N Structure in Exclusive Electroproduction at High Q^2*
University of South Carolina
August 13, 2012

Quark-Hadron Duality & Transition Form Factors

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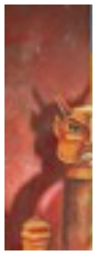
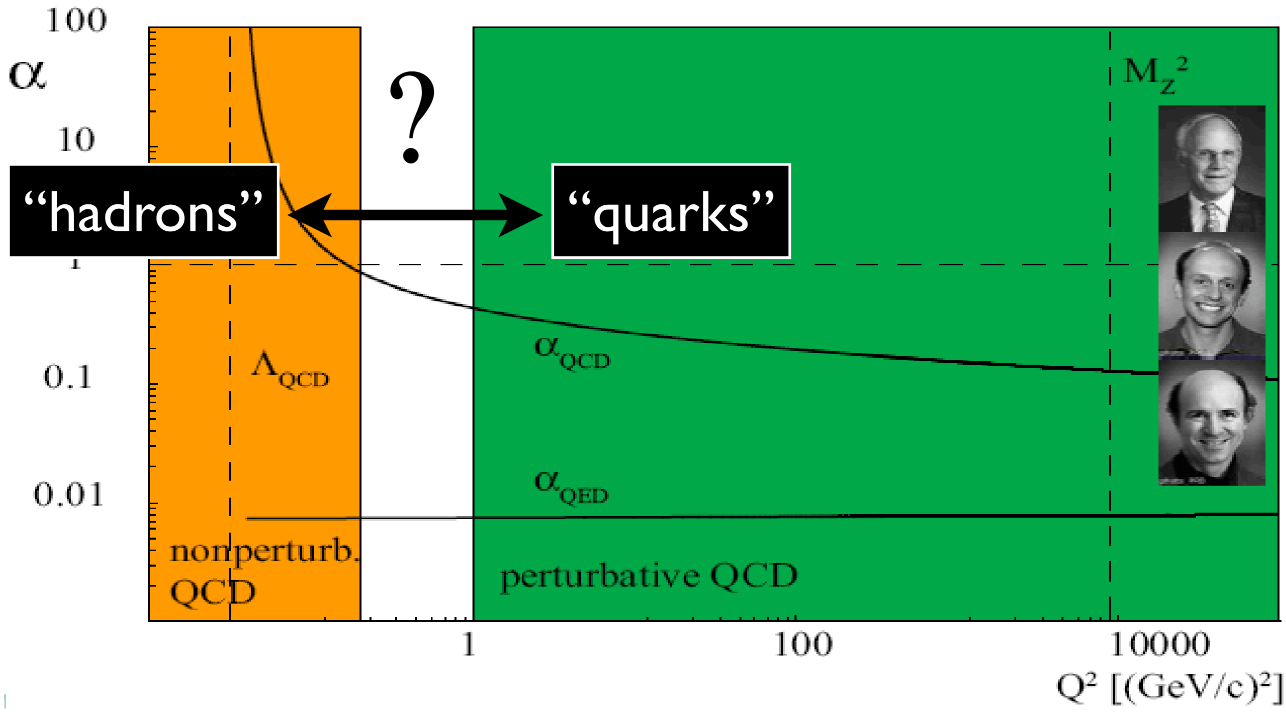
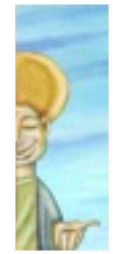
Jefferson Lab



low energy
long distance



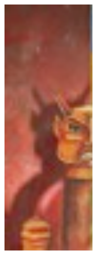


high energy
short distance

- Duality hypothesis: complementarity between *quark* and *hadron* descriptions of observables

$$\sum_{hadrons} = \sum_{quarks}$$

→ can use either set of *complete* basis states to describe physical phenomena



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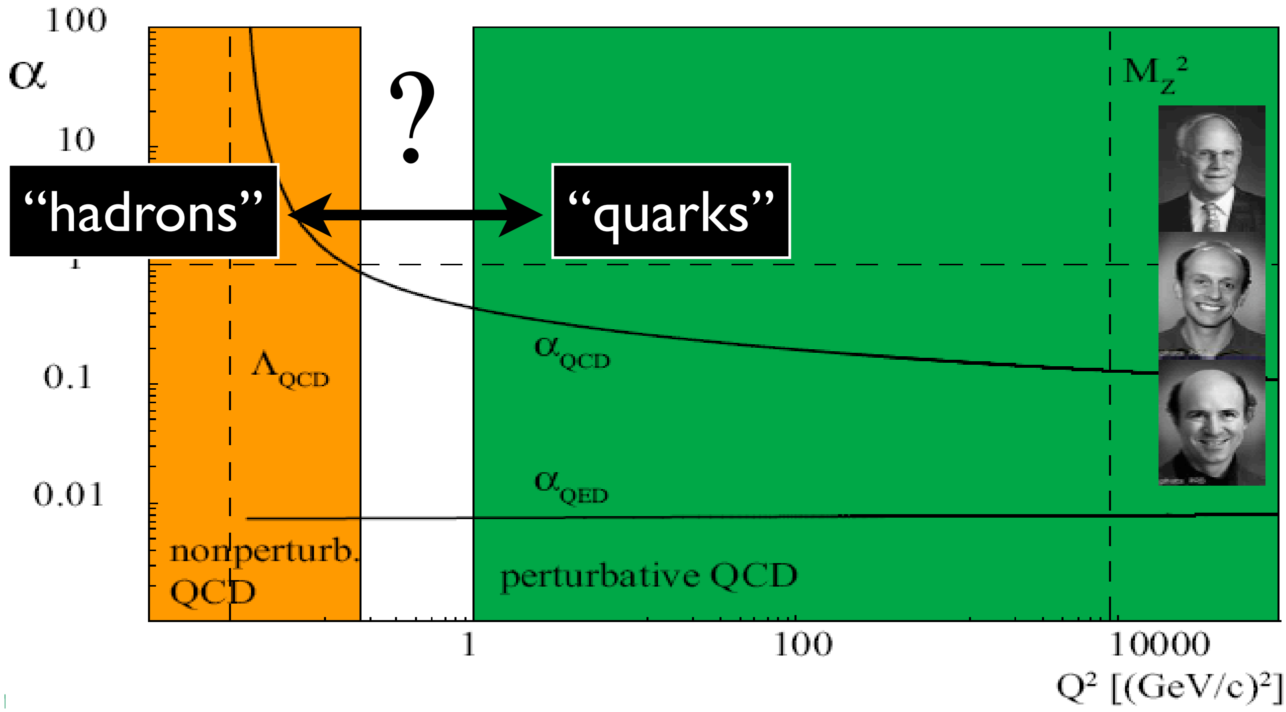
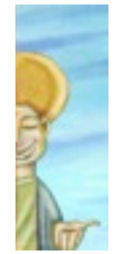
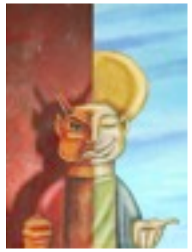




high energy
short distance

- In practice, at finite energy typically have access only to *limited* set of basis states



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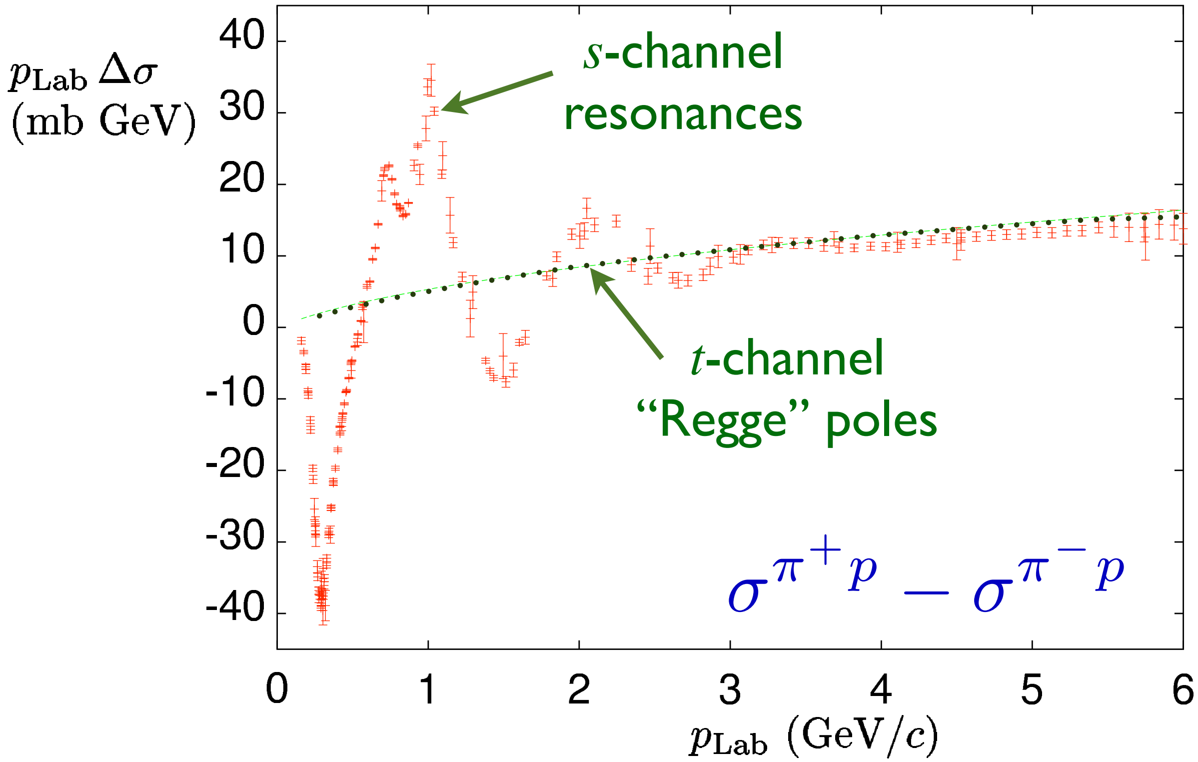
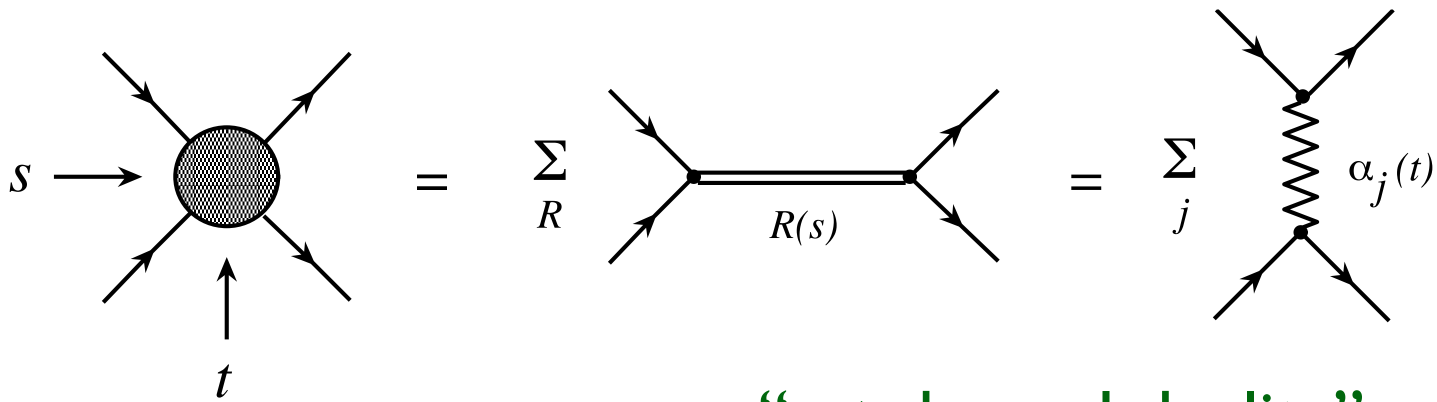
- Question is not *why* duality exists, but *how* it arises where it exists, and how we can *make use of it*

Duality in hadron-hadron scattering



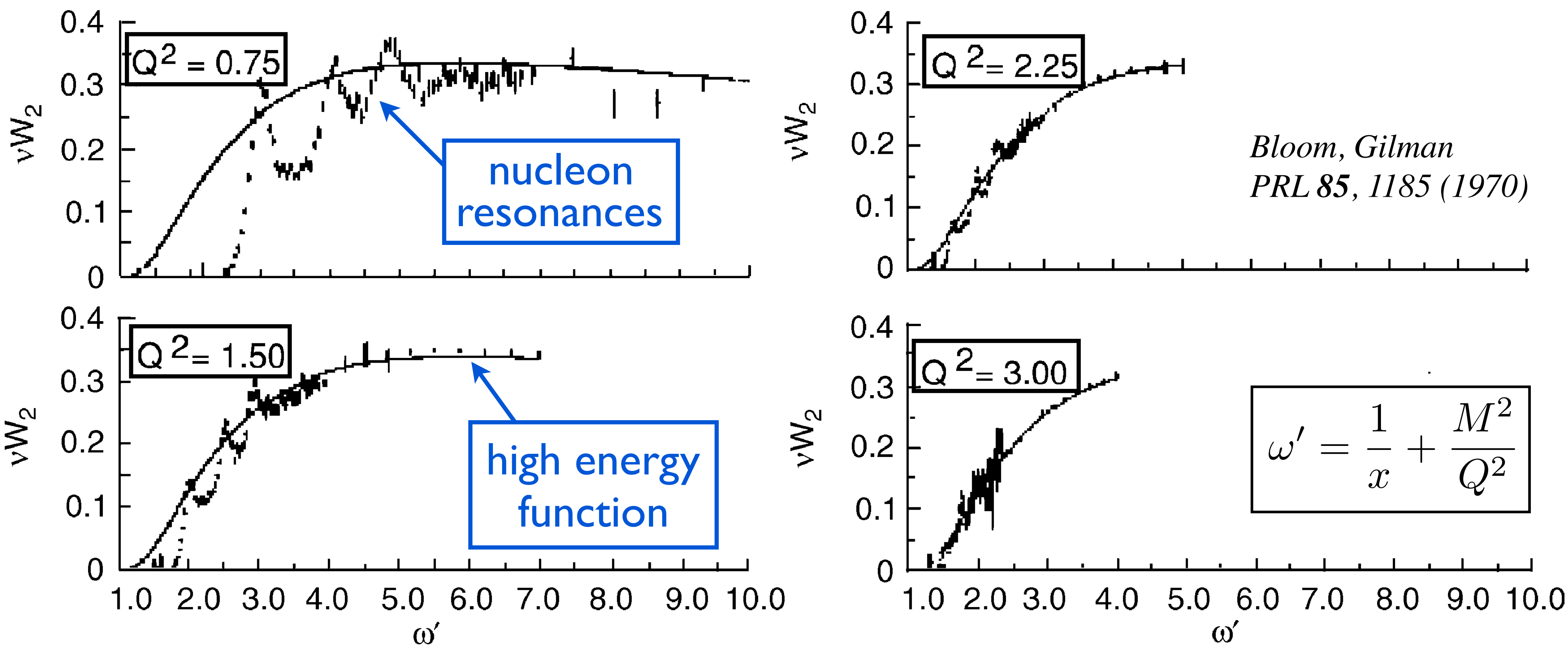
Igi (1962)
Dolen, Horn, Schmidt (1968)



"*s*-*t* channel duality"

Duality in electron-nucleon scattering

“Bloom-Gilman duality”

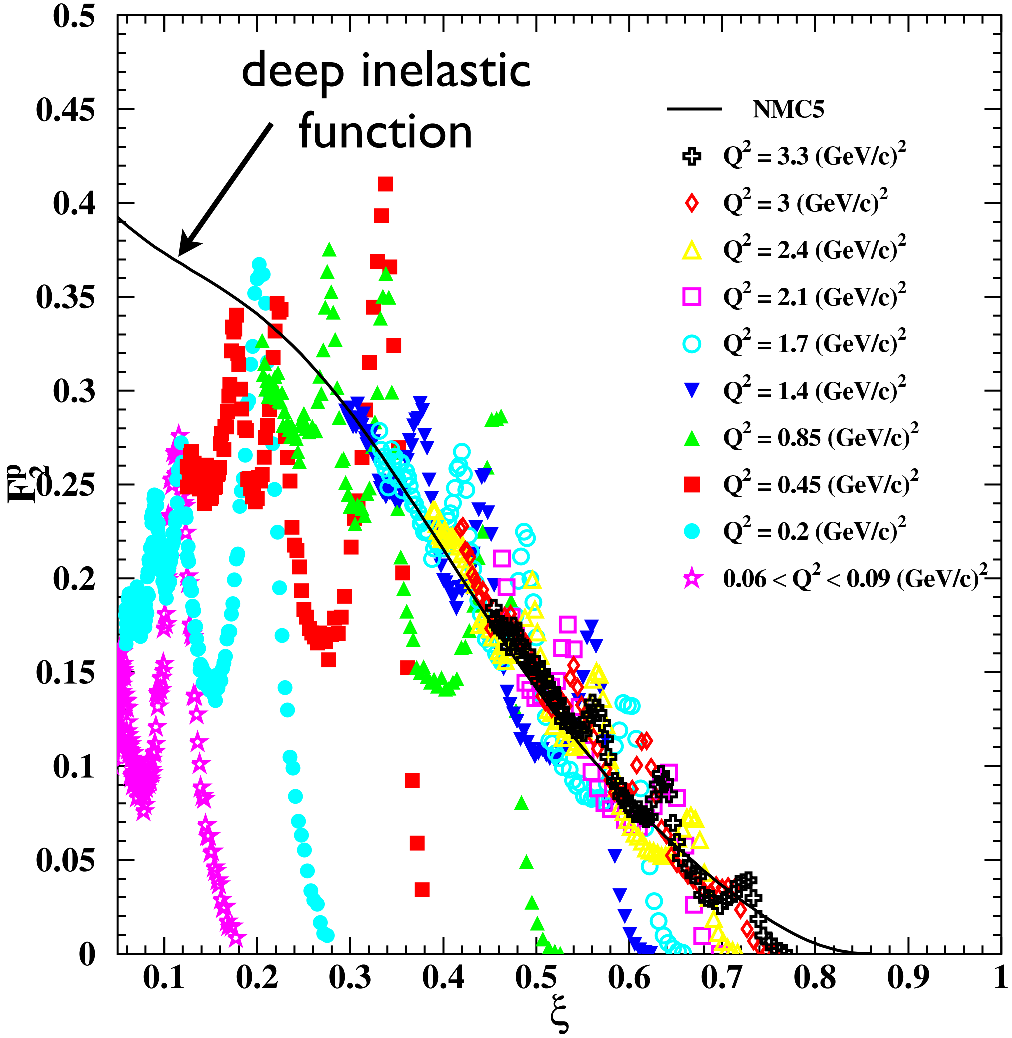


$$\frac{2M}{Q^2}\int_0^{\nu_m} d\nu\ \nu W_2(\nu, Q^2) = \int_1^{\omega'_m} d\omega'\ \nu W_2(\omega')$$

“hadrons” “quarks”

finite-energy sum rules

Duality in electron-nucleon scattering



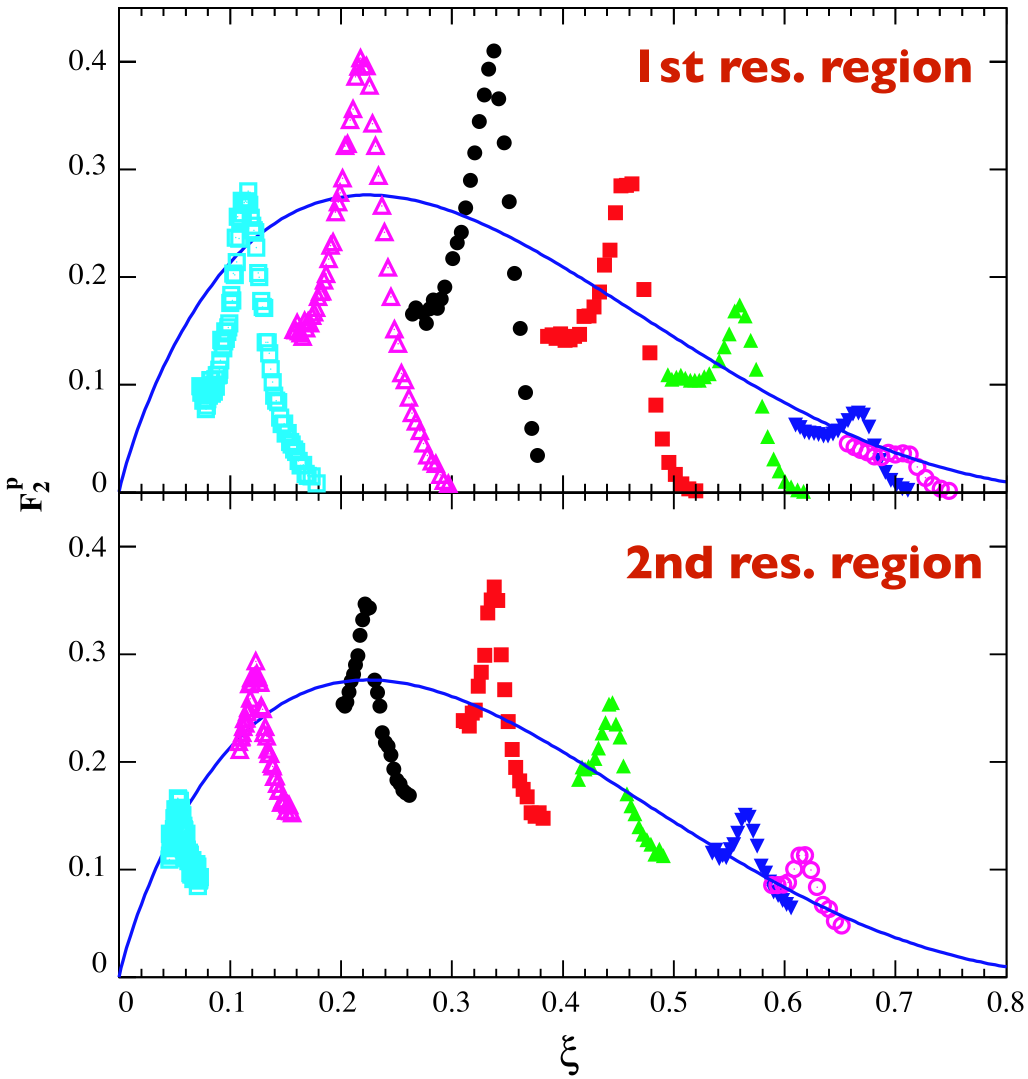
average over
(strongly Q^2 dependent)
resonances
$\approx Q^2$ independent
scaling function

"Nachtmann" scaling variable

$$\xi = \frac{2x}{1+\sqrt{1+4M^2x^2/Q^2}}$$

Niculescu et al., PRL ***85****, 1182 (2000)*
WM, Ent, Keppel, PRep. ***406****, 127 (2005)*

Duality in electron-nucleon scattering

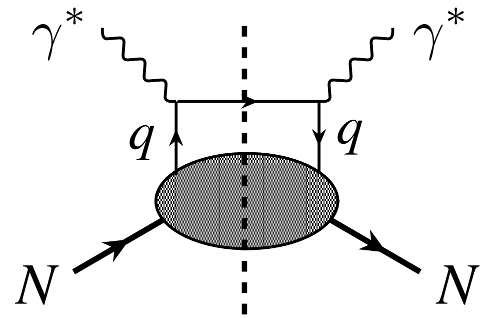


→ also exists *locally* in individual resonance regions

Duality in electron-nucleon scattering

- In *deep-inelastic* region $(W \gtrsim 2\ \mathrm{GeV},\ Q^2 \gtrsim 1\ \mathrm{GeV}^2)$ structure function given by parton distributions

$$F_2(x, Q^2) = x \sum_q e_q^2\ q(x, Q^2)$$



- In *resonance* region $(W \lesssim 2\ \mathrm{GeV})$, or at low Q^2 $(Q^2 \lesssim 1\ \mathrm{GeV}^2)$ can no longer resolve individual quark structure

- Resonance and DIS regions intimately connected

$\longrightarrow$ resonances an *integral* part of scaling structure function

e.g. in large-N_c limit, spectrum of zero-width resonances is "maximally dual" to quark-level (smooth) structure function

How to build up a scaling structure function from $\gamma^* N N^*$ transitions?

- Earliest attempts predate QCD

→ *e.g.* harmonic oscillator spectrum $M_n^2 = (n+1)\Lambda^2$
including states with spin = 1/2, ..., n+1/2
(n even: I = 1/2, n odd: I = 3/2)

*Domokos et al., PRD **3**, 1184 (1971)*

→ at large Q^2 magnetic coupling dominates

$$G_n(Q^2) = \frac{\mu_n}{(1 + Q^2 r^2 / M_n^2)^2} \qquad r^2 \approx 1.41$$

→ in Bjorken limit, $\sum_n \longrightarrow \int dz\,, \quad z \equiv M_n^2/Q^2$

$$F_2 \sim (\omega' - 1)^{1/2} (\mu_{1/2}^2 + \mu_{3/2}^2) \int_0^\infty dz\, \frac{z^{3/2}(1 + r^2/z)^{-4}}{z + 1 - \omega' + \Gamma_0^2 z^2}$$

→ scaling function of $\omega' = \omega + M^2/Q^2 \qquad (\omega = 1/x)$

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Domokos et al., PRD ***3****, 1184 (1971)*

→ in $\Gamma_n \to 0$ limit

$$F_2 \sim \left(\mu_{1/2}^2 + \mu_{3/2}^2\right) \frac{(\omega' - 1)^3}{(\omega' - 1 + r^2)^4}$$

cf. Drell-Yan-West relation

$$G(Q^2) \sim \left(\frac{1}{Q^2}\right)^m \quad \Longleftrightarrow \quad F_2(x) \sim (1-x)^{2m-1}$$

→ similar behavior found in many models

Einhorn, PRD ***14****, 3451 (1976)* ('t Hooft model)
Greenberg, PRD ***47****, 331 (1993)* (NR scalar quarks in HO potential)
Pace, Salme, Lev, PRC ***57****, 2655 (1995)* (relativistic HO with spin)
Isgur et al., PRD ***64****, 054005 (2001)* (transition to scaling)
....

How to build up a scaling structure function from $\gamma^* N N^*$ transitions?

- More recent phenomenological analyses at finite Q^2

→ additional constraints from threshold behavior at $\mathbf{q} \to 0$ and asymptotic behavior at $Q^2 \to \infty$

Davidovsky, Struminsky, Phys. Atom. Nucl. ***66****, 1328 (2003)*

$$\left(1 + \frac{\nu^2}{Q^2}\right) F_2^R = M\nu \ \left[|G_+^R|^2 + 2|G_0^R|^2 + |G_-^R|^2\right] \ \delta(W^2 - M_R^2)$$

→ 21 isospin-1/2 & 3/2 resonances (with mass < 2 GeV)

$$\left|G_\pm^R(Q^2)\right|^2 = \left|G_\pm^R(0)\right|^2 \left(\frac{|\vec{q}|}{|\vec{q}|_0} \frac{\Lambda'^2}{Q^2 + \Lambda'^2}\right)^{\gamma_1} \left(\frac{\Lambda^2}{Q^2 + \Lambda^2}\right)^{m_\pm}$$

$$m_{+,0,-} = 3, 4, 5$$

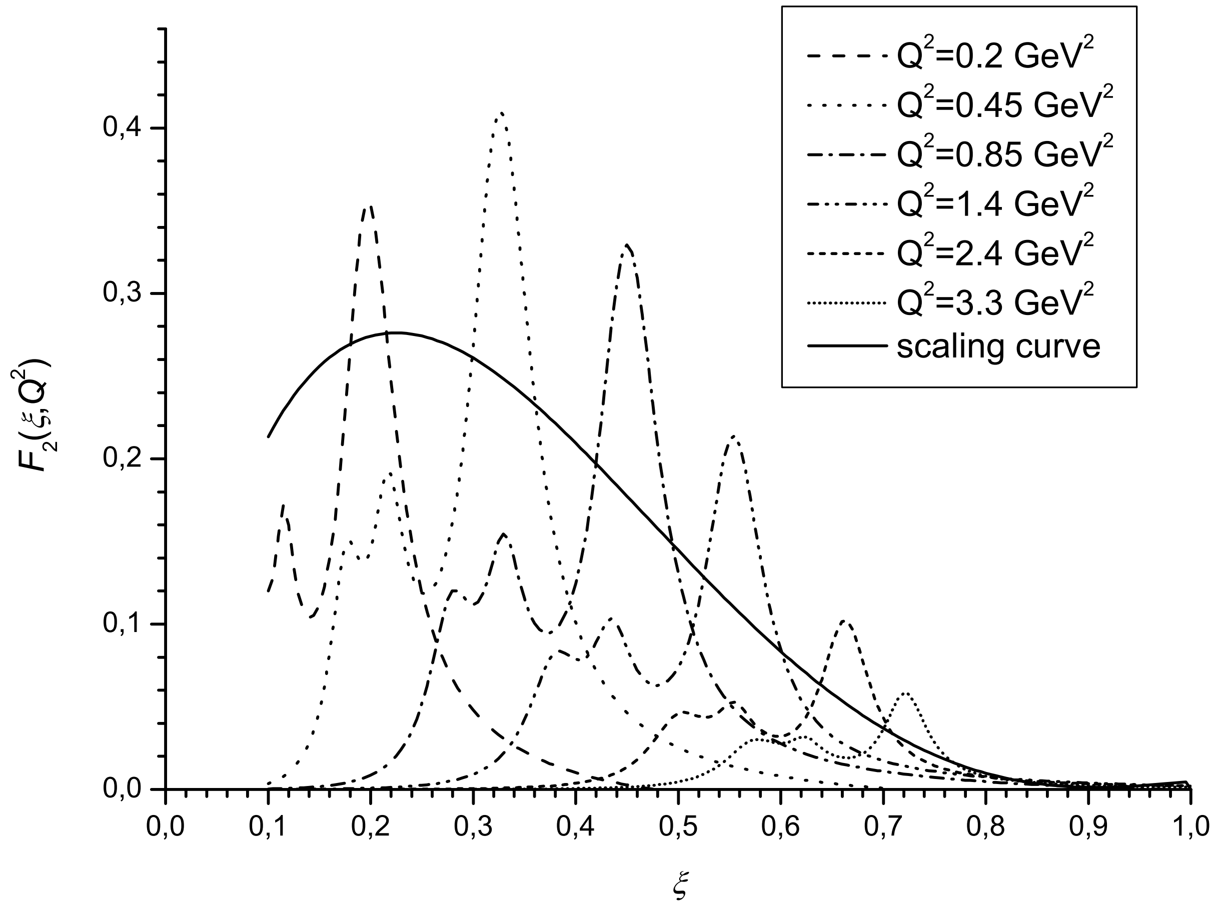
$$\left|G_0^R(Q^2)\right|^2 = C^2 \left(\frac{Q^2}{Q^2 + \Lambda''^2}\right)^{2a} \frac{q_0^2}{|\vec{q}|^2} \left(\frac{|\vec{q}|}{|\vec{q}|_0} \frac{\Lambda'^2}{Q^2 + \Lambda'^2}\right)^{\gamma_2} \left(\frac{\Lambda^2}{Q^2 + \Lambda^2}\right)^{m_0}$$

→ in $x \to 1$ limit,

$$F_2(x) \sim (1 - x)^{m_+}$$

How to build up a scaling structure function from $\gamma^* N N^*$ transitions?

- More recent phenomenological analyses at finite Q^2



Davidovsky, Struminsky,
Phys. Atom. Nucl. ***66****, 1328 (2003)*

→ duality visible for low-W resonances;
at higher W need *nonresonant background*

Duality and QCD

- Operator product expansion

→ expand *moments* of structure functions in powers of $1/Q^2$

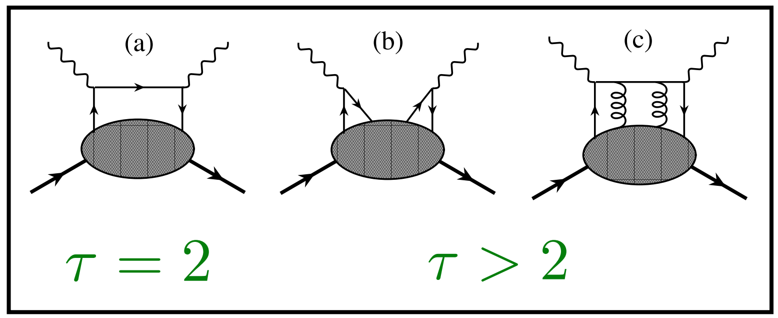
$x \to 1 \iff W \to M$

$$M_n(Q^2) = \int_0^1 dx\ x^{n-2}\ F_2(x, Q^2)$$

$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

matrix elements of operators with specific "twist" τ

$$\tau = \text{dimension} - \text{spin}$$



Duality and QCD

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$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

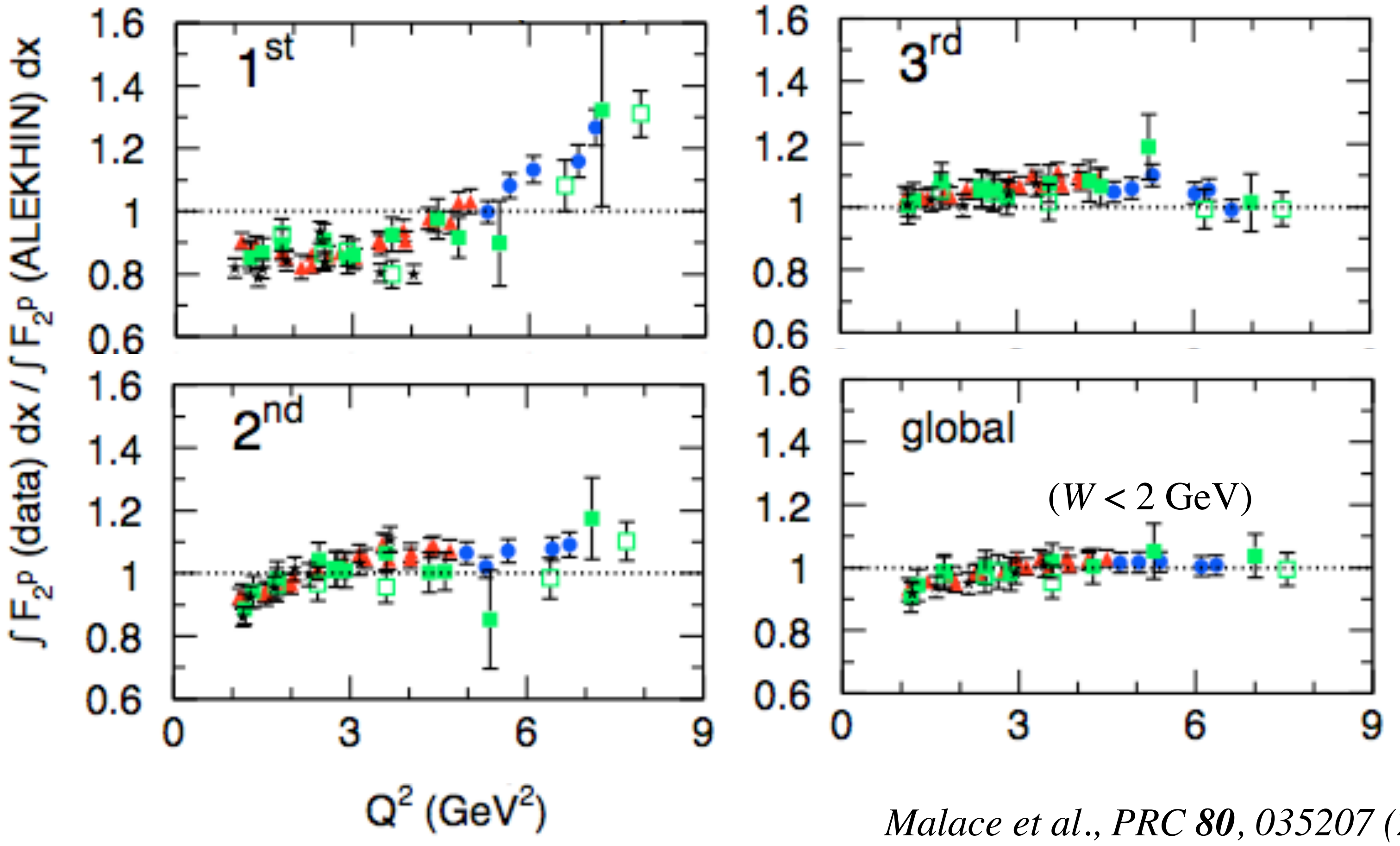
de Rujula, Georgi, Politzer
Ann. Phys. ***103****, 315 (1975)*

- If moment $\approx$ independent of Q^2

  → "higher twist" terms $A_n^{(\tau>2)}$ small

- Duality ⟷ suppression of higher twists

Analysis of (latest) JLab F_2^p resonance region data



*Malace et al., PRC **80**, 035207 (2009)*

→ higher twists $< 10\text{–}15\%$ for $Q^2 > 1\ \mathrm{GeV}^2$

Resonances & twists

- Total "higher twist" is *small* at scales $Q^2 \sim \mathcal{O}(1\ \mathrm{GeV}^2)$
- On average, nonperturbative interactions between quarks and gluons not dominant (at these scales)
  - $\longrightarrow$ nontrivial interference between resonances

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- Can we understand this dynamically, at quark level?
  - $\longrightarrow$ is duality an accident?
- Can we use resonance region data to learn about *leading twist* structure functions (and *vice versa*)?
  - $\longrightarrow$ expanded data set has potentially significant implications for global quark distribution studies

- Consider simple quark model with spin-flavor symmetric wave function

<u>low energy</u>

→ *coherent* scattering from quarks $d\sigma \sim \left(\sum_i e_i \right)^2$

<u>high energy</u>

→ *incoherent* scattering from quarks $d\sigma \sim \sum_i e_i^2$

- For duality to work, these must be equal

→ how can <u>*square of a sum*</u> become <u>*sum of squares*</u>?

Dynamical cancellations

→ *e.g.* for toy model of two quarks bound in a harmonic oscillator potential, structure function given by

$$F(\nu, \mathbf{q}^2) \sim \sum_n \left| G_{0,n}(\mathbf{q}^2) \right|^2 \delta(E_n - E_0 - \nu)$$

→ charge operator $\Sigma_i \; e_i \exp(i\mathbf{q} \cdot \mathbf{r}_i)$ excites
even partial waves with strength $\propto (e_1 + e_2)^2$
odd partial waves with strength $\propto (e_1 - e_2)^2$

→ resulting structure function

$$F(\nu, \mathbf{q}^2) \sim \sum_n \left\{ (e_1 + e_2)^2 \; G_{0,2n}^2 + (e_1 - e_2)^2 \; G_{0,2n+1}^2 \right\}$$

→ if states degenerate, *cross terms* $(\sim e_1 e_2)$ *cancel* when averaged over nearby *even and odd parity* states

Close, Isgur, PLB ***509****, 81 (2001)*

Dynamical cancellations

→ duality is realized by summing over at least one complete set of *even* and *odd* parity resonances

*Close, Isgur, PLB **509**, 81 (2001)*

→ in NR Quark Model, even & odd parity states generalize to **56** (L=0) and **70** (L=1) multiplets of spin-flavor SU(6)

representation	$^2\mathbf{8}[\mathbf{56}^+]$	$^4\mathbf{10}[\mathbf{56}^+]$	$^2\mathbf{8}[\mathbf{70}^-]$	$^4\mathbf{8}[\mathbf{70}^-]$	$^2\mathbf{10}[\mathbf{70}^-]$	Total
F_1^p	$9\rho^2$	$8\lambda^2$	$9\rho^2$	0	λ^2	$18\rho^2+9\lambda^2$
F_1^n	$(3\rho+\lambda)^2/4$	$8\lambda^2$	$(3\rho-\lambda)^2/4$	$4\lambda^2$	λ^2	$(9\rho^2+27\lambda^2)/2$

λ (ρ) = (anti) symmetric component of ground state wave function

*Close, WM, PRC **68**, 035210 (2003)*

Dynamical cancellations

→ in SU(6) limit $\lambda = \rho$, with relative strengths of $N \rightarrow N^*$ transitions

$SU(6):$	$[\mathbf{56},\mathbf{0}^+]^\mathbf{2}\mathbf{8}$	$[\mathbf{56},\mathbf{0}^+]^\mathbf{4}\mathbf{10}$	$[\mathbf{70},\mathbf{1}^-]^\mathbf{2}\mathbf{8}$	$[\mathbf{70},\mathbf{1}^-]^\mathbf{4}\mathbf{8}$	$[\mathbf{70},\mathbf{1}^-]^\mathbf{2}\mathbf{10}$	$total$
F_1^p	9	8	9	0	1	27
F_1^n	4	8	1	4	1	18

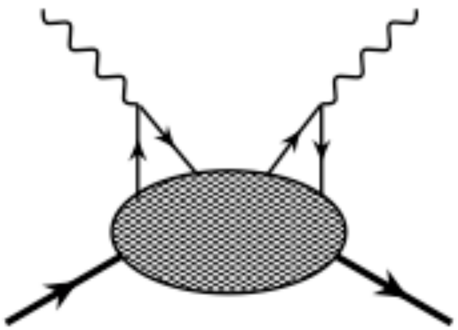
→ summing over all resonances in $\mathbf{56}^+$ and $\mathbf{70}^-$ multiplets

$$\frac{F_1^n}{F_1^p} = \frac{18}{27} = \frac{2}{3}$$

→ at the quark level, n/p ratio is

$$\frac{F_1^n}{F_1^p} = \frac{4d+u}{d+4u} = \frac{6}{9} = \frac{2}{3} \quad ! \qquad \text{if } u = 2d$$

Accidental cancellations of charges?



<u>cat's ears diagram</u> (4-fermion higher twist $\sim 1/Q^2$)

$$\propto \sum_{i \neq j} e_i \, e_j \sim \Big(\sum_i e_i\Big)^2 - \sum_i e_i^2$$

coherent ($\left(\sum_i e_i\right)^2$) incoherent ($\sum_i e_i^2$)

proton $\quad \mathrm{HT} \sim 1 - \left(2 \times \frac{4}{9} + \frac{1}{9}\right) = 0\ !$

neutron $\quad \mathrm{HT} \sim 0 - \left(\frac{4}{9} + 2 \times \frac{1}{9}\right) \neq 0$

Brodsky hep-ph/0006310

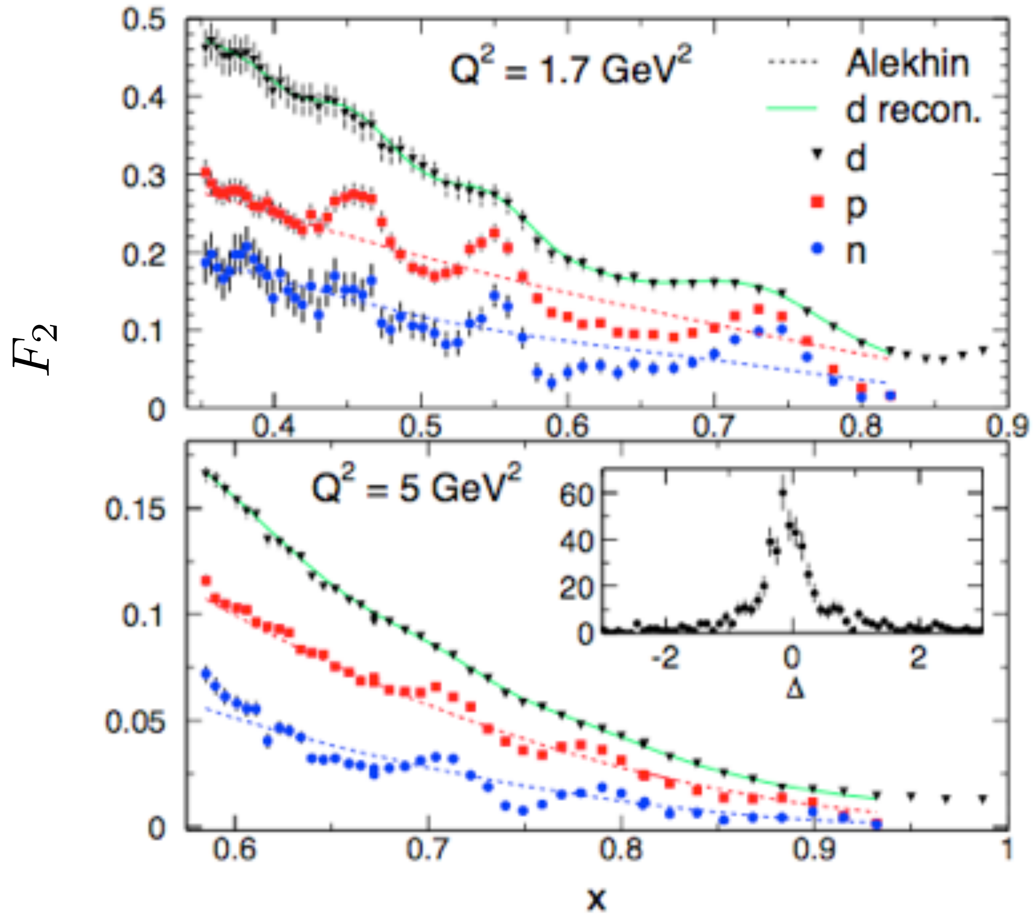
→ duality in proton a *coincidence!*

→ should *not* hold for neutron

Neutron: the smoking gun

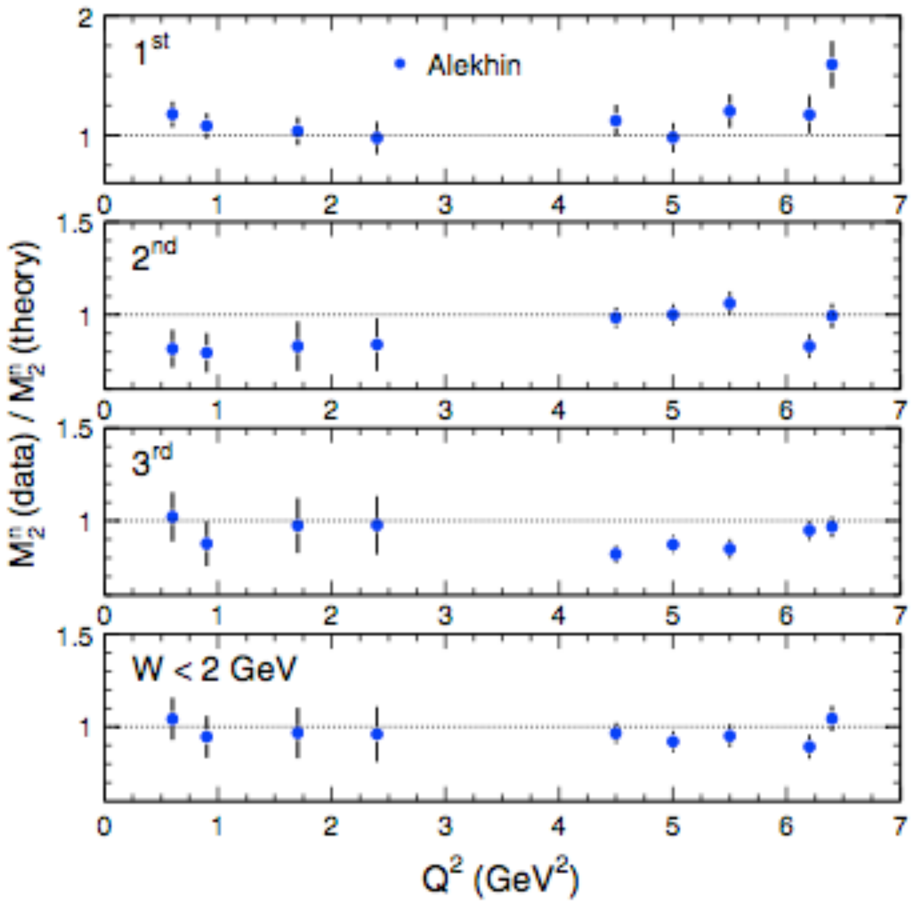
- Duality in *neutron* more difficult to test because of absence of free neutron targets

- New extraction method (using iterative procedure for solving integral convolution equations) has allowed first determination of F_2^n in resonance region & test of neutron duality



Malace, Kahn, WM, Keppel
*PRL **104**, 102001 (2010)*

Neutron: the smoking gun



→ "theory": fit to $W > 2$ GeV data

Alekhin et al., 0908.2762 [hep-ph]

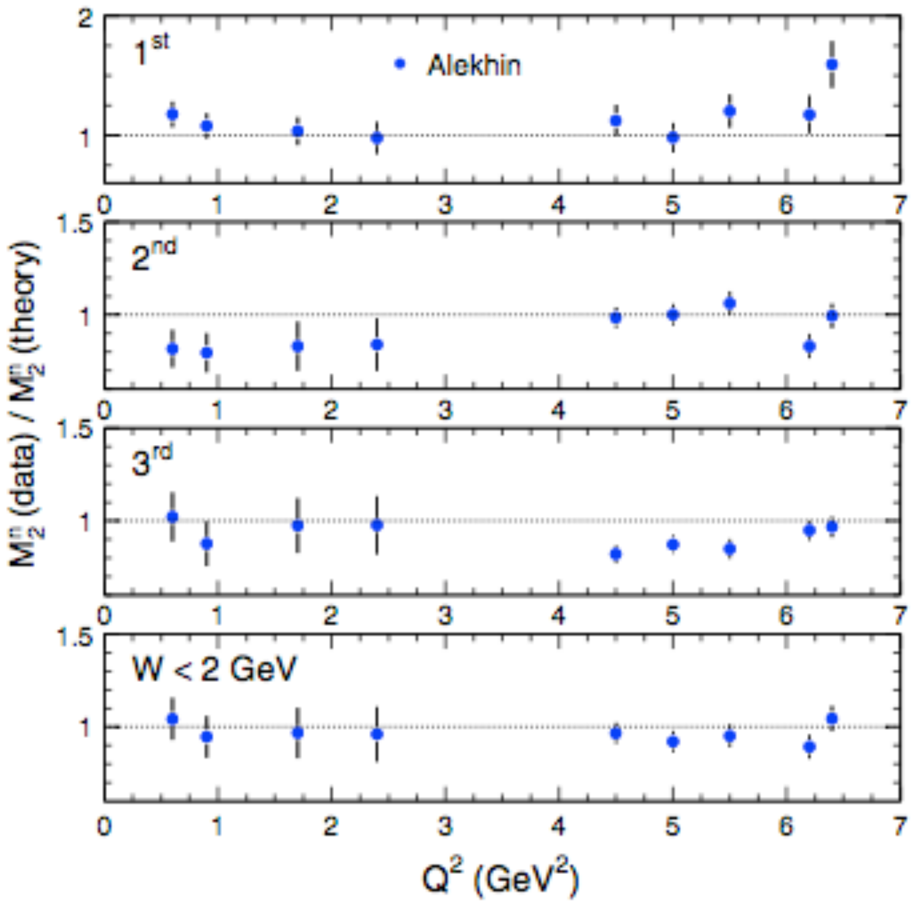
→ *locally*, violations of duality in resonance regions $< 15-20\%$ (largest in Δ region)

→ *globally*, violations $< 10\%$

Malace, Kahn, WM, Keppel
PRL ***104****, 102001 (2010)*

⇨ duality is *not accidental*, but a general feature of resonance−scaling transition!

Neutron: the smoking gun



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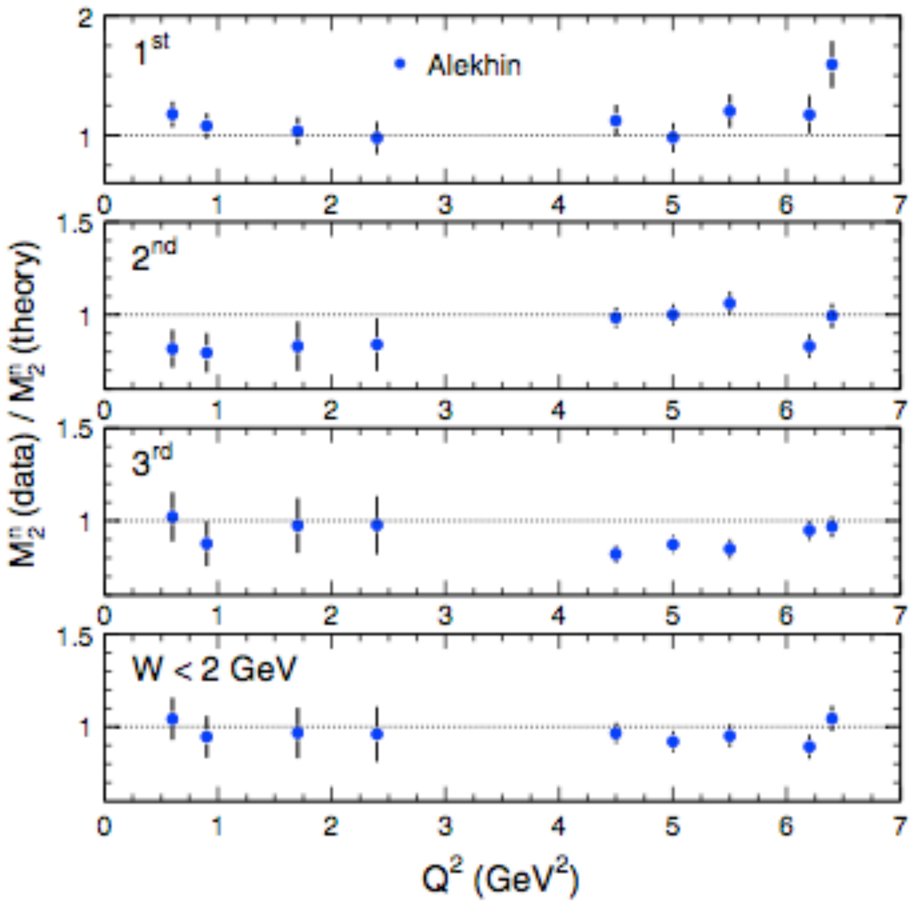
⇒ analysis using recent (model-independent) BoNuS data in progress

Neutron: the smoking gun



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PRL ***104****, 102001 (2010)*

⇒ use resonance region data to learn about *leading twist* structure functions?

CTEQ-JLab (CJ) global PDF analysis *

- New global NLO analysis of expanded set of p and d data (DIS, pp, pd) including large-x, low-Q^2 region

- Systematically study effects of Q^2 & W cuts

  → down to $Q \sim m_c$ and $W \sim 1.7$ GeV

cut0: $Q^2 > 4\ \mathrm{GeV}^2,\quad W^2 > 12.25\ \mathrm{GeV}^2$

cut1: $Q^2 > 3\ \mathrm{GeV}^2,\quad W^2 > 8\ \mathrm{GeV}^2$

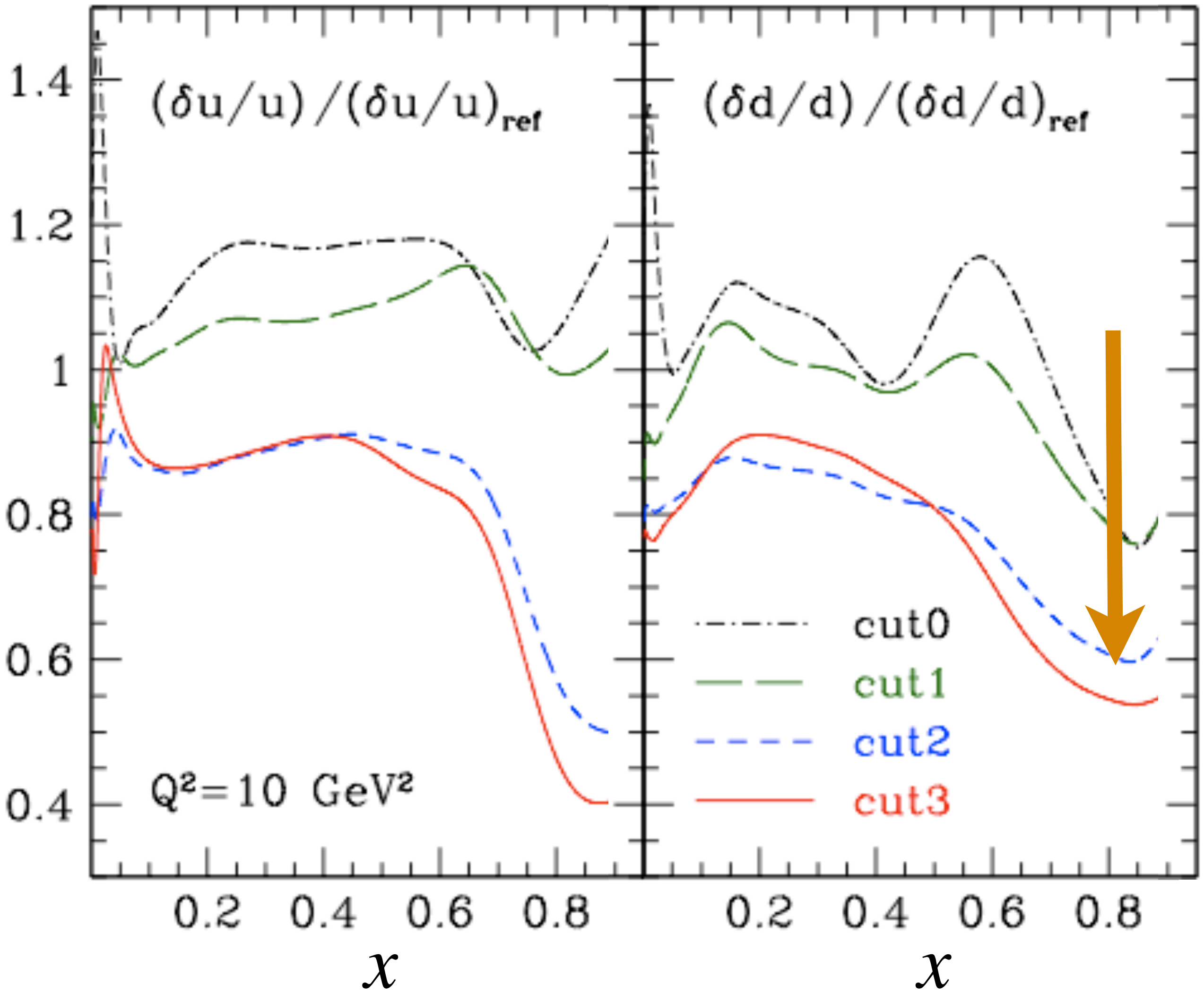
cut2: $Q^2 > 2\ \mathrm{GeV}^2,\quad W^2 > 4\ \mathrm{GeV}^2$

cut3: $Q^2 > m_c^2\ ,\quad W^2 > 3\ \mathrm{GeV}^2$

factor 2 increase in DIS data from cut0 → cut3

* CJ collaboration: http://www.jlab.org/CJ

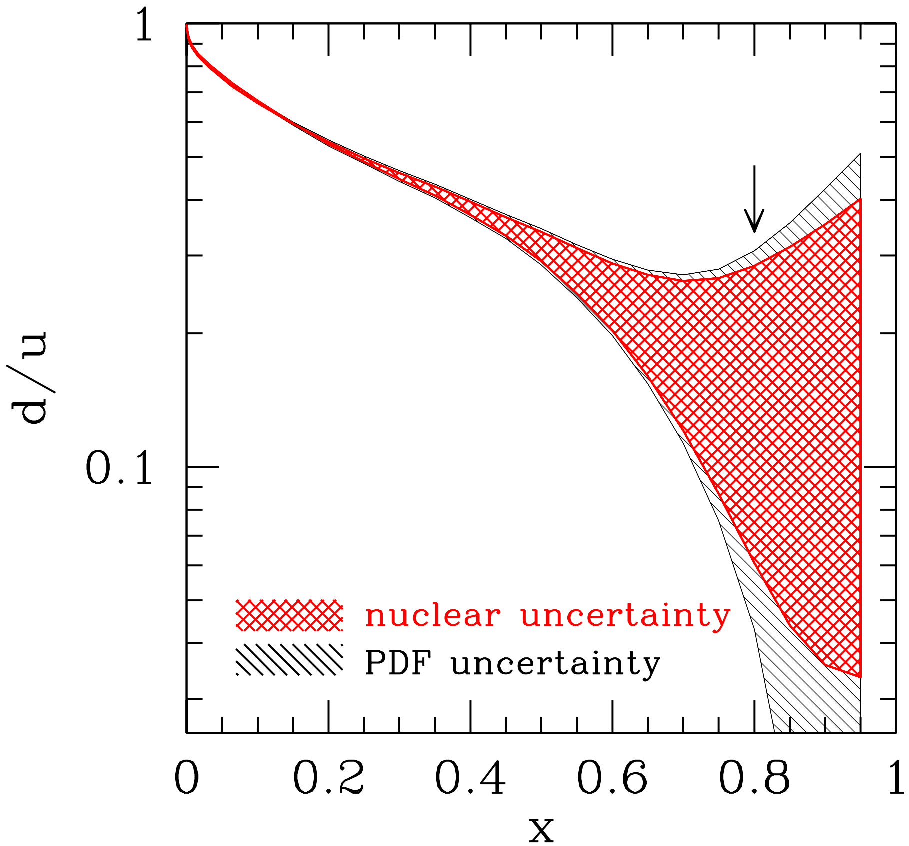
- Larger database with weaker cuts leads to significantly reduced errors, especially at large x

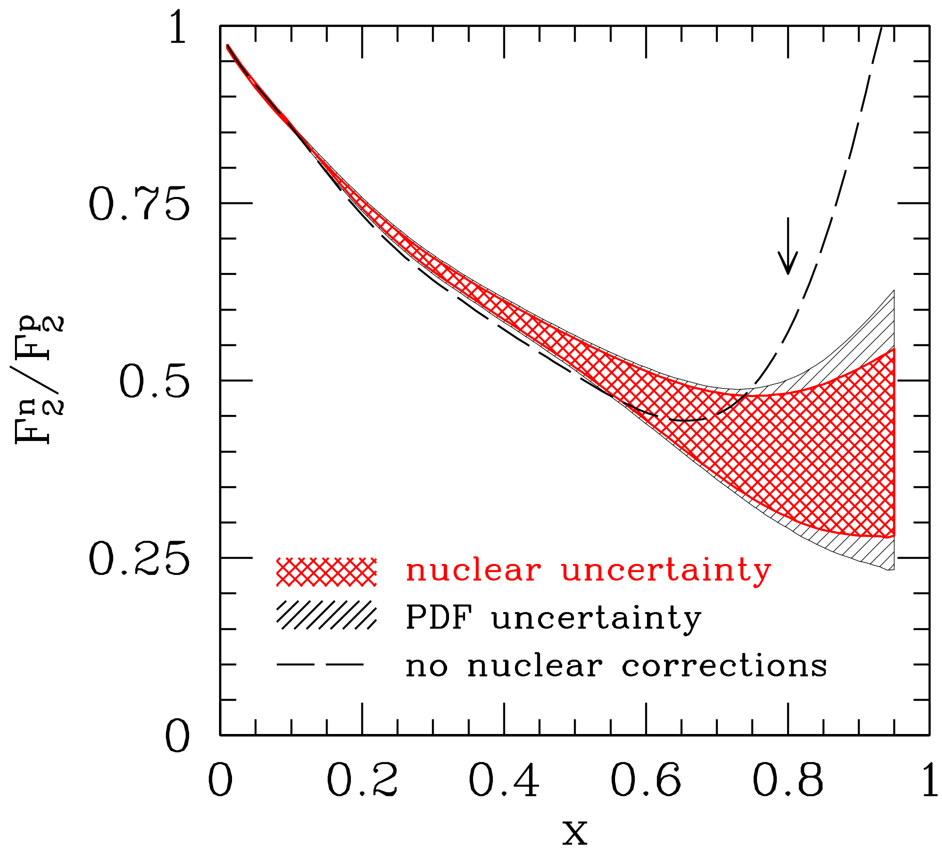


Accardi et al.
PRD ***81****, 034016 (2010)*

→ up to 40–60% error reduction when cuts extended into resonance region

- Vital for large-x analysis, which currently suffers from large uncertainties (mostly due to nuclear corrections)



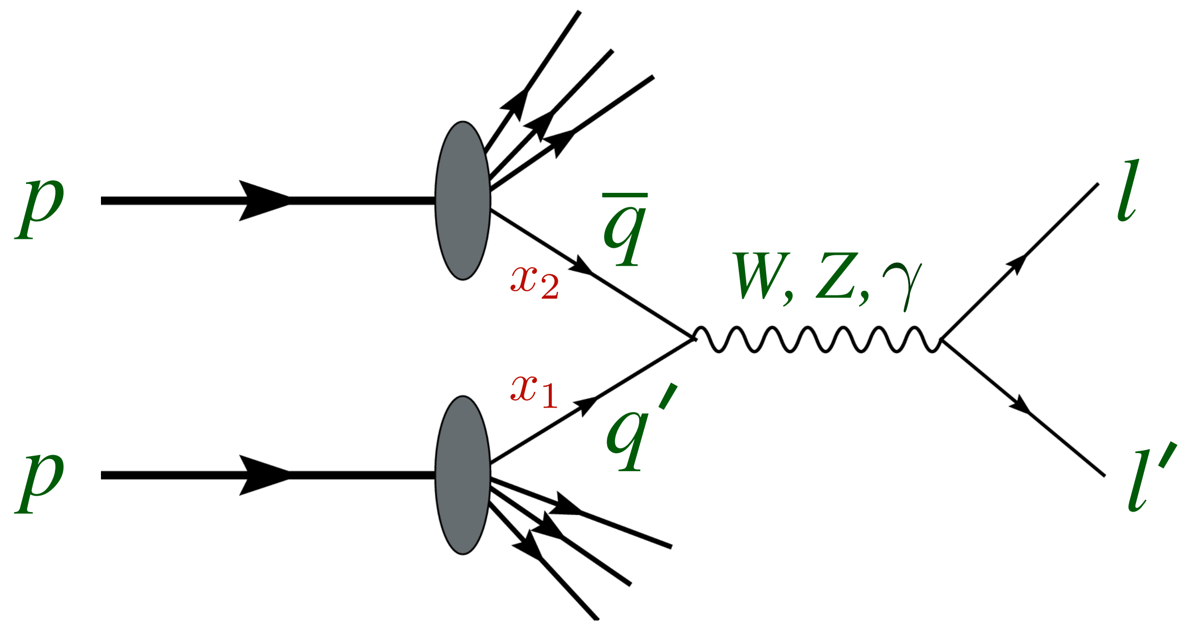


Accardi et al., PRD ***84****, 014008 (2011)*

→ uncertainty in d feeds into larger uncertainty in g at high x (important for LHC physics!)

Large Hadron Collider (CERN)

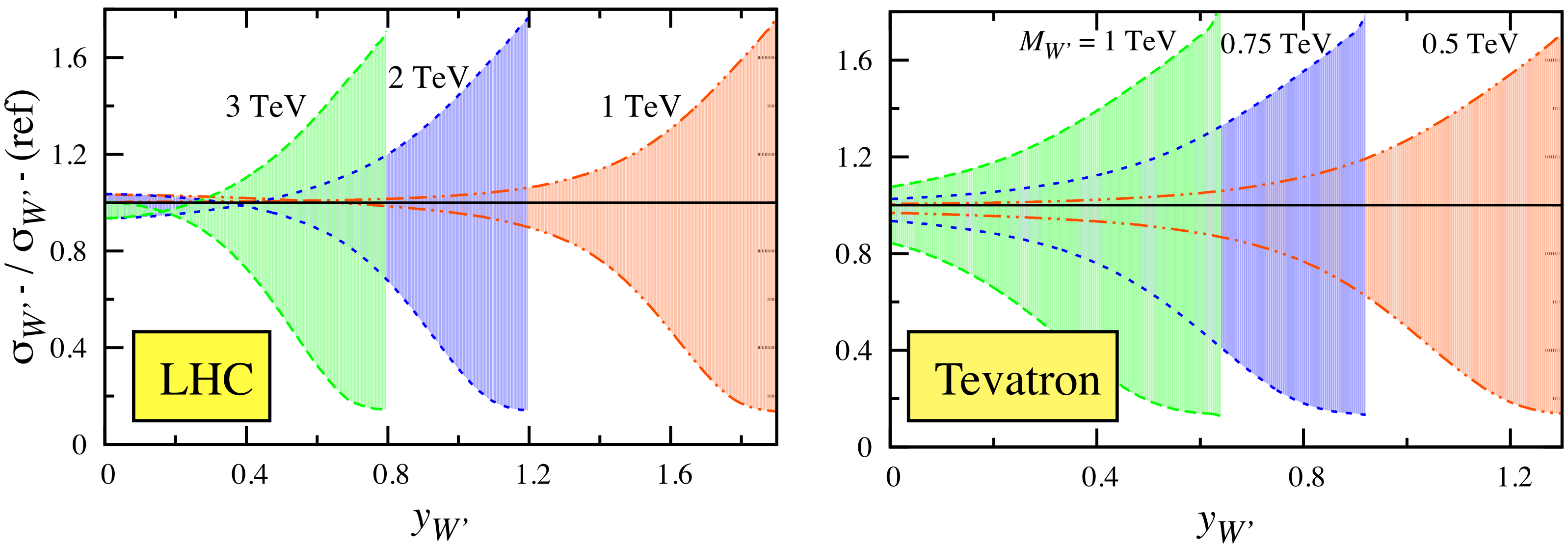
pp collisions
at $\sqrt{s} = 7$ TeV



Heavy Z', W' boson production

- Observation of new physics signals requires accurate determination of QCD backgrounds — depend on PDFs!
  (since $x_{1,2} \sim M_{Z',W'}$, large-x uncertainties scale with mass!)

  - for W'^- production



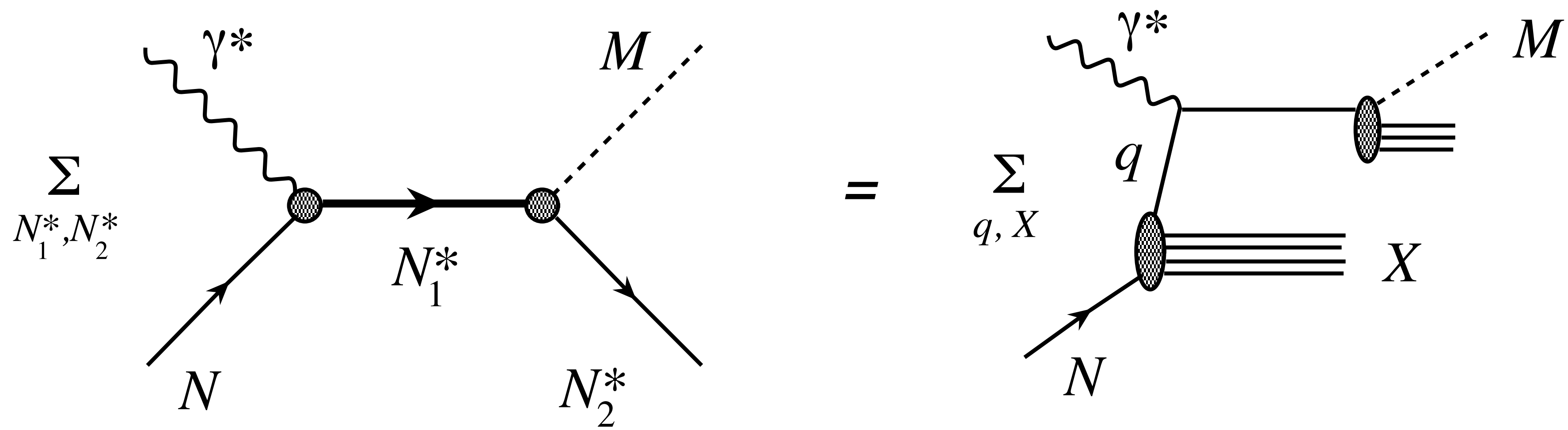
$\longrightarrow$ dominated by $d * \bar{u}$ $\longrightarrow$ dominated by $d * u + u * d$

$> 100\%$ uncertainties at large y !

Brady et al., JHEP ***1206****, 019 (2012)*

Duality in (semi-inclusive) meson production

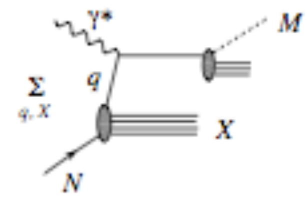
- Can duality be extended to less inclusive processes, such as meson production?



s-channel resonance excitation and decay

parton level scattering and fragmentation

Partonic description

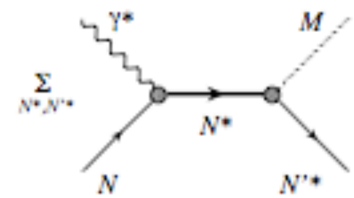


$$\mathcal{N}_N^\pi(x,z) = e_u^2\, u^N(x)\, D_u^\pi(z) + e_d^2\, d^N(x)\, D_d^\pi(z)$$

$q \longrightarrow \pi$ fragmentation function

$z = E_\pi/\nu$ fractional energy carried by pion

Hadronic description

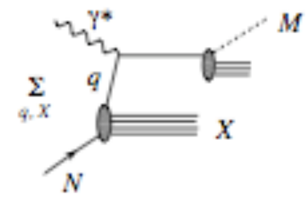


$$\mathcal{N}_N^\pi(x,z) = \sum_{N_2^*} \left| \sum_{N_1^*} F_{\gamma N\to N_1^*}(Q^2, M_1^*)\; \mathcal{D}_{N_1^*\to N_2^*\pi}(M_1^*, M_2^*) \right|^2$$

transition form factor

decay function

Partonic description

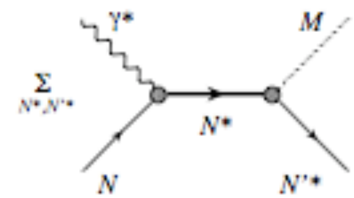


$$\mathcal{N}_N^\pi(x,z) = e_u^2\, u^N(x)\, D_u^\pi(z) + e_d^2\, d^N(x)\, D_d^\pi(z)$$

→ ratios given by quark charges

$$\frac{\mathcal{N}_n^{\pi^+}}{\mathcal{N}_p^{\pi^-}} = \frac{\mathcal{N}_p^{\pi^+}}{\mathcal{N}_n^{\pi^-}} = \frac{e_u^2}{e_d^2} = 4$$

Hadronic description

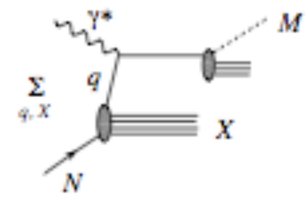


$$\mathcal{N}_N^\pi(x,z) = \sum_{N_2^*} \left| \sum_{N_1^*} F_{\gamma N \to N_1^*}(Q^2, M_1^*)\ \mathcal{D}_{N_1^* \to N_2^* \pi}(M_1^*, M_2^*) \right|^2$$

transition form factor

decay function

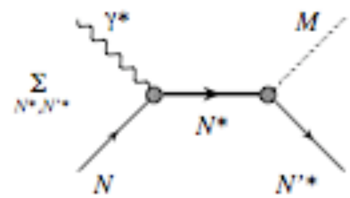
Partonic description



$$\mathcal{N}_N^{\pi}(x,z) = e_u^2\, u^N(x)\, D_u^{\pi}(z) + e_d^2\, d^N(x)\, D_d^{\pi}(z)$$

→ ratios given by quark charges

$$\frac{\mathcal{N}_n^{\pi^+}}{\mathcal{N}_p^{\pi^-}} = \frac{\mathcal{N}_p^{\pi^+}}{\mathcal{N}_n^{\pi^-}} = \frac{e_u^2}{e_d^2} = 4$$



Hadronic description

→ magnetic interaction operator for $\gamma N \to N_1^*$

$$\sum_i e_i\, \sigma_i^+$$

→ pion emission operator for $N_1^* \to N_2^*\, \pi^{\pm}$

$$\sum_i \tau_i^{\mp}\, \sigma_{zi}$$

- Relative probabilities $\mathcal{N}_N^{\pi}$ in $\mathrm{SU(6)}$ symmetric quark model (summed over N_1^*)

N_2^*	$^2 8, 56^+$	$^4 10, 56^+$	$^2 8, 70^-$	$^4 8, 70^-$	$^2 10, 70^-$	sum
$\gamma p \to \pi^+ N_2^*$	100 (100)	32 (–16)	64 (64)	16 (–8)	4 (4)	216 (144)
$\gamma p \to \pi^- N_2^*$	0 (0)	24 (–12)	0 (0)	0 (0)	3 (3)	27 (–9)
$\gamma n \to \pi^+ N_2^*$	0 (0)	96 (–48)	0 (0)	0 (0)	12 (12)	108 (–36)
$\gamma n \to \pi^- N_2^*$	25 (25)	8 (–4)	16 (16)	4 (–2)	1 (1)	54 (36)

spin-averaged → (values outside parentheses); spin-dependent → (values in parentheses)

Close, WM, PRC ***79****, 055202 (2009)*

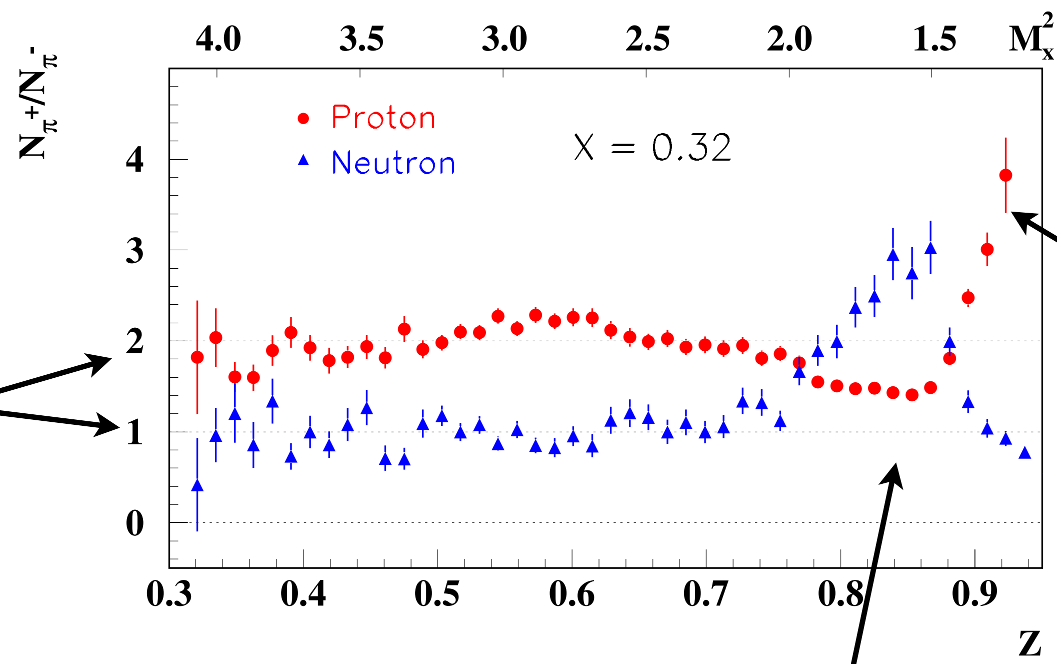
- π^-/π^+ ratios for p and n targets (summing over N_2^*)

$$\frac{\mathcal{N}_p^{\pi^-}}{\mathcal{N}_p^{\pi^+}} = \frac{1}{8}, \quad \frac{\mathcal{N}_n^{\pi^-}}{\mathcal{N}_n^{\pi^+}} = \frac{1}{2} \qquad \frac{\mathcal{N}_n^{\pi^+}}{\mathcal{N}_p^{\pi^+}} = \frac{\mathcal{N}_p^{\pi^-}}{\mathcal{N}_n^{\pi^-}} = \frac{1}{2}, \quad \frac{\mathcal{N}_n^{\pi^+}}{\mathcal{N}_p^{\pi^-}} = \frac{\mathcal{N}_p^{\pi^+}}{\mathcal{N}_n^{\pi^-}} = 4$$

- Consistent with parton model in $\mathrm{SU(6)}$ limit, $d/u = 1/2$

→ <u>inclusive</u> results recovered by summing over π^+, π^-

Comparison with data (JLab Hall C)



expect p ratio $\gg n$ ratio

N elastic

p ratio $\gg n$ ratio

Δ region

n ratio $\gg p$ ratio

	N_2^*					
	$^2\mathbf{8},\mathbf{56}^+$	$^4\mathbf{10},\mathbf{56}^+$	$^2\mathbf{8},\mathbf{70}^-$	$^4\mathbf{8},\mathbf{70}^-$	$^2\mathbf{10},\mathbf{70}^-$	sum
$\gamma p \to \pi^+ N_2^*$	100 (100)	32 (−16)	64 (64)	16 (−8)	4 (4)	216 (144)
$\gamma p \to \pi^- N_2^*$	0 (0)	24 (−12)	0 (0)	0 (0)	3 (3)	27 (−9)
$\gamma n \to \pi^+ N_2^*$	0 (0)	96 (−48)	0 (0)	0 (0)	12 (12)	108 (−36)
$\gamma n \to \pi^- N_2^*$	25 (25)	8 (−4)	16 (16)	4 (−2)	1 (1)	54 (36)

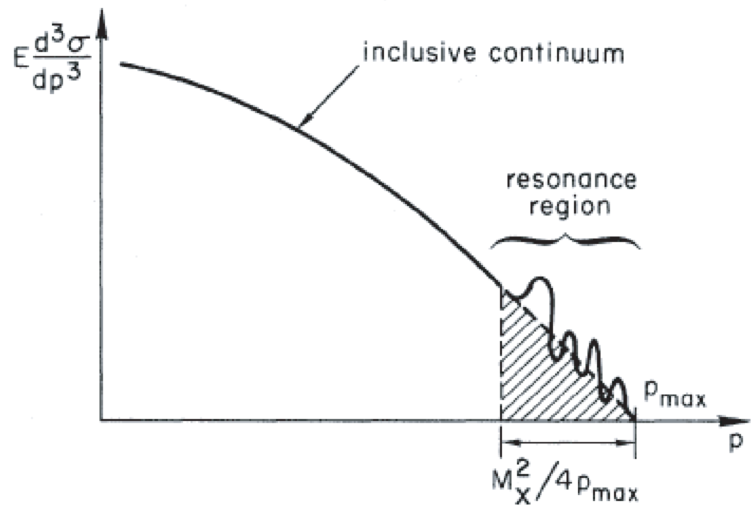
Duality in exclusive reactions

- **Exclusive–inclusive correspondence principle:**

→ continuity of dynamics from one (known) region to another (poorly known)

$$\int_{p_{\max}-M_X^2/4p_{\max}}^{p_{\max}} dp \; E\frac{d^3\sigma}{dp^3}\bigg|_{\text{incl}} \sim \sum_{\text{res}} E\frac{d\sigma}{dp_T^2}\bigg|_{\text{excl}}$$

$\gamma^* N \to M \; X$ $\qquad$ $\gamma^* N \to M \; N^*$



→ resonance contribution to $d\sigma$ should be comparable to the continuum contribution extrapolated from high energy

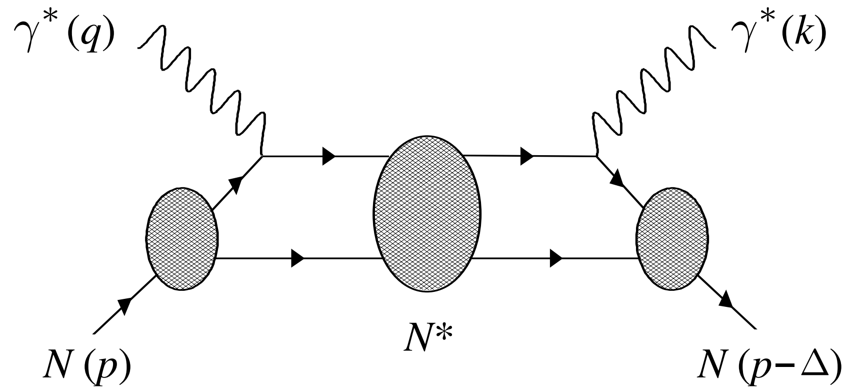
$$\frac{E}{\sigma}\frac{d^3\sigma}{dp^3} \equiv f(x, p_T^2, sQ^2) \longrightarrow f(x, p_T^2, sQ^2) \overset{s\to\infty}{\longrightarrow} f(x, p_T^2)$$

Bjorken, Kogut, PRD **8***, 1341 (1973)*

Duality in (D)VCS

- If duality applies to DVCS, partonic (GPD) interpretation may be valid down to low Q^2

→ generalized response function in scalar CQM with harmonic oscillator potential, for N even $(=2n)$ or N odd $(=2n+1)$



$$R_L = \sum_{N(n)} \frac{1}{4E_0 E_N}(E_0 \pm E_N)^2 \delta(\nu + E_0 \mp E_N)$$
$$\times \left\{ \sum_{l=0(1)}^{N} \Big[(e_1 + e_2)^2 F^{(l)}_{0,2n}(\vec{q}) F^{(l)}_{0,2n}(\vec{k}) \right.$$
$$\left. + (e_1 - e_2)^2 F^{(l)}_{0,2n+1}(\vec{q}) F^{(l)}_{0,2n+1}(\vec{k}) \Big] \sqrt{\frac{4\pi}{(2l+1)}} Y_{l0}(\theta) \right\}$$

Close, Zhao, PRD ***66****, 054001 (2002)*

→ summing over l, sum or difference over all states N gives

$$\left(\sum_{N=\text{even}} \pm \sum_{N=\text{odd}}\right) F_{0,N}(\vec{q})\; F_{N,0}(\vec{k}) = \exp\left(-\frac{(\vec{q}\mp\vec{k})^2}{4\beta^2}\right) \;\equiv\; F_{0,0}(|\vec{q}\mp\vec{k}|)$$

- Integrating over energy, obtain sum rule

$$S \;\equiv \int_{-\infty}^{+\infty} d\nu\; R_L \;=\; (e_1^2+e_2^2)F_{0,0}(|\vec{q}-\vec{k}|) \;+\; 2e_1e_2F_{0,0}(|\vec{q}+\vec{k}|)$$

→ for $Q^2 \gg |\vec{\Delta}^2|$, first term dominates

$$S \;\longrightarrow\; (e_1^2+e_2^2)\; F_{0,0}(\vec{\Delta}^2)$$

→ partonic (incoherent scattering) result!

- Generalize to spin-$1/2$ quarks, non-degenerate multiplets, flavor *non-diagonal* transitions ($\gamma^* N N^*$ form factors)

Summary

- Confirmation of duality (experimentally & theoretically) suggests origin in dynamical cancelations between resonances
  - → explore more realistic descriptions based on phenomenological $\gamma^* N N^*$ form factors
  - → incorporate *nonresonant* background in same framework

- Practical application of duality
  - → use resonance region data to constrain PDFs at high x
    (better knowledge of resonances could be relevant for LHC!)

- Models for exclusive/inclusive π production show similar duality as for inclusive DIS
  - → application to DVCS / GPDs

The End

GAUSTEQ
Germany and U.S. Nuclear Theory Exchange Program for QCD Studies of Hadrons and Nuclei

- Newly approved DOE program for US−Germany exchange in hadron/nuclear theory, centered around JLab and GSI−FAIR (& Helmholz Institut Mainz)

- Fully funds US-based physicists for up to 2−4 week collaborative visits to Germany (~ 5 visits planned already)

- See http://www.jlab.org/GAUSTEQ or contact one of the PIs (Jo Dudek, WM, Christian Weiss) at gausteq@jlab.org

  ⟶ for reciprocal German program contact Klaus Peters K.Peters@gsi.de