Florida International University
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Duality: Towards Bridging the Quark-Hadron Chasm

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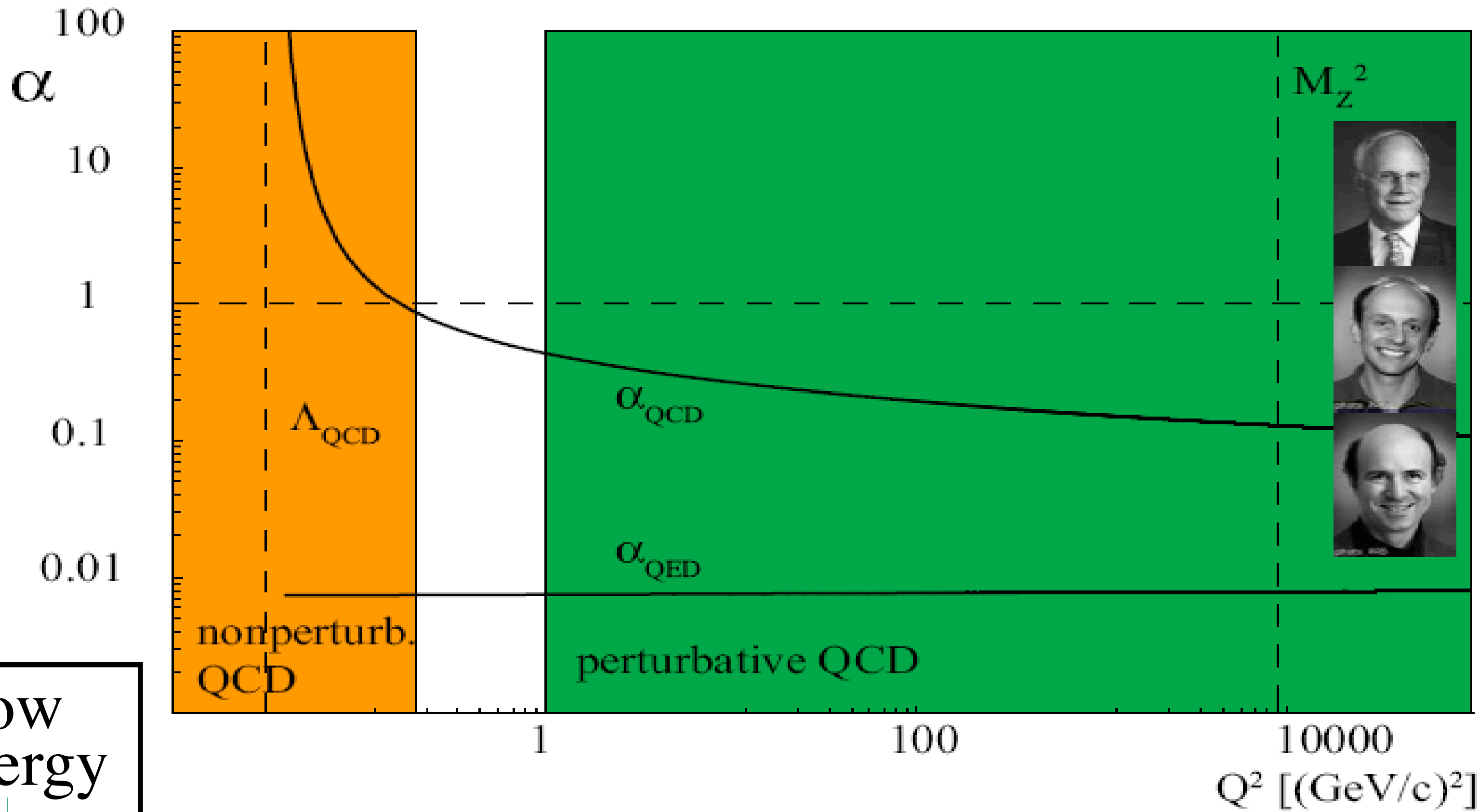
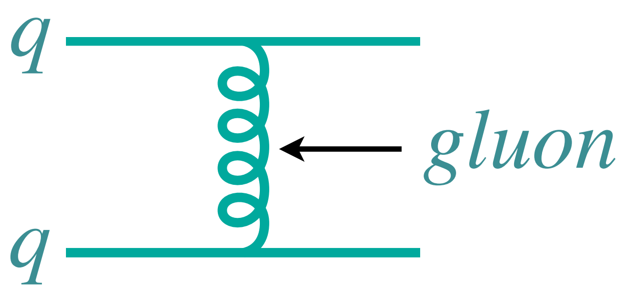
Outline

- Quark-hadron duality: historical perspective
- Duality and Quantum ChromoDynamics (QCD)
  - $\longrightarrow$ twists and moments
  - $\longrightarrow$ insights from nonperturbative models
- Implications of duality for quark distributions
- Outlook

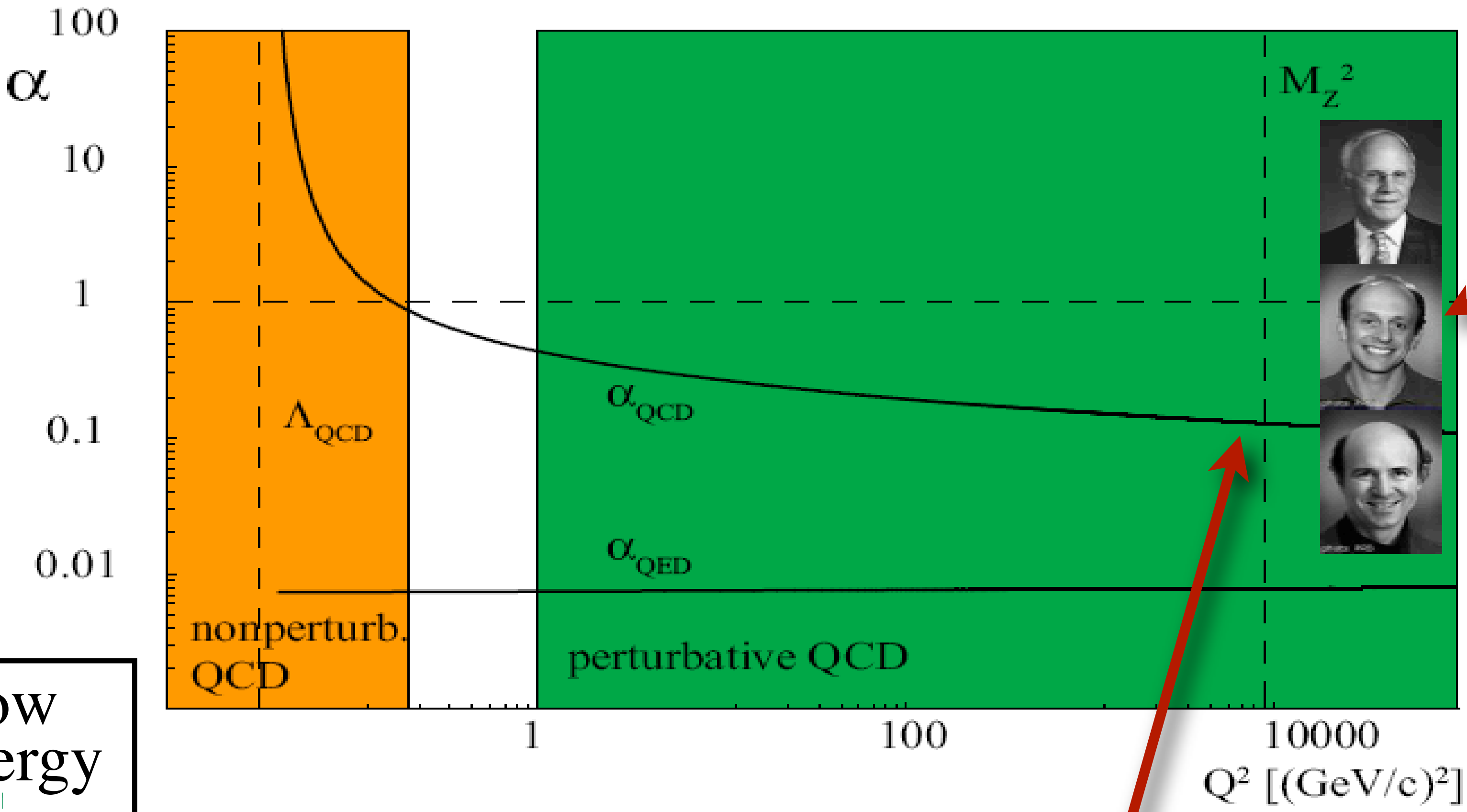
low energy
long distance



high energy
short distance



strength of nuclear force acting between quarks given by α_{QCD} (or α_s)



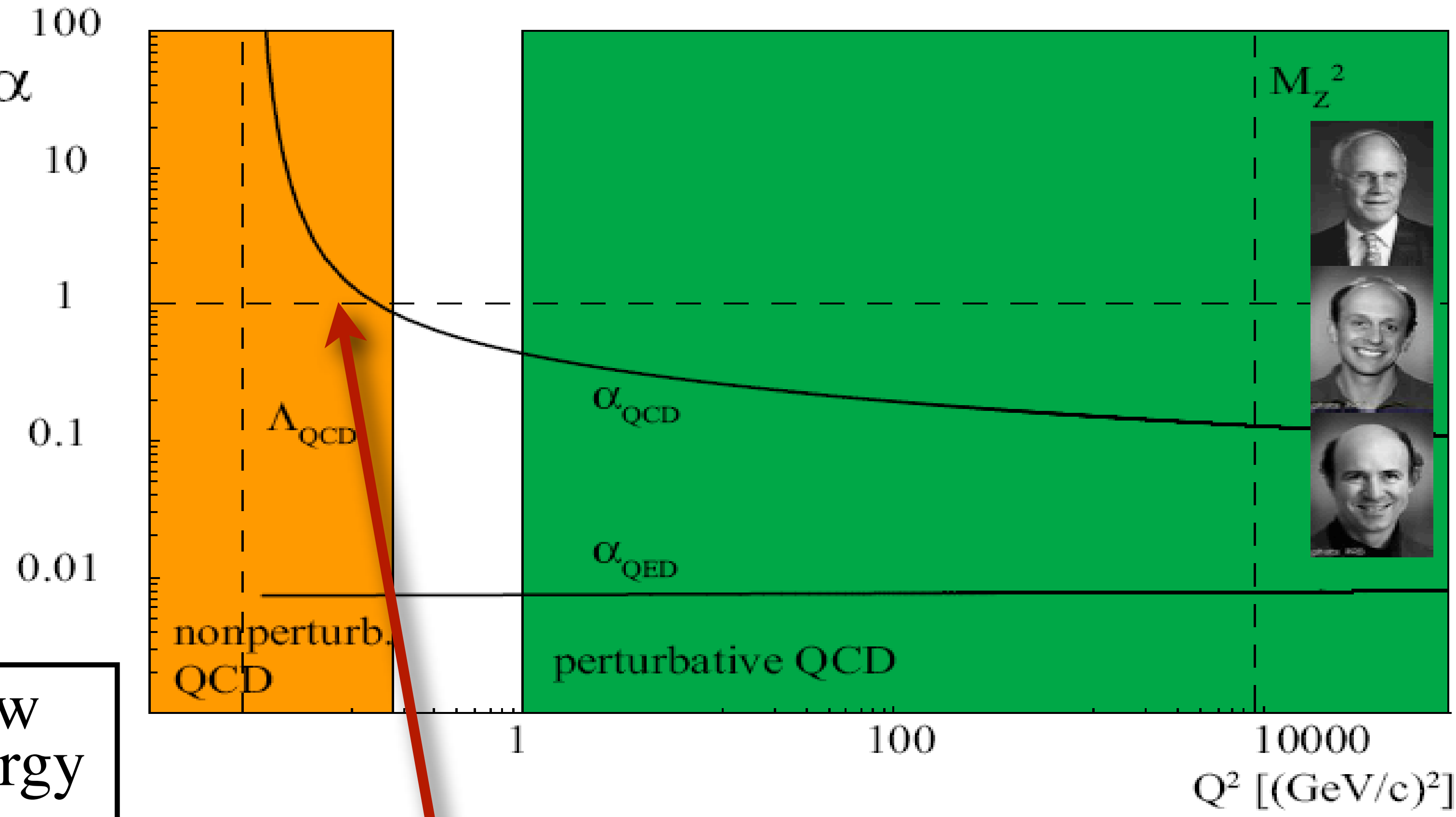
Nobel Prize (2004) for discovery of "asymptotic freedom"

low energy *long distance*

high energy *short distance*

physical observables described in terms of quarks and gluons

→ "strong" coupling constant α_s small, calculate using perturbation theory



low energy *long distance*

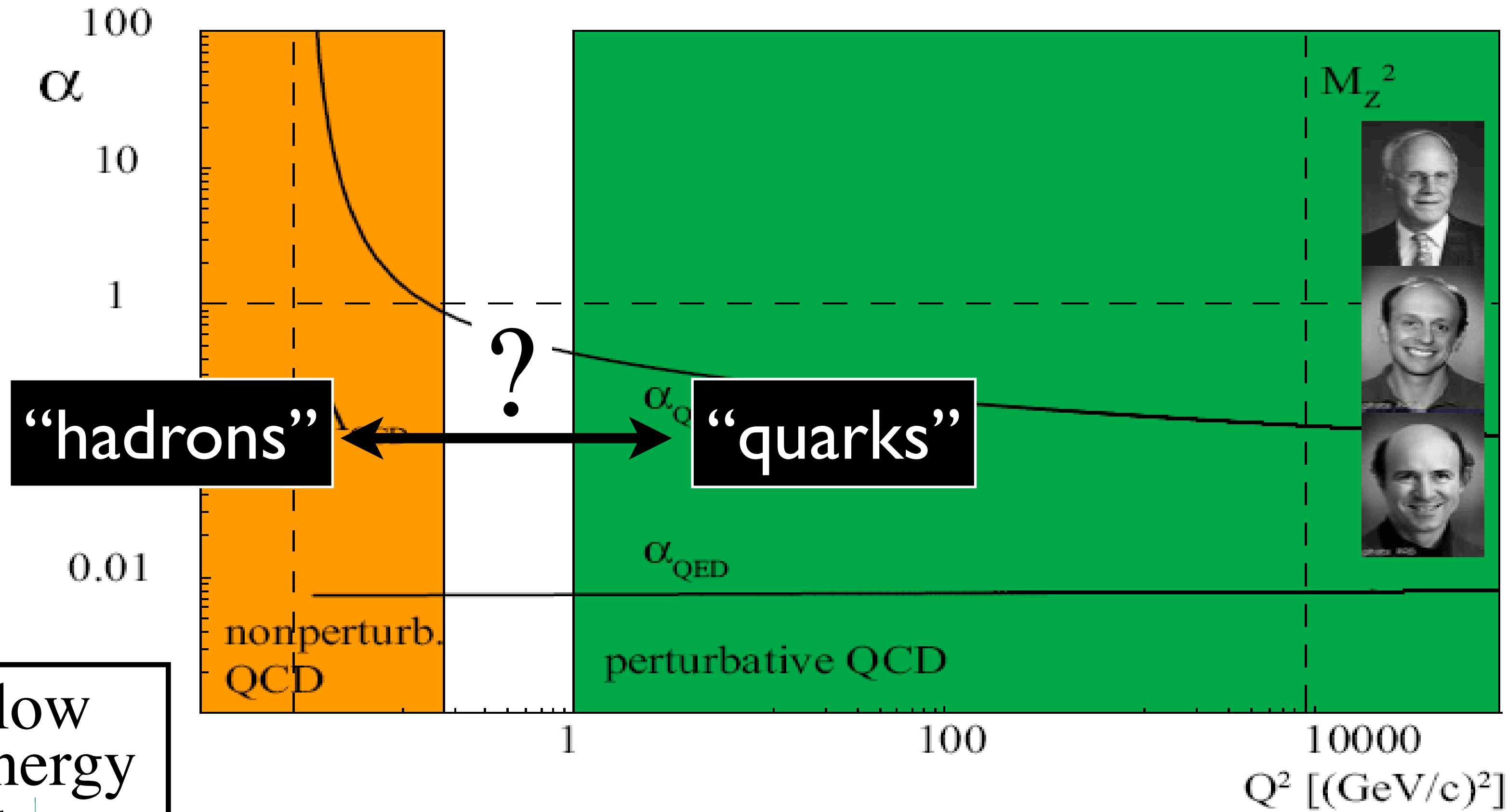
high energy *short distance*

α_s large, cannot describe observables in terms of quarks perturbatively

(need nonperturbative methods *e.g.* lattice QCD)

→ meson & baryon (hadron) degrees of freedom prominent

(*e.g.* chiral effective field theory)



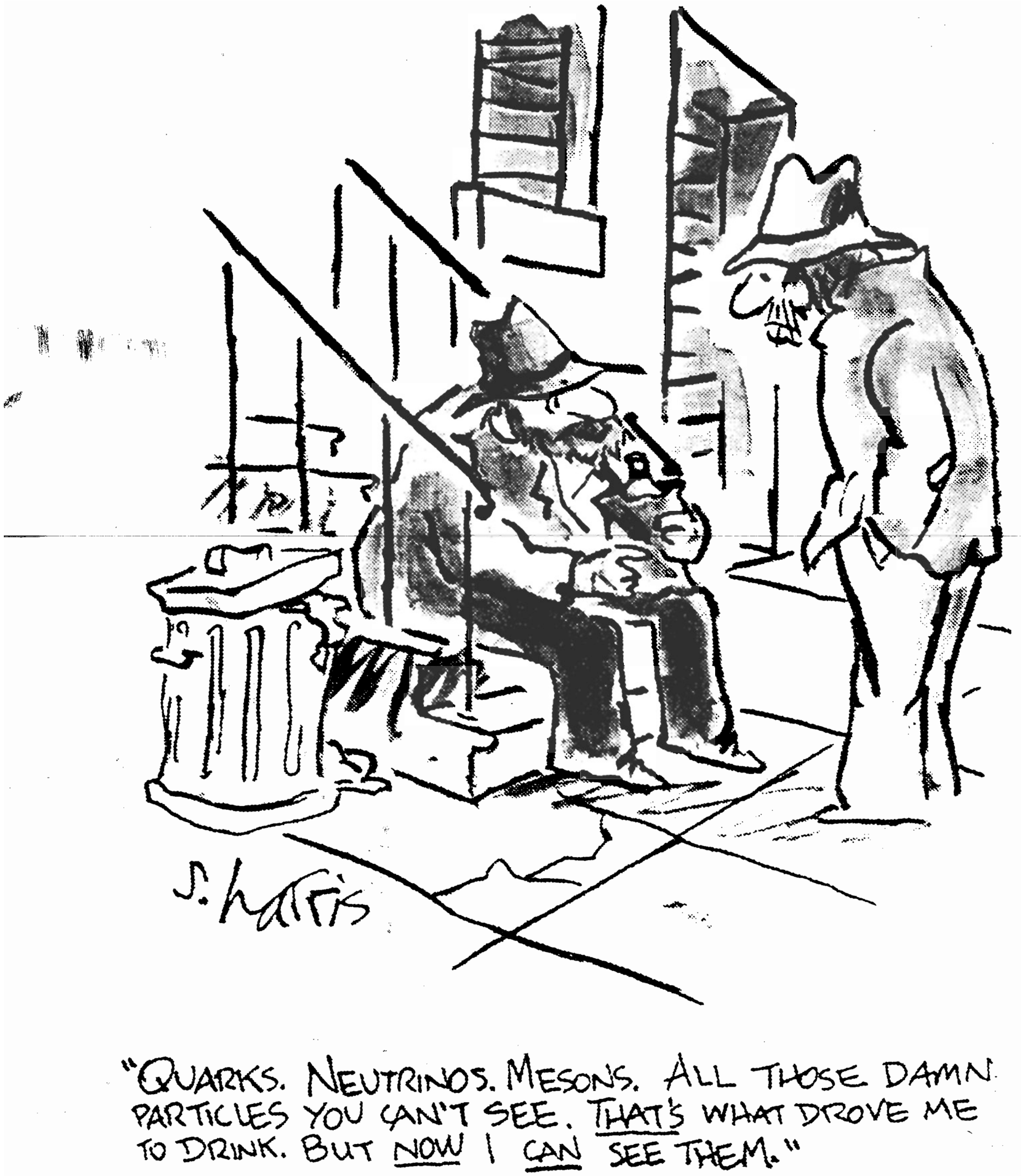
low
energy
long
distance

high
energy
short
distance

Looking for quarks in hadrons
is like looking for the Mafia in Sicily –
everybody *knows* they're there,
but it's hard to find the evidence!

Anonymous

one way of seeing connections...



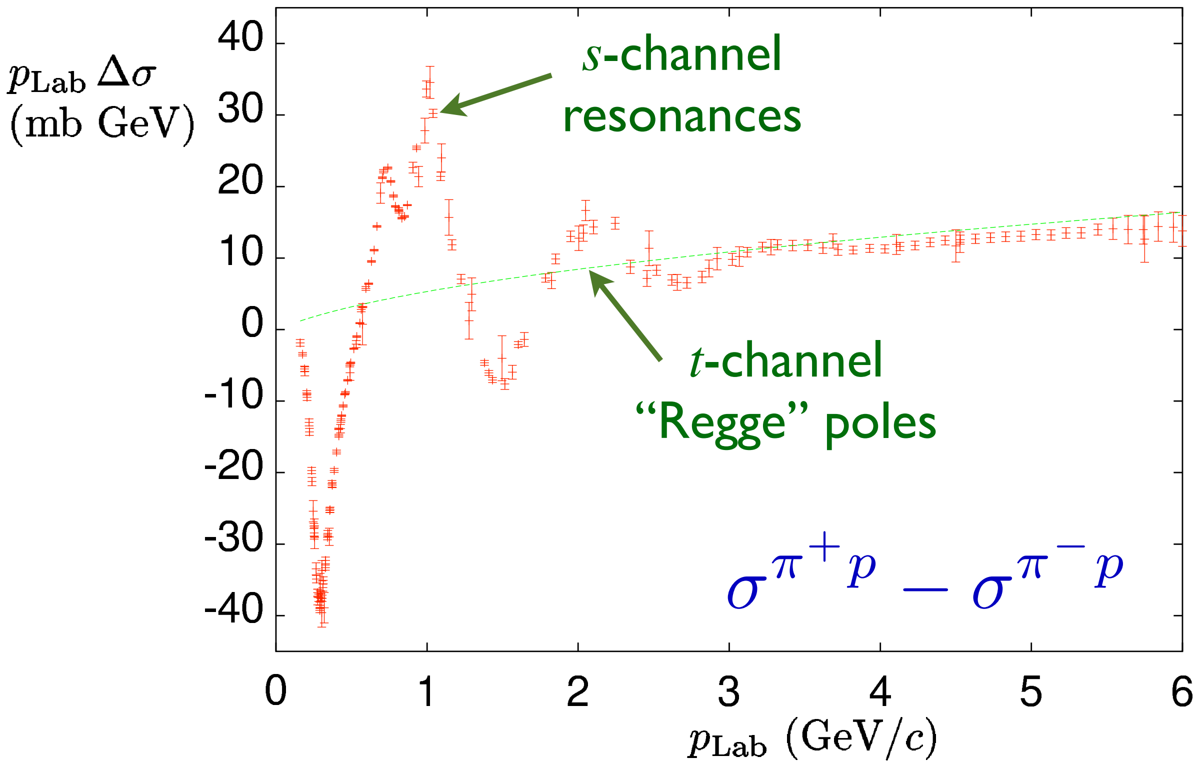
Quark-hadron duality

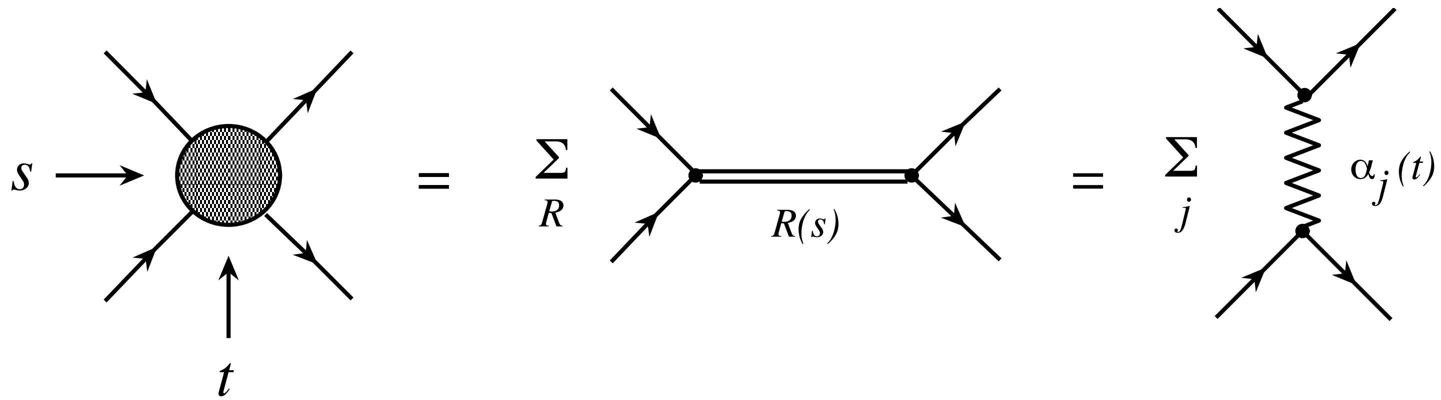
Complementarity between *quark* and *hadron* descriptions of observables

$$\sum_{hadrons} = \sum_{quarks}$$

Can use either set of *complete* basis states to describe physical phenomena

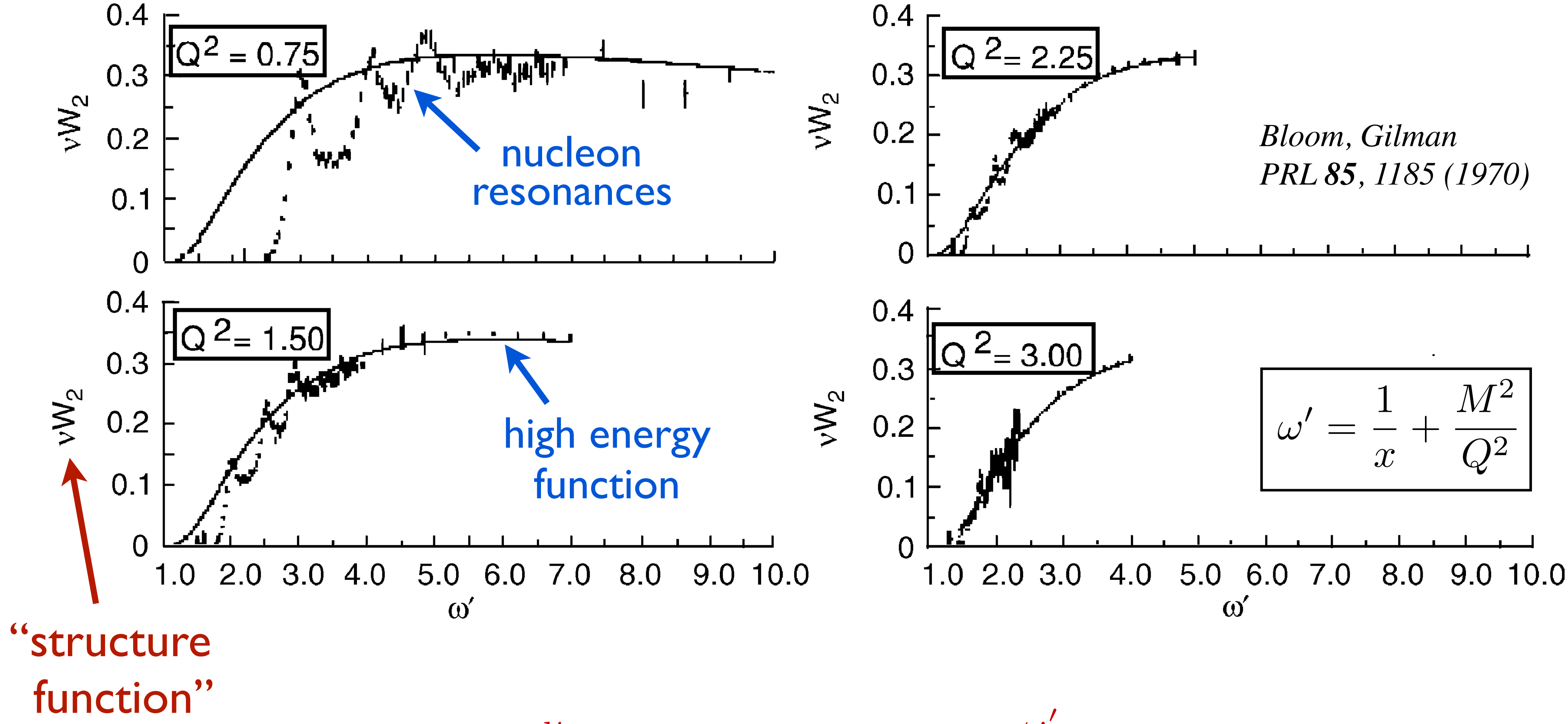
Duality in hadron-hadron scattering





Duality in electron-hadron scattering

"Bloom-Gilman duality"



Bloom, Gilman
PRL ***85****, 1185 (1970)*

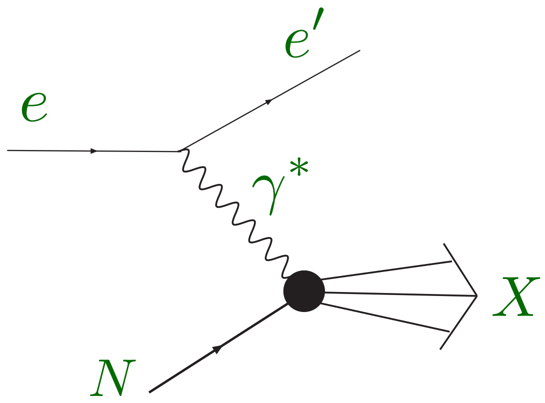
$$\boxed{\omega' = \frac{1}{x} + \frac{M^2}{Q^2}}$$

$$\frac{2M}{Q^2}\int_0^{\nu_m} d\nu\ \nu W_2(\nu, Q^2) = \int_1^{\omega'_m} d\omega'\ \nu W_2(\omega')$$

"hadrons" "quarks"

Electron-nucleon scattering

- Inclusive cross section for $eN \to eX$



$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2 \cos^2\frac{\theta}{2}}{Q^4}\left(2\tan^2\frac{\theta}{2}\frac{F_1}{M} + \frac{F_2}{\nu}\right)$$

$$\left.\begin{aligned} \nu &= E - E' \\ Q^2 &= \vec{q}^{\,2} - \nu^2 = 4EE'\sin^2\frac{\theta}{2} \end{aligned}\right\} \quad x = \frac{Q^2}{2M\nu}$$

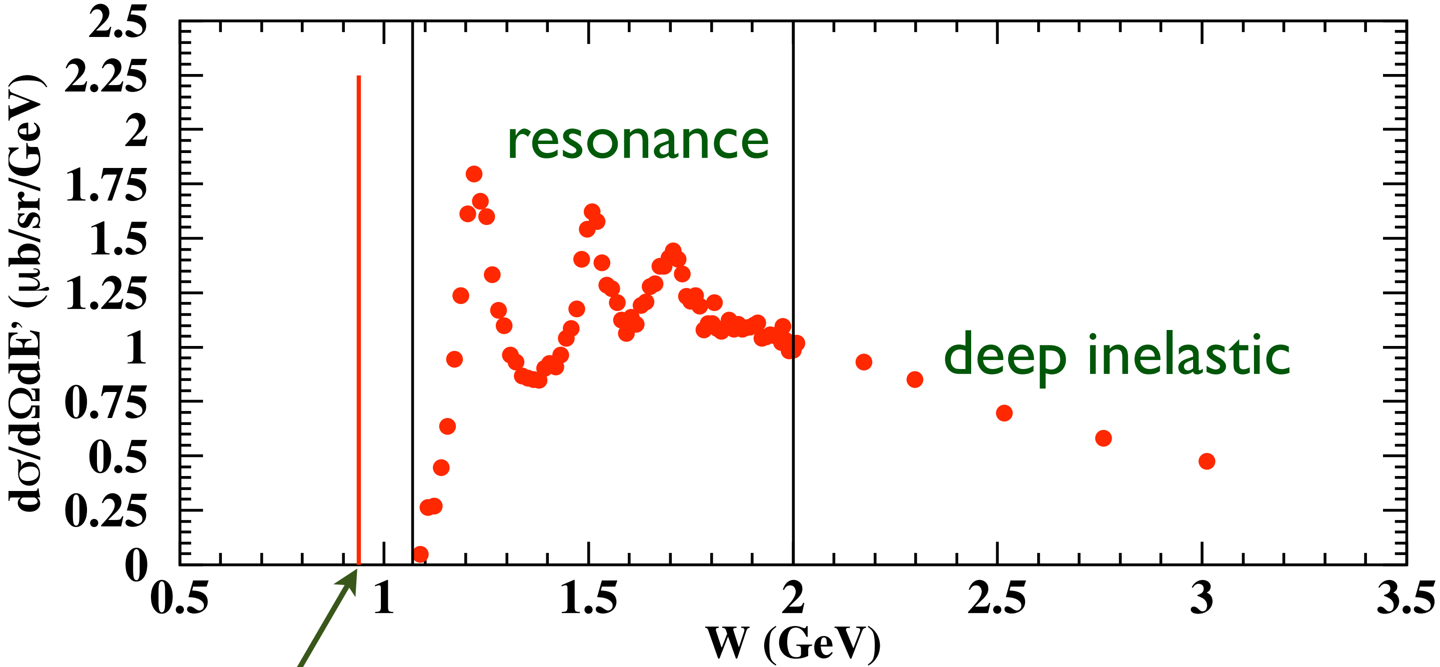
Bjorken scaling variable

- F_1 , F_2 "structure functions"

  → contain all information about structure of nucleon

  → functions of x, Q^2 in general

Electron-nucleon scattering



$\sim F_2$

elastic
$(W{=}M_p)$

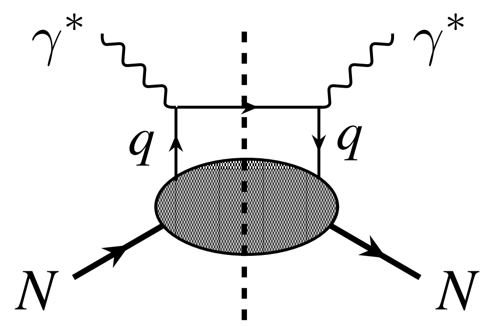
mass of hadron final state X

Bjorken variable in terms of Q^2 & W : $\quad x = \dfrac{Q^2}{W^2 - M^2 + Q^2}$

- In deep inelastic region ($W \gtrsim 2\ \mathrm{GeV},\ Q^2 \gtrsim 1\ \mathrm{GeV}^2$), structure function given by quark and antiquark ("parton") distributions

$$F_2(x,Q^2) = x\sum_q e_q^2\, q(x,Q^2)$$
$$= \frac{4}{9}x(u+\bar{u}) + \frac{1}{9}x(d+\bar{d}) + \frac{1}{9}x(s+\bar{s}) + \cdots$$

→ $q(x,Q^2)$ = probability to find quark type "q" in nucleon, carrying momentum fraction x



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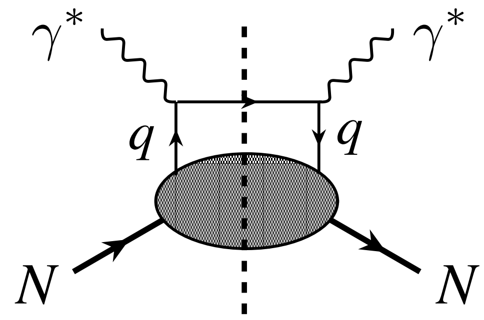
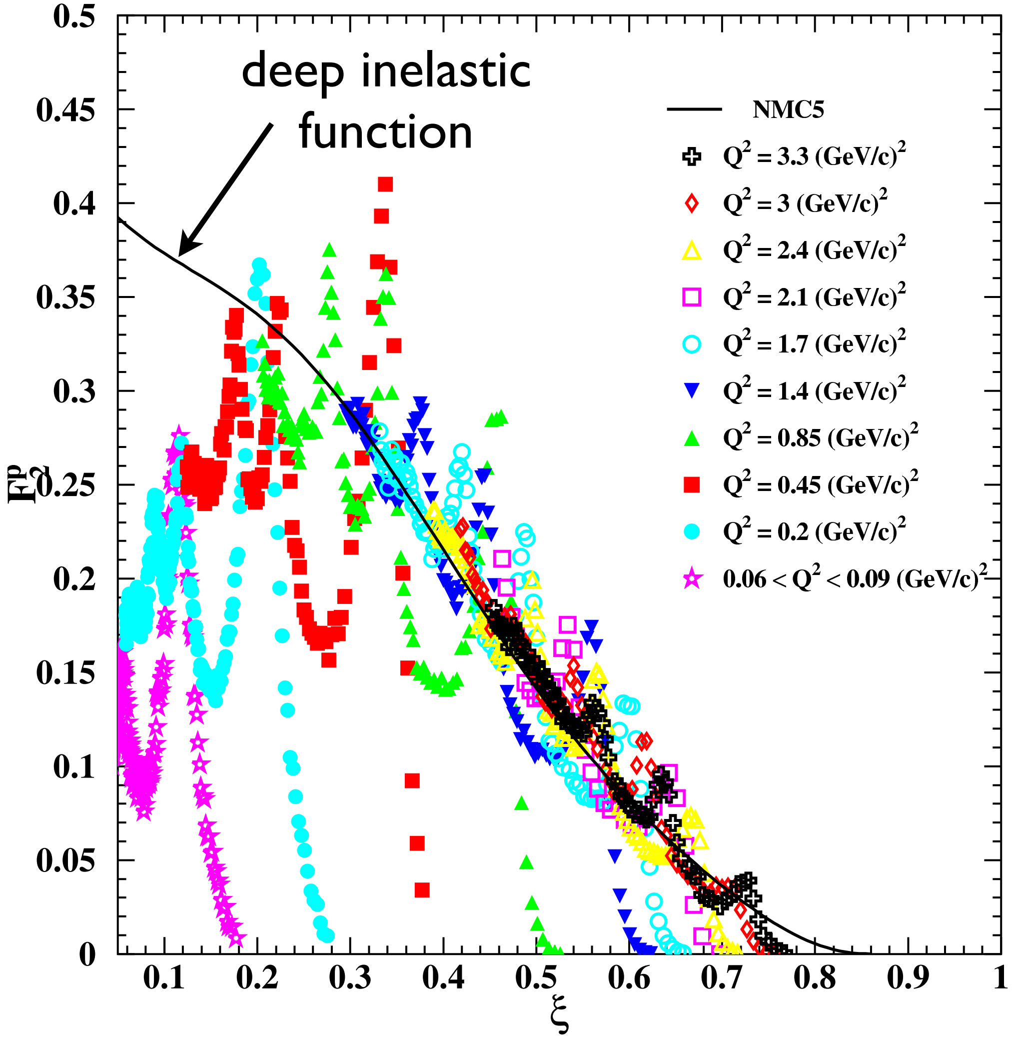


- In resonance region ($W \lesssim 2~\mathrm{GeV}$), or at low Q^2 ($Q^2 \lesssim 1~\mathrm{GeV}^2$) can no longer resolve individual quark structure

→ see *gross features* of hadron (complex, multi-parton effects)

Duality in electron-hadron scattering



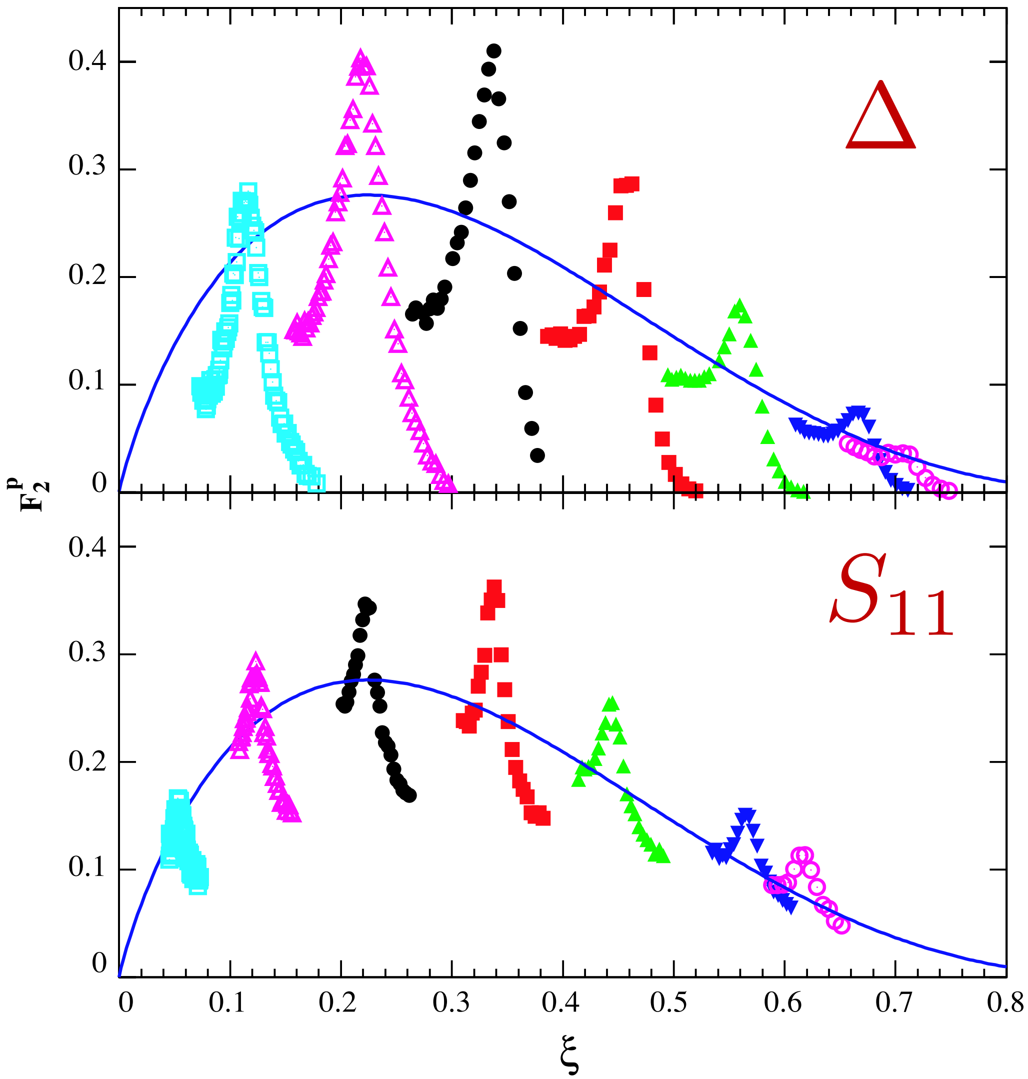
Niculescu et al., PRL ***85****, 1182 (2000)*

average over
(strongly Q^2 dependent)
resonances
$\approx Q^2$ independent
scaling function

"Nachtmann" scaling variable

$$\xi = \frac{2x}{1+\sqrt{1+4M^2x^2/Q^2}}$$

Duality in electron-hadron scattering



→ also exists *locally* in individual resonance regions

Duality in QCD era

- Operator product expansion

  → expand *moments* of structure functions in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx\ x^{n-2}\ F_2(x, Q^2)$$

$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

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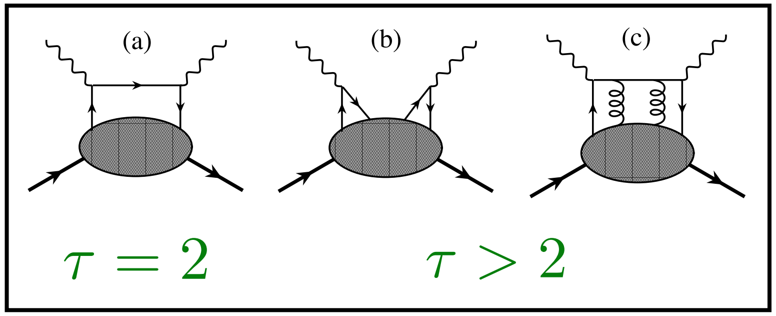
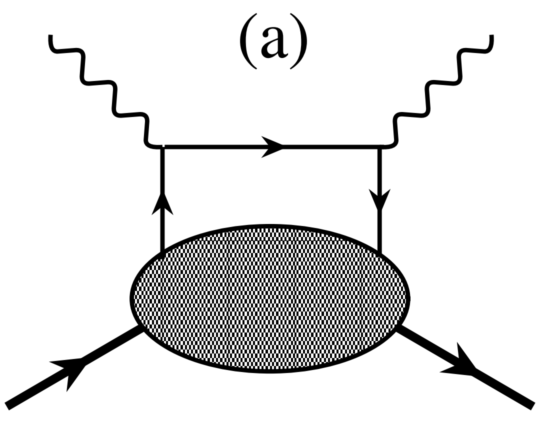
matrix elements of operators with specific "twist" τ

$$\tau = \text{dimension} - \text{spin}$$



Duality in QCD era



$\tau = 2$

single quark scattering

$e.g. \quad \bar{\psi}\ \gamma_\mu\ \psi$

$\tau > 2$

qq and qg correlations

$e.g. \quad \bar{\psi}\ \gamma_\mu\ \psi\ \bar{\psi}\ \gamma_\nu\ \psi$

or $\quad \bar{\psi}\ \tilde{G}_{\mu\nu}\gamma^\nu\ \psi$

Duality in QCD era

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  $\longrightarrow$ expand *moments* of structure functions in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx\ x^{n-2}\ F_2(x,Q^2)$$

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de Rujula, Georgi, Politzer
Ann. Phys. ***103****, 315 (1975)*

- If moment $\approx$ independent of Q^2

  $\longrightarrow$ higher twist terms $A_n^{(\tau>2)}$ small

- Duality $\longleftrightarrow$ suppression of higher twists

Truncated moments

- Seldom have sufficient data to form complete moments

  $\longrightarrow$ usually require $x \to 0$ and $x \to 1$ extrapolations

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- *Truncated* moments allow study of restricted regions in x (or W) within pQCD in well-defined, systematic way

$$\overline{M}_n(\Delta x, Q^2) = \int_{\Delta x} dx\ x^{n-2}\ F_2(x, Q^2)$$

Truncated moments

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- *Truncated* moments allow study of restricted regions in x (or W) within pQCD in well-defined, systematic way

$$\overline{M}_n(\Delta x, Q^2) = \int_{\Delta x} dx\ x^{n-2}\ F_2(x, Q^2)$$

- Obey DGLAP-like evolution equations, similar to PDFs

$$\frac{d\overline{M}_n(\Delta x, Q^2)}{d \log Q^2} = \frac{\alpha_s}{2\pi} \left(P'_{(n)} \otimes \overline{M}_n \right) (\Delta x, Q^2)$$

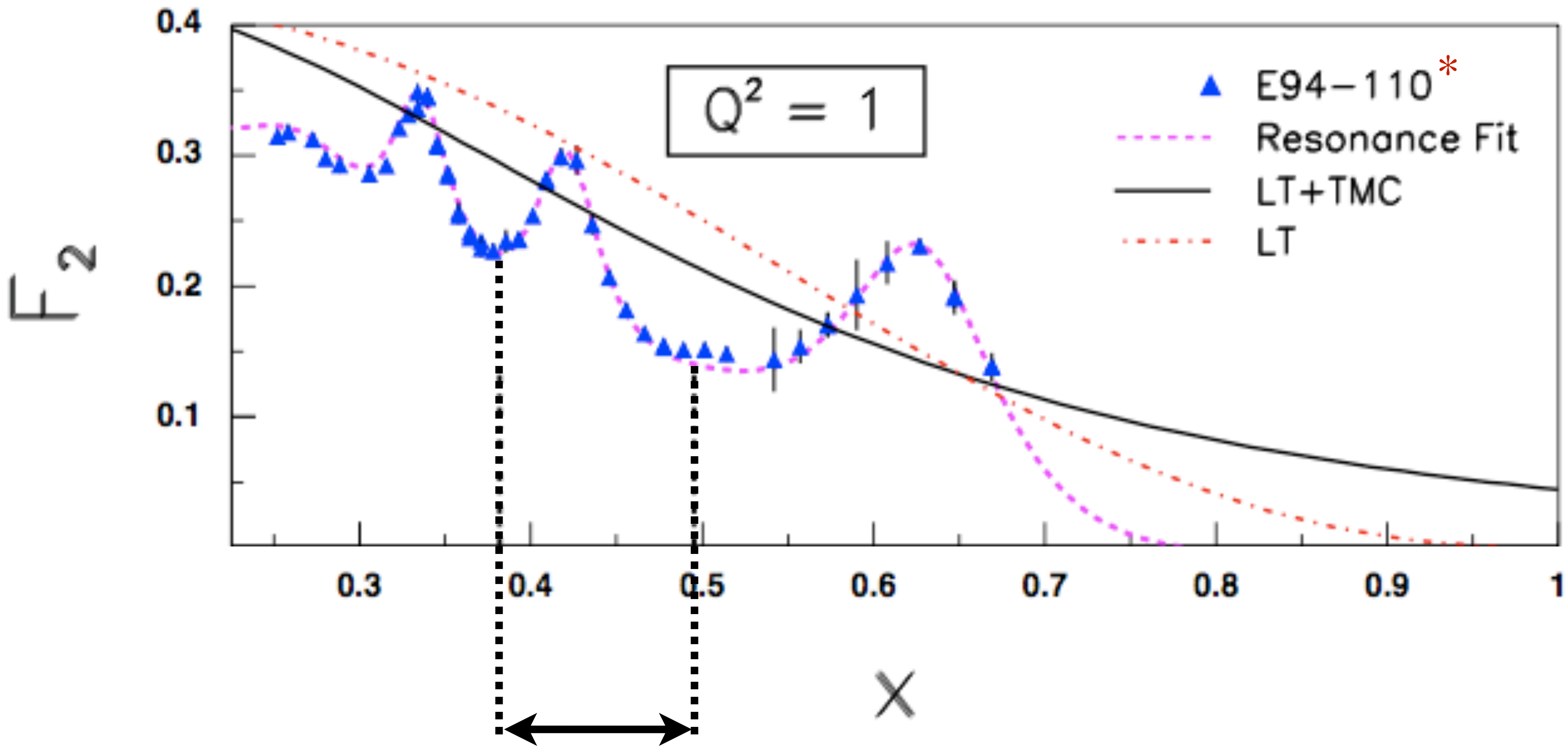
$$P'_{(n)}(z, \alpha_s) = z^n\ P_{NS,S}(z, \alpha_s)$$

truncated splitting function

*Forte, Magnea, PLB **448**, 295 (1999)*
*Kotlorz, Kotlorz, PLB **644**, 284 (2007)*

Truncated moments

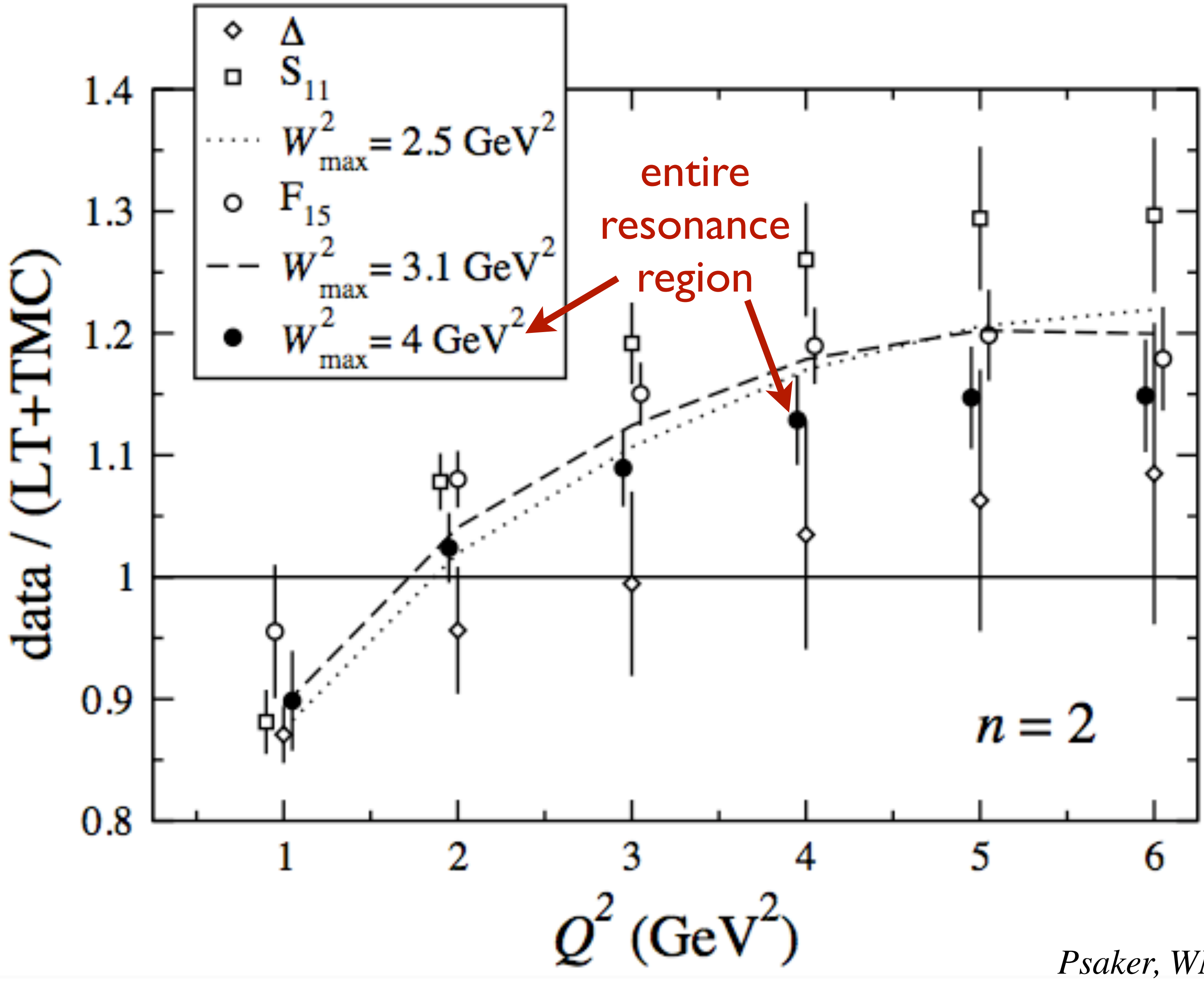
- Follow evolution of *specific resonance (region)* with Q^2 in pQCD framework



* JLab Hall C

how much of this region is <u>*leading twist*</u> ?

Analysis of JLab F_2^p resonance region data



Psaker, WM, Christy, Keppel
*PRC **78**, 025206 (2008)*

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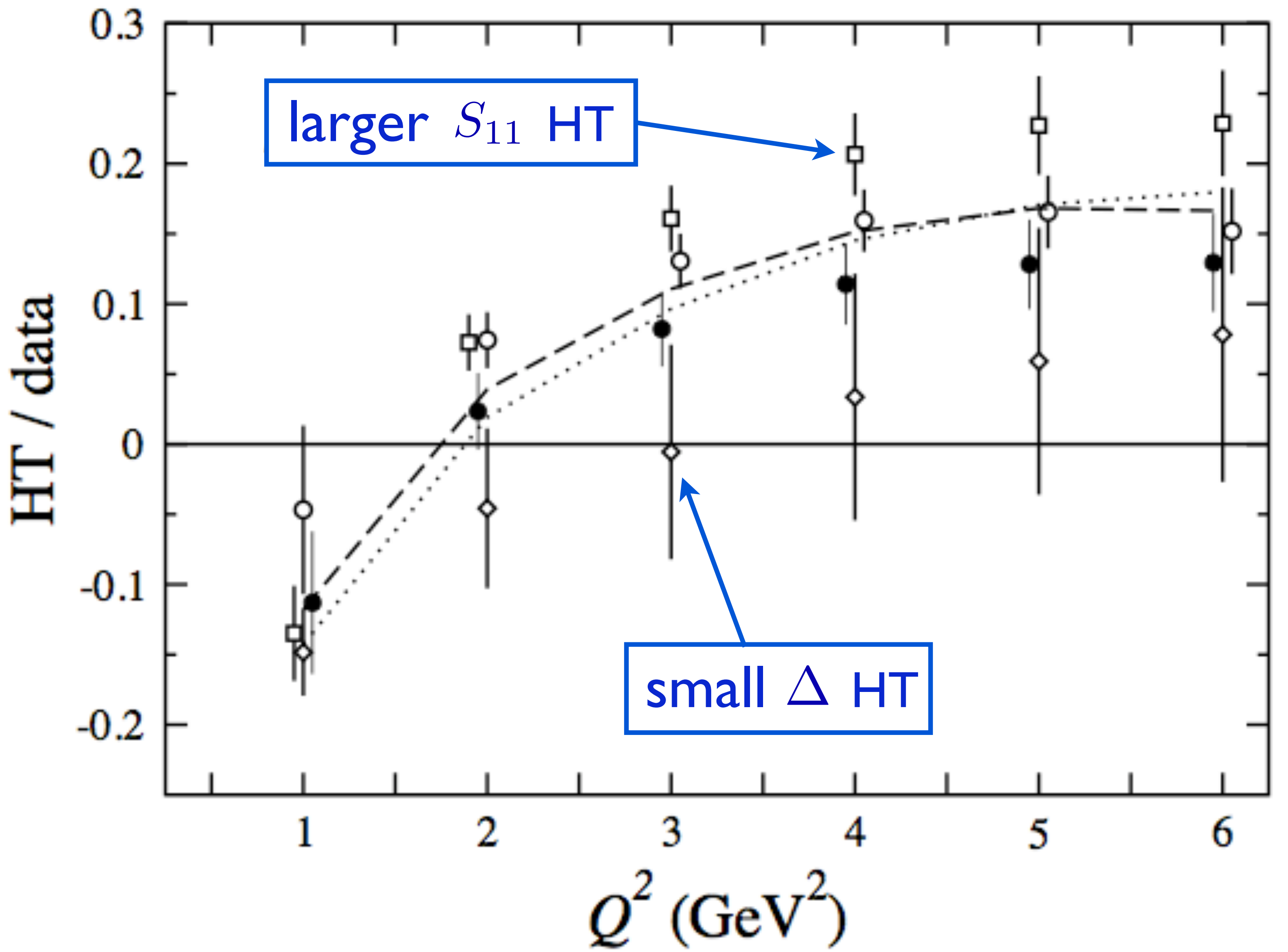

$\longrightarrow$ higher twists $< 10\text{–}15\%$ for $Q^2 > 1\ \mathrm{GeV}^2$

Resonances & twists

- Total higher twist "*small*" at scales $Q^2 \sim \mathcal{O}(1\ \mathrm{GeV}^2)$
- On average, nonperturbative interactions between quarks and gluons not dominant (at these scales)
  $\longrightarrow$ nontrivial interference between resonances

Resonances & twists

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- Can we understand this dynamically, at quark level?

  $\longrightarrow$ is duality an accident?

- Can we use resonance region data to learn about *leading twist* structure functions?

  $\longrightarrow$ expanded data set has potentially significant implications for global PDF studies

- Consider simple quark model with spin-flavor symmetric wave function

<u>form factors</u>

$\longrightarrow$ *coherent* scattering from quarks $\quad d\sigma \sim \left(\sum_i e_i\right)^2$

<u>structure functions</u>

$\longrightarrow$ *incoherent* scattering from quarks $\quad d\sigma \sim \sum_i e_i^2$

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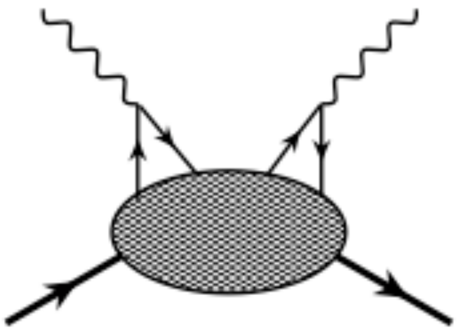
- For duality to work, these must be equal

$\longrightarrow$ how can <u>*square of a sum*</u> become <u>*sum of squares*</u>?

Accidental cancellations of charges?



<u>cat's ears diagram</u> (4-fermion higher twist $\sim 1/Q^2$)

$$\propto \sum_{i \neq j} e_i \, e_j \sim \Big(\sum_i e_i\Big)^2 - \sum_i e_i^2$$

coherent *incoherent*

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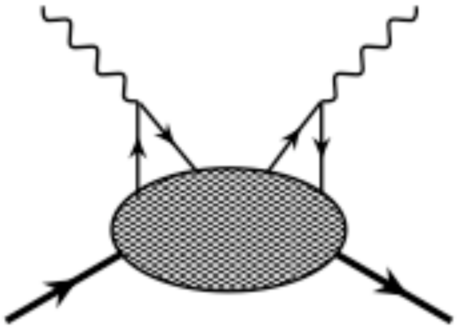

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coherent *incoherent*

<u>*proton*</u> $\quad \mathrm{HT} \sim 1 - \left(2 \times \frac{4}{9} + \frac{1}{9}\right) = 0 \;!$

<u>*neutron*</u> $\quad \mathrm{HT} \sim 0 - \left(\frac{4}{9} + 2 \times \frac{1}{9}\right) \neq 0$

Brodsky hep-ph/0006310

→ duality in proton a *coincidence!*

→ should <u>*not*</u> hold for neutron

Dynamical cancellations?

→ *e.g.* for toy model of two quarks bound in a harmonic oscillator potential, structure function given by

$$F(\nu, \mathbf{q}^2) \sim \sum_n \left| G_{0,n}(\mathbf{q}^2) \right|^2 \delta(E_n - E_0 - \nu)$$

→ charge operator $\Sigma_i \; e_i \exp(i\mathbf{q} \cdot \mathbf{r}_i)$ excites
even partial waves with strength $\propto (e_1 + e_2)^2$
odd partial waves with strength $\propto (e_1 - e_2)^2$

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→ charge operator $\Sigma_i\ e_i \exp(i\mathbf{q} \cdot \mathbf{r}_i)$ excites *even* partial waves with strength $\propto (e_1 + e_2)^2$ *odd* partial waves with strength $\propto (e_1 - e_2)^2$

→ resulting structure function

$$F(\nu, \mathbf{q}^2) \sim \sum_n \left\{(e_1 + e_2)^2\ G_{0,2n}^2 + (e_1 - e_2)^2\ G_{0,2n+1}^2\right\}$$

→ if states degenerate, *cross terms* $(\sim e_1 e_2)$ *cancel* when averaged over nearby *even and odd parity* states

- Dynamical cancellations?

→ duality is realized by summing over at least one complete set of *even* and *odd* parity resonances *

Close, Isgur, PLB ***509****, 81 (2001)*

→ in NR Quark Model, even & odd parity states generalize to **56** (L=0) and **70** (L=1) multiplets of spin-flavor SU(6)

- assume magnetic coupling of photon to quarks (better approximation at high Q^2)
- in this limit Callan-Gross relation valid $F_2 = 2xF_1$
- structure function given by squared sum of transition FFs

$$F_1(\nu,\vec{q}^{\,2}) \sim \sum_R |F_{N\to R}(\vec{q}^{\,2})|^2 \delta(E_R - E_N - \nu)$$

* realized in many models: 't Hooft model, large N_c, RQM, ... see *WM et al., Phys. Rep.* ***406****, 127 (2005)*

Dynamical cancellations?

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representation	$^2\mathbf{8}[\mathbf{56}^+]$	$^4\mathbf{10}[\mathbf{56}^+]$	$^2\mathbf{8}[\mathbf{70}^-]$	$^4\mathbf{8}[\mathbf{70}^-]$	$^2\mathbf{10}[\mathbf{70}^-]$	Total
F_1^p	$9\rho^2$	$8\lambda^2$	$9\rho^2$	0	λ^2	$18\rho^2+9\lambda^2$
F_1^n	$(3\rho+\lambda)^2/4$	$8\lambda^2$	$(3\rho-\lambda)^2/4$	$4\lambda^2$	λ^2	$(9\rho^2+27\lambda^2)/2$

$\lambda\ (\rho) =$ (anti) symmetric component of ground state wfn.

*Close, WM, PRC **68**, 035210 (2003)*

- SU(6) limit ➡ $\lambda = \rho$

$\longrightarrow$ relative strengths of $N \to N^*$ transitions:

$SU(6):$	$[\mathbf{56}, \mathbf{0}^+]^{\mathbf{2}}\mathbf{8}$	$[\mathbf{56}, \mathbf{0}^+]^{\mathbf{4}}\mathbf{10}$	$[\mathbf{70}, \mathbf{1}^-]^{\mathbf{2}}\mathbf{8}$	$[\mathbf{70}, \mathbf{1}^-]^{\mathbf{4}}\mathbf{8}$	$[\mathbf{70}, \mathbf{1}^-]^{\mathbf{2}}\mathbf{10}$	$total$
F_1^p	9	8	9	0	1	27
F_1^n	4	8	1	4	1	18

- summing over all resonances in $\mathbf{56}^+$ and $\mathbf{70}^-$ multiplets

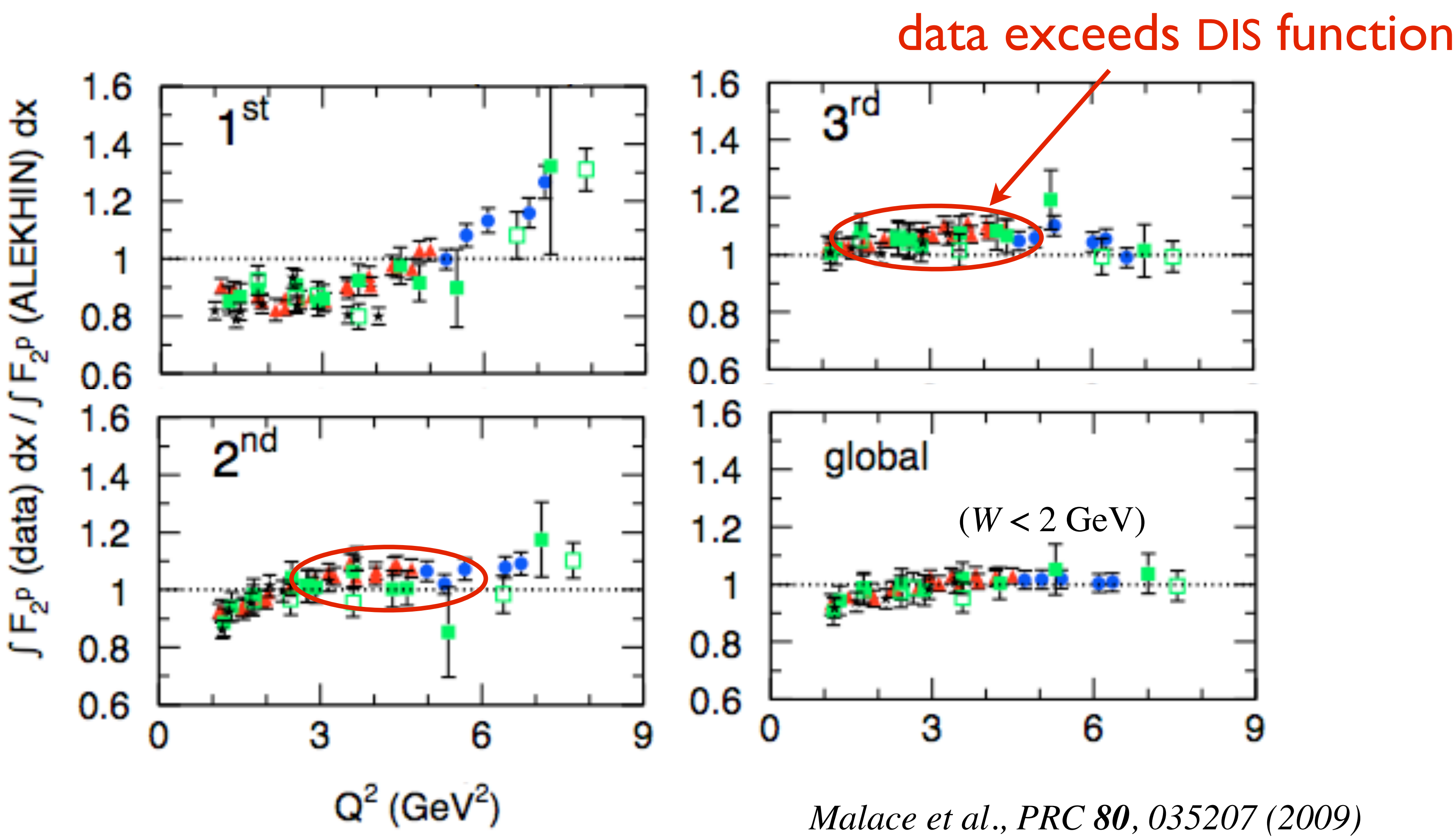
$$\longrightarrow \frac{F_1^n}{F_1^p} = \frac{18}{27} = \frac{2}{3}$$

- at the quark level, n/p ratio is

$$\longrightarrow \frac{F_1^n}{F_1^p} = \frac{4d+u}{d+4u} = \frac{6}{9} = \frac{2}{3} \quad ! \quad \text{if } u = 2d$$

Comparison with data

- Proton data expected to *overestimate* DIS function in 2nd and 3rd resonance regions (odd parity states)



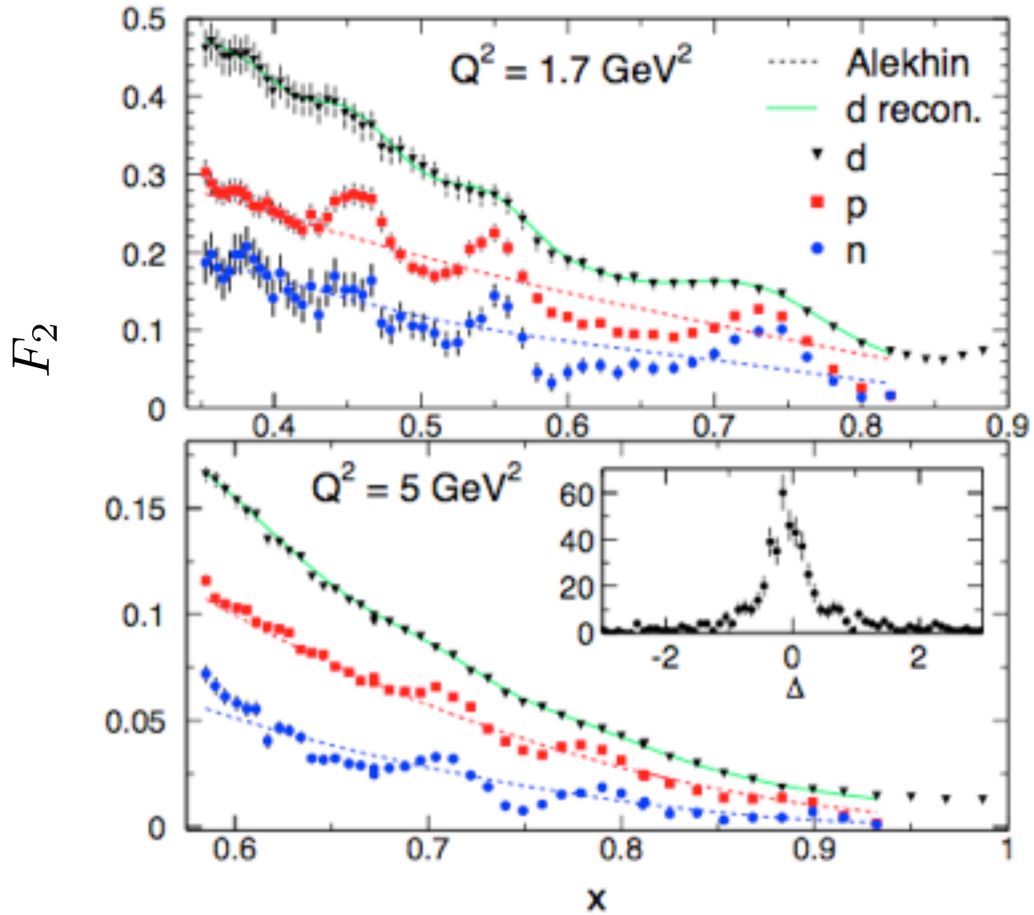
Malace et al., PRC ***80****, 035207 (2009)*

→ duality violation for proton $\lesssim 10\%$, integrated over x

Comparison with data

- Duality in neutron not tested because of absence of free neutron targets
- New extraction method (using iterative procedure for solving integral convolution equations) has allowed first determination of F_2^n in resonance region & test of neutron duality

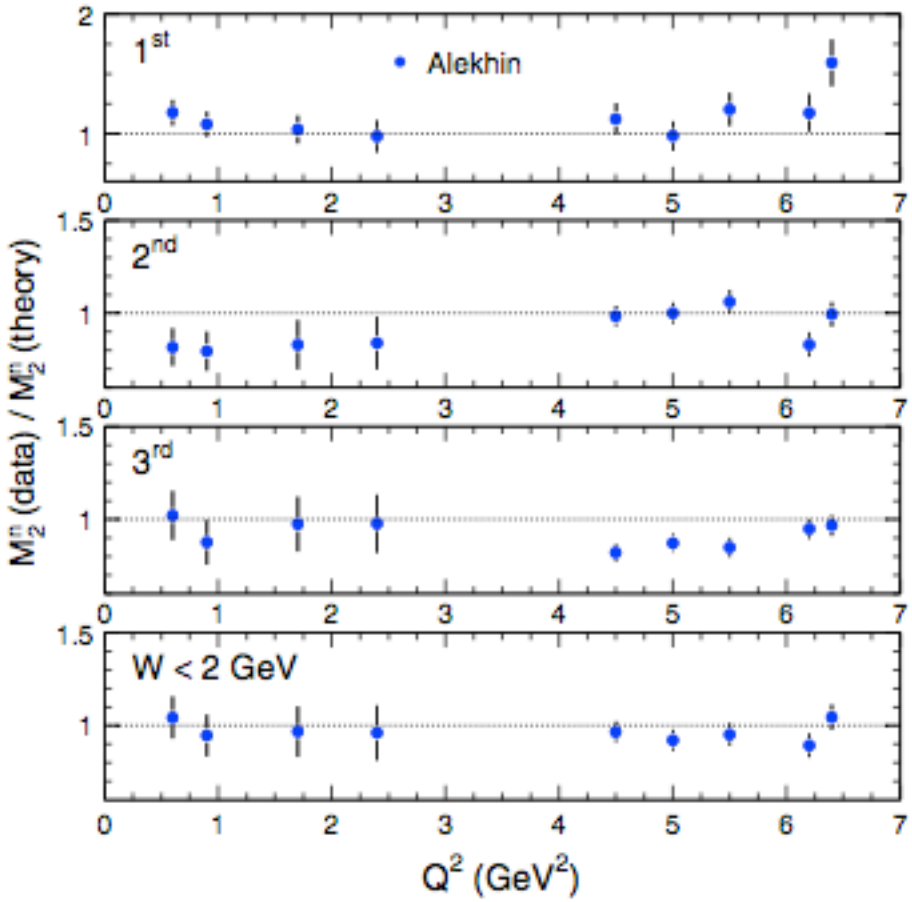
Kahn, WM, Kulagin
PRC ***79****, 035205 (2009)*



Malace, Kahn, WM, Keppel
PRL ***104****, 102001 (2010)*

Comparison with data

- Neutron data expected to lie *below* DIS function in 2nd region



→ "theory": fit to $W > 2$ GeV data

Alekhin et al., 0908.2762 [hep-ph]

→ *locally*, violations of duality in resonance regions $< 15-20\%$ (largest in Δ region)

→ *globally*, violations $< 10\%$

Malace, Kahn, WM, Keppel
PRL ***104****, 102001 (2010)*

⇨ duality is *not accidental*, but a general feature of resonance−scaling transition!

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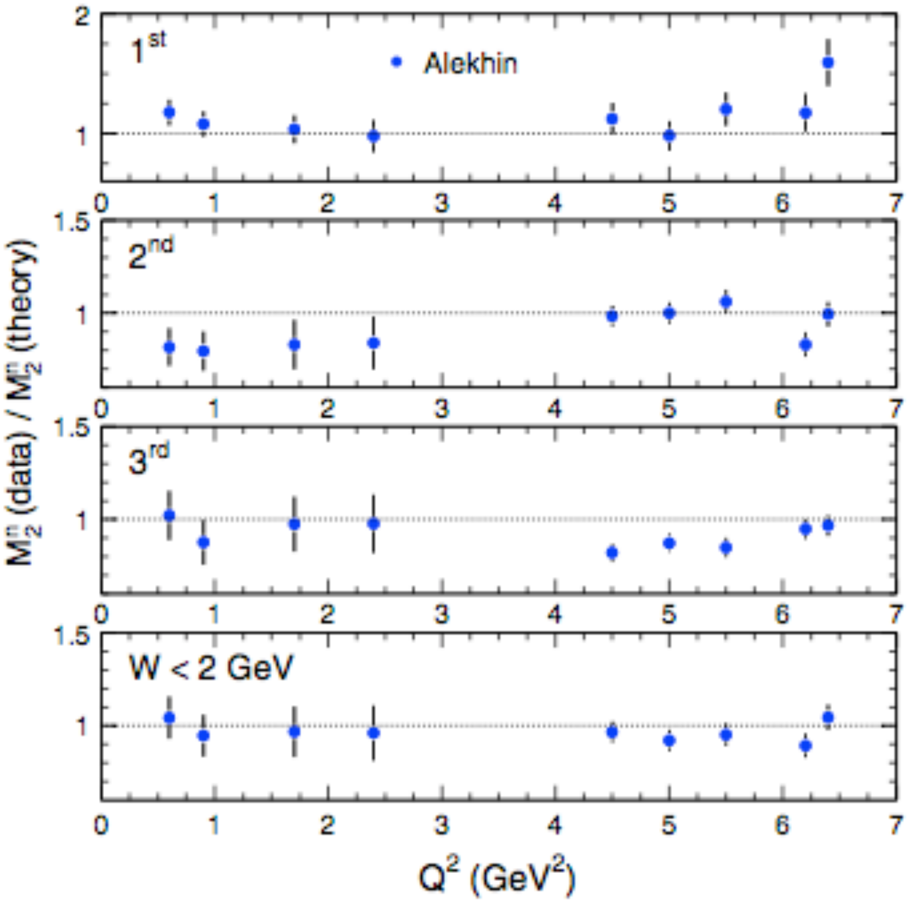

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⇨ use resonance region data to learn about *leading twist* structure functions?

CTEQ6X global PDF analysis

- New global QCD (next-to-leading order) analysis of expanded set of p and d data, including large-x, low-Q^2 region

  $\longrightarrow$ joint JLab-CTEQ theory/experiment collaboration
  (with Hampton, FSU, FNAL, Duke)

- Systematically study effects of Q^2 & W cuts

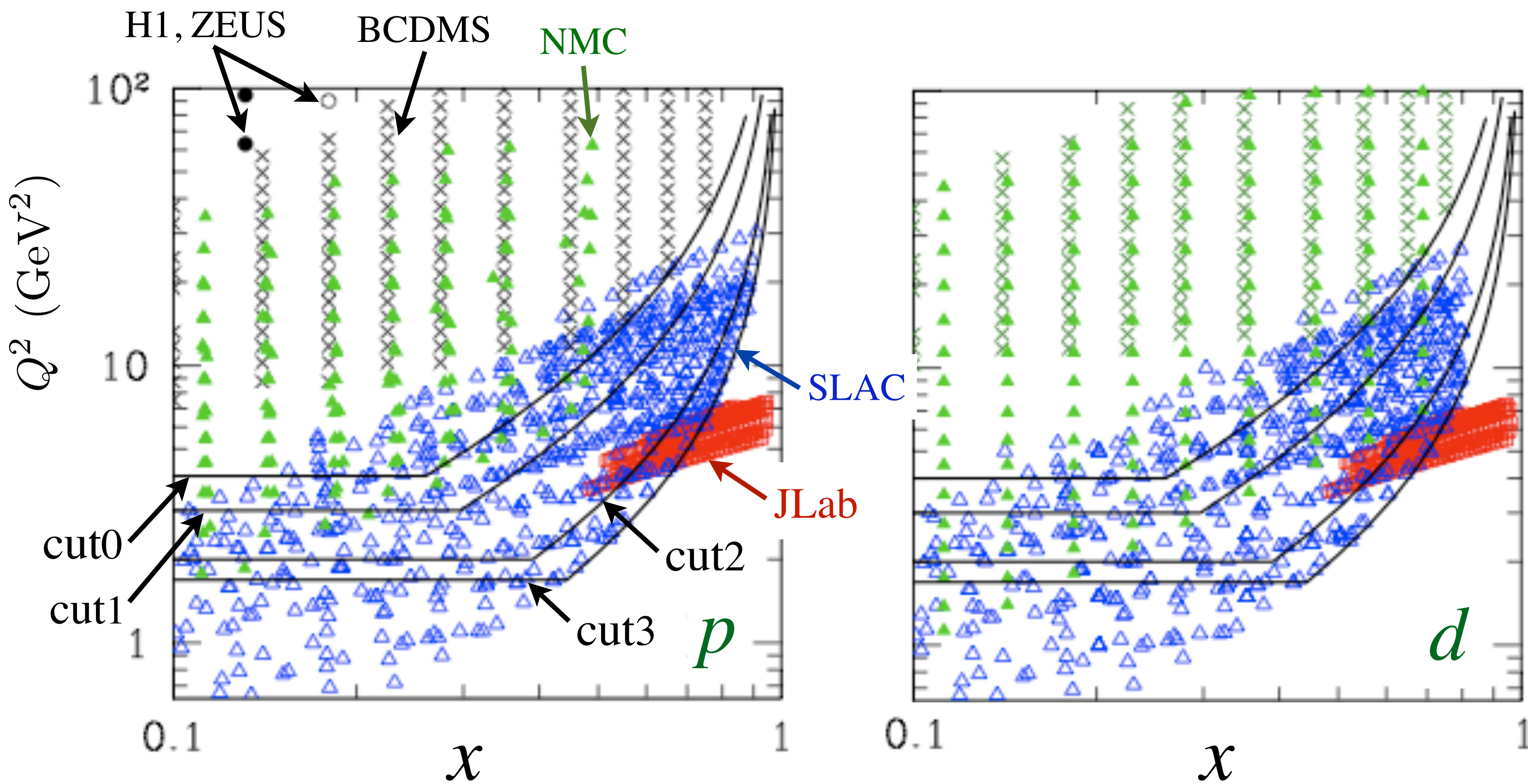
  $\longrightarrow$ as low as $Q \sim m_c$ and $W \sim 1.7$ GeV

- Include large-x corrections

  $\longrightarrow$ TMCs & higher twists $F_2(x, Q^2) = F_2^{\mathrm{LT}}(x, Q^2)(1 + C(x)/Q^2)$

  $\longrightarrow$ realistic nuclear effects in deuteron (binding + off-shell)
  (most analyses assume no nuclear corrections)

CTEQ6X – kinematic cuts



cut0: $Q^2 > 4\ \mathrm{GeV}^2, \quad W^2 > 12.25\ \mathrm{GeV}^2$

cut1: $Q^2 > 3\ \mathrm{GeV}^2, \quad W^2 > 8\ \mathrm{GeV}^2$

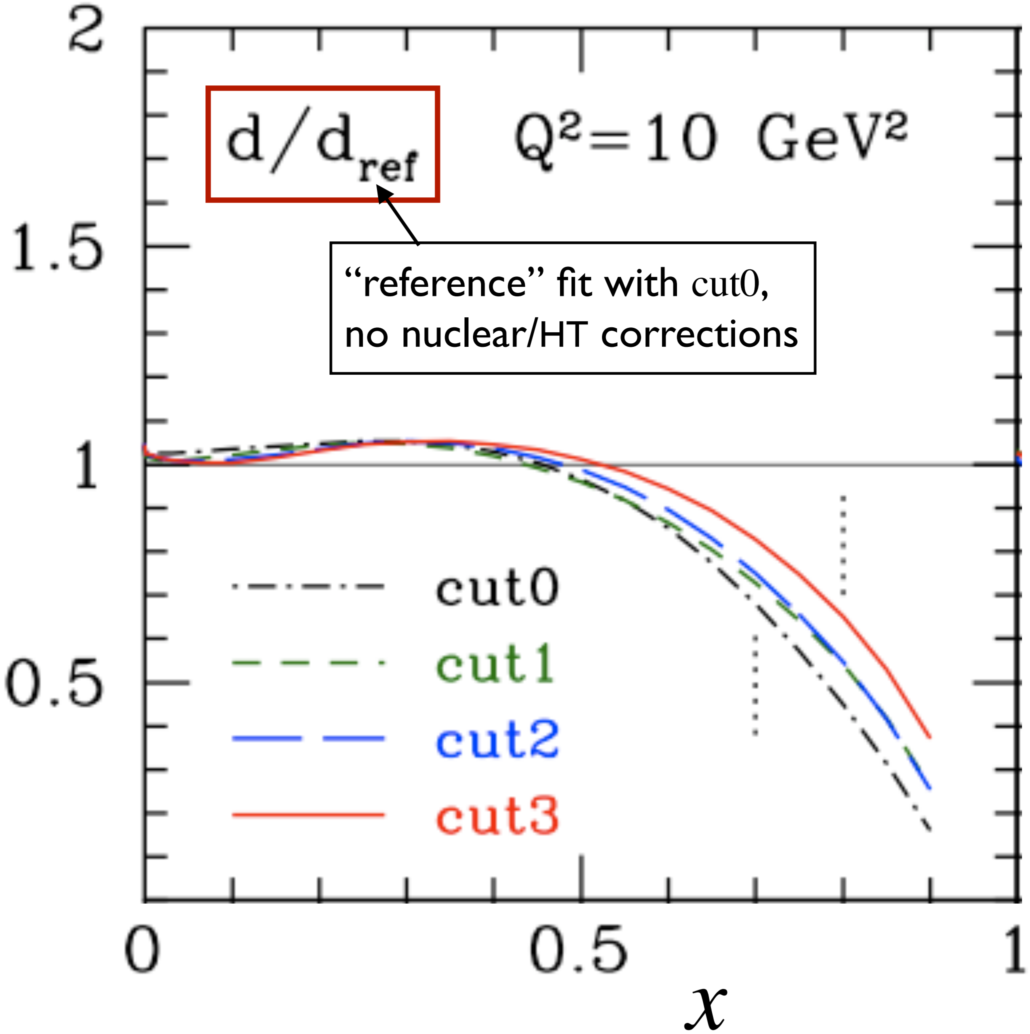
cut2: $Q^2 > 2\ \mathrm{GeV}^2, \quad W^2 > 4\ \mathrm{GeV}^2$

cut3: $Q^2 > m_c^2\ , \quad W^2 > 3\ \mathrm{GeV}^2$

factor 2 increase in DIS data from cut0 $\longrightarrow$ cut3

CTEQ6X – kinematic cuts

- Systematically reduce Q^2 and W cuts, including TMC, HT & nuclear corrections

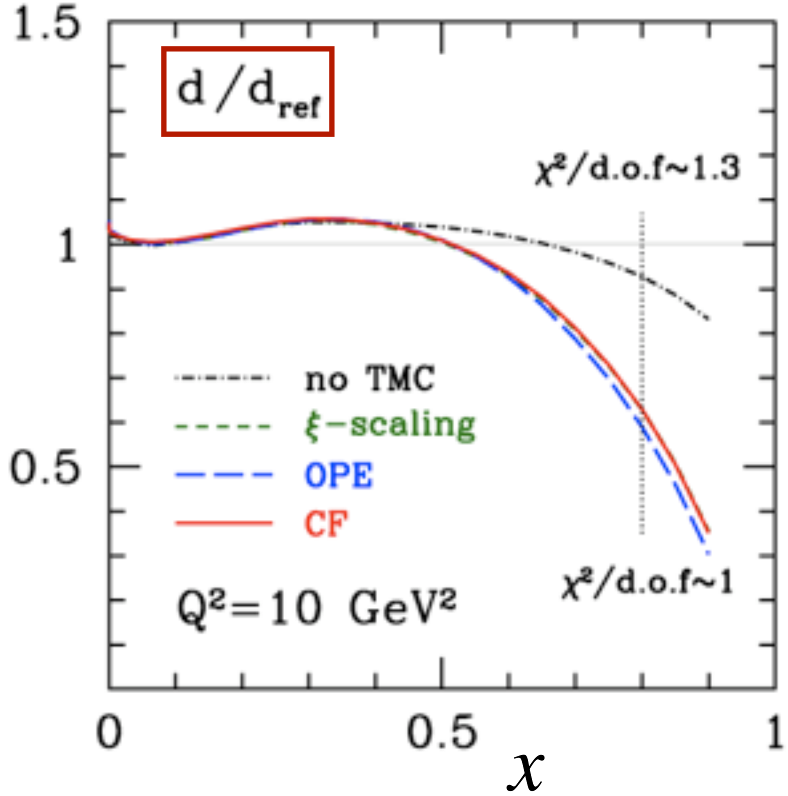


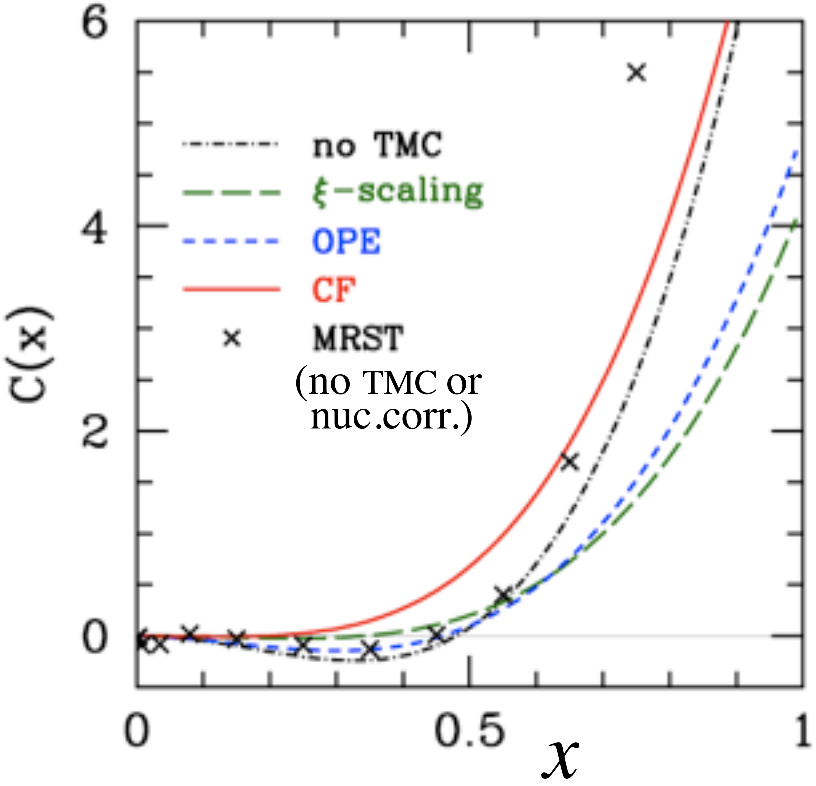
→ *stable* with respect to cut reduction

→ d quark suppressed by $\sim 50\%$ for $x > 0.5$ (driven by nuclear corrections)

Accardi et al., Phys. Rev. D ***81****, 034016 (2010)*

CTEQ6X – $1/Q^2$ corrections

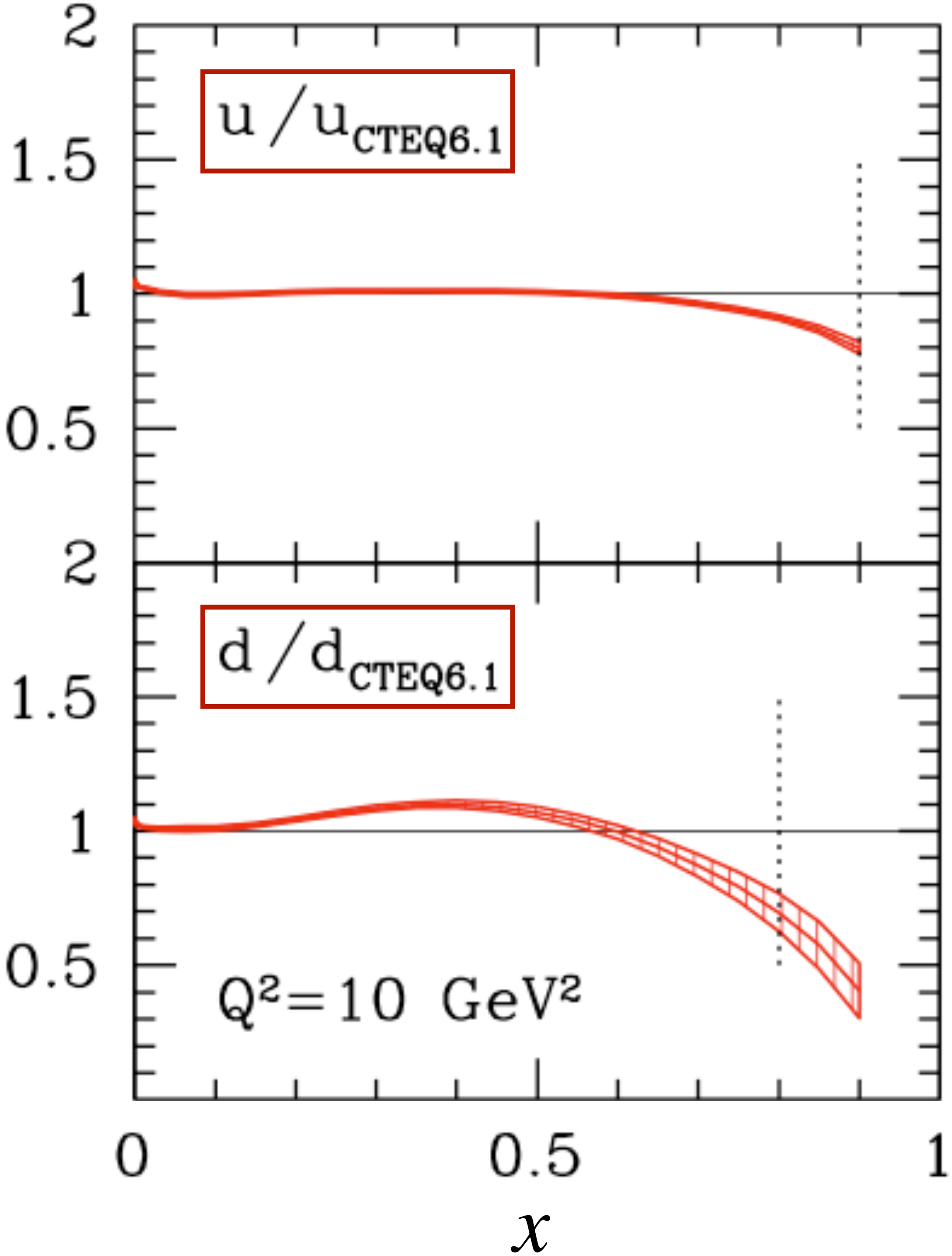




→ important interplay between TMCs and higher twist: HT alone *cannot* accommodate full Q^2 dependence

→ stable leading twist when *both* TMCs and HTs included

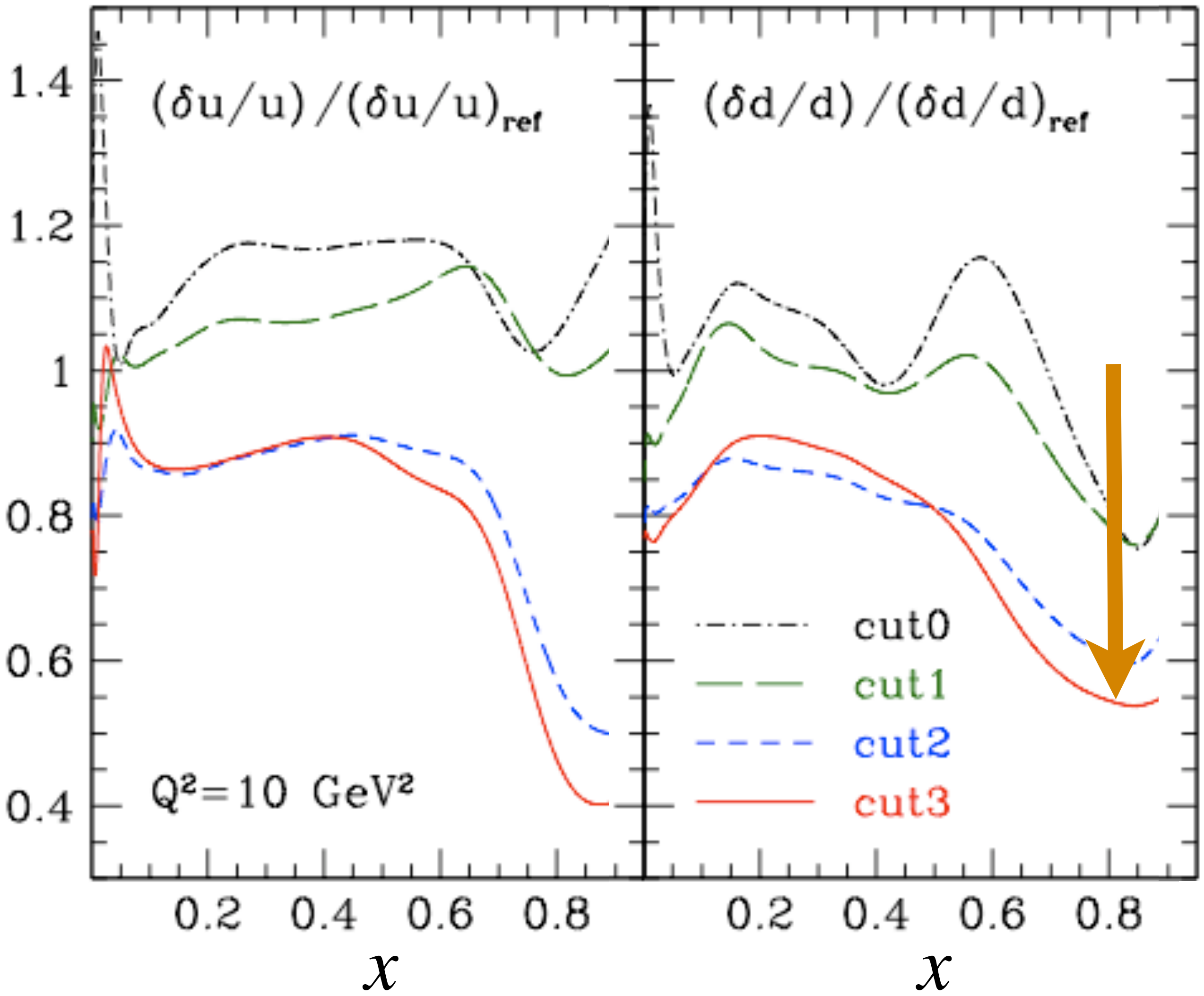
CTEQ6X – final PDF results



→ **full fits favors *smaller d/u* ratio**

(CTEQ6.1 had no nuclear or TMC/HT corrections)

CTEQ6X – final PDF results



→ full fits favors *smaller d/u* ratio

(CTEQ6.1 had no nuclear or TMC/HT corrections)

→ up to 40-60% *reduced errors* with weaker cuts extending into resonance region

Accardi et al., Phys. Rev. D ***81****, 034016 (2010)*

Summary

- Remarkable confirmation of quark-hadron duality in *proton* and *neutron* structure functions

  → duality-violating higher twists $\sim 10{-}15\%$ in few-GeV range

- Confirmation of duality in *neutron* suggests origin in dynamical cancellations of higher twists

  → duality *not* due to accidental cancellations of quark charges

- Practical application of duality

  → use resonance region data to constrain *leading twist* PDFs

  → stable fits at low Q^2 and large x with significantly reduced uncertainties

The End