Qweak Collaboration Meeting
William & Mary, June 24, 2011

Electroweak radiative corrections to the weak charge of the proton

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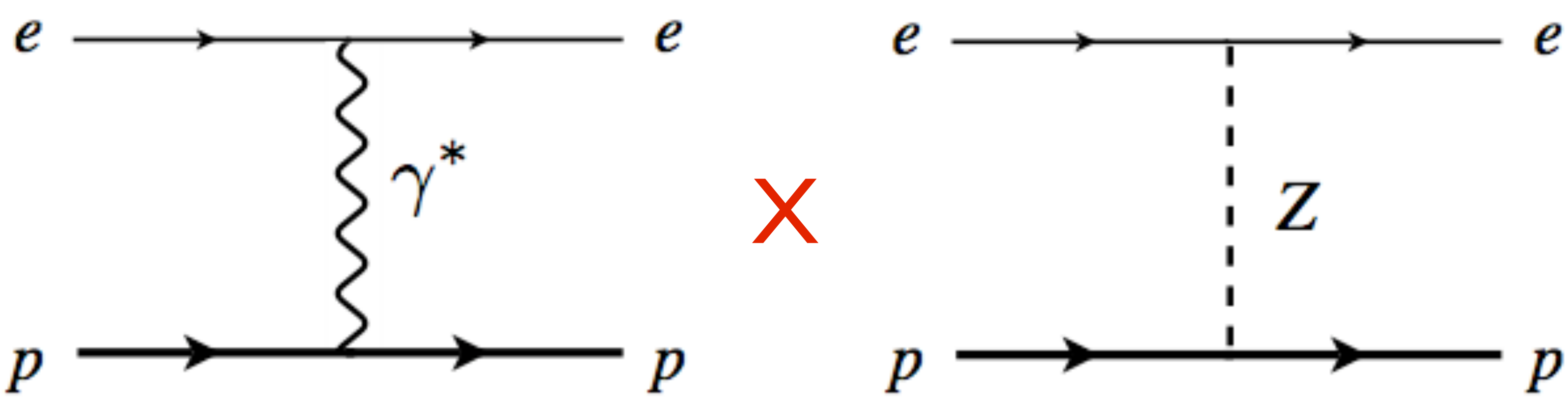
Refs: PRD **82** (2010) 013011 (vector hadron correction)
arXiv:1102.5334 (axial-vector hadron correction)
arXiv:1105.0951 (review article)

Parity-violating e scattering

- Left-right polarization asymmetry in $\vec{e}\, p \to e\, p$ scattering

$$A_{\rm PV} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\left(\frac{G_F Q^2}{4\sqrt{2}\alpha}\right)(A_V + A_A + A_s)$$

$\longrightarrow$ measure interference between e.m. and weak currents



Born (tree) level

Parity-violating e scattering

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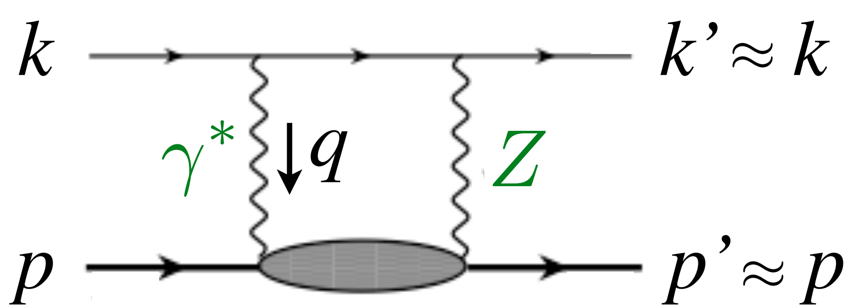
$$A_{\rm PV} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\left(\frac{G_F Q^2}{4\sqrt{2}\alpha}\right)(A_V + A_A + A_s)$$

→ in <u>forward</u> limit measures weak charge of proton Q_W^p

$$A_{\rm PV} \;\to\; \frac{G_F\, Q_W^p}{4\sqrt{2}\pi\alpha}\, t$$



forward limit

$$t = (k - k')^2 \to 0$$
$$s = (k+p)^2$$
$$= M(M + 2E)$$

Proton weak charge

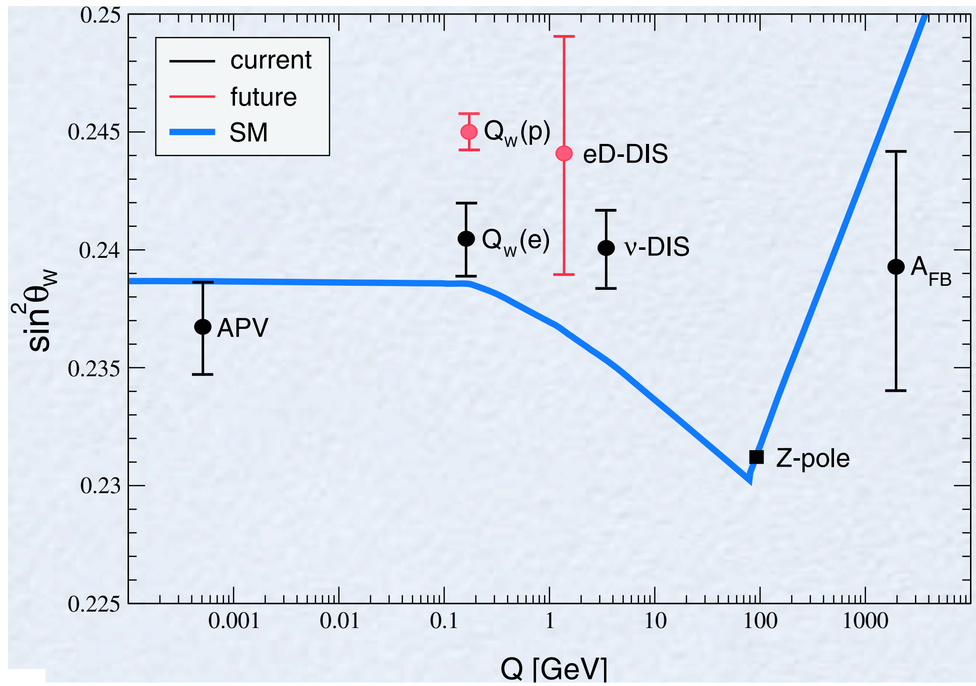
- At tree level Q_W^p gives weak mixing angle

$$Q_W^p = 1 - 4\sin^2\theta_W$$

→ current best values

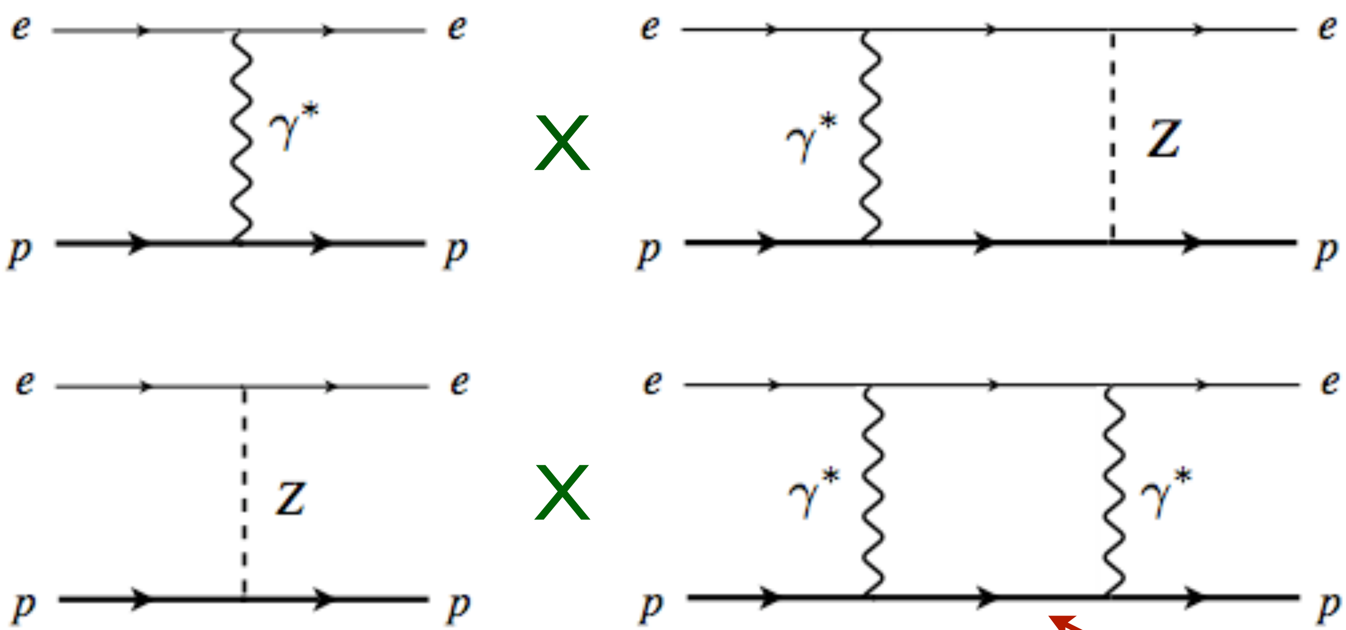
$$\sin^2\theta_W(M_Z^2) = 0.23116(16)$$
$$\sin^2\theta_W(0) = 0.23867(13)$$

scale dependence from radiative effects



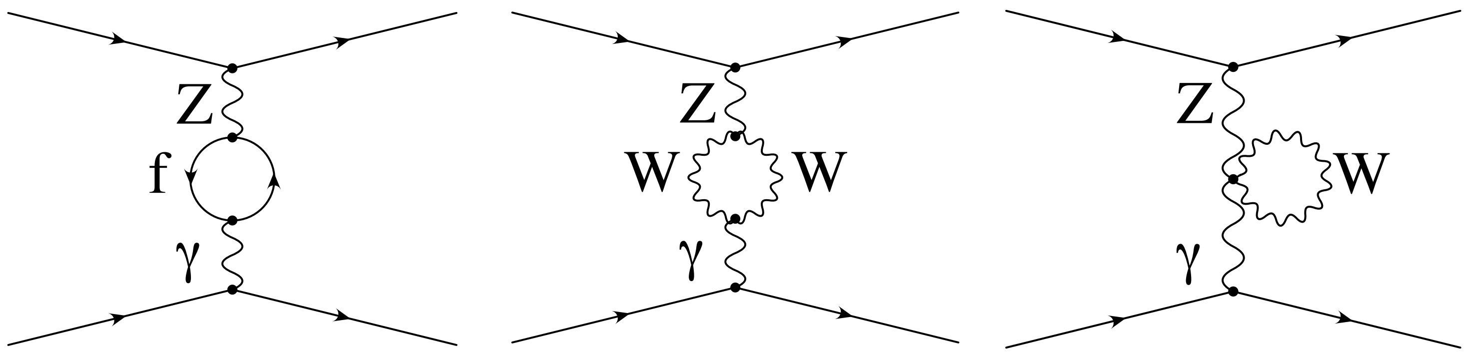
→ Q_W^p small number – sensitive to higher-order corrections

Corrections to proton weak charge



two-boson exchange

two-photon exchange vanishes at $t = 0$



vacuum polarization

+ ...

Corrections to proton weak charge

- including higher order radiative corrections

$$\begin{aligned} Q_W^p &= (1+\Delta\rho+\Delta_e)(1-4\sin^2\theta_W(0)+\Delta_e') \\ &+ \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z} \quad \longleftarrow \text{ box diagrams} \\ &= 0.0713 \pm 0.0008 \end{aligned}$$

Erler et al., PRD ***72*** *(2005) 073003*

→ WW and ZZ box diagrams dominated by short distances, evaluated perturbatively (*WW* box gives ~ 25% correction!)

→ γZ box diagram sensitive to long distance physics, has two contributions $\Box_{\gamma Z} = \Box_{\gamma Z}^A + \Box_{\gamma Z}^V$

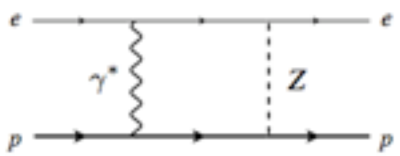
vector e – axial h (finite at E=0)

axial e – vector h (vanishes at E=0)

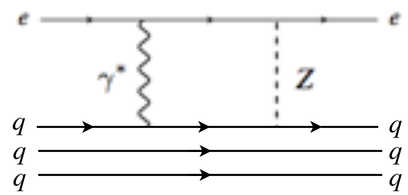
Axial h correction

- axial h correction $\square^{A}_{\gamma Z}$ dominant γZ correction in atomic parity violation at very low (zero) energy

$\longrightarrow$ computed by Marciano & Sirlin (1980s) as sum of two parts:



★ low-energy part approximated by *Born* contribution (elastic intermediate state)



★ high-energy part (above scale $\Lambda \sim 1$ GeV) computed in terms of scattering from *free quarks*

$$\square^{A}_{\gamma Z} = \frac{5\alpha}{2\pi}(1 - 4\sin^2\theta_W)\left[\ln\frac{M_Z^2}{\Lambda^2} + C_{\gamma Z}(\Lambda)\right]$$

$$\approx \; 0.0052(5)$$

short−distance

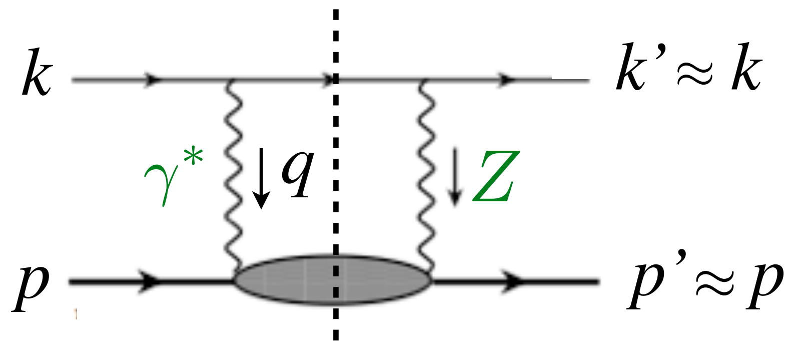
long−distance $\approx 3/2 \pm 1$

*Marciano, Sirlin, PRD **29** (1984) 75; Erler et al., PRD **68** (2003) 016006*

Axial h correction

- axial h correction $\Box^A_{\gamma Z}$ dominant γZ correction in atomic parity violation at very low (zero) energy

  → repeat calculation using forward dispersion relations with realistic (structure function) input



- ★ axial h contribution *anti*symmetric under $E' \leftrightarrow -E'$:

$$\Re e\, \Box^A_{\gamma Z}(E) = \frac{2}{\pi} \int_0^\infty dE' \frac{E'}{E'^2 - E^2}\, \Im m\, \Box^A_{\gamma Z}(E')$$

- ★ negative energy part corresponds to crossed box (crossing symmetry $s \to u$)

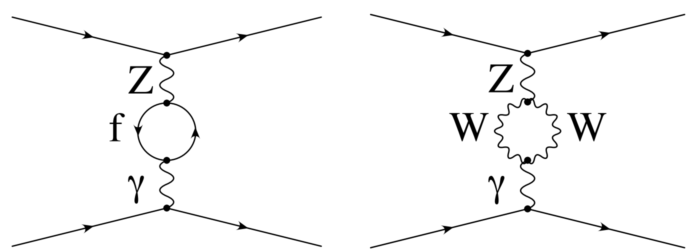
Axial h correction

- imaginary part given by interference $F_3^{\gamma Z}$ structure function

$$\mathcal{I}m\,\Box^A_{\gamma Z}(E) = \frac{1}{(2ME)^2}\int_{M^2}^{s} dW^2 \int_0^{Q^2_{\max}} dQ^2\, \frac{v_e(Q^2)\,\alpha(Q^2)}{1+Q^2/M_Z^2} \times \left(\frac{2ME}{W^2-M^2+Q^2}-\frac{1}{2}\right) F_3^{\gamma Z}$$

with $v_e(Q^2) = 1 - 4\kappa(Q^2)\sin^2\theta_W(Q^2)$

→ scale dependence of v_e, α given by vacuum polarization corrections, *e.g.*



$$\frac{\alpha}{\alpha(Q^2)} = 1 - \Delta\alpha_{\rm lep}(Q^2) - \Delta\alpha^{(5)}_{\rm had}(Q^2)$$

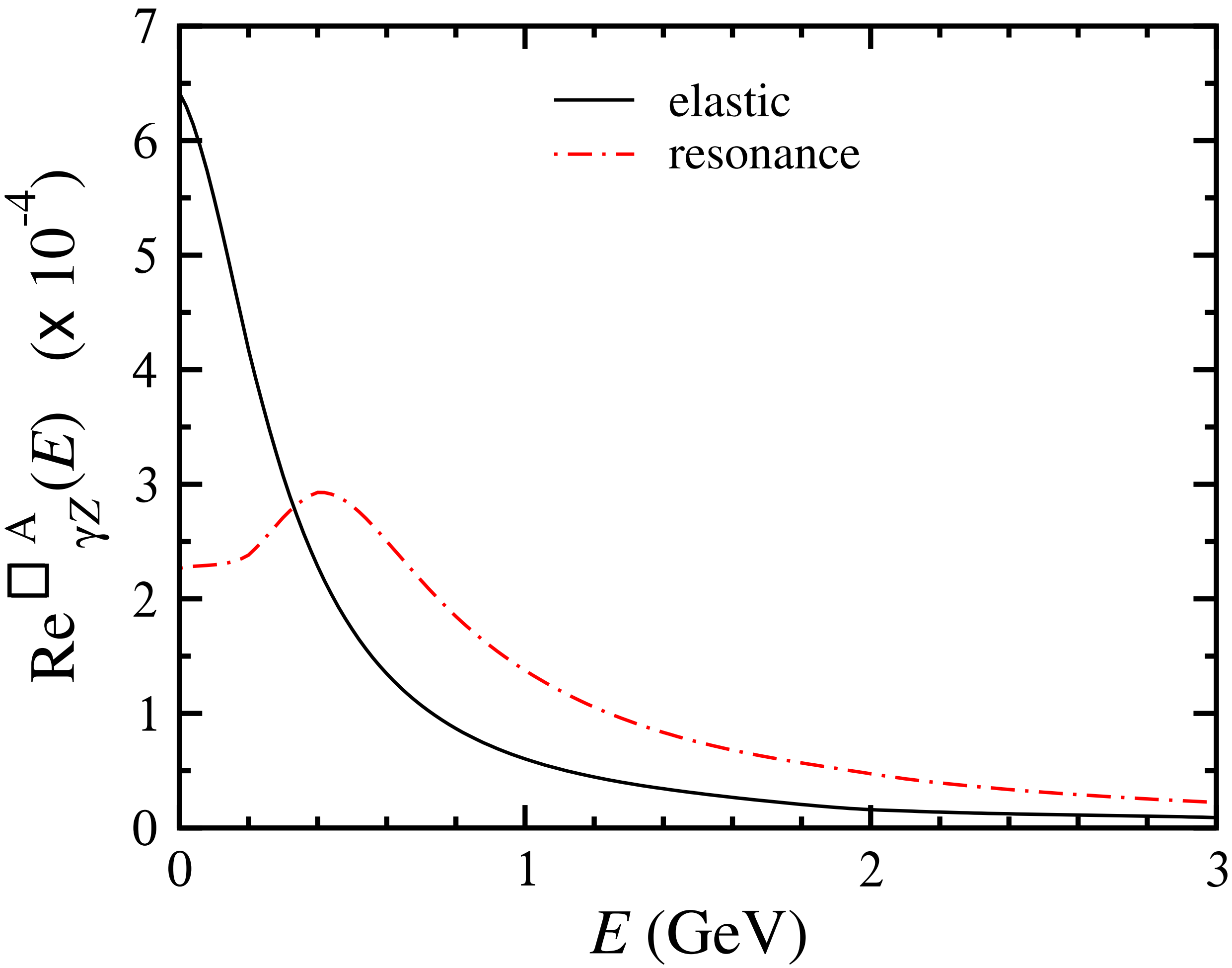
$$\alpha^{-1}(M_Z^2) = 128.94$$

... similarly for weak charges

Axial h correction

- <u>elastic</u> part $F_3^{\gamma Z(\mathrm{el})} = -Q^2\, G_M^p(Q^2)\, G_A^Z(Q^2)\, \delta(W^2 - M^2)$

- <u>resonance</u> part from parametrization of ν scattering data (includes lowest four spin-$1/2$ and $3/2$ states) *Lalakulich, Paschos (2006)*



Axial h correction

- DIS part dominated by leading twist PDFs at high W (small x)

  $e.g.$ at LO, $F_3^{\gamma Z(\mathrm{DIS})} = \sum_q 2e_q\, g_A^q \left(q(x,Q^2) - \bar{q}(x,Q^2)\right)$

→ switching order of integration (energy integral analytic!), expand integrand in $1/Q^2$ in DIS region ($Q^2 \gtrsim 1\ \mathrm{GeV}^2$)

$$\mathcal{R}e\,\Box_{\gamma Z}^{\mathrm{A(DIS)}}(E) = \frac{3}{2\pi}\int_{Q_0^2}^{\infty} dQ^2\, \frac{v_e(Q^2)\,\alpha(Q^2)}{1+Q^2/M_Z^2} \times \left[M_3^{\gamma Z(1)} - \frac{2M^2}{9Q^4}(5E^2 - 3Q^2)M_3^{\gamma Z(3)}\right]$$

with moments $M_3^{\gamma Z(n)}(Q^2) = \int_0^1 dx\, x^{n-1} F_3^{\gamma Z}(x,Q^2)$

Axial h correction

- structure function moments

<u>$n = 1$</u> $$M_3^{\gamma Z(1)}(Q^2) = \frac{5}{3}\left(1 - \frac{\alpha_s(Q^2)}{\pi}\right)$$

$\longrightarrow$ γZ analog of Gross-Llewellyn Smith sum rule

$$\mathcal{R}e\,\Box_{\gamma Z}^{A(\mathrm{DIS})} \approx (1 - 4\hat{s}^2)\frac{5\alpha}{2\pi}\int_{Q_0^2}^{\infty}\frac{dQ^2}{Q^2(1+Q^2/M_Z^2)}\left(1 - \frac{\alpha_s(Q^2)}{\pi}\right)$$

$\longrightarrow$ precisely result from Marciano & Sirlin!
(works because result depends on lowest moment of *valence* PDF, with <u>model-independent normalization</u>!)

<u>$n = 3$</u> $$M_3^{\gamma Z(3)}(Q^2) = \frac{1}{3}\left(2\langle x^2\rangle_u + \langle x^2\rangle_d\right)\left(1 + \frac{5\alpha_s(Q^2)}{12\pi}\right)$$

$\longrightarrow$ related to x^2 -weighted moment of valence quarks

Axial h correction

- "<u>DIS</u>" region at $Q^2 < 1 \ \mathrm{GeV}^2$ does not afford PDF description

  $\longrightarrow$ in absence of data, consider models with general constraints

  - ★ $F_3^{\gamma Z}(x_{\max}, Q^2)$ should not diverge in limit $Q^2 \to 0$
  - ★ $F_3^{\gamma Z}(x, Q^2)$ should match PDF description at $Q^2 = 1\,\mathrm{GeV}^2$

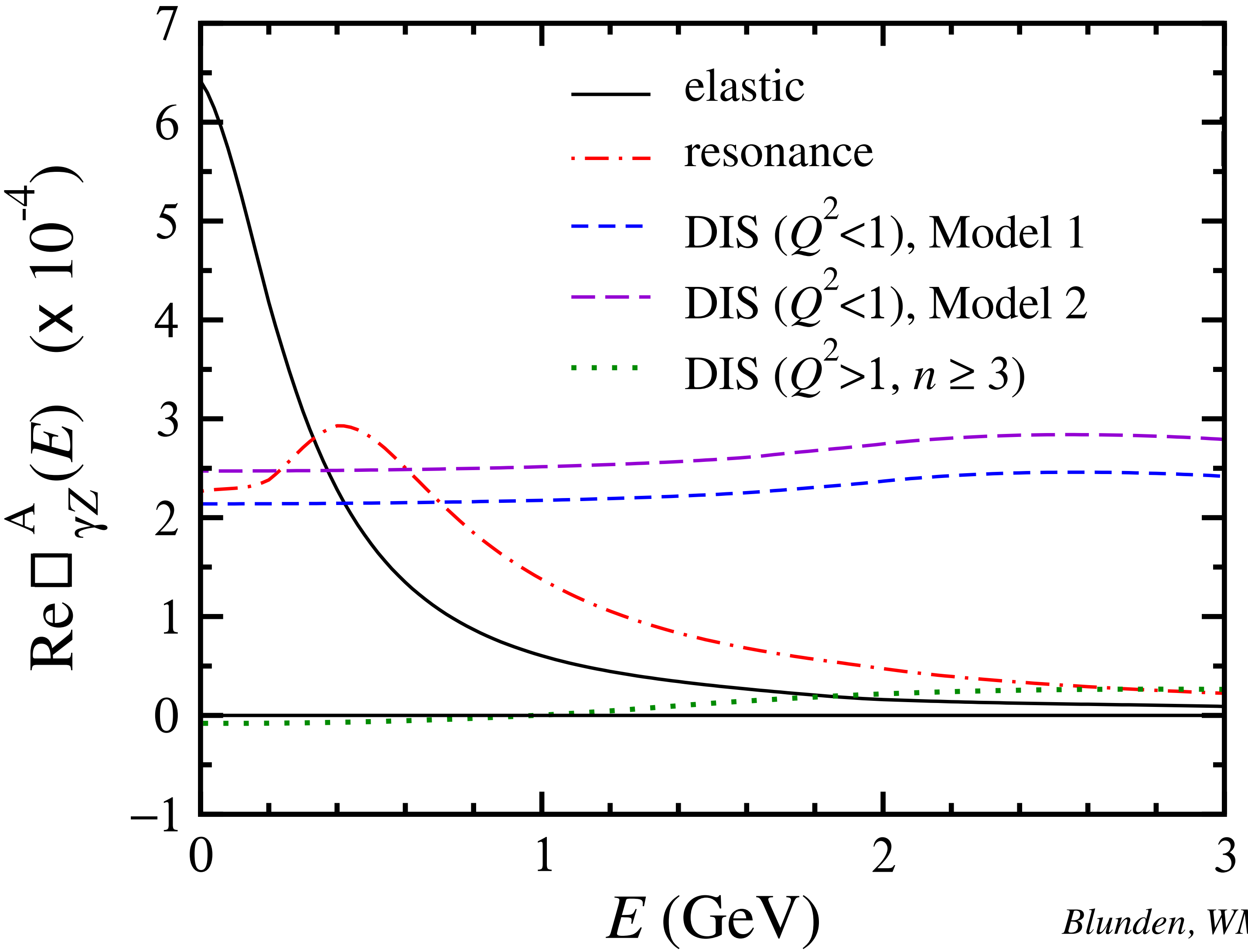
<u>Model 1</u> $$F_3^{\gamma Z}(x, Q^2) = \left(\frac{1 + \Lambda^2/Q_0^2}{1 + \Lambda^2/Q^2}\right) F_3^{\gamma Z}(x, Q_0^2)$$

$$F_3^{\gamma Z} \sim (Q^2)^{0.3} \text{ as } Q^2 \to 0$$

<u>Model 2</u> $F_3^{\gamma Z}$ frozen at $Q^2 = 1$ value for all W^2

$$F_3^{\gamma Z} \text{ finite as } Q^2 \to 0$$

Axial h correction



Blunden, WM, Thomas (2011)

→ dominated by $n = 1$ DIS moment: $\underline{32.8 \times 10^{-4}}$
(weak E dependence)

Axial h correction

→ correction at $\underline{E = 0}$

$$\Re e\,\Box^{A}_{\gamma Z} = \underset{\substack{\uparrow \\ \text{elastic}}}{0.00064} + \underset{\substack{\uparrow \\ \text{resonance}}}{0.00023} + \underset{\substack{\uparrow \\ \text{DIS}}}{0.00350} \rightarrow \underline{0.0044(4)}$$

→ correction at $\underline{E = 1.165\ \text{GeV}}$ (Qweak)

$$\Re e\,\Box^{A}_{\gamma Z} = 0.00005 + 0.00011 + 0.00352 = \underline{0.0037(4)}$$

cf. MS value: $\underline{0.0052(5)}$ (~1% shift in Q_W^p)

→ shifts Q_W^p from $\underline{0.0713(8)} \longrightarrow \underline{0.0705(8)}$

Vector h correction

- vector h correction $\Box^V_{\gamma Z}$ vanishes at $E = 0$, but experiment has $E \sim 1\ \mathrm{GeV}$ – what is energy dependence?

→ forward dispersion relation

★ $\Re e\, \Box^V_{\gamma Z}(E) = \frac{2E}{\pi} \int_0^\infty dE' \frac{1}{E'^2 - E^2}\, \Im m\, \Box^V_{\gamma Z}(E')$

★ integration over $E' < 0$ corresponds to crossed-box, vector h contribution symmetric under $E' \leftrightarrow -E'$

→ imaginary part given by

$$\Im m\, \Box^V_{\gamma Z}(E) = \frac{\alpha}{(s - M^2)^2} \int_{W_\pi^2}^{s} dW^2 \int_0^{Q^2_{\mathrm{max}}} \frac{dQ^2}{1 + Q^2/M_Z^2}$$

$$\times \left(F_1^{\gamma Z} + F_2^{\gamma Z} \frac{s\,(Q^2_{\mathrm{max}} - Q^2)}{Q^2(W^2 - M^2 + Q^2)} \right)$$

factor 2 larger than GH;
confirmed by Rislow & Carlson,
arXiv:1011:2397 [hep-ph]

*Gorchtein, Horowitz, PRL **102** (2009) 091806*
Gorchtein, Horowitz, Ramsey-Musolf, arXiv:1003.4300

Vector h correction

→ $F_{1,2}^{\gamma Z}$ structure functions

- parton model for DIS region $F_2^{\gamma Z} = 2x \sum_q e_q \, g_V^q \, (q + \bar{q}) = 2x F_1^{\gamma Z}$
  - → $F_2^{\gamma Z} \approx F_2^{\gamma}$ good approximation at *low* x
  - → provides upper limit at *large* x $(F_2^{\gamma Z} \lesssim F_2^{\gamma})$

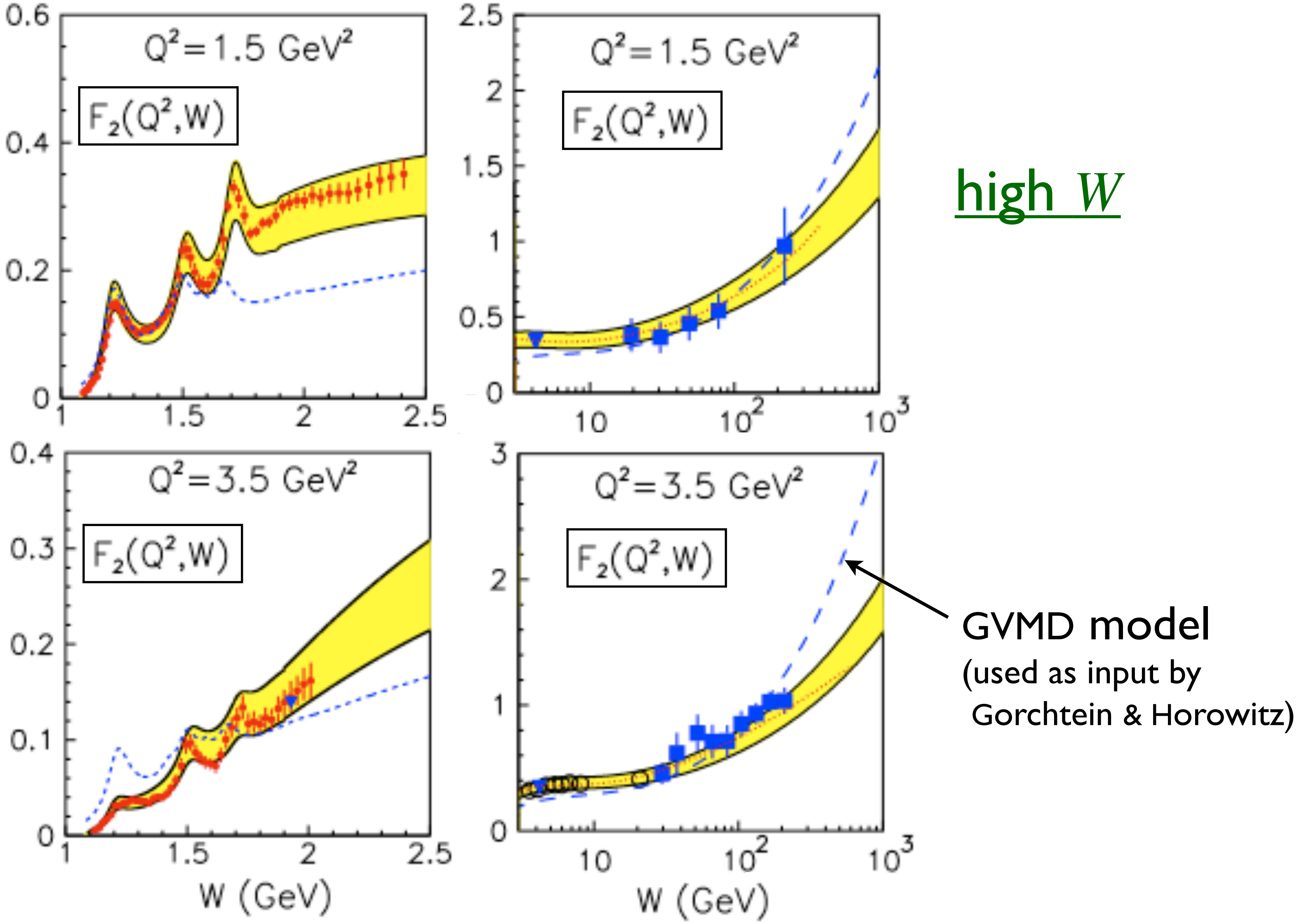
- in resonance region use phenomenological input for F_2, empirical (SLAC) fit for R
  - → for transitions to $I = 3/2$ states ($e.g.$ Δ), CVC and isospin symmetry give $F_i^{\gamma Z} = (1 + Q_W^p) F_i^{\gamma}$
  - → for transitions to $I = 1/2$ states, SU(6) wave functions predict Z & γ transition couplings equal to a few %

Vector h correction

→ compare structure function input with data

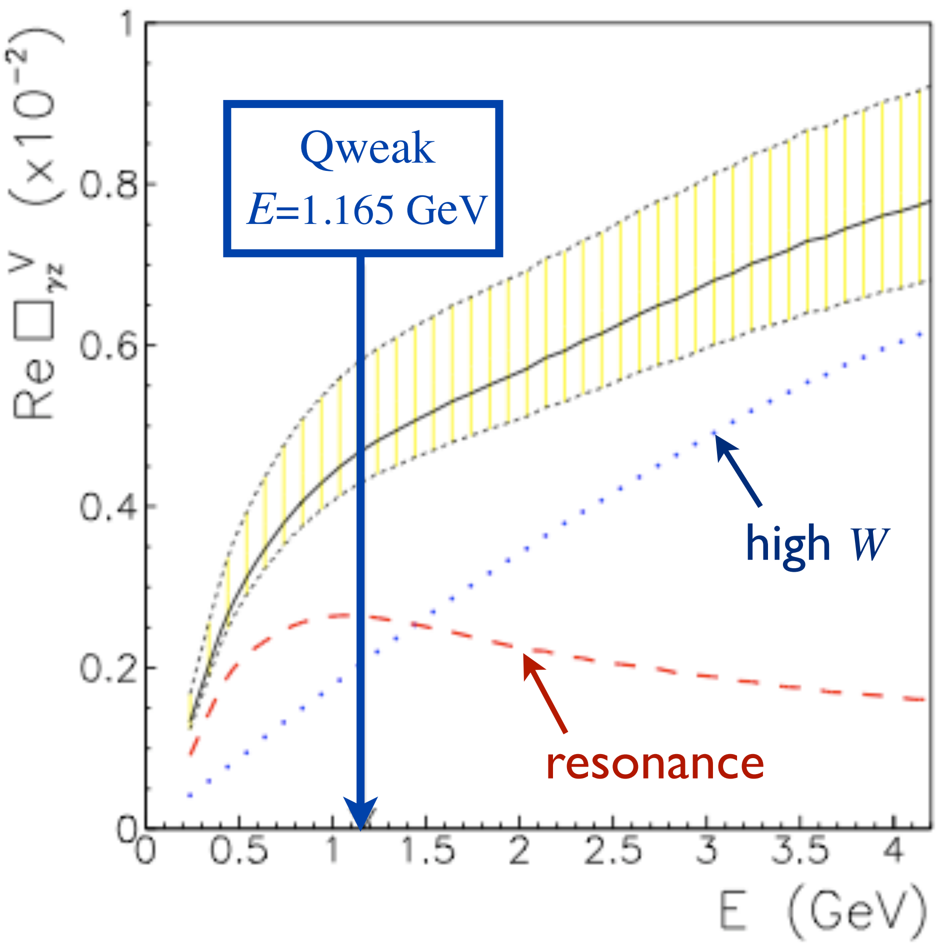
low W

high W



Vector h correction

→ total $\Box_{\gamma Z}^{V}$ correction



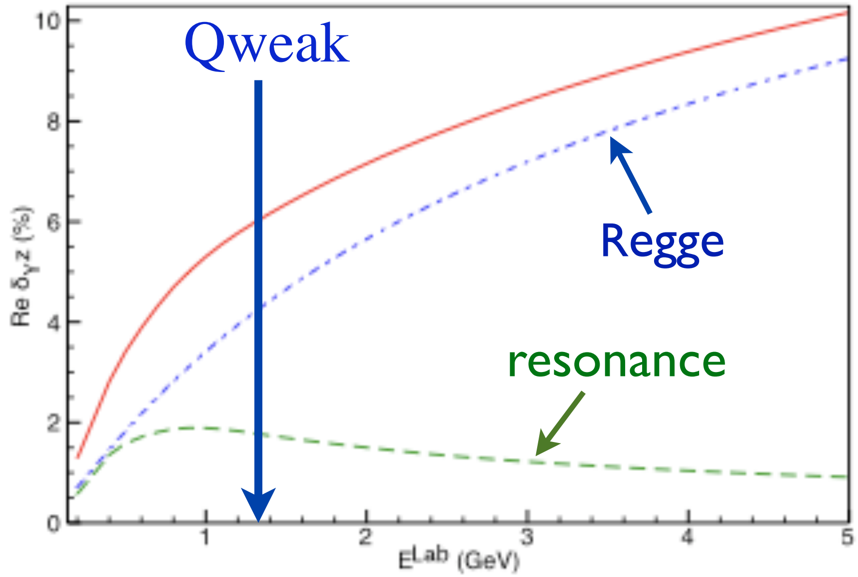
$$\Re e\, \Box_{\gamma Z}^{V} = 0.0047^{+0.0011}_{-0.0004}$$

or $6.6^{+1.5}_{-0.6}\,\%$ of uncorrected Q_W^p

Sibirtsev, Blunden, WM, Thomas
PRD ***82*** *(2010) 013011*

Gorchtein, Horowitz, PRL ***102*** *(2009) 091806*

(see also Gorchtein, Horowitz, Ramsey-Musolf, AIP Conf. Proc. ***1265*** *(2010) 328)*



$$\Re e\, \delta_{\gamma Z} = \Re e \,\Box^{V}_{\gamma Z} / Q^{p}_{W} \approx 6\%$$

mostly from high-W ("Regge") contribution

→ our formula for $\Im m \,\Box^{V}_{\gamma Z}$ factor 2 larger*

("nuclear physics" *vs.* "particle physics" conventions for weak charges in structure function definitions?)

* confirmed by Rislow/Carlson *arXiv:1011.2397*

→ GH omit factor $(1-x)$ in definition of $F_{1,2}$

($\sim 30\%$ enhancement)

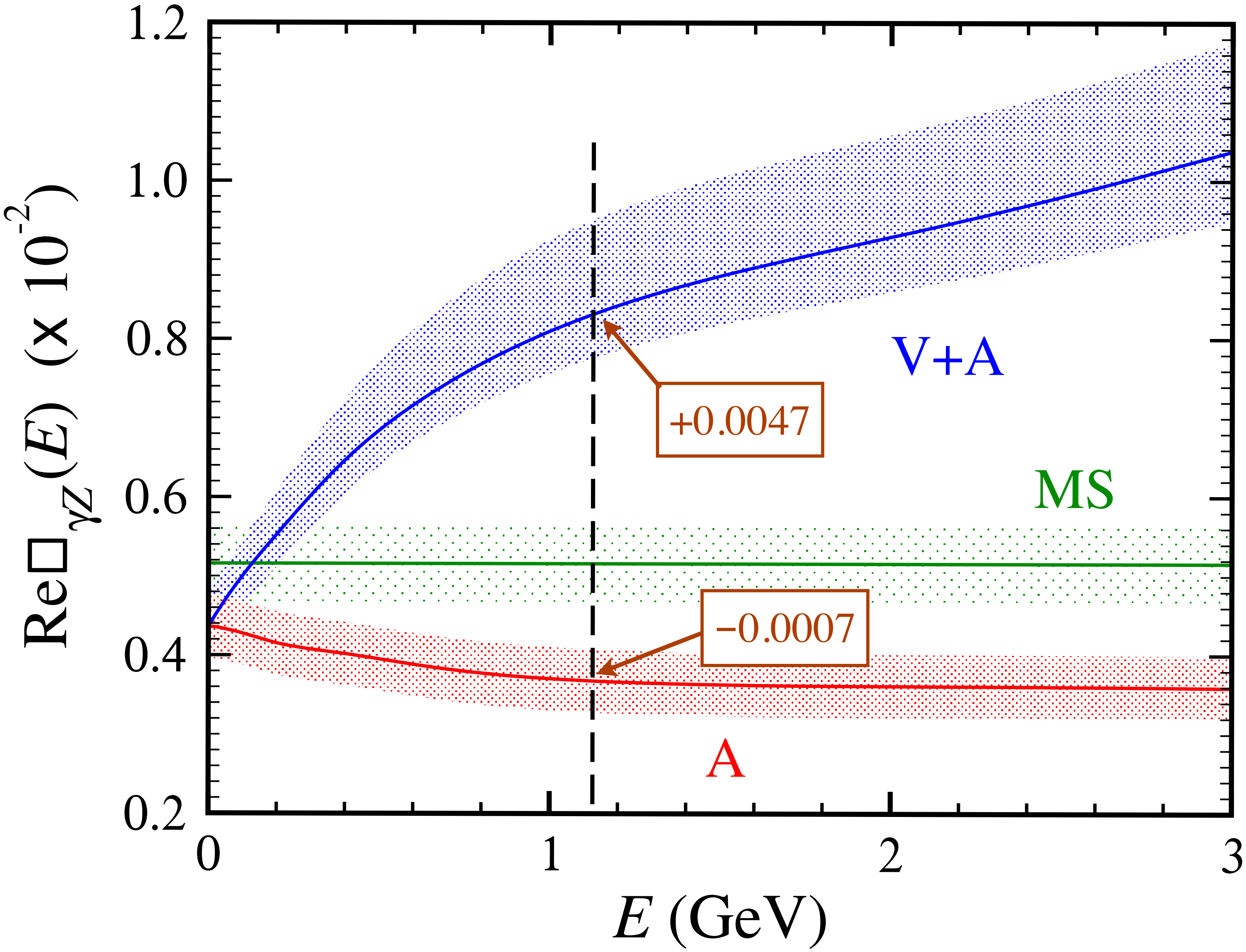
→ GH use $Q^{p}_{W} \sim 0.05$ *cf.* ~ 0.07

($\sim 40\%$ enhancement)

→ numerical agreement for $\delta^{V}_{\gamma Z}$ coincidental (?)

Combined vector and axial h correction

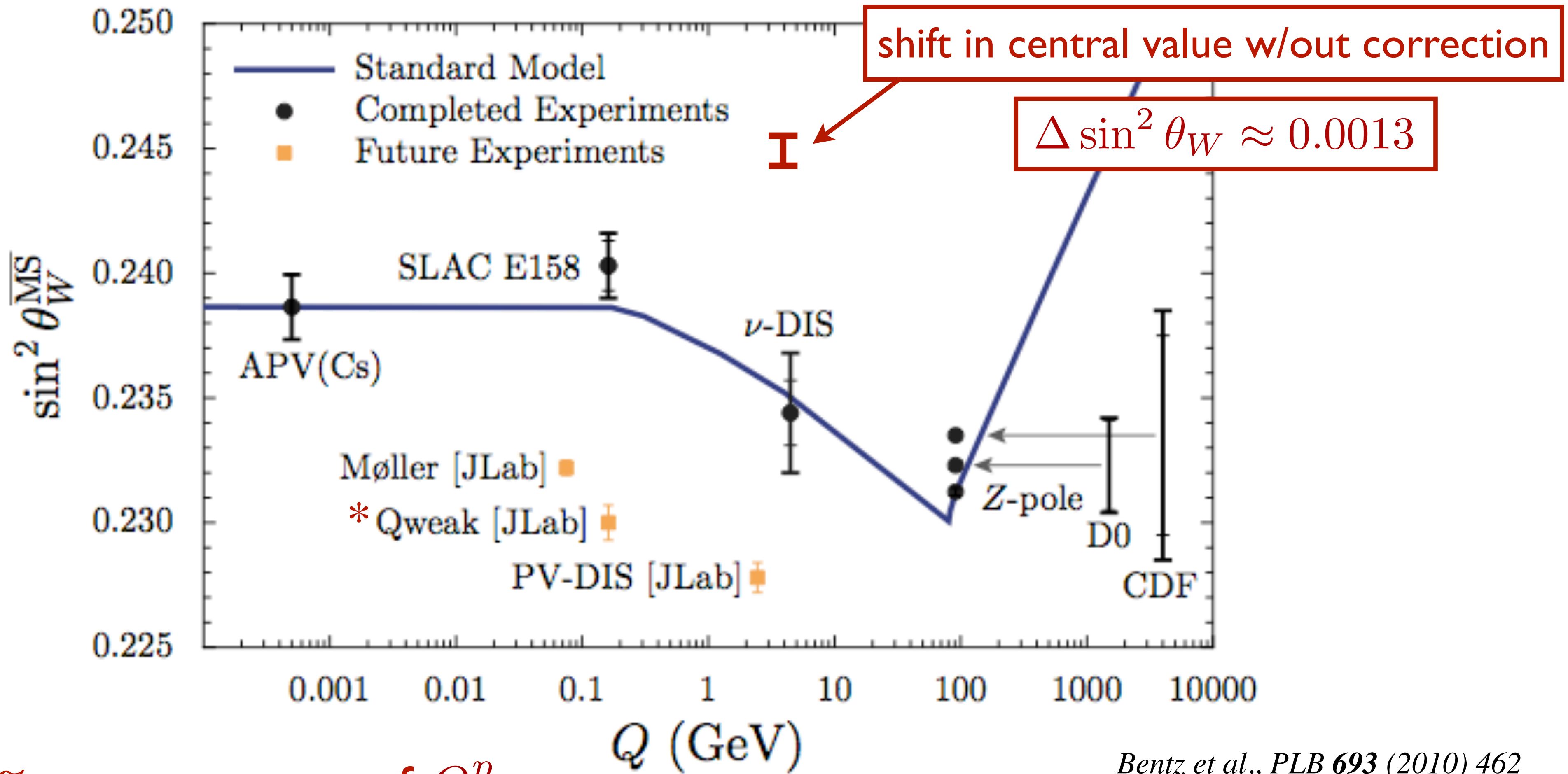
$$\boxed{Q_W^p = 0.0713 \rightarrow 0.0705}$$ (at E=0)



At E=1.165 GeV E-dependent correction is +0.0040

Combined vector and axial h correction

→ significant shift in central value, errors within projected experimental uncertainty $\Delta Q_W^p = \pm 0.003$



*Bentz et al., PLB **693** (2010) 462*

* 4% measurement of Q_W^p

t dependence

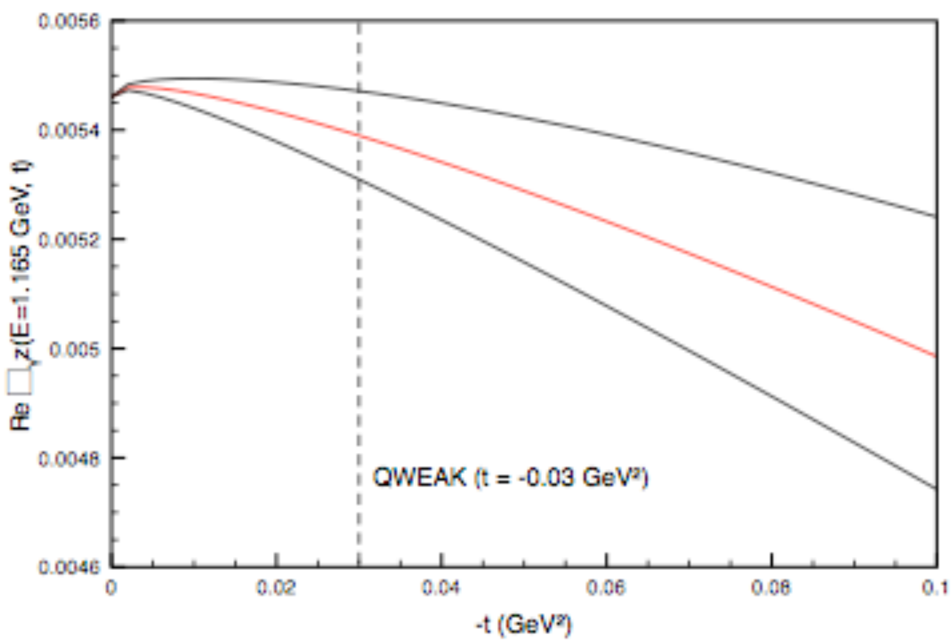
- Extrapolation from $t = -0.03\ \mathrm{GeV}^2$ to $t = 0$

  $\longrightarrow$ phenomenological *ansatz*

$$\Box_{\gamma Z}(E,t) = \Box_{\gamma Z}(0,0)\,\frac{e^{-B|t|/2}}{F_1^{\gamma p}(t)}$$

with $B = (7 \pm 1)\,\mathrm{GeV}^{-2}$ from forward Compton scattering

Gorchtein, Horowitz, PRL ***102*** *(2009) 091806*



$\longrightarrow$ $\sim 2\%$ reduction of $\Box_{\gamma Z}$

Summary

- Dramatic effect of $\gamma(Z\gamma)$ corrections at forward angles on proton weak charge, $\Delta Q_W^p \sim 6\%$, *cf.* PDG

  → would significantly shift extracted weak angle

  → better constraints from direct measurement of $F_{1,2,3}^{\gamma Z}$ (*e.g.* in PVDIS at JLab)

- New formulation in terms of *moments* of structure functions

  → places on firm footing earlier derivation of Marciano/Sirlin in "free quark model"

  → may have some effect on atomic PV predictions (*e.g.* Cs, Fr)

The End