*QCD: History & Perspectives*
*Oberwoelz, Sep. 5, 2012*

# Hadronic effects in extractions of the weak mixing angle

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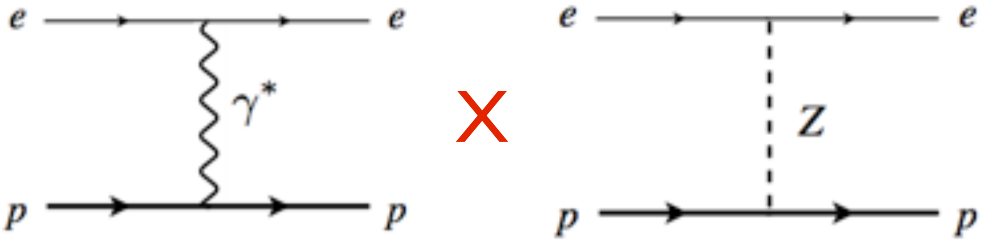
# Parity-violating $e$ scattering

- Left-right polarization asymmetry in $\vec{e}\, p \to e\, p$ scattering

$$A_{\rm PV} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \longrightarrow \frac{G_F\, Q_W^p}{4\sqrt{2}\pi\alpha}\, t$$

$$t = (k_e - k_e')^2 \\ \to 0$$

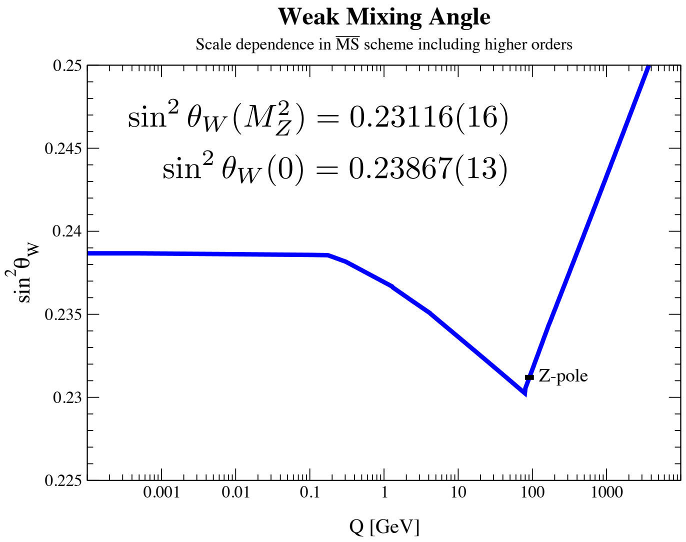
→ measures interference between e.m. and weak currents



→ in forward limit, gives proton weak charge

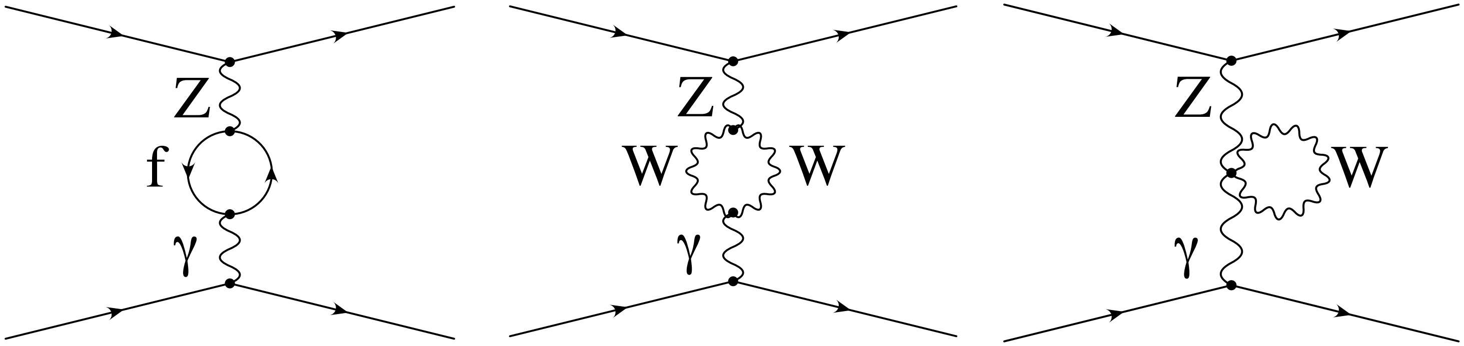
$$Q_W^p = 1 - 4\sin^2\theta_W$$

(tree level)

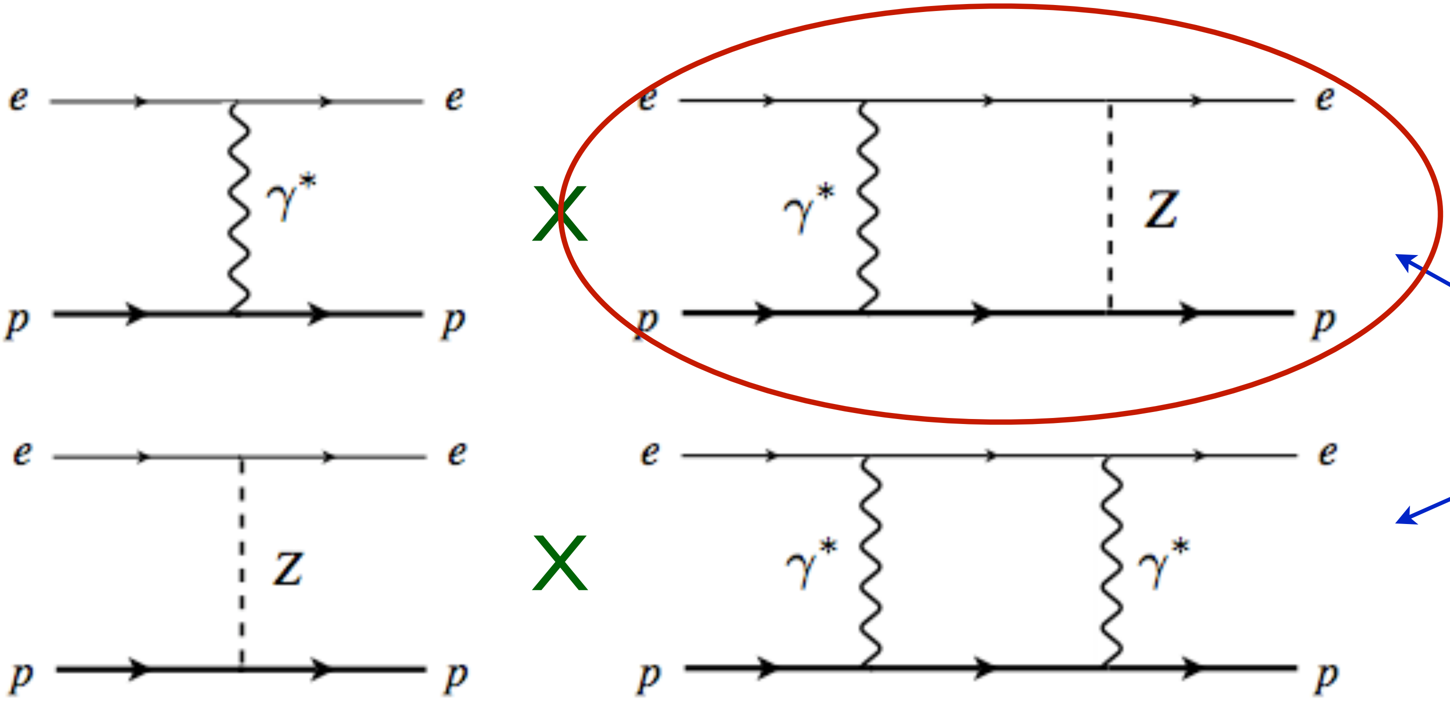

Weak Mixing Angle

Scale dependence in $\overline{\rm MS}$ scheme including higher orders

# Corrections to proton weak charge



vacuum polarization



two-boson exchange

+ ...

# Corrections to proton weak charge

- Including higher order radiative corrections

$$
\begin{aligned}
Q_W^p &= (1+\Delta\rho+\Delta_e)(1-4\sin^2\theta_W(0)+\Delta'_e) \\
&\quad + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z} \\
&= 0.0713 \pm 0.0008
\end{aligned}
$$

box diagrams

*Erler et al., PRD* ***72****, 073003 (2005)*

→ *WW* and *ZZ* box diagrams dominated by short distances, evaluated perturbatively (*WW* box gives ~ 25% correction!)

→ $\gamma Z$ box diagram sensitive to long distance physics, has two contributions $\Box_{\gamma Z} = \Box^A_{\gamma Z} + \Box^V_{\gamma Z}$

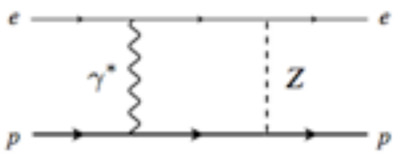
vector $e$ – axial $h$
(finite at $E$=0)

axial $e$ – vector $h$
(vanishes at $E$=0)

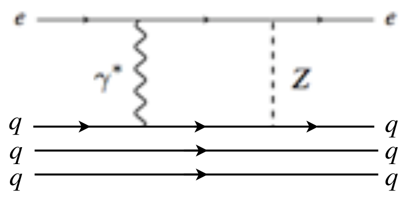
# Axial $h$ correction

- Axial $h$ correction $\Box^A_{\gamma Z}$ dominant $\gamma Z$ correction in atomic parity violation at very low (zero) energy

→ computed by Marciano & Sirlin (1980s) as sum of two parts:



★ low-energy part approximated by *Born* contribution (elastic intermediate state)



★ high-energy part (above scale $\Lambda \sim 1$ GeV) computed in terms of scattering from *free quarks*

$$\Box^A_{\gamma Z} = \frac{5\alpha}{2\pi}(1-4\sin^2\theta_W)\left[\underbrace{\ln\frac{M_Z^2}{\Lambda^2}}_{\boxed{\text{short−distance}}} + \underbrace{C_{\gamma Z}(\Lambda)}_{\boxed{\text{long−distance} \approx 3/2 \pm 1}}\right]$$

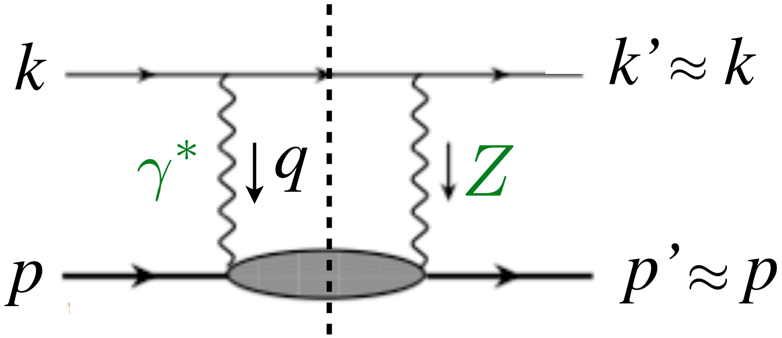
$$\approx \ 0.0052(5)$$

*Marciano, Sirlin, PRD* ***29****, 75 (1984); Erler et al., PRD* ***68****, 016006 (2003)*

# Axial $h$ correction

- Axial $h$ correction $\square^{A}_{\gamma Z}$ dominant $\gamma Z$ correction in atomic parity violation at very low (zero) energy

→ evaluate using *forward dispersion relations* with realistic input (inclusive structure function)



forward limit

$$t = (k - k')^2 \to 0$$

$$\begin{aligned} s &= (k + p)^2 \\ &= M(M + 2E) \end{aligned}$$

★ axial $h$ contribution *anti*symmetric under $E' \leftrightarrow -E'$ :

$$\Re e \; \square^{A}_{\gamma Z}(E) = \frac{2}{\pi} \int_0^\infty dE' \frac{E'}{E'^2 - E^2} \; \Im m \; \square^{A}_{\gamma Z}(E')$$

★ negative energy part corresponds to crossed box (crossing symmetry $s \to u$)

# Axial $h$ correction

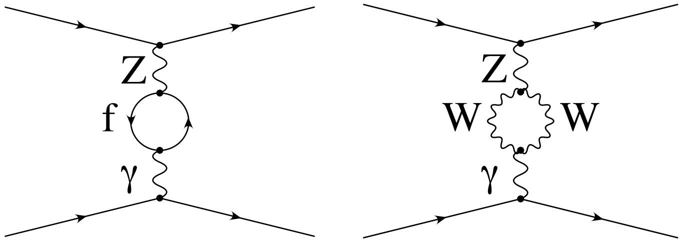
- Imaginary part given by interference $F_3^{\gamma Z}$ structure function

$$\mathcal{I}m\ \Box^{A}_{\gamma Z}(E) = \frac{1}{(2ME)^2}\int_{M^2}^{s} dW^2 \int_0^{Q^2_{\max}} dQ^2\, \frac{v_e(Q^2)\,\alpha(Q^2)}{1+Q^2/M_Z^2} \times \left(\frac{2ME}{W^2-M^2+Q^2}-\frac{1}{2}\right) F_3^{\gamma Z}$$

with $v_e(Q^2) = 1 - 4\kappa(Q^2)\sin^2\theta_W(Q^2)$

*Gorchtein, Horowitz*
*PRL **102** (2009) 091806*

→ scale dependence of $v_e, \alpha$ given by vacuum polarization corrections, $e.g.$



$$\frac{\alpha}{\alpha(Q^2)} = 1 - \Delta\alpha_{\rm lep}(Q^2) - \Delta\alpha^{(5)}_{\rm had}(Q^2)$$

*Jegerlehner, arXiv:1107.4683 [hep-ph]*

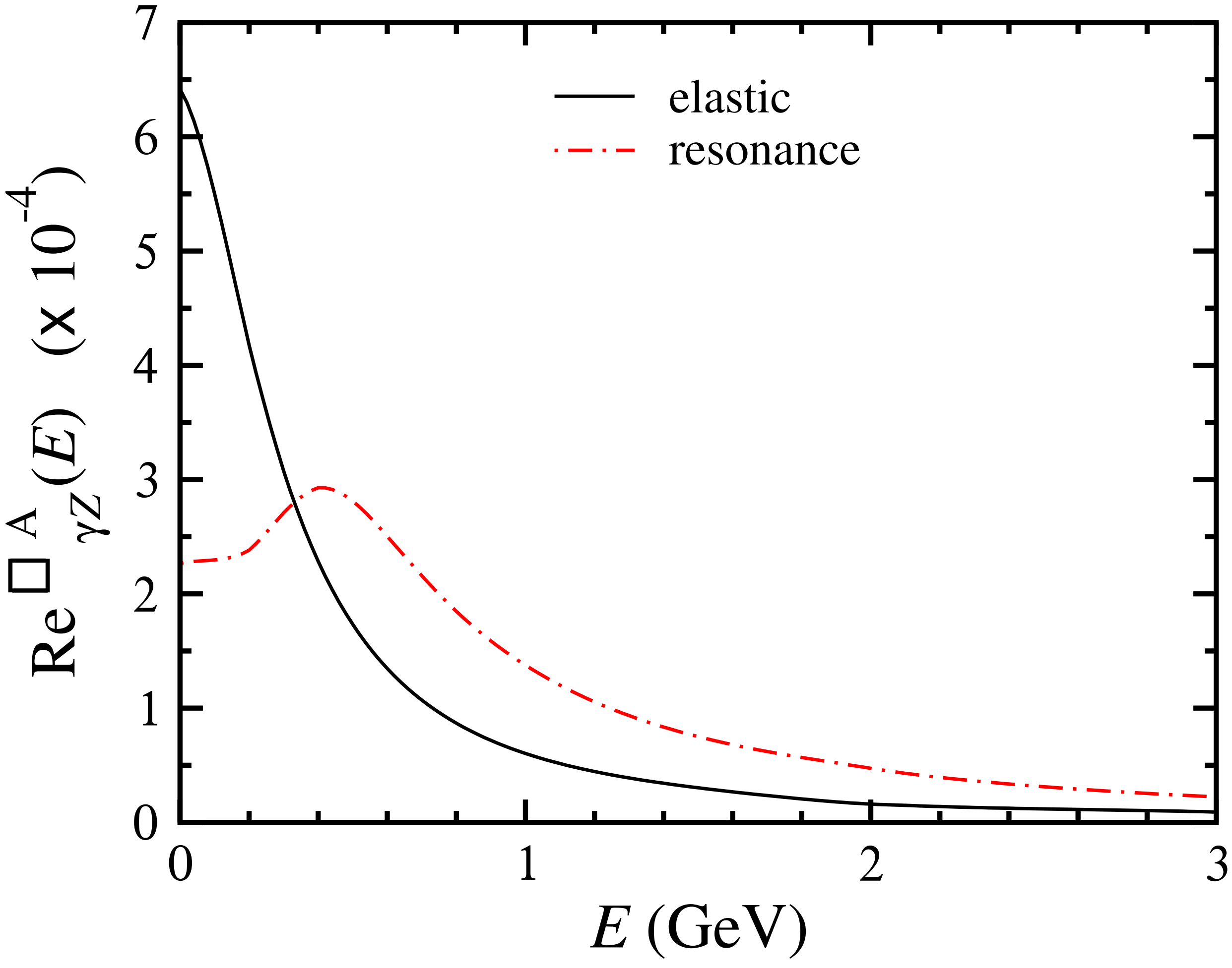
$$\alpha^{-1}(M_Z^2) = 128.94$$

... similarly for weak charges

# Axial $h$ correction

- **elastic** part $F_3^{\gamma Z(\mathrm{el})} = -Q^2\, G_M^p(Q^2)\, G_A^Z(Q^2)\, \delta(W^2 - M^2)$
- **resonance** part from parametrization of $\nu$ scattering data

*Lalakulich, Paschos*
*PRD **74**, 014009 (2006)*



*Blunden, WM, Thomas*
*PRL **107**, 081801 (2011)*

# Axial $h$ correction

- ★ DIS part dominated by leading twist PDFs at high $W$ (small $x$)

$e.g.$ at LO, $F_3^{\gamma Z(\mathrm{DIS})} = \sum_q 2e_q\, g_A^q \left(q(x,Q^2) - \bar{q}(x,Q^2)\right)$

→ expand integrand in $1/Q^2$ in DIS region $(Q^2 \gtrsim 1\ \mathrm{GeV}^2)$

$$\mathcal{R}e\ \Box_{\gamma Z}^{\mathrm{A(DIS)}}(E) = \frac{3}{2\pi}\int_{Q_0^2}^{\infty} dQ^2\, \frac{v_e(Q^2)\,\alpha(Q^2)}{1+Q^2/M_Z^2} \times \left[M_3^{\gamma Z(1)} - \frac{2M^2}{9Q^4}(5E^2-3Q^2)M_3^{\gamma Z(3)}\right]$$

moments $M_3^{\gamma Z(n)}(Q^2) = \int\limits_0^1 dx\, x^{n-1} F_3^{\gamma Z}(x,Q^2)$

# Axial $h$ correction

- Structure function moments

<u>$n = 1$</u> $\quad M_3^{\gamma Z(1)}(Q^2) = \frac{5}{3}\left(1 - \frac{\alpha_s(Q^2)}{\pi}\right)$

→ $\gamma Z$ analog of Gross-Llewellyn Smith sum rule

$$\mathcal{R}e\ \Box_{\gamma Z}^{A(\mathrm{DIS})} \approx (1 - 4\hat{s}^2)\frac{5\alpha}{2\pi}\int_{Q_0^2}^{\infty}\frac{dQ^2}{Q^2(1 + Q^2/M_Z^2)}\left(1 - \frac{\alpha_s(Q^2)}{\pi}\right)$$

→ precisely result from Marciano & Sirlin!

(works because result depends on lowest moment of *valence* PDF, with <u>model-independent normalization</u>!)

<u>$n = 3$</u> $\quad M_3^{\gamma Z(3)}(Q^2) = \frac{1}{3}\left(2\langle x^2\rangle_u + \langle x^2\rangle_d\right)\left(1 + \frac{5\alpha_s(Q^2)}{12\pi}\right)$

→ related to $x^2$ -weighted moment of valence PDFs

# Axial $h$ correction

- ★ "DIS" region at $Q^2 < 1\ \mathrm{GeV}^2$ does not afford PDF description

  ⟶ in absence of data, consider models with general constraints

  - ★ $F_3^{\gamma Z}(x_{\max}, Q^2)$ should not diverge in limit $Q^2 \to 0$
  - ★ $F_3^{\gamma Z}(x, Q^2)$ should match PDF description at $Q^2 = 1\,\mathrm{GeV}^2$

Model 1

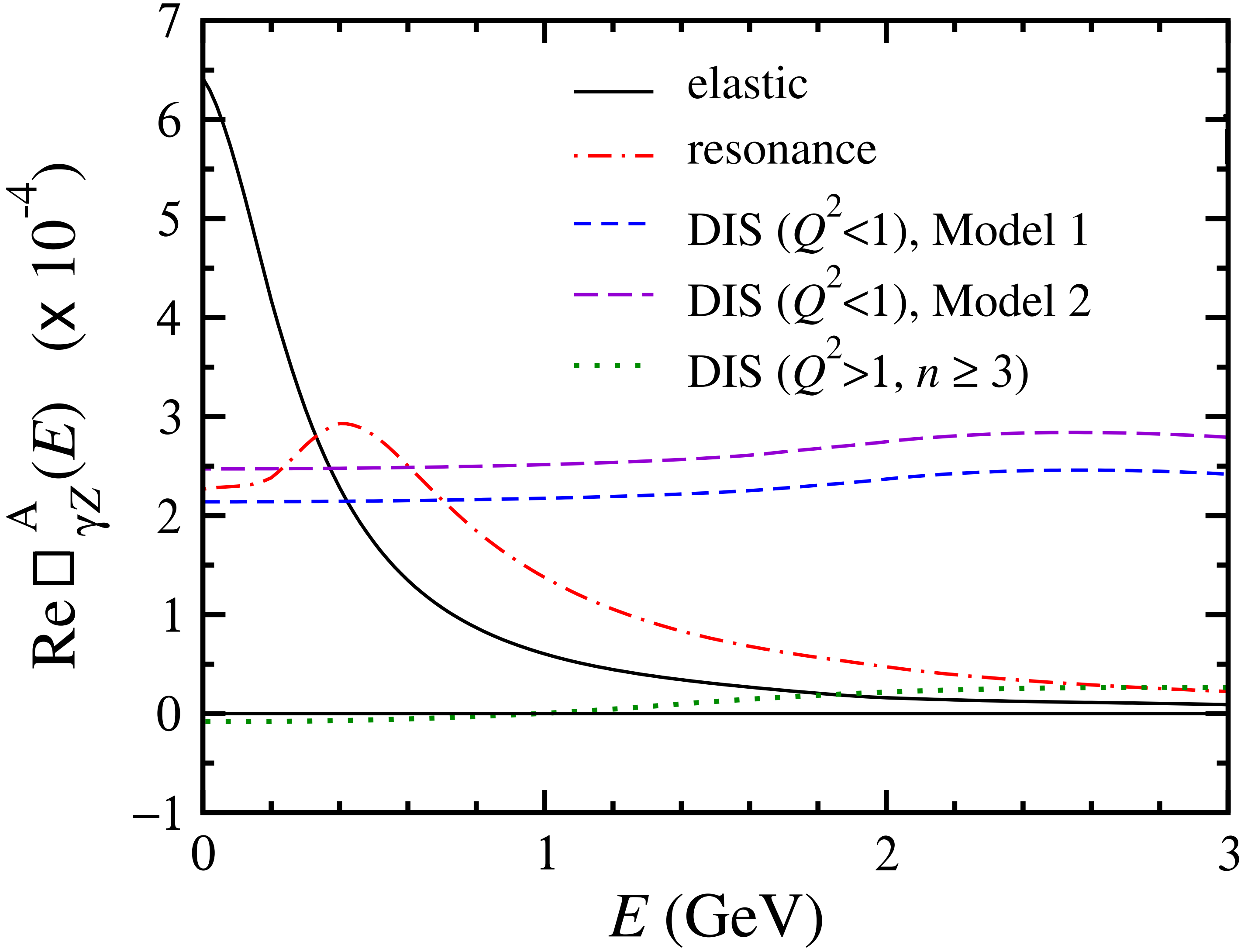
$$F_3^{\gamma Z}(x, Q^2) = \left(\frac{1+\Lambda^2/Q_0^2}{1+\Lambda^2/Q^2}\right) F_3^{\gamma Z}(x, Q_0^2)$$

$$F_3^{\gamma Z} \sim (Q^2)^{0.3} \text{ as } Q^2 \to 0$$

Model 2 $F_3^{\gamma Z}$ frozen at $Q^2 = 1$ value for all $W^2$

$$F_3^{\gamma Z} \text{ finite as } Q^2 \to 0$$

# Axial $h$ correction



*Blunden, WM, Thomas*
*PRL* ***107****, 081801 (2011)*

→ dominated by $n = 1$ DIS moment: $\underline{32.8 \times 10^{-4}}$
(weak $E$ dependence)

# Axial $h$ correction

- correction at $\underline{E = 0}$

$$\Re e\, \Box^{A}_{\gamma Z} = \underset{\substack{\uparrow \\ \boxed{\text{elastic}}}}{0.00064} + \underset{\substack{\uparrow \\ \boxed{\text{resonance}}}}{0.00023} + \underset{\substack{\uparrow \\ \boxed{\text{DIS}}}}{0.00350} \to \underline{0.0044(4)}$$

- correction at $\underline{E = 1.165 \text{ GeV}}$ (Qweak)

$$\Re e\, \Box^{A}_{\gamma Z} = 0.00005 + 0.00011 + 0.00352 = \underline{0.0037(4)}$$

*cf.* MS* value: $\underline{0.0052(5)}$ (~1% shift in $Q_W^p$)

* *Marciano, Sirlin, PRD* ***29****, 75 (1984)*

- shifts $Q_W^p$ from $\underline{0.0713(8)} \longrightarrow \underline{0.0705(8)}$

# Vector $h$ correction

- Vector $h$ correction $\Box^V_{\gamma Z}$ vanishes at $E = 0$, but experiment has $E \sim 1\ \mathrm{GeV}$ – what is energy dependence?

→ forward dispersion relation

★ $$\Re e\, \Box^V_{\gamma Z}(E) = \frac{2E}{\pi} \int_0^\infty dE' \frac{1}{E'^2 - E^2}\, \Im m\, \Box^V_{\gamma Z}(E')$$

★ integration over $E' < 0$ corresponds to crossed-box, vector $h$ contribution symmetric under $E' \leftrightarrow -E'$

→ imaginary part given by

$$\Im m\, \Box^V_{\gamma Z}(E) = \frac{\alpha}{(s - M^2)^2} \int_{W_\pi^2}^{s} dW^2 \int_0^{Q^2_{\max}} \frac{dQ^2}{1 + Q^2/M_Z^2} \times \left( F_1^{\gamma Z} + F_2^{\gamma Z} \frac{s\,(Q^2_{\max} - Q^2)}{Q^2(W^2 - M^2 + Q^2)} \right)$$

*Gorchtein, Horowitz, PRL* ***102***, *091806 (2009)*

# Vector $h$ correction

- $F_{1,2}^{\gamma Z}$ structure functions
  - parton model for DIS region

$$F_2^{\gamma Z} = 2x \sum_q e_q \, g_V^q \, (q + \bar{q}) = 2x F_1^{\gamma Z}$$

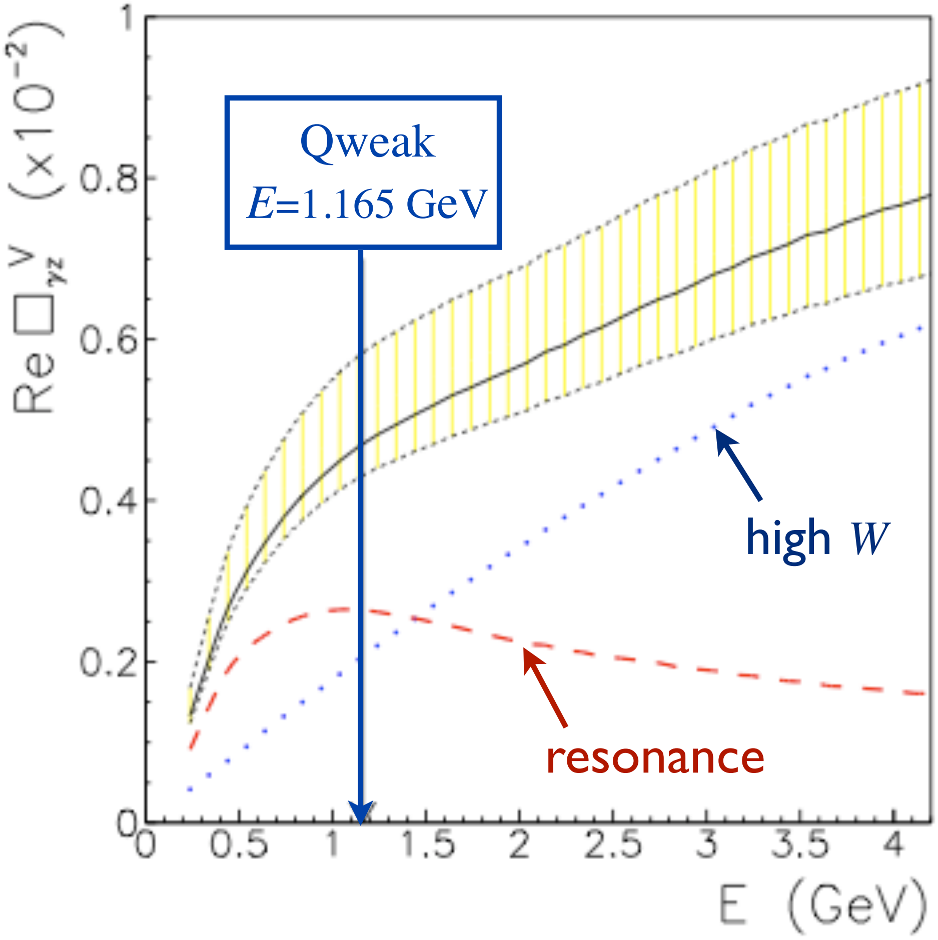
  - in resonance region use phenomenological input for $F_2$ ($e.g.$ Christy-Bosted), empirical (SLAC) fit for $R$
    - → for transitions to $I = 3/2$ states ($e.g.$ $\Delta$), CVC and isospin symmetry give $F_i^{\gamma Z} = (1 + Q_W^p) F_i^{\gamma}$
    - → for transitions to $I = 1/2$ states, $\gamma\gamma \to \gamma Z$ rotations fixed by CVC and $p$, $n$ helicity amplitudes

# Vector *h* correction

- Total $\Box^V_{\gamma Z}$ correction

$$\Re e\,\Box^V_{\gamma Z} = 0.0047^{+0.0011}_{-0.0004}$$

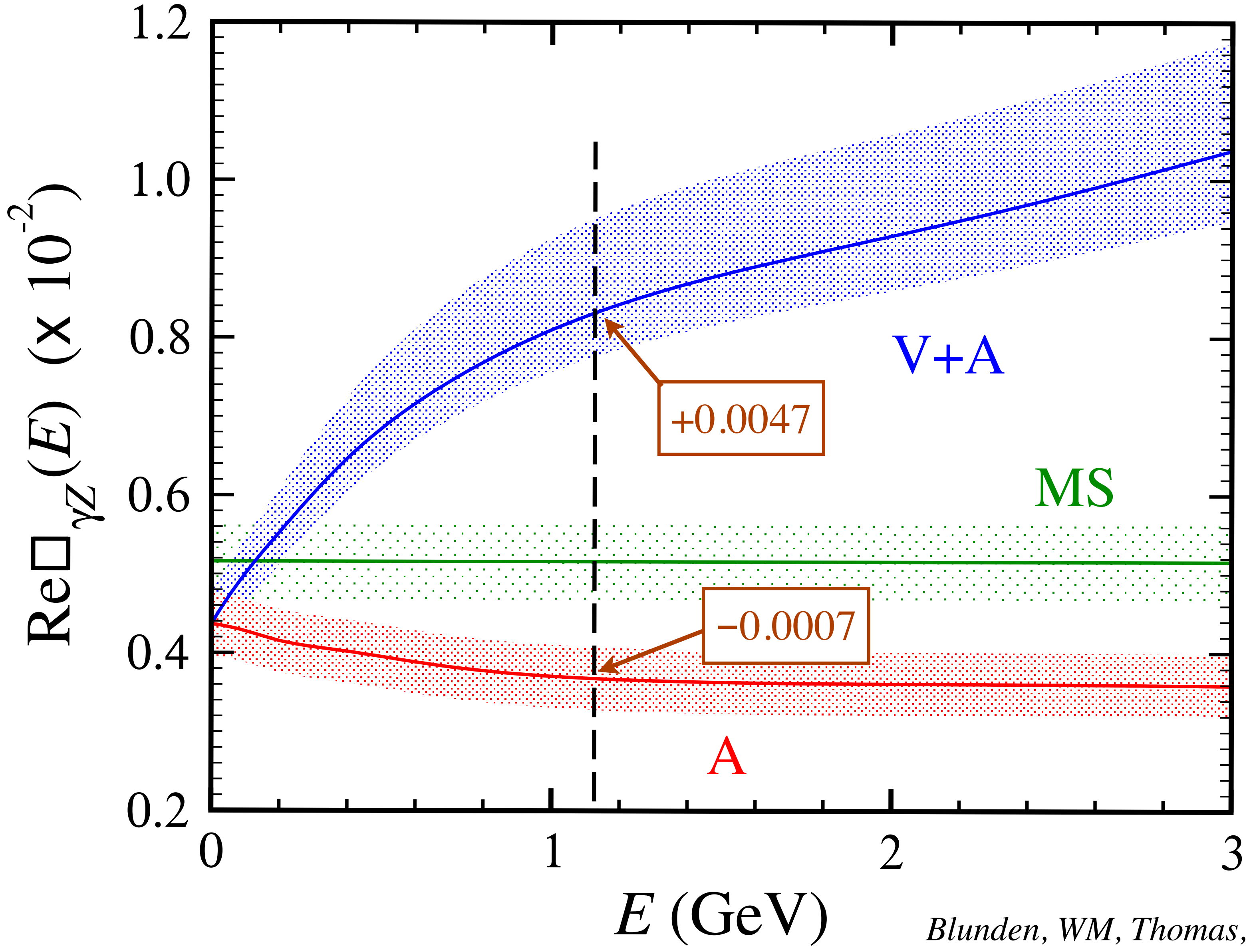
or $6.6^{+1.5}_{-0.6}\,\%$ of uncorrected $Q^p_W$



*Sibirtsev, Blunden, WM, Thomas*
*PRD **82**, 013011 (2010)*

# Combined vector and axial *h* correction

$$\boxed{Q_W^p = 0.0713 \rightarrow 0.0705}$$ (at *E*=0)



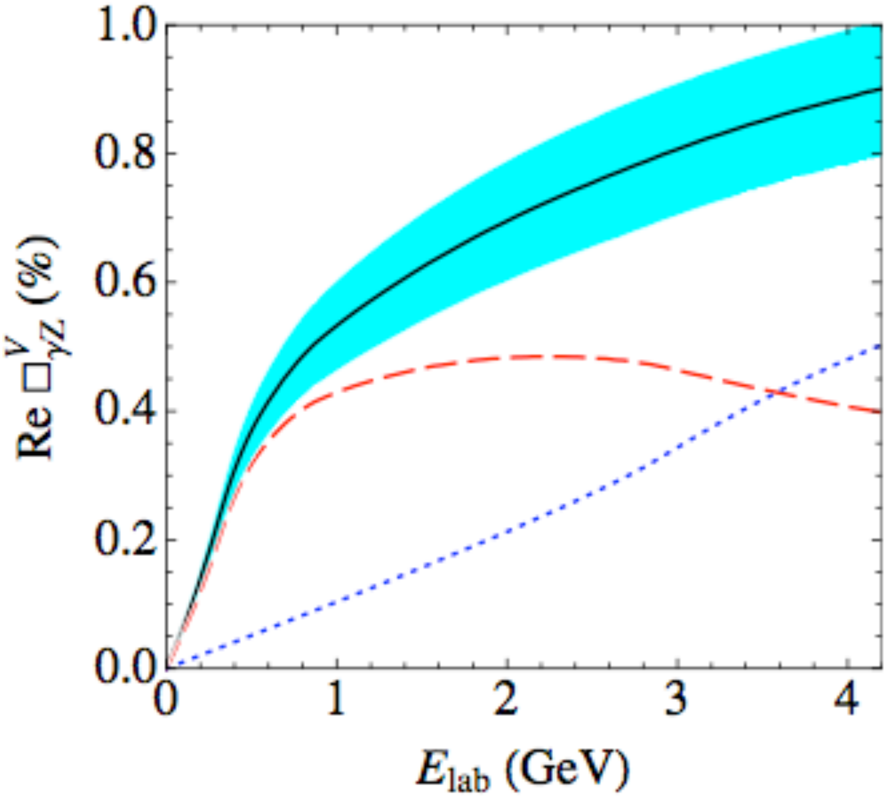
At *E*=1.165 GeV,
*E*-dependent
correction is
+ 0.0040

*Blunden, WM, Thomas, PRL* ***107****, 081801 (2011)*

# Other calculations

## Rislow & Carlson

$$\Re e \, \Box^{V}_{\gamma Z} = 0.0057 \pm 0.0009$$



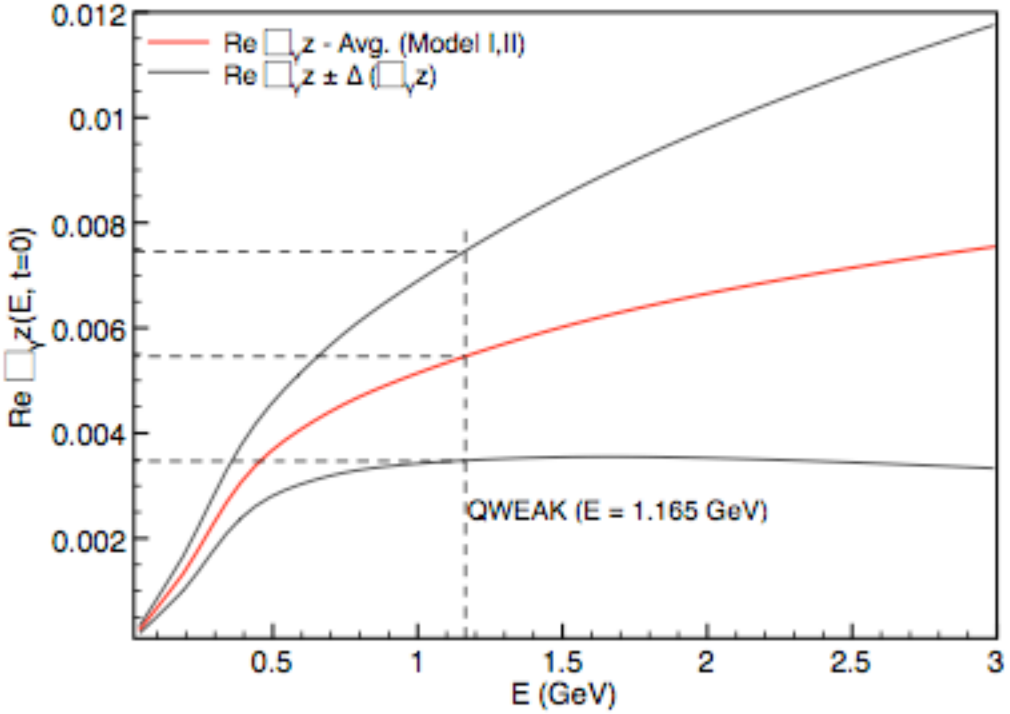
*Rislow, Carlson*
*PRD* ***83****, 113007 (2011)*

→ compatible with SBMT within errors

# Other calculations

## Gorchtein, Horowitz & Ramsey-Musolf

$$\Re e\ \Box^V_{\gamma Z} = 0.0054 \pm 0.0020$$



*Gorchtein, Horowitz,*
*Ramsey-Musolf*
*PRC* ***84****, 015502 (2011)*

→ central value consistent with SBMT and RC, but 2 x larger uncertainty

# Other calculations

## Gorchtein, Horowitz & Ramsey-Musolf

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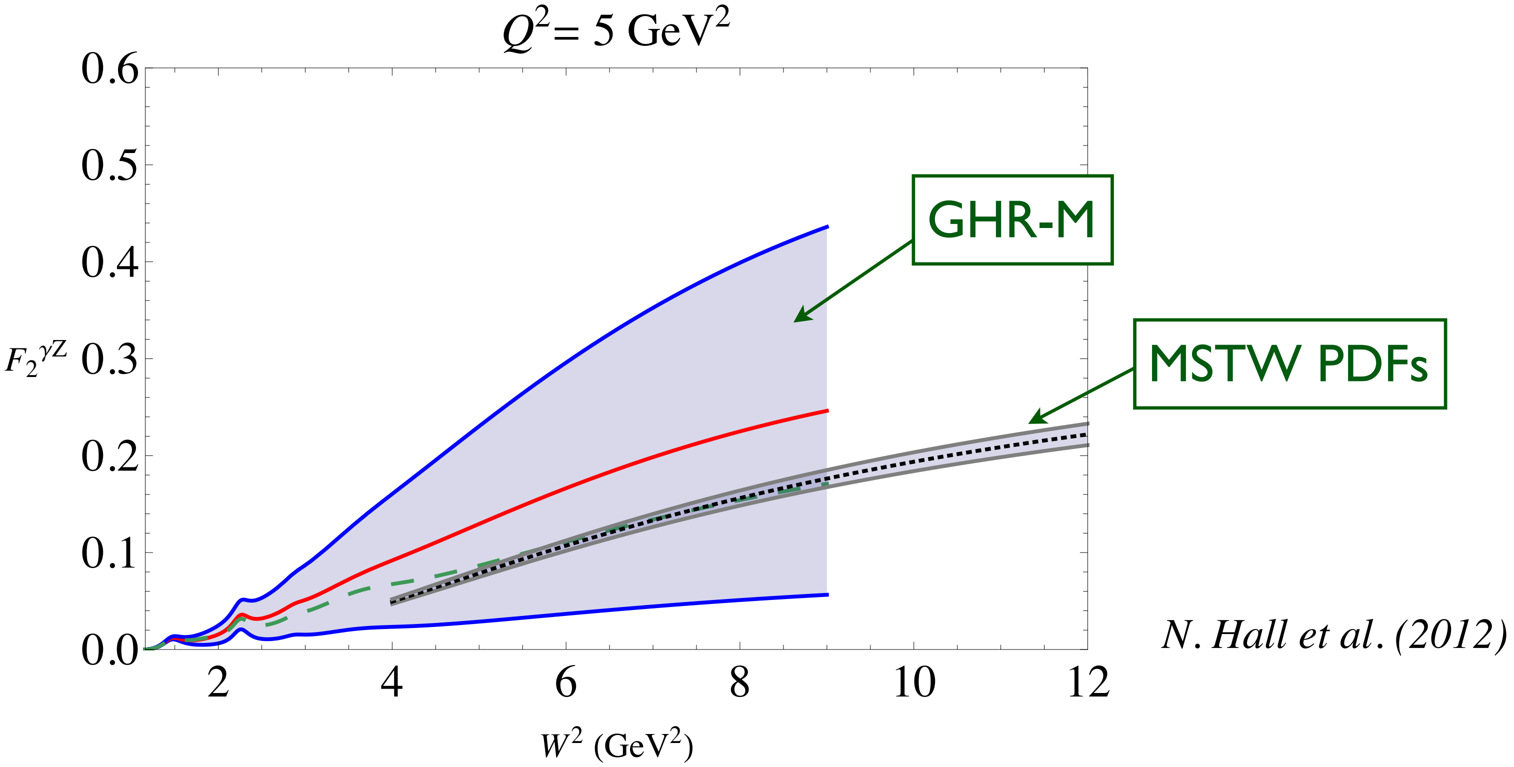
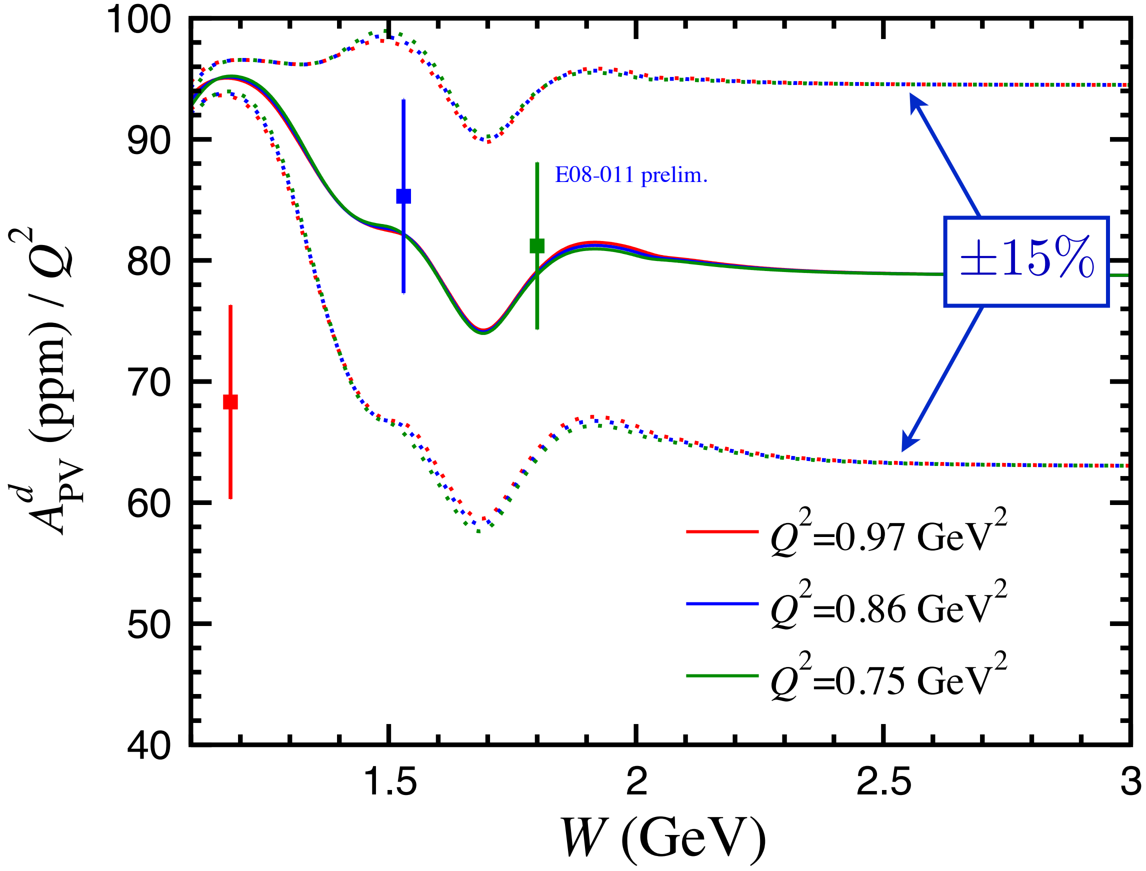


→ large uncertainty on non-resonant background – significantly larger than expected from PDFs

# Parity-violating DIS

- Constraints from PVDIS asymmetries (E08-011 on deuterium)

$$A_{\rm PV} \;\propto\; \frac{xy^2 F_1^{\gamma Z} + (1-y)F_2^{\gamma Z} + \frac{g_V^e}{g_A^e}(y - y^2/2)F_3^{\gamma Z}}{xy^2 F_1^{\gamma\gamma} + (1-y)F_2^{\gamma\gamma}}$$



$$F_i^{\gamma Z} = \left(\frac{F_i^{\gamma Z}}{F_i^{\gamma\gamma}}\right)^{\rm LT} F_i^{\gamma\gamma}$$

*Hall, Blunden, WM et al. (2012)*

# Parity-violating DIS

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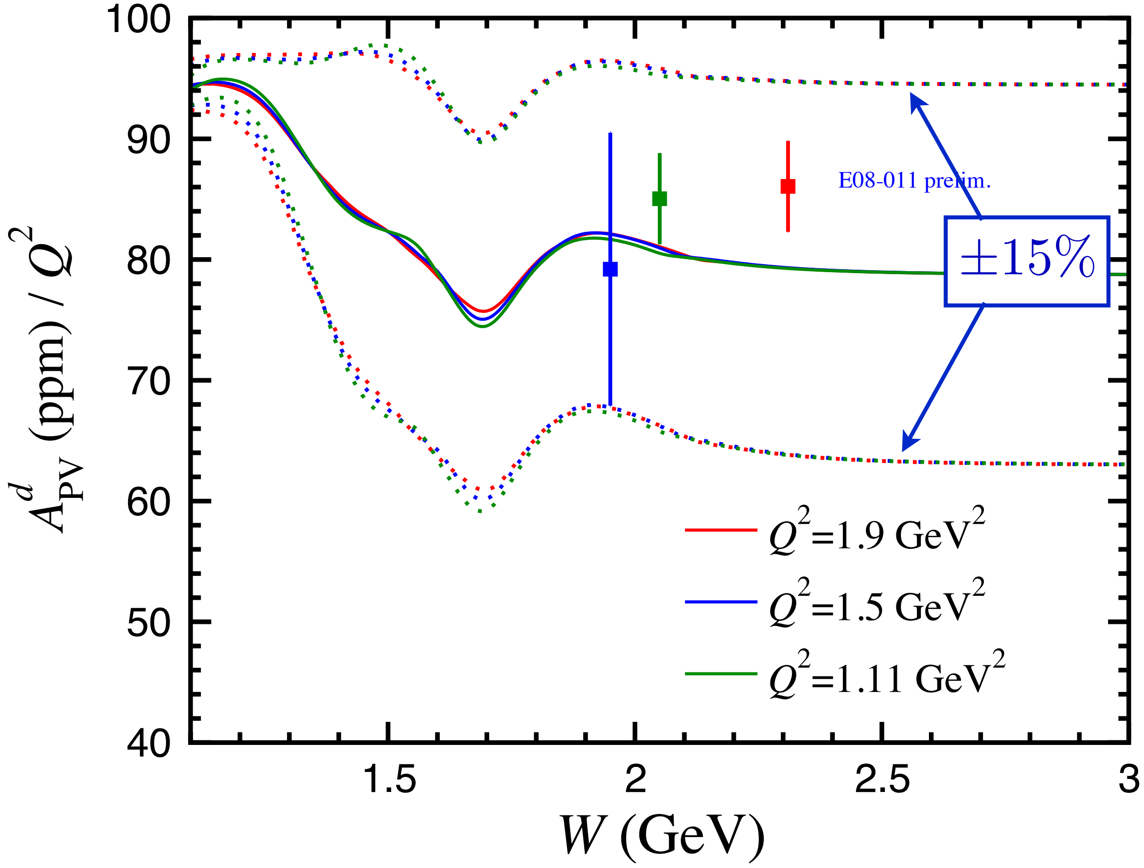

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*Hall, Blunden, WM et al. (2012)*

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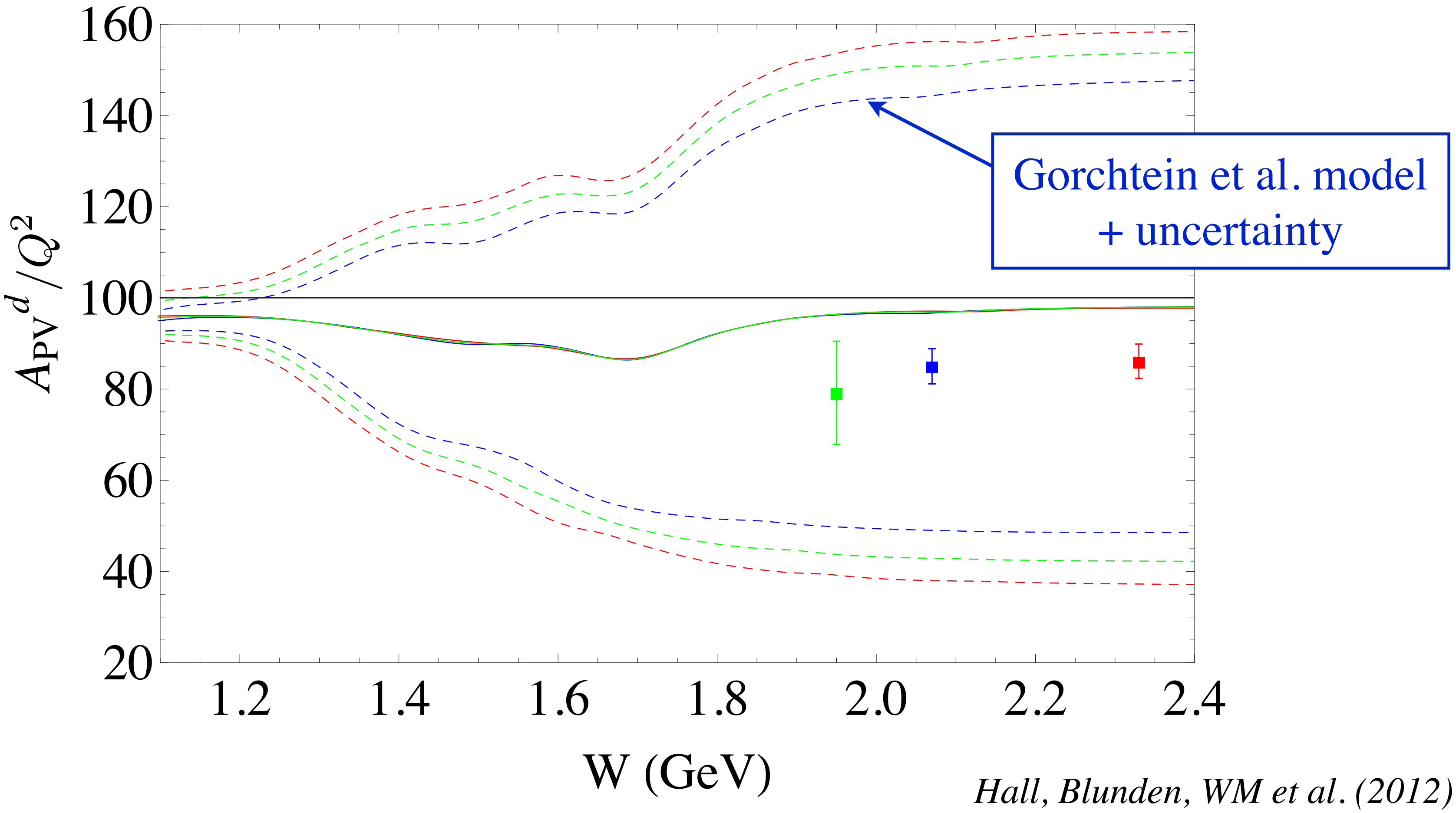
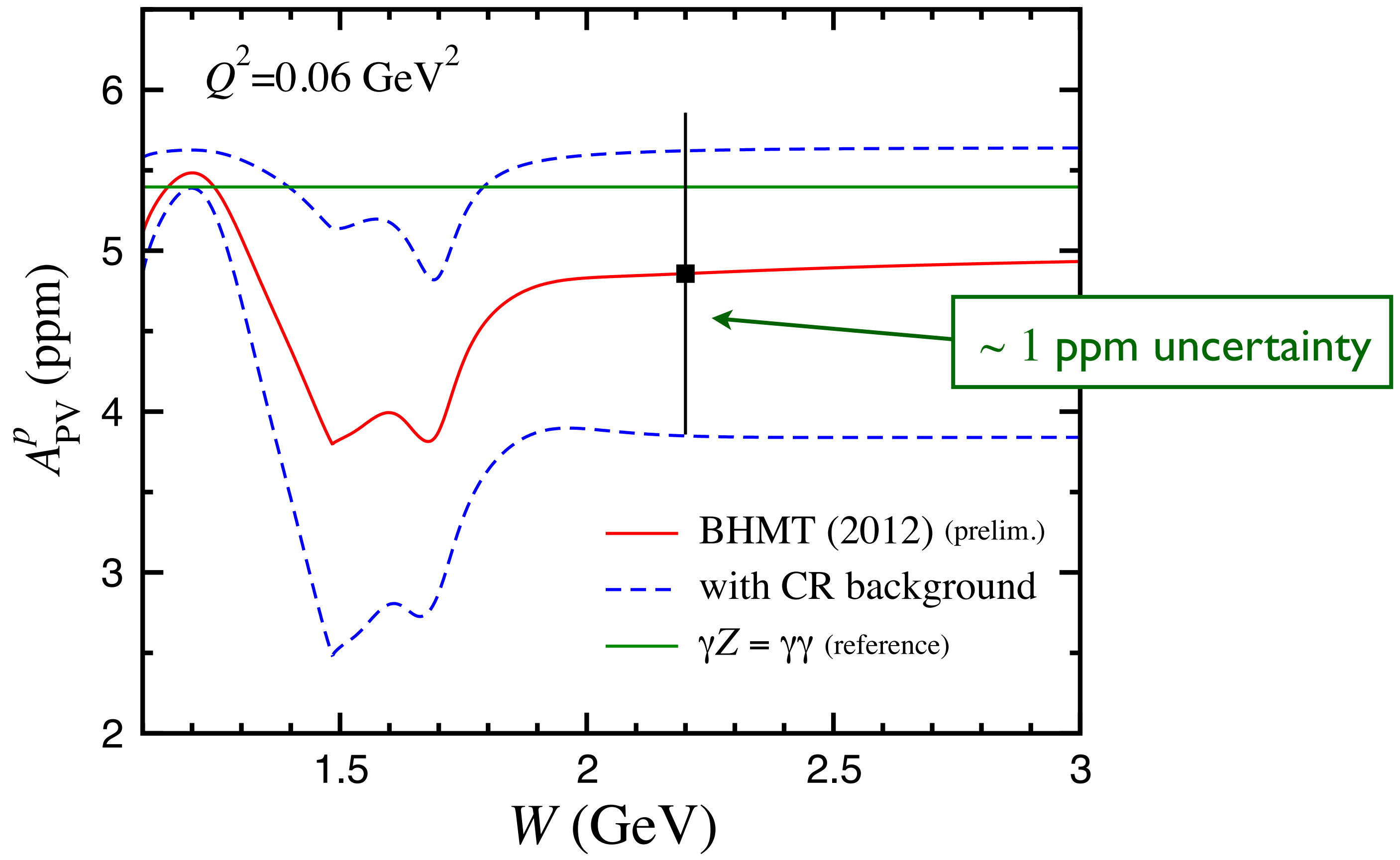


Gorchtein et al. model
+ uncertainty

*Hall, Blunden, WM et al. (2012)*

# Parity-violating DIS

- Expected inelastic asymmetry data from Qweak



~ 1 ppm uncertainty

→ constrain input $F_i^{\gamma Z}$ structure functions for $\mathcal{R}e\,\Box_{\gamma Z}$ (updated analysis in progress)

*Hall, Blunden, WM et al. (2012)*

# APV in $^{133}$Cs

- Parity violating dipole transition $6\,\mathrm{S}_{1/2} - 7\,\mathrm{S}_{1/2}$ sensitive to weak mixing angle $(E \sim 0)$

  → weak charge of Cs

$$Q_W(\mathrm{Cs}) \; = \; 55\,\widetilde{Q}_W^p \; + \; 78\,\widetilde{Q}_W^n$$

weak charge of *bound* $p$ in Cs nucleus

- Nuclear effect on elastic $N$ contribution – Pauli blocking

  → intermediate state $N$ (in target rest frame) must have momentum above Fermi level

$$|\mathbf{q}| > p_F \approx 260\,\mathrm{MeV}$$

$$\Rightarrow \quad Q^2 > Q^2_{\min} = 2M^2\left(\sqrt{1+p_F^2/M^2} - 1\right) \approx p_F^2$$

# APV in $^{133}$Cs

- Significantly reduced elastic contribution

$$\Box_{\gamma Z}^{p\,(\mathrm{el})}:\ 0.00064\ \rightarrow\ 0.00029, \qquad \Box_{\gamma Z}^{n\,(\mathrm{el})}:\ 0.00044\ \rightarrow\ 0.00020$$

- Total $\gamma Z$ corrections dominated by DIS contributions

| | $p$ | $n$ |
|---|---|---|
| total | 0.0040(4) | 0.0032(4) |
| MS | 0.0052(5) | 0.0040(4) |
| $\Delta\widetilde{Q}_W^N$ | −0.0012 | −0.0008 |
| $\Delta Q_W(\mathrm{Cs})$ | −0.065 | −0.060 |

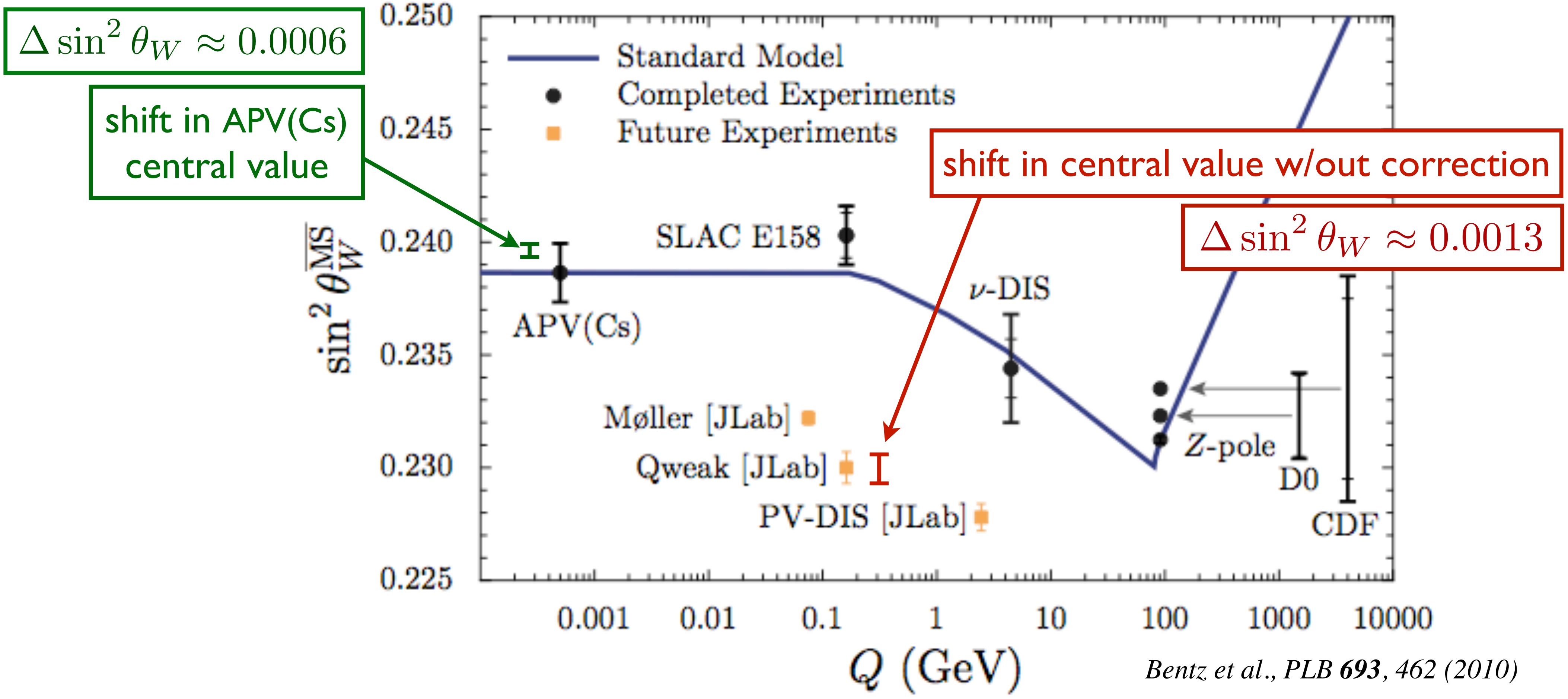
→ overall shift in weak charge (relative to MS)

$$\Delta Q_W(\mathrm{Cs}) = -0.126$$

$$\text{or}\ \boxed{-\,0.16\%\ \ \text{of}\ \ \mathrm{Q_W^{exp}(Cs)} = -73.20(35)}$$

*Blunden, WM, Thomas*
*arXiv:1208.4310*

# Effect on weak mixing angle



$\Delta\sin^2\theta_W \approx 0.0006$

shift in APV(Cs) central value

shift in central value w/out correction

$\Delta\sin^2\theta_W \approx 0.0013$

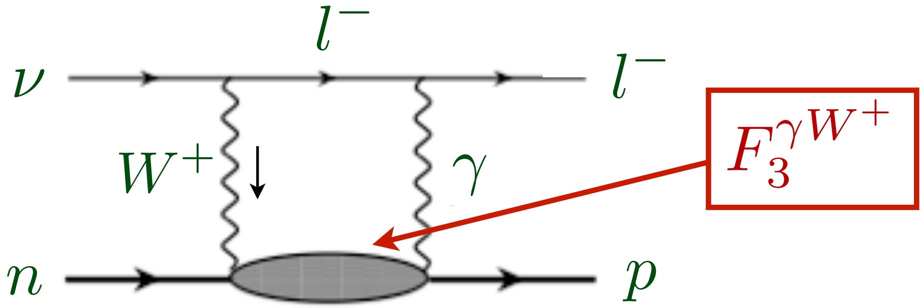
*Bentz et al.*, *PLB* ***693***, *462 (2010)*

→ Qweak: large shift in central value *cf.* MS

→ APV(Cs): shift in central value *cf.* MS by ~ 1/3 of error bar (~ 4 x larger than quoted SM uncertainty)

# Further application: TBE in neutrino scattering

- May expect similar two-boson exchange (TBE) effects in neutrino scattering (QE, DIS)



- Relevant for $n$ beta decay, extraction of CKM matrix element $V_{ud}$

$$F(Q^2) \xrightarrow[\text{high } Q^2]{} \frac{1}{Q^2}\left(1 - \frac{\alpha_s(Q^2)}{\pi}\right)$$

as in Bjorken & GLS sum rules

$$\xrightarrow[\text{low } Q^2]{} \sum_{V=\rho,A,\rho'} \frac{a_V}{Q^2 + m_V^2}$$

vector meson dominance

*Marciano, Sirlin, PRL* ***96****, 032002 (2006)*

# Summary

- $\gamma Z$ box corrections computed via dispersion relations from inclusive $\gamma Z$ interference structure functions

  → new formulation in terms of moments puts on firm footing earlier estimates within "free quark model"

- Significant energy dependent *vector* hadron correction

  → for Qweak kinematics, shifts central value of weak charge

$$Q_W^p = 0.0713 \ \rightarrow \ 0.0705$$

  → significant constraints from new PVDIS asymmetry data

- $\gamma Z$ corrections to APV in $^{133}$Cs

  → shift relative to MS value for $Q_W(\mathrm{Cs})$ of –$0.16\%$

$$\Delta \sin^2\theta_W \ \approx \ 4 \text{ x SM uncertainty}$$