The motion of quarks inside the proton using lattice QCD

Jefferson Lab
Thomas Jefferson National Accelerator Facility

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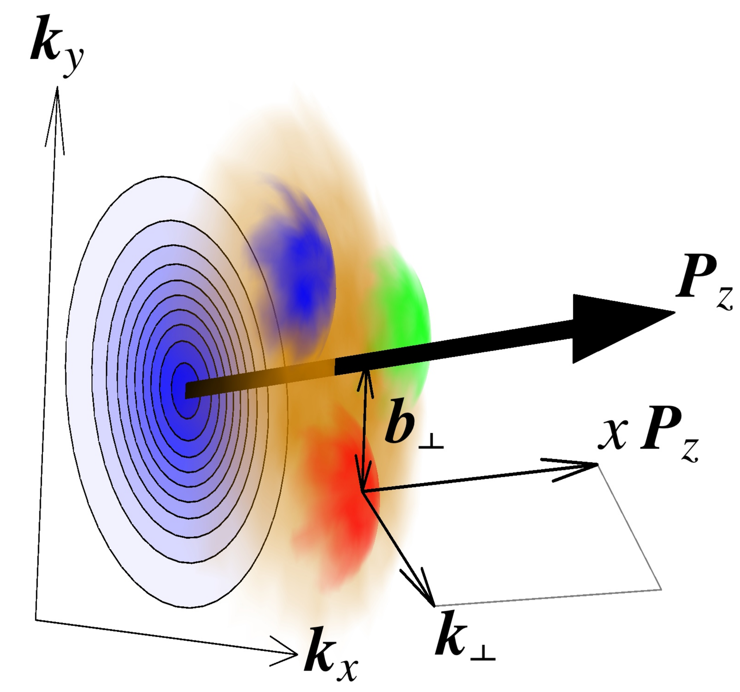
Motivation: our picture of the proton at high velocity

Proton appears flat (Lorentz-contraction). Quarks carry

- a fraction x of the proton momentum $\boldsymbol{P} = (0, 0, \boldsymbol{P}_z)$
- *transverse momentum* $\boldsymbol{k}_\perp = (\boldsymbol{k}_x, \boldsymbol{k}_y)$
  $\leftrightarrow$ intrinsic quark motion!

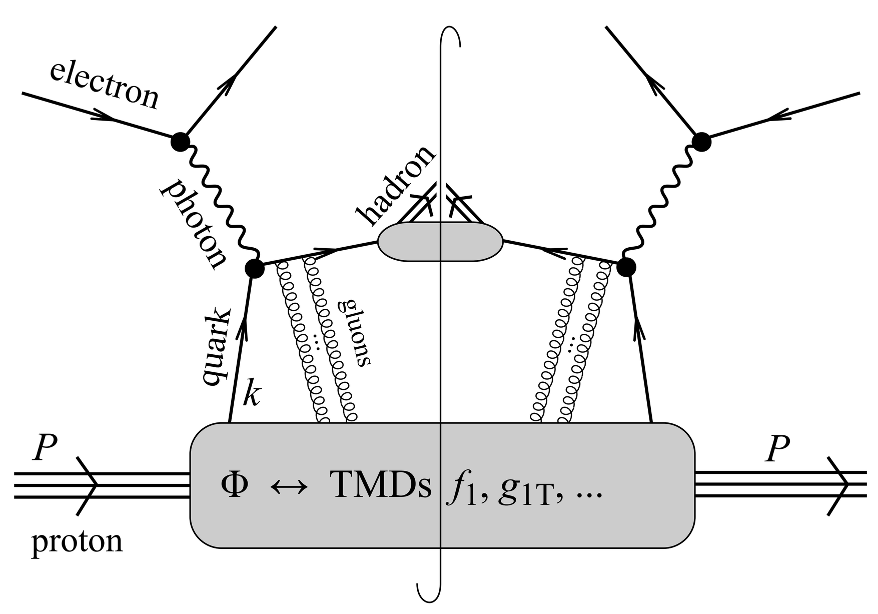
How are the quarks distributed with respect to x and $k_\perp$?

$\Longrightarrow$ **T**ransverse **M**omentum **D**ependent parton distribution functions **(TMDs)**. Example: $f_1(x, \boldsymbol{k}_\perp^2)$.
Intuitively interpretation as 3-dimensional quark densities in momentum space.



Our goal is to make predictions about TMDs from the basic laws of Quantum Chromodynamics (QCD), using lattice QCD computer simulations [HMNS09, Mus09].

TMDs in Experiment



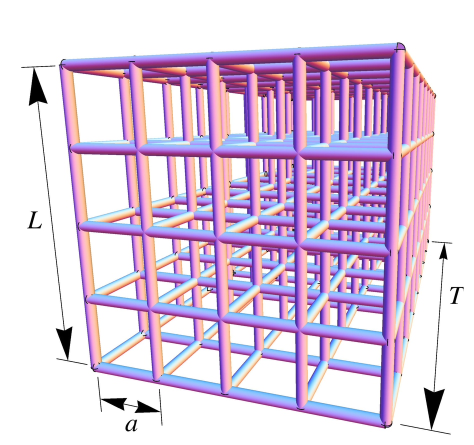
TMDs are explored at COMPASS (CERN), HERMES (DESY), Jefferson Lab, RHIC (BNL), Fermilab and BELLE (KEK). More experiments are planned world wide. TMDs are deduced from angular asymmetries in the particle production in semi-inclusive deep inelastic scattering (SIDIS) or the Drell-Yan process.

Example SIDIS: e+p $\rightarrow$ e$'$+hadron+X, see schematic diagram on the left. The process can be approximately factorized into soft and hard parts [CSS85, JMY05]. The lower gray blob, Φ, characterizes quarks in the proton through TMDs.

Hadron properties from lattice QCD

Properties of the proton or other hadrons cannot be calculated with standard perturbation theory, due to the strong interaction at long distances. In lattice QCD, we put the problem on a grid (lattice) of spacing a inside a four-dimensional box of dimensions $L^3 \times T$. Massive parallel high performance computing is required to solve the QCD path integral with sufficient precision. The resulting "gauge configurations" can be used to calculate many different quantities (hadron masses, form factors, polarizabilities, Generalized Parton Distribution functions, TMDs, ...). To reduce cost, most calculations today are still performed at unrealistically high quark masses, typically specified in terms of m_π.


A computer cluster at Jefferson Lab



Defining TMDs

Some details as to how to define TMDs consistently in field theory are still controversial [Col08, CKS10]. Basic definition [CS82]:

$$\Phi_q^{[\Gamma]}(x, \boldsymbol{k}_\perp; P, S; \mathcal{C}) \equiv \int dk^- \int \frac{d^4 l}{(2\pi)^4}\, e^{-ik\cdot l}\, \underbrace{\frac{1}{2}\langle P, S|\ \bar{q}(l)\, \Gamma\, \mathcal{U}[\mathcal{C}_l]\ q(0)\ |P, S\rangle}_{\widetilde{\Phi}_q^{[\Gamma]}(l, P, S; \mathcal{C})} \Big|_{k^+ = xP^+}$$

$$= \frac{1}{P^+} \underbrace{\int \frac{d(l\cdot P)}{2\pi}\, e^{-i(l\cdot P)x}}_{\int\!\!\!\!\!\mathcal{X}} \underbrace{\int \frac{d^2 \boldsymbol{l}_\perp}{(2\pi)^2}\, e^{i\boldsymbol{l}_\perp \cdot \boldsymbol{k}_\perp}}_{\int\!\!\!\!\!\mathcal{M}} \widetilde{\Phi}_q^{[\Gamma]}(l, P, S; \mathcal{C}) \Big|_{l^+ = 0}$$

where $k^\pm = (k^0 \pm k^3)/\sqrt{2}$. Now $\Phi^{[\Gamma]}$ can be decomposed into TMDs f_1, g_{1T}, etc. [MT96]. Example:

$$\Phi^{[\gamma^+ + \lambda\gamma^+\gamma^5]} = f_1(x, \boldsymbol{k}_\perp^2) + \lambda \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_\perp}{m_N} g_{1T}(x, \boldsymbol{k}_\perp^2) + \left[\frac{\boldsymbol{S}_j \epsilon_{ji} \boldsymbol{k}_i}{m_N} f_{1T}^\perp(x, \boldsymbol{k}_\perp^2) \right]_{\text{odd}} \equiv 2\, \rho_{TL}(x, \boldsymbol{k}_\perp^2; \lambda, \boldsymbol{S}_\perp)$$

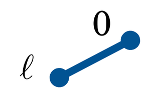
ρ_{TL} is the density of longitudinally polarized quarks (helicity λ) in a transversely polarized proton (spin $\boldsymbol{S}_\perp$).

$\mathcal{U}[\mathcal{C}_l]$ is a Wilson line

$$\mathcal{U}[\mathcal{C}_l] \equiv \mathcal{P} \exp\left(-ig \int_{\mathcal{C}} d\xi^\mu A_\mu(\xi) \right)$$

For SIDIS, the contour $\mathcal{C}$ runs from 0 to ∞ and back to l.

For our calculation, we presently use straight links.



The $[\ ]_{\text{odd}}$ piece has different sign in SIDIS and Drell-Yan [Col02] and vanishes for straight links.

Parametrization with amplitudes

For straight Wilson lines, $\widetilde{\Phi}^{[\Gamma]}$ can be parametrized in terms of Lorentz-invariant amplitudes $\widetilde{A}_i(l^2, l\cdot P)$:

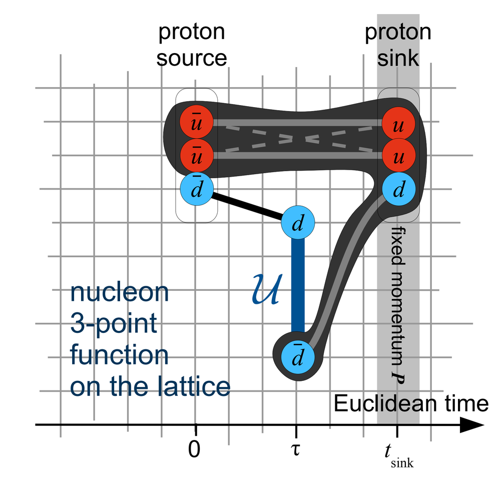
$$\widetilde{\Phi}^{[\gamma^\mu]} = 2\, P^\mu\, \widetilde{A}_2 + 2i\, m_N{}^2\, l^\mu\, \widetilde{A}_3\,,$$

$$\widetilde{\Phi}^{[\gamma^\mu\gamma^5]} = -2\, m_N\, S^\mu\, \widetilde{A}_6 - 2i\, m_N\, P^\mu (l\cdot S)\, \widetilde{A}_7 + 2\, m_N{}^3\, l^\mu (l\cdot S)\, \widetilde{A}_8\,,$$

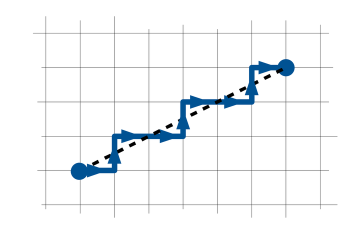
The TMDs are Fourier-transforms of amplitudes. $\int\!\!\!\!\!\mathcal{X}$ transforms $x \leftrightarrow l\cdot P$, $\int\!\!\!\!\!\mathcal{M}$ transforms $\boldsymbol{k}_\perp^2 \leftrightarrow l^2$:

$$f_1(x, \boldsymbol{k}_\perp^2) = 2 \int\!\!\!\!\!\mathcal{X} \int\!\!\!\!\!\mathcal{M}\ \widetilde{A}_2(l^2, l\cdot P)\,, \qquad g_{1T}(x, \boldsymbol{k}_\perp^2) = 4 m_N^2 \partial_{\boldsymbol{k}_\perp^2} \int\!\!\!\!\!\mathcal{X} \int\!\!\!\!\!\mathcal{M}\ \widetilde{A}_7(l^2, l\cdot P)\,, \qquad \text{etc.}$$

Calculating TMDs on the lattice



We calculate the matrix elements $\tilde{\Phi}^{[\Gamma]}(l, P, S)$ directly on the lattice from 3-point functions. Three quark fields $(u,\ u,\ d)$ at the source and sink location generate particles with the right quantum numbers. At Euclidean time τ, we insert the non-local operator $\bar{q}(l)\, \Gamma\, \mathcal{U}[\mathcal{C}_l]\, q(0)$, where q can be u or d. The straight Wilson line is approximated by a steplike product of link variables, see inset on the right. If τ is far enough away from source and sink, only the ground state, the proton, is probed. In practice, the 3-point function is assembled from the gauge link $\mathcal{U}$ and lattice quark propagators (connecting lines), three of which are combined into a "sequential propagator" (dark blob). Disconnected diagrams are neglected.

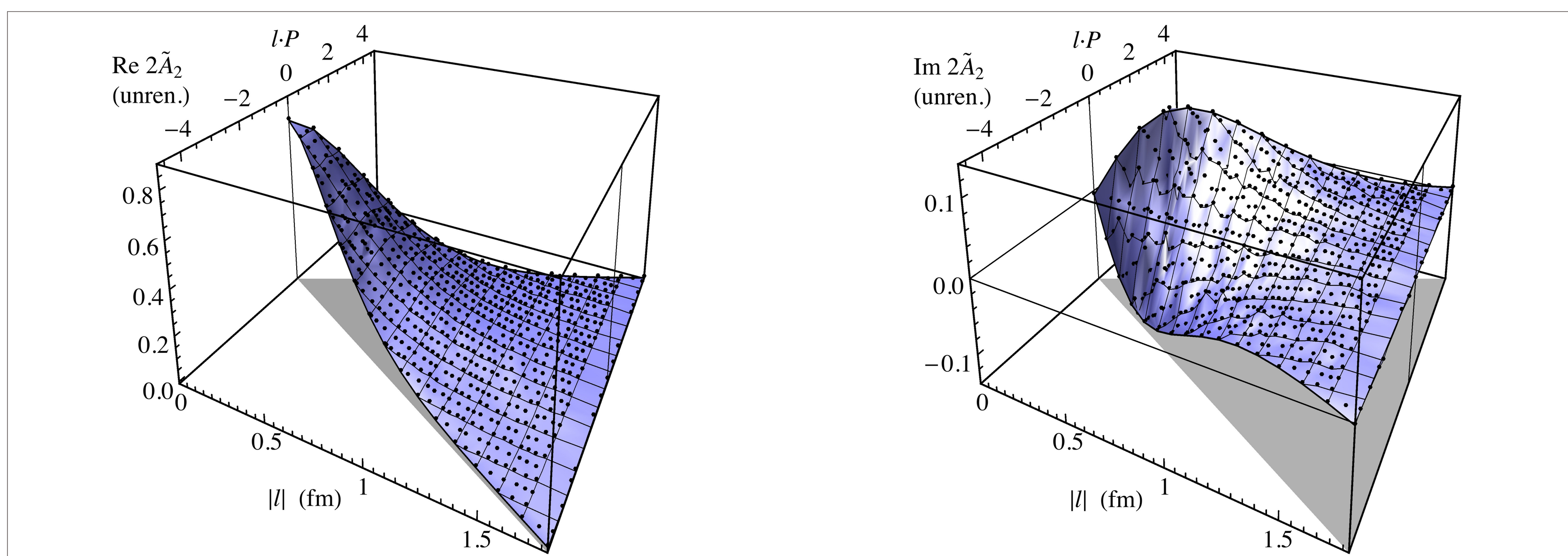


Technically, our method is inspired by and very similar to the GPD analysis by the LHP collaboration [H+08]. We can save computationally extremely expensive steps by reusing existing input data granted to us by MILC and LHPC:

MILC gauge configurations [A+04]
$L^3 \times T = 20^3 \times 64$, $a \approx 0.12\,\text{fm}$,
staggered asqtad action, 2+1 flavors.

LHPC propagators & sequential propagators [H+08]
source-sink-separation: $10a$, two proton momenta:
$\boldsymbol{P} = (0, 0, 0)$ and $\boldsymbol{P} = (-1, 0, 0) 2\pi/L \mathrel{\hat{=}} 500\,\text{MeV}/c$
Domain Wall valence fermions adjusted to staggered sea.

Results with straight Wilson lines

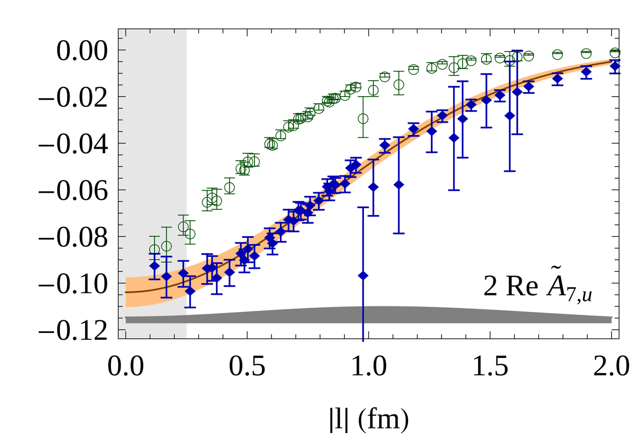


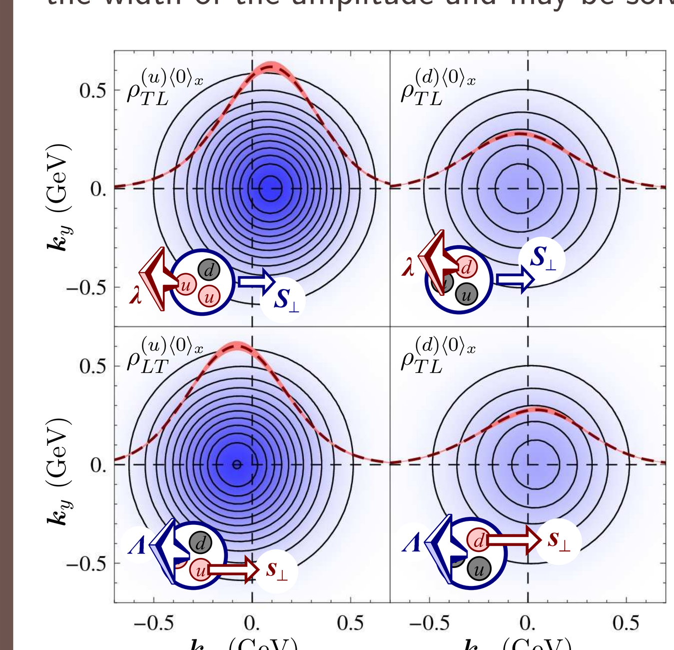
Unrenormailized lattice results for $\widetilde{A}_2(l^2, l\cdot P)$ for $u - d$ quarks at $m_\pi \approx 800\,\text{MeV}$. Each black dot corresponds to a different quark separation $\boldsymbol{l}$. In Euclidean space-time, we must set $l^0 = -il^4 = 0$, confining us to the region $l^2 < 0$, $|l\cdot P| < |\boldsymbol{P}|\sqrt{-l^2}$ (grey triangle). Note $|l| \equiv \sqrt{-l^2}$.

The restricted range of $|l\cdot P|$ prevents us from fully reconstructing TMDs. However, x-integrated distributions are already accessible from the data at $l\cdot P = 0$. Example:

$$g_{1T}^{\langle 0 \rangle x}(\boldsymbol{k}_\perp^2) \equiv \int_{-1}^{1} dx\ g_{1T}(x, \boldsymbol{k}_\perp^2) = 4 m_N^2 \partial_{\boldsymbol{k}_\perp^2} \int\!\!\!\!\!\mathcal{M}\ \widetilde{A}_7(l^2, 0)$$

Lattice results for $\widetilde{A}_7(l^2, 0)$ at $l\cdot P = 0$ for up-quarks at $m_\pi \approx 500\,\text{MeV}$ are shown on the right. The green points are unrenormalized. The blue points are renormalized using the static quark potential [C+08, Mus09]. The orange line and error band are a Gaussian fit. Lattice data below $|l| \leq 0.25\,\text{fm}$ are affected by lattice cutoff effects and have not been included in the fit. Note that the renormalization of the Wilson line introduces a renormalization condition, whose meaning in terms of a physical factorization or renormalization scale is not known, yet. This problem mainly affects the width of the amplitude and may be solved by an improved definition of the TMD correlator.





The Gaussian ansatz for the $\widetilde{A}_i(l^2, 0)$ allows us to perform the Fourier transform $\int\!\!\!\!\!\mathcal{M}$. Combinations of the resulting x-integrated TMDs $f_1^{\langle 0 \rangle x}$, $g_{1T}^{\langle 0 \rangle x}$, etc., have an interpretation as x-integrated quark densities. The upper two plots of the graphics on the left show the x-integrated density $\rho_{TL}^{\langle 0 \rangle x}(\boldsymbol{k}_\perp^2; \lambda = 1, \boldsymbol{S}_\perp = (1, 0))$ for up and for down quarks at $m_\pi = 500\,\text{MeV}$. The insets illustrate the spin orientations of quark and proton. The contribution from $g_{1T}^{\langle 0 \rangle x}$ leads to a dipole deformation of the density. This deformation introduces a non-zero average quark momentum $\langle \boldsymbol{k}_x \rangle_{TL}$, which is largely unaffected by the choice of the renormalization condition. Our results, $\langle \boldsymbol{k}_x \rangle_{TL} = 67(5)\,\text{MeV}$ for up quarks and $\langle \boldsymbol{k}_x \rangle_{TL} = -30(5)\,\text{MeV}$ for down quarks, are remarkably similar to results from a light cone constituent quark model [PCB08]. The lower two plots show x-integrated densities $\rho_{LT}^{\langle 0 \rangle x}$ of transversely polarized quarks in a longitudinally polarized proton. We see deformations in the opposite direction as in $\rho_{TL}^{\langle 0 \rangle x}$, again in qualitative agreement with models.

Outlook

Within a USQCD proposal, we have been granted 1.1 million processor core hours on the Jefferson Lab cluster. In particular, we will analyze staple shaped Wilson lines. This will enable us to study the effect of the link path and naively time-reversal odd phenomena, as they occur, e.g., in SIDIS and Drell-Yan experiments.

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