$*$ 3 π orientation
(Euler angles : α, β, γ)



$\alpha,\ \beta$ fixed ; $0 \leq \gamma \leq 2\pi$ Dalitz plot depends on γ

$\Leftarrow$ Photon excites a polarized M^* that decays to a certain γ more often

Procedure

1. Determine parameters of unitary model with reasonable input
2. Generate *mock data* with the unitary model
3. Fit the data with isobar model
4. Compare M^* properties from the two models

Partial wave, M^*'s in unitary model

[cf. CLAS 6, PRL **102**, 102002 (2009)]

J^{PC}	M^*
1^{++}	$a_1(1230)$, $a_1(1700)$
2^{++}	$a_2(1320)$, $a_2(1700)$
2^{-+}	$\pi_2(1670)$, $\pi_2(1800)$
1^{-+}	$\pi_1(1600)$

Determination of M^* parameters for unitary model

* M^* bare mass
* $M^* \to \pi R$ bare coupling and cutoff

$^3P_0$ model — Barnes et al., PRD **55**, 4157 (1997)

Flux-tube model for $\pi_1(1600)$ — Isgur et al., PRL **54**, 869 (1985)

- Partial width $\Rightarrow$ coupling
- Cutoff is set to 1 GeV

J^{PC}	decay modes		$\Gamma_{q\bar{q}}$	Γ_{hybrid}
1^{++}	$a_1(1230) \rightarrow$	$\pi\rho(770)$	540.	-
	$a_1(1700) \rightarrow$	$\pi f_0(1300)$	2.	6.
		$\pi\rho(770)$	57.	30.
		$\pi\rho(1465)$	41.	0.
		$\pi f_2(1275)$	39.	70.
2^{++}	$a_2(1318) \rightarrow$	$\pi\rho(770)$	55 .	-
	$a_2(1700) \rightarrow$	$\pi\rho(770)$	104.	-
		$\pi f_2(1275)$	20.	-
2^{-+}	$\pi_2(1670) \rightarrow$	$\pi\rho(770)$	118.	-
		$\pi f_2(1275)$	75.	-
	$\pi_2(1800) \rightarrow$	$\pi f_0(1300)$	1.	1.
		$\pi\rho(770)$	162.	8.
		$\pi f_2(1275)$	86.	50.
1^{-+}	$\pi_1(1600) \rightarrow$	$\pi\rho(770)$	-	8.

J^{PC}	decay modes		$\Gamma_{q\bar{q}}$	$\Gamma_{\rm hybrid}$
1^{++}	$a_1(1700) \to$	$\pi f_0(1300)$	2.	6.
		$\pi\rho(770)$	57.	30.
		$\pi\rho(1465)$	41.	0.
		$\pi f_2(1275)$	39.	70.
2^{-+}	$\pi_2(1800) \to$	$\pi f_0(1300)$	1.	1.
		$\pi\rho(770)$	162.	8.
		$\pi f_2(1275)$	86.	50.

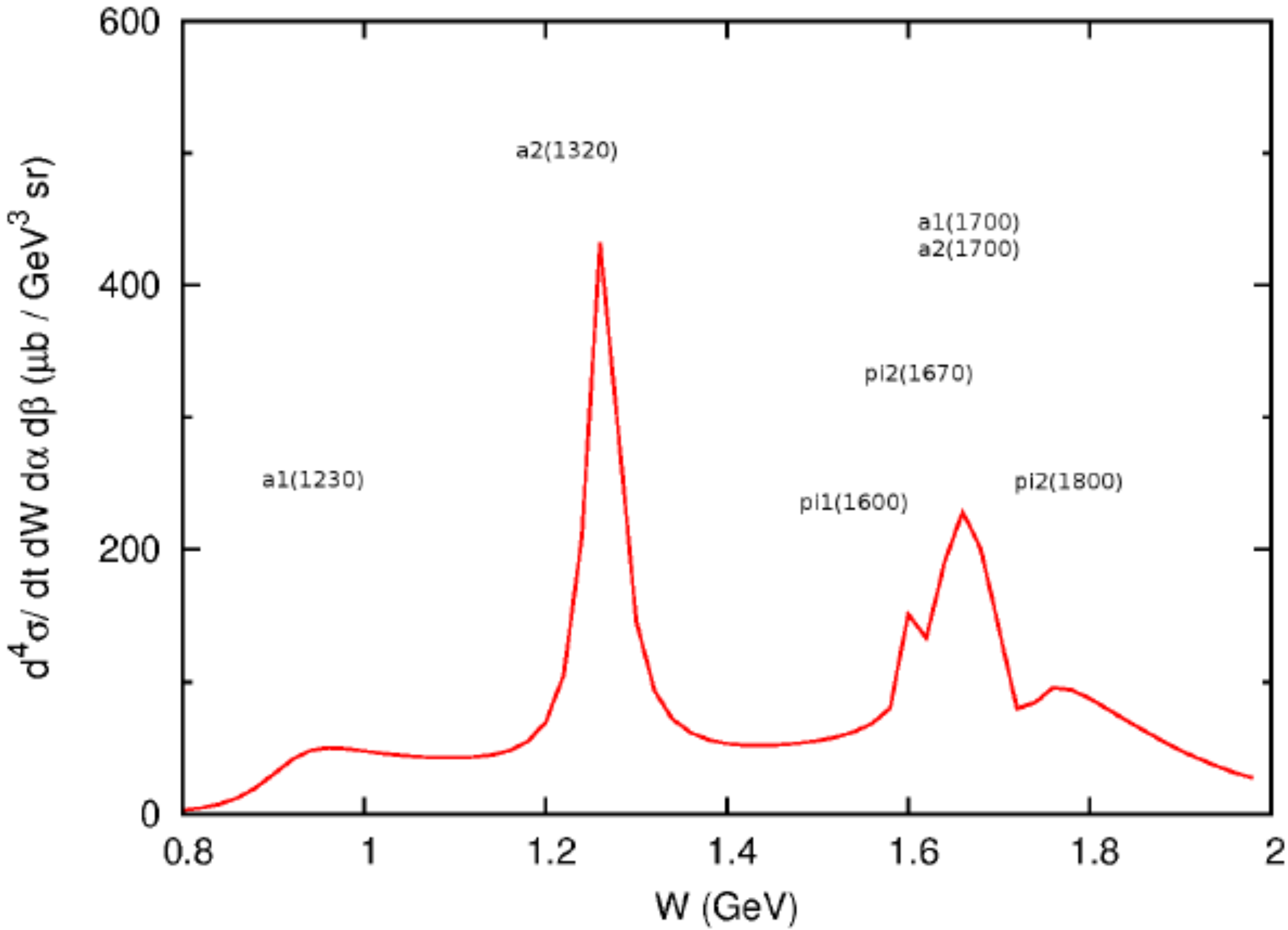
Difference between $\Gamma_{q\bar{q}}$ and $\Gamma_{\rm hybrid}$

$\Rightarrow$ Coupling strength to decay channel is a key information

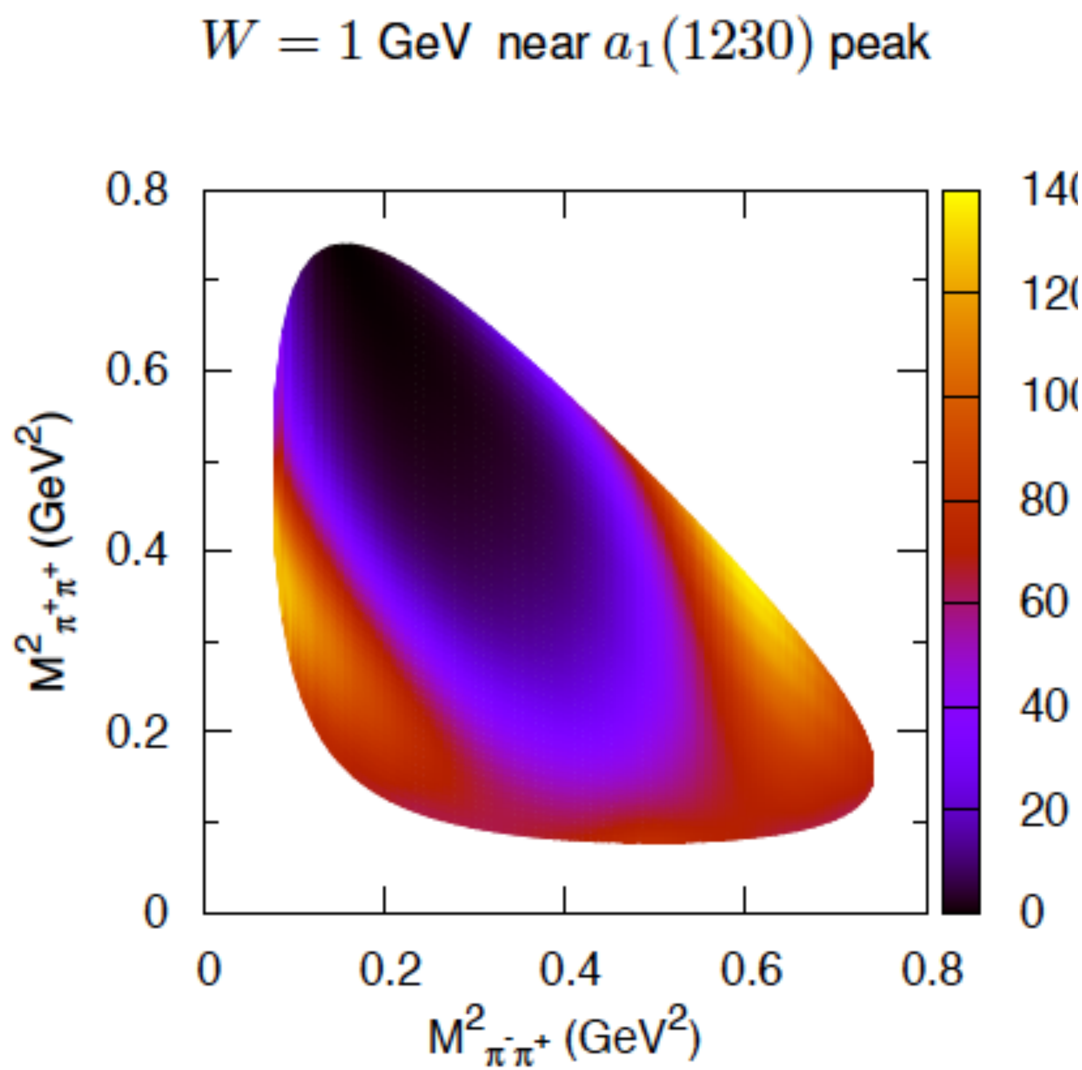
to understand the nature of hadron structure

We use $\Gamma_{q\bar{q}}$ except for $\pi_1(1600)$

W-dependence of integrated Dalitz plots from unitary model

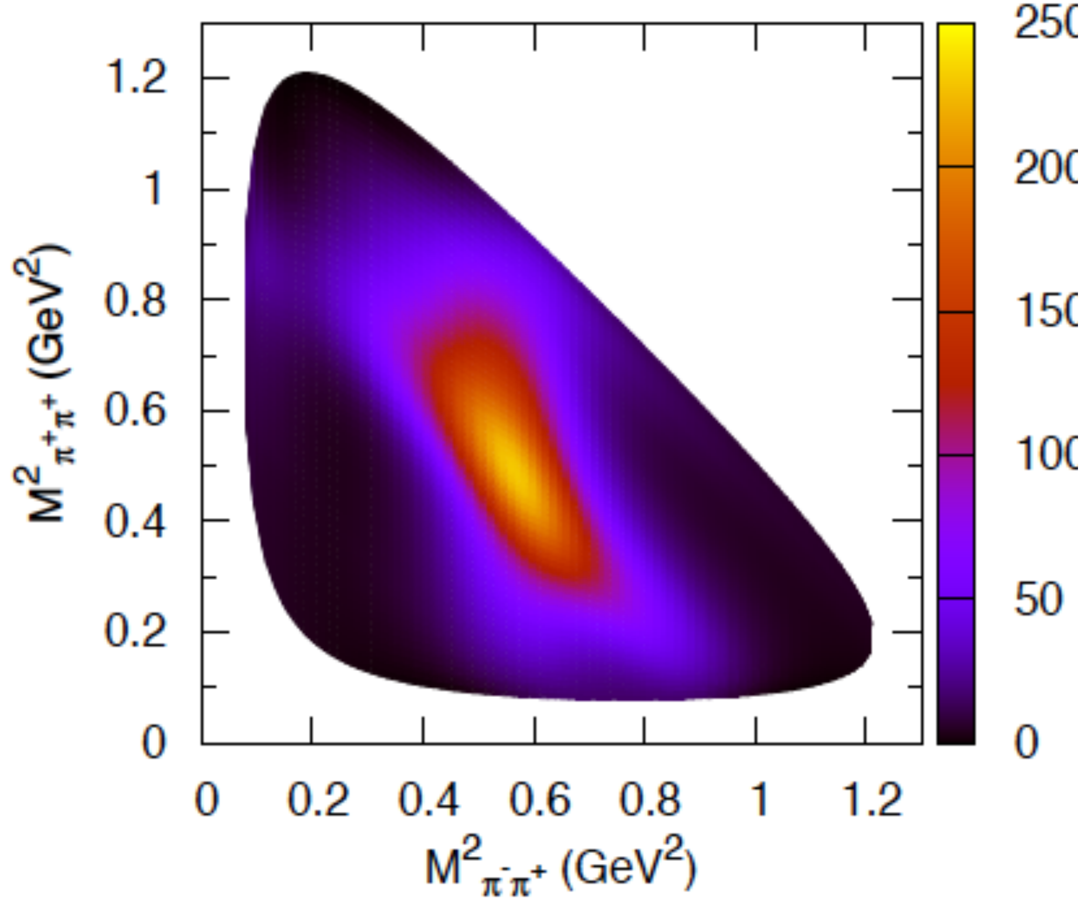


Dalitz plot from unitary model



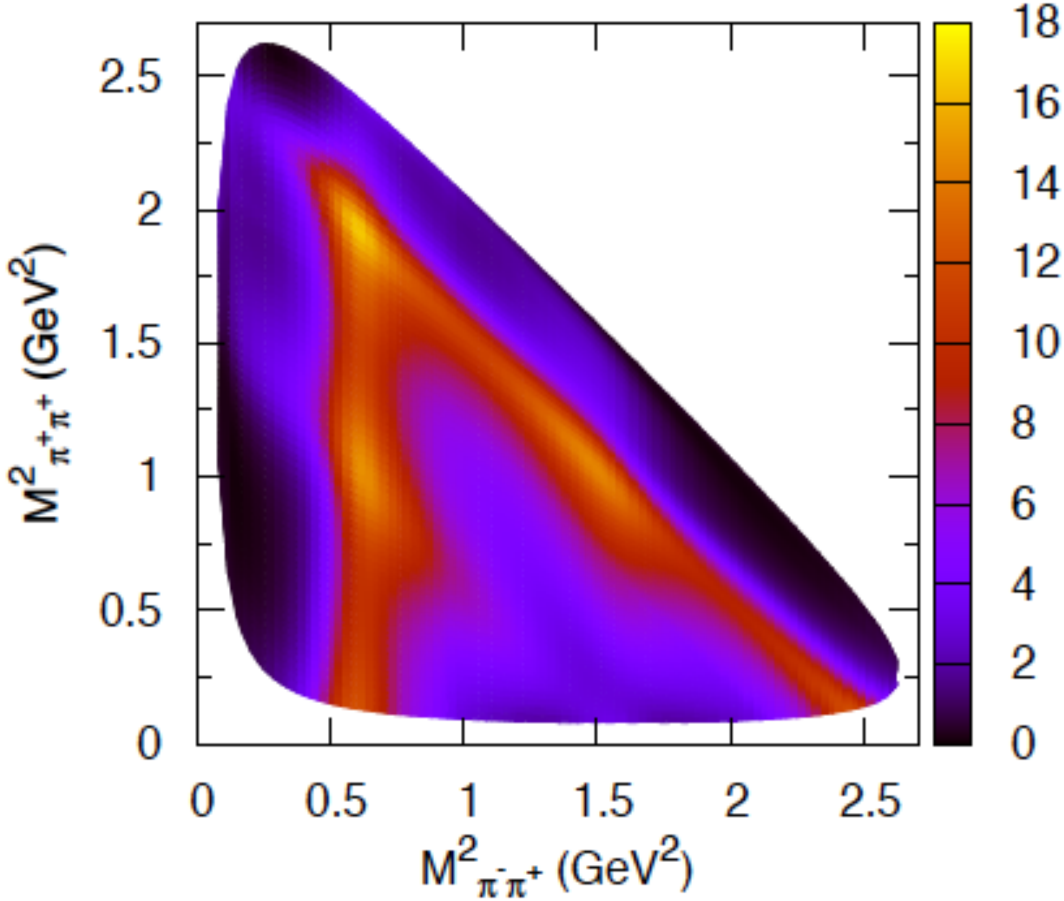
Dalitz plot from unitary model

$W = 1.24$ GeV near $a_2(1320)$ peak



Dalitz plot from unitary model

$W = 1.76$ GeV near $\pi_2(1800)$ peak



Fit with isobar model

Error :

- Data for the same W have the same error
- At each W, the error is assigned by 5% of the highest peak

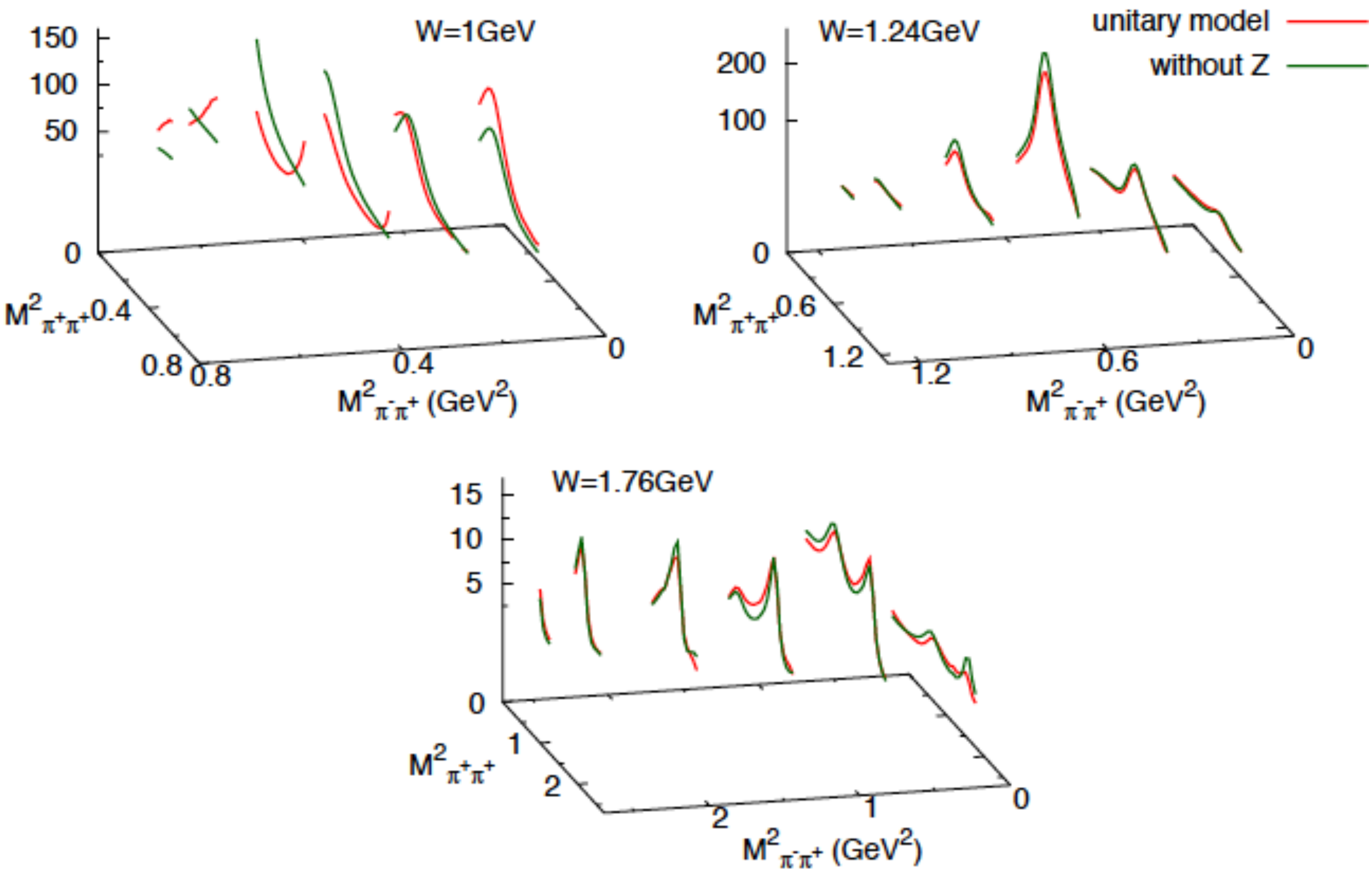
Fit :

1. Fit with real couplings of $M^* \to \pi R$
2. Allow couplings complex
3. Include W-dependent flat non-interfering background

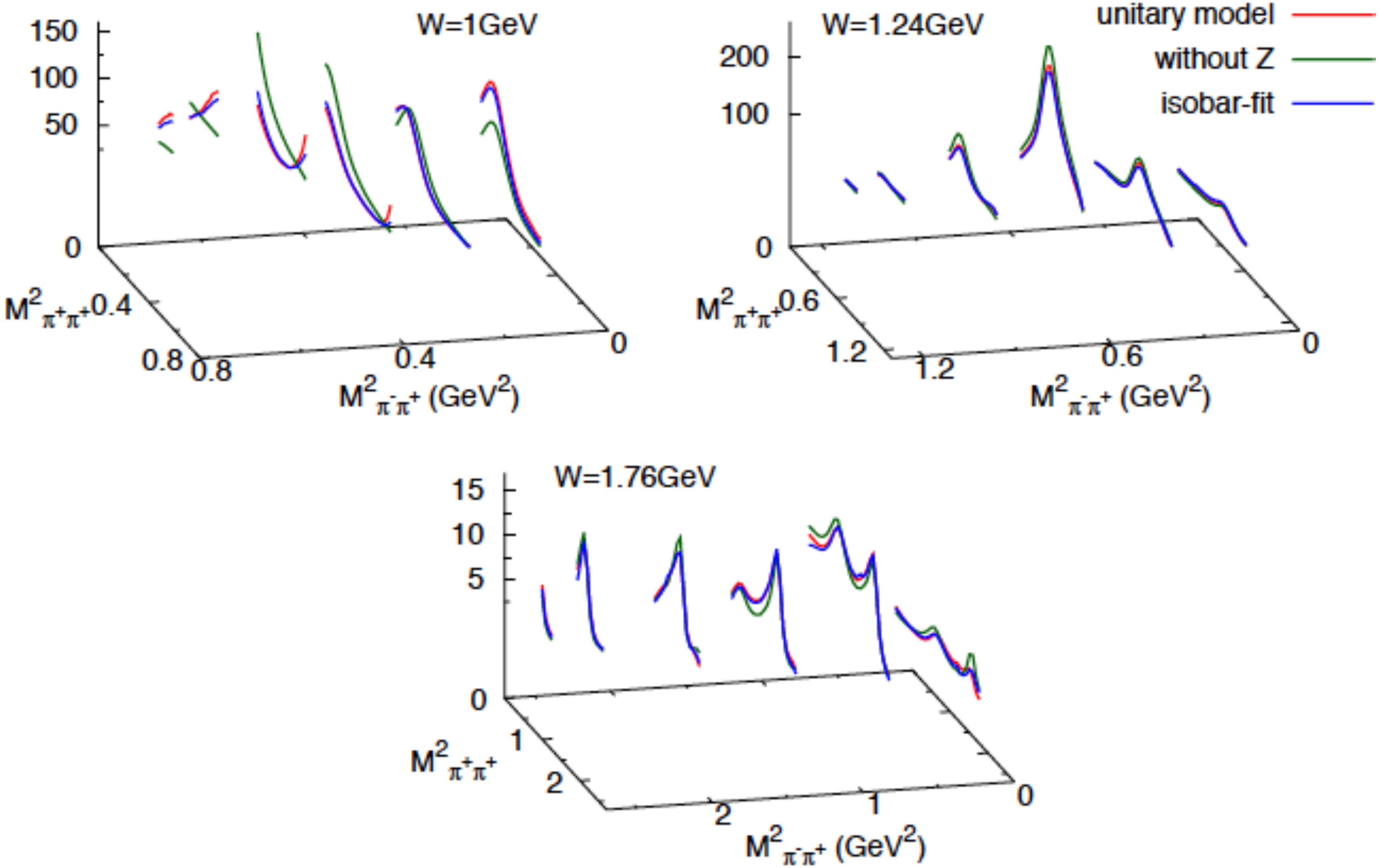
   (2, 3 are common in isobar-model analysis)

$\chi^2/$(# of data) < 0.5 is achieved

Fit with isobar model

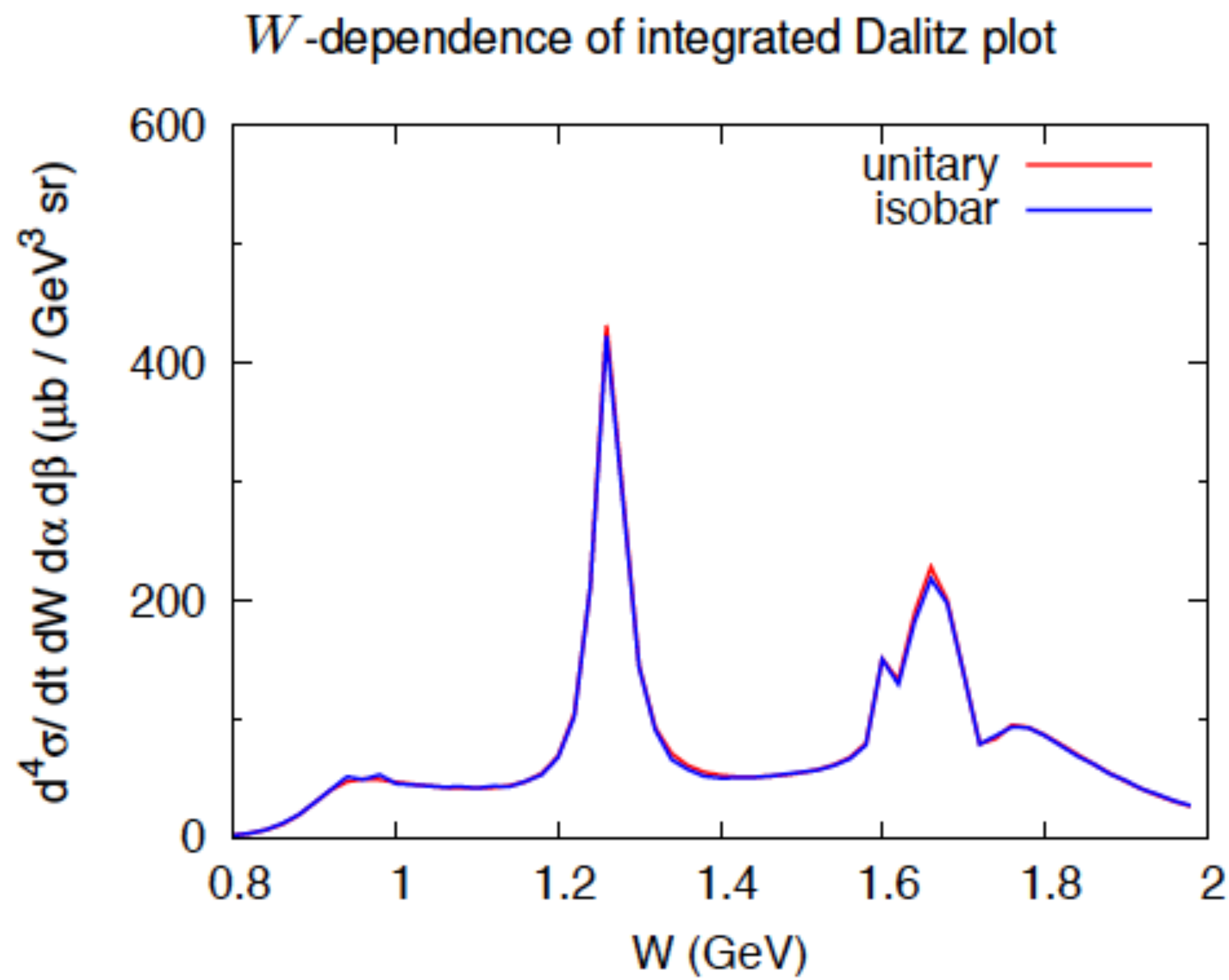


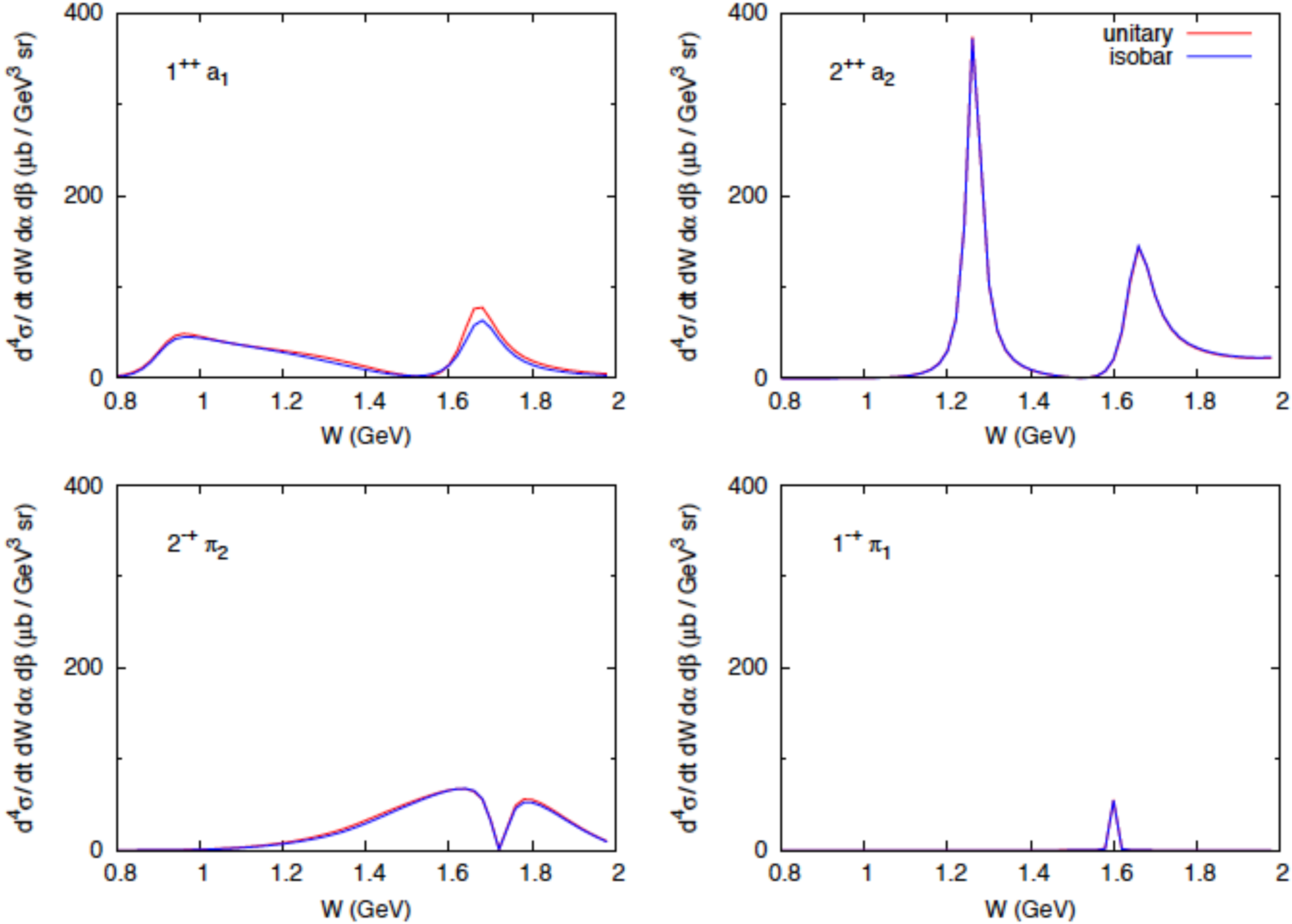
Fit with isobar model



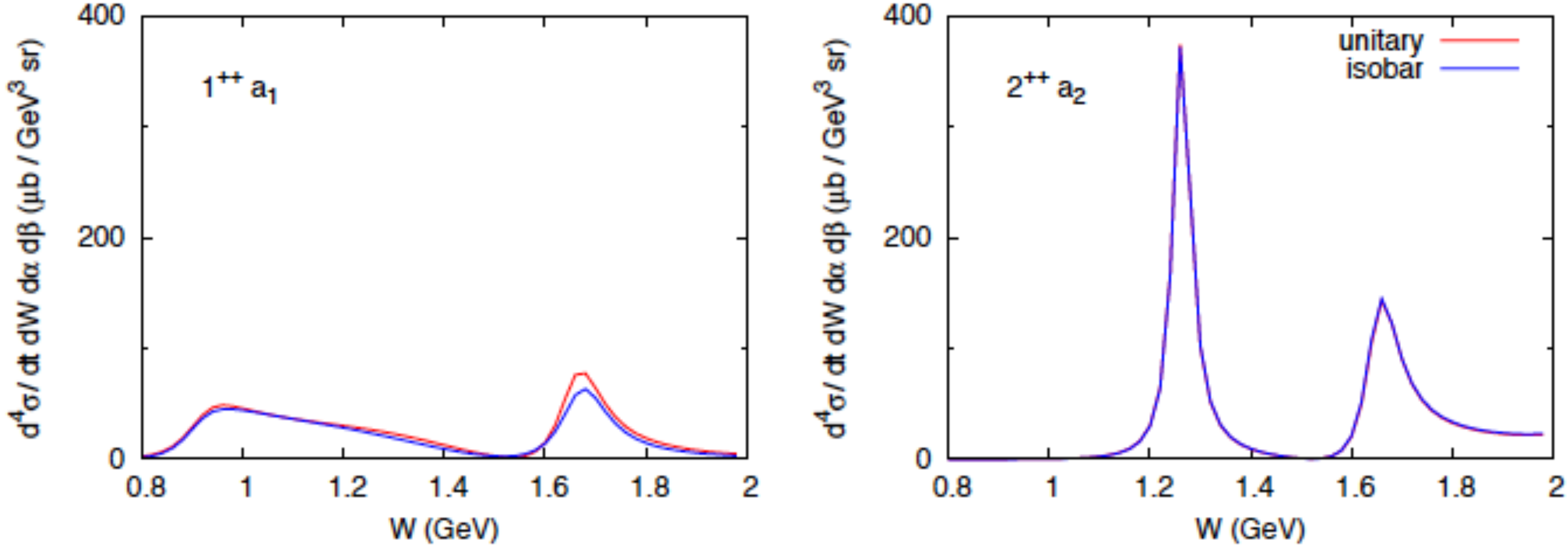
Question I didn't explicitly ask

Can isobar model extract partial wave amplitudes of the unitary model ?





1++ a1
2++ a2
unitary
isobar
2-+ π2
1-+ π1
d4σ/ dt dW dα dβ (μb / GeV3 sr)
W (GeV)
0
200
400
0.8
1
1.2
1.4
1.6
1.8
2



Q : *Can isobar model extract partial wave amplitudes of the unitary model ?*

A : To a good extent, yes.

Comments

- Not so in kinematics where a partial wave amplitude plays a minor role
- Analyzing polarized observables may have helped

M^* pole positions

$\Delta M^0_{M^*}$ (MeV) : pole position shift in the isobar model

$$G^{-1}(W) = W - M^0_{M^*} - \Sigma_{M^*}(W) - \Delta M^0_{M^*}$$

$a_1(1260)$	$a_1(1700)$	$a_2(1320)$	$a_2(1700)$
$-22.37 - 23.21i$	$8.48 - 4.46i$	$0.15 - 0.04i$	$-1.03 - 0.30i$

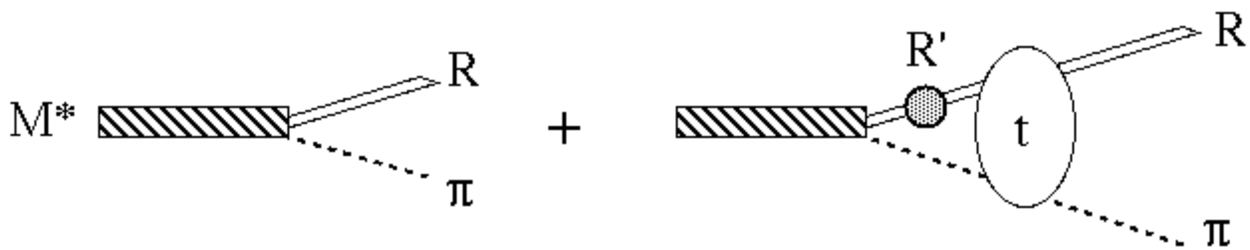
$\pi_2(1670)$	$\pi_2(1800)$	$\pi_1(1600)$
$-0.51 + 0.38i$	$-3.07 - 3.42i$	$0.57 + 0.33i$

Here, 3-body unitarity effect is moderate

Coupling strength to decay channels

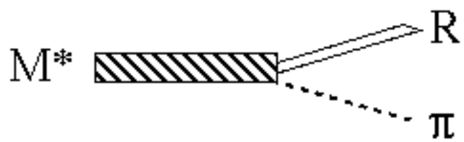
Unitary model

$$F_{M^* \to \pi R} =$$

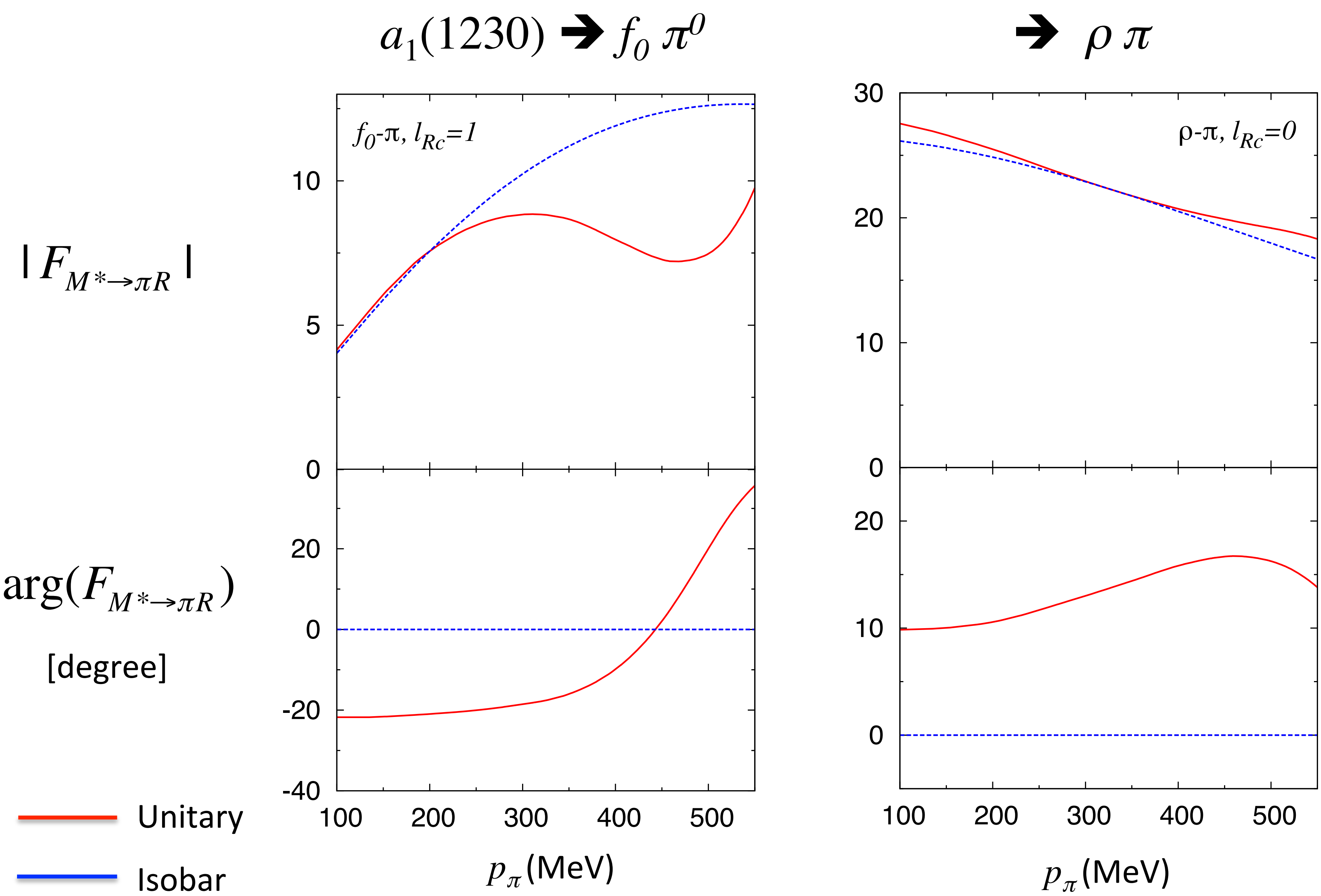


Isobar model

$$F_{M^* \to \pi R} =$$

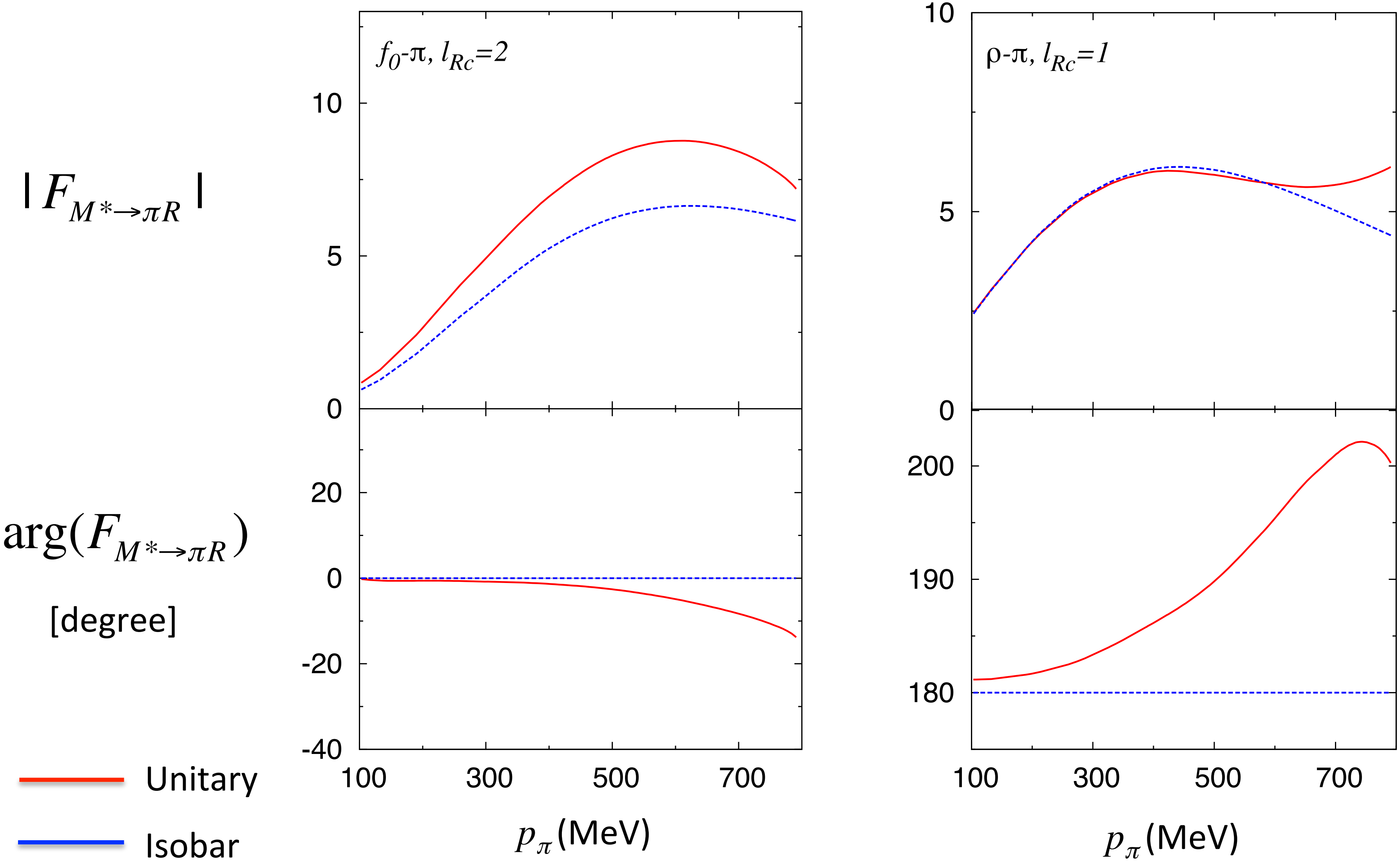


Coupling strength to decay channels



Coupling strength to decay channels

$\pi_2(1670) \rightarrow f_0\, \pi^0$ $\rightarrow \rho\, \pi$



Bare **Couplings of $a_1(1260)$ to decay channels**

		Unitary	Isobar
$a_1(1260)$	$\to \pi f_0(1300)$	-	$-2.0 + 8.2\,i$
	$\to \pi f_0(2400)$	-	$7.5 - 2.3\,i$
	$\to \pi\rho(770)$	24.6	$31.9 - 3.7\,i$
	$\to \pi\rho(1700)$	-	$11.0 + 5.2\,i$
	$\to \pi f_2(1270)$	-	$-2.7 + 4.4\,i$

Rather large change in M^* couplings to decay channels

$\Leftarrow$ Large Z-graph effect in $a_1(1260)$ region

Bare **Couplings of $a_2(1320)$ to decay channels**

		Unitary	Isobar
$a_2(1320)$	$\rightarrow \pi\rho(770)$	1.0	$0.9 - 0.1\,i$
	$\rightarrow \pi f_2(1270)$	-	$1.4 - 0.1\,i$

Still rather large change in M^* couplings to decay channels

even though Z-graph effect on Dalitz plot in $a_2(1320)$ region seems moderate

Conclusion

Q : How 3-body unitary makes a difference in extracting M^* properties from data ?

Method

1. Construct a unitary and an isobar models

2. Fit them to the same Dalitz plot

3. Extract and compare M^* properties from them

Conclusion

Q : How 3-body unitary makes a difference in extracting M^* properties from data ?

A : It (and thus Z-diagrams) makes a significant difference in extracting

dynamical aspect of M^* properties , i.e., coupling strength to decay channel

Key information to understand the hadron structure

Conclusion

Q : How 3-body unitary makes a difference in extracting M^* properties from data ?

A : It (and thus Z-diagrams) makes a significant difference in extracting dynamical aspect of M^* properties , i.e., coupling strength to decay channel

Q : What about pole position ?

A : Moderate. Sometimes non-negligible.