North Carolina State University
February 1, 2011

Weak charge of the proton:
loop corrections to parity-violating electron scattering

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Outline

- Parity-violating elastic ep scattering (PVES)

  $\longrightarrow$ strange form factors of the proton

- Two-boson exchange corrections

  $\longrightarrow$ γZ box diagrams

- Weak charge of the proton Q_W^p

  $\longrightarrow$ dispersive corrections for JLab's "Qweak" experiment

Parity-violating elastic *ep* scattering

Parity-violating e scattering

- Two linear combinations of $G^{u,d,s}$:

$$G^{\gamma p} = \frac{2}{3}G^u - \frac{1}{3}G^d - \frac{1}{3}G^s$$
$$G^{\gamma n} = \frac{2}{3}G^d - \frac{1}{3}G^u - \frac{1}{3}G^s$$

- Third combination from PVES:

$$G^{Zp} = g_V^u G^u + g_V^d G^d + g_V^s G^s$$

$$g_V^u = \frac{1}{2} - \frac{4}{3}\sin^2\theta_W$$
$$g_V^{d,s} = -\frac{1}{2} + \frac{2}{3}\sin^2\theta_W$$

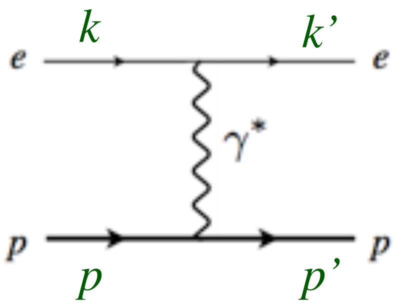
Note: <u>PDG definition</u> (factor $1/2$ *cf*. nuclear physics definition)!

Parity-violating e scattering

- Electromagnetic Born amplitude

$$\mathcal{M}_\gamma = -\frac{e^2}{q^2}\, j_\gamma^\mu\, J_{\gamma\mu}$$

$$e = \sqrt{4\pi\alpha}$$



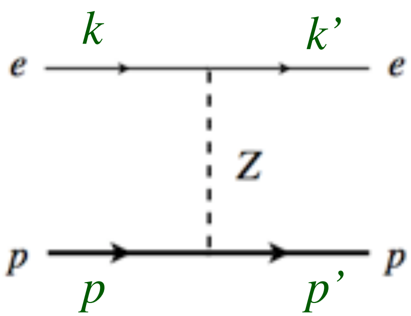
$$q^2 = (k-k')^2 \\ = -t$$

- Weak neutral current Born amplitude

$$\mathcal{M}_Z = -\frac{g^2}{(4\cos\theta_W)^2}\,\frac{1}{M_Z^2 - q^2}\, j_Z^\mu\, J_{Z\mu}$$
$$\approx -\frac{G_F}{2\sqrt{2}}\, j_Z^\mu J_{Z\mu}$$

$$g = \frac{e}{\sin^2\theta_W}$$

$$G_F = \frac{\pi\alpha}{\sqrt{2} M_Z^2 \sin^2\theta_W \cos^2\theta_W}$$



Parity-violating e scattering

- Electroweak lepton currents

$$j_\gamma^\mu = \bar{u}_e(k')\gamma^\mu u_e(k)$$

$$j_Z^\mu = \bar{u}_e(k')(g_V^e \gamma^\mu + g_A^e \gamma_5)u_e(k)$$

$$g_A^e = -\tfrac{1}{2}, \quad g_V^e = -\tfrac{1}{2}(1 - 4\sin^2\theta_W)$$

- Hadronic currents

$$J_{\gamma,Z}^\mu = \bar{u}_N(p')\,\Gamma_{\gamma,Z}^\mu\, u_N(p)$$

$$\Gamma_\gamma^\mu = \gamma^\mu F_1^\gamma + \frac{i\sigma^{\mu\nu}q_\nu}{2M}F_2^\gamma$$

$$\Gamma_Z^\mu = \gamma^\mu F_1^Z + \frac{i\sigma^{\mu\nu}q_\nu}{2M}F_2^Z + \gamma^\mu\gamma_5 G_A^Z$$

Parity-violating e scattering

- Born cross section

$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha}{4MQ^2} \frac{E'}{E} \right)^2 |\mathcal{M}|^2$$

where total squared amplitude is

$$|\mathcal{M}|^2 = |\mathcal{M}_\gamma|^2 + 2\,\Re e\left(\mathcal{M}_\gamma^* \mathcal{M}_Z\right) + |\mathcal{M}_Z|^2$$

P-conserving

P-violating
γZ interference

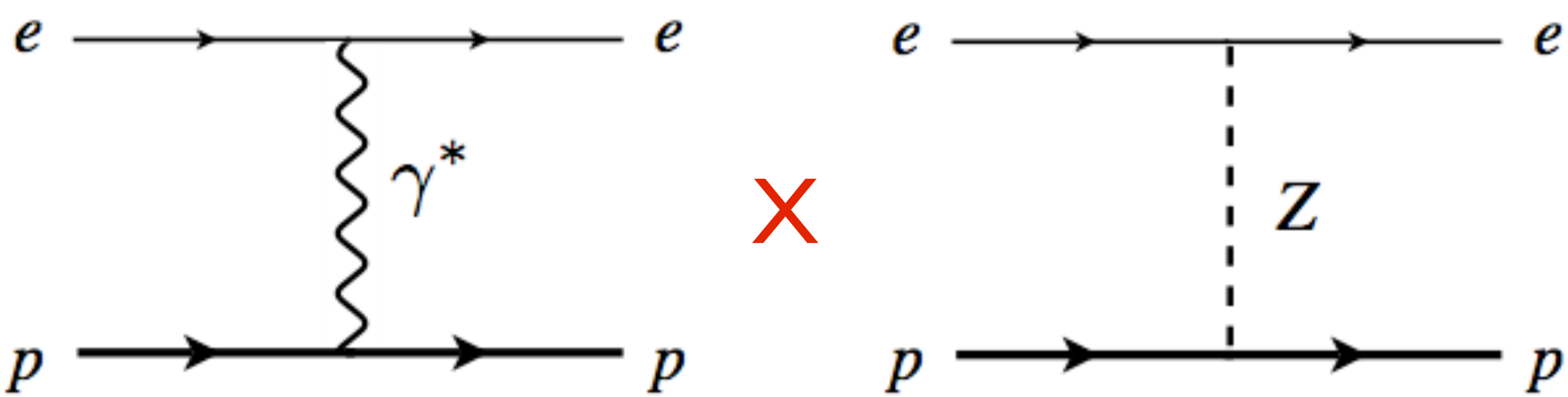
negligible

Parity-violating e scattering

- Left-right polarization asymmetry in $\vec{e}\, p \to e\ p$ scattering

$$A_{\rm PV} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\left(\frac{G_F Q^2}{4\sqrt{2}\alpha}\right)(A_V + A_A + A_s)$$

$\longrightarrow$ measure interference between e.m. and weak currents



Born (tree) level

Parity-violating e scattering

- Left-right polarization asymmetry in $\vec{e}\, p \to e\, p$ scattering

$$A_{\mathrm{PV}} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\left(\frac{G_F Q^2}{4\sqrt{2}\alpha}\right)(A_V + A_A + A_s)$$

$\longrightarrow$ measure interference between e.m. and weak currents

vector asymmetry

$$A_V = g_A^e \rho \left[(1 - 4\kappa \sin^2\theta_W) - (\varepsilon G_E^{\gamma p} G_E^{\gamma n} + \tau G_M^{\gamma p} G_M^{\gamma n})/\sigma^{\gamma p}\right]$$

axial vector asymmetry

$$A_A = g_V^e \sqrt{\tau(1+\tau)(1-\varepsilon^2)}\; \widetilde{G}_A^{Zp} G_M^{\gamma p} / \sigma^{\gamma p}$$

strange asymmetry

$$A_s = -g_A^e \rho \left(\varepsilon G_E^{\gamma p} G_E^s + \tau G_M^{\gamma p} G_M^s\right) / \sigma^{\gamma p}$$

Parity-violating e scattering

- Left-right polarization asymmetry in $\vec{e}\, p \to e\, p$ scattering

$$A_{\rm PV} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\left(\frac{G_F Q^2}{4\sqrt{2}\alpha}\right)(A_V + A_A + A_s)$$

$\longrightarrow$ measure interference between e.m. and weak currents

vector asymmetry

$$A_V = g_A^e \rho \left[(1 - 4\kappa \sin^2\theta_W) - (\varepsilon G_E^{\gamma p} G_E^{\gamma n} + \tau G_M^{\gamma p} G_M^{\gamma n})/\sigma^{\gamma p}\right]$$

radiative corrections, including TBE

using isospin to relate weak and e.m. form factors

$$G_{E,M}^{Zp} = (1 - 4\sin^2\theta_W) G_{E,M}^{\gamma p} - G_{E,M}^{\gamma n} - G_{E,M}^{s}$$

Parity-violating e scattering

- Left-right polarization asymmetry in $\vec{e}\, p \to e\, p$ scattering

$$A_{\mathrm{PV}} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\left(\frac{G_F Q^2}{4\sqrt{2}\alpha}\right)(A_V + A_A + A_s)$$

$\longrightarrow$ measure interference between e.m. and weak currents

axial vector asymmetry

$$A_A = g_V^e \sqrt{\tau(1+\tau)(1-\varepsilon^2)}\; \widetilde{G}_A^{Zp} G_M^{\gamma p} / \sigma^{\gamma p}$$

insensitive to axial contribution at forward angles $(\varepsilon \to 1)$

Parity-violating e scattering

- Left-right polarization asymmetry in $\vec{e}\,p \to e\,p$ scattering

$$A_{\rm PV} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\left(\frac{G_F Q^2}{4\sqrt{2}\alpha}\right)(A_V + A_A + A_s)$$

$\longrightarrow$ measure interference between e.m. and weak currents

extract strange electric & magnetic form factors

strange asymmetry

$$A_s = -g_A^e \rho \left(\varepsilon G_E^{\gamma p} G_E^s + \tau G_M^{\gamma p} G_M^s\right)/\sigma^{\gamma p}$$

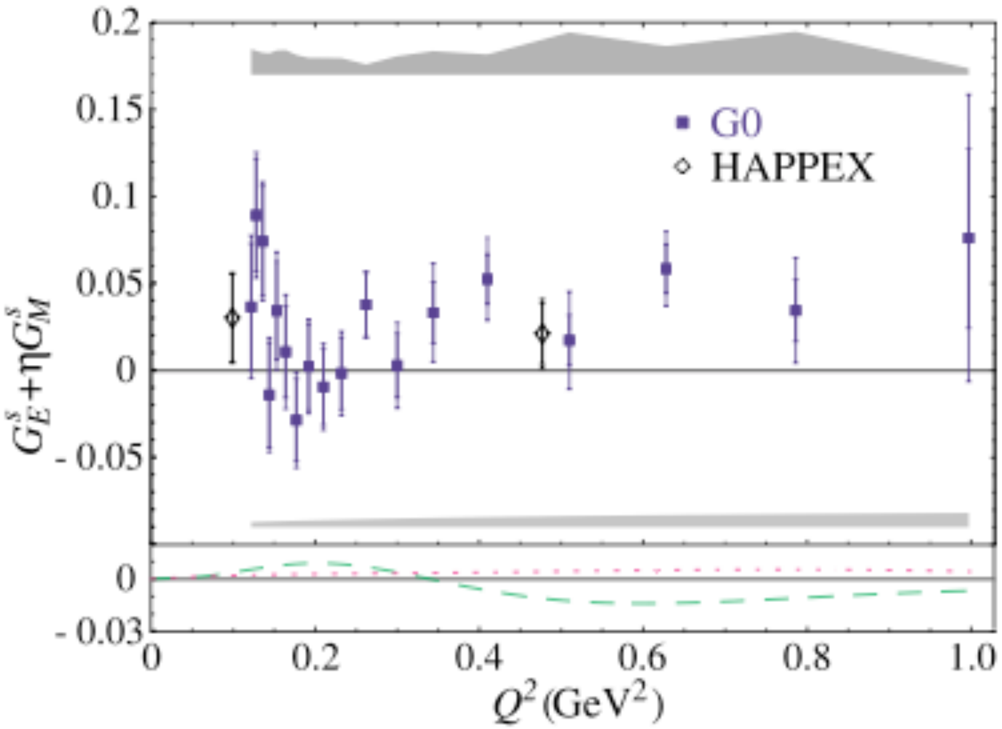
PVES experiments

Collaboration	Q^2	η_0	η_A^p	η_A^n	η_E	η_M	$A^{\rm phys}$	δA	$\delta A_{\rm cor}$	$\tilde{G}_A^p$	$\tilde{G}_A^n$	G_E^s	G_M^s	χ^2	C.L.
SAMPLE	0.038	−2.13	0.46	−0.30	1.16	0.28	−3.51	0.81	0	···	···	···	···	···	
SAMPLE	0.091	−7.02	1.04	−0.65	1.63	0.77	−7.77	1.03	0	···	···	···	···	···	
HAPPEx	0.091	−7.50	0	0	−20.2	0	−6.72	0.87	0	···	···	···	···	···	
HAPPEx	0.099	−1.40	0.04	0	9.55	0.76	−1.14	0.25	0	···	···	···	···	···	
SAMPLE	0.1	−5.47	1.58	0	2.11	3.46	−5.61	1.11	0	−2.6(21)	−0.6(30)	−0.044(47)	1.00(75)	1.0	63
PVA4	0.108	−1.80	0.26	0	10.1	1.05	−1.36	0.32	0	−2.0(20)	0.3(29)	−0.025(43)	0.87(74)	1.0	71
G0	0.122	−1.90	0.06	0	12.0	1.18	−1.51	0.49	0.18	−1.8(19)	0.5(27)	−0.023(43)	0.79(69)	0.7	76
G0	0.128	−2.04	0.06	0	12.6	1.30	−0.97	0.46	0.17	−2.4(18)	−0.1(26)	−0.027(42)	0.99(65)	0.7	96
G0	0.136	−2.24	0.07	0	13.5	1.48	−1.30	0.45	0.17	−2.5(17)	−0.2(26)	−0.028(42)	1.03(63)	0.6	99
G0	0.144	−2.44	0.08	0	14.3	1.67	−2.71	0.47	0.18	−1.6(16)	0.8(25)	−0.021(42)	0.71(61)	1.4	91
G0	0.153	−2.68	0.09	0	15.3	1.89	−2.22	0.51	0.21	−1.4(16)	1.0(25)	−0.020(42)	0.66(60)	1.2	91
G0	0.164	−2.97	0.11	0	16.5	2.19	−2.88	0.54	0.23	−1.1(16)	1.3(25)	−0.018(42)	0.55(60)	1.2	83
G0	0.177	−3.34	0.13	0	18.0	2.58	−3.95	0.50	0.20	−0.4(16)	2.1(24)	−0.012(42)	0.32(59)	1.7	36
G0	0.192	−3.78	0.15	0	19.7	3.07	−3.85	0.53	0.19	−0.2(15)	2.3(24)	−0.010(42)	0.24(58)	1.6	18
G0	0.210	−4.34	0.19	0	21.8	3.72	−4.68	0.54	0.21	0.1(15)	2.7(24)	−0.007(42)	0.14(57)	1.6	1
PVA4	0.230	−5.66	0.89	0	22.6	5.07	−5.44	0.60	0	0.0(15)	2.5(24)	−0.007(42)	0.14(57)	1.5	1
G0	0.232	−5.07	0.23	0	24.4	4.61	−5.27	0.59	0.23	0.2(14)	2.8(23)	−0.005(42)	0.09(57)	1.4	3
G0	0.262	−6.12	0.31	0	28.0	5.99	−5.26	0.53	0.17	−0.2(14)	2.3(23)	−0.010(41)	0.19(56)	1.4	18
G0	0.299	−7.51	0.42	0	32.6	8.00	−7.72	0.80	0.35	0.0(14)	2.6(23)	−0.006(41)	0.12(55)	1.3	5
G0	0.344	−9.35	0.57	0	38.4	10.9	−8.40	1.09	0.52	0.0(14)	2.5(22)	−0.008(41)	0.15(54)	1.2	11
G0	0.410	−12.28	0.87	0	47.3	16.1	−10.25	1.11	0.55	−0.4(13)	2.1(22)	−0.015(40)	0.27(53)	1.2	44
HAPPEx	0.477	−15.46	1.12	0	56.9	22.6	−15.05	1.13	0	0.1(12)	2.7(21)	−0.004(38)	0.10(49)	1.2	28

$Q^2 \sim 0.04 - 0.5 \ {\rm GeV}^2$

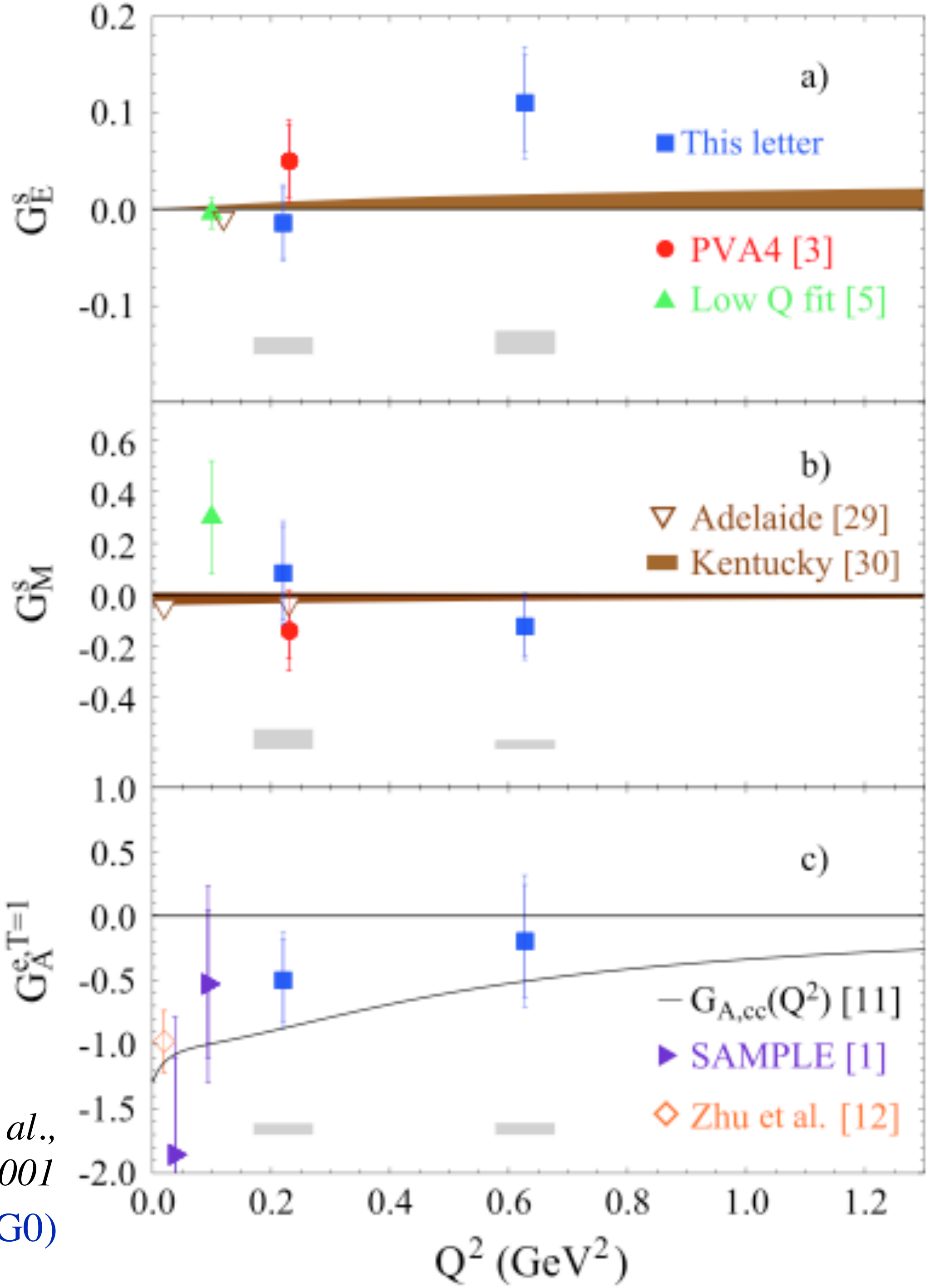
Young et al., PRL ***97*** *(2006) 102002*

PVES experiments



Armstrong et al., PRL ***95*** *(2005) 092001*

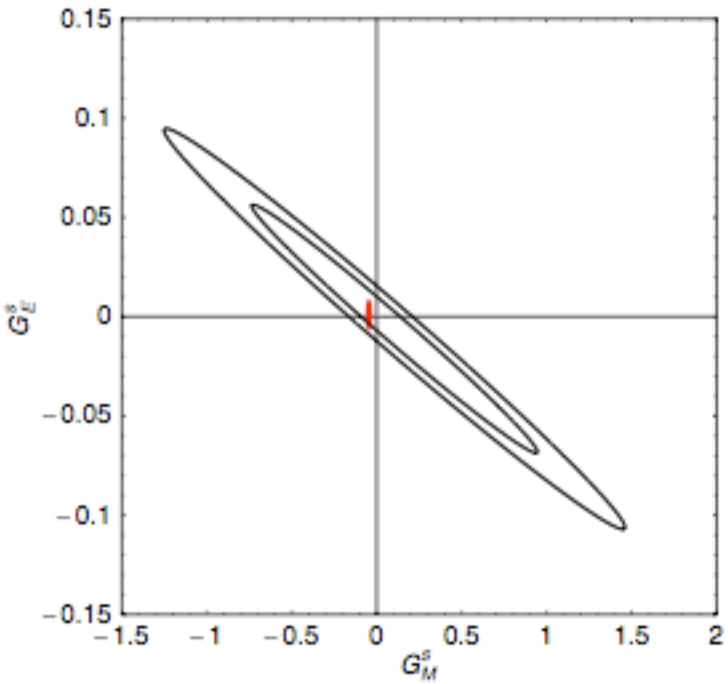
(forward angle G0)



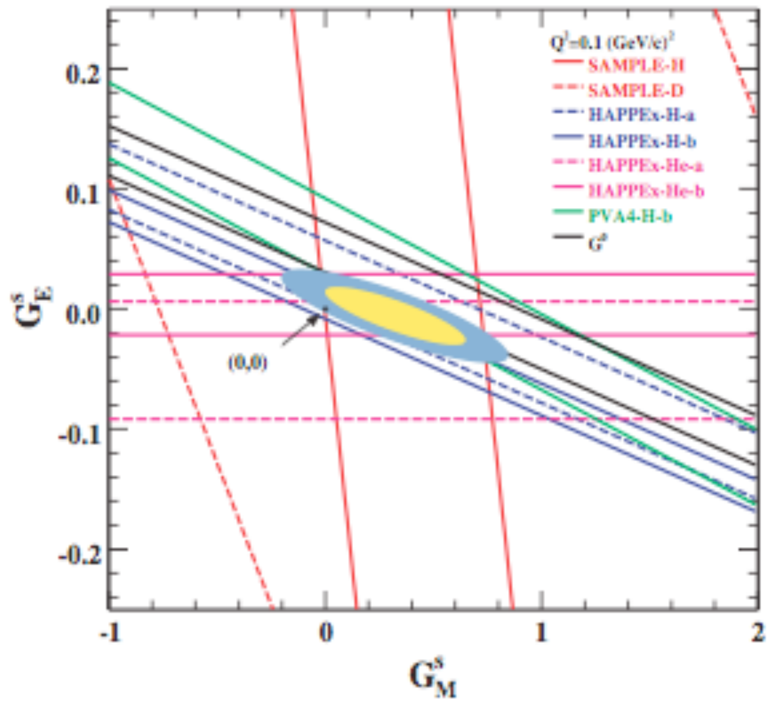
Androic et al.,
PRL ***104*** *(2010) 012001*

(backward angle G0)

PVES global analysis



Young, Roche, Carlini, Thomas
*PRL **97** (2006) 102002*



Liu, McKeown, Ramsey-Musolf
*PRC **76** (2007) 025202*

$$G_E^s = +0.0025 \pm 0.0182$$
$$G_M^s = -0.011 \pm 0.254$$

$$Q^2 = 0.1\ \mathrm{GeV}^2$$

$$G_E^s = -0.008 \pm 0.016$$
$$G_M^s = +0.29 \pm 0.21$$

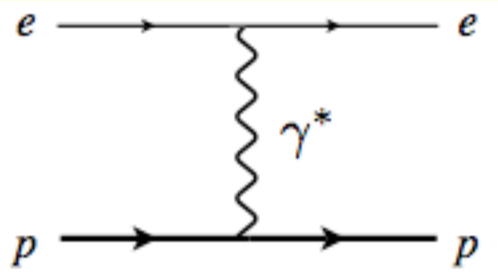
→ strange form factors <u>small</u> (analyses compatible)

→ how important are higher order ($e.g.\ \gamma Z$) corrections?

Two-boson exchange corrections

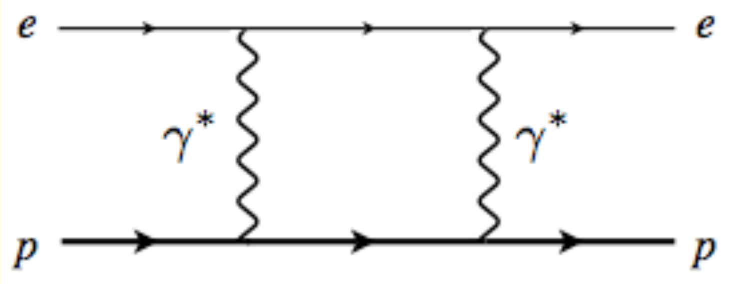
Two-photon exchange corrections

- calculation uses same framework as that for computing *two-photon exchange* corrections to e.m. form factors



$\mathcal{M}_0$

X



$\mathcal{M}_{\gamma\gamma}$

$$\delta^{(2\gamma)} = \frac{2\mathcal{R}e\left\{\mathcal{M}_0^\dagger\,\mathcal{M}_{\gamma\gamma}\right\}}{|\mathcal{M}_0|^2}$$

$$\mathcal{M}_{\gamma\gamma} = e^4 \int \frac{d^4 l}{(2\pi)^4} \frac{N(l)}{D(l)} \quad + \text{crossed box}$$

$$N(l) = \bar{u}(k')\gamma_\mu(\not{k} - \not{l} + m_e)\gamma_\nu u(k)\ \bar{u}(p')\Gamma^\mu(q-l)(\not{p} + \not{l} + M)\Gamma^\nu(l)u(p)$$

$$D(l) = (l^2 - \lambda^2)((l-q)^2 - \lambda^2)((k-l)^2 - m_e^2)((p+l)^2 - M^2)$$

$\lambda(\to 0)$ = infrared regulator

Two-photon exchange corrections

- "exact" evaluation of integrals including form factors (Veltman-Passarino functions)

  → *cf.* soft photon approximation (used in most data analyses!) which assumes pole dominance of TPE amplitude & neglects nucleon structure $N(l) \approx N(0)$

Mo, Tsai (1969)

$$\mathcal{M}_{\gamma\gamma} = e^4 \int \frac{d^4 l}{(2\pi)^4} \frac{N(l)}{D(l)} \quad + \text{crossed box}$$

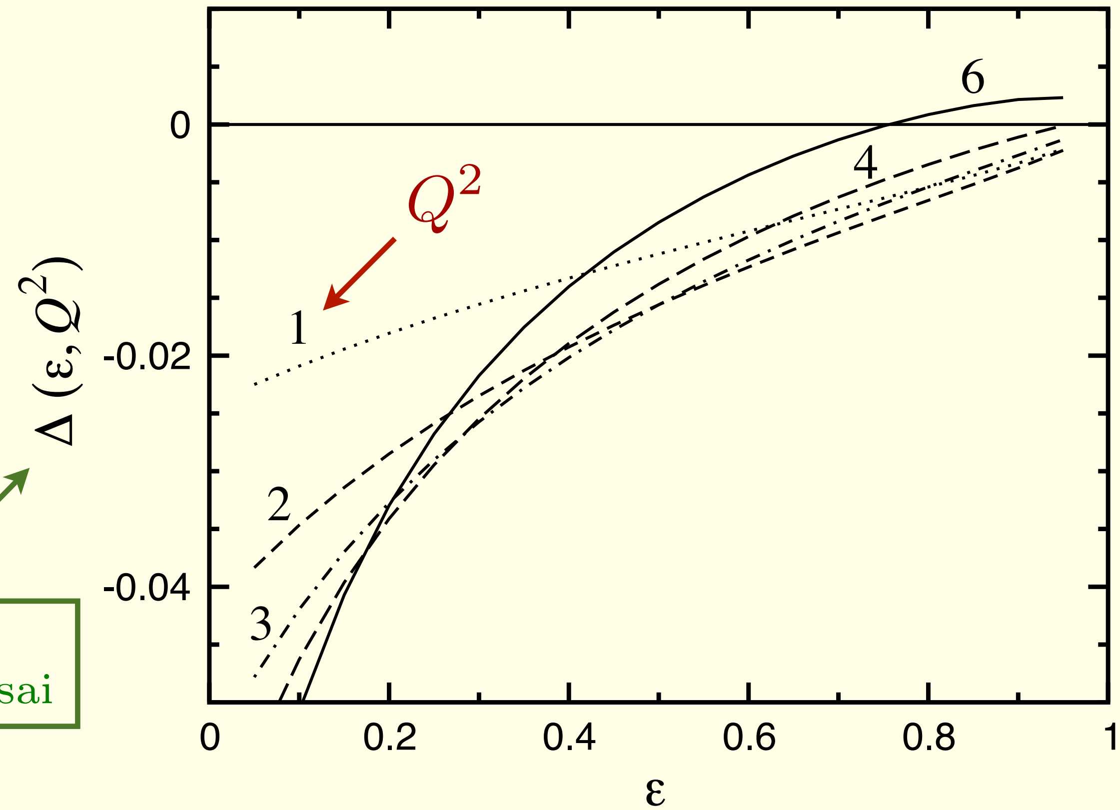
$$N(l) = \bar{u}(k')\gamma_\mu(\not{k} - \not{l} + m_e)\gamma_\nu u(k)\ \bar{u}(p')\Gamma^\mu(q-l)(\not{p} + \not{l} + M)\Gamma^\nu(l)u(p)$$

$$D(l) = (l^2 - \lambda^2)((l-q)^2 - \lambda^2)((k-l)^2 - m_e^2)((p+l)^2 - M^2)$$

$\lambda(\to 0)$ = infrared regulator

Two-photon exchange corrections

- calculation uses same framework as that for computing *two-photon exchange* corrections to e.m. form factors



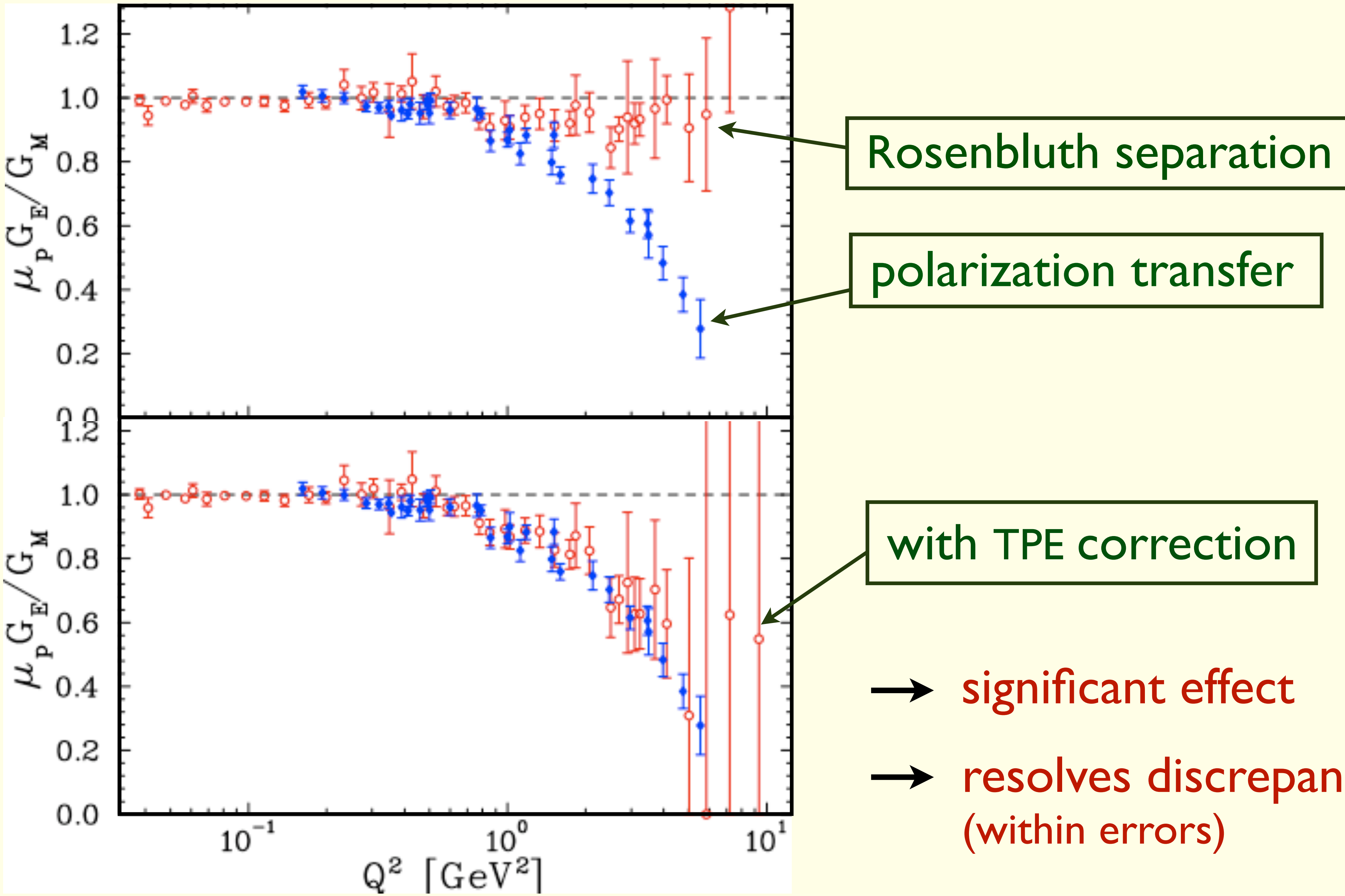
$\delta^{(2\gamma)}_{\rm full} - \delta^{(2\gamma)}_{\rm Mo-Tsai}$

Blunden, WM, Tjon
*PRL **91** (2003) 142304;*
*PRC **72** (2005) 034612*

→ few % magnitude, non-linear in ε, positive slope

→ does not depend strongly on vertex form factors

Two-photon exchange corrections



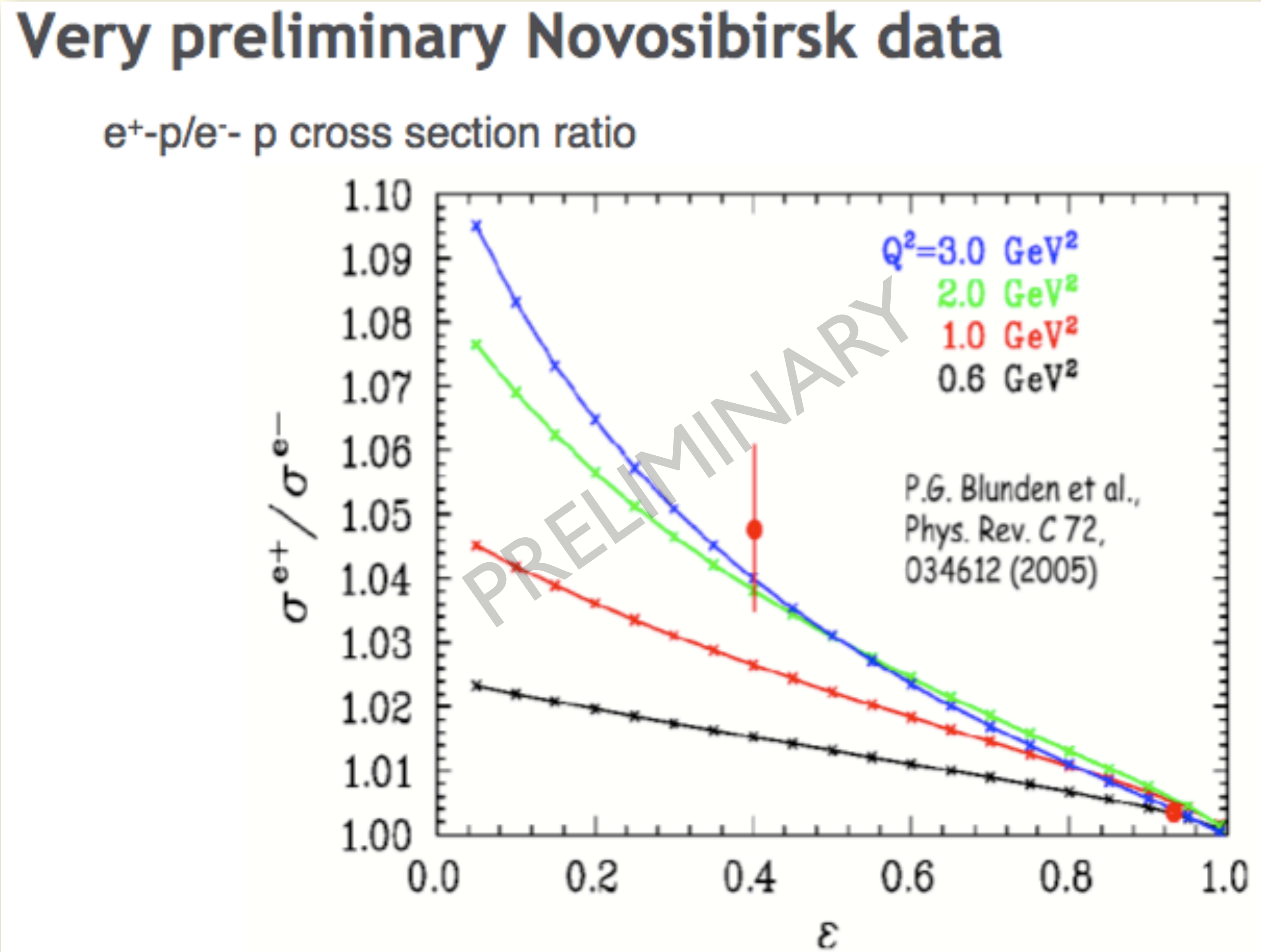
*Arrington, WM, Tjon, PRC **76** (2007) 035205*

→ significant effect

→ resolves discrepancy (within errors)

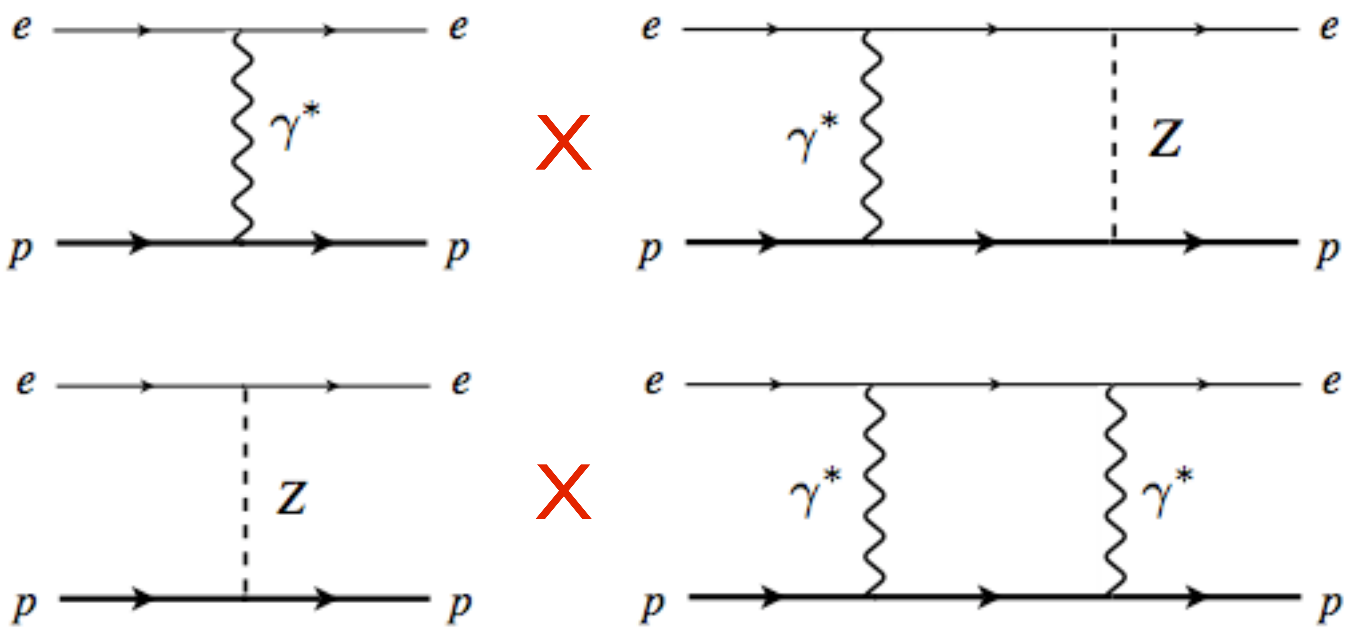
Two-photon exchange corrections

- 1γ (2γ) exchange changes sign (invariant) under $e^+ \leftrightarrow e^-$



Arrington, Holt et al. (2010)

Two-boson exchange corrections



"$\gamma(Z\gamma)$"

"$Z(\gamma\gamma)$"

$$A_{\rm PV} = (1+\delta)A^0_{\rm PV} \equiv \left(\frac{1+\delta_{Z(\gamma\gamma)}+\delta_{\gamma(Z\gamma)}}{1+\delta_{\gamma(\gamma\gamma)}}\right) A^0_{\rm PV}$$

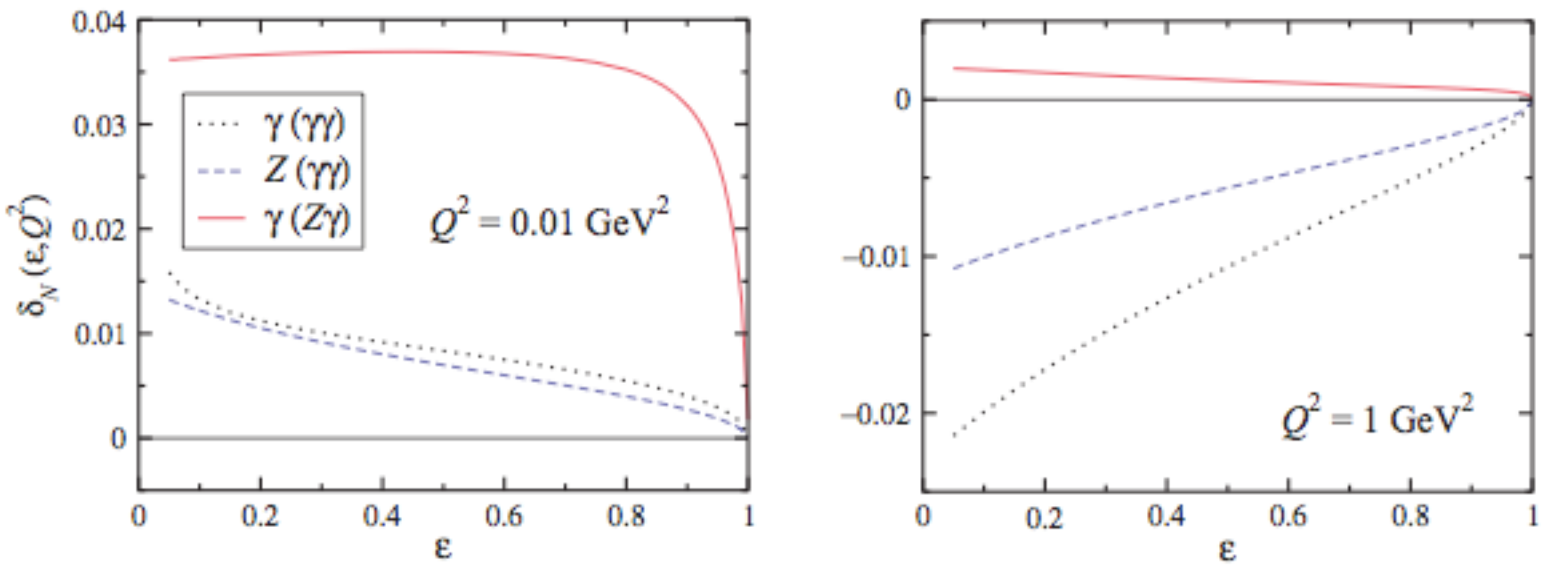
Born asymmetry

$$\delta_{\gamma(Z\gamma)} = \frac{2\Re e(\mathcal{M}^*_\gamma \mathcal{M}_{\gamma Z} + \mathcal{M}^*_\gamma \mathcal{M}_{Z\gamma})}{2\Re e(\mathcal{M}^*_Z \mathcal{M}_\gamma)} \qquad \delta_{\gamma(\gamma\gamma)} = \frac{2\Re e(\mathcal{M}^*_\gamma \mathcal{M}_{\gamma\gamma})}{|\mathcal{M}_\gamma|^2}$$

$$\delta_{Z(\gamma\gamma)} = \frac{2\Re e(\mathcal{M}^*_Z \mathcal{M}_{\gamma\gamma})}{2\Re e(\mathcal{M}^*_Z \mathcal{M}_\gamma)} \qquad \delta \approx \delta_{Z(\gamma\gamma)} + \delta_{\gamma(Z\gamma)} - \delta_{\gamma(\gamma\gamma)}$$

Two-boson exchange corrections

- nucleon intermediate states



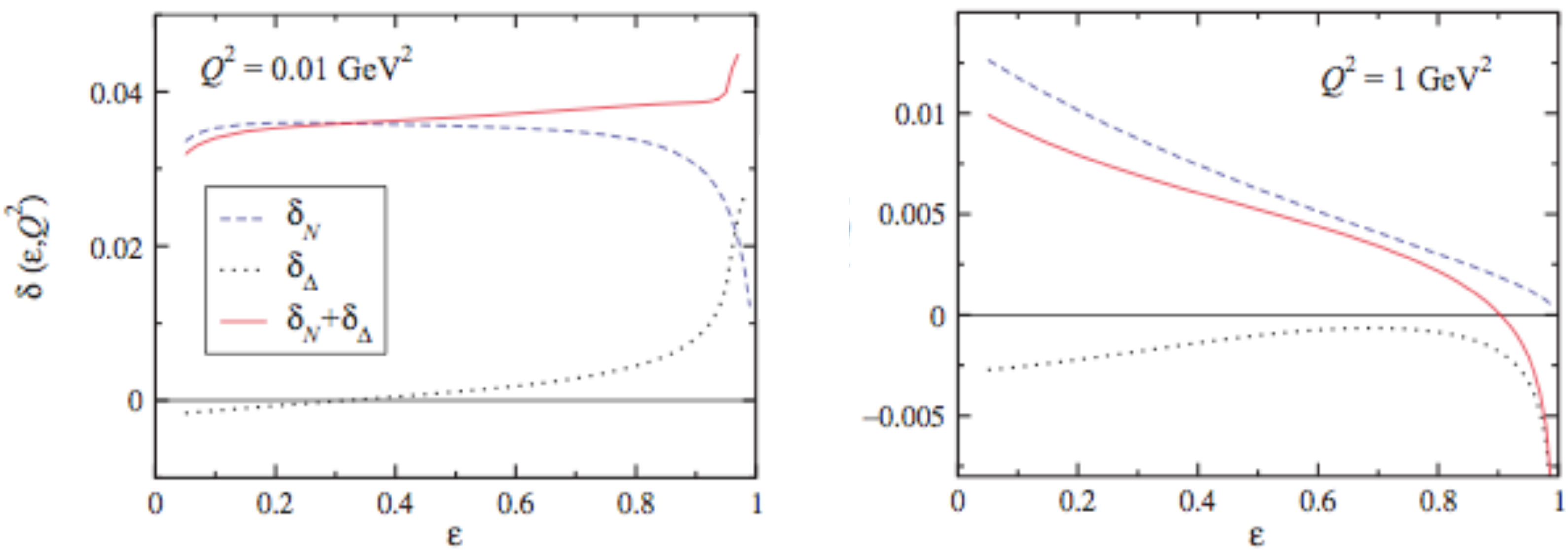
*Tjon, WM, PRL **100** (2008) 082003*
*Tjon, Blunden, WM, PRC **79** (2009) 055201*

→ cancellation between $Z(\gamma\gamma)$ and $\gamma(\gamma\gamma)$ corrections, especially at low Q^2

→ dominated by $\gamma(Z\gamma)$ contribution

Two-boson exchange corrections

- Δ intermediate states

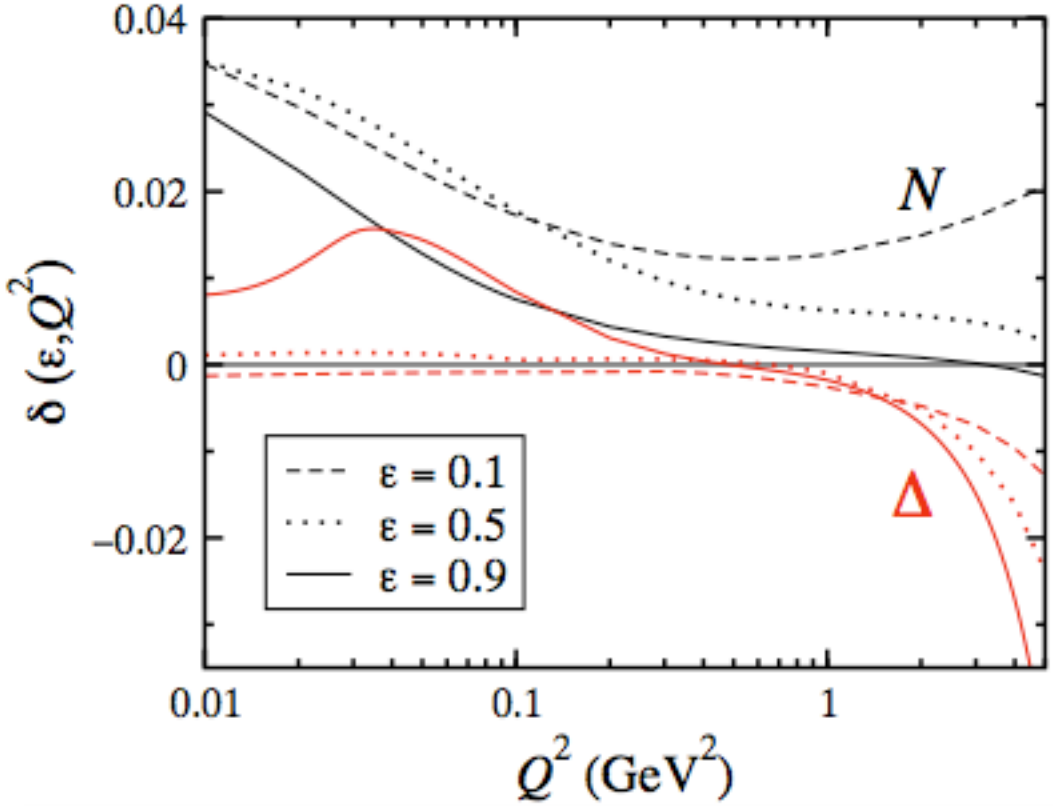


Tjon, WM, PRL ***100*** *(2008) 082003*
Tjon, Blunden, WM, PRC ***79*** *(2009) 055201*

→ Δ contribution small, except at very forward angles
(numerators have higher powers of loop momenta)

→ Δ calculation less reliable for $\varepsilon \to 1$
(grows faster with s than nucleon)

Two-boson exchange corrections



→ $\sim 2\text{–}4\%$ correction for $Q^2 \sim 0.01\text{–}0.1\ \mathrm{GeV}^2$

→ stronger Q^2 dependence at larger Q^2
(especially at forward angles)

TBE corrections at experimental kinematics

Q^2 (GeV^2)	θ	Expt.	δ_N	δ_Δ	$\delta_{N+\Delta}$	$\delta_{\overline{MS}}^{had}$	$\delta_{\overline{MS}}^{tot}$
0.099	6.0°	HAPPEX [1]	0.19	−1.20	−1.01	0.45	2.42
0.477	12.3°	HAPPEX [1]	0.13	−0.44	−0.31	0.16	0.86
0.077	6.0°	HAPPEX [3]	0.22	−1.04	−0.82	0.52	2.78
0.1	144.0°	SAMPLE [5]	1.63	−0.09	1.54	0.06	0.33
0.108	35.37°	PVA4 [7]	1.05	0.78	1.83	0.37	1.98
0.23	35.31°	PVA4 [7]	0.62	0.34	0.96	0.23	1.22
0.122	6.68°	G0 [2]	0.18	−1.06	−0.88	0.40	2.13
0.128	6.84°	G0 [2]	0.18	−1.03	−0.85	0.39	2.07
0.136	7.06°	G0 [2]	0.18	−0.99	−0.81	0.37	1.99
0.144	7.27°	G0 [2]	0.17	−0.96	−0.79	0.36	1.92
0.153	7.5°	G0 [2]	0.17	−0.92	−0.75	0.35	1.85
0.164	7.77°	G0 [2]	0.17	−0.88	−0.71	0.33	1.77
0.177	8.09°	G0 [2]	0.16	−0.83	−0.67	0.32	1.69
0.192	8.43°	G0 [2]	0.16	−0.79	−0.63	0.30	1.60
0.21	8.84°	G0 [2]	0.16	−0.73	−0.57	0.28	1.51
0.232	9.31°	G0 [2]	0.16	−0.68	−0.52	0.26	1.41
0.262	9.92°	G0 [2]	0.15	−0.62	−0.47	0.24	1.30
0.299	10.63°	G0 [2]	0.15	−0.55	−0.40	0.22	1.19
0.344	11.46°	G0 [2]	0.15	−0.48	−0.33	0.20	1.07
0.41	12.59°	G0 [2]	0.15	−0.41	−0.26	0.18	0.95
0.511	14.2°	G0 [2]	0.15	−0.32	−0.17	0.15	0.81
0.631	15.98°	G0 [2]	0.15	−0.26	−0.11	0.13	0.70
0.788	18.16°	G0 [2]	0.16	−0.23	−0.07	0.11	0.60
0.997	20.9°	G0 [2]	0.17	−0.22	−0.05	0.10	0.51
0.23	110.0°	G0 [4]	1.37	−0.10	1.27	0.09	0.47
0.62	110.0°	G0 [4]	1.10	−0.15	0.95	0.07	0.35

partial TBE corrections (γZ at $Q^2 = 0$) need to be removed before adding new results

G0 (fwd): < 1%
(negative)

G0 (bck): ~ 1%
(positive)

Tjon, Blunden, WM
PRC ***79*** *(2009) 055201*

Effect on strange form factors

- include TBE corrections in global analysis

→ *e.g.* Young et al.

$$\boxed{\begin{aligned} G_E^s &= +0.0025 \pm 0.0182 \\ G_M^s &= -0.011 \pm 0.254 \end{aligned}}$$

$$\Downarrow$$

$$\boxed{\begin{aligned} G_E^s &= +0.0023 \pm 0.0182 \\ G_M^s &= -0.020 \pm 0.254 \end{aligned}}$$

at $Q^2 = 0.1\ \mathrm{GeV}^2$

→ small (absolute) shift in strange form factors from TBE (large relative shift to G_M^s), well within experimental errors

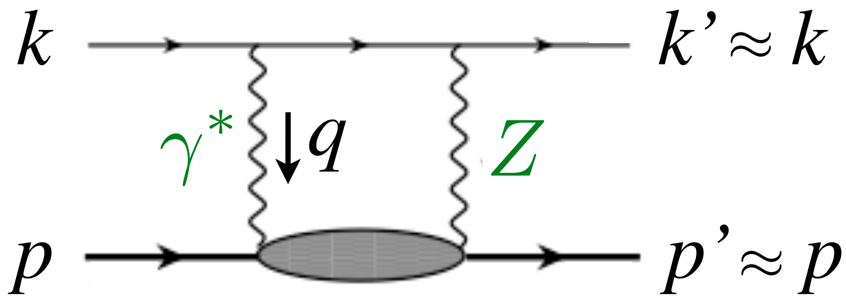
Extraction of proton’s weak charge

– *JLab Qweak experiment* –

Correction to proton weak charge

- in forward limit $A_{\rm PV}$ measures weak charge of proton Q_W^p

$$A_{\rm PV} \;\rightarrow\; \frac{G_F\, Q_W^p}{4\sqrt{2}\pi\alpha}\, t$$



forward limit

$$t = (k - k')^2 \rightarrow 0$$

$$s = (k + p)^2$$
$$= M(M + 2E)$$

- at tree level Q_W^p gives weak mixing angle

$$Q_W^p = 1 - 4\sin^2\theta_W$$

Correction to proton weak charge

- including higher order radiative corrections

$$
\begin{aligned}
Q_W^p &= (1+\Delta\rho+\Delta_e)(1-4\sin^2\theta_W(0)+\Delta'_e) \\
&\quad + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z} \quad \longleftarrow \text{ box diagrams} \\
&= 0.0713 \pm 0.0008\ ^{*}
\end{aligned}
$$

Erler et al., PRD ***72*** *(2005) 073003*

$^{*}\ \sin^2\theta_W(0) = 0.23867(16)$

→ WW and ZZ box diagrams dominated by short distances, evaluated perturbatively

→ γZ box diagram sensitive to long distance physics, has two contributions $\Box_{\gamma Z} = \Box^A_{\gamma Z} + \Box^V_{\gamma Z}$

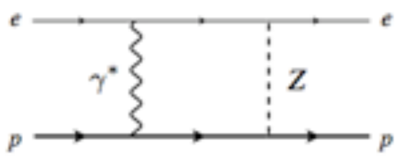
vector e – axial h (finite at E=0)

axial e – vector h (vanishes at E=0)

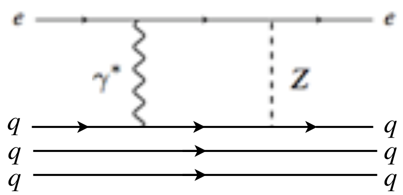
Axial h correction

- axial h correction $\Box^{A}_{\gamma Z}$ dominant γZ correction in atomic parity violation at very low (zero) energy

$\longrightarrow$ computed by Marciano & Sirlin as sum of two parts:



★ low-energy part approximated by *Born* contribution (elastic intermediate state)



★ high-energy part (above scale $\Lambda \sim 1$ GeV) computed in terms of scattering from *free quarks*

$$\Box^{A}_{\gamma Z} = \frac{5\alpha}{2\pi}\left(1 - 4\sin^2\theta_W\right)\left[\ln\frac{M_Z^2}{\Lambda^2} + C_{\gamma Z}(\Lambda)\right]$$

$$\approx \; 0.0048$$

short−distance

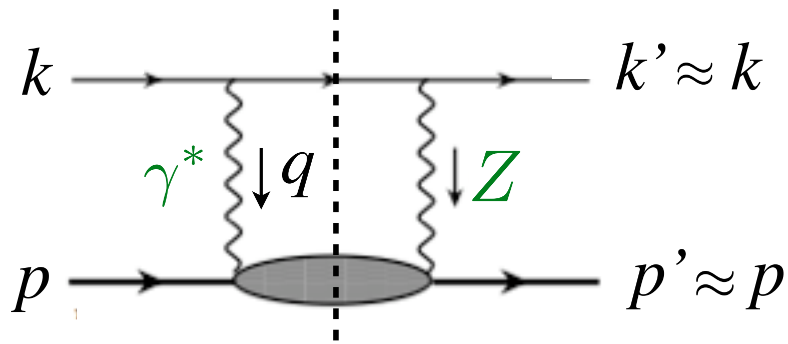
long−distance

Marciano, Sirlin, PRD ***29*** *(1984) 75; Erler et al., PRD* ***68*** *(2003) 016006*

Axial h correction

- axial h correction $\Box^{A}_{\gamma Z}$ dominant γZ correction in atomic parity violation at very low (zero) energy

  → repeat calculation using forward dispersion relations with realistic (structure function) input



★ axial h contribution *anti*symmetric under $E' \leftrightarrow -E'$:

$$\Re e\, \Box^{A}_{\gamma Z}(E) = \frac{2}{\pi} \int_0^\infty dE' \frac{E'}{E'^2 - E^2}\, \Im m\, \Box^{A}_{\gamma Z}(E')$$

★ imaginary part can only grow as $\log E' / E'$

Axial h correction

- imaginary part given by interference $F_3^{\gamma Z}$ structure function

$$\Im m\,\Box^A_{\gamma Z}(E) = \frac{\alpha}{(s-M^2)^2}\int_{W_\pi^2}^{s} dW^2 \int_0^{Q^2_{\max}} \frac{dQ^2}{1+Q^2/M_Z^2}$$
$$\times\ \frac{g_V^e}{2g_A^e}\left(\frac{4ME}{W^2-M^2+Q^2}-1\right)F_3^{\gamma Z}$$

with $g_A^e = -\frac{1}{2}, \quad g_V^e = -\frac{1}{2}(1-4\hat{s}^2)$

$$\boxed{\begin{aligned}\hat{s}^2 &= \sin^2\theta_W^{\overline{\mathrm{MS}}}(M_Z)\\ &= 0.23116(13)\end{aligned}}$$

$\rightarrow$ $F_3^{\gamma Z}$ structure function

- elastic part given by $G_M^p\, G_A^Z$
- resonance part from parametrization of ν scattering data (Lalakulich-Paschos)
- DIS part dominated by leading twist PDFs at small x (MSTW, CTEQ, Alekhin)

Axial h correction

- change integration variable $W^2 \to x$ and switch order of integration

$$\mathcal{I}m\,\Box^{A}_{\gamma Z} = (1-4\hat{s}^2)\frac{\alpha}{2ME}\int\limits_0^{2ME}\frac{dQ^2}{1+Q^2/M_Z^2}\int\limits_{x_{\min}}^{1}\frac{dx}{x}\left(1-\frac{y}{2}\right)F_3^{\gamma Z}$$

where $y = (W^2 - M^2 + Q^2)/2ME$

→ in DIS region ($Q^2 \gtrsim 1\ \mathrm{GeV}^2$), expand integrand in $1/Q^2$

$$\mathcal{R}e\,\Box^{A(\mathrm{DIS})}_{\gamma Z} = (1-4\hat{s}^2)\frac{3\alpha}{2\pi}\int\limits_{Q_0^2}^{\infty}\frac{dQ^2}{Q^2(1+Q^2/M_Z^2)}$$

$$\times\left[M_3^{\gamma Z(1)} - \frac{2M^2}{9Q^4}(5E^2-3Q^2)M_3^{\gamma Z(3)}\right]$$

with moments $M_3^{\gamma Z(n)}(Q^2) = \int\limits_0^1 dx\, x^{n-1} F_3^{\gamma Z}(x,Q^2)$

Axial h correction

- structure function moments

<u>$n = 1$</u> $$M_3^{\gamma Z(1)}(Q^2) = \frac{5}{3}\left(1 - \frac{\alpha_s(Q^2)}{\pi}\right)$$

→ γZ analog of Gross-Llewellyn Smith sum rule

$$\mathcal{R}e\,\Box_{\gamma Z}^{A(\mathrm{DIS})} \approx (1-4\hat{s}^2)\,\frac{5\alpha}{2\pi}\int_{Q_0^2}^{\infty}\frac{dQ^2}{Q^2(1+Q^2/M_Z^2)}\left(1-\frac{\alpha_s(Q^2)}{\pi}\right)$$

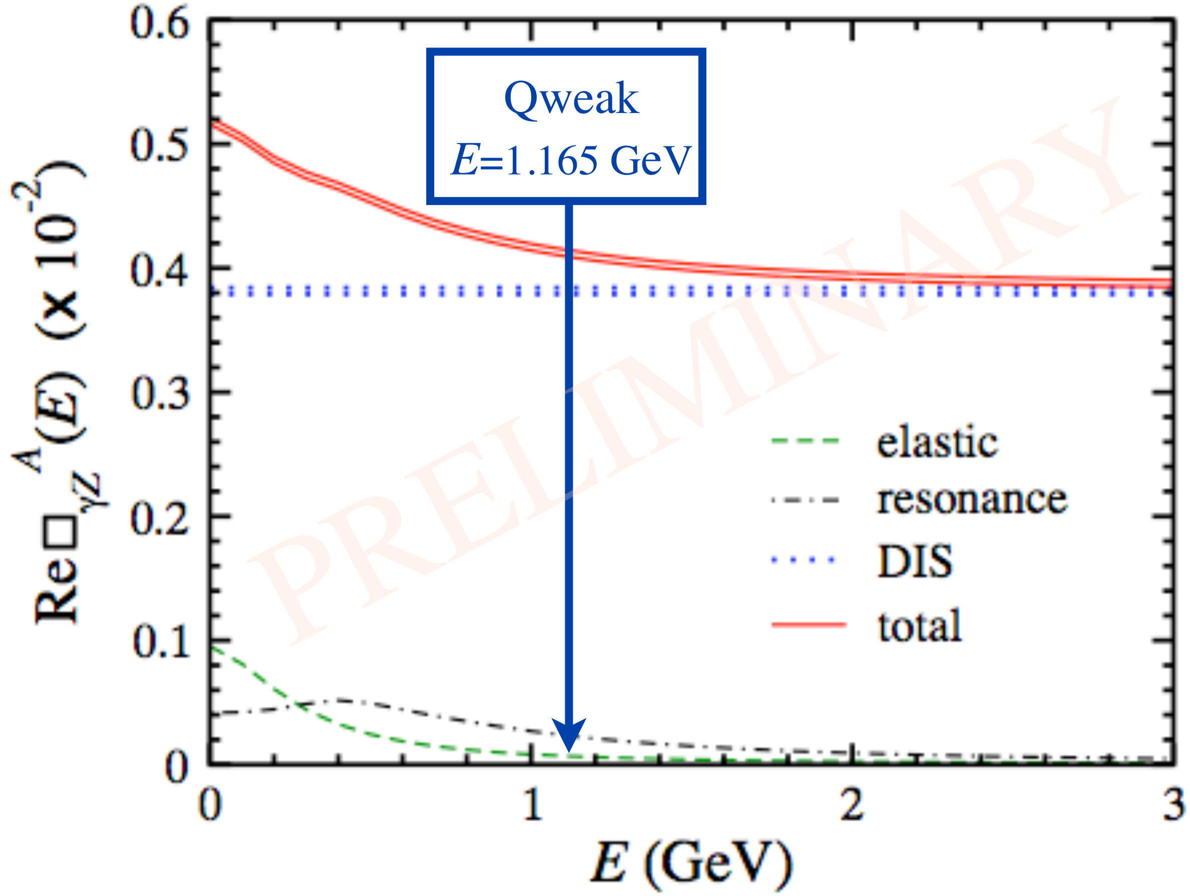
→ precisely result from Marciano & Sirlin!

(works because result depends on lowest moment of *valence* PDF, with <u>model-independent normalization</u>!)

<u>$n = 3$</u> $$M_3^{\gamma Z(3)}(Q^2) = \frac{1}{3}\left(2\langle x\rangle_{u_V} + \langle x\rangle_{d_V}\right)\left(1 + \frac{5\alpha_s(Q^2)}{12\pi}\right)$$

→ related to momentum carried by valence quarks

Axial h correction



Blunden, WM, Thomas (2011)

→ dominated by DIS contribution (weak E dependence)

Axial h correction

→ correction at $\underline{E = 0}$

$$\Re e\,\Box^{A}_{\gamma Z} = \underset{\text{elastic}}{\underset{\uparrow}{0.0009}} + \underset{\text{resonance}}{\underset{\uparrow}{0.0003}} + \underset{\text{DIS}}{\underset{\uparrow}{0.0037}} = \underline{0.0050}$$

→ correction at $\underline{E = 1.165\ \text{GeV}}$ (Qweak)

$$\Re e\,\Box^{A}_{\gamma Z} = 0.00006 + 0.0002 + 0.0037 = \underline{0.0039}$$

cf. MS value: $\underline{0.0048}$ ($\sim$1% shift in Q^p_W)

Vector h correction

- vector h correction $\Box^V_{\gamma Z}$ vanishes at $E = 0$, but experiment has $E \sim 1\ \mathrm{GeV}$ – what is energy dependence?

→ forward dispersion relation

★ $\Re e\,\Box^V_{\gamma Z}(E) = \frac{2E}{\pi}\int_0^\infty dE' \frac{1}{E'^2 - E^2}\,\Im m\,\Box^V_{\gamma Z}(E')$

★ integration over $E' < 0$ corresponds to crossed-box, vector h contribution symmetric under $E' \leftrightarrow -E'$

→ imaginary part given by

$$\Im m\,\Box^V_{\gamma Z}(E) = \frac{\alpha}{(s - M^2)^2}\int_{W_\pi^2}^{s} dW^2 \int_0^{Q^2_{\max}} \frac{dQ^2}{1 + Q^2/M_Z^2}$$

$$\times\left(F_1^{\gamma Z} + F_2^{\gamma Z}\frac{s\,(Q^2_{\max} - Q^2)}{Q^2(W^2 - M^2 + Q^2)}\right)$$

factor 2 larger than GH;
confirmed by Rislow & Carlson,
arXiv:1011:2397 [hep-ph]

*Gorchtein, Horowitz, PRL **102** (2009) 091806*
Gorchtein, Horowitz, Ramsey-Musolf, arXiv:1003.4300

Vector h correction

→ $F_{1,2}^{\gamma Z}$ structure functions

- parton model for <u>DIS</u> region $F_2^{\gamma Z} = 2x \sum_q e_q\, g_V^q\, (q + \bar{q}) = 2x F_1^{\gamma Z}$

  → $F_2^{\gamma Z} \approx F_2^{\gamma}$ good approximation at *low* x

  → provides upper limit at *large* x $(F_2^{\gamma Z} \lesssim F_2^{\gamma})$

- in <u>resonance</u> region use phenomenological input for F_2, empirical (SLAC) fit for R

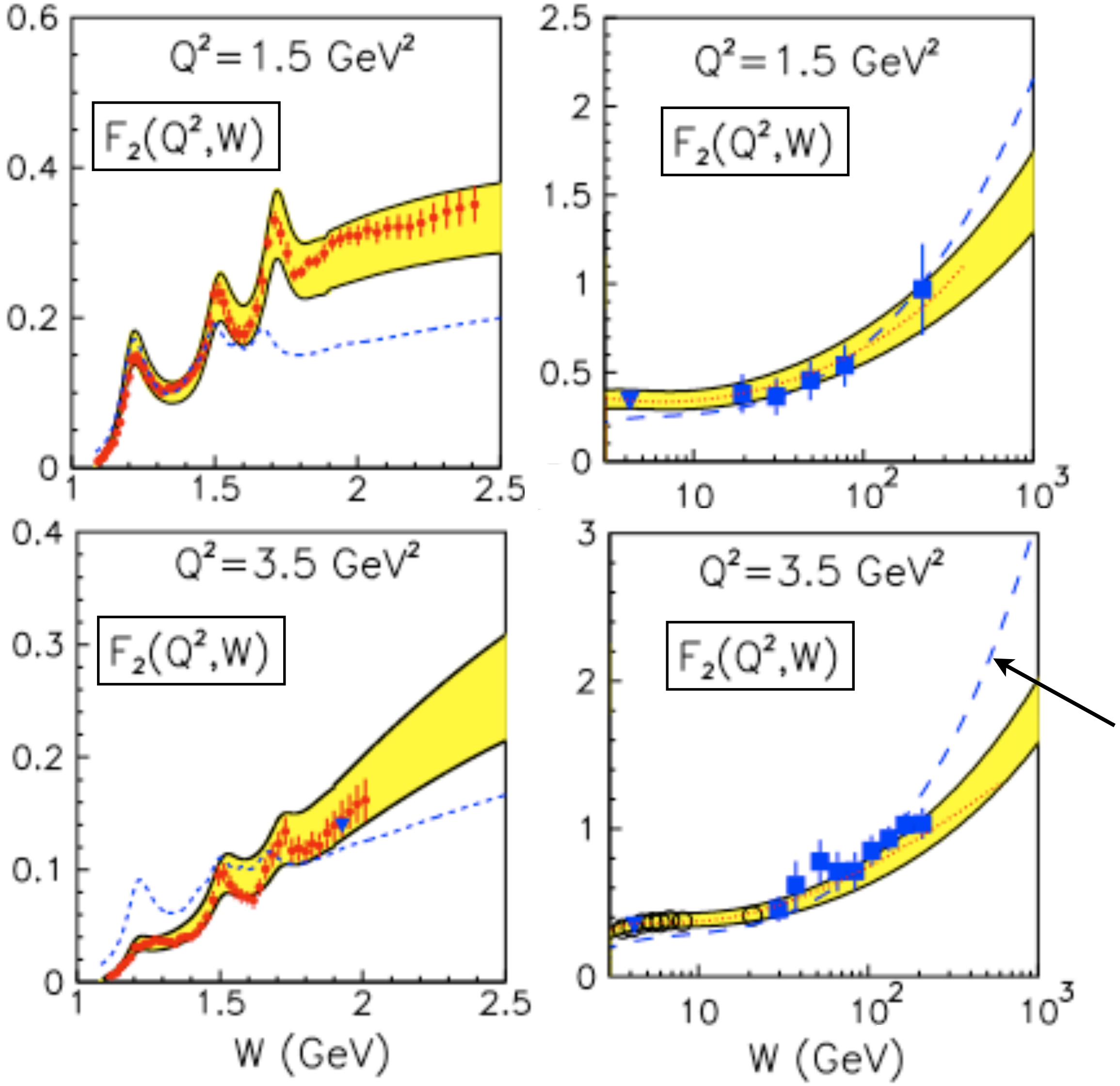
  → for transitions to <u>$I = 3/2$</u> states ($e.g.$ Δ), CVC and isospin symmetry give $F_i^{\gamma Z} = (1 + Q_W^p) F_i^{\gamma}$

  → for transitions to <u>$I = 1/2$</u> states, SU(6) wave functions predict Z & γ transition couplings equal to a few %

Vector h correction

→ compare structure function input with data

low W



high W

GVMD model
(used as input by
Gorchtein & Horowitz)

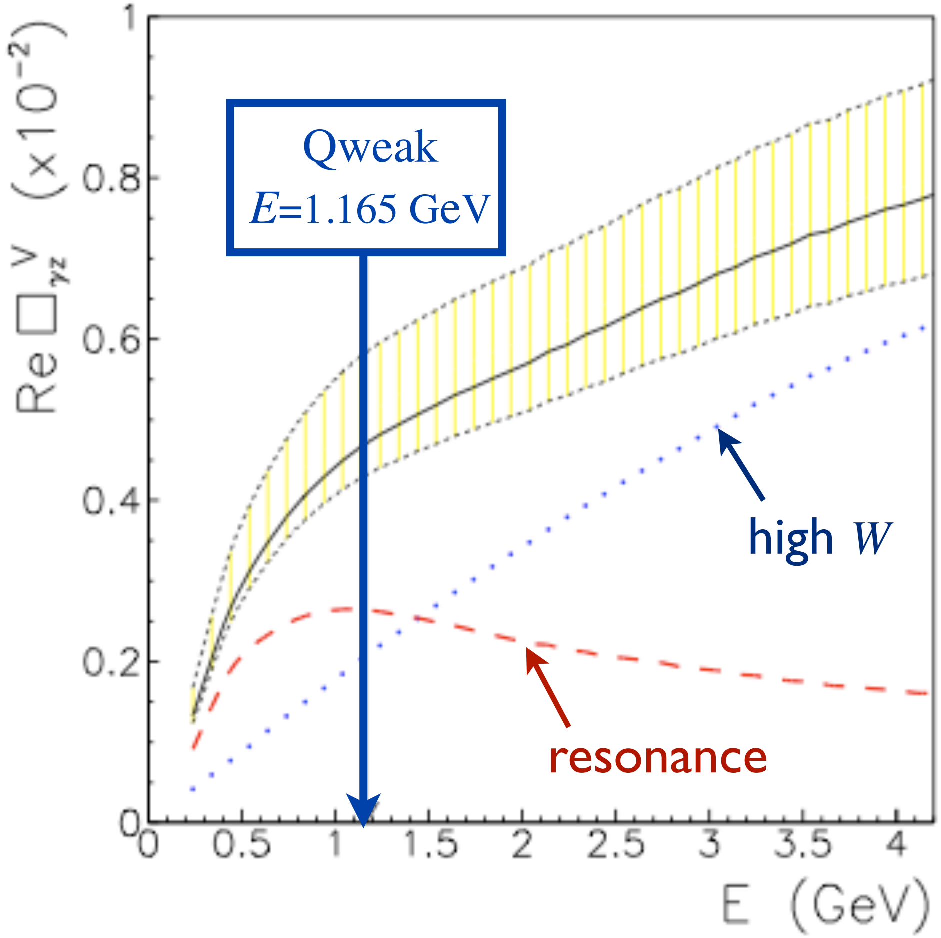
Vector h correction

→ total $\square^V_{\gamma Z}$ correction:

$$\Re e\, \square^V_{\gamma Z} = 0.0047^{+0.0011}_{-0.0004}$$

or $6.6^{+1.5}_{-0.6}\,\%$ of uncorrected Q^p_W

$$\boxed{Q^p_W = 0.0713 \to 0.0760}$$



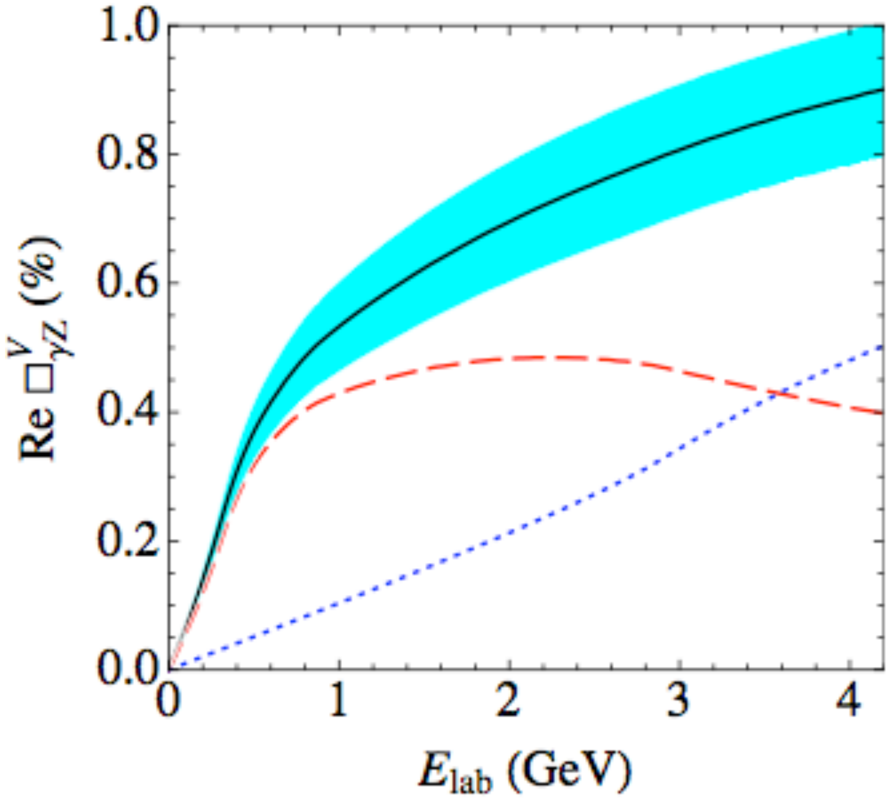
Sibirtsev, Blunden, WM, Thomas
PRD ***82*** *(2010) 013011*

Vector h correction

→ total $\Box^V_{\gamma Z}$ correction:

$$\Re e\,\Box^V_{\gamma Z} = 0.0057 \pm 0.0009$$

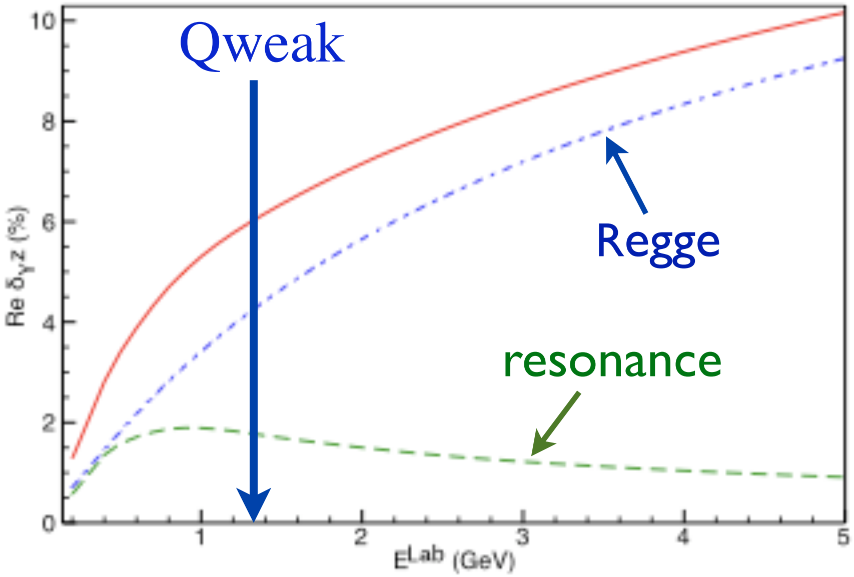
→ compatible with SBMT within errors



Rislow, Carlson, arXiv:1011.2397 [hep-ph]

*Gorchtein, Horowitz, PRL **102** (2009) 091806*

*(see also Gorchtein, Horowitz, Ramsey-Musolf, AIP Conf. Proc. **1265** (2010) 328)*



$$\Re e\,\delta_{\gamma Z} = \Re e\,\Box^V_{\gamma Z}/Q^p_W \approx 6\%$$

mostly from high-W ("Regge") contribution

→ our formula for $\Im m\,\Box^V_{\gamma Z}$ factor 2 larger*
("nuclear physics" *vs.* "particle physics" conventions for weak charges in structure function definitions?)

* confirmed by Rislow/Carlson *arXiv:1011.2397*

→ GH omit factor $(1-x)$ in definition of $F_{1,2}$
($\sim 30\%$ enhancement)

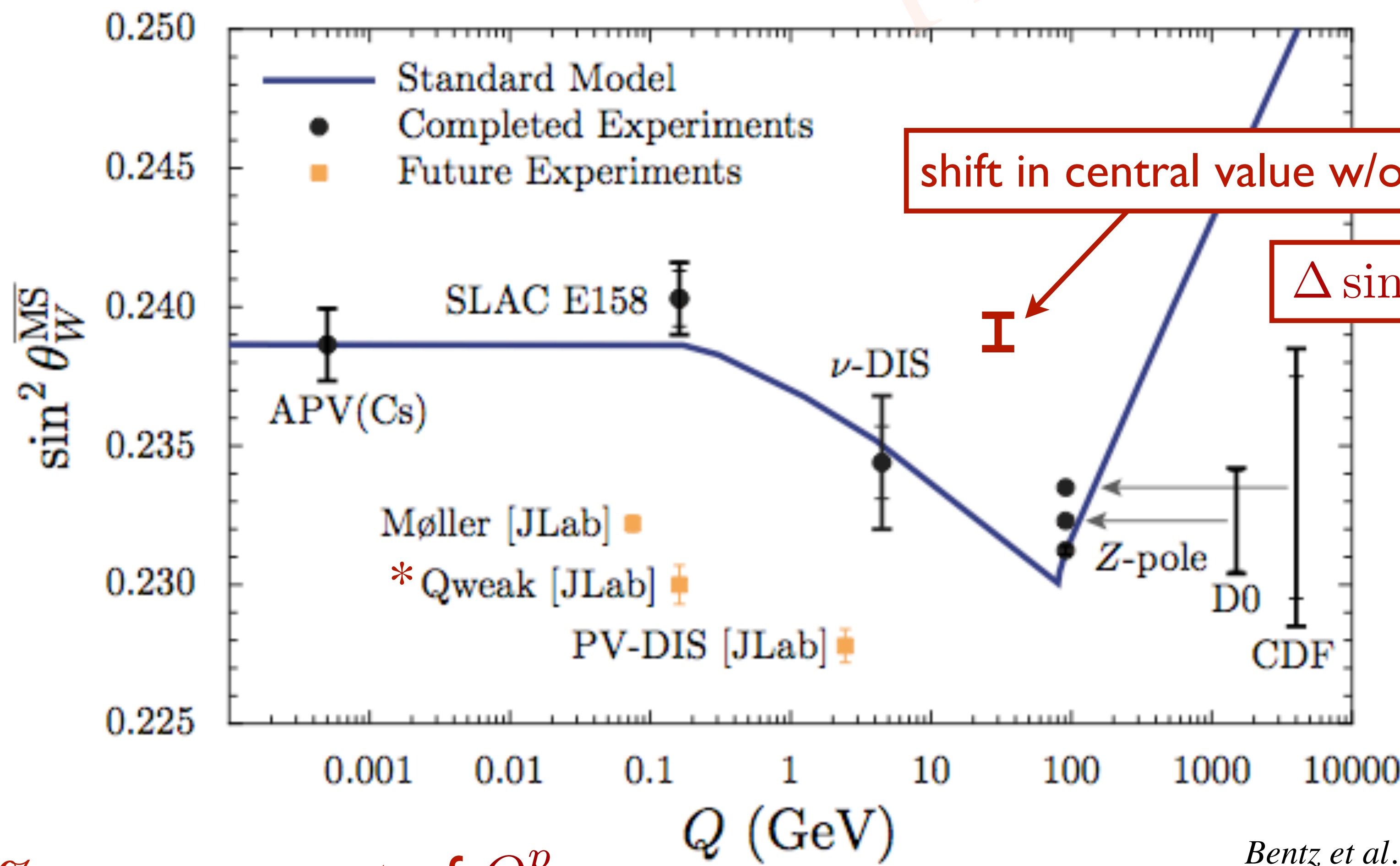
→ GH use $Q^p_W \sim 0.05$ *cf.* ~ 0.07
($\sim 40\%$ enhancement)

→ numerical agreement for $\delta^V_{\gamma Z}$ coincidental (?)

Combined vector and axial h correction

$$\boxed{Q_W^p = 0.0713 \rightarrow \;\approx 0.076}$$

→ significant shift in central value, errors within projected experimental uncertainty $\Delta Q_W^p = \pm 0.003$



Bentz et al., PLB ***693*** *(2010) 462*

* 4% measurement of Q_W^p

Summary

- Two-boson exchange corrections play minor role in *strange form factor* extraction

  $\longrightarrow$ *cf*. significant role of TPE in Rosenbluth extraction of G_E^p

- Dramatic effect of $\gamma(Z\gamma)$ corrections at forward angles on *proton weak charge*, $\Delta Q_W^p \sim 6\%$, *cf*. PDG

  $\longrightarrow$ would significantly shift extracted weak angle

  $\longrightarrow$ better constraints from direct measurement of $F_{1,2,3}^{\gamma Z}$ (*e.g.* in PVDIS at JLab)

- New formulation in terms of *moments* of structure functions

  $\longrightarrow$ places on firm footing earlier derivation of Marciano/Sirlin in "free quark model"

The End