Details on lattice calculations of TMDs

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in collaboration with
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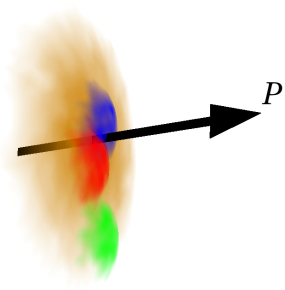
[Hägler et al. EPL88 61001 (2009)]
[Musch arXiv:0907.2381]

Emmy Noether-Programm
Deutsche Forschungsgemeinschaft DFG

Jefferson Lab
Thomas Jefferson National Accelerator Facility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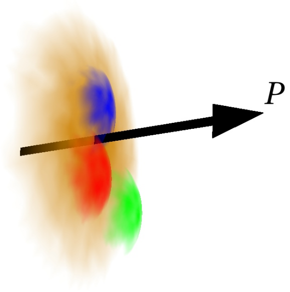
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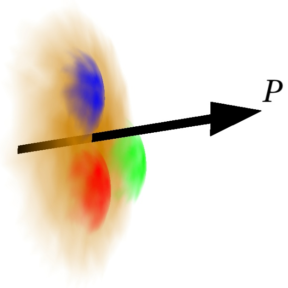
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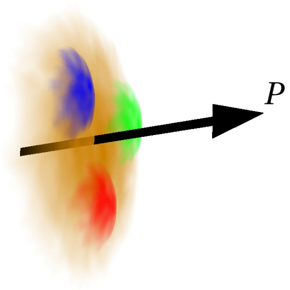
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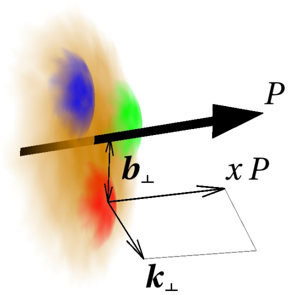
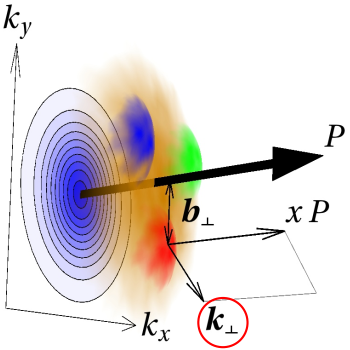

P
$\boldsymbol{b}_\perp$
$x\,P$
$\boldsymbol{k}_\perp$



What is the answer to the question about intrinsic motion of quarks inside the nucleon?

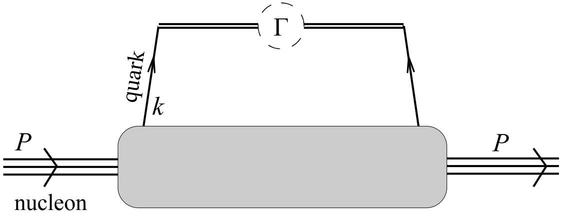
TMDs !
$f_1(x, \boldsymbol{k}_\perp^2)$, ...

- x (longitudinal momentum fraction) $\Rightarrow$ PDFs
- $x, \boldsymbol{b}_\perp$ (impact parameter) $\Rightarrow$ GPDs
- $x, \boldsymbol{k}_\perp$ (intrinsic transverse momentum) $\Rightarrow$ TMD PDFs

prerequisite for quantitative lattice predictions

"To allow non-perturbative methods in QCD to be used to estimate parton densities, operator definitions of parton densities are needed that can be taken literally." [Collins arXiv:0808.2665 (2008)]

- self-energy of the gauge link
  (should be cancelled by a soft factor?)
- short-range divergence / large $\boldsymbol{k}_\perp$-behavior,
  Gaussian "cure",
  probability interpretation
- main lattice-specific restriction:
  The non-local operator lies in a spacelike plane.
  - ⇒ results in a restricted kinematic window
  - ⇒ staple-shaped links not so close to the light cone



$$\Phi^{[\Gamma]}(k,P,S) \;\equiv\; \text{“}\, \langle P,S|\, \bar{q}(k)\, \Gamma\, q(k)\, |P,S\rangle \,\text{”}$$

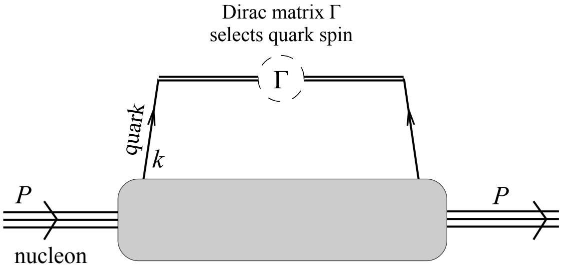
lightcone coor. $w^{\pm} = \frac{1}{\sqrt{2}}(w^0 \pm w^3)$, so $w = w^+ \hat{n}_+ + w^- \hat{n}_- + w_\perp$

proton flies along z-axis: P^+ large, $P_\perp = 0$

parametrization in terms of TMD PDFs, example

$$\int dk^- \;\Phi^{[\gamma^+]}(k,P,S)\Big|_{k^+=xP^+} = f_1(x,\boldsymbol{k}_\perp^2) - \frac{\epsilon_{ij}\boldsymbol{k}_i\boldsymbol{S}_j}{m_N} f_{1T}^{\perp}(x,\boldsymbol{k}_\perp)$$

[Ralston, Soper NPB 1979], [Mulders, Tangerman NPB 1996], [Goeke, Metz, Schlegel PLB 2005]



$$\Phi^{[\Gamma]}(k,P,S) \equiv \frac{1}{2}\int \frac{d^4\ell}{(2\pi)^4}\, e^{-ik\cdot\ell}\, \langle P,S|\; \bar{q}(\ell)\,\Gamma\, \mathcal{U}\, q(0)\; |P,S\rangle$$

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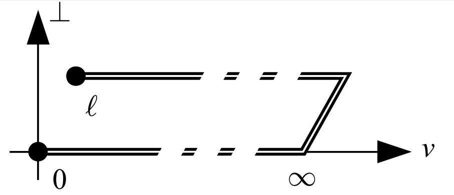
$$\mathcal{U} \equiv \mathcal{P} \exp\left(-ig \int_0^\ell d\xi^\mu A_\mu(\xi)\right) \qquad \text{along path from 0 to } \ell$$

$\Longrightarrow \quad \langle P|\ \bar{q}(\ell)\,\Gamma\,\mathcal{U}\,q(0)\ |P\rangle$ is gauge invariant.

SIDIS / Drell Yan

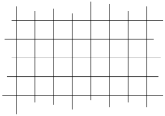


$v = \hat{n}_-$ (lightlike) [Hautmann, Collins, Metz (2000-2007)] [Cherednikov, Stefanis (2008)] [Chay (2007)]

or slightly off, $v^- \gg v^+$ [Collins, Soper, Sterman (1981)] [Ji, Ma, Yuan (2004,2005)]

shape of transverse piece? [Cherednikov, Stefanis]

We employ the Chroma library [Edwards, Joo (2005)] to process



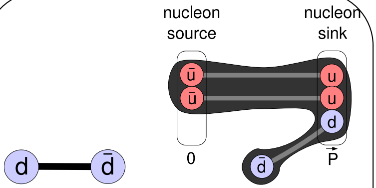
MILC gauge configurations

staggered Asqtad action,
2+1 flavors, $a \approx 0.124$ fm,
$m_\pi \approx 500,\ 610,$ and 760 MeV

[Orginos, Toussaint PRD (1999)]

+ finer MILC lattices
  to test renormalization

[Aubin et al. PRD (2004)]
[Bazavov et al. 0903.3598]

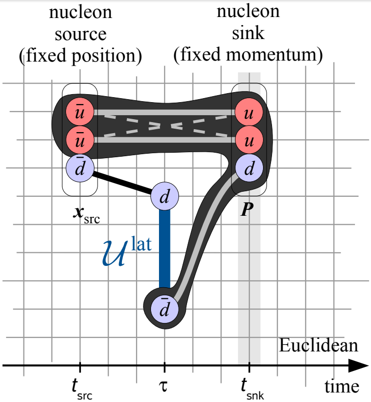


LHPC propagators

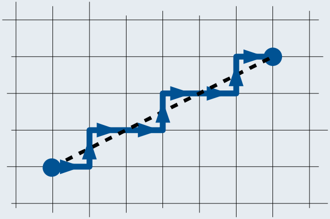
domain wall valence fermions,
m_π adjusted to staggered sea,

nucleon momenta:
$\boldsymbol{P} = 0$ and $|\boldsymbol{P}| = 500$ MeV

e.g., [Hägler et al. PRD (2008)]



gauge link on lattice



For now, approximate **direct** gauge link, no soft factor.
$\Longrightarrow$ no T-odd structuers (Sivers, Boer-Mulders fcn.)

extract Lorentz-invariant amplitudes $\tilde{A}_i(\ell^2, \ell\cdot P)$

$$\langle P,S|\ \bar{q}(\ell)\,\gamma_\mu\,\mathcal{U}\,q(0)\ |P,S\rangle \;=\; 4\,\tilde{A}_2\,P_\mu \;+\; 4i\,m_N{}^2\,\tilde{A}_3\,\ell_\mu$$

$\Rightarrow\ f_1(x, \boldsymbol{k}_\perp^2)$

Amplitudes are complex and fulfill $[\tilde{A}_i(\ell^2, \ell\cdot P)]^* = \tilde{A}_i(\ell^2, -\ell\cdot P)$.
Operator must not have temporal extent: $\ell^0 = \ell_4 = 0$.

$$\Phi^{[\Gamma]}(k,P,S) \;\equiv\; \frac{1}{2}\int \frac{d^4\ell}{(2\pi)^4}\; e^{-ik\cdot\ell}\;\langle P,S|\;\bar{q}(\ell)\,\Gamma\,\mathcal{U}\,q(0)\;|P,S\rangle$$

$$\Phi^{[\Gamma]}(x,\boldsymbol{k}_\perp;P,S) \equiv \int_{-\infty}^{\infty} dk^-\;\Phi^{[\Gamma]}(k;P,S)\;\Big|_{k^+=xP^+}$$

$$= \frac{1}{2(2\pi)^3}\int d\ell^-\; d^2\ell_\perp\; e^{ik\cdot\ell}\;\langle P,S|\;\bar{q}(\ell)\,\Gamma\,\mathcal{U}\,q(0)\;|P,S\rangle\;\Big|_{\ell^+=0}$$

$$= \int \frac{d(\ell\cdot P)}{4\pi P^+}\, e^{ix(\ell\cdot P)} \int \frac{d^2\boldsymbol{\ell}_\perp}{(2\pi)^2}\, e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{\ell}_\perp}\;\langle P,S|\;\bar{q}(\ell)\,\Gamma\,\mathcal{U}\,q(0)\;|P,S\rangle\;\Big|_{\ell^+=0}$$

Note: $\ell^2\Big|_{\ell^+=0} = -\boldsymbol{\ell}_\perp^2.$

$x \longleftrightarrow \ell\cdot P$

$\boldsymbol{k}_\perp^2 \longleftrightarrow \ell^2$

example: unpolarized case

$$f_1(x,\boldsymbol{k}_\perp^2) \equiv \Phi^{[\gamma^+]}(x,\boldsymbol{k}_\perp;P,S)$$

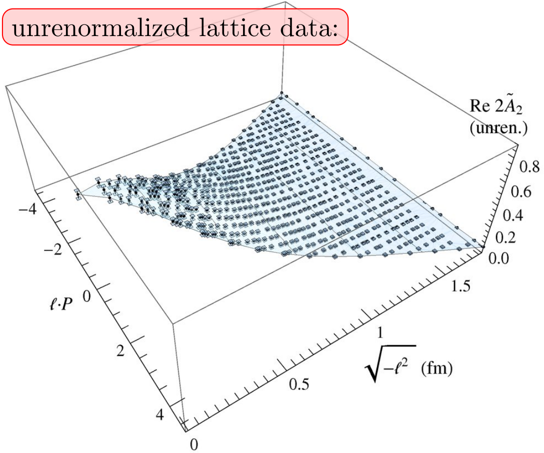
$$= \int \frac{d(\ell\cdot P)}{2\pi}\, e^{ix(\ell\cdot P)} \int \frac{d^2\boldsymbol{\ell}_\perp}{(2\pi)^2}\, e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{\ell}_\perp}\; 2\tilde{A}_2(\ell^2,\ell\cdot P)\;\Big|_{\ell^+=0}$$

extract Lorentz-invariant amplitudes $\tilde{A}_i(\ell^2, \ell\cdot P)$, example :

$$\langle P,S|\ \overline{q}(\ell)\,\gamma_\mu\,\mathcal{U}\,q(0)\ |P,S\rangle\ =\ 4\tilde{A}_2 P_\mu\ +\ 4i\,m_N{}^2\ \tilde{A}_3\ \ell_\mu\ ,$$

$$f_1(x,\boldsymbol{k}_\perp^2) = \int \frac{d(\ell\cdot P)}{2\pi}\ e^{ix(\ell\cdot P)} \int \frac{d^2\boldsymbol{\ell}_\perp}{(2\pi)^2}\ e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{\ell}_\perp}\ 2\tilde{A}_2(\ell^2,\ell\cdot P)\ \Big|_{\ell^+=0}$$

unrenormalized lattice data:



$\ell^2 \overset{\text{FT}}{\longleftrightarrow} \boldsymbol{k}_\perp^2$

$\ell\cdot P \overset{\text{FT}}{\longleftrightarrow} x$

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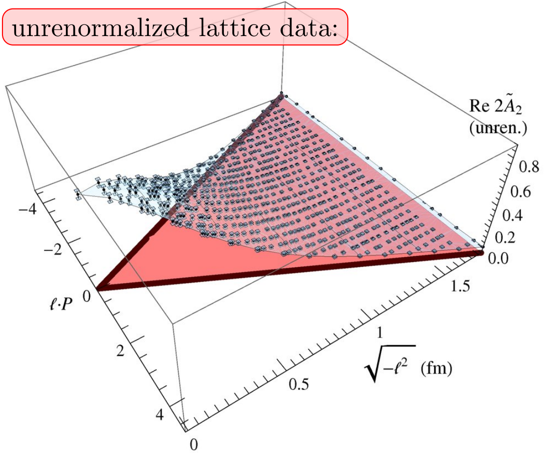

$\ell^2 \overset{\text{FT}}{\longleftrightarrow} \boldsymbol{k}_\perp^2$

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Euclidean lattice

$$\ell^0 = \ell_4 = 0$$

$$\Downarrow$$

$$\ell^2 \le 0,$$
$$|\ell\cdot P| \le |\boldsymbol{P}|\sqrt{-\ell^2}$$

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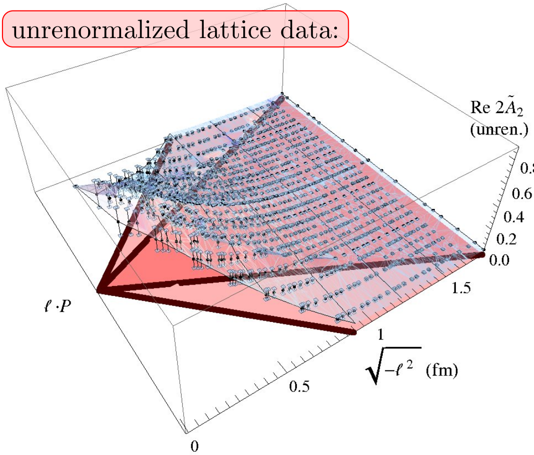

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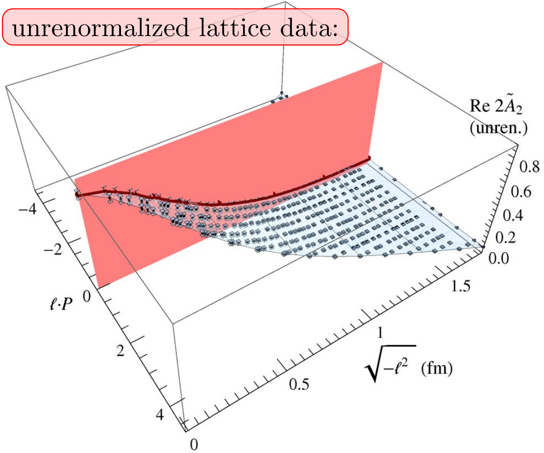
direct link amplitudes from the lattice

extract Lorentz-invariant amplitudes $\tilde{A}_i(\ell^2, \ell\cdot P)$, example :

$$\langle P, S|\ \overline{q}(\ell)\, \gamma_\mu \mathcal{U}\, q(0)\ |P, S\rangle \ =\ 4\tilde{A}_2 P_\mu \ +\ 4i\, m_N{}^2\ \tilde{A}_3\ \ell_\mu\ ,$$

$$f_1(x, \boldsymbol{k}_\perp^2) = \int \frac{d(\ell\cdot P)}{2\pi}\ e^{ix(\ell\cdot P)} \int \frac{d^2\boldsymbol{\ell}_\perp}{(2\pi)^2}\ e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{\ell}_\perp}\ 2\tilde{A}_2(\ell^2, \ell\cdot P)\ \Big|_{\ell^+=0}$$

unrenormalized lattice data:



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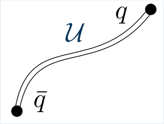
continuum renormalization of gauge links

[Craigie, Dorn NPB185,204 (1981)]

smooth path



$$[\bar{q}\ \mathcal{U}\ q]_{\text{ren}} = Z^{-1} \exp\left(-\delta\hat{m}\,\frac{l}{a} \right) [\bar{q}\ \mathcal{U}\ q]$$

l : the total (Euclidean) length of the gauge link,

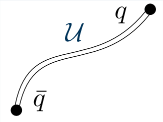
$\delta\hat{m}$: removes the power divergence $\sim 1/a$

$\Rightarrow$ A spacelike link has a self-energy proportional to the length l

- In dim. reg., $\delta m = 0$, but then renormalon ambiguities appear.
- Lattice QCD is a cutoff regularization, in general $\delta m \neq 0$.
- Renormalization condition needed to fix δm.
- (Notice the length l of a lightlike link of infinite extent is $0 \times \infty$.)

continuum renormalization of gauge links [Craigie, Dorn NPB185,204 (1981)]

smooth path



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static quark potential

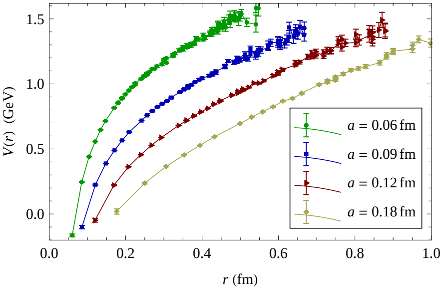
$$V_{\rm ren}(r) = V(r) + 2\,\delta\hat{m}/a$$

string [Lüscher, Symanzik, Weisz (1980)]

at large r: $V_{\rm ren}(r) \approx$
$V_{\rm string}(r) = \sigma r - \pi/12r + C$

method [Cheng PRD77,014511 (2008)]

determine $\delta\hat{m}$ from
$V_{\rm ren}(0.7\text{ fm}) \stackrel{!}{=} V_{\rm string}(0.7\text{ fm})$



continuum renormalization of gauge links [Craigie, Dorn NPB185,204 (1981)]

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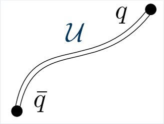

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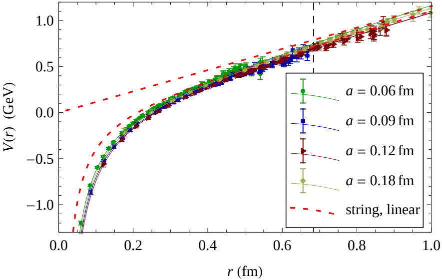

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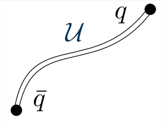

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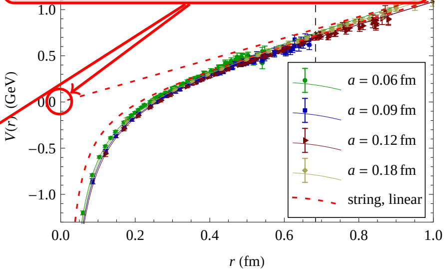
renormalization condition $C^{\text{ren}} = 0$



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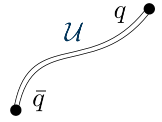

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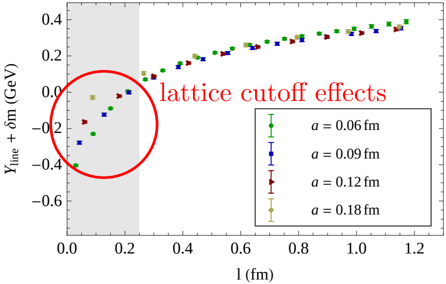
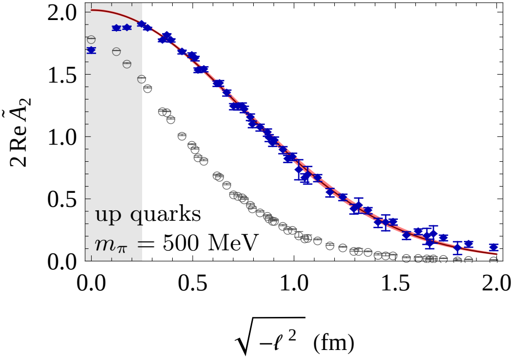


$$Y_{\text{line}}(l) \equiv \frac{d}{dl}\ln\langle\text{tr}\,\mathcal{U}\rangle_{\text{(Landau gauge)}}$$

$$f_1^{(0_x)}(\boldsymbol{k}_\perp^2) \equiv \int_{-1}^{1} dx\ f_1(x, \boldsymbol{k}_\perp^2) = \int \frac{d^2\ell_\perp}{(2\pi)^2}\ e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{\ell}_\perp}\ 2\,\tilde{A}_2(-\ell_\perp^2, 0)$$



fit function

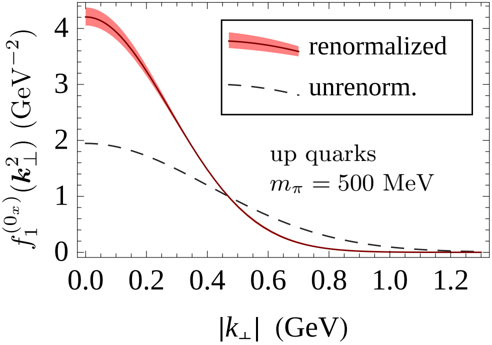
$C_1 \exp(-|\ell|^2/\sigma_1^2)$

Z-factor

$Z^{-1}\ C_1^{\text{up-down}} \stackrel{!}{=} 1$

multiplicative renormalization based on quark counting

$$f_1^{(0_x)}(\boldsymbol{k}_\perp^2) \equiv \int_{-1}^{1} dx\ f_1(x, \boldsymbol{k}_\perp^2) = \int \frac{d^2\ell_\perp}{(2\pi)^2}\ e^{i\boldsymbol{k}_\perp \cdot \boldsymbol{\ell}_\perp}\ 2\,\tilde{A}_2(-\ell_\perp^2, 0)$$



width of the distribution (RMS momentum):

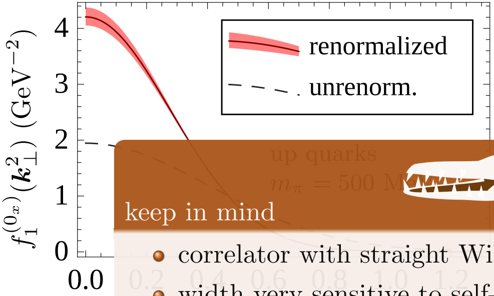
$\langle \boldsymbol{k}_\perp^2 \rangle^{1/2} =$ $(391 \pm 8_{\text{stat}} \pm 27_{\text{sys}})$ MeV

compare phenomenology [Anselmino et al., PRD71, 074006 (2005)]:

$\langle \boldsymbol{k}_\perp^2 \rangle^{1/2} \approx 500$ MeV

(estimate, Gaussian Ansatz)

$$f_1^{(0_x)}(\boldsymbol{k}_\perp^2) \equiv \int_{-1}^{1} dx\ f_1(x,\boldsymbol{k}_\perp^2) = \int \frac{d^2\ell_\perp}{(2\pi)^2}\ e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{\ell}_\perp}\ 2\,\tilde{A}_2(-\ell_\perp^2, 0)$$



width of the distribution (RMS momentum):

$\langle \boldsymbol{k}_\perp^2 \rangle^{1/2} =$ $(391 \pm 8_{\text{stat}} \pm 27_{\text{sys}})$ MeV

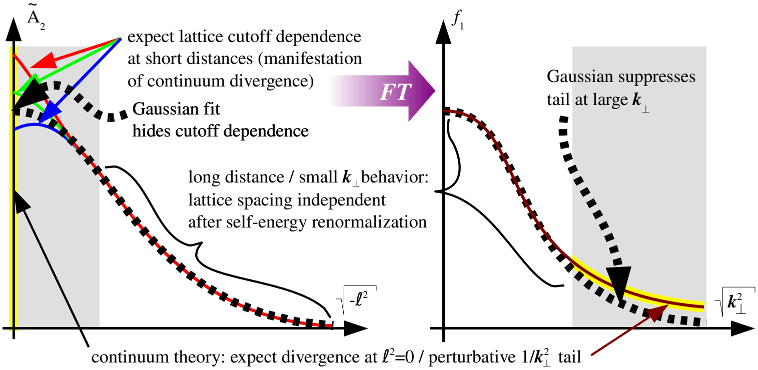
…phenomenology [Anselmino et al., PRD 71, 074006 (2005)]:

$\langle \boldsymbol{k}_\perp^2 \rangle^{1/2} \approx 500$ MeV (estimate, Gaussian Ansatz)

keep in mind

- correlator with straight Wilson line ("sW")
- width very sensitive to self-energy renormalization
- Gaussian fit ansatz ("wrong" at large-$\boldsymbol{k}_\perp$ [Diehl, arXiv:0811.0774])
- $m_\pi \approx 500$ MeV

sketch: behavior at small/large $\boldsymbol{k}_\perp$



Problem with the perturbative tail

$\int d^2\boldsymbol{k}_\perp\, f_1(x, \boldsymbol{k}_\perp^2)$ is undefined,
in conflict with probability interpretation.

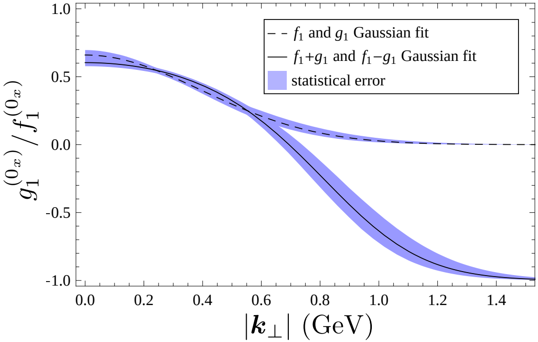
Gaussian is a poor man's solution.

Ideal would be a prescription that maintains
$\int d^2\boldsymbol{k}_\perp\, f_1(x, \boldsymbol{k}_\perp^2; \mu) = f_1(x; \mu)$ at some scale μ.

$$f_1^{(0_x)}(\boldsymbol{k}_\perp^2) = C_0 \exp(-\boldsymbol{k}_\perp^2/\mu_0^2)$$
$$g_1^{(0_x)}(\boldsymbol{k}_\perp^2) = C_2 \exp(-\boldsymbol{k}_\perp^2/\mu_2^2)$$

vs.

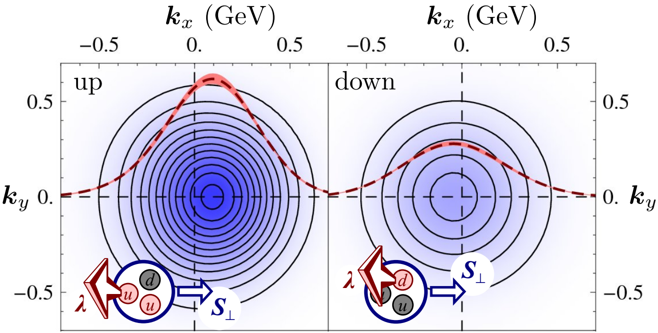
$$\rho_{LL}^{\pm}(\boldsymbol{k}_\perp) \equiv \tfrac{1}{2} f_1^{(0_x)}(\boldsymbol{k}_\perp^2) \pm \tfrac{1}{2} g_1^{(0_x)}(\boldsymbol{k}_\perp^2)$$
$$\rho_{LL}^{+}(\boldsymbol{k}_\perp) = C_+ \exp(-\boldsymbol{k}_\perp^2/\mu_+^2)$$
$$\rho_{LL}^{-}(\boldsymbol{k}_\perp) = C_- \exp(-\boldsymbol{k}_\perp^2/\mu_-^2)$$



⇒ Asymptotic behavior at large $\boldsymbol{k}_\perp$ imposed by Gaussian ansatz; not a “lattice result”. Similar issues in analysis of experimental data.

Density of quarks with positive helicity, $\lambda = 1$,
in a transversely polarized nucleon, $\boldsymbol{S}_\perp = (1,0)$:

$$\rho_{TL}(\boldsymbol{k}_\perp;\boldsymbol{S}_\perp,\lambda) \equiv \frac{1}{2}\int dx \int dk^- \; \Phi^{[\gamma^+ \frac{1}{2}(\mathbb{1}+\gamma^5)]}(k,P,S_\perp)$$
$$= \frac{1}{2} f_1^{(0_x)}(\boldsymbol{k}_\perp^2) + \frac{\lambda}{2}\frac{\boldsymbol{k}_\perp\cdot\boldsymbol{S}_\perp}{m_N}\; g_{1T}^{(0_x)}(\boldsymbol{k}_\perp^2)$$



($m_\pi \approx 500$ MeV, straight gauge link operator, renormalization condition $C^{\mathrm{ren}} = 0$, Gaussian fit)

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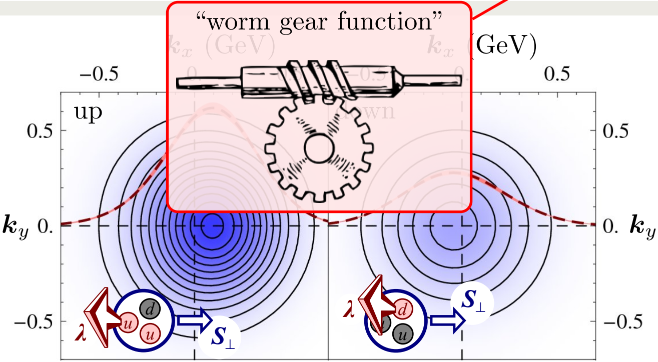

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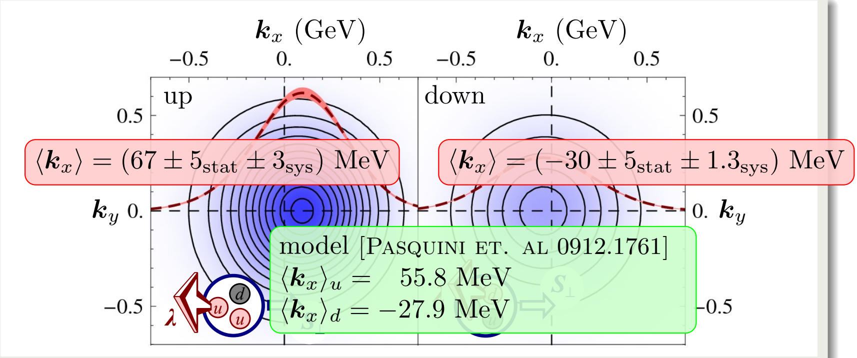


($m_\pi \approx 500$ MeV, straight gauge link operator,
renormalization condition $C^{\text{ren}} = 0$, Gaussian fit)

$$f^{(m_x,n_\perp)} \equiv \int_{-1}^{1} dx\, x^m \int d^2\boldsymbol{k}_\perp \left(\frac{\boldsymbol{k}_\perp^2}{2m_N^2}\right)^n f(x,\boldsymbol{k}_\perp^2)$$

Let us assume the amplitudes $\tilde{A}_i$ are regular at $\ell^2 = 0$.

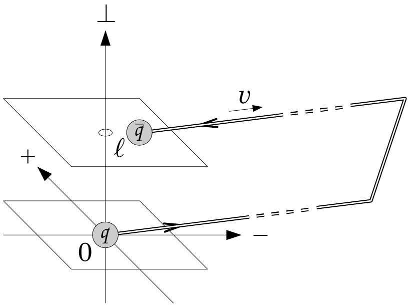
$$\langle \boldsymbol{k}_\perp \rangle_{\rho_{TL}} \;=\; \lambda \boldsymbol{S}_\perp m_N \frac{g_{1T}^{(0_x,1_\perp)}}{f_1^{(0_x,0_\perp)}} \;=\; \lambda \boldsymbol{S}_\perp m_N \frac{\tilde{A}_7(0,0)}{\tilde{A}_2(0,0)}$$

$\Rightarrow$ estimates for certain $\boldsymbol{k}_\perp$-moments:

$$\langle \boldsymbol{k}_\perp \rangle_{\rho_{TL}} \;\approx \lambda \boldsymbol{S}_\perp m_N \frac{\tilde{A}_7(\ell^2_{\text{min}},0)}{\tilde{A}_2(\ell^2_{\text{min}},0)}$$

with ℓ^2_{min} large enough to avoid strong lattice artefacts.
All self-energies from the gauge link cancel on the RHS
($\Rightarrow$ no dependence on the renormalization condition).

In the presence of divergences at $\ell^2 = 0$, we assume ℓ^2_{min} represents a regularization scale.



- gauge link = effective representation of struck quark ("final state interaction")
- ⇒ (almost lightlike)

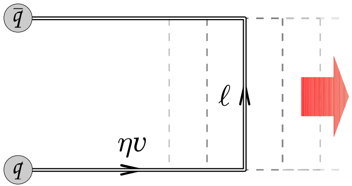
$$\zeta \equiv \frac{(v\cdot P)^2}{v^2} \to \pm\infty$$

- keep ζ finite to avoid "rapidity divergences"

now 32 Lorentz-invariant amplitudes [Goeke,Metz,Schlegel PLB618,90 (2005)]

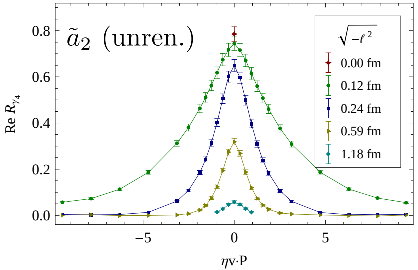
$$A_i\left(k^2, k\cdot P, \frac{v\cdot k}{|v\cdot P|}, \frac{v^2}{|v\cdot P|^2}, \frac{v\cdot P}{|v\cdot P|}\right) = A_i\left(k^2, k\cdot P, \underbrace{\frac{v\cdot k}{|v\cdot P|}}_{\approx x}, \zeta^{-1}, \mathrm{sgn}(v\cdot P)\right)$$

Large ζ: evolution with ζ known [Collins,Soper NPB194,445 (1981)].



- v spatial $\Rightarrow$ $|\zeta| = \frac{(v\cdot P)^2}{|v|^2} \leq |\boldsymbol{P}_{\text{lat.}}|^2$
- look for plateaus at large $|\eta|$
- now 32 amplitudes $\tilde{a}_i(\ell^2, \ell\cdot P, v\cdot P; \eta, \zeta)$

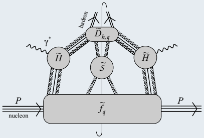
Problem: need to subtract gauge link self-energy ($\rightarrow$ η-independence)



Unmodified/unrenormalized amplitude vanishes for $\eta \rightarrow \infty$.

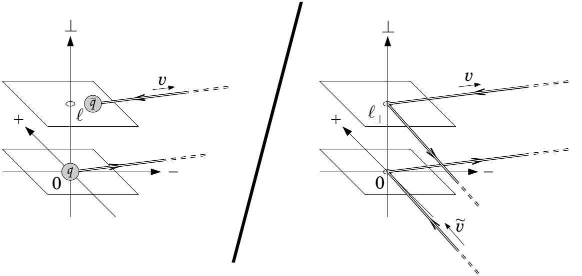
e.g., [JI, MA, YUAN PRD (2005)] :

$$W^{\mu\nu}_{\text{unpol.,LO}} \quad \propto \quad H \times f_1 \otimes D_h \otimes \underbrace{S}_{\text{soft factor}}$$



modified definition of TMD PDF correlator:

$$\Phi^{[\Gamma]}(k,P,S) \;\equiv\; \frac{1}{2}\int \frac{d^4\ell}{(2\pi)^4}\; e^{-ik\cdot\ell}\; \frac{\langle P,S|\;\bar{q}(\ell)\,\Gamma\,\mathcal{U}\,q(0)\;|P,S\rangle}{\widetilde{S}(\boldsymbol{\ell}_\perp,\ldots)}$$

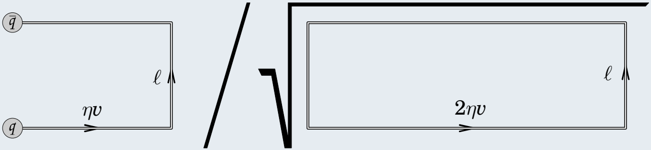


$$\Phi^{[\Gamma]}(k,P,S) \equiv \frac{1}{2}\int \frac{d^4\ell}{(2\pi)^4}\, e^{-ik\cdot\ell}\, \frac{\langle P,S|\ \bar{q}(\ell)\,\Gamma\,\mathcal{U}\,q(0)\ |P,S\rangle}{\widetilde{S}(\boldsymbol{\ell}_\perp,\ldots)}$$

with $\widetilde{S}$ obtained from a vacuum expectation value of gauge links

Adjust soft factor to cancel $\exp(\delta m\, L)$

Suggestion [Collins PoS LC]2008 :



Is this a meaningful definition of TMD PDFs?

example Sivers function

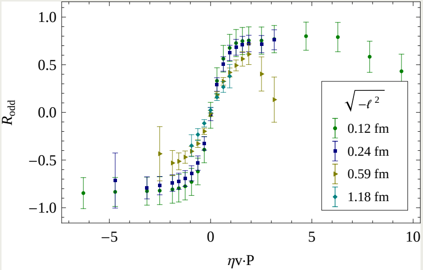
$$\langle \boldsymbol{k}_y \rangle_{TU} \overset{\text{formally}}{=} -2m_N \boldsymbol{S}_x \lim_{\eta\to\infty} \frac{\tilde{a}_{12}(0,0,0;\eta,\zeta)+\ldots}{\tilde{a}_2(0,0,0;\eta,\zeta)} \propto \frac{\int dx \int d^2\boldsymbol{k}_\perp\, \boldsymbol{k}_\perp^2\, f_{1T}^\perp}{\int dx \int d^2\boldsymbol{k}_\perp\, f_1}$$

On the lattice, we can try to compute

$$\langle \boldsymbol{k}_y \rangle_{TU} \underset{\eta \text{ large}}{\approx} -2m_N \boldsymbol{S}_x \frac{\tilde{a}_{12}(\ell^2_{\text{min}},0,0;\eta,\zeta)+\ldots}{\tilde{a}_2(\ell^2_{\text{min}},0,0;\eta,\zeta)}$$

Self-energy cancels!

Test calculation: a time reversal odd ratio of ampitudes



$$R_{\text{odd}} = -\frac{\tilde{a}_{12} - (\eta \frac{m_N^2 v_1}{P_1})\, \tilde{b}_8}{\tilde{a}_2}$$

Plateaus visible at large $|\eta|$.

"Time-reversal odd" $\leftrightarrow$ odd in $\eta v{\cdot}P$.

Part of the effect comes from the Sivers function $f_{1T}^\perp$!

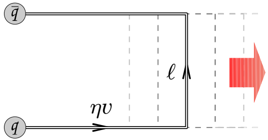
Conclusion

Summary:

- Lattice exploration of intrinsic quark momentum distributions in the nucleon. "Full QCD testbed".
- Manifestly non-local operators on the lattice.
- First results based on a a simplified operator geometry (direct gauge link) and a Gaussian fit model, at $m_\pi \approx 500$ MeV:
  - Obtaind x-integrated leading twist T-even TMD PDFs $f_1^{(0_x)}(\boldsymbol{k}_\perp^2)$, $g_{1T}^{(0_x)}(\boldsymbol{k}_\perp^2)$, $h_{1L}^{\perp(0_x)}(\boldsymbol{k}_\perp^2)$, ...
  - Observed deformed quark densities due to worm-gear functions.

Outlook:

- Study of non-straight gauge links similar as in SIDIS.



- Higher statistics needed to discuss factorization $f_1(x, \boldsymbol{k}_\perp^2) \approx f_1(x)\ f_1^{(0_x)}(\boldsymbol{k}_\perp^2)/\mathcal{N}$.
- Beyond Gaussian fits: Matching to perturbative behavior at small ℓ, i.e., large $\boldsymbol{k}_\perp$.

Backup Slides

$$\Phi^{[\Gamma]}(k,P,S) \equiv \frac{1}{2}\int \frac{d^4\ell}{(2\pi)^4}\, e^{-ik\cdot\ell}\, \langle P,S|\ \bar{q}(\ell)\,\Gamma\,\mathcal{U}\,q(0)\ |P,S\rangle$$

isolation of Lorentz-invariant amplitudes

compare [Mulders, Tangerman NPB (1996)]

$$\langle P,S|\ \bar{q}(\ell)\,\gamma_\mu\,\mathcal{U}\,q(0)\ |P,S\rangle = 4\,\tilde{A}_2\,P_\mu + 4i\,m_N{}^2\,\tilde{A}_3\,\ell_\mu$$

$$\begin{aligned}\langle P,S|\ \bar{q}(\ell)\,\gamma_\mu\gamma^5\,\mathcal{U}\,q(0)\ |P,S\rangle = &-4\,m_N\,\tilde{A}_6\,S_\mu\\ &-4i\,m_N\,\tilde{A}_7\,P_\mu(\ell\cdot S)\\ &+4\,m_N{}^3\,\tilde{A}_8\,\ell_\mu(\ell\cdot S)\end{aligned}$$

$$\langle P,S|\ \bar{q}(\ell)\,\ldots\,\mathcal{U}\,q(0)\ |P,S\rangle = \text{further structures (9 amplitudes in total)}$$

Transformation properties of the matrix element ($\dagger$, $\mathcal{P}$, $\mathcal{T}$) limit number of allowed structures. No $\mathcal{T}$-odd structures (Sivers function, ...) with straight gauge link.

The amplitudes fulfill $\tilde{A}_i(\ell^2,\ell\cdot P) = \left[\tilde{A}_i(\ell^2,-\ell\cdot P)\right]^*$.

$$\Phi^{[\Gamma]}(k,P,S) \equiv \frac{1}{2}\int \frac{d^4\ell}{(2\pi)^4}\, e^{-ik\cdot\ell}\, \langle P,S|\ \bar{q}(\ell)\,\Gamma\,\mathcal{U}\,q(0)\ |P,S\rangle$$

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$\Rightarrow f_1(x,\boldsymbol{k}_\perp^2)$ (from $\tilde{A}_2$)

$$\langle P,S|\ \bar{q}(\ell)\,\gamma_\mu\gamma^5\,\mathcal{U}\,q(0)\ |P,S\rangle = -4\,m_N\,\tilde{A}_6\,S_\mu - 4i\,m_N\,\tilde{A}_7\,P_\mu(\ell\cdot S) + 4\,m_N{}^3\,\tilde{A}_8\,\ell_\mu(\ell\cdot S)$$

$\Rightarrow g_{1T}(x,\boldsymbol{k}_\perp^2)$ (from $\tilde{A}_7$)

$\langle P,S|\ \bar{q}(\ell)\ \ldots\ \mathcal{U}\,q(0)\ |P,S\rangle =$ further structures (9 amplitudes in total)

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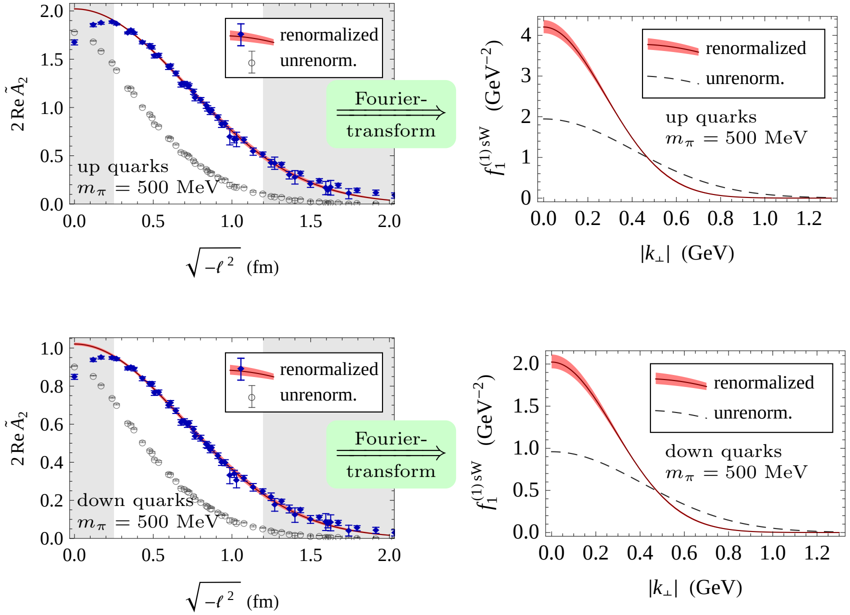
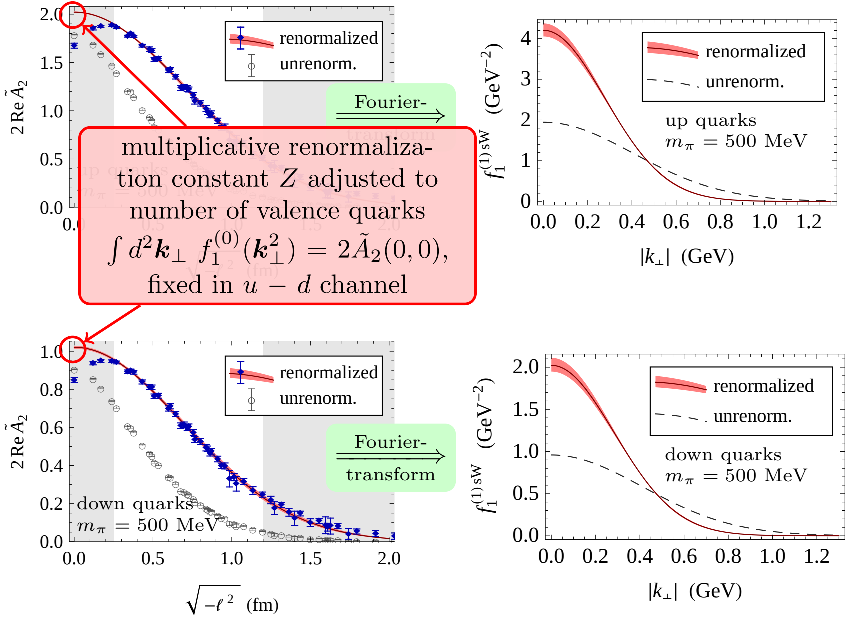
$f_1^{(0_x)}$ from $\tilde{A}_2$

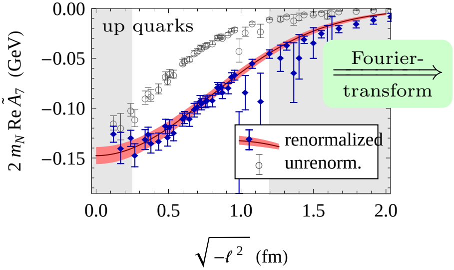


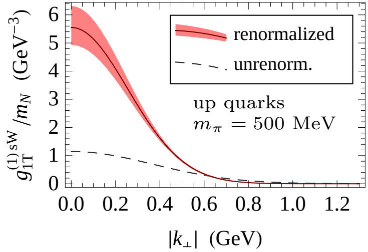
$f_1^{(0_x)}$ from $\tilde{A}_2$

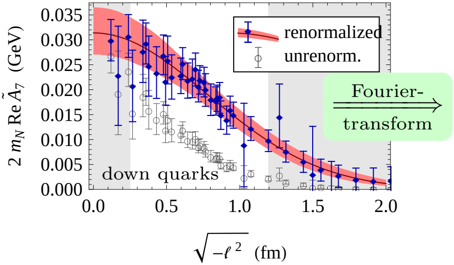


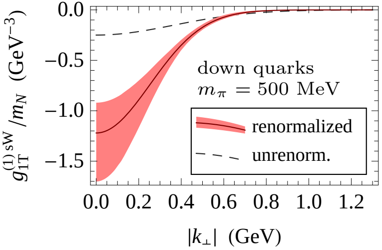
multiplicative renormalization constant Z adjusted to number of valence quarks $\int d^2\boldsymbol{k}_\perp\ f_1^{(0)}(\boldsymbol{k}_\perp^2) = 2\tilde{A}_2(0,0)$, fixed in $u-d$ channel

$g_{1T}^{(0_x)}$ from $\tilde{A}_7$







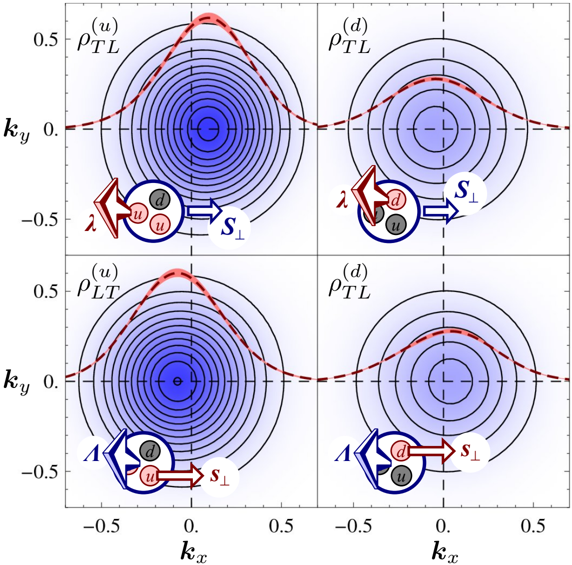


N \ q	U	L	T
U	f_1		$h_1^{\perp}$
L		g_1	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	h_1 $h_{1T}^{\perp}$

time-reversal odd

N \ q	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

time-reversal odd



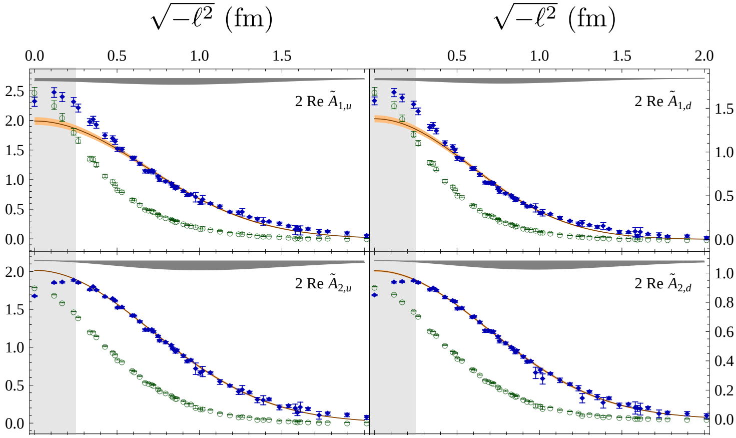
Diplole deformations

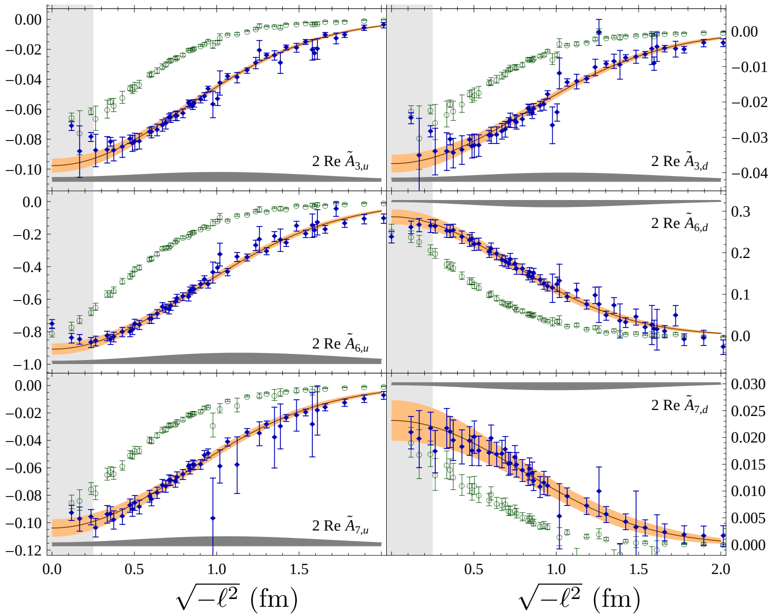
$\rho_{TL} : \sim \lambda\, \boldsymbol{k}_\perp \cdot \boldsymbol{S}_\perp \; g_{1T}$

$\rho_{TL} : \sim \Lambda\, \boldsymbol{k}_\perp \cdot \boldsymbol{s}_\perp \; h^\perp_{1L}$

The corresponding dipole structures $\sim \lambda\, \boldsymbol{b}_\perp \cdot \boldsymbol{S}_\perp$, $\sim \Lambda\, \boldsymbol{b}_\perp \cdot \boldsymbol{s}_\perp$ for impact parameter densities (from GPDs) are ruled out by symmetries.

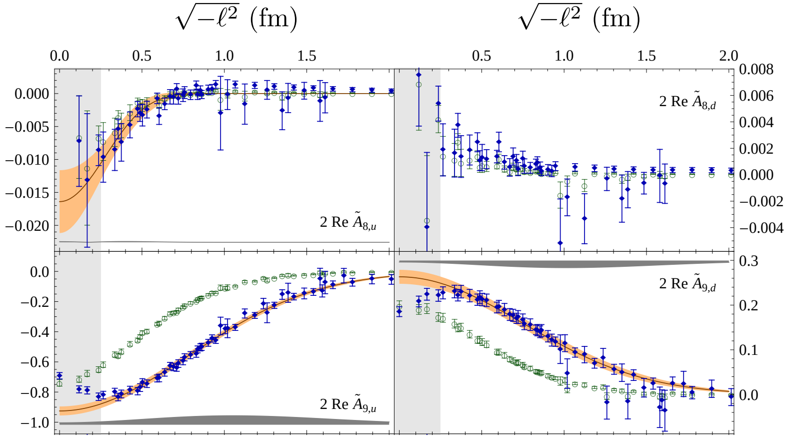
[Hägler, Musch, Negele, Schäfer EPL 88, 61001 (2009)]

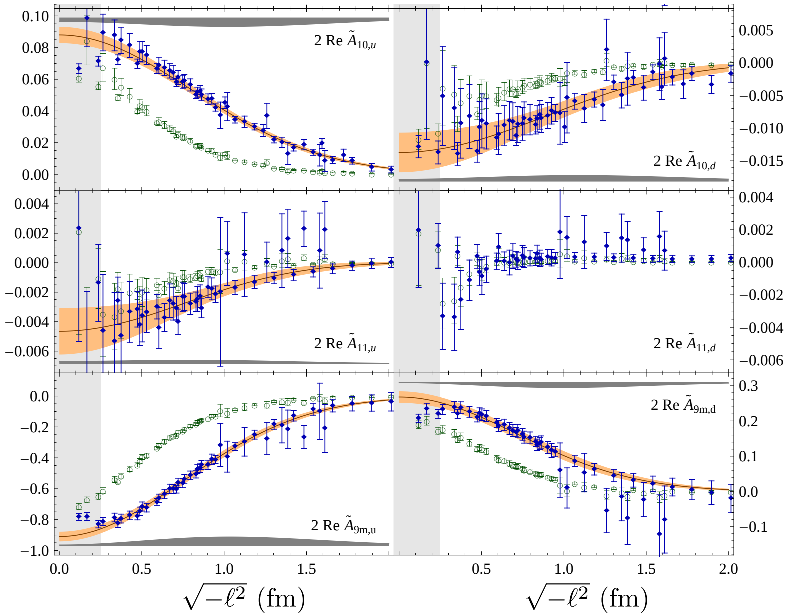

$\sqrt{-\ell^2}$ (fm)
$\sqrt{-\ell^2}$ (fm)
0.0
0.5
1.0
1.5
2.0
2.5
2 Re $\tilde{A}_{1,u}$
2 Re $\tilde{A}_{1,d}$
2 Re $\tilde{A}_{2,u}$
2 Re $\tilde{A}_{2,d}$
0.2
0.4
0.6
0.8



0.00
−0.02
−0.04
−0.06
−0.08
−0.10
2 Re $\tilde{A}_{3,u}$
2 Re $\tilde{A}_{3,d}$
−0.01
−0.02
−0.03
−0.04
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
2 Re $\tilde{A}_{6,u}$
2 Re $\tilde{A}_{6,d}$
0.3
0.2
0.1
−0.12
2 Re $\tilde{A}_{7,u}$
2 Re $\tilde{A}_{7,d}$
0.030
0.025
0.020
0.015
0.010
0.005
0.000
0.5
1.0
1.5
2.0
$\sqrt{-\ell^2}$ (fm)
$\sqrt{-\ell^2}$ (fm)

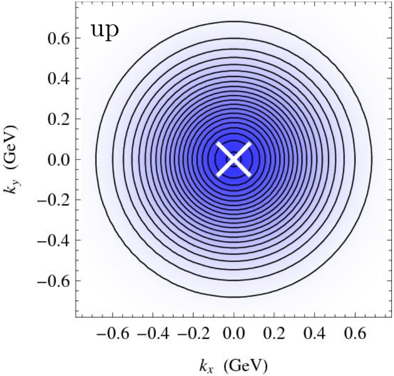
more amplitudes … (preliminary)




2 Re $\tilde{A}_{10,u}$
2 Re $\tilde{A}_{10,d}$
2 Re $\tilde{A}_{11,u}$
2 Re $\tilde{A}_{11,d}$
2 Re $\tilde{A}_{9m,u}$
2 Re $\tilde{A}_{9m,d}$
$\sqrt{-\ell^2}$ (fm)
$\sqrt{-\ell^2}$ (fm)

Density of unpolarized quarks (minus antiquarks)
in an unpolarized nucleon as a function of transverse momentum $\boldsymbol{k}_\perp$:

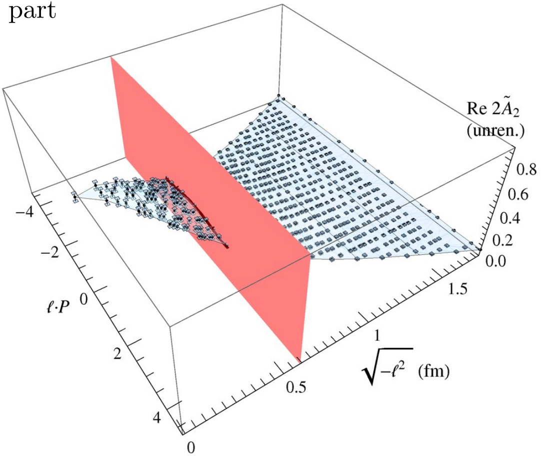
$$\rho_{UU}(\boldsymbol{k}_\perp) = \int_{-1}^{1} dx\ f_1(x, \boldsymbol{k}_\perp^2)$$



axially symmetric

($m_\pi \approx 500$ MeV, straight gauge link operator,
renormalization condition $C^{\text{ren}} = 0$, Gaussian fit)

real part



$\ell^2 \xleftrightarrow{\text{FT}} \boldsymbol{k}_\perp^2$

$\ell\cdot P \xleftrightarrow{\text{FT}} x$

factorization hypothesis

$$f_1(x, \boldsymbol{k}_\perp^2) \;\approx\; f_1(x) \;\; f_1^{(0_x)}(\boldsymbol{k}_\perp^2) \;/\; \mathcal{N}$$

as in phenomenological applications,
e.g., Monte Carlo event generators

Then $\tilde{A}_2$ factorizes, too:

$$\tilde{A}_2(\ell^2, \ell\cdot P) = \tilde{A}_2^{\mathrm{norm}}(\ell\cdot P)\; \tilde{A}_2(\ell^2, 0).$$

To test this, we define

$$\boxed{\tilde{A}_2^{\mathrm{norm}}(\ell^2, \ell\cdot P) \equiv \frac{\tilde{A}_2(\ell^2, \ell\cdot P)}{\mathrm{Re}\;\tilde{A}_2(\ell^2, 0)}}$$

(needs no renormalization!)

If factorization holds, $\tilde{A}_2^{\mathrm{norm}}$ should be ℓ^2-independent.

factorization hypothesis

$$f_1(x,\boldsymbol{k}_\perp^2) \;\approx\; f_1(x)\;\; f_1^{(0_x)}(\boldsymbol{k}_\perp^2)\;/\;\mathcal{N}$$

as in phenomenological applications,
e.g., Monte Carlo event generators

Then $\tilde{A}_2$ factorizes, too:

$$\tilde{A}_2(\ell^2,\ell\cdot P) = \tilde{A}_2^{\text{norm}}(\ell\cdot P)\;\tilde{A}_2(\ell^2,0).$$

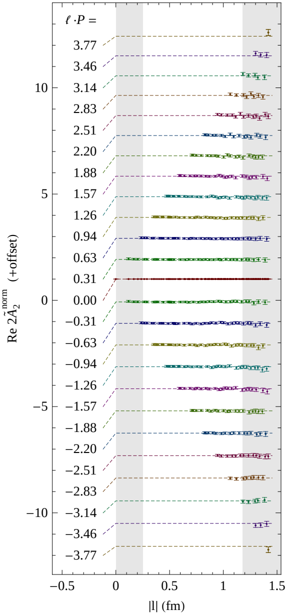
To test this, we define

$$\tilde{A}_2^{\text{norm}}(\ell^2,\ell\cdot P) \equiv \frac{\tilde{A}_2(\ell^2,\ell\cdot P)}{\text{Re}\;\tilde{A}_2(\ell^2,0)}$$

(needs no renormalization!)

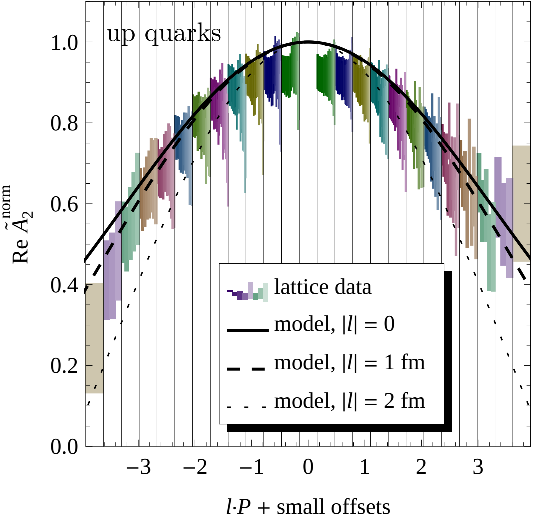
If factorization holds, $\tilde{A}_2^{\text{norm}}$ should be ℓ^2-independent.

✓ within statistics



All our data for $\tilde{A}_2^{\mathrm{norm}}(\ell^2, \ell \cdot P)$ at $m_\pi \approx 600$ MeV

qualitative comparison to a scalar diquark model
[Bacchetta, Conti, Radici PRD (2008)] at $\sqrt{-\ell^2} = 0$, 1 and 2 fm



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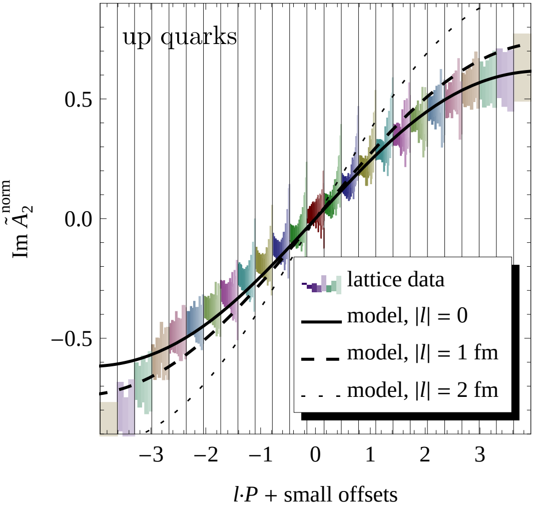


ratio of correlators far away from nucleon source and sink

$$\frac{C_{3\mathrm{pt}}(\tau;\Gamma,\ell,P)}{C_{2\mathrm{pt}}(P)} \xrightarrow{t_{\mathrm{src}}\ll\tau\ll t_{\mathrm{sink}}} \text{const. (“plateau value”)},$$

$$\Downarrow$$

access to $\langle P,S|\ \bar{q}(\ell)\,\Gamma\,\mathcal{U}\,q(0)\ |P,S\rangle$

Γ	$\frac{1}{2}C^{\mathrm{ren}}_{3\mathrm{pt}}(\tau;\Gamma,\boldsymbol{\ell},\boldsymbol{P})/C_{2\mathrm{pt}}(\boldsymbol{P})$ (LHPC projectors)
$\mathbb{1}$	$\frac{m_N}{E(P)}\,\tilde{A}_1$
$-\gamma_4\gamma_5$	$im_N\,\tilde{A}_7\,\ell_z$
γ_4	$\tilde{A}_2$
$\frac{1}{2}[\gamma_2,\gamma_4]$	$\frac{1}{E(P)}\,\tilde{A}_9\,P_x + \frac{im_N^2}{E(P)}\,\tilde{A}_{10}\,\ell_x + \frac{m_N^2}{E(P)}\,\tilde{A}_{11}\,(\ell_z)^2 P_x$
...	...

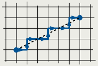
ratio of correlators far away from nucleon source and sink

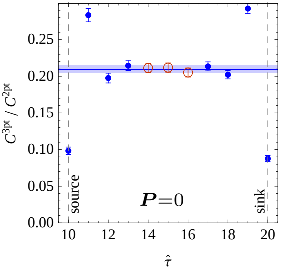
$$\frac{C_{3pt}(\tau;\Gamma,\ell,P)}{C_{2pt}(P)} \xrightarrow{t_{src} \ll \tau \ll t_{sink}} \text{const. ("plateau value")},$$

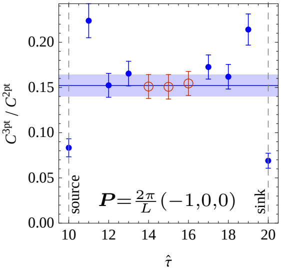
$$\Downarrow$$

access to $\langle P,S|\ \bar{q}(\ell)\,\Gamma\,\mathcal{U}\,q(0)\ |P,S\rangle$

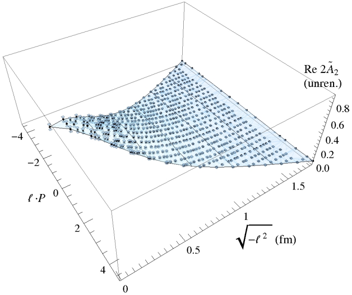
example plateau plots at $m_\pi \approx 600$ MeV

for $\Gamma = \gamma_4$ ($\Rightarrow \tilde{A}_2$), with HYP smeared gauge link $\mathcal{U} =$  :

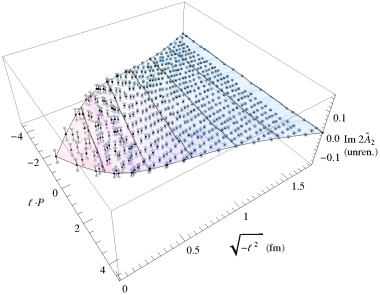




2 Re $\tilde{A}_2(\ell^2, \ell\cdot P)$

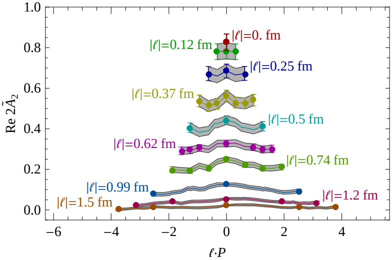


2 Im $\tilde{A}_2(\ell^2, \ell\cdot P)$

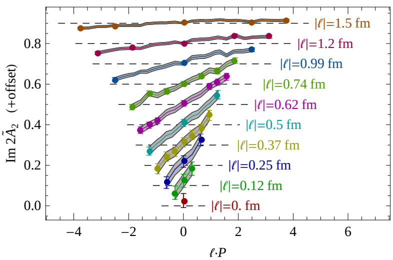


$\ell\cdot P$ - dependence of $\tilde{A}_2(\ell^2, \ell\cdot P)$

$2\ \mathrm{Re}\ \tilde{A}_2(\ell^2, \ell\cdot P)$

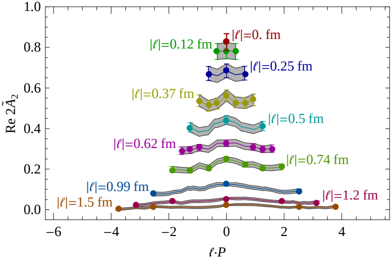


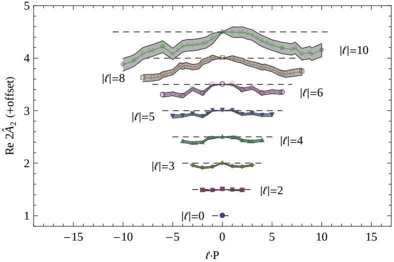
$2\ \mathrm{Im}\ \tilde{A}_2(\ell^2, \ell\cdot P)$



$$2 \text{ Re } \tilde{A}_2(\ell^2, \ell \cdot P)$$

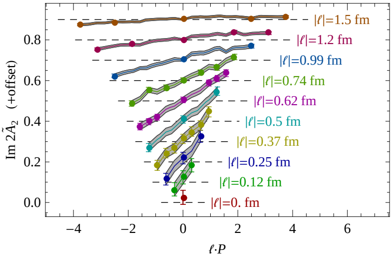
$$\text{Re } \tilde{A}_2^{\text{norm}} = \frac{\text{Re } \tilde{A}_2(\ell^2, \ell \cdot P)}{\text{Re } \tilde{A}_2(\ell^2, 0)}$$

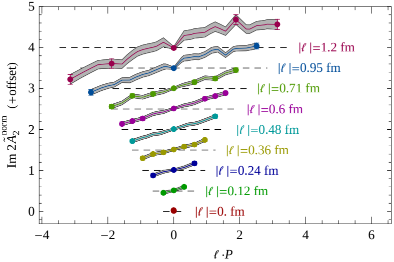




$$2 \ \mathrm{Im} \ \tilde{A}_2(\ell^2, \ell \cdot P)$$

$$\mathrm{Im} \ \tilde{A}_2^{\mathrm{norm}} = \frac{\mathrm{Im} \ \tilde{A}_2(\ell^2, \ell \cdot P)}{\mathrm{Re} \ \tilde{A}_2(\ell^2, 0)}$$





$$\left.\begin{array}{r}(v^0, v^1, v^2, v^3) \\ \text{future pointing } v \\ \text{TMD PDFs for SIDIS}\end{array}\right\} \xrightarrow{\mathcal{T}} \left\{\begin{array}{l}(-v^0, v^1, v^2, v^3) \\ \text{past pointing } v \\ \text{TMD PDFs for Drell-Yan}\end{array}\right.$$

The transformation property of the matrix elements under time reversal provides relations:

Example of a $\mathcal{T}$-even amplitude:

$$A_2\Big(k^2, k\cdot P, \tfrac{v\cdot k}{v\cdot P}, \zeta^{-1}, 1\Big) = A_2\Big(k^2, k\cdot P, \tfrac{v\cdot k}{v\cdot P}, \zeta^{-1}, -1\Big)$$

$$\Downarrow$$

$$f_1^{(\text{SIDIS})}(x, \boldsymbol{k}_\perp; \zeta, \ldots) = f_1^{(\text{Drell-Yan})}(x, \boldsymbol{k}_\perp; \zeta, \ldots)$$

Example of a $\mathcal{T}$-odd amplitude: ($\rightarrow$ Sivers function $f_{1T}^\perp$)

$$A_{12}\Big(k^2, k\cdot P, \tfrac{v\cdot k}{v\cdot P}, \zeta^{-1}, 1\Big) = -A_{12}\Big(k^2, k\cdot P, \tfrac{v\cdot k}{v\cdot P}, \zeta^{-1}, -1\Big)$$

$$\Downarrow$$

$$f_{1T}^{\perp(\text{SIDIS})}(x, \boldsymbol{k}_\perp; \zeta, \ldots) = -f_{1T}^{\perp(\text{Drell-Yan})}(x, \boldsymbol{k}_\perp; \zeta, \ldots)$$