

**Dynamical Model of  
Coherent Pion Production  
in Neutrino-Nucleus Scattering**

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Satoshi Nakamura (JLab)

Collaborators

T. Sato (Osaka U.)

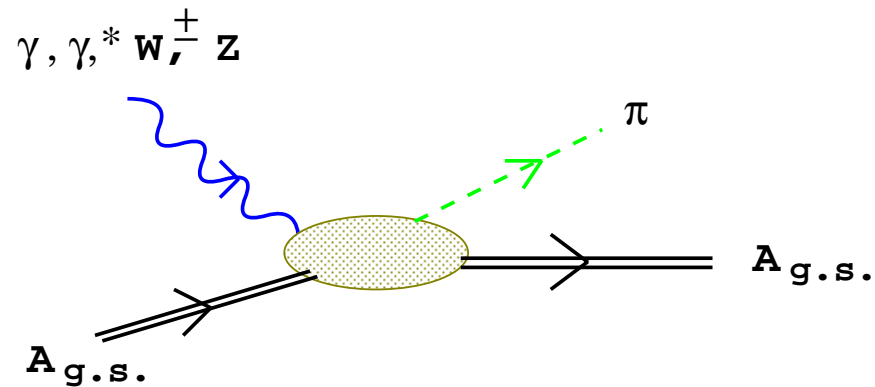
T. S.-H. Lee (ANL)

B. Szczerbinska (Dakota State U.)

K. Kubodera (U. South Carolina)

## Introduction

What is coherent pion production ?



- \* Amplitude is approximately proportional to nuclear form factor  
 $\implies$  Forward scattering (small momentum transfer) is favored
- \*  $\Delta$ -excitation and its propagation in nuclei  
 $\implies$  Good laboratory to study  $\Delta$  in nuclei

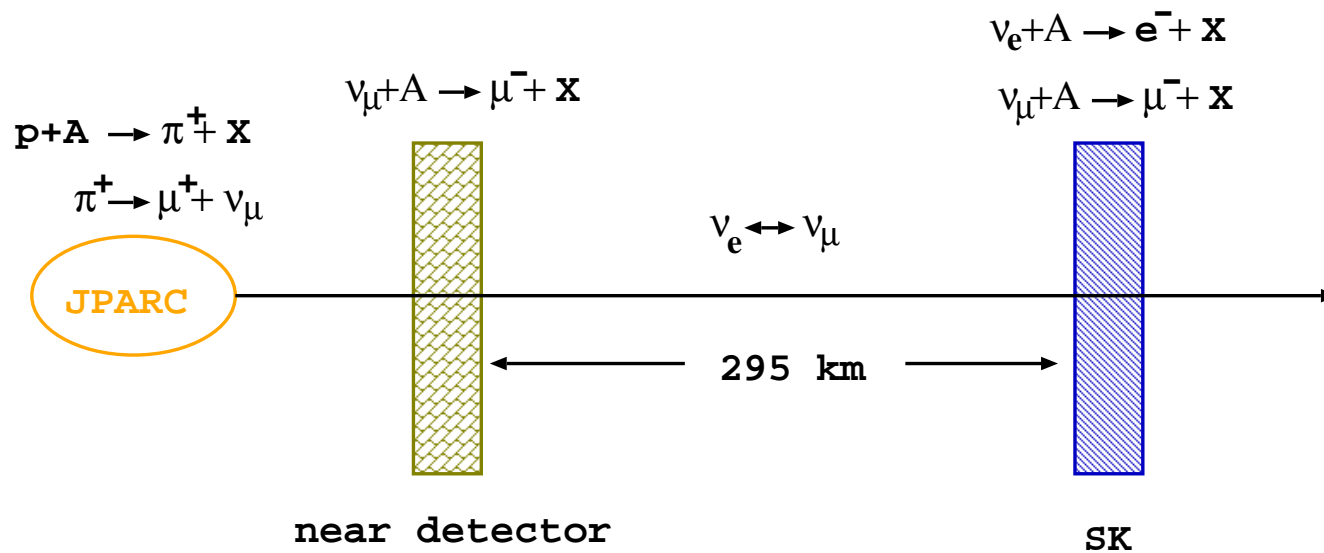
# Recent interest in coherent $\pi$ production in $\nu$ -nucleus interaction

Neutrino oscillation experiments (K2K, T2K, MiniBooNE ... etc.)

current trend : accelerator neutrino in sub- few-GeV region

precise determination of neutrino mixing angle and  $\Delta m_\nu^2$

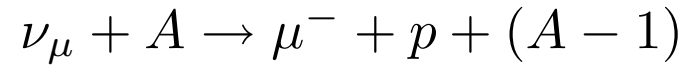
*e.g.* **T2K**



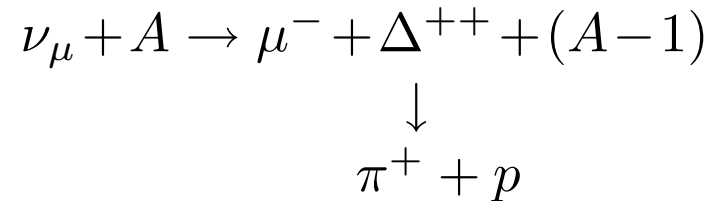
## $\nu$ -nucleus scattering in few-GeV region

\* dominant

- Quasi-elastic

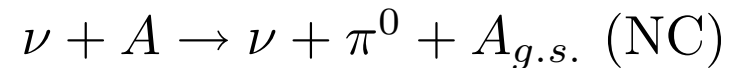
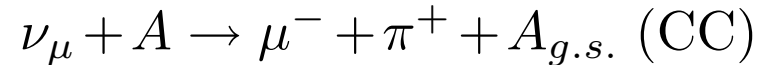


- Quasi-free  $1\pi$  production



\* sub-dominant (still need to be understood)

- Coherent  $\pi$  production



— Improved description of  $\pi$  event in forward direction

— Excess of Electron-like Events in MiniBooNE ( $\nu_\mu \rightarrow \nu_e$ )

PRL **102**, 101802 (2009)

## Recent experiments for $\nu$ -induced coherent $\pi$ production

\* **No evidence for CC** ( $\nu_\mu + {}^{12}\text{C} \rightarrow \mu^- + \pi^+ + {}^{12}\text{C}_{g.s.}$  ,  $E_\nu \sim 1 \text{ GeV}$  )

K2K [PRL **95**, 252301 (2005)], SciBooNE [PRD **78**, 112004 (2008)]

\* **Signature for NC** ( $\nu_\mu + {}^{12}\text{C} \rightarrow \nu_\mu + \pi^0 + {}^{12}\text{C}_{g.s.}$  ,  $E_\nu \sim 1 \text{ GeV}$  )

MiniBooNE [PLB **664**, 41 (2008)], SciBooNE [PRD **81**, 111102 (2010)]

Puzzling result because ...

- Naive expectation from isospin matrix element :  $\sigma_{CC} \sim 2 \sigma_{NC}$
- Finite  $\mu$  mass reduces phase space at low-energy

# Theoretical approaches to coherent $\pi$ production

- \* **PCAC** (Partially Conserved Axial Current)-based model

- Rein, Sehgal, NPB **223**, 29 (1983)
- Paschos, Kartavtsev, Gounaris, PRD **74**, 054007 (2006)
- Berger, Sehgal, PRD **79**, 053003 (2009)

- \* **Dynamical microscopic model**

- Amaro, Hernandez, Nieves, Valverde, PRD **79**, 013002 (2009)
- Alvarez-Ruso et al., PRC **75**, 055501 (2007)
- Martini, Ericson, Chanfray, Marteau, PRC **80**, 065501 (2009)

## PCAC-based model

Rein and Sehgal, NPB **223**, 29 (1983)

- For  $q^2 \rightarrow 0$  ( $q_\mu$  : momentum transfer from lepton)

$$\mathcal{M}_{coh} \sim q_\mu (F_{\pi N \Delta} G_{\Delta} A_{\Delta N}^\mu + F_{\pi N N} G_N A_{N N}^\mu) F_A F_{abs}$$

- PCAC relation and Klein-Gordon Equation

$$\begin{aligned} q_\mu A^\mu &= f_\pi m_\pi^2 \pi && \text{(PCAC)} \\ (-q^2 + m_\pi^2) \pi &= S_\pi && \text{(K - G eq.)} \end{aligned}$$

$$\implies \mathcal{M}_{coh} \sim f_{\pi N \rightarrow \pi N} F_A F_{abs}$$

Good approximation for

$$* E_\nu \gtrsim 2 \text{ GeV}$$

\* medium and heavy nuclei

## Dynamical model for coherent $\pi$ production

\* Elementary amplitudes ( $\nu N \rightarrow \mu^- \pi^+ N$  ,  $\nu N \rightarrow \nu \pi^0 N$ )

\* Medium effects

Final state interaction ( $\pi$ -nucleus scattering)

$\Delta$ -properties in nucleus (mass, width, non-locality)



## Dynamical model for coherent $\pi$ production

\* Elementary amplitudes ( $\nu N \rightarrow \mu^- \pi^+ N$  ,  $\nu N \rightarrow \nu \pi^0 N$ )

$\Rightarrow$  **SL model** [Sato and Lee, PRC **54**, 2660 (1996); **67**, 065201 (2003)]

\* Medium effects

Final state interaction ( $\pi$ -nucleus scattering)

$\Delta$ -properties in nucleus (mass, width, **non-locality**)

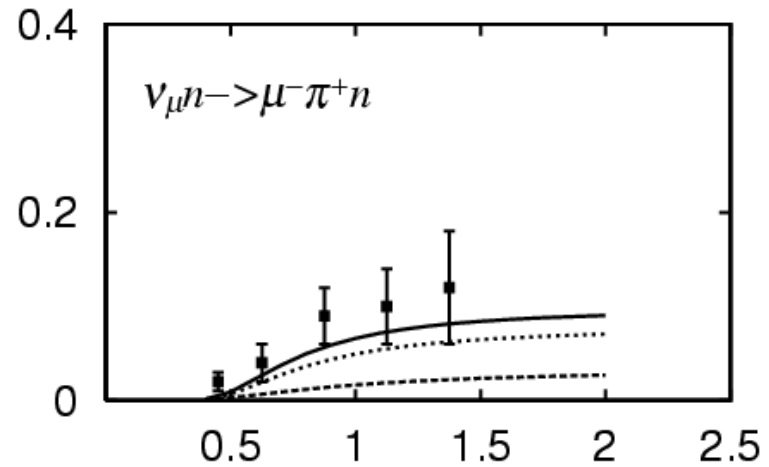
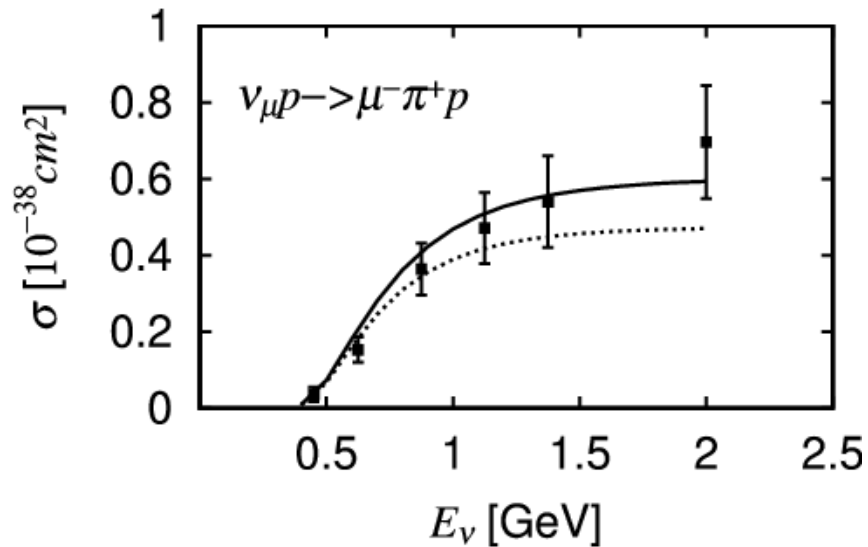
$\Rightarrow$   **$\Delta$ -hole model** [*e.g.*, Koch and Moniz, PRC **27**, 751 (1983)]

### **This work**

*Combine SL and  $\Delta$ -hole model and study coherent  $\pi$  production !*

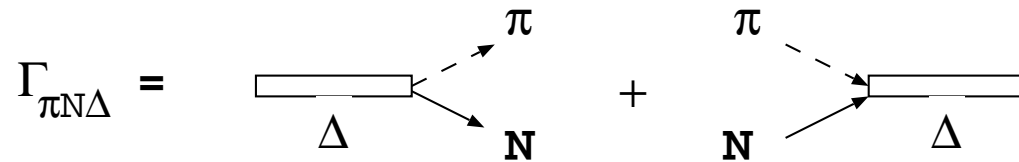
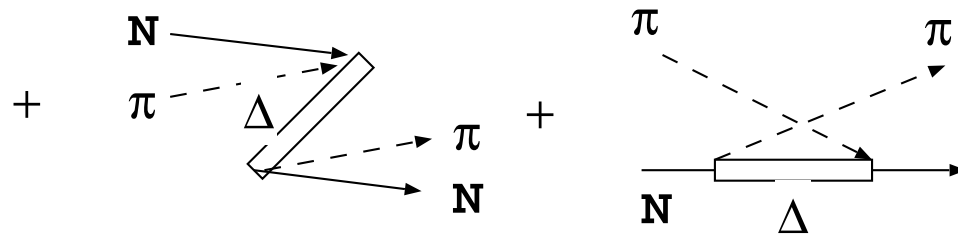
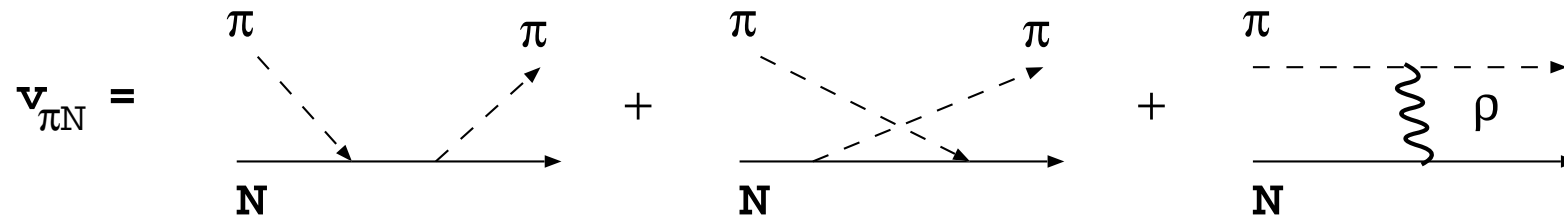
**Model**SL model[PRC **54**, 2660 (1996); **67**, 065201 (2003)]

- \* Model for electroweak  $\pi$  production off nucleon in  $\Delta$  region
- \* Non-perturbative resonant and non-resonant amplitudes ( $\pi$  cloud)
- \* Consistent description of JLab, BNL data for  $(\gamma, \pi^0)$  and  $(e, e'\pi^0)$

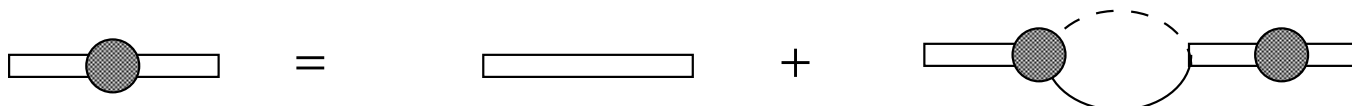
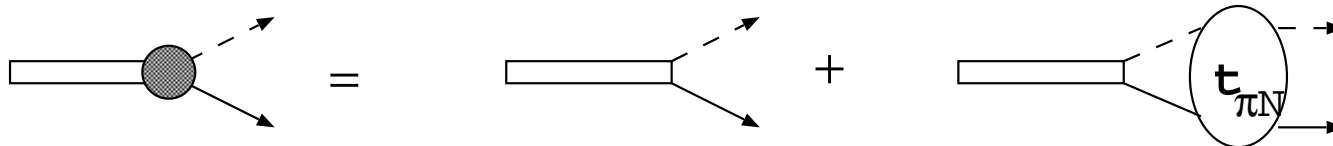
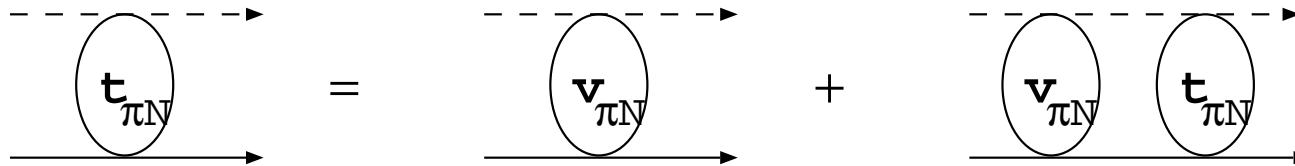
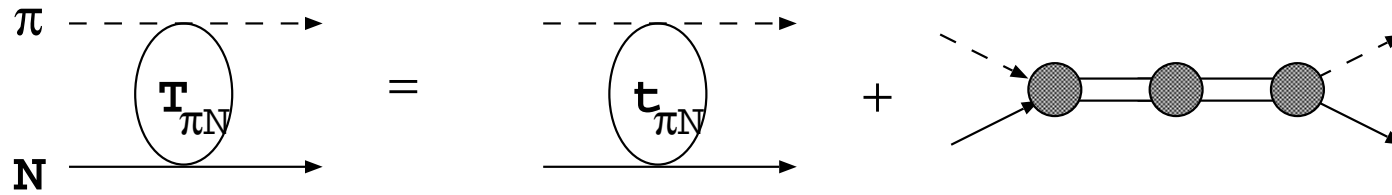


# example : $\pi N$ scattering in SL model

## Kernels



# T-matrix and Lippman-Schwinger Equation



## $\pi$ - $A$ (nucleus) optical potential from $\Delta$ -hole model

[Karaoglu *et al.*, PRC **33**, 974 (1986)]

$$t_{\pi N}^{SL} = t_{\Delta} + t_{non-R} \quad \Rightarrow \quad U_{\pi A} = U_{\Delta} + U_{non-R} + c\rho^2$$

$$t_{\Delta} = \frac{F_{\pi N \Delta}(\tilde{k}') F_{\pi N \Delta}(\tilde{k})}{D(W)}, \quad D(W) = W - m_{\Delta}^0 - \Sigma_{self}(W)$$

$$\Rightarrow \quad U_{\Delta}(k', k) \sim \sum_n^{occupied} \int d\vec{p}_{\Delta} \phi_n^* \frac{F_{\pi N \Delta}(\tilde{k}') F_{\pi N \Delta}(\tilde{k})}{D(E - H_{\Delta}) - \Sigma_{Pauli} - \Sigma_{spr}} \phi_n$$

$$H_{\Delta} = T_{\Delta} + V_{\Delta} + H_{A-1}, \quad T_{\Delta} \Rightarrow \text{non-local effect}$$

$$\Sigma_{spr} = V_C \rho(r) + V_{LS}(r) \vec{L}_{\Delta} \cdot \vec{S}_{\Delta}$$

Parameters (complex) :  $V_C, V_{LS}, c_s, c_p \rightarrow \pi$ -nucleus scattering data

## Optical potential for $\pi$ - $A$ scattering (contn'd)

$$\begin{aligned}
 U_{\Delta}(k', k) &\sim \sum_n^{\text{occupied}} \int d\vec{p} \phi_n^*(\vec{p}') \frac{F_{\pi N\Delta}(\tilde{k}') F_{\pi N\Delta}(\tilde{k})}{D(E - H_{\Delta}) - \Sigma_{\text{Pauli}} - \Sigma_{\text{spr}}} \phi_n(\vec{p}) \\
 &= \int d\vec{r} d\vec{r}' \int d\vec{p} \frac{F_{\pi N\Delta}(\tilde{k}') F_{\pi N\Delta}(\tilde{k})}{D(E - H_{\Delta}) - \Sigma_{\text{Pauli}} - \Sigma_{\text{spr}}} e^{-i\vec{p}' \cdot \vec{r}'} \rho(\vec{r}, \vec{r}') e^{i\vec{p} \cdot \vec{r}}
 \end{aligned}$$

\* Negele's local density approx. [PRC **5**, 1472 (1972)]

$$\rho(\vec{r}, \vec{r}') \sim \rho(R) j_1(k_F s) \frac{3}{k_F s}, \quad [ \rho(R) \Leftarrow \text{electron scattering data} ]$$

$$R = |\vec{r} + \vec{r}'|/2, \quad s = |\vec{r} - \vec{r}'|, \quad k_F^3 = 3\pi^2 \rho(R)/2$$

\* Local approx. (*not* used in this work)

$$T_{\Delta} = p_{\Delta}^2/2m_{\Delta} \rightarrow 0 \quad \Longrightarrow \quad \rho(\vec{r}, \vec{r}') \rightarrow \rho(R)$$

Transition amplitude for  $\lambda A \rightarrow \pi A$      [ $\lambda$  : (axial-)vector current]

$$a_{\lambda N \rightarrow \pi N}^{SL} = a_{\Delta} + a_{non-R} \quad \Rightarrow \quad A_{\lambda A \rightarrow \pi A} = A_{\Delta} + A_{non-R}$$

$$a_{\Delta} = \frac{N(\tilde{k}, \tilde{q}_{\lambda})}{D(W)}, \quad D(W) = W - m_{\Delta}^0 - \Sigma_{self}(W)$$

$$\Rightarrow \quad A_{\Delta}(k, q) \sim \sum_n^{occupied} \int d\vec{p}_{\Delta} \phi_n^* \frac{N(\tilde{k}, \tilde{q}_{\lambda})}{D(E - H_{\Delta}) - \Sigma_{Pauli} - \Sigma_{spr}} \phi_n$$

$$\Rightarrow \quad A_{\lambda A \rightarrow \pi A}^{DW} = \int d\vec{k}' \psi_{\pi A}(\vec{k}') A_{\lambda A \rightarrow \pi A}(\vec{k}', \vec{q}_{\lambda})$$

$\psi_{\pi A}$  :  $\pi$  wave function (final state interaction,  $U_{\pi A}$ )

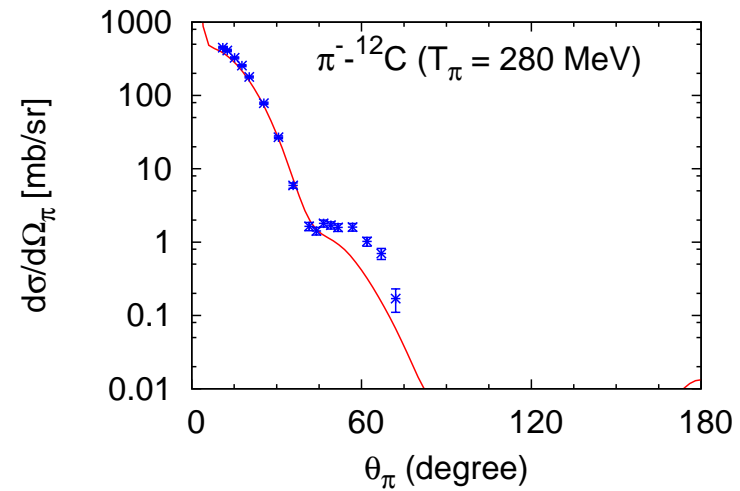
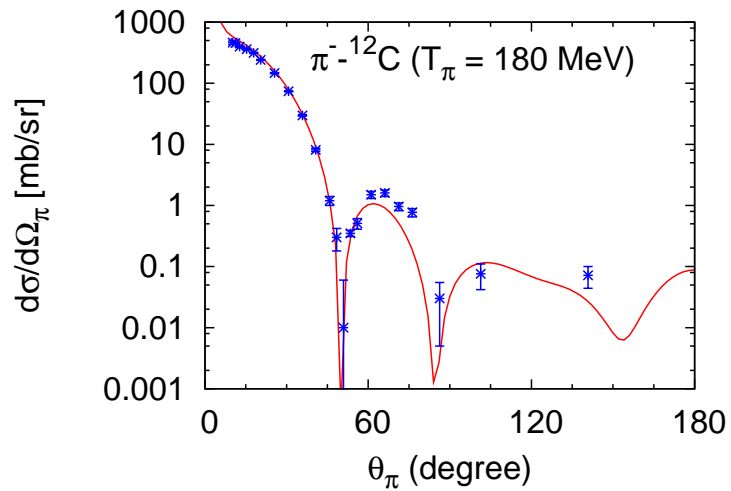
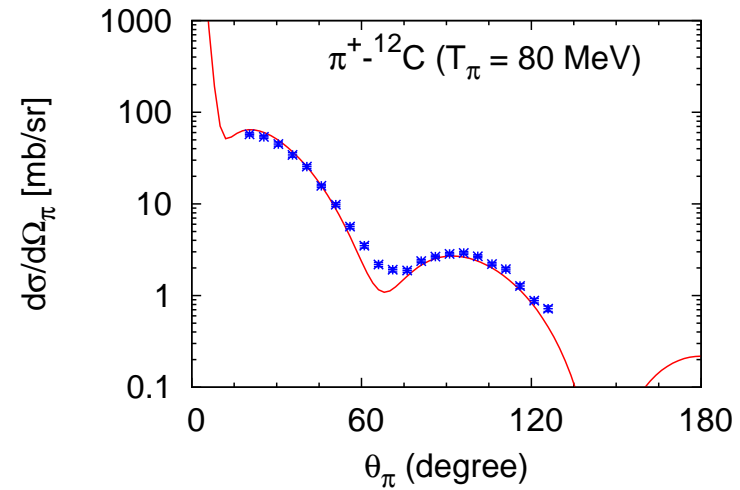
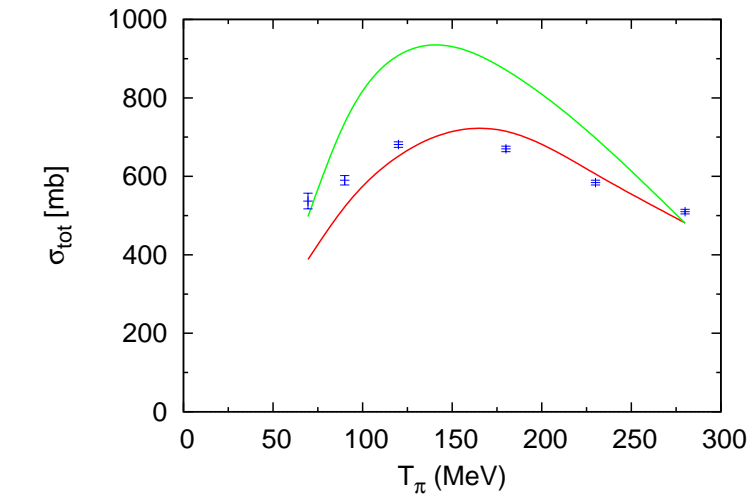
All parameters fixed by  $\pi$ -nucleus scattering data

$\Rightarrow$  **parameter-free prediction** for  $\lambda A \rightarrow \pi A$  (consistent model-building)

# Results

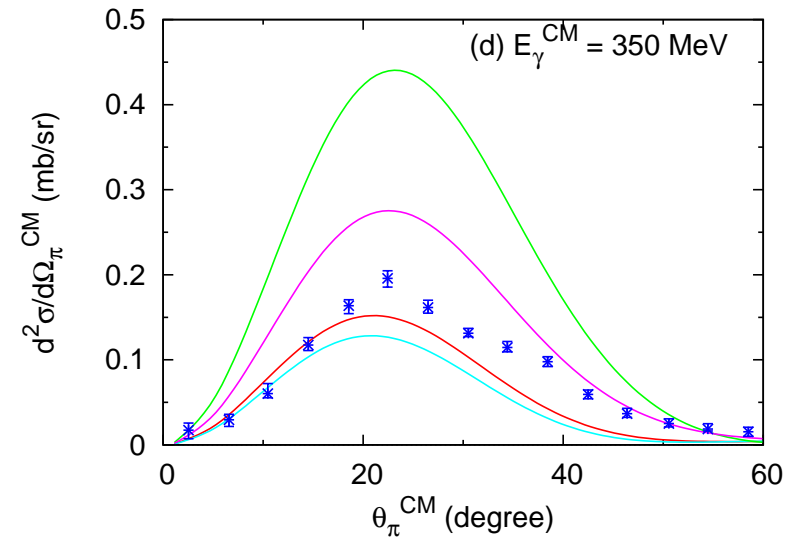
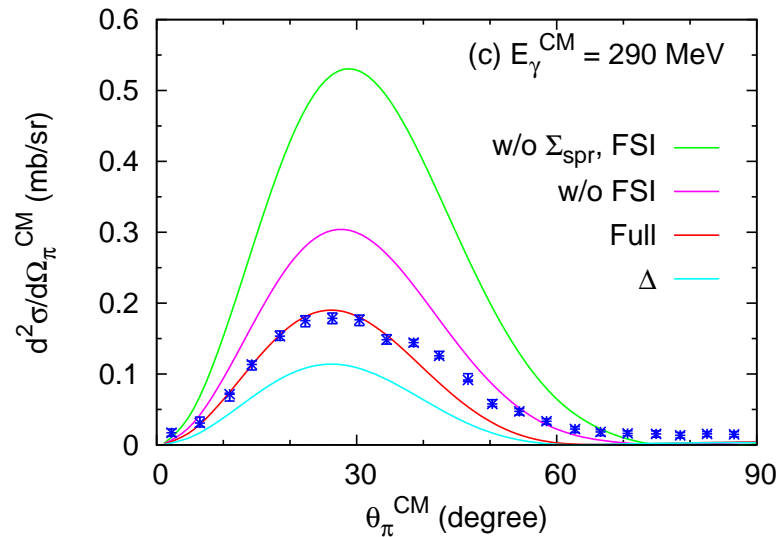
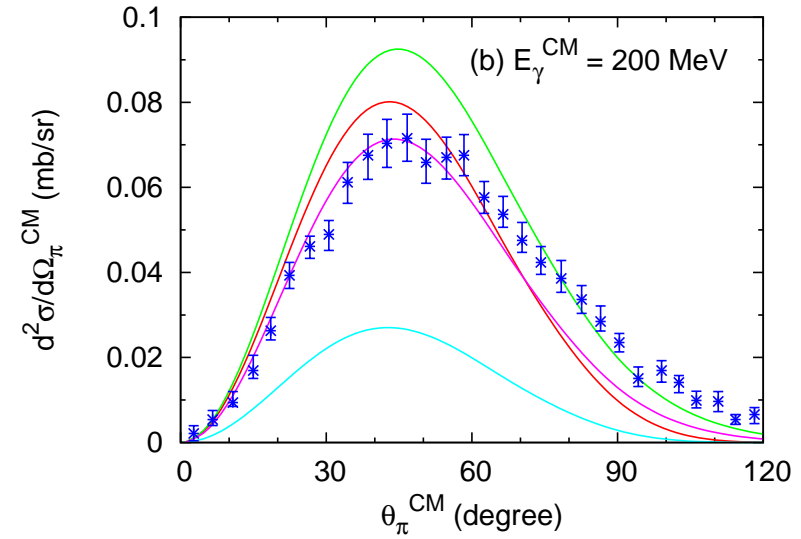
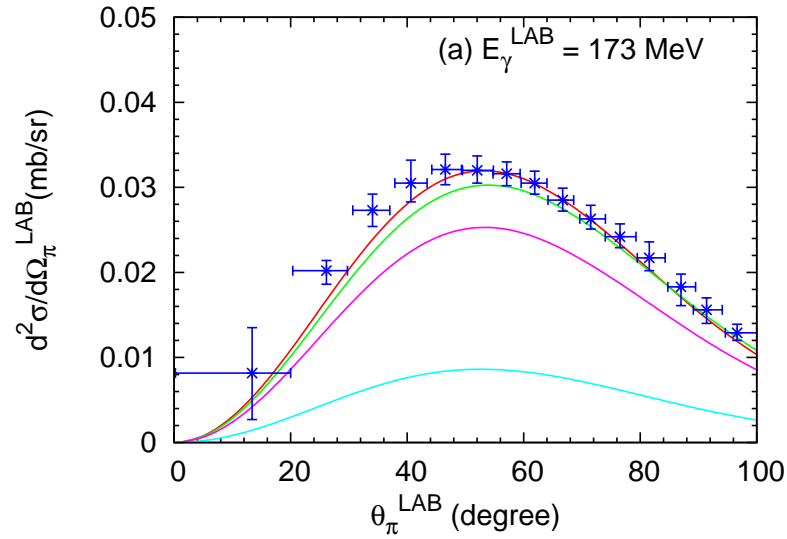
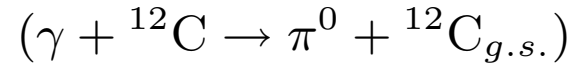
$\pi^\pm - {}^{12}\text{C}$  scattering

[Data : NPB **17**, 168 (1970), PRC **29**, 561 (1984)]





# Photo coherent $\pi$ production

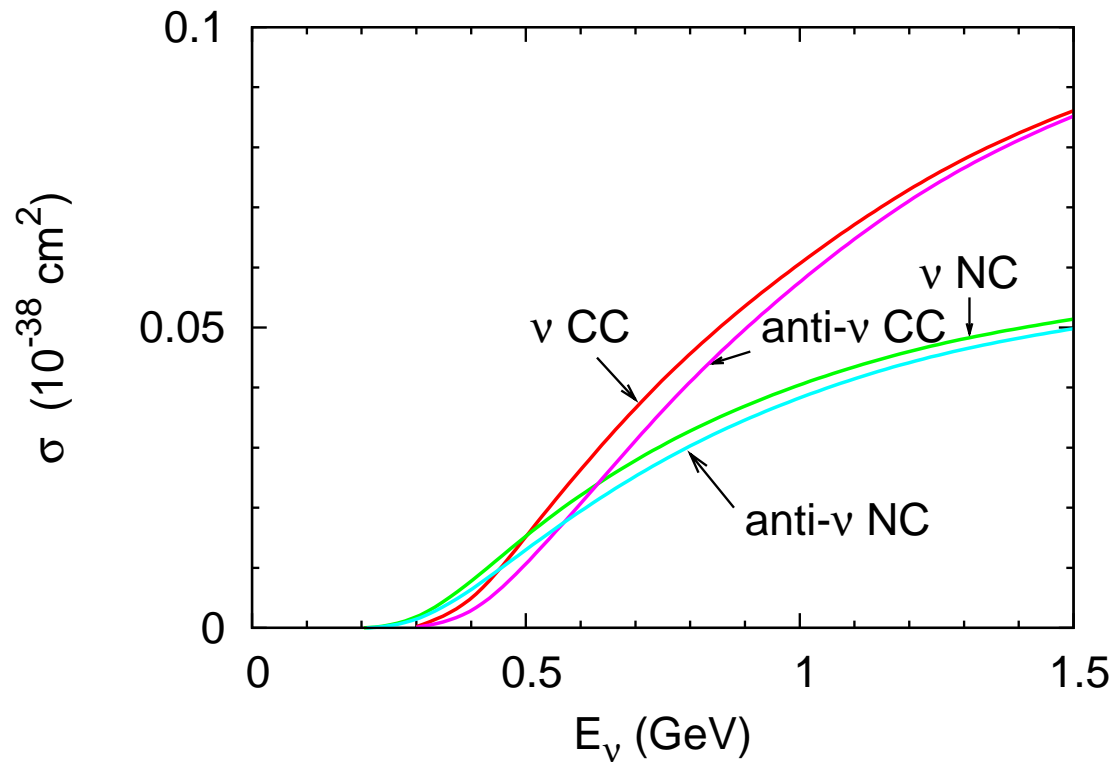


[Data:(a) Gothe et al., PLB**355**,59(1995); (b)-(d) Krusche et al., PLB**526**,287(2002)]

# Discussion

- Parameter-free prediction
- Important medium effects
- Important non-resonant mechanism
- Discrepancy in larger angle region  
⇒ contamination from incoherent processes

# $\nu$ -induced coherent $\pi$ production on $^{12}\text{C}$



## CC

$$\sigma_{\text{ave}} = 6.3 \times 10^{-40} \text{cm}^2$$

( $p_\mu < 450$  MeV cut applied)

$$\sigma_{\text{K2K}} < 7.7 \times 10^{-40} \text{cm}^2$$

[ $\nu$ -flux,  $\sigma_{\text{exp}}$ : PRD **74**, 072003 (2006);  
PRL **95**, 252301 (2005)]

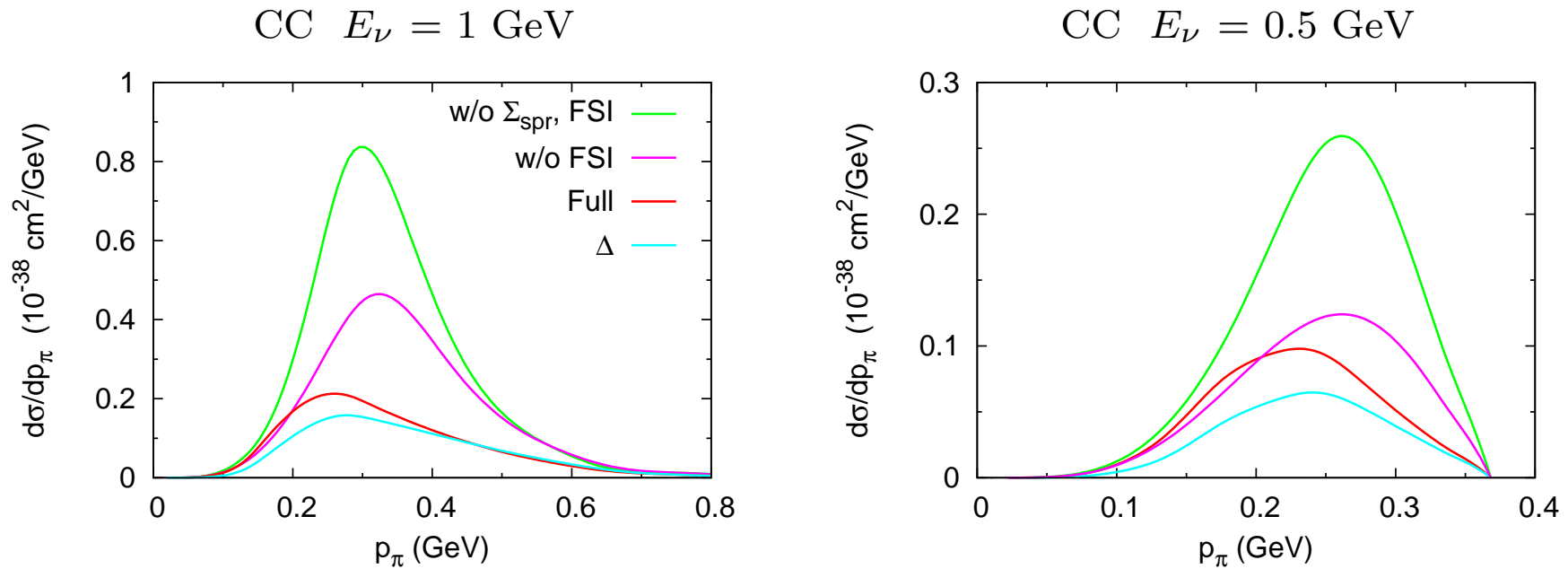
## NC

$$\sigma_{\text{ave}} = 3.9 \times 10^{-40} \text{cm}^2$$

$$\sigma_{\text{MiniBooNE}} = 7.7 \pm 1.6 \pm 3.6 \\ \times 10^{-40} \text{cm}^2$$

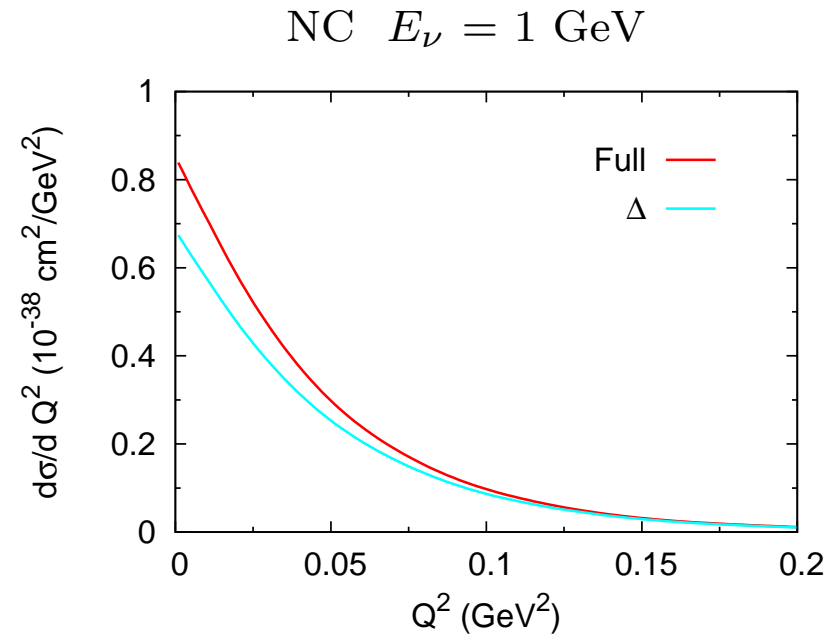
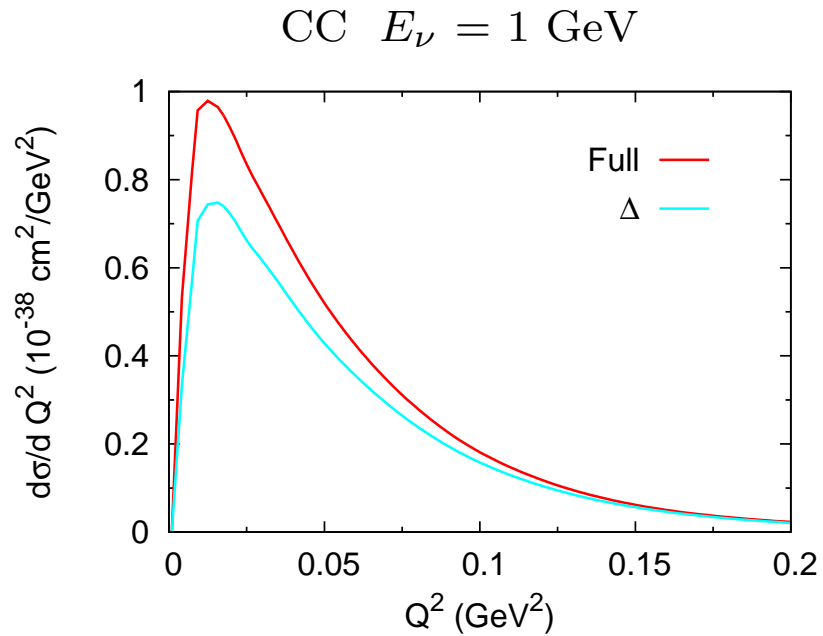
[ $\nu$ -flux,  $\sigma_{\text{exp}}$ : PLB **664**, 41 (2008);  
Raaf, PhD thesis]

## $\pi$ momentum spectra



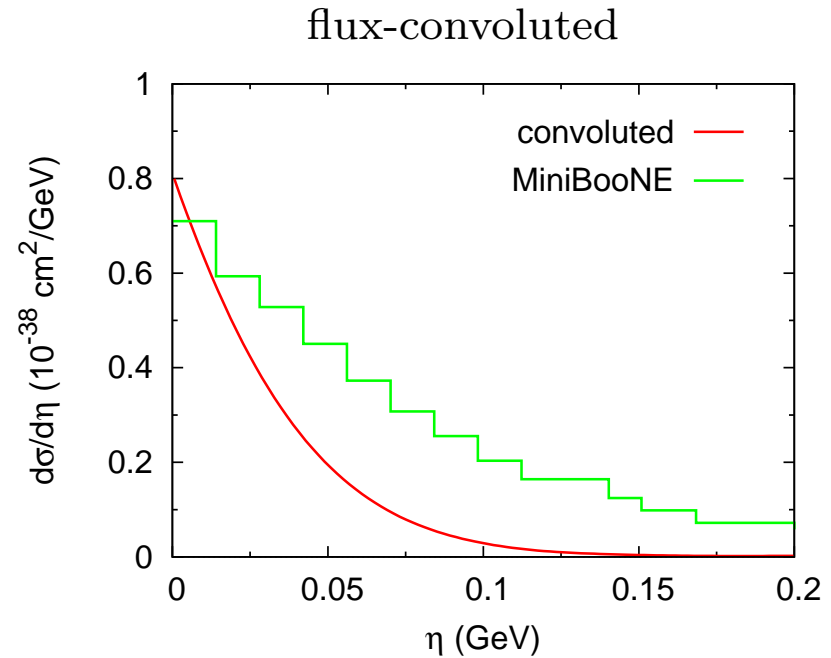
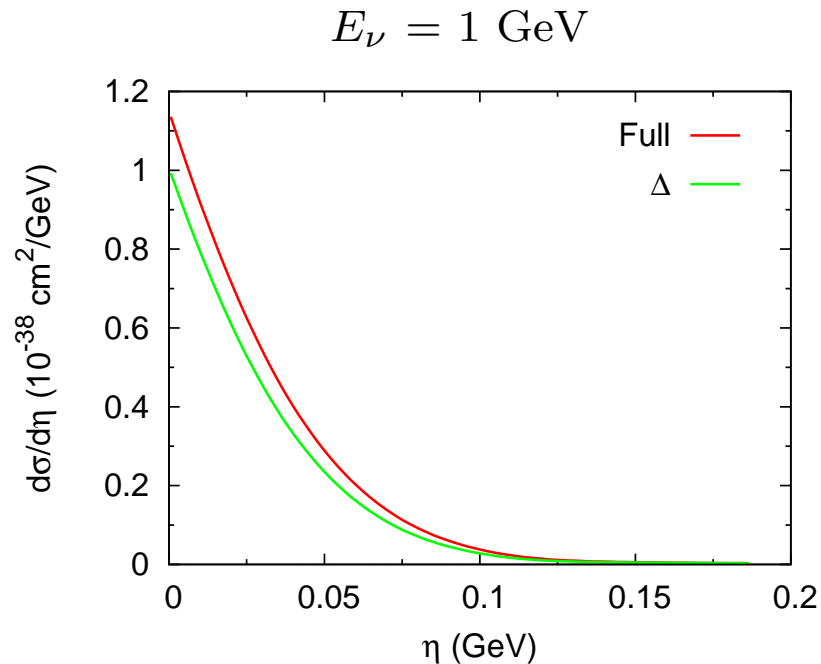
- Large medium effects in  $\Delta$  region
- Enhancement due to non-resonance (interference with  $\Delta$ )  
32 (10) % at  $E_\nu = 0.5$  (1) GeV
- No contribution from (tree-level) non-resonant mechanism in previous calculations

## $Q^2$ distribution



- Sharp peak at  $Q^2 \sim 0$  due to strong damping by nuclear form factor
- Non-resonant mechanism sharpens the peak

$\eta \equiv E_\pi(1 - \cos \theta_\pi)$  distribution for NC

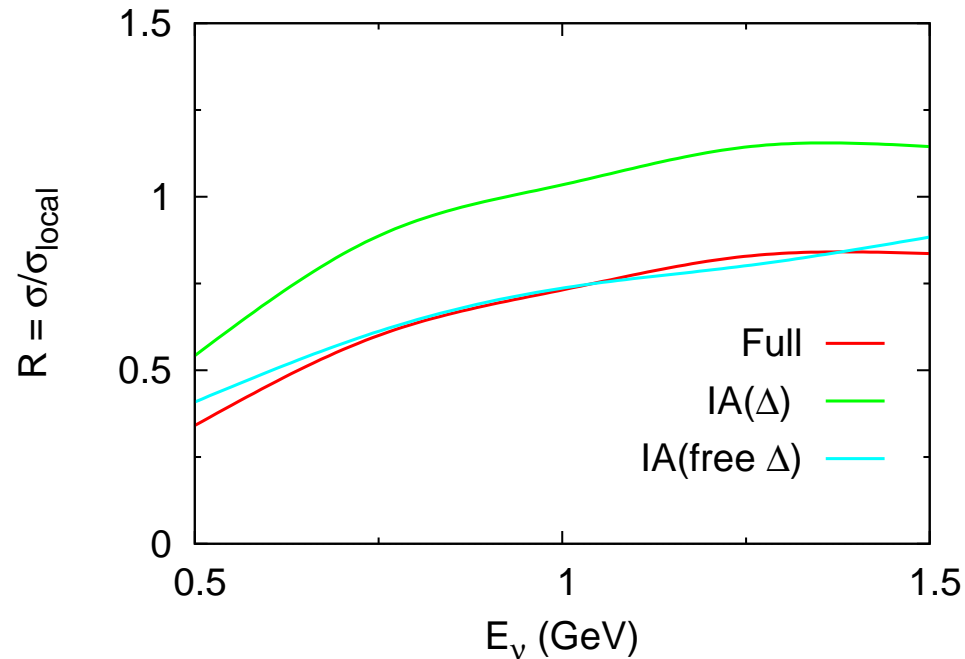


[flux and MC result of MiniBooNE  
PLB **664**, 41 (2008)]

- $\eta$  is useful to break degeneracy of several pion productions in data
- Discrepancy between Monte Carlo (Rein-Sehgal model) and ours  
⇒ possible overestimation of NC cross section

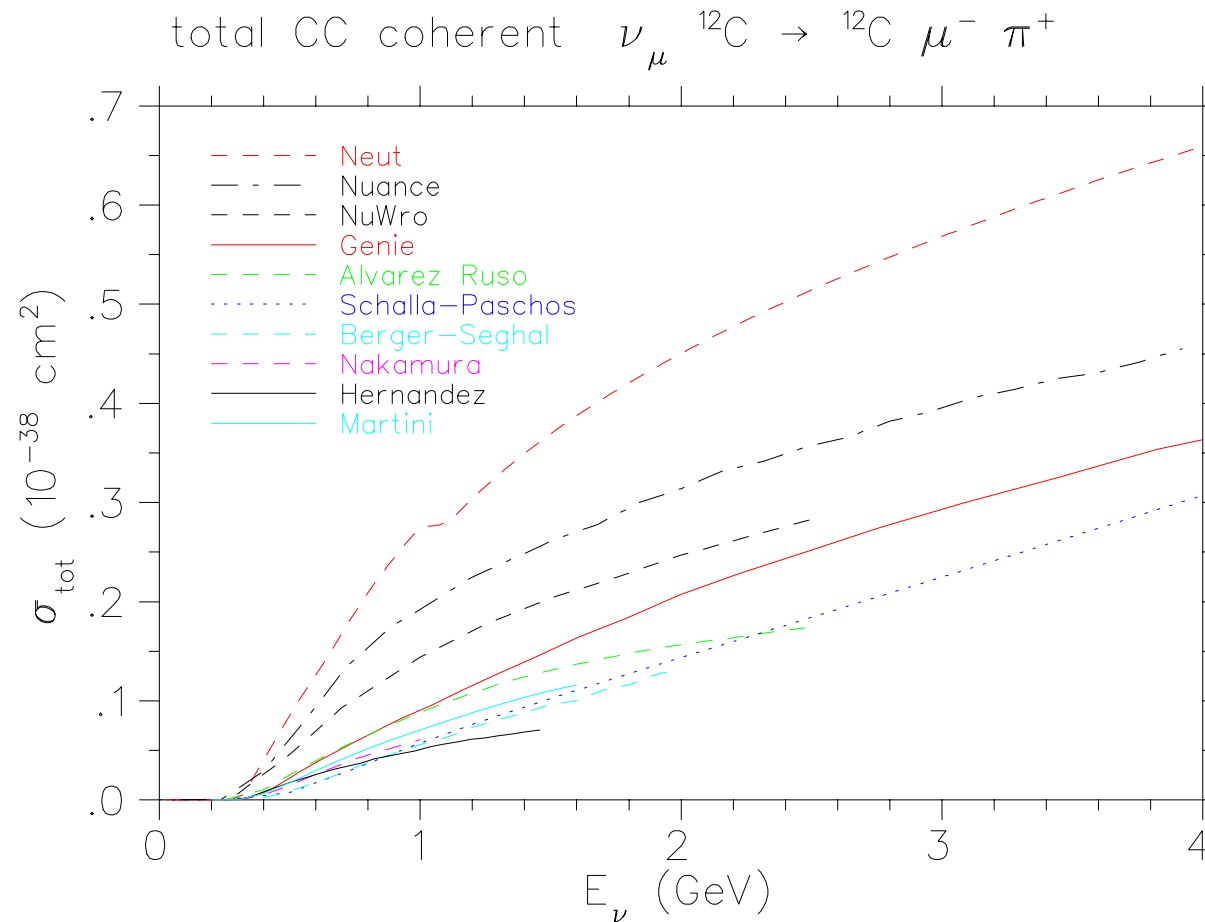
# Non-locality of $\Delta$ -propagation

(Leitner et al., PRC **79**,057601 (2009))



- 60, 30, 20 % reduction for  $E_\nu = 0.5, 1, 1.5$  GeV for free  $\Delta$  and Full
- Non-local effect is still important after including medium effects
- All previous microscopic calculations used local approximation

# Comparison of models

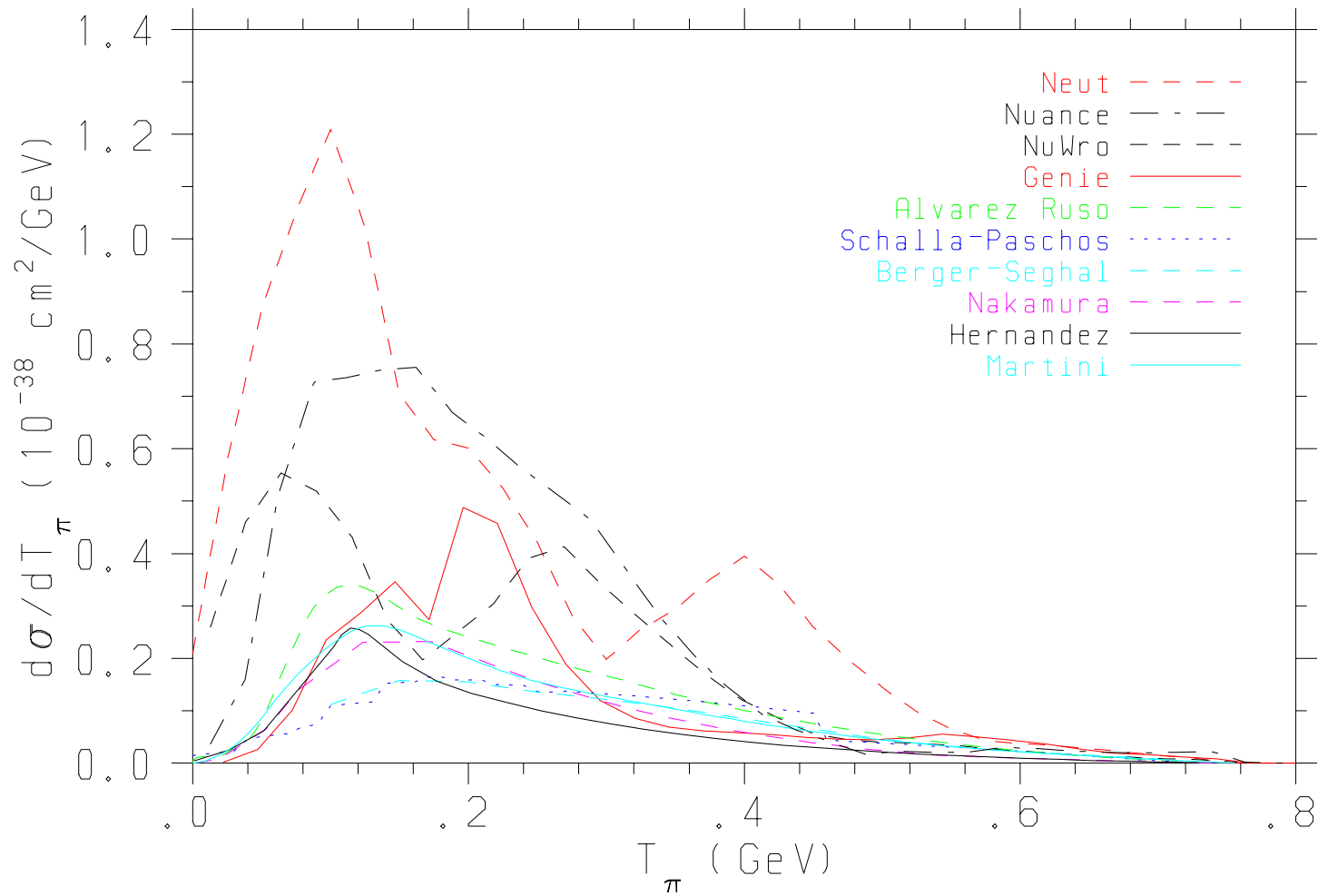


(Figure from NUINT09 joint project)

- Large discrepancy between MC code (Rein-Sehgal model) and recent calculations



CC coherent  $\pi$  KE distribution at  $E_\nu=1.0$  GeV  $\nu_\mu$   $^{12}\text{C} \rightarrow$   $^{12}\text{C} \mu^- \pi^+$



(Figure from NUINT09 joint project)

# Discussion

## PCAC-based model

Basic assumptions are questionable for  $E_\nu \lesssim 2$  GeV and light nuclei

- $Q^2 = 0$  kinematics
- $p_\pi = q$  kinematics
- local approximation

[critical review : Amaro et al., PRD **79**, 013002 (2009)]

## Microscopic model

- Elementary amplitude, tree-level or unitarized, scarce data
- Medium effect, local approximation
- Final state interaction around  $\Delta$  region
- Comparison with photo-reaction data

## Summary

### Development of microscopic model for coherent $\pi$ production

*SL model +  $\Delta$ -hole model* which enables a **unified** description of:

$$\pi A \rightarrow \pi A \quad \gamma A \rightarrow \pi^0 A \quad \nu A \rightarrow \ell^- \pi^+ A \quad \nu A \rightarrow \nu \pi^0 A$$

- Reproduce  $\pi - A$  and  $\gamma A \rightarrow \pi^0 A$  data reasonably
- Consistent with K2K data for CC coherent  $\pi$  production
- Numerical results relevant to neutrino experiments presented
- Comparison and review of models made

## Future development

- \* Elementary amplitude with **higher resonances beyond  $\Delta$**
- \* Implementation of more **detailed nuclear structure**

QMC wave function significantly improves B(E2) over shell-model

$\implies$  Improved description of  $\pi - {}^{6,7}\text{Li}$  inelastic scattering

[Lee and Wiringa, PRC **63**,014006 (2000)]

$$\rho_N(\vec{r}, \vec{r}') = \langle \Psi_{\text{QMC}} | \frac{1}{A} \sum_{i=1}^A \delta(\vec{r}' - \vec{r}'_i) \delta(\vec{r} - \vec{r}_i) \frac{1 + 2t_N \tau_z(i)}{2} | \Psi_{\text{QMC}} \rangle$$

Fit to data improves ?      Spreading potential changes ?

## Future development (contn'd)

Implementation of detailed nuclear structure

⇒ Fully utilize **pion-nucleus processes as laboratory** to study

- **$\Delta$  in nucleus**

Quantitative microscopic evaluation of  $\Delta$  spreading potential

( $\Leftrightarrow$  Phenomenological parameterization)

- **Role of many-body correlations**

How two-body correlation contributes ?

More than two-body correlation ?