Transverse Momentum Distributions of Quarks in the Nucleon from Lattice QCD

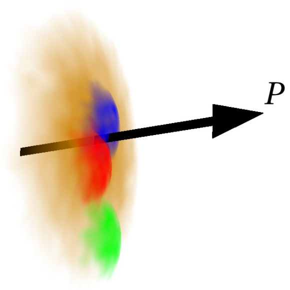
Bernhard Musch

in collaboration with
Philipp Hägler (TUM), John Negele (MIT),
Andreas Schäfer (Univ. Regensburg),
and the LHP Collaboration

Emmy Noether-Programm
Deutsche Forschungsgemeinschaft DFG

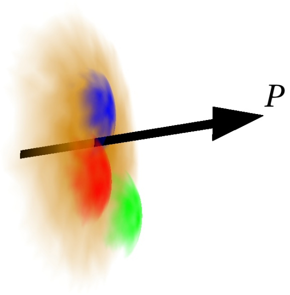
Technische Universität München

Jefferson Lab
Thomas Jefferson National Accelerator Facility



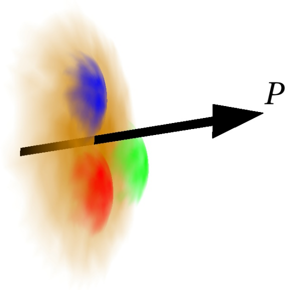
Nucleon (= proton or neutron) moving close to speed of light:

- appears flat like a disk (Lorentz contraction),
- intrinsic motion of quarks inside the nucleon.



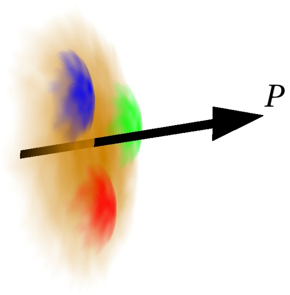
Nucleon (= proton or neutron) moving close to speed of light:

- appears flat like a disk (Lorentz contraction),
- intrinsic motion of quarks inside the nucleon.



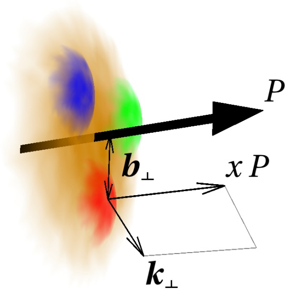
Nucleon (= proton or neutron) moving close to speed of light:

- appears flat like a disk (Lorentz contraction),
- intrinsic motion of quarks inside the nucleon.



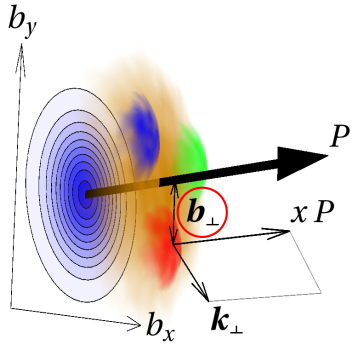
Nucleon (= proton or neutron) moving close to speed of light:

- appears flat like a disk (Lorentz contraction),
- intrinsic motion of quarks inside the nucleon.



$\Longrightarrow$ quark distributions with respect to

- momentum fraction x of the nucleon momentum P, $\Rightarrow$ PDFs
- transverse position $\boldsymbol{b}_\perp$ (impact parameter), $\Rightarrow$ GPDs
- intrinsic transverse momentum $\boldsymbol{k}_\perp$. $\Rightarrow$ TMD PDFs



Charge density $\rho(\boldsymbol{b}_\perp)$

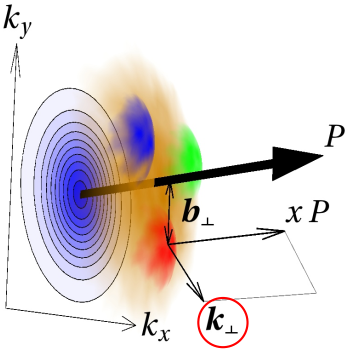
“Nucleon tomography”

spatial image of the nucleon

has been successfully calculated using GPDs from lattice QCD.

$\Longrightarrow$ quark distributions with respect to

- momentum fraction x of the nucleon momentum P, $\Rightarrow$ PDFs
- transverse position $\boldsymbol{b}_\perp$ (impact parameter), $\Rightarrow$ GPDs
- intrinsic transverse momentum $\boldsymbol{k}_\perp$. $\Rightarrow$ TMD PDFs



new: lattice study of

TMD PDFs

transverse **m**omentum **d**ependent **p**arton **d**istribution **f**unctions

e.g., $f_1(x, \boldsymbol{k}_\perp^2)$

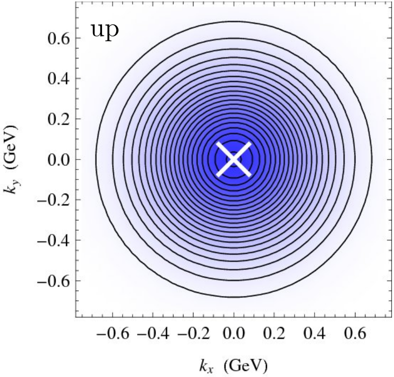
$\Rightarrow$ quark density $\rho(\boldsymbol{k}_\perp)$.

$\Longrightarrow$ quark distributions with respect to

- momentum fraction x of the nucleon momentum P, $\Rightarrow$ PDFs
- transverse position $\boldsymbol{b}_\perp$ (impact parameter), $\Rightarrow$ GPDs
- intrinsic transverse momentum $\boldsymbol{k}_\perp$. $\Rightarrow$ TMD PDFs

Density of unpolarized quarks (minus antiquarks)
in an unpolarized nucleon as a function of transverse momentum $\boldsymbol{k}_\perp$:

$$\rho_{UU}(\boldsymbol{k}_\perp) = \int_{-1}^{1} dx\ f_1(x, \boldsymbol{k}_\perp^2)$$



axially symmetric

($m_\pi \approx 500$ MeV, straight gauge link operator, renormalization condition $C^{\text{ren}} = 0$, Gaussian fit)

N \ q	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

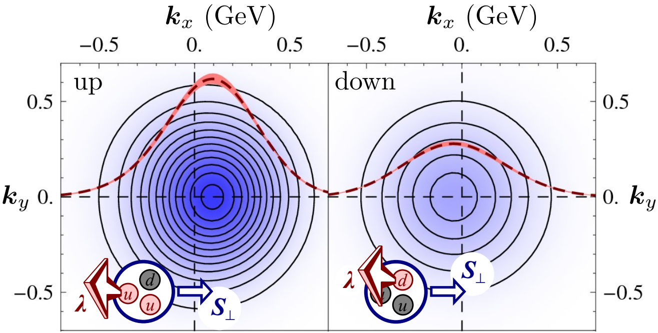
time-reversal odd

N \ q	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

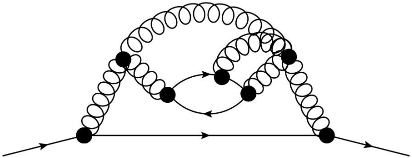
time-reversal odd

Density of quarks with positive helicity, $\lambda = 1$,
in a transversely polarized nucleon, $\boldsymbol{S}_\perp = (1,0)$:

$$\rho_{TL}(\boldsymbol{k}_\perp;\boldsymbol{S}_\perp,\lambda) \equiv \frac{1}{2}\int dx \int dk^- \; \Phi^{[\gamma^+ \frac{1}{2}(\mathbb{1}+\gamma^5)]}(k,P,S_\perp)$$
$$= \frac{1}{2} f_1^{(0_x)}(\boldsymbol{k}_\perp^2) + \frac{\lambda}{2}\frac{\boldsymbol{k}_\perp\cdot\boldsymbol{S}_\perp}{m_N}\; g_{1T}^{(0_x)}(\boldsymbol{k}_\perp^2)$$



($m_\pi \approx 500$ MeV, straight gauge link operator,
renormalization condition $C^{\text{ren}} = 0$, Gaussian fit)



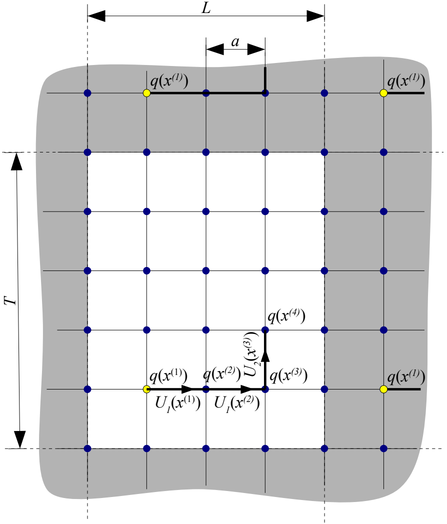
action

$$S_{\rm QCD}[\overline{q}, q, A] = \int d^4x \, \{\bar{q}(i\gamma^\mu D_\mu - m)q\} + S_{\rm gauge}[A]$$

$q, \bar{q}$: quark fields,
A_μ : gluon fields ("gauge fields"),
$D_\mu = \partial_\mu - igA_\mu$

path integral

$$\langle\!\langle O[\bar{q}, q, A] \rangle\!\rangle = \frac{\int \mathcal{D}\overline{q}\,\mathcal{D}q\,\mathcal{D}A \; O[\bar{q}, q, A] \; \exp(iS_{\rm QCD})}{\int \mathcal{D}\overline{q}\,\mathcal{D}q\,\mathcal{D}A \; \exp(iS_{\rm QCD})}$$



periodic/antiperiodic lattice
$L \times L \times L \times T$

gluon fields as "links variables"
$U_\mu(x) \ = \ \exp(igaA_\mu(x))$

derivatives → finite differences
$\partial_\mu q(x) \approx \dfrac{q(x+ae_\mu) - q(x-ae_\mu)}{2a}$

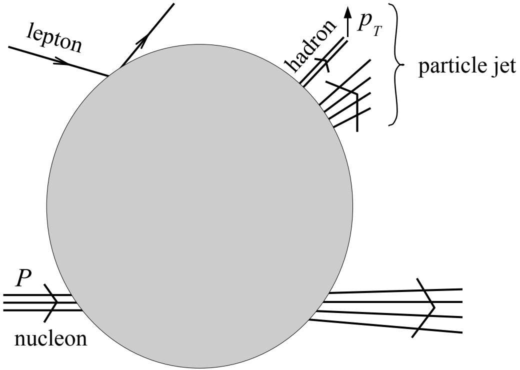
- Wick rotation: $x^0 \to -ix^4$
  $\Rightarrow$ Euclidean space, $iS_{\text{QCD}} \to -S_{\text{lat}}$ (real)
- integrate out fermions $q, \bar{q}$ analytically:
  $$\langle\!\langle O \rangle\!\rangle = \frac{\int \mathcal{D}U\, \tilde{O}[U]\, \exp(-S_{\text{gauge}}[U])K[U]}{\int \mathcal{D}U\, \exp(-S_{\text{lat}})K[U]}$$
  $\tilde{O}[U]$ is an expression in terms of
  - link variables $U_\mu(x)$ (gauge field),
  - full quark propagators on a gauge background $\{\overbrace{q(y)\bar{q}}(x)\}[U]$

importance sampling

1. generate a set of N gauge configurations $\{U\}$ with a method ensuring probability$[U] \sim \exp(-S_{\text{gauge}})K[U]$
2. $\langle\!\langle O \rangle\!\rangle = \dfrac{1}{N} \sum_{\{U\}} \tilde{O}[U]$

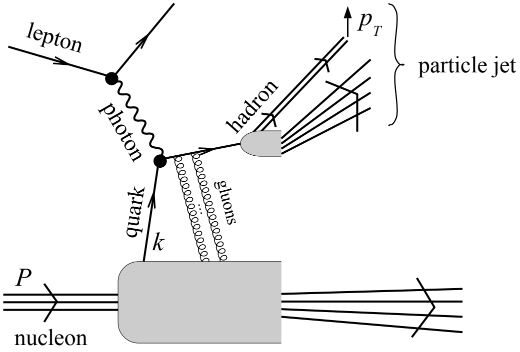
Example: semi-inclusive deeply inelastic scattering (**SIDIS**)

Intrinsic transverse quark momentum responsible for azimuthal asymmetries of the cross section (Cahn- and Sivers-effect, ...).



Example: semi-inclusive deeply inelastic scattering (**SIDIS**)

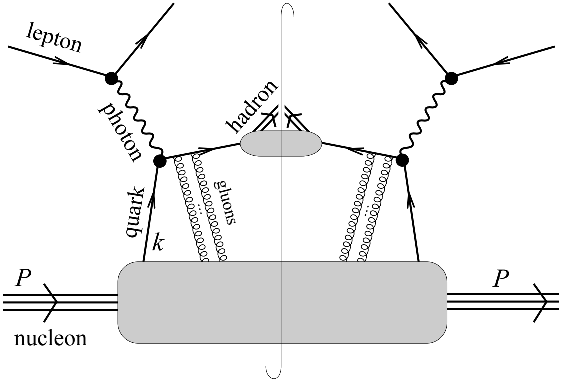
Intrinsic transverse quark momentum responsible for azimuthal asymmetries of the cross section (Cahn- and Sivers-effect, ...).



factorization:

hard process + soft blobs (non-perturbative)

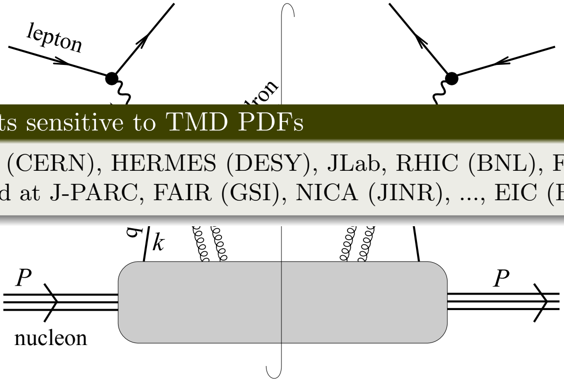
[Collins, Soper, Sterman PLB 83, NPB 85]
[Ji, Ma, Yuan PRD (2005)]
[Mulders, Tangerman NPB (1996)]



factorization:

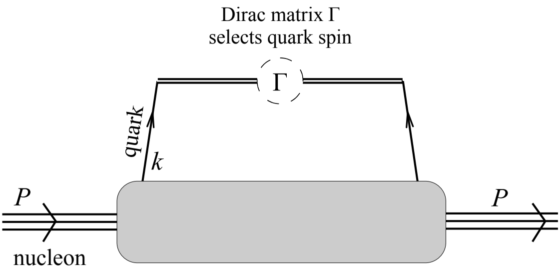
hard process + soft blobs (non-perturbative)

[Collins, Soper, Sterman PLB 83, NPB 85]
[Ji, Ma, Yuan PRD (2005)]
[Mulders, Tangerman NPB (1996)]



experiments sensitive to TMD PDFs

COMPASS (CERN), HERMES (DESY), JLab, RHIC (BNL), Fermilab, also planned at J-PARC, FAIR (GSI), NICA (JINR), ..., EIC (BNL/JLab?)



$$\Phi^{[\Gamma]}(k,P,S) \;\equiv\; \text{“}\, \langle P,S|\,\bar{q}(k)\,\Gamma\,q(k)\,|P,S\rangle\, \text{”}$$

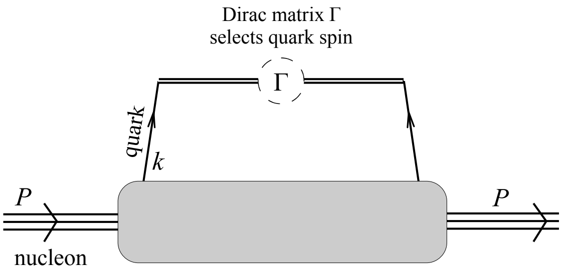
lightcone coor. $w^{\pm} = \frac{1}{\sqrt{2}}(w^0 \pm w^3)$, so $w = w^+ \hat{n}_+ + w^- \hat{n}_- + w_\perp$

proton flies along z-axis: P^+ large, $P_\perp = 0$

parametrization in terms of TMD PDFs, example

$$\int dk^- \;\Phi^{[\gamma^+]}(k,P,S)\Big|_{k^+=xP^+} = f_1(x,\boldsymbol{k}_\perp^2) - \frac{\epsilon_{ij}\boldsymbol{k}_i\boldsymbol{S}_j}{m_N} f_{1T}^{\perp}(x,\boldsymbol{k}_\perp)$$

[Ralston, Soper NPB 1979], [Mulders, Tangerman NPB 1996], [Goeke, Metz, Schlegel PLB 2005]



$$\Phi^{[\Gamma]}(k,P,S) \equiv \frac{1}{2}\int \frac{d^4\ell}{(2\pi)^4}\, e^{-ik\cdot\ell}\ \langle P,S|\ \bar{q}(\ell)\,\Gamma\,\mathcal{U}\,q(0)\ |P,S\rangle$$

lightcone coor. $w^{\pm} = \frac{1}{\sqrt{2}}(w^0 \pm w^3)$, so $w = w^+\hat{n}_+ + w^-\hat{n}_- + w_\perp$

proton flies along z-axis: P^+ large, $P_\perp = 0$

parametrization in terms of TMD PDFs, example

$$\int dk^-\ \Phi^{[\gamma^+]}(k,P,S)\Big|_{k^+=xP^+} = f_1(x,\boldsymbol{k}_\perp^2) - \frac{\boldsymbol{\epsilon}_{ij}\boldsymbol{k}_i\boldsymbol{S}_j}{m_N} f_{1T}^\perp(x,\boldsymbol{k}_\perp)$$

[Ralston, Soper NPB 1979], [Mulders, Tangerman NPB 1996], [Goeke, Metz, Schlegel PLB 2005]

$\langle P| \ \overline{q}(\ell)\,\Gamma\,\mathcal{U}\,q(0) \ |P\rangle$ is gauge invariant.

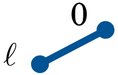
continuum

$$\mathcal{U} \equiv \mathcal{P}\exp\left(-ig\int_0^\ell d\xi^\mu A_\mu(\xi)\right)$$

along path from 0 to ℓ

- factorization in SIDIS :
  path runs to infinity and back

- ***here*** (up to now):
  straight path



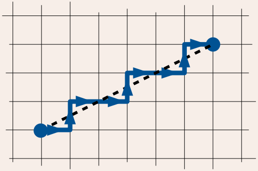
$\langle P|\ \bar{q}(\ell)\,\Gamma\,\mathcal{U}\,q(0)\ |P\rangle$ is gauge invariant.

continuum

$$\mathcal{U} \equiv \mathcal{P}\exp\left(-ig\int_0^\ell d\xi^\mu A_\mu(\xi)\right)$$

along path from 0 to ℓ

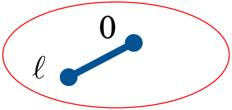
lattice



product of link variables

- factorization in SIDIS : path runs to infinity and back

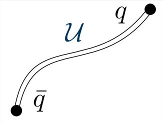
- ***here*** (up to now): straight path



continuum renormalization of gauge links

[Craigie, Dorn NPB185,204 (1981)]

smooth path



$$[\bar{q}\,\mathcal{U}\,q]_{\rm ren} = Z^{-1} \exp\left(-\delta\hat{m}\,\frac{l}{a} \right) [\bar{q}\,\mathcal{U}\,q]$$

l : the total length of the gauge link,

$\delta\hat{m}$: removes the power divergence $\sim 1/a$

static quark potential

$$V_{\rm ren}(r) = V(r) + 2\,\delta\hat{m}/a$$

string [Lüscher,Symanzik,Weisz (1980)]

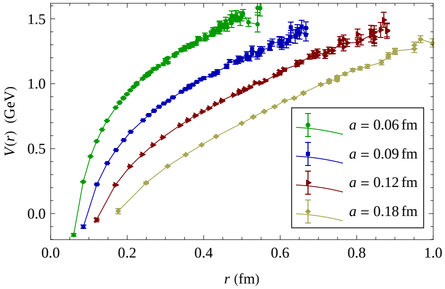
at large r: $V_{\rm ren}(r) \approx$

$V_{\rm string}(r) = \sigma r - \pi/12r + C$

method [Cheng PRD77,014511 (2008)]

determine $\delta\hat{m}$ from

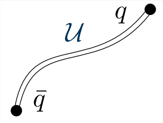
$V_{\rm ren}(0.7\text{ fm}) \stackrel{!}{=} V_{\rm string}(0.7\text{ fm})$



continuum renormalization of gauge links

[Craigie, Dorn NPB185,204 (1981)]

smooth path



$$[\bar{q}\,\mathcal{U}\,q]_{\text{ren}} = Z^{-1} \exp\left(-\delta\hat{m}\,\frac{l}{a} \right) [\bar{q}\,\mathcal{U}\,q]$$

l : the total length of the gauge link,

$\delta\hat{m}$: removes the power divergence $\sim 1/a$

static quark potential

$$V_{\text{ren}}(r) = V(r) + 2\,\delta\hat{m}/a$$

string [Lüscher,Symanzik,Weisz (1980)]

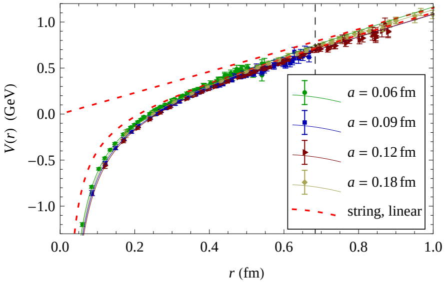
at large r: $V_{\text{ren}}(r) \approx$

$$V_{\text{string}}(r) = \sigma r - \pi/12r + C$$

method [Cheng PRD77,014511 (2008)]

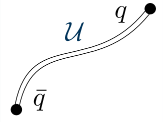
determine $\delta\hat{m}$ from

$$V_{\text{ren}}(0.7\text{ fm}) \overset{!}{=} V_{\text{string}}(0.7\text{ fm})$$



continuum renormalization of gauge links [Craigie, Dorn NPB185,204 (1981)]

smooth path



$$[\bar{q}\,\mathcal{U}\,q]_{\text{ren}} = Z^{-1} \exp\left(-\delta\hat{m}\,\frac{l}{a} \right) [\bar{q}\,\mathcal{U}\,q]$$

l : the total length of the gauge link,

$\delta\hat{m}$: removes the power divergence $\sim 1/a$

static quark potential

$$V_{\text{ren}}(r) = V(r) + 2\,\delta\hat{m}/a$$

string [Lüscher, Symanzik, Weisz (1980)]

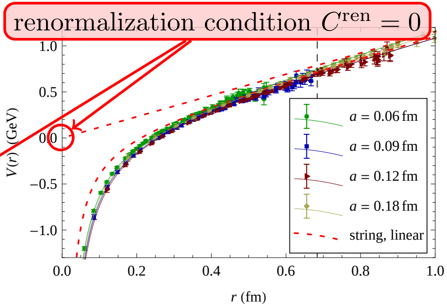
at large r: $V_{\text{ren}}(r) \approx$

$V_{\text{string}}(r) = \sigma r - \pi/12r + 0$

method [Cheng PRD77,014511 (2008)]

determine $\delta\hat{m}$ from

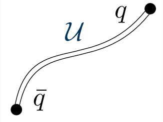
$V_{\text{ren}}(0.7\text{ fm}) \overset{!}{=} V_{\text{string}}(0.7\text{ fm})$



continuum renormalization of gauge links

[Craigie, Dorn NPB185,204 (1981)]

smooth path



$$[\bar{q}\,\mathcal{U}\,q]_{\text{ren}} = Z^{-1} \exp\left(-\delta\hat{m}\,\frac{l}{a}\right)[\bar{q}\,\mathcal{U}\,q]$$

l : the total length of the gauge link,

$\delta\hat{m}$: removes the power divergence $\sim 1/a$

static quark potential

$$V_{\text{ren}}(r) = V(r) + 2\,\delta\hat{m}/a$$

string [Lüscher,Symanzik,Weisz (1980)]

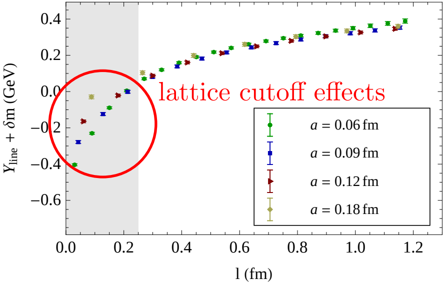
at large r: $V_{\text{ren}}(r) \approx$

$$V_{\text{string}}(r) = \sigma r - \pi/12r + 0$$

method [Cheng PRD77,014511 (2008)]

determine $\delta\hat{m}$ from

$$V_{\text{ren}}(0.7\text{ fm}) \stackrel{!}{=} V_{\text{string}}(0.7\text{ fm})$$



$$Y_{\text{line}}(l) \equiv \frac{d}{dl}\ln\langle \text{tr}\,\mathcal{U}\rangle_{\text{(Landau gauge)}}$$

$$\Phi^{[\Gamma]}(k,P,S) \equiv \frac{1}{2}\int \frac{d^4\ell}{(2\pi)^4}\, e^{-ik\cdot\ell}\ \langle P,S|\ \bar{q}(\ell)\,\Gamma\,\mathcal{U}\,q(0)\ |P,S\rangle$$

isolation of Lorentz-invariant amplitudes compare [Mulders, Tangerman NPB (1996)]

$$\langle P,S|\ \bar{q}(\ell)\,\gamma_\mu\,\mathcal{U}\,q(0)\ |P,S\rangle = 4\,\tilde{A}_2\,P_\mu + 4i\,{m_N}^2\,\tilde{A}_3\,\ell_\mu$$

$$\begin{aligned}\langle P,S|\ \bar{q}(\ell)\,\gamma_\mu\gamma^5\,\mathcal{U}\,q(0)\ |P,S\rangle = & -4\,m_N\,\tilde{A}_6\,S_\mu \\ & -4i\,m_N\,\tilde{A}_7\,P_\mu(\ell\cdot S) \\ & +4\,{m_N}^3\,\tilde{A}_8\,\ell_\mu(\ell\cdot S)\end{aligned}$$

$$\langle P,S|\ \bar{q}(\ell)\ \ldots\ \mathcal{U}\,q(0)\ |P,S\rangle = \text{further structures (9 amplitudes in total)}$$

Transformation properties of the matrix element ($\dagger$, $\mathcal{P}$, $\mathcal{T}$) limit number of allowed structures. No $\mathcal{T}$-odd structures (Sivers function, ...) with straight gauge link.

The amplitudes fulfill $\tilde{A}_i(\ell^2,\ell\cdot P) = \left[\tilde{A}_i(\ell^2,-\ell\cdot P)\right]^*$.

$$\Phi^{[\Gamma]}(k,P,S) \equiv \frac{1}{2}\int \frac{d^4\ell}{(2\pi)^4}\, e^{-ik\cdot\ell}\, \langle P,S|\ \bar{q}(\ell)\,\Gamma\,\mathcal{U}\,q(0)\ |P,S\rangle$$

isolation of Lorentz-invariant amplitudes

compare [Mulders, Tangerman NPB (1996)]

$$\langle P,S|\ \bar{q}(\ell)\,\gamma_\mu\,\mathcal{U}\,q(0)\ |P,S\rangle = 4\,\tilde{A}_2\,P_\mu + 4i\,m_N{}^2\,\tilde{A}_3\,\ell_\mu$$

$\tilde{A}_2 \Rightarrow f_1(x,\boldsymbol{k}_\perp^2)$

$$\langle P,S|\ \bar{q}(\ell)\,\gamma_\mu\gamma^5\,\mathcal{U}\,q(0)\ |P,S\rangle = -4\,m_N\,\tilde{A}_6\,S_\mu - 4i\,m_N\,\tilde{A}_7\,P_\mu(\ell\cdot S) + 4\,m_N{}^3\,\tilde{A}_8\,\ell_\mu(\ell\cdot S)$$

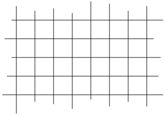
$\tilde{A}_7 \Rightarrow g_{1T}(x,\boldsymbol{k}_\perp^2)$

$\langle P,S|\ \bar{q}(\ell)\ \ldots\ \mathcal{U}\,q(0)\ |P,S\rangle =$ further structures (9 amplitudes in total)

Transformation properties of the matrix element ($\dagger$, $\mathcal{P}$, $\mathcal{T}$) limit number of allowed structures. No $\mathcal{T}$-odd structures (Sivers function, ...) with straight gauge link.

The amplitudes fulfill $\tilde{A}_i(\ell^2,\ell\cdot P) = \left[\tilde{A}_i(\ell^2,-\ell\cdot P)\right]^*$.

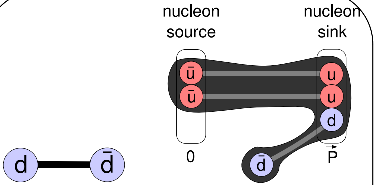
We employ the Chroma library [Edwards, Joo (2005)] to process



MILC gauge configurations

staggered Asqtad action,
2+1 flavors, $a \approx 0.124$ fm,
$m_\pi \approx 500,\ 610,\ \text{and } 760$ MeV

[Orginos, Toussaint PRD (1999)]



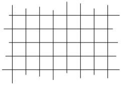
LHPC propagators

domain wall valence fermions,
m_π adjusted to staggered sea,

nucleon momenta:
$\boldsymbol{P} = 0$ and $|\boldsymbol{P}| = 500$ MeV

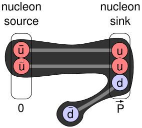
e.g., [Hägler et al. PRD (2008)]

Ingredients	Output : 3-point correlator $C_{3\text{pt}}$

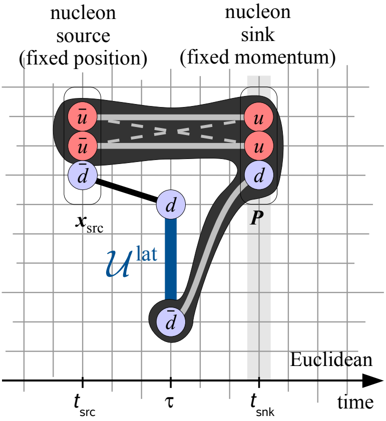


gauge configs.

quark propagators



nucleon sequential propagators



[We neglect “disconnected contributions” (absent for up minus down).]

ratio of correlators far away from nucleon source and sink

$$\frac{C_{3\text{pt}}(\tau;\Gamma,\ell,P)}{C_{2\text{pt}}(P)} \xrightarrow{t_{\text{src}}\ll\tau\ll t_{\text{sink}}} \text{const. ("plateau value")},$$

$$\Downarrow$$

access to $\langle P,S|\ \overline{q}(\ell)\,\Gamma\,\mathcal{U}\,q(0)\ |P,S\rangle$

Γ	$\frac{1}{2}C^{\text{ren}}_{3\text{pt}}(\tau;\Gamma,\boldsymbol{\ell},\boldsymbol{P})/C_{2\text{pt}}(\boldsymbol{P})$ (LHPC projectors)
$\mathbb{1}$	$\frac{m_N}{E(P)}\,\tilde{A}_1$
$-\gamma_4\gamma_5$	$i m_N\,\tilde{A}_7\,\ell_z$
γ_4	$\tilde{A}_2$
$\frac{1}{2}[\gamma_2,\gamma_4]$	$\frac{1}{E(P)}\,\tilde{A}_9\,P_x + \frac{i m_N^2}{E(P)}\,\tilde{A}_{10}\,\ell_x + \frac{m_N^2}{E(P)}\,\tilde{A}_{11}\,(\ell_z)^2 P_x$
...	...

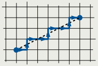
ratio of correlators far away from nucleon source and sink

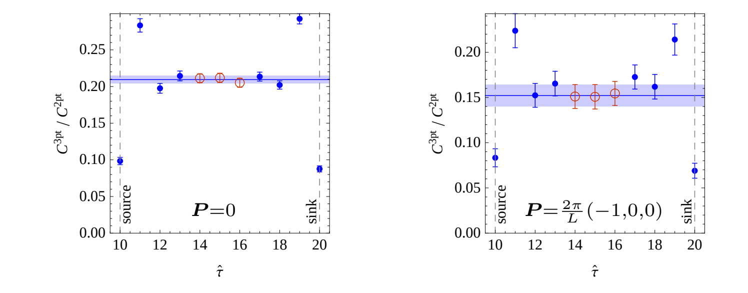
$$\frac{C_{3\mathrm{pt}}(\tau;\Gamma,\ell,P)}{C_{2\mathrm{pt}}(P)} \xrightarrow{t_{\mathrm{src}}\ll\tau\ll t_{\mathrm{sink}}} \text{const. (“plateau value”)},$$

$$\Downarrow$$

$$\text{access to } \langle P,S|\ \bar{q}(\ell)\,\Gamma\,\mathcal{U}\,q(0)\ |P,S\rangle$$

example plateau plots at $m_\pi \approx 600$ MeV

for $\Gamma = \gamma_4$ ($\Rightarrow \tilde{A}_2$), with HYP smeared gauge link $\mathcal{U} =$  :



$$\Phi^{[\Gamma]}(k,P,S) \;\equiv\; \frac{1}{2}\int \frac{d^4\ell}{(2\pi)^4}\; e^{-ik\cdot\ell}\;\langle P,S|\;\bar{q}(\ell)\,\Gamma\,\mathcal{U}\,q(0)\;|P,S\rangle$$

$$\Phi^{[\Gamma]}(x,\boldsymbol{k}_\perp;P,S) \equiv \int_{-\infty}^{\infty} dk^-\;\Phi^{[\Gamma]}(k;P,S)\,\Big|_{k^+=xP^+}$$

$$= \frac{1}{2(2\pi)^3}\int d\ell^-\;d^2\ell_\perp\;e^{ik\cdot\ell}\;\langle P,S|\;\bar{q}(\ell)\,\Gamma\,\mathcal{U}\,q(0)\;|P,S\rangle\,\Big|_{\ell^+=0}$$

$$= \int \frac{d(\ell\cdot P)}{4\pi P^+}\,e^{ix(\ell\cdot P)}\int \frac{d^2\boldsymbol{\ell}_\perp}{(2\pi)^2}\,e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{\ell}_\perp}\;\langle P,S|\;\bar{q}(\ell)\,\Gamma\,\mathcal{U}\,q(0)\;|P,S\rangle\,\Big|_{\ell^+=0}$$

Note: $\ell^2\Big|_{\ell^+=0} = -\boldsymbol{\ell}_\perp^2.$ $x \longleftrightarrow \ell\cdot P$ $\boldsymbol{k}_\perp^2 \longleftrightarrow \ell^2$

example: unpolarized case

$$f_1(x,\boldsymbol{k}_\perp^2) \equiv \Phi^{[\gamma^+]}(x,\boldsymbol{k}_\perp;P,S)$$

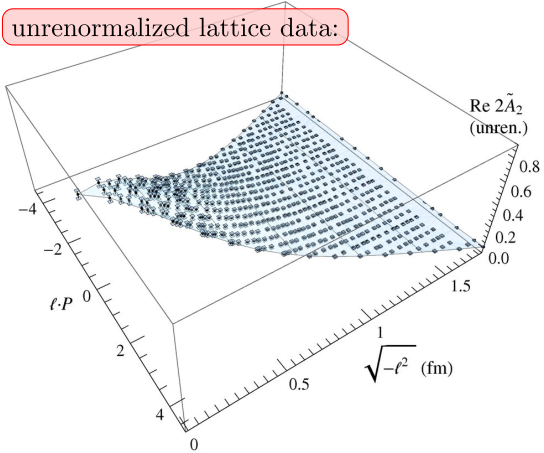
$$= \int \frac{d(\ell\cdot P)}{2\pi}\,e^{ix(\ell\cdot P)}\int \frac{d^2\boldsymbol{\ell}_\perp}{(2\pi)^2}\,e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{\ell}_\perp}\;2\tilde{A}_2(\ell^2,\ell\cdot P)\,\Big|_{\ell^+=0}$$

extract Lorentz-invariant amplitudes $\tilde{A}_i(\ell^2, \ell\cdot P)$, example :

$$\langle P,S|\ \bar{q}(\ell)\,\gamma_\mu\,\mathcal{U}\,q(0)\ |P,S\rangle \;=\; 4\tilde{A}_2 P_\mu \;+\; 4i\,m_N{}^2\ \tilde{A}_3\ \ell_\mu\ ,$$

$$f_1(x,\boldsymbol{k}_\perp^2) = \int \frac{d(\ell\cdot P)}{2\pi}\ e^{ix(\ell\cdot P)} \int \frac{d^2\boldsymbol{\ell}_\perp}{(2\pi)^2}\ e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{\ell}_\perp}\ 2\tilde{A}_2(\ell^2,\ell\cdot P)\ \Big|_{\ell^+=0}$$

unrenormalized lattice data:



$$\ell^2 \ \overset{\text{FT}}{\longleftrightarrow}\ \boldsymbol{k}_\perp^2$$

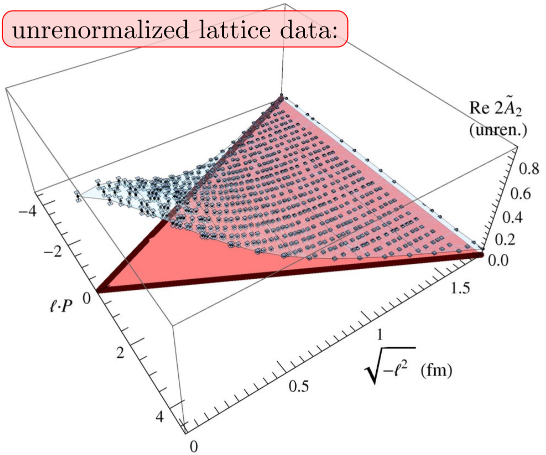
$$\ell\cdot P \ \overset{\text{FT}}{\longleftrightarrow}\ x$$

extract Lorentz-invariant amplitudes $\tilde{A}_i(\ell^2, \ell\cdot P)$, example :

$$\langle P,S|\ \overline{q}(\ell)\,\gamma_\mu\,\mathcal{U}\,q(0)\ |P,S\rangle \ = \ 4\tilde{A}_2 P_\mu \ + \ 4i\,m_N{}^2\ \tilde{A}_3\ \ell_\mu\ ,$$

$$f_1(x,\boldsymbol{k}_\perp^2) = \int \frac{d(\ell\cdot P)}{2\pi}\ e^{ix(\ell\cdot P)} \int \frac{d^2\boldsymbol{\ell}_\perp}{(2\pi)^2}\ e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{\ell}_\perp}\ 2\tilde{A}_2(\ell^2,\ell\cdot P)\ \Big|_{\ell^+=0}$$

unrenormalized lattice data:



$$\ell^2 \ \overset{\text{FT}}{\longleftrightarrow}\ \boldsymbol{k}_\perp^2$$

$$\ell\cdot P \ \overset{\text{FT}}{\longleftrightarrow}\ x$$

Euclidean lattice

$$\ell^0 = \ell_4 = 0$$

$$\Downarrow$$

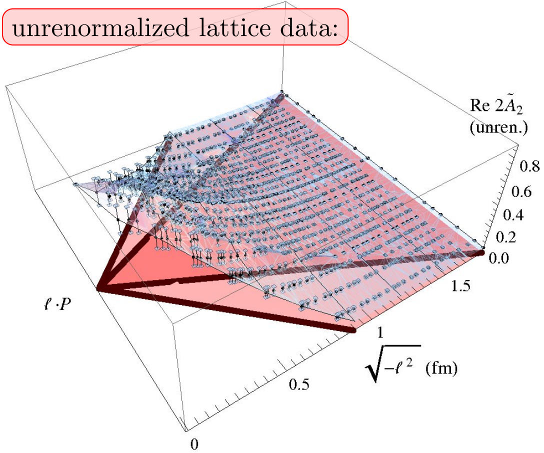
$$\ell^2 \leq 0,$$
$$|\ell\cdot P| \leq |\boldsymbol{P}|\sqrt{-\ell^2}$$

extract Lorentz-invariant amplitudes $\tilde{A}_i(\ell^2, \ell\cdot P)$, example :

$$\langle P,S|\ \overline{q}(\ell)\,\gamma_\mu\,\mathcal{U}\,q(0)\ |P,S\rangle \ =\ 4\tilde{A}_2 P_\mu \ +\ 4i\,m_N{}^2\ \tilde{A}_3\ \ell_\mu\ ,$$

$$f_1(x,\boldsymbol{k}_\perp^2) = \int \frac{d(\ell\cdot P)}{2\pi}\ e^{ix(\ell\cdot P)} \int \frac{d^2\boldsymbol{\ell}_\perp}{(2\pi)^2}\ e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{\ell}_\perp}\ 2\tilde{A}_2(\ell^2,\ell\cdot P)\ \Big|_{\ell^+=0}$$

unrenormalized lattice data:



$\ell^2 \overset{\text{FT}}{\longleftrightarrow} \boldsymbol{k}_\perp^2$

$\ell\cdot P \overset{\text{FT}}{\longleftrightarrow} x$

Euclidean lattice

$$\ell^0 = \ell_4 = 0$$

$$\Downarrow$$

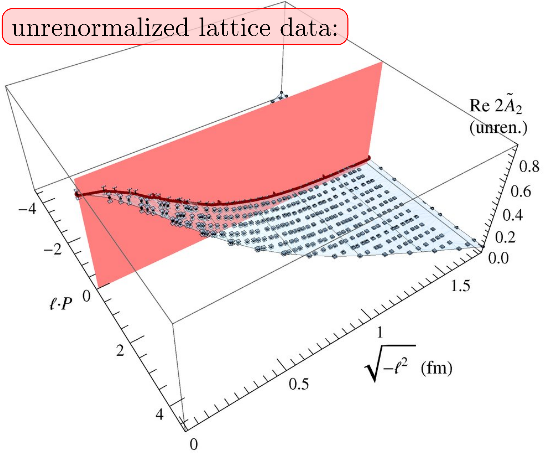
$$\ell^2 \leq 0,$$
$$|\ell\cdot P| \leq |\boldsymbol{P}|\sqrt{-\ell^2}$$

extract Lorentz-invariant amplitudes $\tilde{A}_i(\ell^2, \ell\cdot P)$, example :

$$\langle P, S|\ \bar{q}(\ell)\,\gamma_\mu\,\mathcal{U}\,q(0)\ |P, S\rangle\ =\ 4\tilde{A}_2 P_\mu\ +\ 4i\,m_N{}^2\ \tilde{A}_3\ \ell_\mu\ ,$$

$$f_1(x, \boldsymbol{k}_\perp^2) = \int \frac{d(\ell\cdot P)}{2\pi}\ e^{ix(\ell\cdot P)} \int \frac{d^2\boldsymbol{\ell}_\perp}{(2\pi)^2}\ e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{\ell}_\perp}\ 2\tilde{A}_2(\ell^2, \ell\cdot P)\,\Big|_{\ell^+=0}$$

unrenormalized lattice data:



$$\ell^2 \overset{\text{FT}}{\longleftrightarrow} \boldsymbol{k}_\perp^2$$

$$\ell\cdot P \overset{\text{FT}}{\longleftrightarrow} x$$

Euclidean lattice

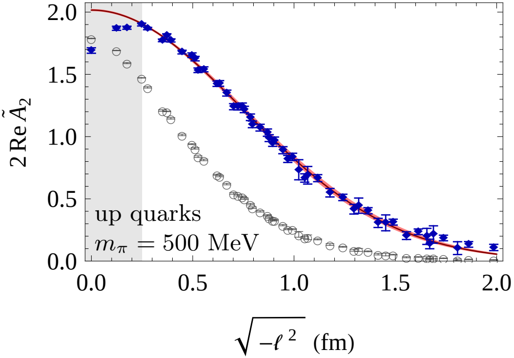
$$\ell^0 = \ell_4 = 0$$

$$\Downarrow$$

$$\ell^2 \le 0,$$
$$|\ell\cdot P| \le |\boldsymbol{P}|\sqrt{-\ell^2}$$

Lowest x-moment of TMD PDFs

$$f_1^{(0_x)}(\boldsymbol{k}_\perp^2) \equiv \int_{-1}^{1} dx\ f_1(x, \boldsymbol{k}_\perp^2) = \int \frac{d^2\ell_\perp}{(2\pi)^2}\ e^{i\boldsymbol{k}_\perp \cdot \boldsymbol{\ell}_\perp}\ 2\,\tilde{A}_2(-\ell_\perp^2, 0)$$



fit function

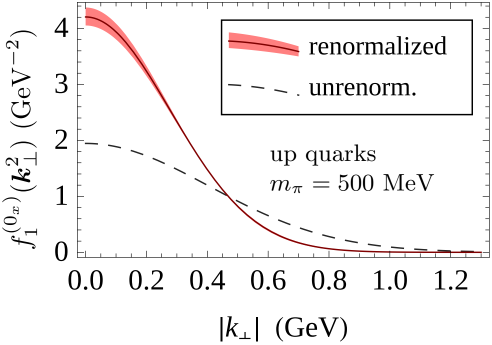
$C_1\ \exp(-|\ell|^2/\sigma_1^2)$

Z-factor

$Z^{-1}\ C_1^{\text{up-down}} \stackrel{!}{=} 1$

multiplicative renormalization based on quark counting

$$f_1^{(0_x)}(\boldsymbol{k}_\perp^2) \equiv \int_{-1}^{1} dx \; f_1(x, \boldsymbol{k}_\perp^2) = \int \frac{d^2\ell_\perp}{(2\pi)^2} \; e^{i\boldsymbol{k}_\perp \cdot \boldsymbol{\ell}_\perp} \; 2\,\tilde{A}_2(-\ell_\perp^2, 0)$$



width of the distribution (RMS momentum):

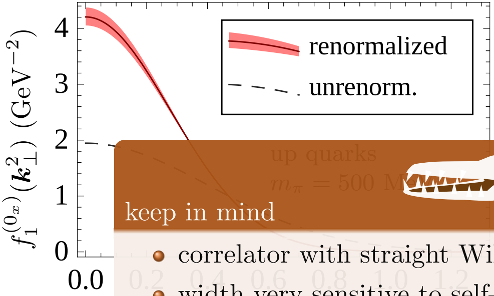
$\langle \boldsymbol{k}_\perp^2 \rangle^{1/2} =$
$(391 \pm 8_{\text{stat}} \pm 27_{\text{sys}})$ MeV

compare phenomenology [Anselmino et al., PRD71, 074006 (2005)]:

$\langle \boldsymbol{k}_\perp^2 \rangle^{1/2} \approx 500$ MeV

(estimate, Gaussian Ansatz)

$$f_1^{(0_x)}(\boldsymbol{k}_\perp^2) \equiv \int_{-1}^{1} dx\ f_1(x, \boldsymbol{k}_\perp^2) = \int \frac{d^2\ell_\perp}{(2\pi)^2}\ e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{\ell}_\perp}\ 2\,\tilde{A}_2(-\ell_\perp^2, 0)$$



width of the distribution (RMS momentum):

$\langle \boldsymbol{k}_\perp^2 \rangle^{1/2} =$ $(391 \pm 8_{\text{stat}} \pm 27_{\text{sys}})$ MeV

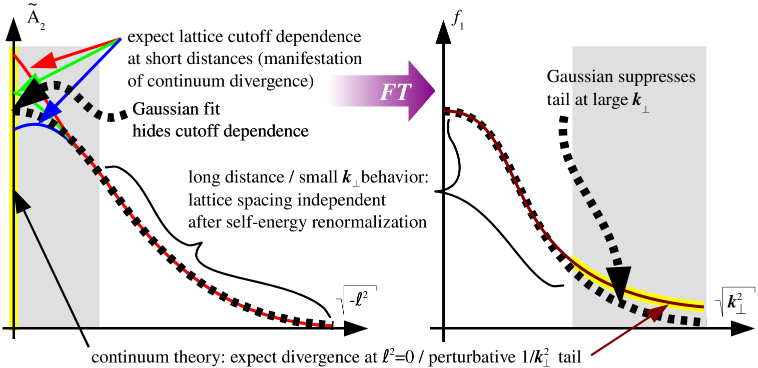
...nenology ...AL., (2005)]:

...MeV

...Ansatz)

keep in mind

- correlator with straight Wilson line (“sW”)
- width very sensitive to self-energy renormalization
- Gaussian fit ansatz (“wrong” at large-$\boldsymbol{k}_\perp$ [Diehl, arXiv:0811.0774])
- $m_\pi \approx 500$ MeV



Problem with the perturbative tail

$\int d^2\boldsymbol{k}_\perp\, f_1(x,\boldsymbol{k}_\perp^2)$ is undefined,
in conflict with probability interpretation.

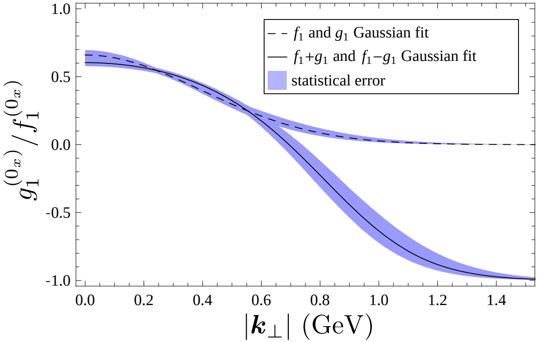
Gaussian is a poor man's solution.

Ideal would be a prescription that maintains
$\int d^2\boldsymbol{k}_\perp\, f_1(x,\boldsymbol{k}_\perp^2;\mu) = f_1(x;\mu)$ at some scale μ.

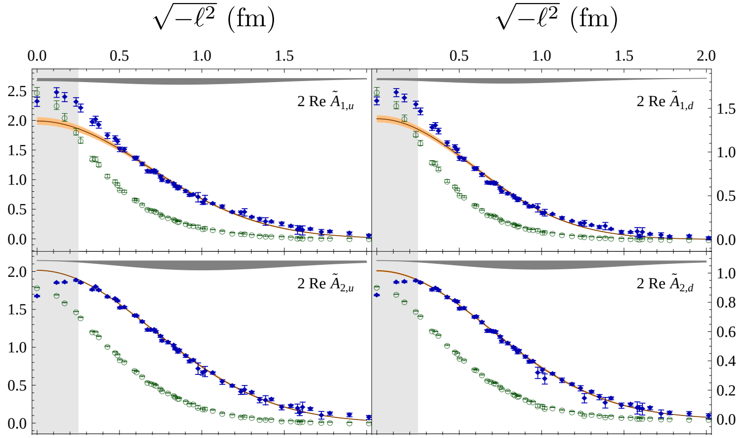
$$f_1^{(0_x)}(\boldsymbol{k}_\perp^2) = C_0 \exp(-\boldsymbol{k}_\perp^2/\mu_0^2)$$
$$g_1^{(0_x)}(\boldsymbol{k}_\perp^2) = C_2 \exp(-\boldsymbol{k}_\perp^2/\mu_2^2)$$

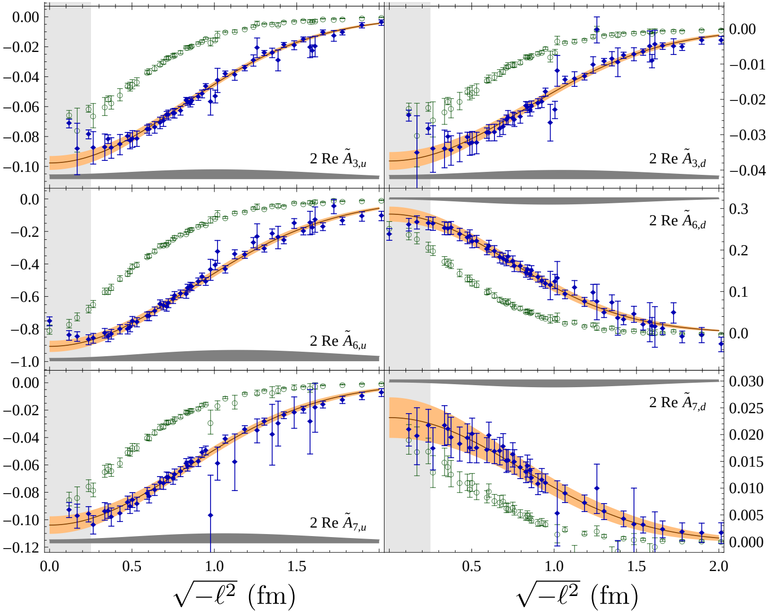
vs.

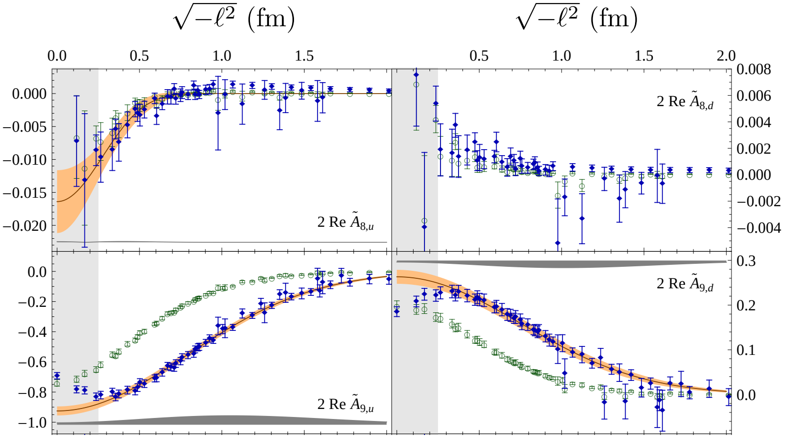
$$\rho_{LL}^{\pm}(\boldsymbol{k}_\perp) \equiv \tfrac{1}{2} f_1^{(0_x)}(\boldsymbol{k}_\perp^2) \pm \tfrac{1}{2} g_1^{(0_x)}(\boldsymbol{k}_\perp^2)$$
$$\rho_{LL}^{+}(\boldsymbol{k}_\perp) = C_+ \exp(-\boldsymbol{k}_\perp^2/\mu_+^2)$$
$$\rho_{LL}^{-}(\boldsymbol{k}_\perp) = C_- \exp(-\boldsymbol{k}_\perp^2/\mu_-^2)$$

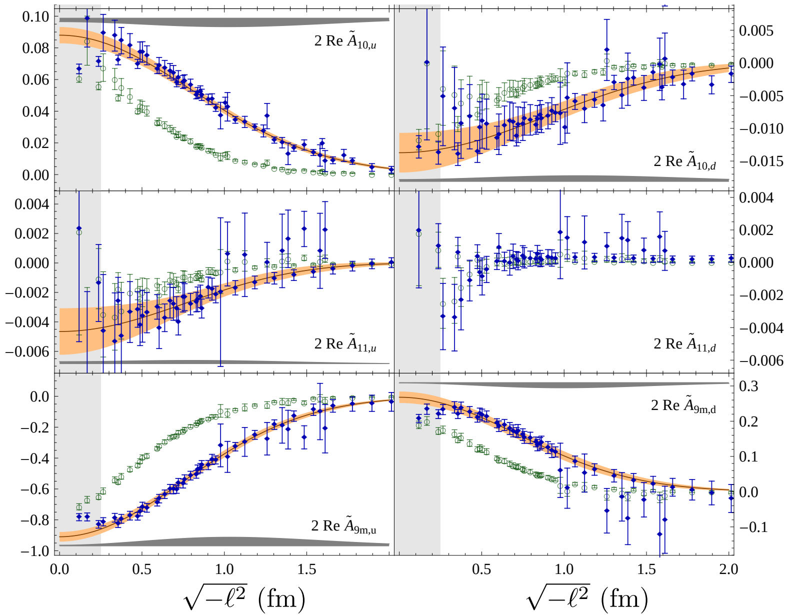


⇒ Asymptotic behavior at large $\boldsymbol{k}_\perp$ imposed by Gaussian ansatz; not a “lattice result”. Similar issues in analysis of experimental data.


$\sqrt{-\ell^2}$ (fm)
$\sqrt{-\ell^2}$ (fm)
0.0
0.5
1.0
1.5
2.0
2.5
2 Re $\tilde{A}_{1,u}$
2 Re $\tilde{A}_{1,d}$
2 Re $\tilde{A}_{2,u}$
2 Re $\tilde{A}_{2,d}$
0.2
0.4
0.6
0.8


0.00
−0.02
−0.04
−0.06
−0.08
−0.10
2 Re $\tilde{A}_{3,u}$
0.00
−0.01
−0.02
−0.03
−0.04
2 Re $\tilde{A}_{3,d}$
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
2 Re $\tilde{A}_{6,u}$
0.3
0.2
0.1
0.0
2 Re $\tilde{A}_{6,d}$
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
2 Re $\tilde{A}_{7,u}$
0.030
0.025
0.020
0.015
0.010
0.005
0.000
2 Re $\tilde{A}_{7,d}$
0.0
0.5
1.0
1.5
2.0
$\sqrt{-\ell^2}$ (fm)
$\sqrt{-\ell^2}$ (fm)


$\sqrt{-\ell^2}$ (fm)
$\sqrt{-\ell^2}$ (fm)
0.0
0.5
1.0
1.5
2.0
0.000
−0.005
−0.010
−0.015
−0.020
$2\,\mathrm{Re}\,\tilde{A}_{8,u}$
0.008
0.006
0.004
0.002
0.000
−0.002
−0.004
$2\,\mathrm{Re}\,\tilde{A}_{8,d}$
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
$2\,\mathrm{Re}\,\tilde{A}_{9,u}$
0.3
0.2
0.1
0.0
$2\,\mathrm{Re}\,\tilde{A}_{9,d}$

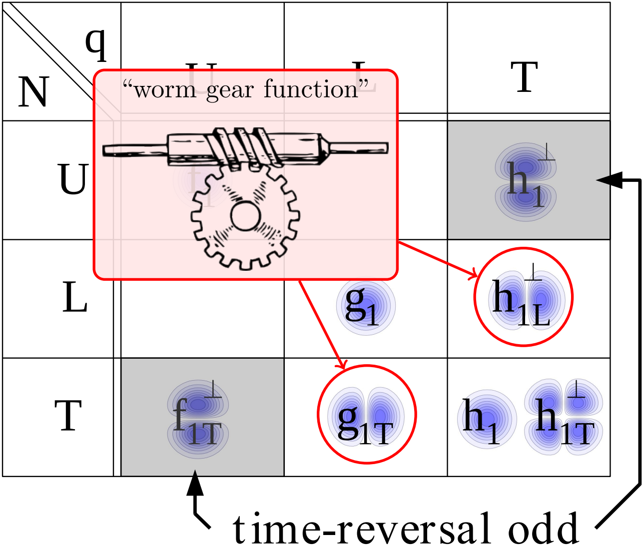


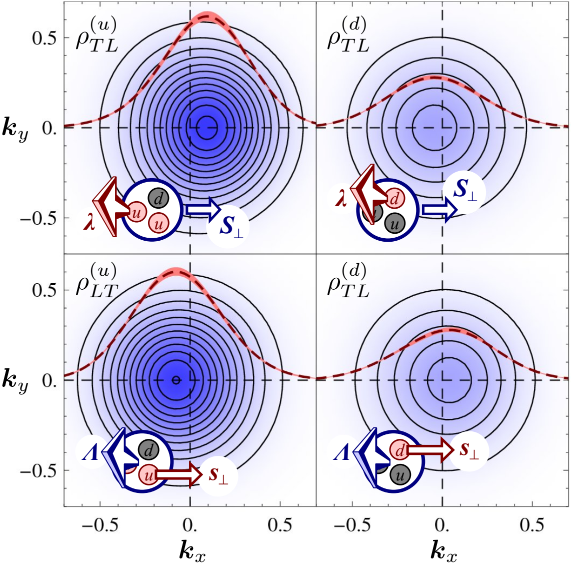
2 Re $\tilde{A}_{10,u}$
2 Re $\tilde{A}_{10,d}$
2 Re $\tilde{A}_{11,u}$
2 Re $\tilde{A}_{11,d}$
2 Re $\tilde{A}_{9m,u}$
2 Re $\tilde{A}_{9m,d}$
$\sqrt{-\ell^2}$ (fm)
$\sqrt{-\ell^2}$ (fm)

Leading-twist TMD PDFs

N \ q	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

time-reversal odd


q
N
T
U
L
T
“worm gear function”
$h_1^\perp$
g_1
$h_{1L}^\perp$
$f_{1T}^\perp$
g_{1T}
h_1
$h_{1T}^\perp$
time-reversal odd



"worm gear functions"

Diplole deformations

$\rho_{TL} : \sim \lambda\, \boldsymbol{k}_\perp \cdot \boldsymbol{S}_\perp \;\; g_{1T}$

$\rho_{TL} : \sim \Lambda\, \boldsymbol{k}_\perp \cdot \boldsymbol{s}_\perp \;\; h_{1L}^\perp$

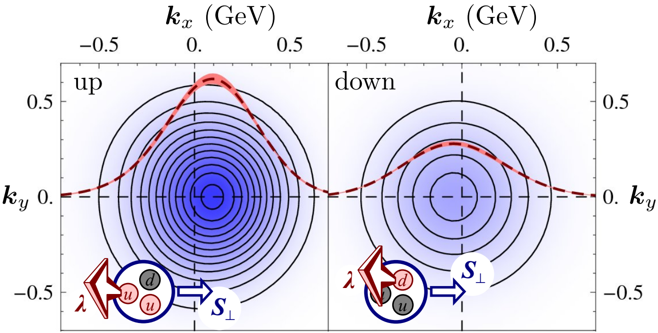
The corresponding dipole structures $\sim \lambda\, \boldsymbol{b}_\perp \cdot \boldsymbol{S}_\perp$, $\sim \Lambda\, \boldsymbol{b}_\perp \cdot \boldsymbol{s}_\perp$ for impact parameter densities (from GPDs) are ruled out by symmetries.

[Hägler, Musch, Negele, Schäfer EPL 88, 61001 (2009)]

Density of quarks with positive helicity, $\lambda = 1$,
in a transversely polarized nucleon, $\boldsymbol{S}_\perp = (1,0)$:

$$\rho_{TL}(\boldsymbol{k}_\perp; \boldsymbol{S}_\perp, \lambda) \equiv \frac{1}{2} \int dx \int dk^- \ \Phi^{[\gamma^+ \frac{1}{2}(\mathbb{1}+\gamma^5)]}(k, P, S_\perp)$$

$$= \frac{1}{2} f_1^{(0_x)}(\boldsymbol{k}_\perp^2) + \frac{\lambda}{2} \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_\perp}{m_N} g_{1T}^{(0_x)}(\boldsymbol{k}_\perp^2)$$

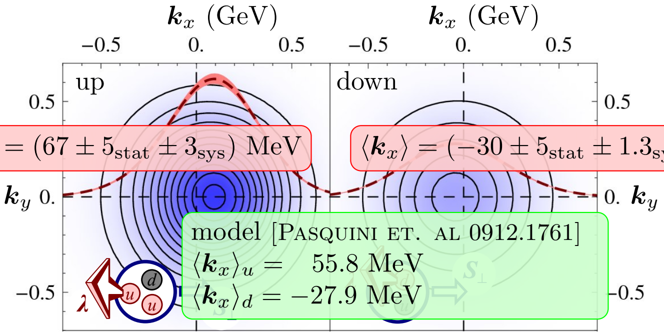


($m_\pi \approx 500$ MeV, straight gauge link operator, renormalization condition $C^{\text{ren}} = 0$, Gaussian fit)

Density of quarks with positive helicity, $\lambda = 1$,
in a transversely polarized nucleon, $\boldsymbol{S}_\perp = (1, 0)$:

$$\rho_{TL}(\boldsymbol{k}_\perp; \boldsymbol{S}_\perp, \lambda) \equiv \frac{1}{2} \int dx \int dk^- \; \Phi^{[\gamma^+ \frac{1}{2}(\mathbb{1}+\gamma^5)]}(k, P, S_\perp)$$

$$= \frac{1}{2} f_1^{(0_x)}(\boldsymbol{k}_\perp^2) + \frac{\lambda}{2} \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_\perp}{m_N} \; g_{1T}^{(0_x)}(\boldsymbol{k}_\perp^2)$$



$\langle \boldsymbol{k}_x \rangle = (67 \pm 5_{\text{stat}} \pm 3_{\text{sys}})$ MeV

$\langle \boldsymbol{k}_x \rangle = (-30 \pm 5_{\text{stat}} \pm 1.3_{\text{sys}})$ MeV

model [Pasquini et. al 0912.1761]
$\langle \boldsymbol{k}_x \rangle_u = \quad 55.8$ MeV
$\langle \boldsymbol{k}_x \rangle_d = -27.9$ MeV

($m_\pi \approx 500$ MeV, straight gauge link operator,
renormalization condition $C^{\text{ren}} = 0$, Gaussian fit)

$$f^{(m_x,n_\perp)} \equiv \int_{-1}^{1} dx\, x^m \int d^2\boldsymbol{k}_\perp \left(\frac{\boldsymbol{k}_\perp^2}{2m_N^2}\right)^n f(x,\boldsymbol{k}_\perp^2)$$

Let us assume the amplitudes $\tilde{A}_i$ are regular at $\ell^2 = 0$.

$$\langle \boldsymbol{k}_\perp \rangle_{\rho_{TL}} \;=\; \lambda \boldsymbol{S}_\perp m_N \frac{g_{1T}^{(0_x,1_\perp)}}{f_1^{(0_x,0_\perp)}} \;=\; \lambda \boldsymbol{S}_\perp m_N \frac{\tilde{A}_7(0,0)}{\tilde{A}_2(0,0)}$$

$\Rightarrow$ estimates for certain $\boldsymbol{k}_\perp$-moments:

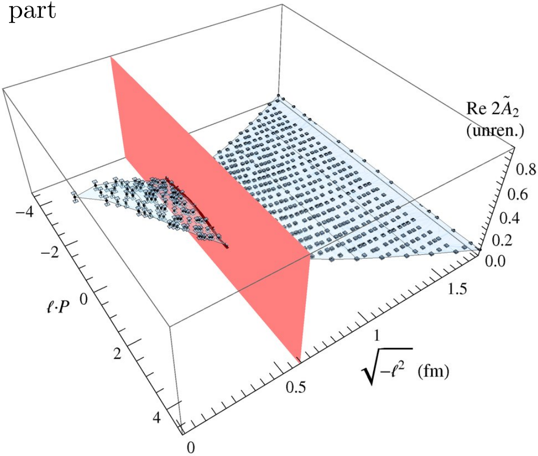
$$\langle \boldsymbol{k}_\perp \rangle_{\rho_{TL}} \;\approx \lambda \boldsymbol{S}_\perp m_N \frac{\tilde{A}_7(\ell^2_{\mathrm{min}},0)}{\tilde{A}_2(\ell^2_{\mathrm{min}},0)}$$

with ℓ^2_{min} large enough to avoid strong lattice artefacts.
All self-energies from the gauge link cancel on the RHS
($\Rightarrow$ no dependence on the renormalization condition).

In the presence of divergences at $\ell^2 = 0$, we assume ℓ^2_{min} represents a regularization scale.

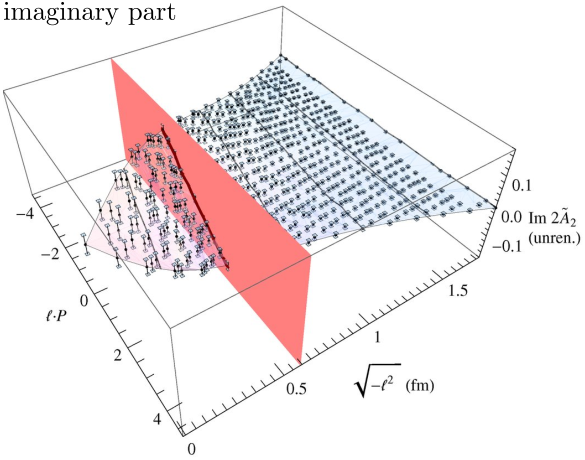
x-dependence

real part



$$\ell^2 \overset{\text{FT}}{\longleftrightarrow} \boldsymbol{k}_\perp^2$$

$$\ell \cdot P \overset{\text{FT}}{\longleftrightarrow} x$$



$\ell^2 \xleftrightarrow{\text{FT}} \boldsymbol{k}_\perp^2$

$\ell \cdot P \xleftrightarrow{\text{FT}} x$

factorization hypothesis

$$f_1(x, \boldsymbol{k}_\perp^2) \;\approx\; f_1(x) \;\; f_1^{(0_x)}(\boldsymbol{k}_\perp^2) \; / \; \mathcal{N}$$

as in phenomenological applications,
e.g., Monte Carlo event generators

Then $\tilde{A}_2$ factorizes, too:

$$\tilde{A}_2(\ell^2, \ell\cdot P) = \tilde{A}_2^{\mathrm{norm}}(\ell\cdot P) \; \tilde{A}_2(\ell^2, 0).$$

To test this, we define

$$\boxed{\tilde{A}_2^{\mathrm{norm}}(\ell^2, \ell\cdot P) \equiv \frac{\tilde{A}_2(\ell^2, \ell\cdot P)}{\mathrm{Re}\; \tilde{A}_2(\ell^2, 0)}}$$

(needs no renormalization!)

If factorization holds, $\tilde{A}_2^{\mathrm{norm}}$ should be ℓ^2-independent.

factorization hypothesis

$$f_1(x, \boldsymbol{k}_\perp^2) \;\approx\; f_1(x) \;\; f_1^{(0_x)}(\boldsymbol{k}_\perp^2) \;/\; \mathcal{N}$$

as in phenomenological applications,
e.g., Monte Carlo event generators

Then $\tilde{A}_2$ factorizes, too:

$$\tilde{A}_2(\ell^2, \ell\cdot P) = \tilde{A}_2^{\text{norm}}(\ell\cdot P)\; \tilde{A}_2(\ell^2, 0).$$

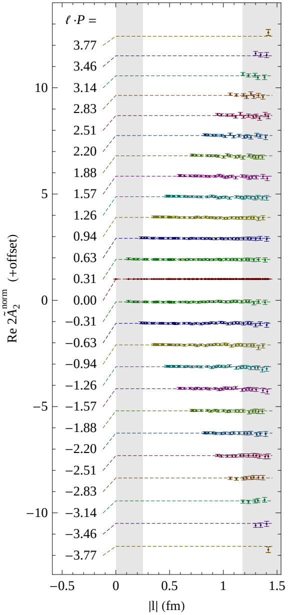
To test this, we define

$$\boxed{\tilde{A}_2^{\text{norm}}(\ell^2, \ell\cdot P) \equiv \frac{\tilde{A}_2(\ell^2, \ell\cdot P)}{\text{Re}\; \tilde{A}_2(\ell^2, 0)}}$$

(needs no renormalization!)

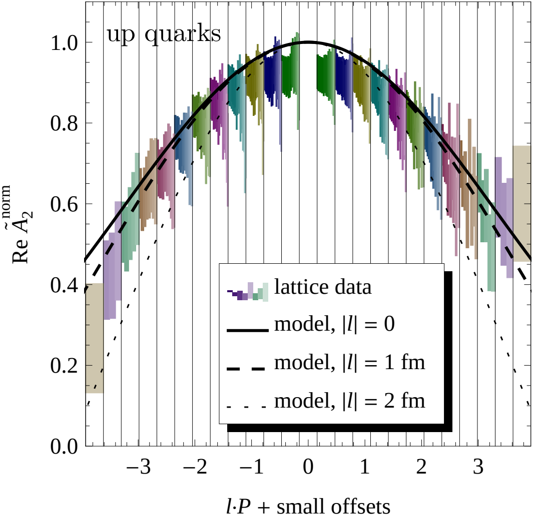
If factorization holds, $\tilde{A}_2^{\text{norm}}$ should be ℓ^2-independent.

✓ within statistics



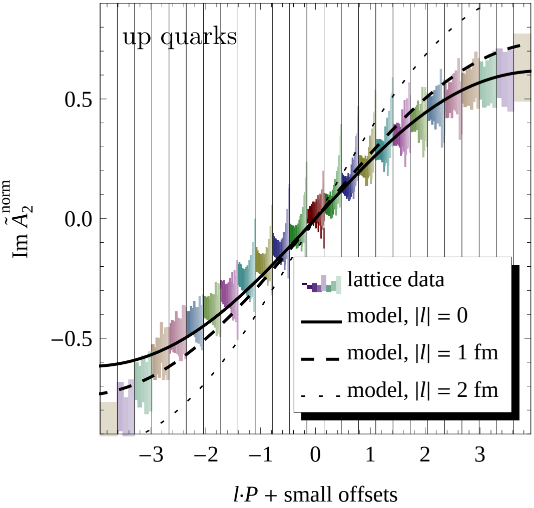
All our data for $\tilde{A}_2^{\text{norm}}(\ell^2, \ell \cdot P)$ at $m_\pi \approx 600$ MeV

qualitative comparison to a scalar diquark model
[Bacchetta, Conti, Radici PRD (2008)] at $\sqrt{-\ell^2} = 0$, 1 and 2 fm

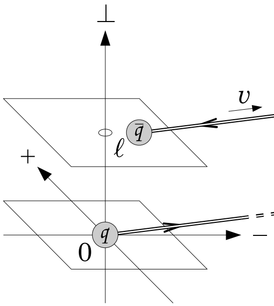


All our data for $\tilde{A}_2^{\text{norm}}(\ell^2, \ell \cdot P)$ at $m_\pi \approx 600$ MeV

qualitative comparison to a scalar diquark model
[Bacchetta, Conti, Radici PRD (2008)] at $\sqrt{-\ell^2} = 0$, 1 and 2 fm



Extended Gauge Links



- gauge link = effective representation of struck quark ("final state interaction")
- ⇒ (almost lightlike)

$$\zeta \equiv \frac{(v\cdot P)^2}{v^2} \to \pm\infty$$

- keep ζ finite to avoid "rapidity divergences"

now 32 Lorentz-invariant amplitudes [Goeke,Metz,Schlegel PLB618,90 (2005)]

$$A_i\left(k^2, k\cdot P, \frac{v\cdot k}{|v\cdot P|}, \frac{v^2}{|v\cdot P|^2}, \frac{v\cdot P}{|v\cdot P|}\right) = A_i\Bigg(k^2, k\cdot P, \underbrace{\frac{v\cdot k}{|v\cdot P|}}_{\approx\, x}, \zeta^{-1}, \mathrm{sgn}(v\cdot P)\Bigg)$$

Large ζ: evolution with ζ known [Collins,Soper NPB194,445 (1981)].

$$\left.\begin{array}{r}(v^0, v^1, v^2, v^3)\\ \text{future pointing } v\\ \text{TMD PDFs for SIDIS}\end{array}\right\} \xrightarrow{\mathcal{T}} \left\{\begin{array}{l}(-v^0, v^1, v^2, v^3)\\ \text{past pointing } v\\ \text{TMD PDFs for Drell-Yan}\end{array}\right.$$

The transformation property of the matrix elements under time reversal provides relations:

Example of a $\mathcal{T}$-even amplitude:

$$A_2\left(k^2, k\cdot P, \tfrac{v\cdot k}{v\cdot P}, \zeta^{-1}, 1\right) = A_2\left(k^2, k\cdot P, \tfrac{v\cdot k}{v\cdot P}, \zeta^{-1}, -1\right)$$

$$\Downarrow$$

$$f_1^{(\text{SIDIS})}(x, \boldsymbol{k}_\perp; \zeta, \ldots) = f_1^{(\text{Drell-Yan})}(x, \boldsymbol{k}_\perp; \zeta, \ldots)$$

Example of a $\mathcal{T}$-odd amplitude: ($\rightarrow$ Sivers function $f_{1T}^\perp$)

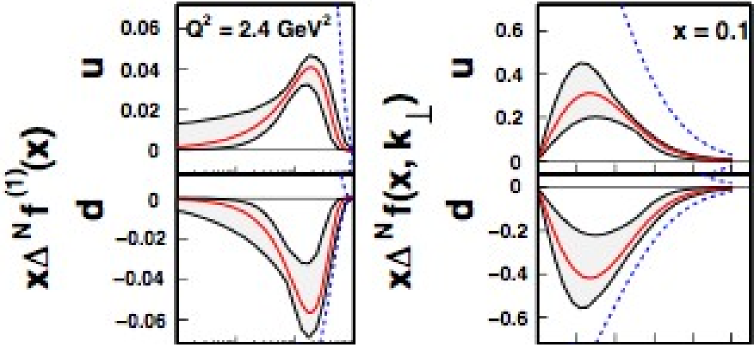
$$A_{12}\left(k^2, k\cdot P, \tfrac{v\cdot k}{v\cdot P}, \zeta^{-1}, 1\right) = -A_{12}\left(k^2, k\cdot P, \tfrac{v\cdot k}{v\cdot P}, \zeta^{-1}, -1\right)$$

$$\Downarrow$$

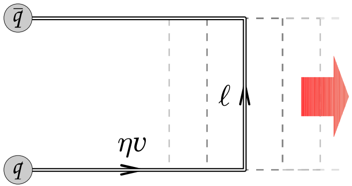
$$f_{1T}^{\perp(\text{SIDIS})}(x, \boldsymbol{k}_\perp; \zeta, \ldots) = -f_{1T}^{\perp(\text{Drell-Yan})}(x, \boldsymbol{k}_\perp; \zeta, \ldots)$$

HERMES and COMPASS data

[Anselmino *et. al.* EPJ A (2009)]

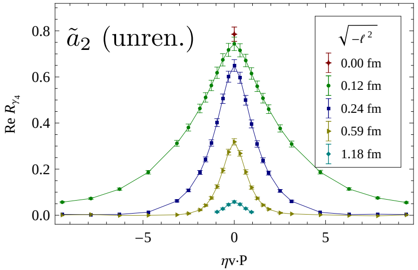


$2\,\langle k_\perp \rangle_{TU} = 96^{+60}_{-28}$ MeV (up) $\quad \langle k_\perp \rangle_{TU} = -113^{+45}_{-51}$ MeV (down)



- v spatial $\Rightarrow$ $|\zeta| = \frac{(v\cdot P)^2}{|v|^2} \leq |\boldsymbol{P}_{\text{lat.}}|^2$
- look for plateaus at large $|\eta|$
- now 32 amplitudes $\tilde{a}_i(\ell^2, \ell\cdot P, v\cdot P; \eta, \zeta)$

Problem: need to subtract gauge link self-energy ($\to$ η-independence)

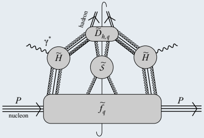


Unmodified/unrenormalized amplitude vanishes for $\eta \to \infty$.

SIDIS beyond the "basic" ansatz

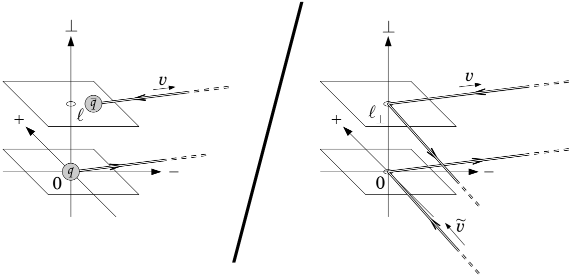
e.g., [Ji, Ma, Yuan PRD (2005)] :

$$W^{\mu\nu}_{\text{unpol.,LO}} \quad \propto \quad H \;\times\; f_1 \;\otimes\; D_h \;\otimes\; \underbrace{S}_{\text{soft factor}}$$



modified definition of TMD PDF correlator:

$$\Phi^{[\Gamma]}(k,P,S) \;\equiv\; \frac{1}{2}\int \frac{d^4\ell}{(2\pi)^4}\, e^{-ik\cdot\ell}\, \frac{\langle P,S|\;\bar{q}(\ell)\,\Gamma\,\mathcal{U}\,q(0)\;|P,S\rangle}{\widetilde{S}(\boldsymbol{\ell}_\perp,\ldots)}$$



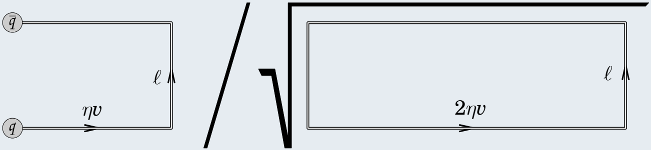
idea #1: modify definition of TMD PDFs

$$\Phi^{[\Gamma]}(k, P, S) \;\equiv\; \frac{1}{2} \int \frac{d^4\ell}{(2\pi)^4}\; e^{-ik\cdot\ell}\; \frac{\langle P, S|\;\; \bar{q}(\ell)\, \Gamma\, \mathcal{U}\, q(0)\;\; |P, S\rangle}{\widetilde{S}(\boldsymbol{\ell}_\perp, \ldots)}$$

with $\widetilde{S}$ obtained from a vacuum expectation value of gauge links

Adjust soft factor to cancel $\exp(\delta m\, L)$

Suggestion [Collins PoS LC]2008 :



Is this a meaningful definition of TMD PDFs?

example Sivers function

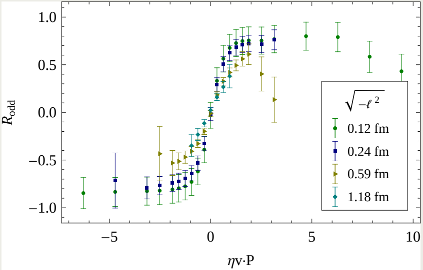
$$\langle \boldsymbol{k}_y \rangle_{TU} \overset{\text{formally}}{=} -2m_N \boldsymbol{S}_x \lim_{\eta\to\infty} \frac{\tilde{a}_{12}(0,0,0;\eta,\zeta) + \ldots}{\tilde{a}_2(0,0,0;\eta,\zeta)} \propto \frac{\int dx \int d^2\boldsymbol{k}_\perp\, \boldsymbol{k}_\perp^2\, f_{1T}^\perp}{\int dx \int d^2\boldsymbol{k}_\perp\, f_1}$$

On the lattice, we can try to compute

$$\langle \boldsymbol{k}_y \rangle_{TU} \underset{\eta \text{ large}}{\approx} -2m_N \boldsymbol{S}_x \frac{\tilde{a}_{12}(\ell^2_{\text{min}},0,0;\eta,\zeta) + \ldots}{\tilde{a}_2(\ell^2_{\text{min}},0,0;\eta,\zeta)}$$

Self-energy cancels!

Test calculation: a time reversal odd ratio of ampitudes



$$R_{\text{odd}} = -\frac{\tilde{a}_{12} - (\eta \frac{m_N^2 v_1}{P_1})\, \tilde{b}_8}{\tilde{a}_2}$$

Plateaus visible at large $|\eta|$.

"Time-reversal odd" $\leftrightarrow$ odd in $\eta v\cdot P$.

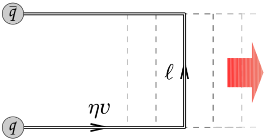
Part of the effect comes from the Sivers function $f_{1T}^\perp$!

Summary:

- Lattice exploration of intrinsic quark momentum distributions in the nucleon. "Full QCD testbed".
- Manifestly non-local operators on the lattice.
- First results based on a a simplified operator geometry (direct gauge link) and a Gaussian fit model, at $m_\pi \approx 500$ MeV:
  - Obtaind x-integrated leading twist T-even TMD PDFs $f_1^{(0_x)}(\boldsymbol{k}_\perp^2)$, $g_{1T}^{(0_x)}(\boldsymbol{k}_\perp^2)$, $h_{1L}^{\perp(0_x)}(\boldsymbol{k}_\perp^2)$, ...
  - Observed deformed quark densities due to worm-gear functions.

Outlook:

- Study of non-straight gauge links similar as in SIDIS.



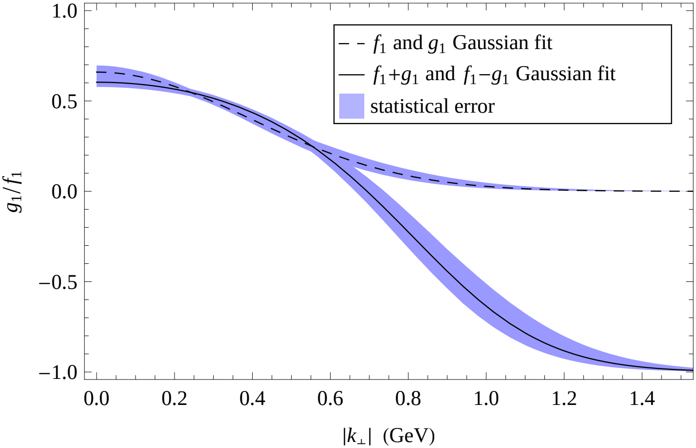
- Higher statistics needed to discuss factorization $f_1(x, \boldsymbol{k}_\perp^2) \approx \; f_1(x)\; f_1^{(0_x)}(\boldsymbol{k}_\perp^2)/\mathcal{N}$.
- Beyond Gaussian fits: Matching to perturbative behavior at small ℓ, i.e., large $\boldsymbol{k}_\perp$.

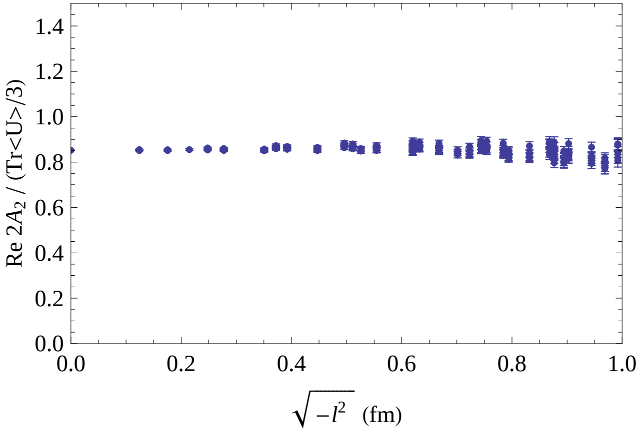
Backup Slides

$$f_1^{(0)}(\boldsymbol{k}_\perp^2) = C_0 \exp(-\boldsymbol{k}_\perp^2/\mu_0^2)$$
$$g_1^{(0)}(\boldsymbol{k}_\perp^2) = C_2 \exp(-\boldsymbol{k}_\perp^2/\mu_2^2)$$

vs.

$$\rho_{LL}^{\pm}(\boldsymbol{k}_\perp^2) \equiv \tfrac{1}{2} f_1^{(0)}(\boldsymbol{k}_\perp^2) \pm \tfrac{1}{2} g_1^{(0)}(\boldsymbol{k}_\perp^2)$$
$$\rho_{LL}^{\pm}(\boldsymbol{k}_\perp) = C_+ \exp(-\boldsymbol{k}_\perp^2/\mu_+^2)$$
$$\rho_{LL}^{\pm}(\boldsymbol{k}_\perp) = C_- \exp(-\boldsymbol{k}_\perp^2/\mu_-^2)$$





$$\frac{\langle \boldsymbol{P}=0|\ \bar{q}(\ell)\,\gamma^{+}\,\mathcal{U}\,q(0)\ |\boldsymbol{P}=0\rangle}{\langle 0|\ \frac{1}{3}\,\mathrm{Tr}\,\mathcal{U}\ |0\rangle}$$ turns out constant

	a (fm)	$\hat{L}$	$\hat{T}$	# configurations used
super-fine	0.06	48	144	15
fine	0.09	28	96	79
coarse	0.12	20	64	264
super-coarse	0.18	16	48	200

For all ensembles, $m_{u,d}/m_s = 0.4$, $m_\pi \approx 500$ MeV

I have HYP-smeared all configurations.

as outlined by Alexei Bazavov, *priv. commun.*

renormalization of potential

$$a\,V_{\text{ren}}(r) = \ln\left[W_{\text{ren}}(r,t)/W_{\text{ren}}(r,t+a)\right] = a\,V(r) + 2\,\delta\hat{m}$$

$\Rightarrow$ Extract $\delta\hat{m}$ from the shift of the static potential?

fit Ansatz

$$V(r) = V_0 + \sigma r - \alpha/r + \lambda\,[1/r]$$

We need a renormalization condition!

potential of a Nambu-Goto string [Lüscher, Symanzik, Weisz]

$$V_{\text{string}}(r) = \sigma r - \pi/12r$$

We obtain $\delta\hat{m}$ by matching static potential and string potential at large enough distance: $V_{\text{ren}}(0.75\text{ fm}) = V_{\text{string}}(0.75\text{ fm})$.

Results from preliminary analysis on our smeared ensembles:

a (fm)	0.06	0.09	0.12	0.18
$\delta\hat{m}$	-0.158	-0.164	-0.155	-0.10

as outlined by ALEXEI BAZAVOV, *priv. commun.*

renormalization of potential

$$a\,V_{\text{ren}}(r) = \ln\left[W_{\text{ren}}(r,t)/W_{\text{ren}}(r,t+a)\right] = a\,V(r) + 2\,\delta\hat{m}$$

⇒ Extract $\delta\hat{m}$ from the shift of the static potential?

fit Ansatz

$$V(r) = V_0 + \sigma r - \alpha/r + \lambda\,[1/r]$$

We need a renormalization condition!

potential of a Nambu-Goto string [LÜSCHER, SYMANZIK, WEISZ]

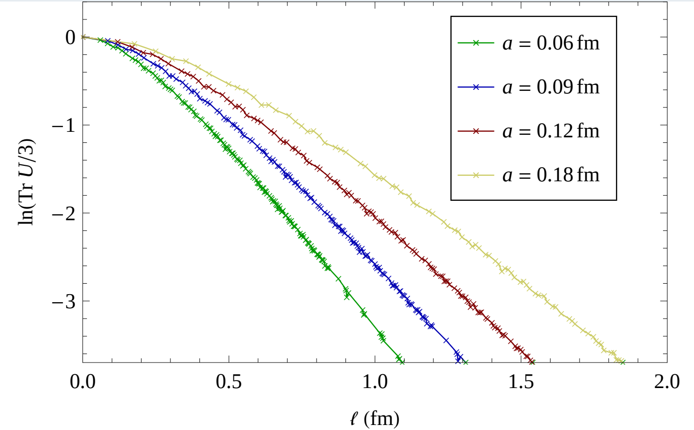
$$V_{\text{string}}(r) = \sigma r - \pi/12r$$

We obtain $\delta\hat{m}$ by matching static potential and string potential at large enough distance: $V_{\text{ren}}(0.75\ \text{fm}) = V_{\text{string}}(0.75\ \text{fm})$.

Results from preliminary analysis on our smeared ensembles:

a (fm)	0.06	0.09	0.12	0.18
$\delta\hat{m}$	-0.158	-0.164	-0.155	-0.10

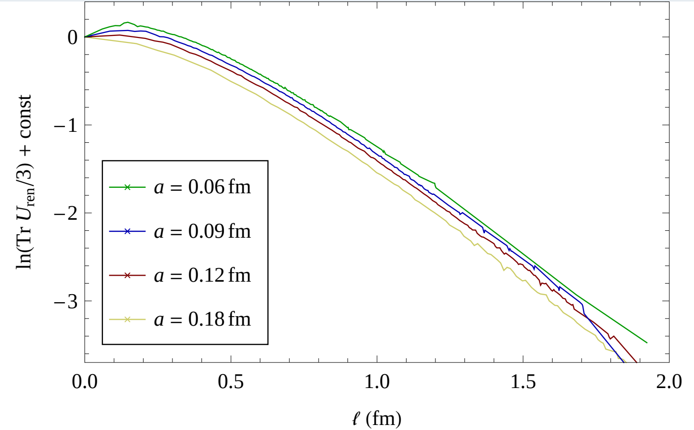
Vacuum expectation value of Wilson line $\langle \frac{1}{3} \mathrm{Tr}\, \mathcal{U}_{[0,\ell]} \rangle$
on Landau gauge fixed ensemble:



unrenormalized

$\ln\langle \mathrm{Tr}\, \mathcal{U}^{\mathrm{ren}}_{[0,\ell]} \rangle = \ln\langle \mathrm{Tr}\, \mathcal{U}_{[0,\ell]} \rangle - \delta\hat{m}\frac{\ell}{a} - \ln Z_z, \quad Z_z$ responsible for shift.

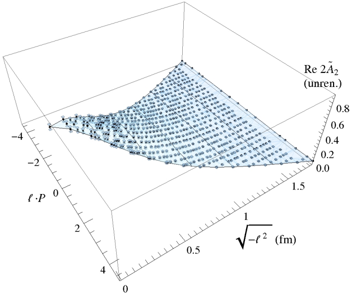
Vacuum expectation value of Wilson line $\langle \frac{1}{3} \mathrm{Tr}\, \mathcal{U}_{[0,\ell]} \rangle$
on Landau gauge fixed ensemble:



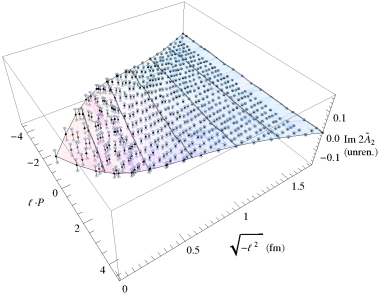
renormalization constants from static potential

$\ln\langle \mathrm{Tr}\, \mathcal{U}^{\mathrm{ren}}_{[0,\ell]} \rangle = \ln\langle \mathrm{Tr}\, \mathcal{U}_{[0,\ell]} \rangle - \delta\hat{m}\frac{\ell}{a} - \ln Z_z,$ Z_z responsible for shift.

2 Re $\tilde{A}_2(\ell^2, \ell\cdot P)$

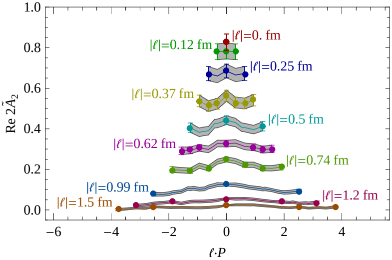


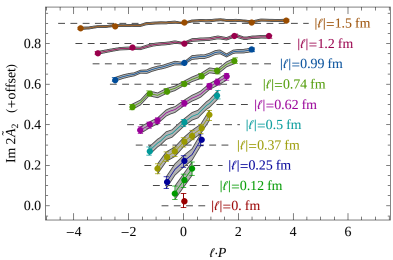
2 Im $\tilde{A}_2(\ell^2, \ell\cdot P)$



2 Re $\tilde{A}_2(\ell^2, \ell \cdot P)$

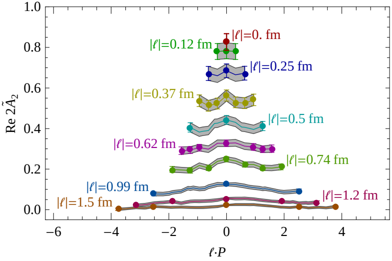
2 Im $\tilde{A}_2(\ell^2, \ell \cdot P)$

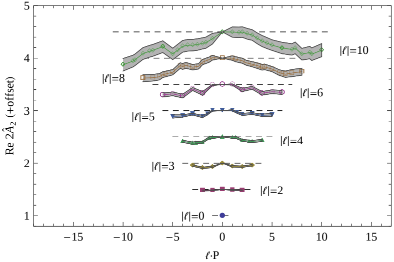




$$2\ \mathrm{Re}\ \tilde{A}_2(\ell^2, \ell\cdot P)$$

$$\mathrm{Re}\ \tilde{A}_2^{\mathrm{norm}} = \frac{\mathrm{Re}\ \tilde{A}_2(\ell^2, \ell\cdot P)}{\mathrm{Re}\ \tilde{A}_2(\ell^2, 0)}$$





$$2\ \mathrm{Im}\ \tilde{A}_2(\ell^2, \ell\cdot P)$$

$$\mathrm{Im}\ \tilde{A}_2^{\mathrm{norm}} = \frac{\mathrm{Im}\ \tilde{A}_2(\ell^2, \ell\cdot P)}{\mathrm{Re}\ \tilde{A}_2(\ell^2, 0)}$$