Hall A A_1^n Collaboration Meeting
May 24, 2012

Quark spin distributions at large x

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Outline

- Why large-x quarks are important

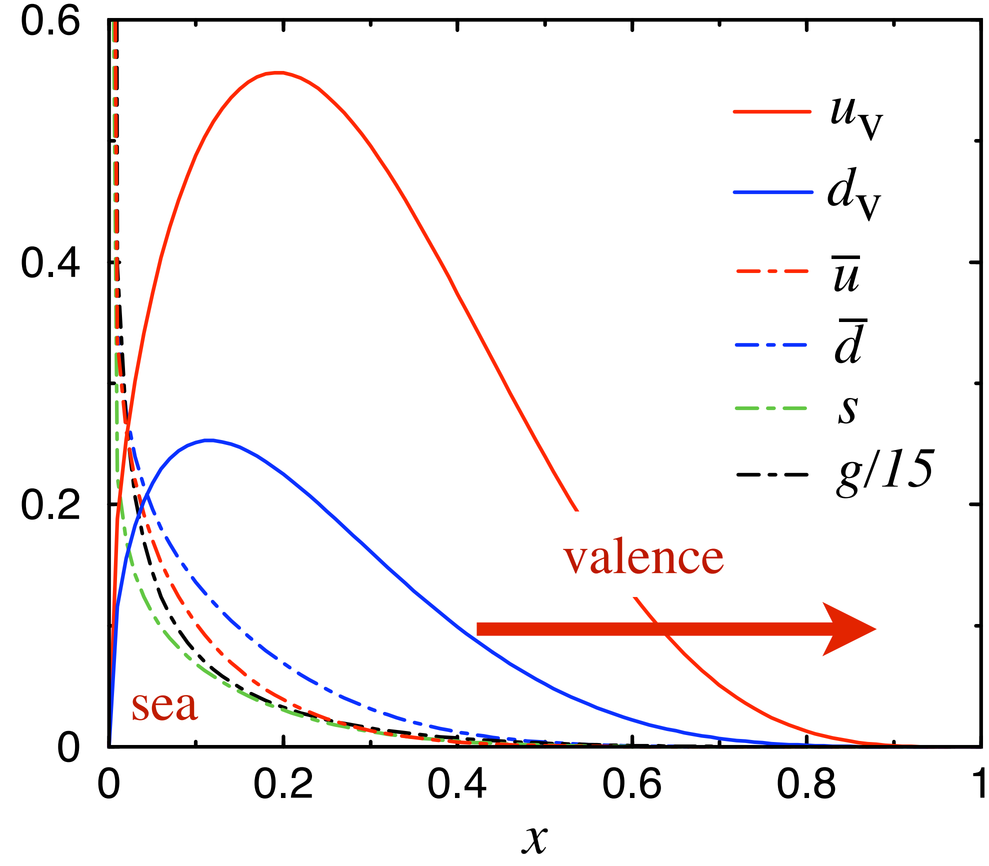
- $x \rightarrow 1$ behavior from perturbative QCD
  - → role of orbital angular momentum

- Nuclear effects in ^{3}He
  - → limitations of "effective polarizations"

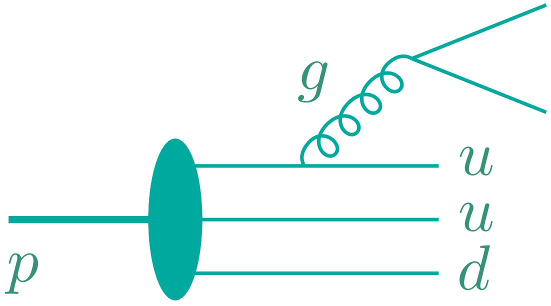
Why large x?

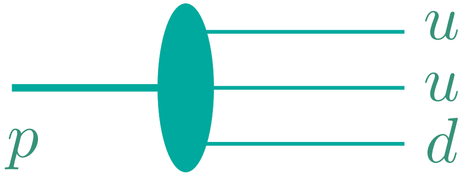
- Most direct connection between quark distributions and models of nucleon structure is via *valence* quarks

→ most cleanly revealed at $x > 0.4$



structure of *hadron*
or structure of *probe*?



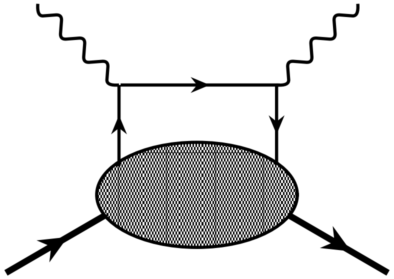


- Ideal testing ground for nonperturbative & perturbative models of the nucleon

→ *e.g.* ratio d/u or $\Delta d/d$ sensitive to spin-flavor dynamics

SU(6) proton wave function

$$\begin{aligned} p^\uparrow &= -\frac{1}{3}d^\uparrow(uu)_1 - \frac{\sqrt{2}}{3}d^\downarrow(uu)_1 \\ &\quad + \frac{\sqrt{2}}{6}u^\uparrow(ud)_1 - \frac{1}{3}u^\downarrow(ud)_1 + \frac{1}{\sqrt{2}}u^\uparrow(ud)_0 \end{aligned}$$



interacting quark

spectator "diquark"

diquark spin

- 🟥 SU(6) symmetry

50% S=0 (qq)
50% S=1 (qq)

→ $u(x) = 2\, d(x)$ for all x

- $$\frac{d}{u} = \frac{1}{2} \qquad \frac{F_2^n}{F_2^p} = \frac{2}{3}$$

- $$\frac{\Delta u}{u} = \frac{2}{3} \qquad A_1^p = \frac{5}{9}$$

  $$\frac{\Delta d}{d} = -\frac{1}{3} \qquad A_1^n = 0$$

e.g. Close, "An Introduction to Quarks and Partons" (1979)

Scalar diquark dominance

$M_\Delta > M$ ⟶ $(qq)_1$ has larger energy than $(qq)_0$

⟶ scalar diquark dominant in $x \to 1$ limit

⟶ since only u quarks couple to scalar diquarks

- $\dfrac{d}{u} \to 0$ $\qquad\qquad$ $\dfrac{F_2^n}{F_2^p} \to \dfrac{1}{4}$

- $\dfrac{\Delta u}{u} \to 1$ $\qquad\qquad$ $A_1^p \to 1$

  $\dfrac{\Delta d}{d} \to -\dfrac{1}{3}$ $\qquad\qquad$ $A_1^n \to 1$

Feynman, "Photon-Hadron Interactions" (1972)
*Close, PLB **43**, 422 (1973)*
*Close, Thomas, PLB **212**, 227 (1988)*

Local duality models

→ duality in quark models realized by summing over complete sets of *even* and *odd* parity resonances, *e.g.* **56** (L=0) and **70** (L=1) multiplets of SU(6)

representation	$^2\mathbf{8}[\mathbf{56}^+]$	$^4\mathbf{10}[\mathbf{56}^+]$	$^2\mathbf{8}[\mathbf{70}^-]$	$^4\mathbf{8}[\mathbf{70}^-]$	$^2\mathbf{10}[\mathbf{70}^-]$	Total
F_1^p	$9\rho^2$	$8\lambda^2$	$9\rho^2$	0	λ^2	$18\rho^2+9\lambda^2$
F_1^n	$(3\rho+\lambda)^2/4$	$8\lambda^2$	$(3\rho-\lambda)^2/4$	$4\lambda^2$	λ^2	$(9\rho^2+27\lambda^2)/2$
g_1^p	$9\rho^2$	$-4\lambda^2$	$9\rho^2$	0	λ^2	$18\rho^2-3\lambda^2$
g_1^n	$(3\rho+\lambda)^2/4$	$-4\lambda^2$	$(3\rho-\lambda)^2/4$	$-2\lambda^2$	λ^2	$(9\rho^2-9\lambda^2)/2$

$\lambda\ (\rho) =$ (anti) symmetric component of ground state wave function

→ summing over all resonances in $\mathbf{56}^+$ and $\mathbf{70}^-$ multiplets

$$F_1^n/F_1^p \to 2/3, \quad A_1^p \to 5/9, \quad A_1^n \to 0$$

as in parton model (if u=$2d$)!

■ Local duality models

→ various ways of breaking $\mathrm{SU}(6)$ while respecting duality

- spin-$1/2$ dominance (Δ suppression)

$$A_1^p = \frac{19 - 23\sin^2\theta_s}{19 - 11\sin^2\theta_s} \qquad A_1^n = \frac{1 - 2\sin^2\theta_s}{1 + \sin^2\theta_s}$$

- helicity-$1/2$ dominance

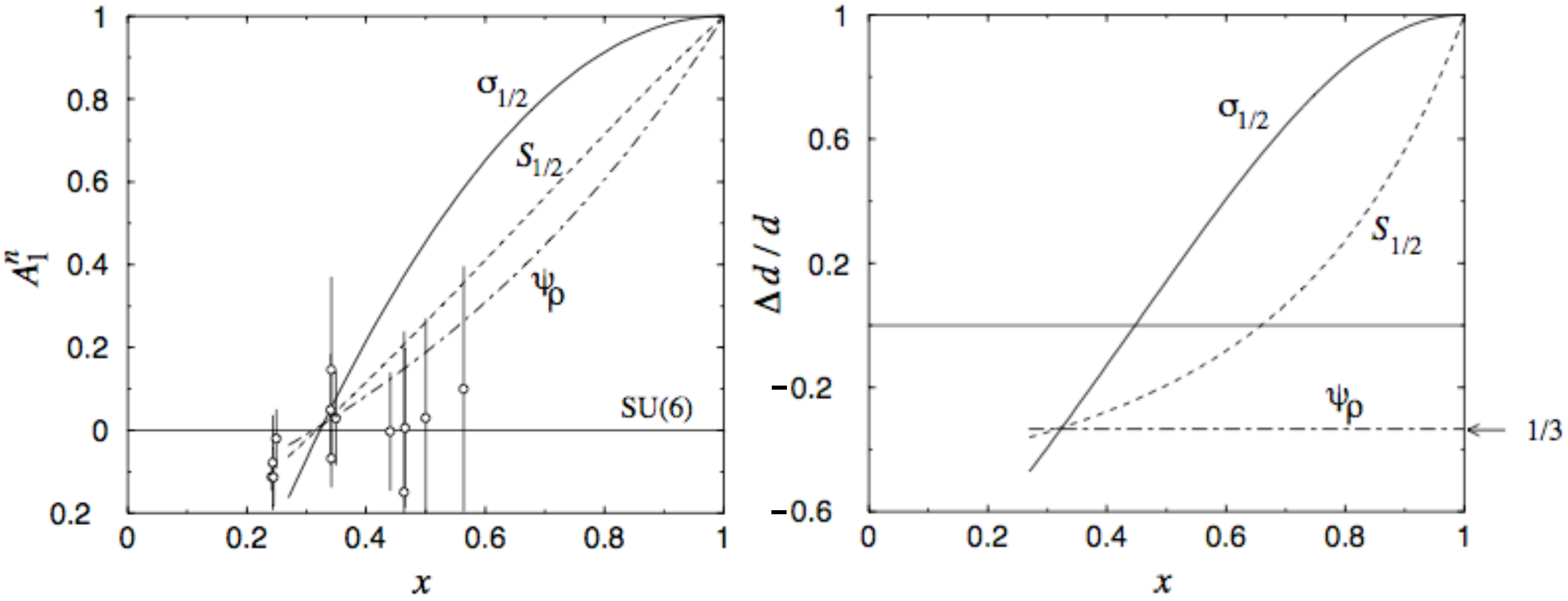
$$A_1^p = \frac{7 - 9\sin^2\theta_h}{7 - 5\sin^2\theta_h} \qquad A_1^n = 1 - 2\sin^2\theta_h$$

- antisymmetric wave function (ρ) dominance

$$A_1^p = \frac{6 - 7\sin^2\theta_w}{6 - 3\sin^2\theta_w} \qquad A_1^n = \frac{1 - 2\sin^2\theta_w}{1 + 2\sin^2\theta_w}$$

Local duality models

→ various ways of breaking SU(6) while respecting duality



Close, WM, PRC ***68****, 035210 (2003)*

Local duality models

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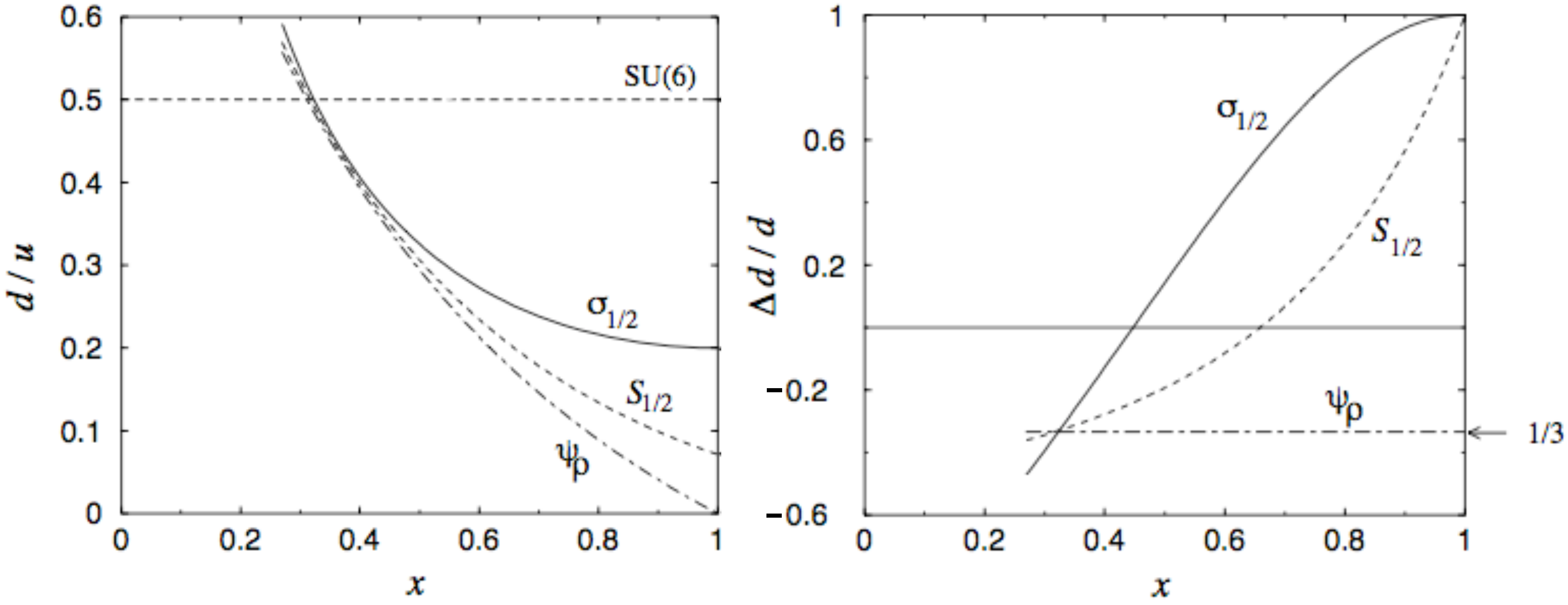


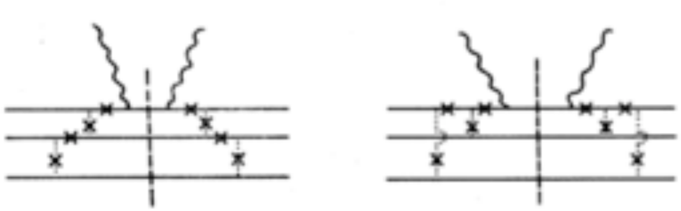
*Close, WM, PRC **68**, 035210 (2003)*

→ x dependence of polarized & unpolarized PDFs correlated

Perturbative QCD

Perturbative QCD

- In QCD, "exceptional" $x \rightarrow 1$ configurations of proton wave function generated from "typical" wave function (for which $x_i \sim 1/3$) by exchange of ≥ 2 hard gluons, with mass $k^2 \sim -\langle k_\perp^2 \rangle/(1-x)$



Farrar, Jackson, PRL ***35****, 1416 (1975)*

- Since $|k^2|$ is large, coupling at q-g vertex is small

  ⟶ use lowest-order perturbation theory!

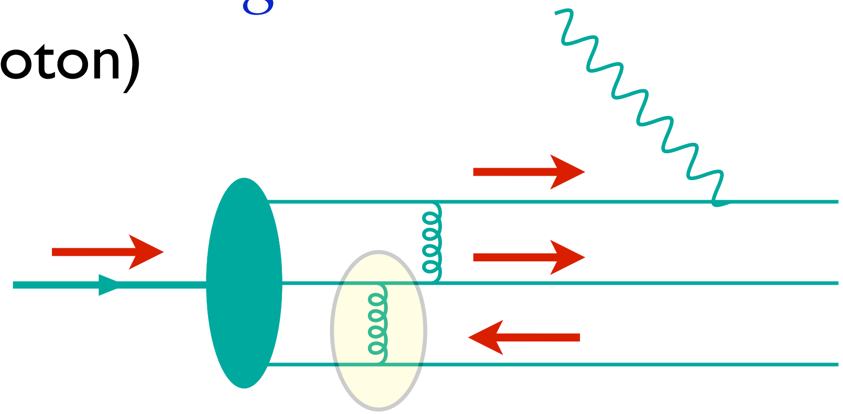
- Assume wave function vanishes sufficiently fast as $|k^2| \to \infty$ and *unperturbed* wave function dominated by 3-quark Fock component with $\mathrm{SU}(2) \times \mathrm{SU}(3)$ symmetry

Perturbative QCD

- If spectator "diquark" spins are *anti-aligned*

  (helicity of struck quark = helicity of proton)

  ⟶ can exchange *transverse or longitudinal* gluon



- If spectator "diquark" spins are *aligned*

  (helicity of struck quark ≠ helicity of proton)

  ⟶ can exchange *only longitudinal* gluon

- Coupling of (large-k^2) longitudinal gluon to (small-p^2) quark is suppressed by $(p^2/k^2)^{1/2} \sim (1-x)^{1/2}$ w.r.t. transverse

  ⟶ $q^{\downarrow} \sim (1-x)^2 \, q^{\uparrow} \sim (1-x)^5$

Perturbative QCD

- Phenomenological consequences of $S_z = 0$ qq dominance*
  (dominance of helicity-$1/2$ photoproduction cross section)

→ assuming unperturbed $\mathrm{SU(6)}$ wave function

- $\frac{d}{u} \to \frac{1}{5}$ $\qquad$ $\frac{F_2^n}{F_2^p} \to \frac{3}{7}$

- $\frac{\Delta u}{u} \to 1$ $\qquad$ $A_1^p \to 1$

  $\frac{\Delta d}{d} \to 1$ $\qquad$ $A_1^n \to 1$

→ dramatically different predictions for $\Delta d/d$
cf. nonperturbative models

* valid in Abelian & non-Abelian theories

Role of orbital angular momentum

- Above results assume quarks in lowest Fock state are in relative s-wave

  → higher Fock states and nonzero quark OAM will in general introduce additional suppression in $(1-x)$

- BUT nonzero OAM can provide logarithmic enhancement of *helicity-flip* amplitudes!

  → quark OAM modifies asymptotic behavior of nucleon's Pauli form factor

$$F_2(Q^2) \ \sim \ \log^2(Q^2/\Lambda^2)\frac{1}{Q^6}$$

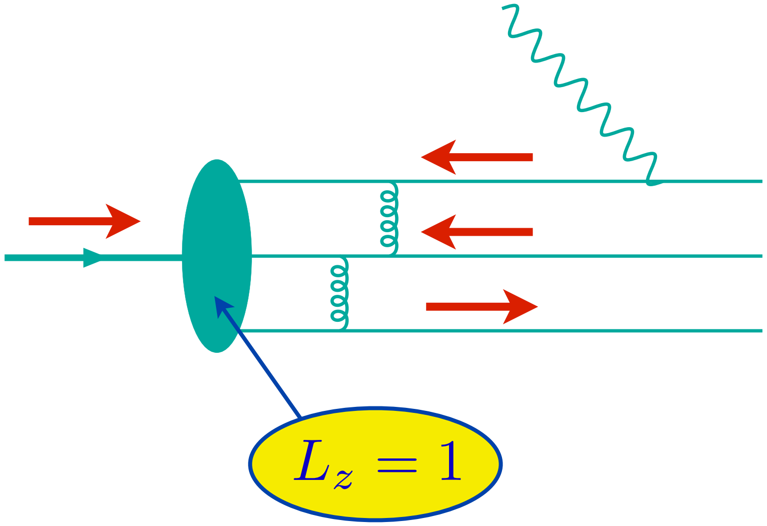
Belitsky, Ji, Yuan
PRL ***91****, 092003 (2003)*

  → consistent with surprising Q^2 dependence of proton's G_E/G_M form factor ratio

Role of orbital angular momentum

- For $L_z = 1$ Fock state, expand hard scattering amplitude in powers of $k_\perp$ (“collinear expansion”)

→ logarithmic singularities arise when integrating over longitudinal momentum fractions x_i of soft quarks



→ leads to additional $\log^2(1{-}x)$ enhancement of $q^\downarrow$

$$q^\downarrow \sim (1-x)^5 \log^2(1-x)$$

*Avakian, Brodsky, Deur, Yuan, PRL **99**, 082001 (2007)*

(similar contributions to positive helicity $q^\uparrow$ are power-suppressed)

Role of orbital angular momentum

- $k_\perp$-odd transverse momentum dependent (TMD) distributions (vanish after $k_\perp$ integration)

  ⟶ arise from *interference* between $L_z = 0$ and $L_z = 1$ states

- T-even TMDs

  ⟶ g_{1T} (*longitudinally* polarized q in a *transversely* polarized N)

  h_{1L} (*transversely* polarized q in a *longitudinally* polarized N)

- T-odd TMDs

  ⟶ $f_{1T}^{\perp}$ (*unpolarized* q in a *transversely* polarized N – "Sivers")

  $h_1^{\perp}$ (*transversely* polarized q in an *unpolarized* N – "Boer-Mulders")

- Each behaves in $x \rightarrow 1$ limit as

$$\mathrm{TMD} \ \sim \ (1-x)^4$$

Brodsky, Yuan
PRD ***74****, 094018 (2006)*

Phenomenological implications

- Power counting rule constraints used in exploratory fit to limited set of inclusive DIS spin structure function data

$$q^{\uparrow} = x^{\alpha}\left[A(1-x)^3 + B(1-x)^4\right]$$

$$q^{\downarrow} = x^{\alpha}\left[C(1-x)^5 + D(1-x)^6\right]$$

Brodsky, Burkardt, Schmidt
NPB ***441****, 197 (1995)*

Phenomenological implications

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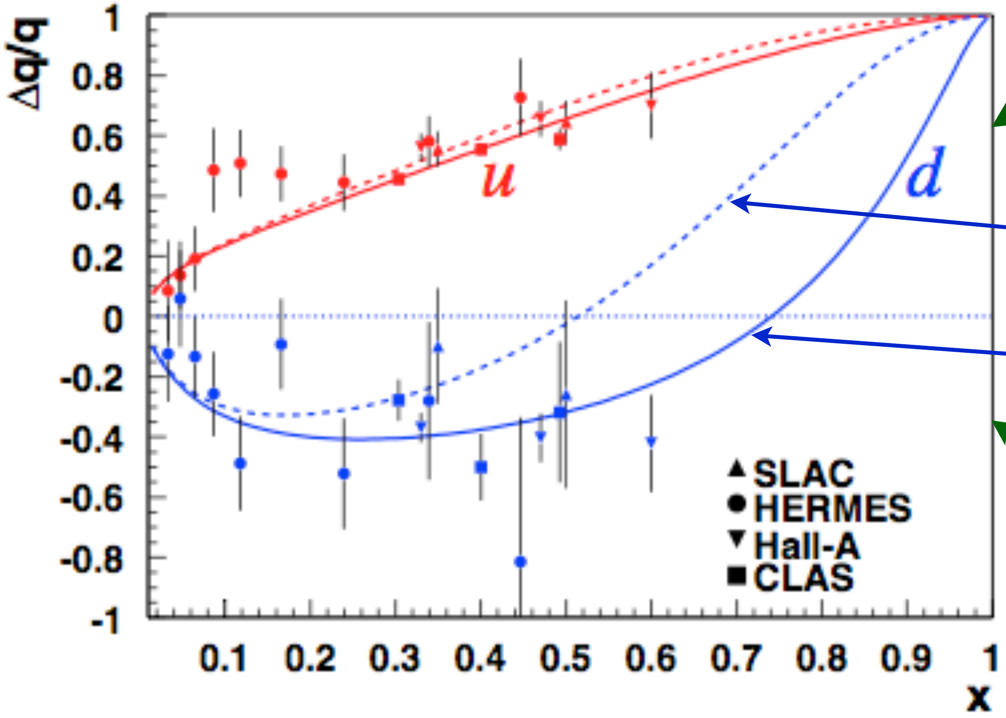
$$q^{\downarrow} = x^{\alpha}\left[C(1-x)^5 + D(1-x)^6 + C'(1-x)^5\log^2(1-x)\right]$$

additional $L_z = 1$ term



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LSS'98 (pQCD-inspired)

ABDY'07 (including OAM)

−1/3

*Avakian et al., PRL **99**, 082001 (2007)*

→ improved fit for $\Delta d/d$

Phenomenological implications

- Determining $x \to 1$ behavior experimentally is problematic

→ simple $x^\alpha (1-x)^\beta$ parametrizations inadequate for describing *high-precision* data, and global fits typically require more complicated x dependence, *e.g.*

$$q \sim x^\alpha (1-x)^\beta \, (1 + \gamma \sqrt{x} + \eta \, x)$$

→ recent global fits of spin-dependent PDFs find (at $Q^2 \sim 5 \text{ GeV}^2$)

$$\beta \approx 3.3 \, (\Delta u_V), \; 3.9 \, (\Delta d_V)$$

de Florian et al.
PRD ***80****, 034030 (2009)*

but with $\gamma, \eta \sim \mathcal{O}(10\text{–}100)$

- Challenge to perform constrained *global* fit to all DIS, SIDIS & $\vec{p}\,\vec{p}$ scattering data

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Leader, Sidorov, Stamenov
PRD **82**, *114018 (2010)*

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→ recent global fits of spin-dependent PDFs find (at $Q^2 \sim 5\ \mathrm{GeV}^2$)

$$\beta \;\approx\; 3.0\ (\Delta u_V),\ 4.1\ (\Delta d_V)$$

Bluemlein, Boettcher
*NPB **841**, 205 (2010)*

but with $\gamma, \eta \;\sim\; \mathcal{O}(10\text{–}100)$

- Challenge to perform constrained *global* fit to all DIS, SIDIS & $\vec{p}\,\vec{p}$ scattering data

Phenomenological implications

- Challenges for large-x PDF analysis

  → at fixed Q^2, increasing x corresponds to decreasing W – eventually run into nucleon *resonance* region as $x \to 1$

  → subleading $1/Q^2$ corrections (target mass, higher twists)

  → nuclear corrections in extraction of *neutron* information from nuclear (deuterium, ^{3}He) data

- New "JAM" (JLab Angular Momentum) global PDF analysis* dedicated to describing large-x, moderate-Q^2 region

  → preliminary results expected this summer

  → global spin asymmetry / structure function database currently being compiled

* JAM collaboration: P. Jimenez-Delgado, A. Accardi, WM (theory) + Halls A, B, C (expt.)
http://www.jlab.org/jam

Nuclear corrections to spin structure functions

Nuclear structure functions

- Incoherent scattering from nucleons in nucleus $(x \gg 0)$
  + expansion in powers of $\boldsymbol{p}^2/M^2$ & binding energy

$\longrightarrow$ Weak Binding Approximation ("WBA")

$$x g_i^A(x, Q^2) = \sum_N \int \frac{dy}{y} \, f_{ij}^N(y, \gamma) \, x g_j^N(x/y, Q^2) \qquad i, j = 1, 2$$

photon "velocity" $\gamma = |\mathbf{q}|/q_0 = \sqrt{1 + 4M^2x^2/Q^2}$

light-cone momentum fraction $y = \dfrac{p \cdot q}{P \cdot q} = \dfrac{p_0 + \gamma p_z}{M}$

$\longrightarrow$ spin-dependent smearing functions

$$f_{ij}^N(y, \gamma) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} D_{ij}^N(\varepsilon, \mathbf{p}, \gamma) \, \delta\left(y - 1 - \frac{\varepsilon + \gamma p_z}{M}\right)$$

^{3}He structure functions

- For ^{3}He, nuclear functions D_{ij} given in terms of components of ^{3}He spectral function

$$D_{11} = \mathcal{F}_1 + \frac{3-\gamma^2}{6\gamma^2}(3\widehat{p}_z^2 - 1)\mathcal{F}_2 + \frac{p_z}{3\gamma}(3\mathcal{F}_1 + 2\mathcal{F}_2)$$
$$+\frac{\mathbf{p}^2}{M^2}\frac{(3-\gamma^2)\widehat{p}_z^2 - 1 - \gamma^2}{12\gamma^2}(3\mathcal{F}_1 - \mathcal{F}_2) \qquad etc.$$

where spectral function is defined as

$$\mathcal{P}(\varepsilon, \mathbf{p}, \mathbf{S}) = \frac{1}{2}\left[\mathcal{F}_0 I + \mathcal{F}_1 \sigma \cdot \mathbf{S} + \mathcal{F}_2 \big(\hat{p}_i \hat{p}_j - \frac{1}{3}\delta_{ij}\big) S_i \sigma_j\right]$$

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$$
\begin{aligned}
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$$

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$$

spin-averaged

spin-dependent

^{3}He structure functions

- Proton and neutron contributions differ qualitatively

$$\mathcal{F}^p_{1,2} = \mathcal{F}^{p(\text{cont})}_{1,2}(E,\mathbf{p}) + \mathcal{F}^{p(d)}_{1,2}(\mathbf{p})\,\delta(E+\varepsilon_{^3\text{He}}-\varepsilon_d)$$

$$\mathcal{F}^n_{1,2} = \mathcal{F}^{n(\text{cont})}_{1,2}(E,\mathbf{p})$$

deuteron pole contributes
~ 60% to normalization!

- Normalizations

$$\int \frac{d^3\mathbf{p}}{(2\pi)^3}\,\mathcal{F}^{p(n)}_0 = 2\ (1)$$

number sum rules

$$\int \frac{d^3\mathbf{p}}{(2\pi)^3}\,\mathcal{F}^N_1 = \langle\sigma_z\rangle^N$$

average N polarization

$$\int \frac{d^3\mathbf{p}}{(2\pi)^3}\,\mathcal{F}^N_2 = \frac{9}{2}\langle T_{zi}\sigma_i\rangle^N$$

tensor polarization

^{3}He structure functions

- Proton and neutron contributions differ qualitatively

$$\mathcal{F}^p_{1,2} = \mathcal{F}^{p(\mathrm{cont})}_{1,2}(E,\mathbf{p}) + \mathcal{F}^{p(d)}_{1,2}(\mathbf{p})\,\delta(E+\varepsilon_{^3\mathrm{He}}-\varepsilon_d)$$

$$\mathcal{F}^n_{1,2} = \mathcal{F}^{n(\mathrm{cont})}_{1,2}(E,\mathbf{p})$$

deuteron pole contributes
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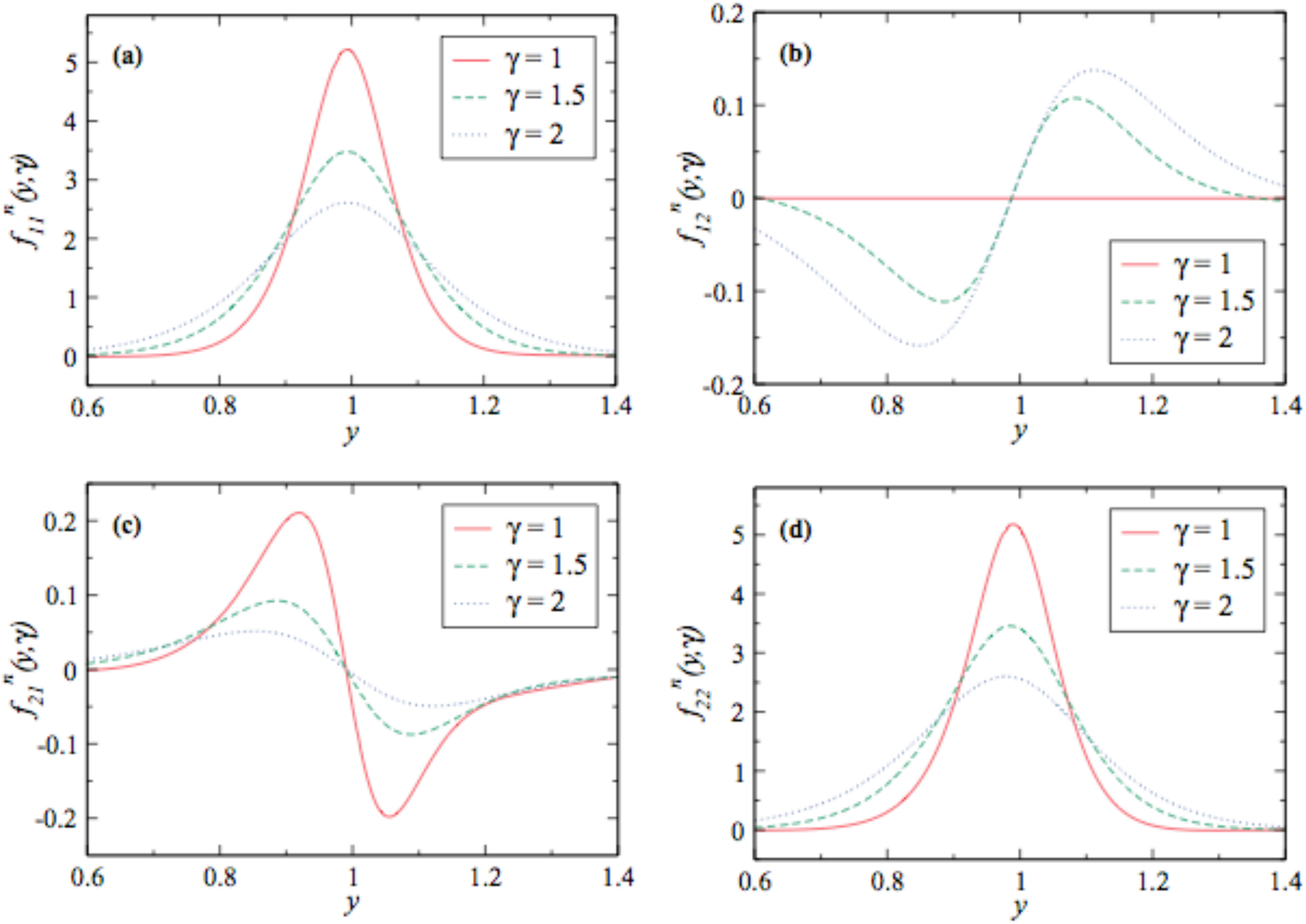
- Nucleon polarizations

$$\langle\sigma_z\rangle^p = -\frac{2}{3}(P_D - P_{S'}) \;\approx\; (-0.04) - (-0.06)$$

$$\langle\sigma_z\rangle^n = P_S - \frac{1}{3}(P_D - P_{S'}) \;\approx\; 0.86 - 0.89$$

Smearing functions

n

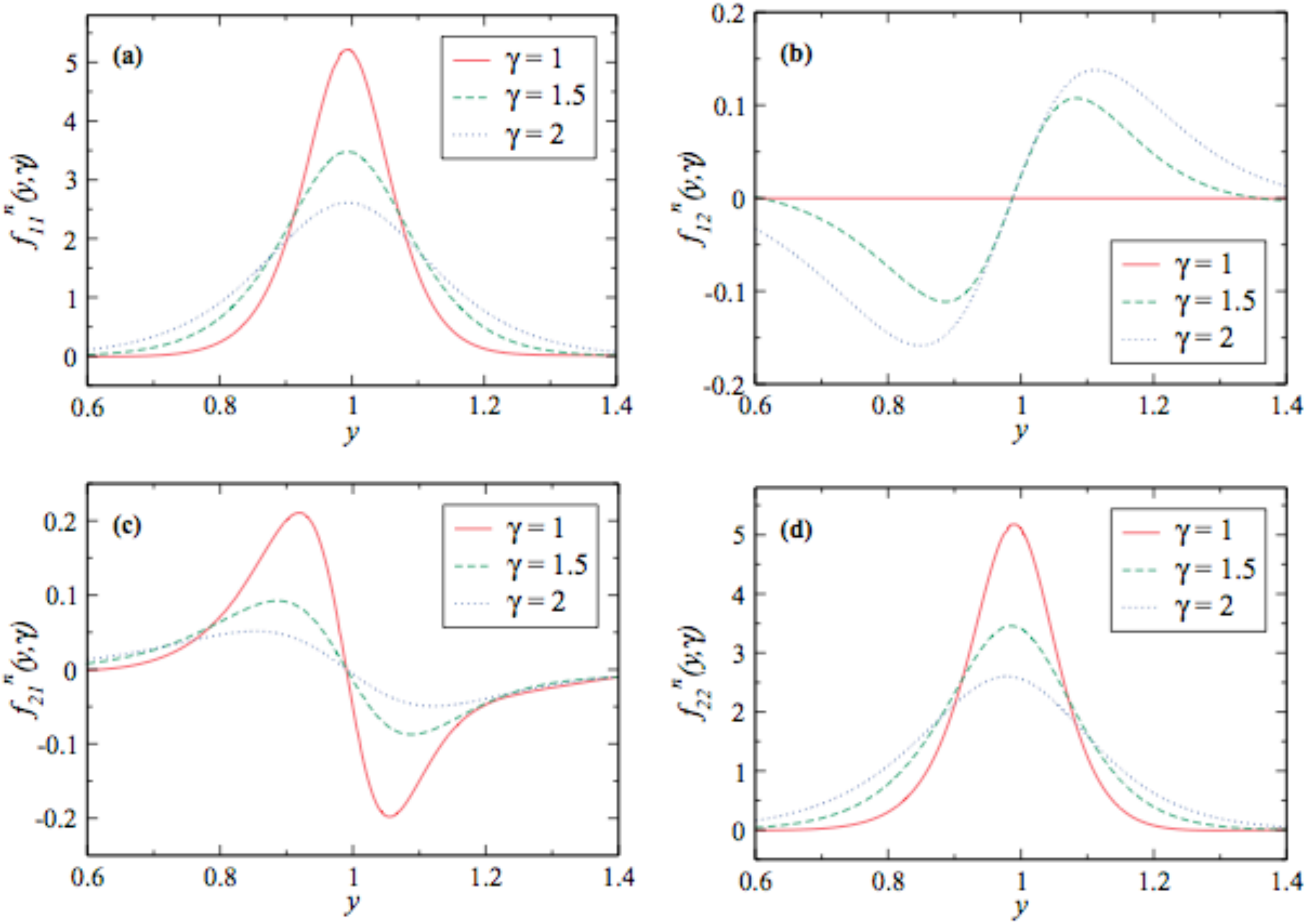


*Kulagin, WM, PRC **78**, 065203 (2008)*

→ effectively more smearing for larger x or lower Q^2

Smearing functions

$\boxed{n}$

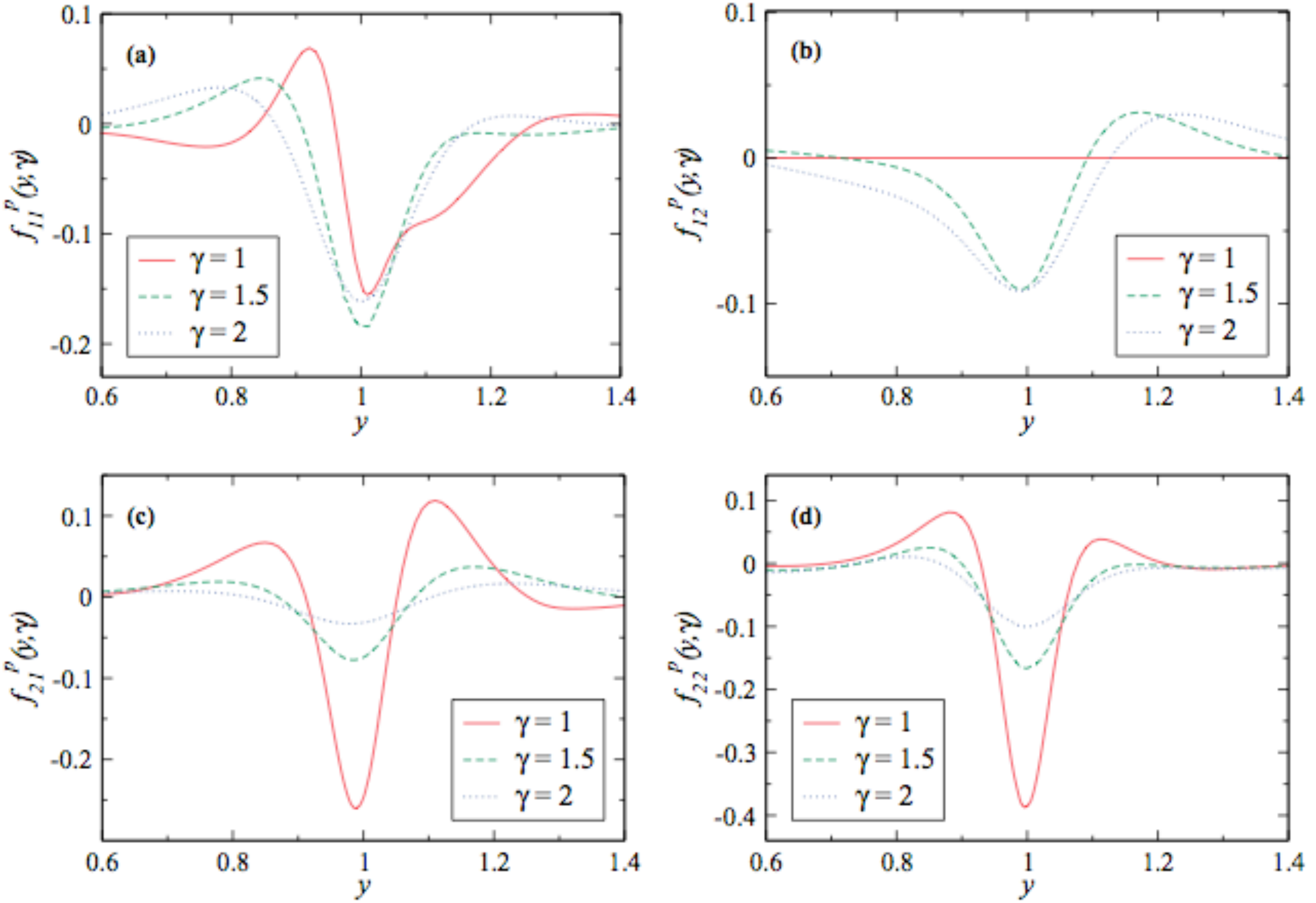


Kulagin, WM, PRC ***78****, 065203 (2008)*

→ diagonal smearing functions $\gg$ off-diagonal

Smearing functions

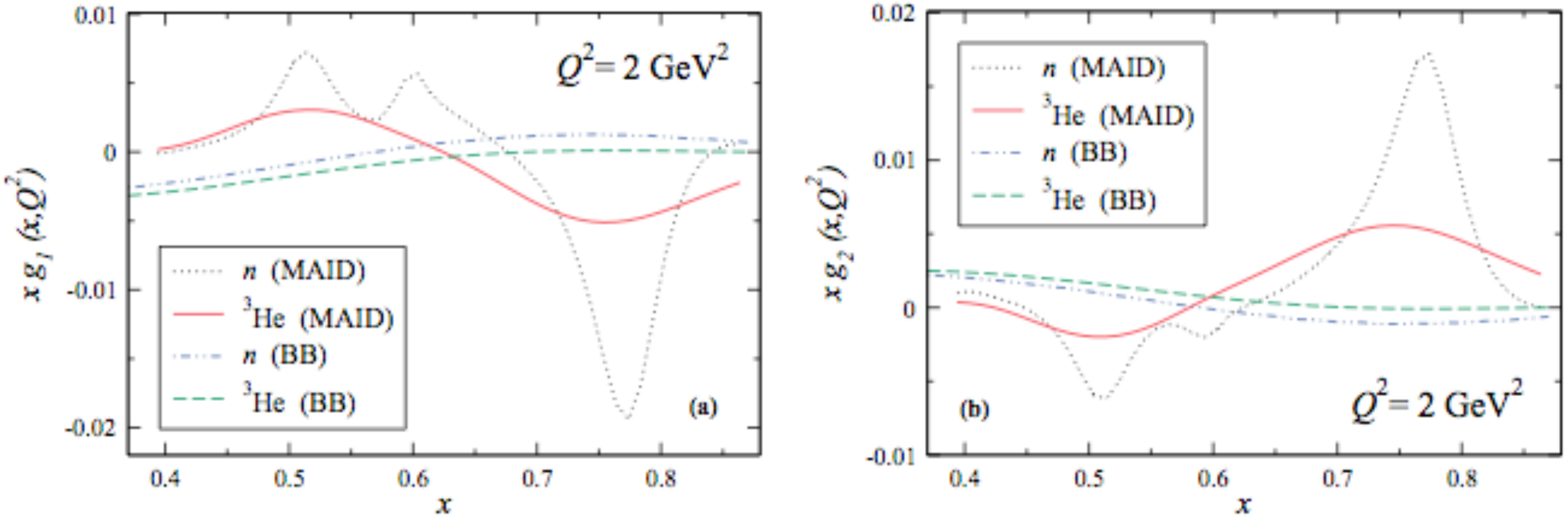
$\boxed{p}$



*Kulagin, WM, PRC **78**, 065203 (2008)*

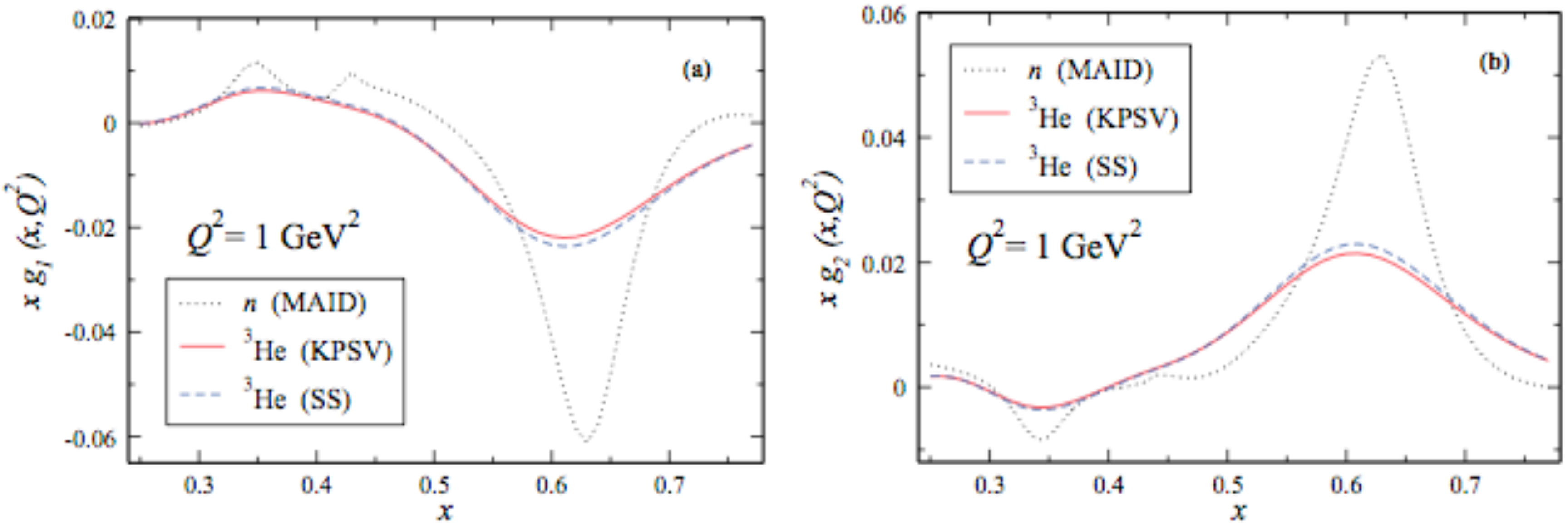
→ proton smearing functions $\ll$ neutron

Nuclear effects in ^{3}He



→ significant smearing, especially in resonance region

Nuclear effects in ^{3}He



→ nuclear wave function model dependence (KPSV[1], SS[2]) not significant

[1] *Kievsky, Pace, Salme, Viviani, PRC **56**, 64 (1997)*

[2] *Schulze, Sauer, PRC **48**, 38 (1993)*

Nuclear effects in $^3\mathrm{He}$

- Effective polarizations

$$f_{ii}^{N}(y,\gamma) \;\to\; \langle\sigma_z\rangle^{N}\;\delta(y-1)$$ (zero width)

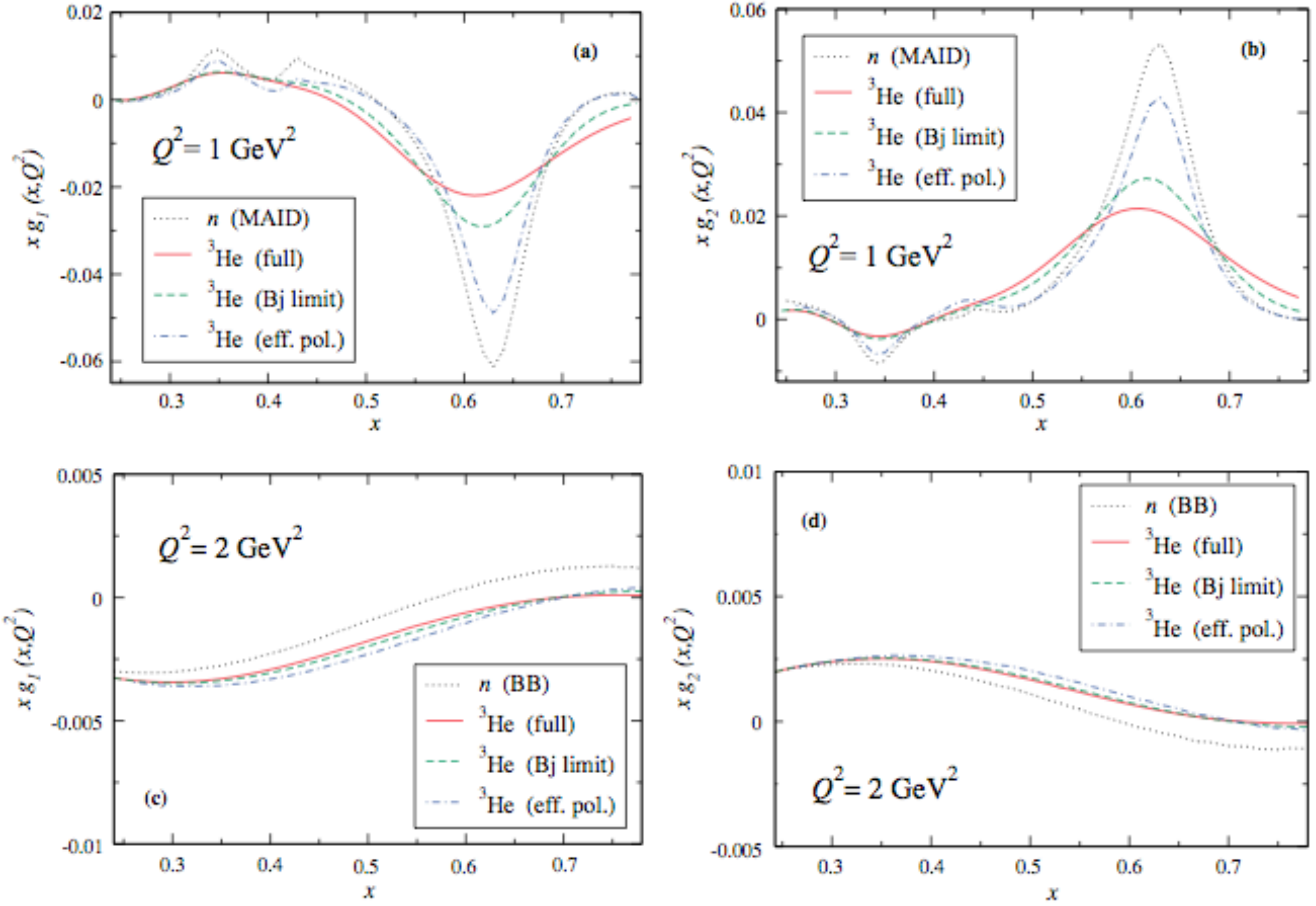
$$f_{i\neq j}^{N}(y,\gamma) \;\to\; 0$$ (no off-diagonal)

⟶ assumes nuclear corrections independent of x and Q^2

$$g_1^{^3\mathrm{He}} \;\to\; \langle\sigma_z\rangle^p\, g_1^p \;+\; \langle\sigma_z\rangle^n\, g_1^n$$

$$g_2^{^3\mathrm{He}} \;\to\; \langle\sigma_z\rangle^p\, g_2^p \;+\; \langle\sigma_z\rangle^n\, g_2^n$$

Nuclear effects in ^{3}He

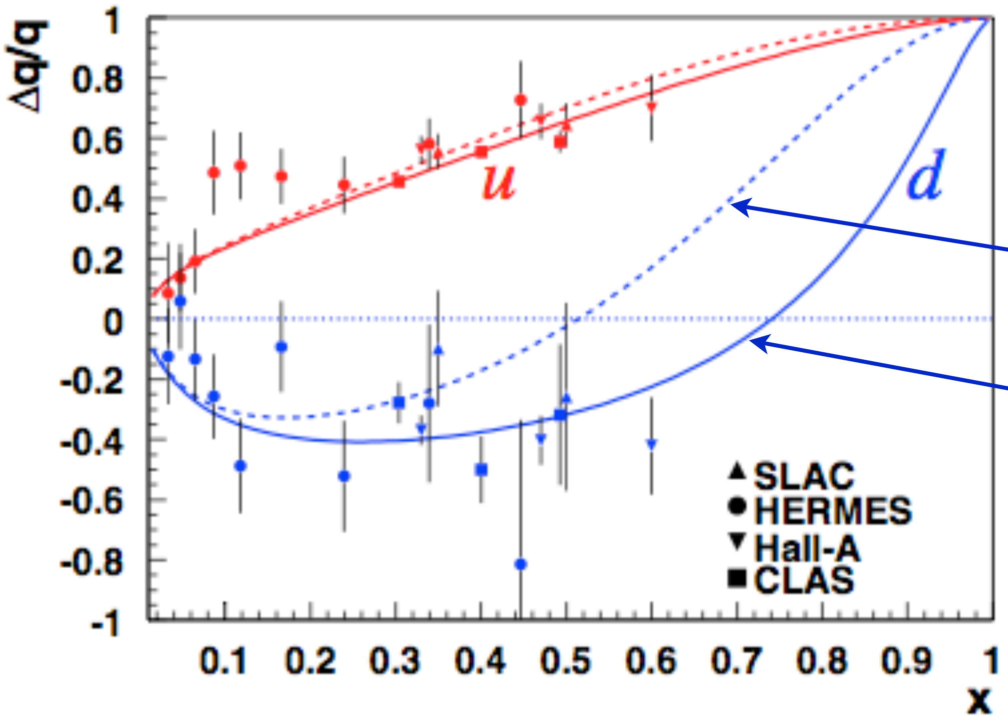


→ significant differences between “effective polarizations” and full results, especially at low W

Nuclear effects in ^{3}He

- At large x, correct treatment of nuclear corrections essential for extraction of free-n information from ^{3}He

→ difficult to observe $\log^2(1-x)$ enhancement of $q^{\downarrow}$ predicted from $L_z = 1$ component of wave function



$L_z = 0$ only

with $L_z = 1$

*Avakian et al., PRL **99**, 082001 (2007)*

Summary

- New JLab $12\ \mathrm{GeV}$ measurements of $A_1^{^3\mathrm{He}}$ will provide vital information on $\Delta d/d$ at $x \to 1$

  → test applicability of pQCD vs. nonperturbative models, and role of OAM

- Nuclear effects in $^3\mathrm{He}$ important at large x

  → "effective polarization" method insufficient for $x \gtrsim 0.6$, and especially low W
  (could distort information extracted on $\Delta d/d$)

- New "JAM" global analysis of spin-dependent PDFs dedicated to large-x, moderate-Q^2 region

  → initial focus on helicity PDFs; later expand scope to TMDs
  (first results soon)