Prospects of TMD PDF predictions from Lattice QCD

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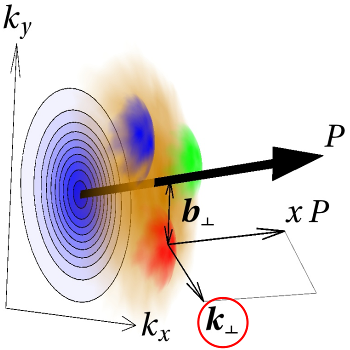


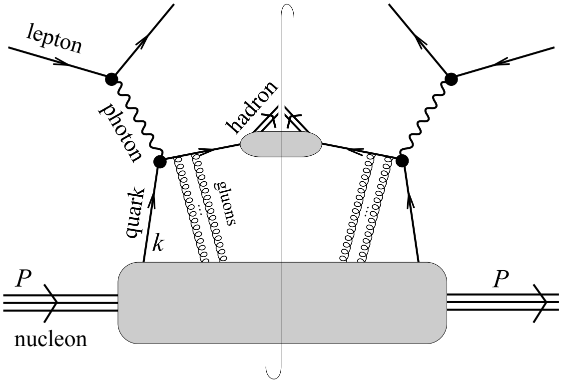
TMD PDFs

transverse **m**omentum **d**ependent **p**arton **d**istribution **f**unctions

e.g., $f_1(x, \boldsymbol{k}_\perp^2)$

$\Rightarrow$ quark density $\rho(\boldsymbol{k}_\perp)$.

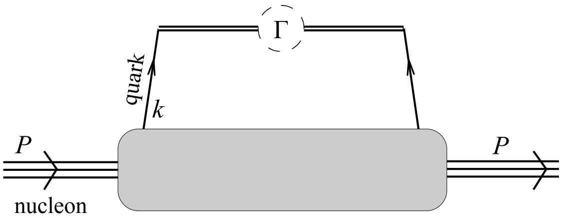
- x, $\Rightarrow$ PDFs
- $\boldsymbol{b}_\perp$ (impact parameter), $\Rightarrow$ GPDs
- $\boldsymbol{k}_\perp$ (intrinsic quark transverse momentum) $\Rightarrow$ TMD PDFs



$$\frac{d\sigma}{d^3P_h\ d^3P_{l'}} \propto \underbrace{L_{\mu\nu}}_{\substack{\text{lepton}\\ \text{tensor}}} \quad \underbrace{W^{\mu\nu}}_{\substack{\text{hadron}\\ \text{tensor}}}$$

$$W^{\mu\nu}_{\text{unpol.,LO}} \propto \int d\boldsymbol{\ell}_\perp\ e^{i\boldsymbol{\ell}_\perp\cdot \boldsymbol{P}_{h\perp}}\ \underbrace{\tilde{f}_1(x, z\boldsymbol{\ell}_\perp, \ldots)}_{\text{TMD PDF}}\ \underbrace{\widetilde{D}_h(z, \boldsymbol{\ell}_\perp, \ldots)}_{\text{fragmentation f.}}\ \underbrace{\widetilde{H}(Q^2, \ldots)}_{\text{hard part}}$$

definition of TMD PDFs ("simplistic")



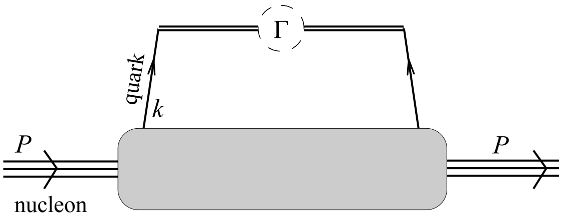
$$\Phi^{[\Gamma]}(k,P,S) \;\equiv\; \text{“}\,\langle P,S|\,\bar{q}(k)\,\Gamma\,q(k)\,|P,S\rangle\,\text{”}$$

lightcone coor. $w^\pm = \frac{1}{\sqrt{2}}(w^0 \pm w^3)$, so $w = w^+\hat{n}_+ + w^-\hat{n}_- + w_\perp$

proton flies along z-axis: P^+ large, $P_\perp = 0$

parametrization in terms of TMD PDFs, example

$$\int dk^- \;\Phi^{[\gamma^+]}(k,P,S)\Big|_{k^+=xP^+} = f_1(x,\boldsymbol{k}_\perp^2) + \langle \text{spin dep. terms}\rangle$$



$$\Phi^{[\Gamma]}(k,P,S) \equiv \frac{1}{2}\int \frac{d^4\ell}{(2\pi)^4}\, e^{-ik\cdot\ell}\ \langle P,S|\ \bar{q}(\ell)\,\Gamma\,\mathcal{U}\,q(0)\ |P,S\rangle$$

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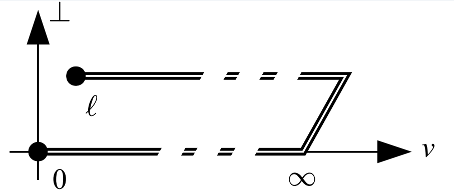
$$\mathcal{U} \equiv \mathcal{P} \exp\left(-ig \int_0^{\ell} d\xi^\mu A_\mu(\xi)\right) \qquad \text{along path from 0 to } \ell$$

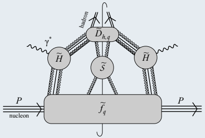
$\Longrightarrow \quad \langle P| \; \overline{q}(\ell)\, \Gamma\, \mathcal{U}\, q(0) \; |P\rangle$ is gauge invariant.

SIDIS / Drell Yan



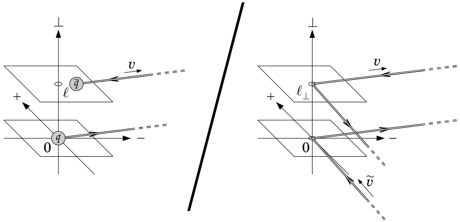
$v = \hat{n}_-$ (lightlike), or slightly off $v^- \gg v^+$

$$W^{\mu\nu}_{\text{unpol.,LO}} \propto \int d\boldsymbol{\ell}_\perp \; e^{i\boldsymbol{\ell}_\perp\cdot \boldsymbol{P}_{h\perp}}$$
$$\times\; \tilde{f}_1(\ldots)\; \widetilde{D}_h(\ldots)\; \widetilde{H}(\ldots)\; \underbrace{\widetilde{S}(\boldsymbol{\ell}_\perp,\ldots)}_{\text{soft factor}}$$



modified definition of TMD PDF correlator:

$$\Phi^{[\Gamma]}(k,P,S) \;\equiv\; \frac{1}{2}\int \frac{d^4\ell}{(2\pi)^4}\; e^{-ik\cdot\ell}\; \frac{\langle P,S|\;\bar{q}(\ell)\,\Gamma\,\mathcal{U}\,q(0)\;|P,S\rangle}{\widetilde{S}(\boldsymbol{\ell}_\perp,\ldots)}$$



- gauge links slightly off lightcone: $v \neq \hat{n}_-$

$\Rightarrow$ evolution eqn. in $\zeta \equiv (v\cdot P)^2/v^2$

- soft factor $\tilde{S}$: vacuum expectation value of gauge link structure

solved

- factorization shown.
- "rapidity divergences" removed by use of non-lightlike gauge link.

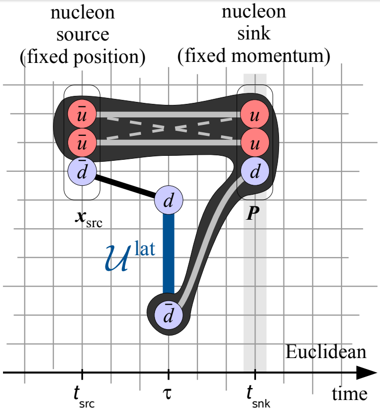
open issues

- UV divergence from gauge link self energy:

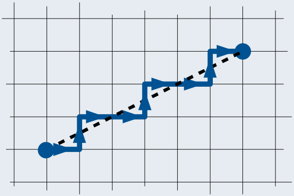
$$\mathcal{U}^{\text{ren}} = \mathcal{U}\ \exp\left(\ \delta m\ \langle\text{link length}\rangle\ \right)$$

wish list

- $f_1(x) = \int d^2\boldsymbol{k}_\perp\ f_1(x, \boldsymbol{k}_\perp^2)$
- probability interpretation



gauge link on lattice



For now, approximate **straight** gauge link.
$\Longrightarrow$ no T-odd structuers (Sivers, Boer-Mulders fcn.)

extract Lorentz-invariant amplitudes $\tilde{A}_i(\ell^2, \ell\cdot P)$

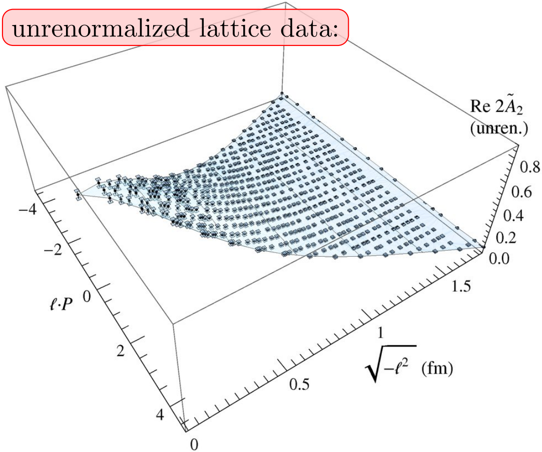
$$\langle P,S|\ \bar{q}(\ell)\,\gamma_\mu \mathcal{U}\, q(0)\ |P,S\rangle \;=\; 4\,\tilde{A}_2\, P_\mu \;+\; 4i\, m_N{}^2\ \tilde{A}_3\ \ell_\mu$$

$\Rightarrow\ f_1(x, \boldsymbol{k}_\perp^2)$

Amplitudes are complex and fulfill $[\tilde{A}_i(\ell^2, \ell\cdot P)]^* = \tilde{A}_i(\ell^2, -\ell\cdot P)$.
Operator must not have temporal extent: $\ell^0 = \ell_4 = 0$.

$$f_1(x,\boldsymbol{k}_\perp^2) \equiv \Phi^{[\gamma^+]}(x,\boldsymbol{k}_\perp;P,S)$$
$$= \int \frac{d(\ell\cdot P)}{2\pi}\, e^{ix(\ell\cdot P)} \int \frac{d^2\boldsymbol{\ell}_\perp}{(2\pi)^2}\, e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{\ell}_\perp}\, 2\tilde{A}_2(\ell^2,\ell\cdot P)\Big|_{\ell^+=0}$$

unrenormalized lattice data:



$\ell^2 \overset{\text{FT}}{\longleftrightarrow} \boldsymbol{k}_\perp^2$

$\ell\cdot P \overset{\text{FT}}{\longleftrightarrow} x$

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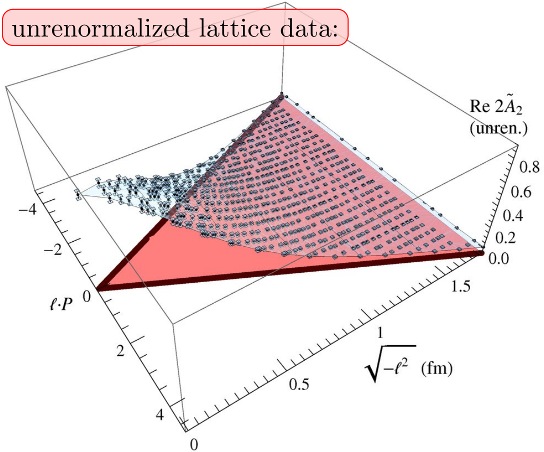

$$\ell^2 \overset{\text{FT}}{\longleftrightarrow} \boldsymbol{k}_\perp^2$$
$$\ell \cdot P \overset{\text{FT}}{\longleftrightarrow} x$$

Euclidean lattice

$$\ell_4 = 0$$
$$\Downarrow$$
$$\ell^2 \leq 0,$$
$$|\ell \cdot P| \leq |\boldsymbol{P}|\sqrt{-\ell^2}$$

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$$= \int \frac{d(\ell \cdot P)}{2\pi}\, e^{ix(\ell\cdot P)} \int \frac{d^2\boldsymbol{\ell}_\perp}{(2\pi)^2}\, e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{\ell}_\perp}\, 2\tilde{A}_2(\ell^2, \ell\cdot P)\Big|_{\ell^+=0}$$

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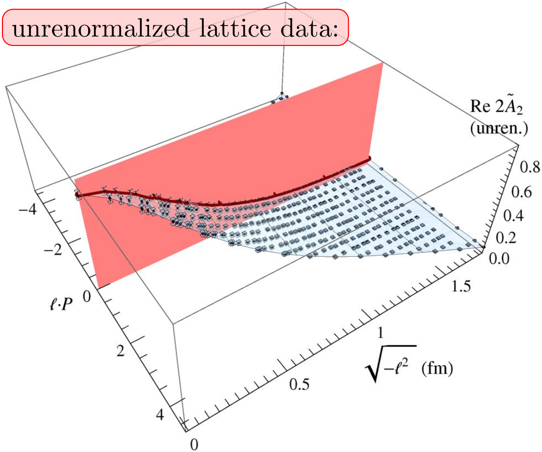


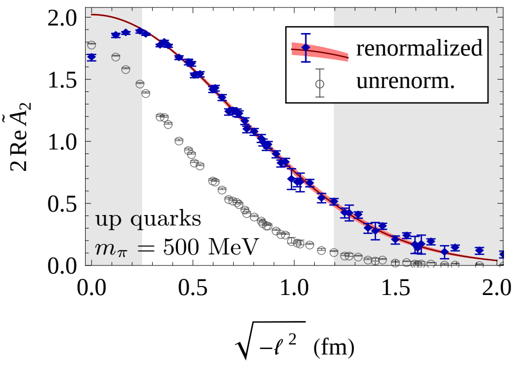
$\ell^2 \overset{\text{FT}}{\longleftrightarrow} \boldsymbol{k}_\perp^2$

$\ell\cdot P \overset{\text{FT}}{\longleftrightarrow} x$

Euclidean lattice

$$\ell_4 = 0$$
$$\Downarrow$$
$$\ell^2 \le 0,$$
$$|\ell\cdot P| \le |\boldsymbol{P}|\sqrt{-\ell^2}$$

lowest x-moment $$\int_{-1}^{1} dx\ f_1(x, \boldsymbol{k}_\perp^2) \equiv \int dx \int dk^-\ \Phi^{[\gamma^+]}(k,P,S) = \int \frac{d^2\ell_\perp}{(2\pi)^2}\, e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{\ell}_\perp}\ 2\,\tilde{A}_2(-\ell_\perp^2, 0)$$



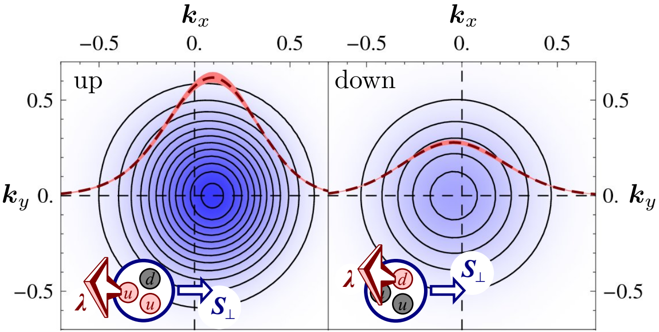
fit function

$C_1\ \exp(-|\ell|^2/\sigma_1^2)$

Exclude data below $\sqrt{-\ell^2} \leq 0.25$ fm: lattice cutoff effects. Expect continuum divergence at $\ell = 0$. Gaussian $\hat{=}$ regulator.

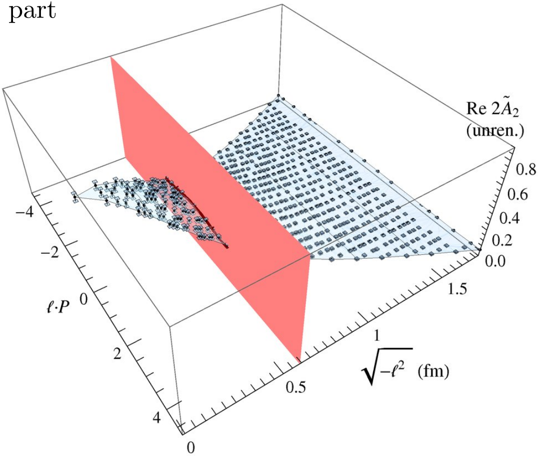
Density of quarks with positive helicity, $\lambda = 1$,
in a transversely polarized nucleon, $\boldsymbol{S}_\perp = (1,0)$:

$$\rho_{TL}(\boldsymbol{k}_\perp;\boldsymbol{S}_\perp,\lambda) \equiv \frac{1}{2}\int dx \int dk^- \; \Phi^{[\gamma^+ \frac{1}{2}(1\!\!1+\gamma^5)]}(k,P,S_\perp)$$
$$= \frac{1}{2} f_1^{(1)\mathrm{sW}}(\boldsymbol{k}_\perp^2) + \frac{\lambda}{2}\frac{\boldsymbol{k}_\perp\cdot\boldsymbol{S}_\perp}{m_N} g_{1T}^{(1)\mathrm{sW}}(\boldsymbol{k}_\perp^2)$$



($m_\pi \approx 500$ MeV, straight gauge link operator,
renormalization condition $C^{\mathrm{ren}} = 0$, Gaussian fit)

real part

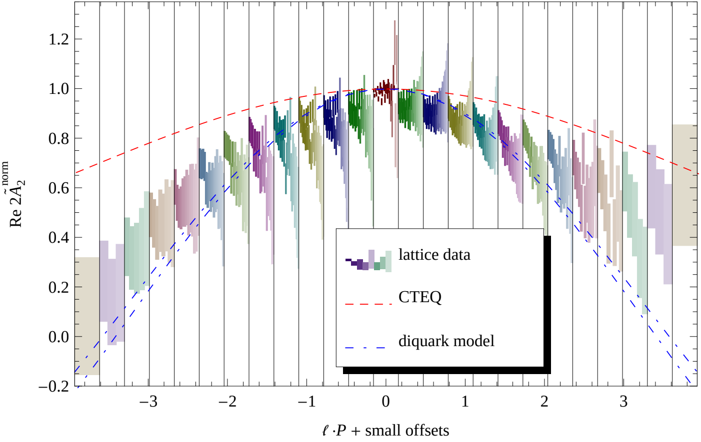


$$\ell^2 \xleftrightarrow{\text{FT}} \boldsymbol{k}_\perp^2$$

$$\ell \cdot P \xleftrightarrow{\text{FT}} x$$

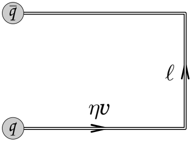
Within our limited $(\ell^2, \ell\cdot P)$ window,
our lattice data are compatible with the assumption

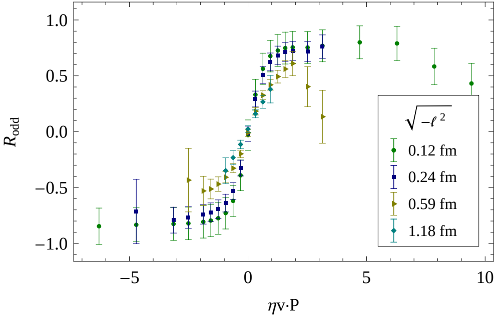
$$f_1(x, \boldsymbol{k}_\perp^2) = f_1(x)\, \hat{f}_1(\boldsymbol{k}_\perp^2)$$



ratio of amplitudes with staple like gauge links

$$R_{\text{odd}} \xrightarrow{\eta \text{ large}} \frac{\left[\tilde{A}_{12} + \frac{m_N^2}{{P_1}^2}\,\tilde{B}_8\right](\ell^2, 0, 0, \zeta^{-1}, \pm 1)}{\tilde{A}_2(\ell^2, 0, 0, \zeta^{-1}, \pm 1)}$$





Part of the effect comes from the Sivers function $f_{1T}^{\perp}$ via $\tilde{A}_{12}$!

$\zeta = (v \cdot P)^2 / v^2$

$\zeta \to \infty$ for lightlike links.

Need to evolve lattice results to high ζ.

Presently $\zeta^{\text{lat}} \approx (0.5\ \text{GeV})^2$.

$\zeta^{\text{lat}}_{\text{max}} \propto (\boldsymbol{P}^{\text{lat}})^2$

Need high nucleon momenta on lattice. Difficult (noisy).

present: straight gauge links

- not directly comparable to experimental situation

$\Rightarrow$ qualitative statements

- comparison to models, test of phenomenological assumptions
- easy access to spin-dependence

near future: staple shaped links, ratios of amplitudes

- question: ζ large enough on lattice?
- size estimates of spin-dependent and T-odd effects (Sivers,...)

prerequisite for quantitative lattice predictions

"To allow non-perturbative methods in QCD to be used to estimate parton densities, operator definitions of parton densities are needed that **can be taken literally**." [Collins arXiv:0808.2665 (2008)]

known fundamental limitations of (our) lattice TMD PDF calculations

- Euclidean space $\Rightarrow$ limited $(\ell \cdot P)$-range $\Rightarrow$ limited access to x-depend.
- evolution parameter $\zeta \propto$ (lattice nucleon momentum)^2