Workshop on Chiral Dynamics
Jefferson Lab
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Equivalence of pion loops in equal time and light-front dynamics

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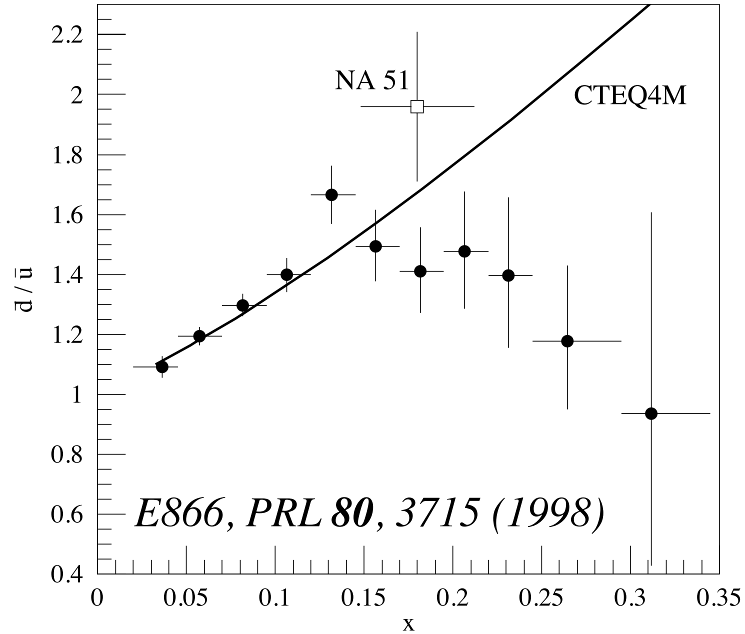
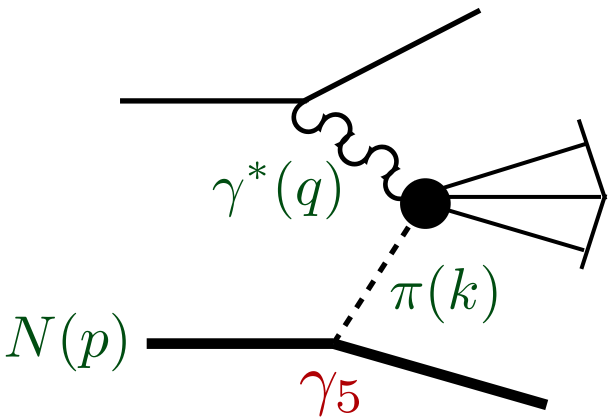
Background

- Large flavor asymmetry in proton sea suggests important role of π cloud even in high-energy reactions



→ Sullivan process



*Sullivan, PRD **5**, 1732 (1972)*
*Thomas, PLB **126**, 97 (1983)*

$$(\bar{d} - \bar{u})(x) = \int_x^1 \frac{dy}{y} \, f_\pi(y) \; \bar{q}^\pi(x/y)$$

pion light-cone momentum distribution in nucleon

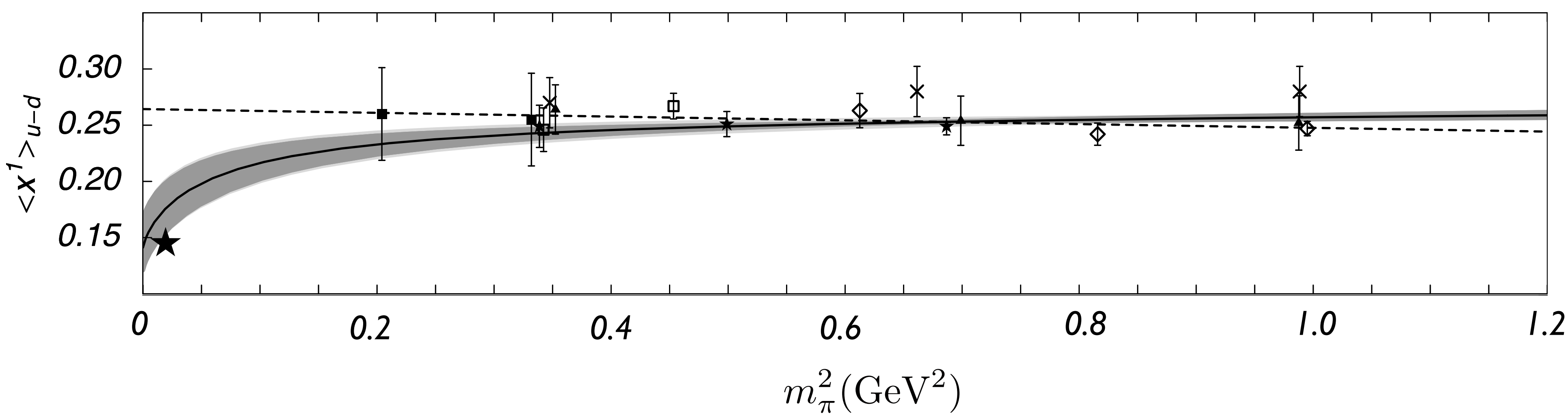
- Chiral expansion of moments of $f_\pi(y)$

$\longrightarrow$ model-independent leading nonanalytic (LNA) behavior

$$\langle x^0 \rangle_{\bar{d}-\bar{u}} \equiv \int_0^1 dx(\bar{d}-\bar{u}) = \frac{2g_A^2}{(4\pi f_\pi)^2} m_\pi^2 \log(m_\pi^2/\mu^2)$$

Thomas, WM, Steffens, PRL ***85****, 2892 (2000)*

- Nonanalytic behavior vital for chiral extrapolation of lattice data



Detmold, WM et al., PRL ***87****, 172001 (2001)*

- Direct calculation of matrix elements of local twist-2 operators in ChPT disagrees with "Sullivan" result

$$\langle x^n \rangle_{u-d} = a_n \left(1 + \frac{(3g_A^2+1)}{(4\pi f_\pi)^2} m_\pi^2 \log(m_\pi^2/\mu^2) \right) + \mathcal{O}(m_\pi^2)$$

*Chen, X. Ji, PLB **523**, 107 (2001)*
*Arndt, Savage, NPA **692**, 429 (2002)*

$\longrightarrow$ is there a problem with application of ChPT or "Sullivan process" to DIS?

$\longrightarrow$ is light-front treatment of pion loops (zero modes) problematic?

$\longrightarrow$ investigate relation between *covariant*, *instant-form*, and *light-front* formulations

$\longrightarrow$ consider simple test case: nucleon self-energy

Self-energy

- From lowest order PV Lagrangian

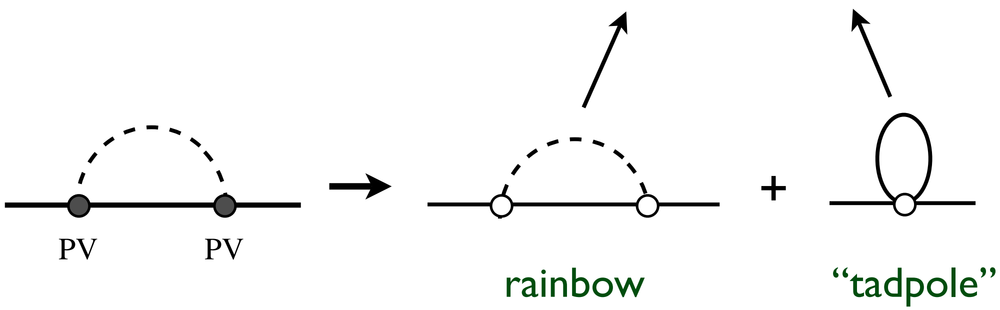
$$\Sigma = i\left(\frac{g_{\pi NN}}{2M}\right)^2 \overline{u}(p)\int\frac{d^4k}{(2\pi)^4}\,(\not{k}\gamma_5\vec{\tau})\,\frac{i\,(\not{p}-\not{k}+M)}{D_N}\,(\gamma_5\not{k}\vec{\tau})\,\frac{i}{D_\pi^2}\,u(p)$$

Goldberger-Treiman $\quad \dfrac{g_A}{f_\pi} = \dfrac{g_{\pi NN}}{M}$

$$D_\pi \equiv k^2 - m_\pi^2 + i\varepsilon$$
$$D_N \equiv (p-k)^2 - M^2 + i\varepsilon$$

$\longrightarrow$ rearrange in more transparent "reduced" form

$$\Sigma = -\frac{3ig_A^2}{4f_\pi^2}\int\frac{d^4k}{(2\pi)^4}\,\frac{1}{2M}\left[4M^2\left(\frac{m_\pi^2}{D_\pi D_N}+\frac{1}{D_N}\right)+\cancel{\frac{2p\cdot k}{D_\pi}}\right]$$



*C.-R. Ji, WM, Thomas, PRD **80**, 054018 (2009)*

Self-energy

- Covariant (dimensional regularization)

$$\int d^{4-2\varepsilon}k \frac{1}{D_\pi D_N} = -i\pi^2 \left(\gamma + \log\pi - \frac{1}{\varepsilon} + \int_0^1 dx \log\frac{(1-x)^2 M^2 + x m_\pi^2}{\mu^2} + \mathcal{O}(\varepsilon) \right)$$

$$\int d^{4-2\varepsilon}k \frac{1}{D_N} = -i\pi^2 M^2 \left(\gamma + \log\pi - \frac{1}{\varepsilon} + \log\frac{\mu^2}{M^2} + \mathcal{O}(\varepsilon) \right)$$

$\longrightarrow$ combining terms gives well-known m_π^3 LNA behavior (from $1/D_\pi D_N$ term)

$$\Sigma_{\rm cov}^{\rm LNA} = -\frac{3g_A^2}{32\pi f_\pi^2} \left(m_\pi^3 + \frac{1}{2\pi}\frac{m_\pi^4}{M}\log m_\pi^2 + \mathcal{O}(m_\pi^5) \right)$$

Self-energy

- Equal time (rest frame)

$$\int d^4k \frac{1}{D_\pi D_N} = \int d^3k \int_{-\infty}^{\infty} dk_0 \frac{1}{(-2)(\omega_k - i\varepsilon)} \left(\frac{1}{k_0 - \omega_k + i\varepsilon} - \frac{1}{k_0 + \omega_k - i\varepsilon} \right)$$
$$\times \frac{1}{2(E' - i\varepsilon)} \left(\frac{1}{k_0 - E + E' - i\varepsilon} - \frac{1}{k_0 - E - E' + i\varepsilon} \right)$$

$$\omega_k = \sqrt{\mathbf{k}^2 + m_\pi^2}\,, \quad E' = \sqrt{\mathbf{k}^2 + M^2}$$

→ four time-orderings

$$\Sigma_{\rm ET}^{(+-)}, \quad \Sigma_{\rm ET}^{(-+)}, \quad \Sigma_{\rm ET}^{(++)}, \quad \Sigma_{\rm ET}^{(--)}$$

positive energy

"Z-graph"

= 0

$$\Sigma_{\rm ET}^{(+-)\rm LNA} = -\frac{3g_A^2}{32\pi f_\pi^2} \left(m_\pi^3 + \frac{3}{4\pi} \frac{m_\pi^4}{M} \log m_\pi^2 + \mathcal{O}(m_\pi^5) \right)$$
$$\Sigma_{\rm ET}^{(-+)\rm LNA} = -\frac{3g_A^2}{32\pi f_\pi^2} \left(- \frac{1}{4\pi} \frac{m_\pi^4}{M} \log m_\pi^2 + \mathcal{O}(m_\pi^5) \right)$$

Self-energy

- Equal time (infinite momentum frame)

$$\Sigma_{\mathrm{IMF}}^{(+-)} = -\frac{3g_A^2 M}{16\pi^3 f_\pi^2}\int_{-\infty}^{\infty} dy \int d^2k_\perp \frac{P}{2E'}\frac{1}{2\omega_k}\frac{m_\pi^2}{(E-E'-\omega_k)}$$

$$= \frac{3g_A^2 M}{32\pi^2 f_\pi^2}\int_0^1 dy \int_0^{\Lambda^2} dk_\perp^2 \frac{m_\pi^2}{k_\perp^2 + M^2(1-y)^2 + m_\pi^2 y}$$

$$p_z \equiv P \to \infty$$

$$y = p_z'/p_z$$

$$\Sigma_{\mathrm{IMF}}^{(-+)} = \frac{3g_A^2 M}{16\pi^3 f_\pi^2}\int_{-\infty}^{\infty} dy \int d^2k_\perp \frac{P}{2E'}\frac{1}{2\omega_k}\frac{m_\pi^2}{(E+E'+\omega_k)} = \mathcal{O}(1/P^2)$$

→ nonanalytic behavior as for rest frame expression

$$\Sigma_{\mathrm{IMF}}^{\mathrm{LNA}} = \Sigma_{\mathrm{IMF}}^{(+-)\mathrm{LNA}} = -\frac{3g_A^2}{32\pi f_\pi^2}\left(m_\pi^3 + \frac{1}{2\pi}\frac{m_\pi^4}{M}\log m_\pi^2 + \mathcal{O}(m_\pi^5)\right)$$

Self-energy

- **Light-front**

$$\int dk^+ dk^- d^2k_\perp \frac{1}{D_\pi D_N} = \frac{1}{p^+}\int_{-\infty}^{\infty} \frac{dx}{x(x-1)} d^2k_\perp \int dk^- \left(k^- - \frac{k_\perp^2 + m_\pi^2}{xp^+} + \frac{i\varepsilon}{xp^+}\right)^{-1}$$
$$\times \left(k^- - \frac{M^2}{p^+} - \frac{k_\perp^2 + M^2}{(x-1)p^+} + \frac{i\varepsilon}{(x-1)p^+}\right)^{-1}$$
$$= 2\pi^2 i \int_0^1 dx\ dk_\perp^2\ \frac{1}{k_\perp^2 + (1-x)m_\pi^2 + x^2 M^2}$$

$$x = k^+/p^+$$

$\longrightarrow$ identical nonanalytic results as covariant & instant form

$$\Sigma_{\mathrm{LF}}^{\mathrm{LNA}} = -\frac{3g_A^2}{32\pi f_\pi^2}\left(m_\pi^3 + \frac{1}{2\pi}\frac{m_\pi^4}{M}\log m_\pi^2 + \mathcal{O}(m_\pi^5)\right)$$

*C.-R. Ji, WM, Thomas, PRD **80**, 054018 (2009)*

Self-energy

■ Light-front

→ $1/D_N$ "tadpole" term has k^- pole that depends on k^+ and moves to infinity as $k^+ \to 0$
("treacherous" in LF dynamics)

→ use LF cylindrical coordinates $k^+ = r\cos\phi,\ k^- = r\sin\phi$

$$\int d^4k \frac{1}{D_N} = \frac{1}{2}\int d^2k_\perp \int \frac{dk^+}{k^+} \int dk^- \left(k^- - \frac{k_\perp^2 + M^2}{k^+} + \frac{i\varepsilon}{k^+}\right)^{-1}$$

$$= -2\pi \int d^2k_\perp \left[\int_0^{r_0} dr \frac{r}{\sqrt{r_0^4 - r^4}} + i \lim_{R\to\infty} \int_{r_0}^{R} dr \frac{r}{\sqrt{r^4 - r_0^4}}\right]$$

$$= \frac{1}{2}\int d^2k_\perp \lim_{R\to\infty} \left(-\pi^2 + 2\pi i \ \log\frac{r_0^2}{R^2} + \mathcal{O}(1/R^4)\right)$$

$$r_0 = \sqrt{2(k_\perp^2 + M^2)}$$

contains $\log(k_\perp^2 + M^2)$
term as required

Self-energy

- Pseudoscalar interaction

$$\Sigma^{\rm PS} = i g_{\pi NN}^2 \; \overline{u}(p) \int \frac{d^4k}{(2\pi)^4} \, (\gamma_5 \vec{\tau}) \, \frac{i \, (\not{p} - \not{k} + M)}{D_N} (\gamma_5 \vec{\tau}) \frac{i}{D_\pi^2} \, u(p)$$

$$= -\frac{3 i g_A^2 M}{2 f_\pi^2} \int \frac{d^4k}{(2\pi)^4} \left[\frac{m_\pi^2}{D_\pi D_N} + \frac{1}{D_N} - \frac{1}{D_\pi} \right]$$

→ contains additional ("treacherous") pion "tadpole" term

→ similar evaluation as for $1/D_N$ term

$$\Sigma_{\rm LNA}^{\rm PS} = \frac{3 g_A^2}{32\pi f_\pi^2} \left(\frac{M}{\pi} m_\pi^2 \log m_\pi^2 - m_\pi^3 - \frac{m_\pi^4}{2\pi M^2} \log \frac{m_\pi^2}{M^2} + \mathcal{O}(m_\pi^5) \right)$$

additional *lower order* term in PS theory!

Self-energy

- Alberg & Miller claim on light-front $\Sigma^{\rm PS} = \Sigma^{\rm PV}$
  – form factor removes $k^+ = 0$ contribution *PRL* ***108***, *172001 (2012)*

- In practice, AM drop "treacherous" $k^+ = 0$ (end-point) term

$$\Sigma^{\rm PS} = \Sigma^{\rm PV} + \Sigma^{\rm PS}_{\rm end-pt}$$

after which PS result happens to coincide with PV

→ but, *even with* form factors, end-point term is non-zero

$$\Sigma^{\rm PS}_{\rm end-pt} = \frac{3g_A^2 M}{16\pi^2 f_\pi^2} \int_0^\infty dt \frac{\sqrt{t}\, F^2(m_\pi^2, -t)}{\sqrt{t + m_\pi^2}} \quad \xrightarrow{\rm LNA} \quad \frac{3g_A^2}{32\pi f_\pi^2} \frac{M}{\pi} m_\pi^2 \log m_\pi^2$$

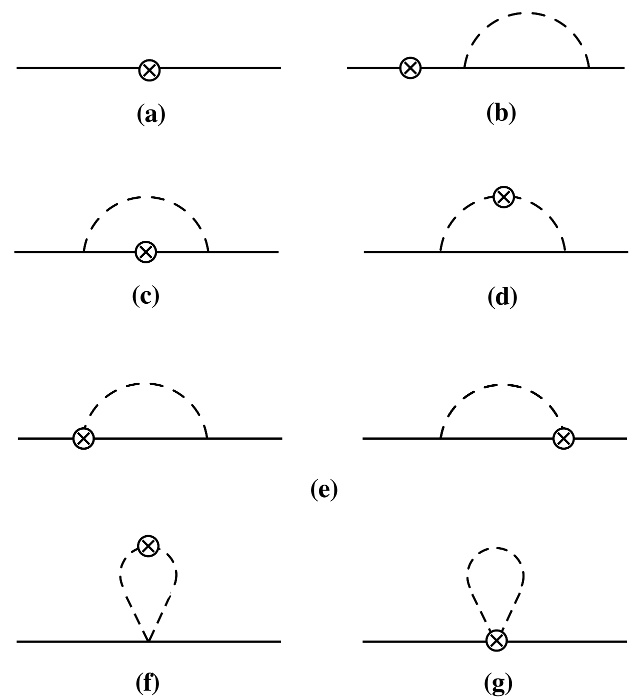
Ji, WM, Thomas, arXiv:1206.3671

→ *ansatz* does not work for other quantities
e.g. vertex renormalization

Vertex corrections

- Pion cloud corrections to electromagnetic N coupling

  → N rainbow **(c)**, π rainbow **(d)**, Weinberg-Tomozawa **(e)**, π tadpole **(f)**, N tadpole **(g)**



- Vertex renormalization

$$(Z_1^{-1} - 1)\,\bar{u}(p)\,\gamma^\mu\,u(p) = \bar{u}(p)\,\Lambda^\mu\,u(p)$$

→ taking "+" components: $Z_1^{-1} - 1 \approx 1 - Z_1 = \dfrac{M}{p^+}\,\bar{u}(p)\,\Lambda^+\,u(p)$

→ $e.g.$ for N rainbow contribution,

$$\Lambda_\mu^N = -\frac{\partial\hat{\Sigma}}{\partial p^\mu}$$

Vertex corrections

- Define light-cone momentum distributions $f_i(y)$

$$1 - Z_1^i = \int dy\, f_i(y)$$

where
$$\begin{aligned} f_\pi(y) &= f^{(\mathrm{on})}(y) - f^{(\delta)}(y) \\ f_N(y) &= f^{(\mathrm{on})}(y) - f^{(\mathrm{off})}(y) + f^{(\delta)}(y) \\ f_{\mathrm{WT}}(y) &= -f^{(\mathrm{off})}(y) + 2 f^{(\delta)}(y) \\ f_{\pi(\mathrm{tad})}(y) &= -f_{N(\mathrm{tad})}(y) \;=\; f^{(\mathrm{tad})}(y) \end{aligned}$$

with components
$$\begin{aligned} f^{(\mathrm{on})}(y) &= \frac{g_A^2 M^2}{(4\pi f_\pi)^2} \int dk_\perp^2\, \frac{y(k_\perp^2 + y^2 M^2)}{\left[k_\perp^2 + y^2 M^2 + (1-y) m_\pi^2\right]^2} \\ f^{(\mathrm{off})}(y) &= \frac{g_A^2 M^2}{(4\pi f_\pi)^2} \int dk_\perp^2\, \frac{y}{k_\perp^2 + y^2 M^2 + (1-y) m_\pi^2} \\ f^{(\delta)}(y) &= -\frac{g_A^2}{4(4\pi f_\pi)^2} \int dk_\perp^2\, \log\left(\frac{k_\perp^2 + m_\pi^2}{\mu^2}\right) \delta(y) \\ f^{(\mathrm{tad})}(y) &= -\frac{1}{2(4\pi f_\pi)^2} \int dk_\perp^2\, \log\left(\frac{k_\perp^2 + m_\pi^2}{\mu^2}\right) \delta(y) \end{aligned}$$

- Pion distribution $f_\pi(y)$ contains *on-shell* contribution $f^{(\mathrm{on})}(y)$ equivalent to PS result

- Nucleon distribution $f_N(y)$ contains in addition new *off-shell* contribution $f^{(\mathrm{off})}(y)$

- Both contain $\delta(y)$ components $f^{(\delta)}(y)$ which are present only in PV theory

- Weinberg-Tomozawa term $f^{(\mathrm{WT})}(y)$ needed for gauge invariance

$$(1 - Z_1^N) = (1 - Z_1^\pi) + (1 - Z_1^{\mathrm{WT}})$$

- Nucleon and pion tadpole terms equal & opposite

$$(1 - Z_1^{\pi\,(\mathrm{tad})}) + (1 - Z_1^{N\,(\mathrm{tad})}) = 0$$

Nonanalytic behavior of vertex renormalization factors

$$1 - Z_1^N \xrightarrow{\rm NA} \frac{3g_A^2}{4(4\pi f_\pi)^2}\left\{ m_\pi^2 \log m_\pi^2 - \pi \frac{m_\pi^3}{M} - \frac{2m_\pi^4}{3M^2}\log m_\pi^2 + \mathcal{O}(m_\pi^5)\right\}$$

$$1 - Z_1^\pi \xrightarrow{\rm NA} \frac{3g_A^2}{4(4\pi f_\pi)^2}\left\{ m_\pi^2 \log m_\pi^2 - \frac{5\pi}{3}\frac{m_\pi^3}{M} - \frac{m_\pi^4}{M^2}\log m_\pi^2 + \mathcal{O}(m_\pi^5)\right\}$$

$$1 - Z_1^{\rm WT} \xrightarrow{\rm NA} \frac{3g_A^2}{4(4\pi f_\pi)^2}\left\{ \frac{2\pi}{3}\frac{m_\pi^3}{M} - \frac{m_\pi^4}{3M^2}\log m_\pi^2 + \mathcal{O}(m_\pi^5)\right\}$$

$$1 - Z_1^{N\,({\rm tad})} \xrightarrow{\rm NA} -\frac{1}{2(4\pi f_\pi)^2}\, m_\pi^2 \log m_\pi^2$$

$$1 - Z_1^{\pi\,({\rm tad})} \xrightarrow{\rm NA} \frac{1}{2(4\pi f_\pi)^2}\, m_\pi^2 \log m_\pi^2$$

→ cancellation of $m_\pi^2 \log m_\pi^2$ terms in WT contribution

→ demonstration of gauge invariance condition
(in fact, to *all* orders!)

Nonanalytic behavior of vertex renormalization factors

	$1/D_\pi D_N^2$	$1/D_\pi^2 D_N$	$1/D_\pi D_N$	$1/D_\pi$ or $1/D_\pi^2$	sum (PV)	sum (PS)
$1 - Z_1^N$	g_A^2 *	0	$-\frac{1}{2}g_A^2$	$\frac{1}{4}g_A^2$	$\frac{3}{4}g_A^2$	g_A^2
$1 - Z_1^\pi$	0	g_A^2 *	0	$-\frac{1}{4}g_A^2$	$\frac{3}{4}g_A^2$	g_A^2
$1 - Z_1^{\mathrm{WT}}$	0	0	$-\frac{1}{2}g_A^2$	$\frac{1}{2}g_A^2$	0	0
$1 - Z_1^{N\,\mathrm{tad}}$	0	0	0	$-1/2$	$-1/2$	0
$1 - Z_1^{\pi\,\mathrm{tad}}$	0	0	0	$1/2$	$1/2$	0

* also in PS

in units of $\dfrac{1}{(4\pi f_\pi)^2}\, m_\pi^2 \log m_\pi^2$

→ origin of ChPT *vs*. Sullivan process difference clear

$$\left(1 - Z_1^{N\,(\mathrm{PV})}\right)_{\mathrm{LNA}} = \frac{3}{4}\left(1 - Z_1^{N\,(\mathrm{PS})}\right)_{\mathrm{LNA}}$$

Moments of PDFs

- PDF moments related to nucleon matrix elements of local twist-2 operators

$$\langle N|\widehat{\mathcal{O}}_q^{\mu_1\cdots\mu_n}|N\rangle \;=\; 2\,\langle x^{n-1}\rangle_q\; p^{\{\mu_1}\cdots p^{\mu_n\}}$$

$\longrightarrow$ n-th moment of (spin-averaged) PDF $q(x)$

$$\langle x^{n-1}\rangle_q \;=\; \int_0^1 dx\, x^{n-1}\,(q(x)+(-1)^n\bar{q}(x))$$

$\longrightarrow$ operator is $\widehat{\mathcal{O}}_q^{\mu_1\cdots\mu_n} \;=\; \bar{\psi}\gamma^{\{\mu_1}\,iD^{\mu_2}\cdots iD^{\mu_n\}}\psi \;-\; \text{traces}$

- Lowest $(n{=}1)$ moment $\langle x^0\rangle_q \equiv \mathcal{M}_N+\mathcal{M}_\pi$ given by vertex renormalization factors $\sim 1-Z_1^i$

- For couplings involving nucleons

$$\mathcal{M}_N^{(p)} = Z_2 + (1 - Z_1^N) + (1 - Z_1^{N\,(\mathrm{tad})})$$
$$\mathcal{M}_N^{(n)} = 2(1 - Z_1^N) - (1 - Z_1^{N\,(\mathrm{tad})})$$

$\longrightarrow$ wave function renormalization

$$1 - Z_2 = (1 - Z_1^p) + (1 - Z_1^n) \equiv 3(1 - Z_1^N)$$

- For couplings involving only pions

$$\mathcal{M}_\pi^{(p)} = 2(1 - Z_1^\pi) + 2(1 - Z_1^{\mathrm{WT}}) + (1 - Z_1^{\pi\,(\mathrm{tad})})$$
$$\mathcal{M}_\pi^{(n)} = -2(1 - Z_1^\pi) - 2(1 - Z_1^{\mathrm{WT}}) - (1 - Z_1^{\pi\,(\mathrm{tad})})$$

Nonanalytic behavior

$$\mathcal{M}_N^{(p)} \xrightarrow{\text{LNA}} 1 - \frac{(3g_A^2+1)}{2(4\pi f_\pi)^2}\, m_\pi^2 \log m_\pi^2 \qquad \mathcal{M}_\pi^{(p)} \xrightarrow{\text{LNA}} \frac{(3g_A^2+1)}{2(4\pi f_\pi)^2}\, m_\pi^2 \log m_\pi^2$$

$$\mathcal{M}_N^{(n)} \xrightarrow{\text{LNA}} \frac{(3g_A^2+1)}{2(4\pi f_\pi)^2}\, m_\pi^2 \log m_\pi^2 \qquad \mathcal{M}_\pi^{(n)} \xrightarrow{\text{LNA}} -\frac{(3g_A^2+1)}{2(4\pi f_\pi)^2}\, m_\pi^2 \log m_\pi^2$$

→ no pion corrections to isosclar moments

→ isovector correction agrees with ChPT calculation

$$\mathcal{M}_N^{(p-n)} \xrightarrow{\text{LNA}} 1 - \frac{\left(4g_A^2 + [1-g_A^2]\right)}{(4\pi f_\pi)^2}\, m_\pi^2 \log m_\pi^2$$

$$\mathcal{M}_\pi^{(p-n)} \xrightarrow{\text{LNA}} \frac{\left(4g_A^2 + [1-g_A^2]\right)}{(4\pi f_\pi)^2}\, m_\pi^2 \log m_\pi^2$$

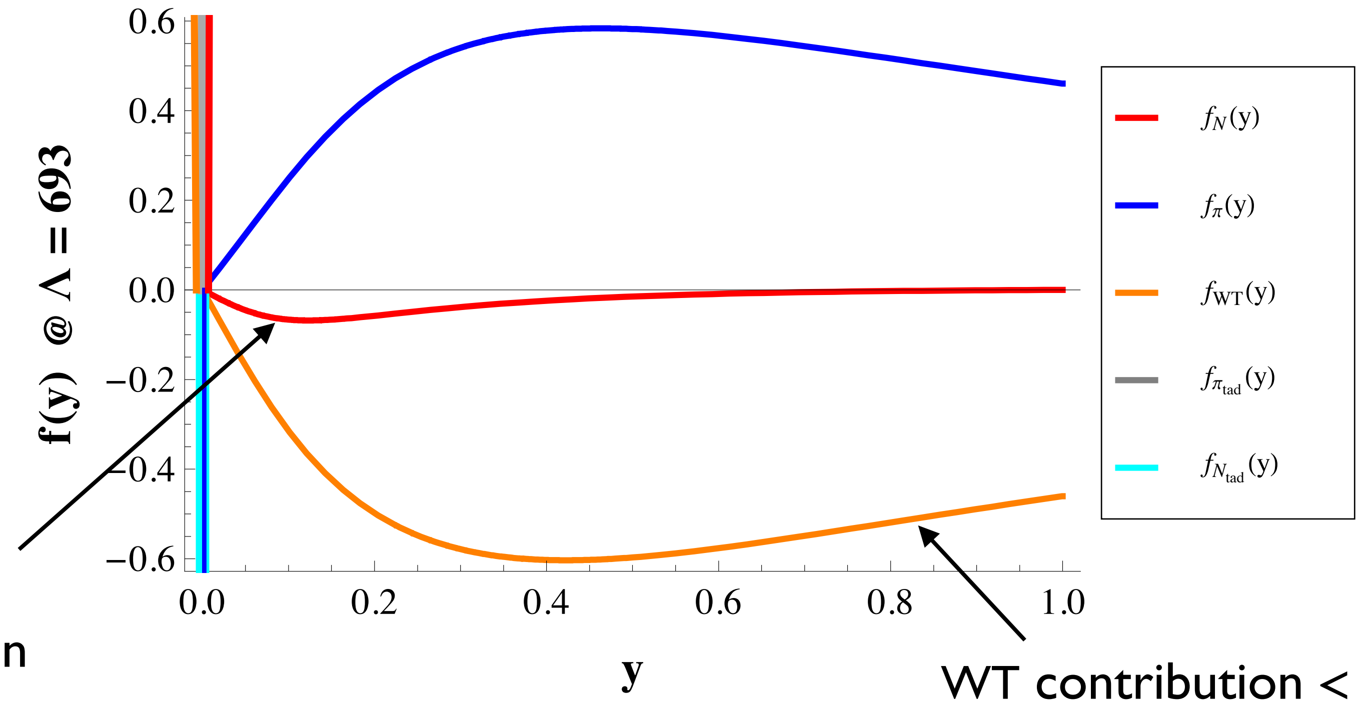
PS (“on-shell”) contribution

δ-function contribution

Pion distribution functions

- Using phenomenological form factors, compute functions $f_i(y)$ numerically

→ for transverse momentum cut-off $F(k_\perp) = \Theta(k_\perp^2 - \Lambda^2)$



large cancelations of on- and off-shell terms in N rainbow distribution

WT contribution < 0 because of off-shell term

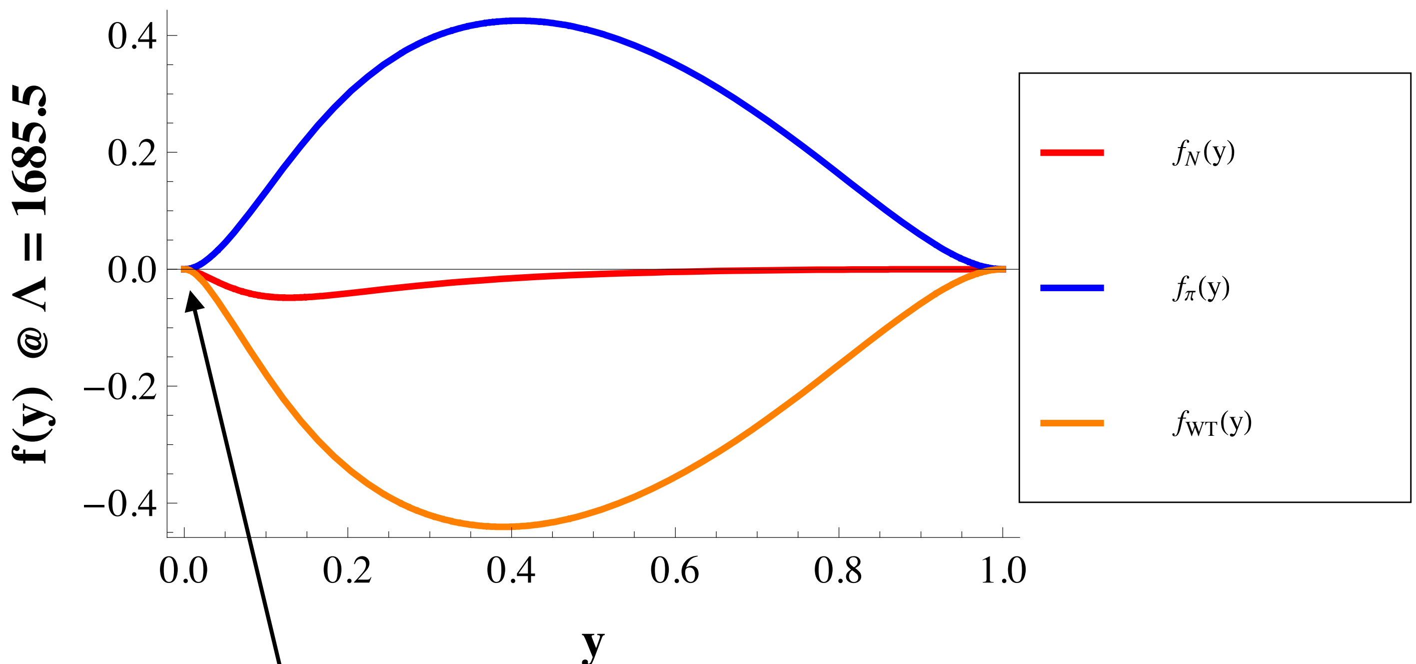
→ symmetry relation respected

$$f_\pi(y) + f_{\rm WT}(y) + \frac{1}{2} f_{\pi({\rm tad})}(y) = f_N(y) - \frac{1}{2} f_{N({\rm tad})}(y)$$

Pion distribution functions

- Using phenomenological form factors, compute functions $f_i(y)$ numerically

→ s-dependent (dipole) form factor $s_{\pi N} = \dfrac{k_\perp^2 + m_\pi^2}{y} + \dfrac{k_\perp^2 + M^2}{1-y}$



suppresses contributions
at $y = 0$ and $y = 1$
– no tadpoles!

Hendricks, Ji, WM, Thomas (2012)

Summary

- Equivalence demonstrated between self-energy in equal-time, covariant, and light-front formalisms

$$\Sigma_{\rm cov}^{\rm LNA} = \Sigma_{\rm ET}^{(+-)\rm LNA} + \Sigma_{\rm ET}^{(-+)\rm LNA} = \Sigma_{\rm IMF}^{(+-)\rm LNA} = \Sigma_{\rm LF}^{\rm LNA}$$

$\longrightarrow$ non-trivial due to end-point singularities

$\longrightarrow$ PV and PS results clearly differ

- Vertex corrections satisfy gauge invariance relations

$$(1 - Z_1^N) = (1 - Z_1^\pi) + (1 - Z_1^{\rm WT})$$

$\longrightarrow$ difference between PDF moments in ChPT (PV) $\&$ "Sullivan" process (PS)

$\longrightarrow$ model-independent constraints on pion light-cone momentum distributions

(impact on $\bar{d} - \bar{u}$ data analysis in progress)