Transverse Spin Phenomena & Their Impact on QCD ("GaryFest")
JLab, October 28, 2010

Electroweak Loops in Elastic *ep* Scattering

Wally Melnitchouk

Jefferson Lab

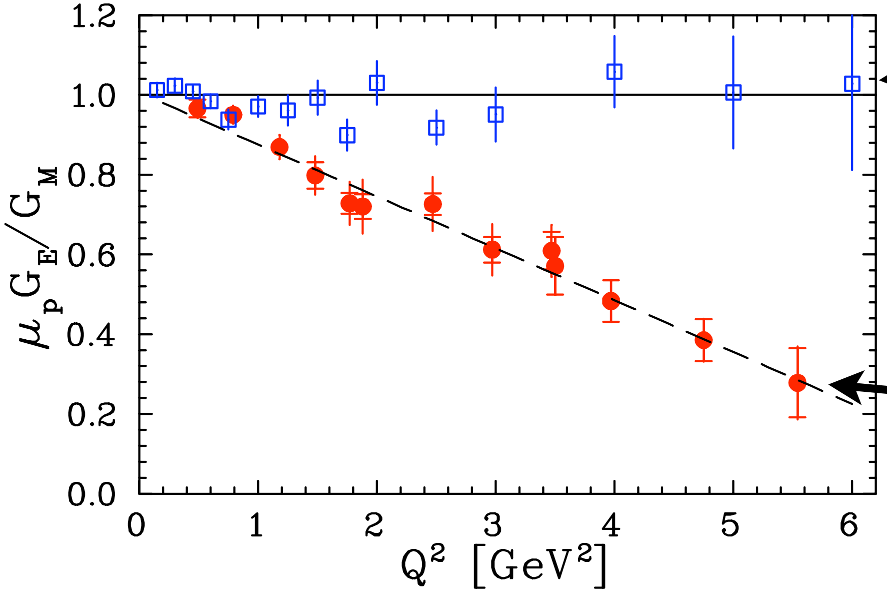
with Peter Blunden (Manitoba), Alex Sibirtsev (Juelich), Tony Thomas (Adelaide), John Tjon[†] *(Utrecht)*

Outline

- *Background:* two-photon exchange in elastic *ep* scattering
  - → electric/magnetic form factor ratio puzzle:
    (Rosenbluth separation *vs.* polarization transfer)
- Parity-violating electron scattering
  - → effect of γZ exchange on strange form factors
  - → dispersive corrections to proton's weak charge
    ("Qweak" experiment at Jefferson Lab)
- Summary

Two-photon exchange in elastic *e*-*p* scattering

Proton G_E/G_M ratio



Rosenbluth (Longitudinal-Transverse) Separation

Arrington et al., PRC ***68****, 034325 (2003)*

Polarization Transfer

Jones et al., PRL ***84****, 1398 (2000)*
Gayou et al., PRL ***88****, 092301 (2002)*

LT method

$$\sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

→ G_E from slope in ε plot

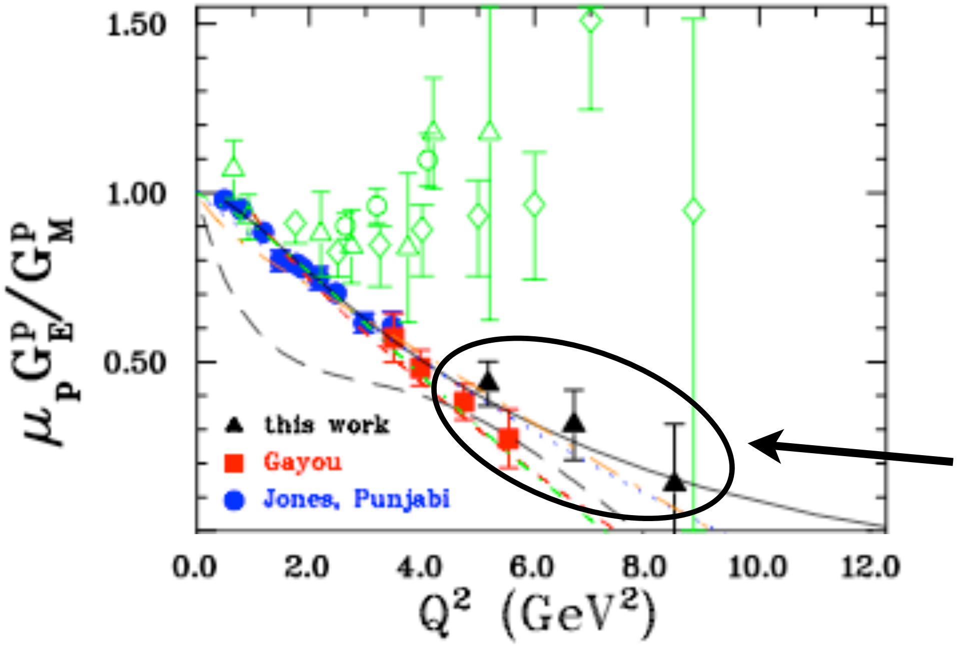
→ suppressed at large Q^2

PT method

$$\frac{G_E}{G_M} = -\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}} \frac{P_T}{P_L}$$

→ $P_{T,L}$ recoil proton polarization in $\vec{e}\, p \to e\, \vec{p}$

Proton G_E/G_M ratio



Polarization Transfer (latest from JLab)

Puckett et al., PRL ***104****, 242301 (2010)*

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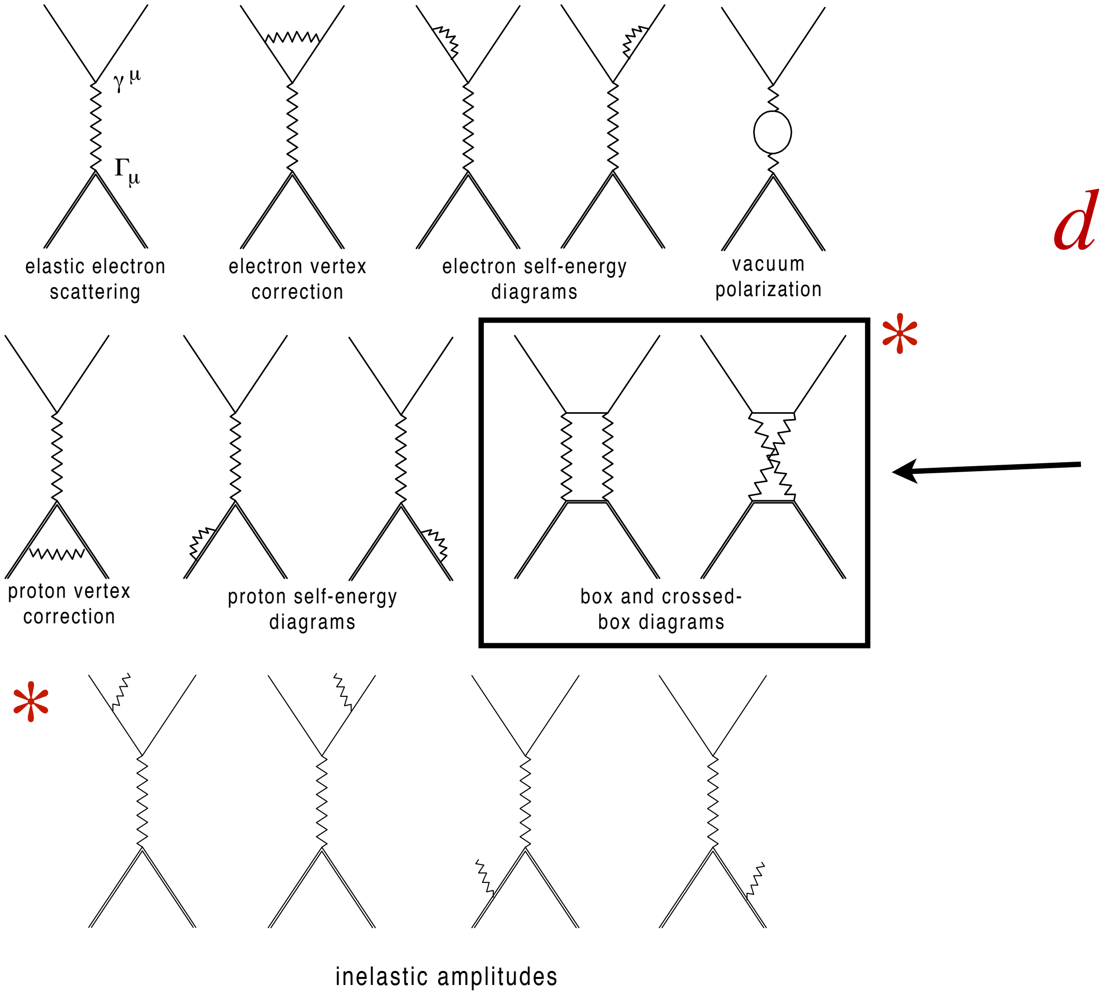
QED radiative corrections

- cross section modified by 1γ loop effects



$$d\sigma = \overset{\text{Born}}{d\sigma_0}\;(1+\overset{\text{TPE}}{\delta})$$

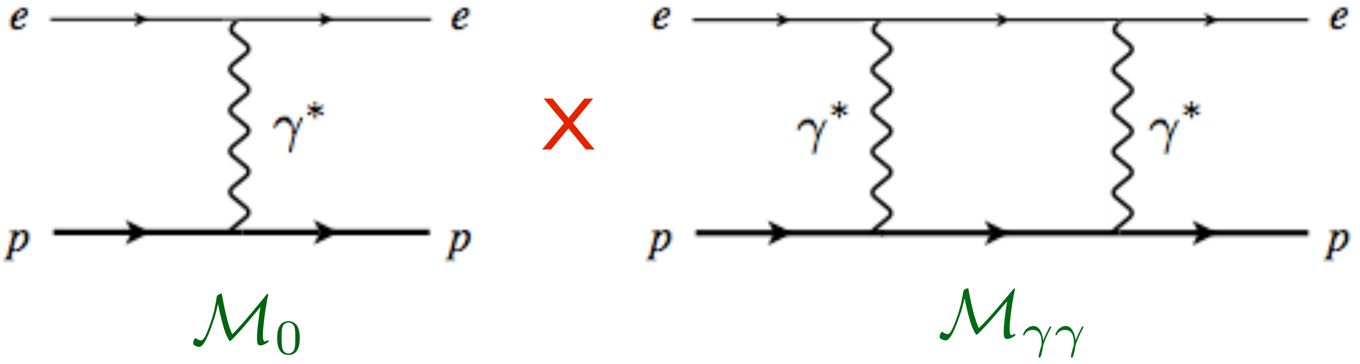
δ contains additional ε dependence, mostly from box diagrams

(most difficult to calculate)

* IR divergences cancel

Two-photon exchange

- interference between Born and TPE amplitudes



- contribution to cross section:

$$\delta^{(2\gamma)} = \frac{2\mathcal{R}e\left\{\mathcal{M}_0^\dagger\;\mathcal{M}_{\gamma\gamma}\right\}}{\left|\mathcal{M}_0\right|^2}$$

- “soft photon approximation” (used in all previous data analyses)

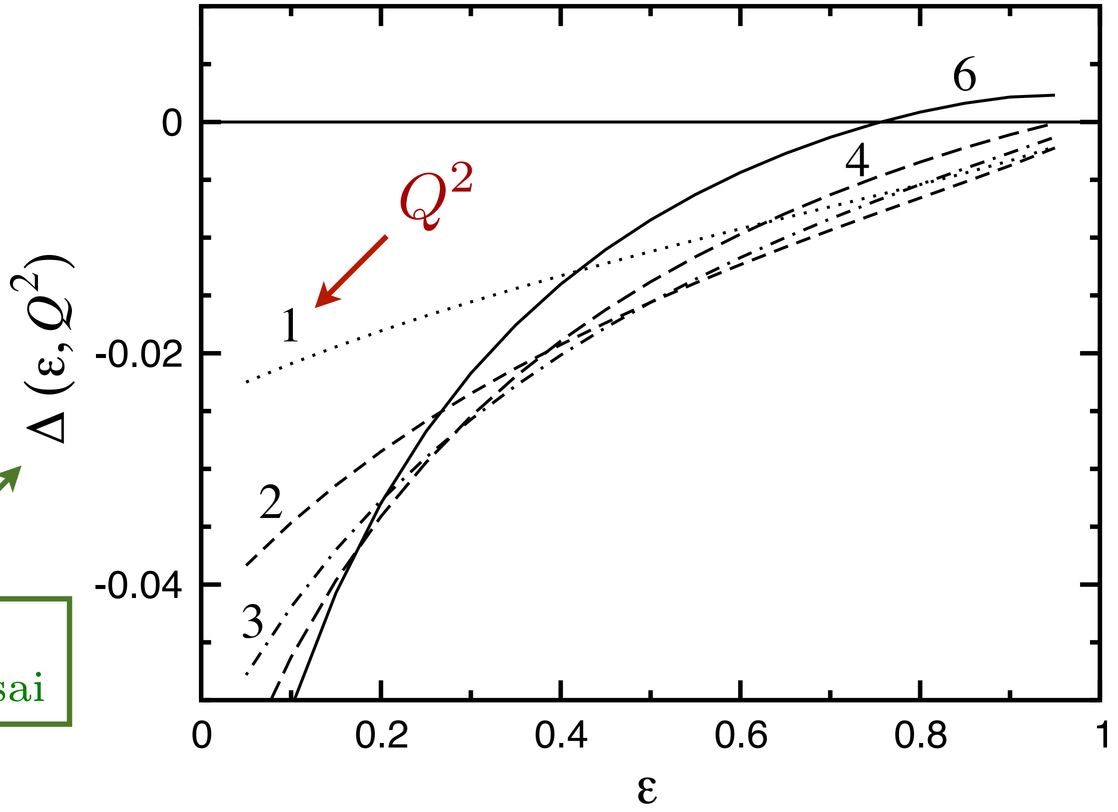
  ⟶ approximate integrand in $\mathcal{M}_{\gamma\gamma}$ by values at γ^* poles

  ⟶ neglect nucleon structure (no form factors)

Mo, Tsai (1969)

Two-photon exchange

- “exact” calculation of loop diagram (including hadron structure)



$\delta^{(2\gamma)}_{\rm full} - \delta^{(2\gamma)}_{\rm Mo-Tsai}$

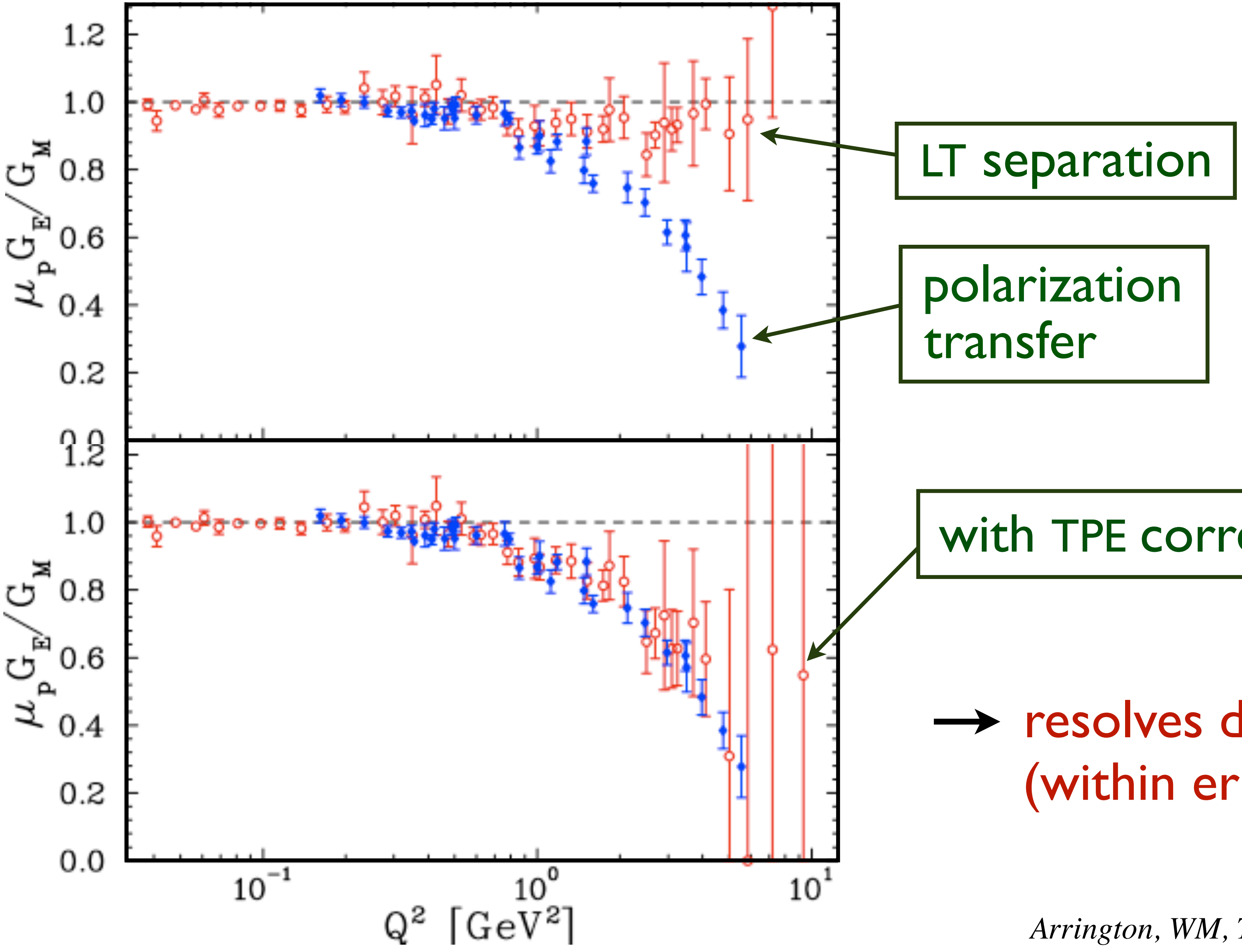
Blunden, WM, Tjon
*PRL **91**, 142304 (2003);*
*PRC **72**, 034612 (2005)*

→ few % magnitude, non-linear in ε, *positive* slope

→ will *reduce* Rosenbluth ratio

→ does not depend strongly on vertex form factors

Two-photon exchange



LT separation

polarization transfer

with TPE correction

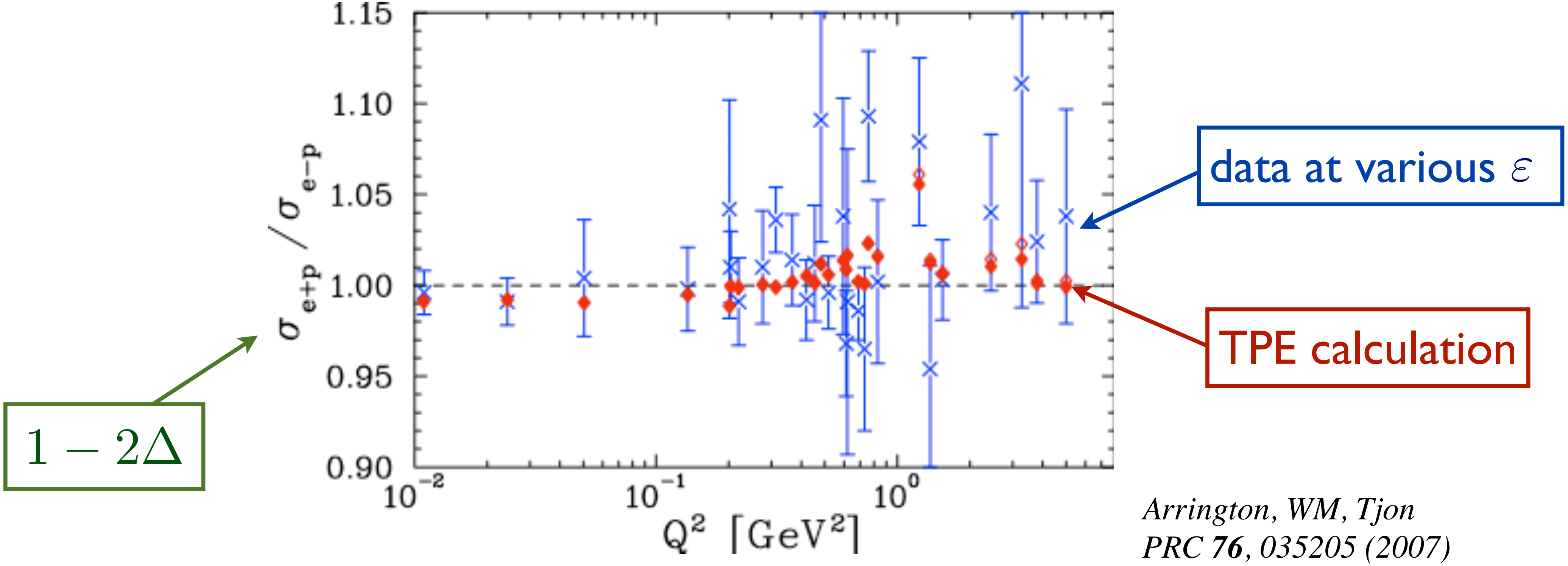
→ resolves discrepancy (within errors)

Arrington, WM, Tjon
*PRC **76**, 035205 (2007)*

Direct evidence?

- 1γ (2γ) exchange changes sign (invariant) under $e^+ \leftrightarrow e^-$

→ ratio of e^+p/e^-p cross sections sensitive to $\Delta(\varepsilon, Q^2)$



Arrington, WM, Tjon
*PRC **76**, 035205 (2007)*

→ simultaneous e^+p/e^-p measurement using tertiary e^+/e^- beam to Q^2 ~ 1-2 GeV^2
(Hall B experiment E04-116)

Direct evidence?

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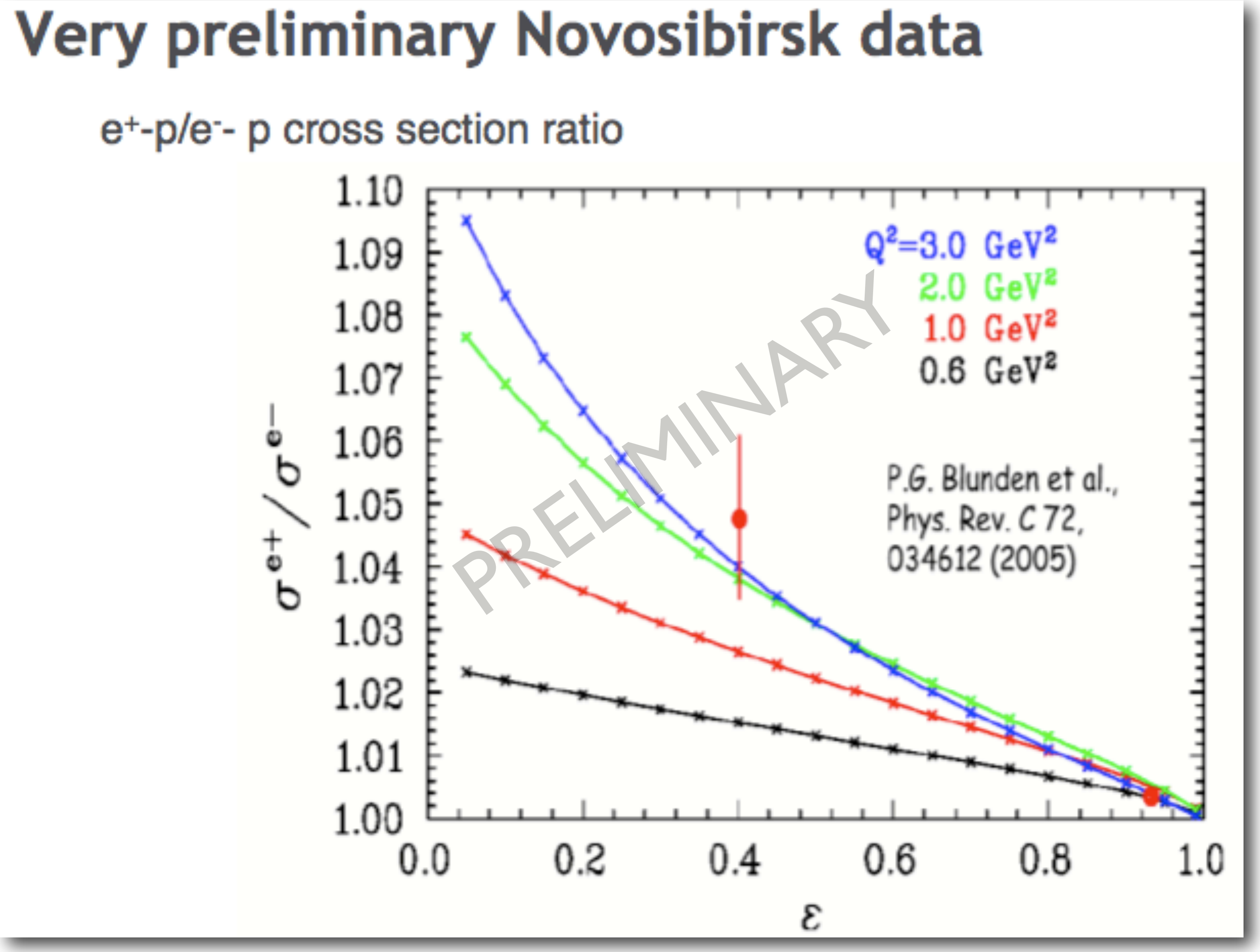
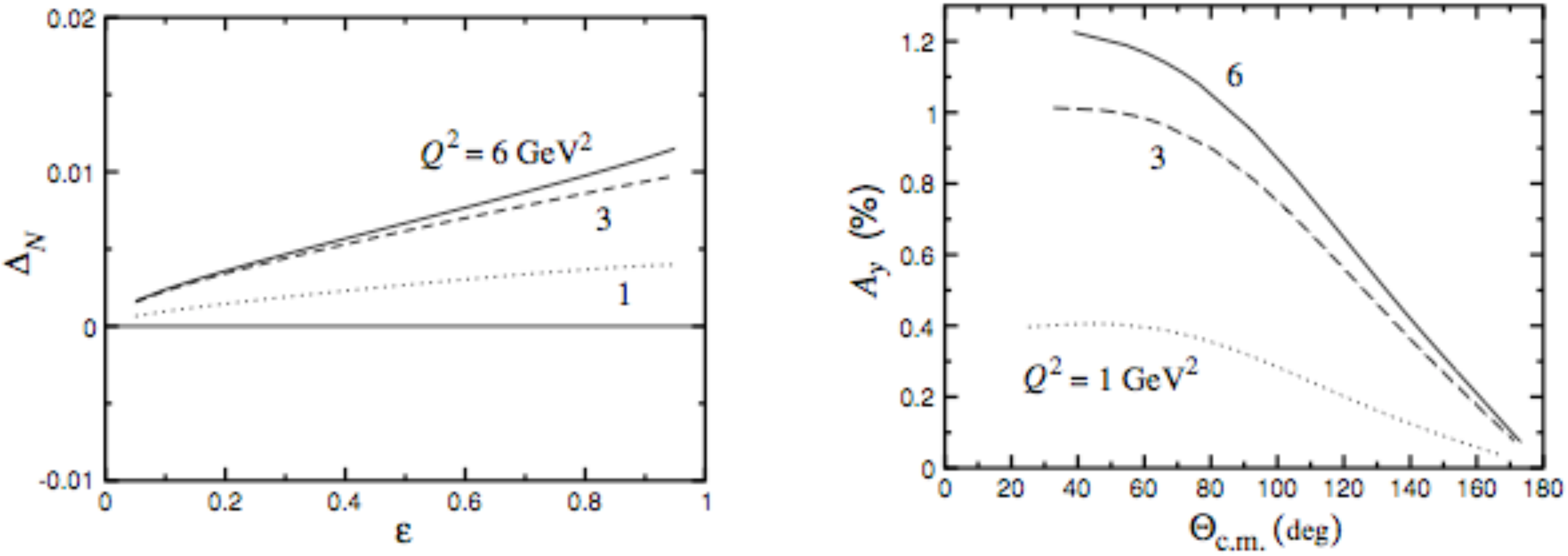


Arrington, Holt et al. (2010)

Direct evidence?

- polarization transfer with recoil proton polarized *normal* to scattering plane

→ purely *imaginary* (does not contribute to form factor), vanishes in Born approximation!



ratio of normal to unpolarized

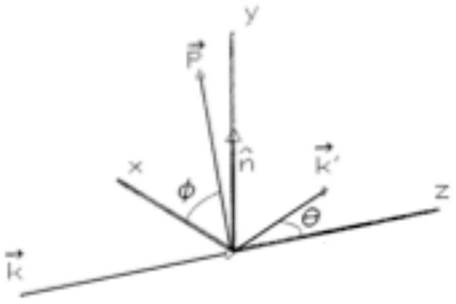
Blunden, WM, Tjon, PRC ***72****, 034612 (2005)*

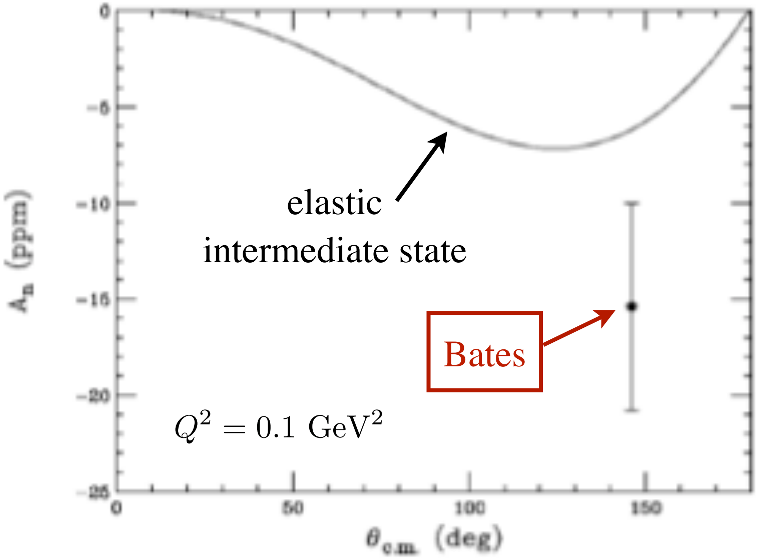
→ effect largest at forward angles, grows with Q^2

Direct evidence?

- beam asymmetry for *e* polarized *normal* to scattering plane

→ also vanishes for one-photon exchange





Wells et al., PRC ***63****, 064001 (2001)*

→ significant inelastic contributions to imaginary part of TPE

Direct evidence?

- beam asymmetry for *e* polarized *normal* to scattering plane

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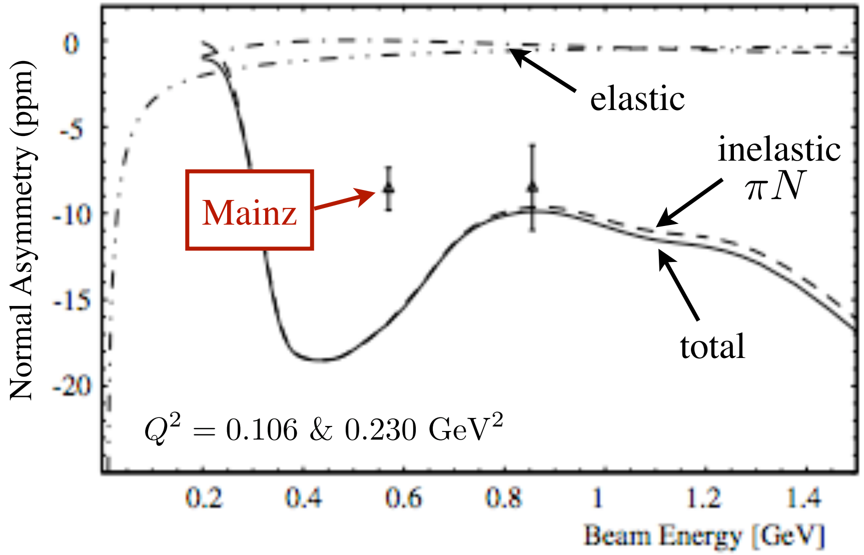
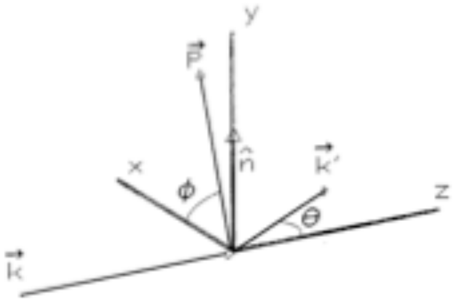




Maas et al., *PRL* ***94***, *082001 (2005)*

→ significant inelastic contributions to imaginary part of TPE

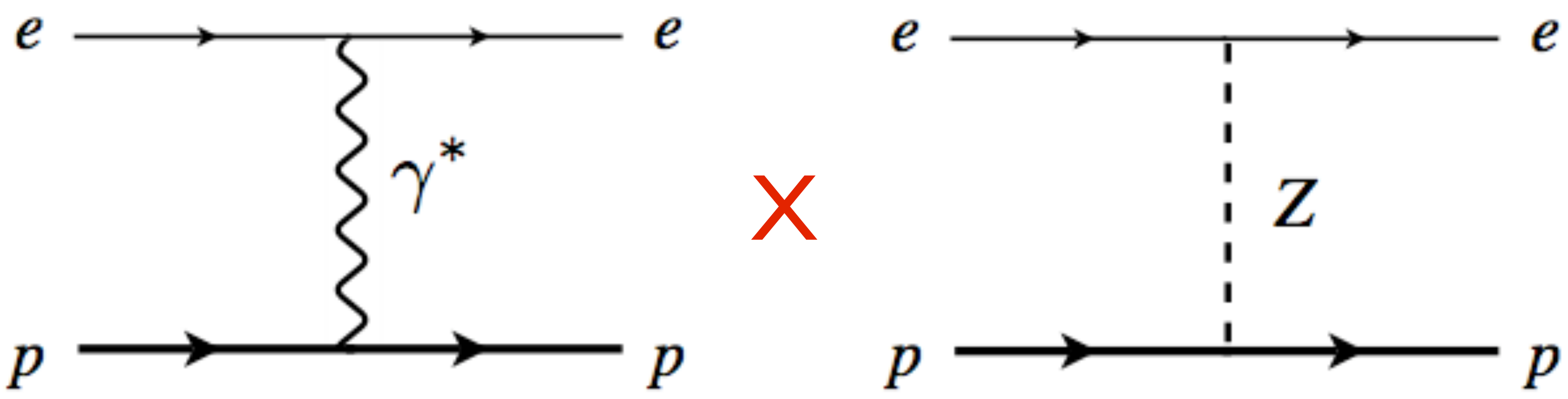
Parity-violating electron scattering

Parity-violating e scattering

- Left-right polarization asymmetry in $\vec{e}\, p \to e\, p$ scattering

$$A_{\mathrm{PV}} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\left(\frac{G_F Q^2}{4\sqrt{2}\alpha}\right)(A_V + A_A + A_s)$$

⟶ measure interference between e.m. and weak currents



Born (tree) level

Parity-violating e scattering

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vector asymmetry

$$A_V = g_A^e \rho \left[(1 - 4\kappa \sin^2\theta_W) - (\varepsilon G_E^{\gamma p} G_E^{\gamma n} + \tau G_M^{\gamma p} G_M^{\gamma n})/\sigma^{\gamma p}\right]$$

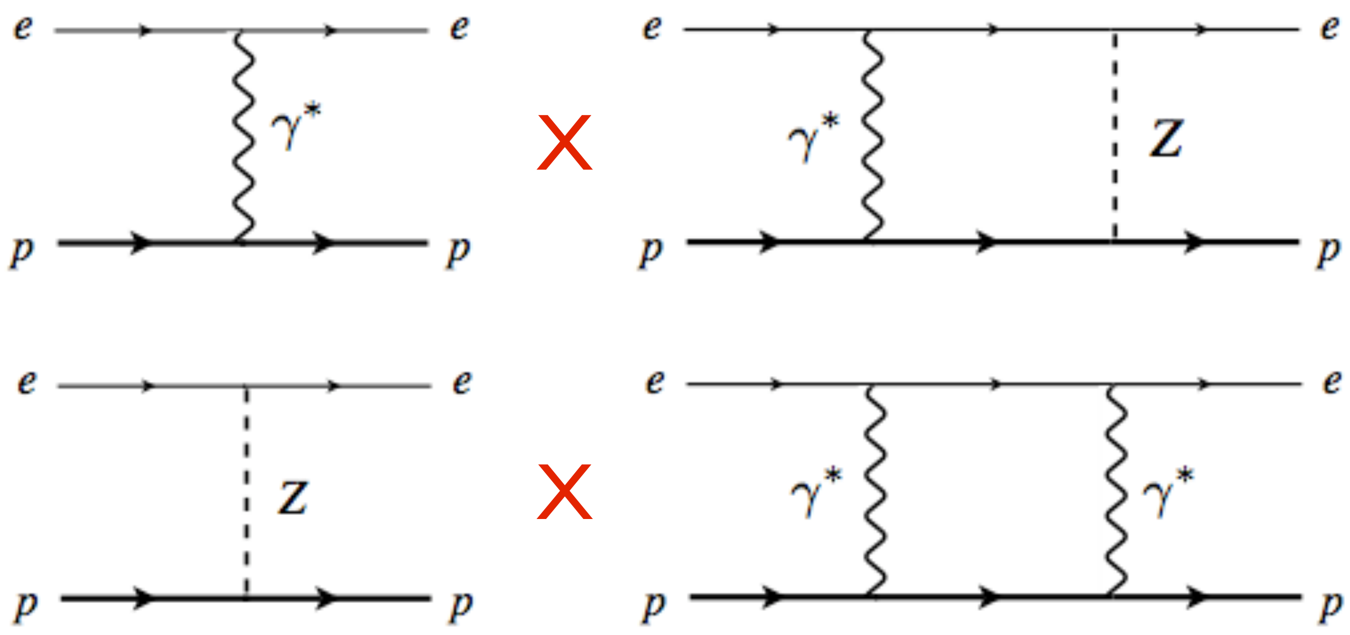
axial vector asymmetry

$$A_A = g_V^e \sqrt{\tau(1+\tau)(1-\varepsilon^2)}\ \widetilde{G}_A^{Zp} G_M^{\gamma p}/\sigma^{\gamma p}$$

strange asymmetry

$$A_s = -g_A^e \rho \left(\varepsilon G_E^{\gamma p} G_E^s + \tau G_M^{\gamma p} G_M^s\right)/\sigma^{\gamma p}$$

Two-boson exchange corrections



"$\gamma(Z\gamma)$"

"$Z(\gamma\gamma)$"

$$A_{\rm PV} = (1+\delta)A^0_{\rm PV} \equiv \left(\frac{1+\delta_{Z(\gamma\gamma)}+\delta_{\gamma(Z\gamma)}}{1+\delta_{\gamma(\gamma\gamma)}}\right)A^0_{\rm PV}$$

Born asymmetry

→ total TBE correction

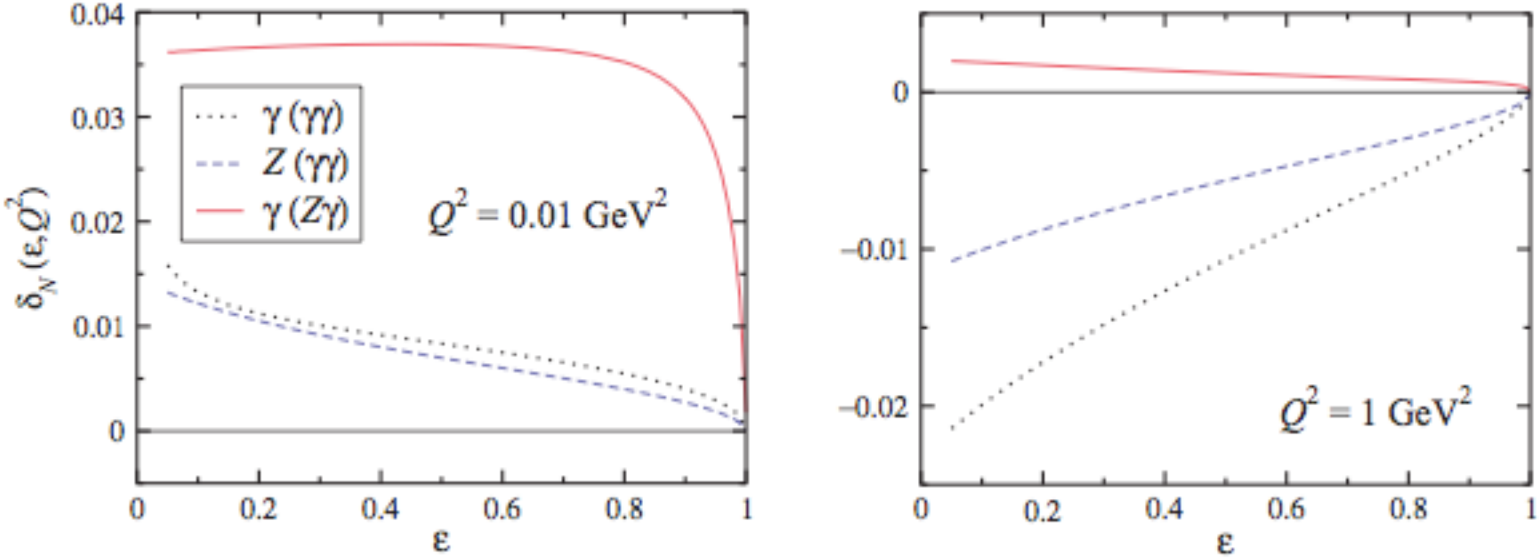
$$\delta \;\approx\; \delta_{Z(\gamma\gamma)} + \delta_{\gamma(Z\gamma)} - \delta_{\gamma(\gamma\gamma)}$$

Two-boson exchange corrections

→ previous estimates computed at $Q^2 = 0$, do not include hadron structure effects

Marciano, Sirlin (1980)

nucleon intermediate states



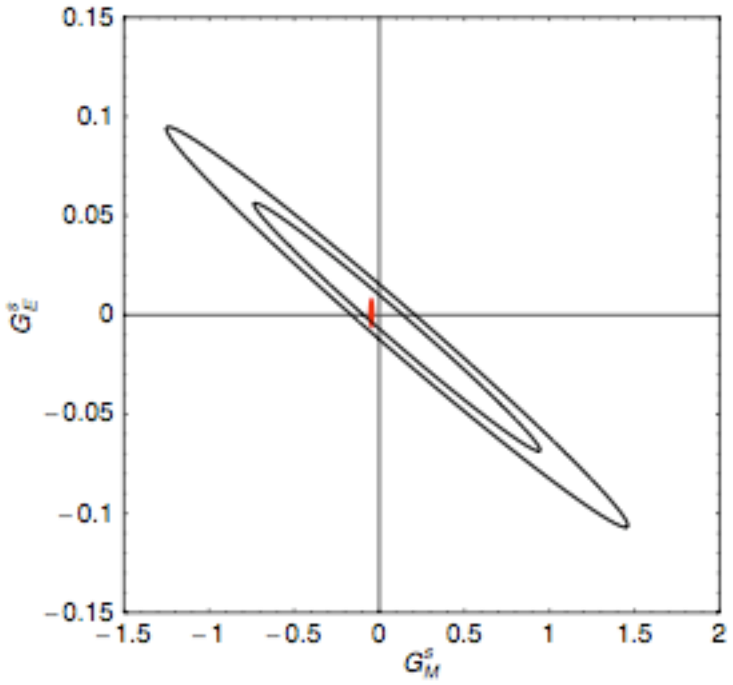
Tjon, Blunden, WM
*PRC **79** (2009) 055201*

→ cancellation between $Z(\gamma\gamma)$ and $\gamma(\gamma\gamma)$ corrections, especially at low Q^2

→ dominated by $\gamma(Z\gamma)$ contribution

Effects on strange form factors

- global analysis of all PVES data at $Q^2 < 0.3\ \mathrm{GeV}^2$



$$G_E^s = 0.0025 \pm 0.0182$$

$$G_M^s = -0.011 \pm 0.254$$

at $Q^2 = 0.1\ \mathrm{GeV}^2$

Young et al., PRL ***97*** *(2006) 102002*

- including TBE corrections:

$$\boxed{\begin{aligned} G_E^s &= 0.0023 \pm 0.0182 \\ G_M^s &= -0.020 \pm 0.254 \end{aligned}}$$

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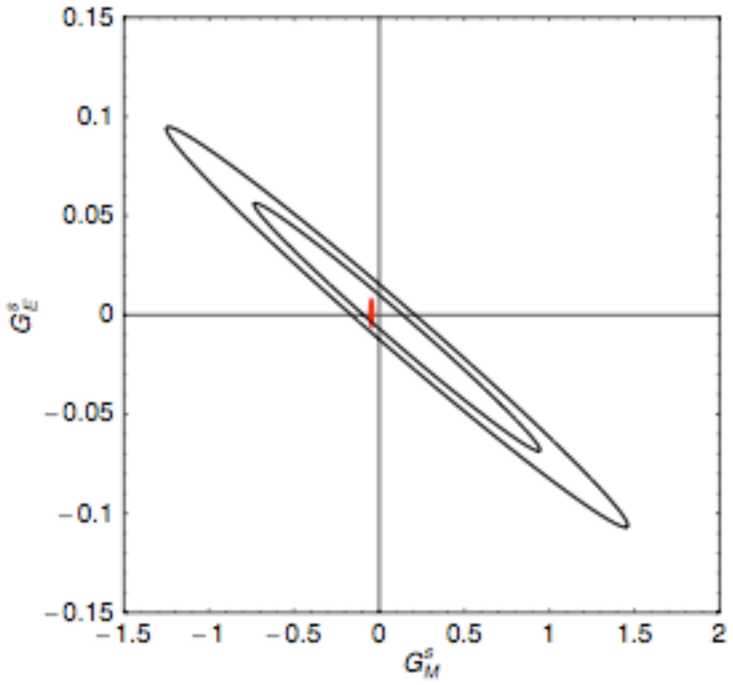

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at $Q^2 = 0.1\ \mathrm{GeV}^2$

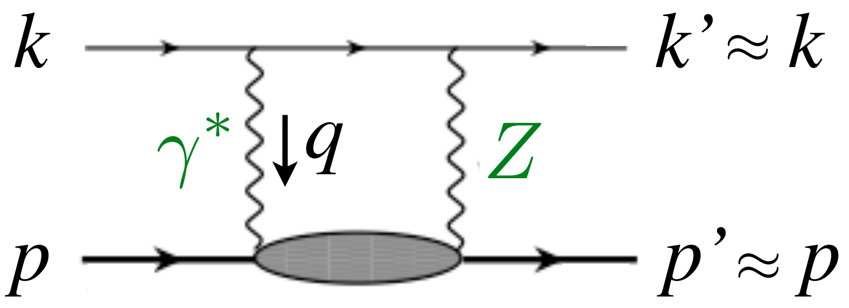
fixed mainly by ^{4}He data ...

... TBE for ^{4}He not yet included

Correction to proton weak charge

- in forward limit $A_{\rm PV}$ measures weak charge of proton Q_W^p

$$A_{\rm PV} \;\to\; \frac{G_F\,Q_W^p}{4\sqrt{2}\pi\alpha}t$$



forward limit

$$t = (k-k')^2 \to 0$$

$$s = (k+p)^2$$
$$= M(M+2E)$$

- at tree level Q_W^p gives weak mixing angle

$$Q_W^p = 1 - 4\sin^2\theta_W$$

Correction to proton weak charge

- including higher order radiative corrections

$$Q_W^p = (1+\Delta\rho+\Delta_e)(1-4\sin^2\theta_W(0)+\Delta'_e) + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z} \longleftarrow \text{box diagrams}$$

$$= 0.0713 \pm 0.0008$$

Erler et al., PRD ***72****, 073003 (2005)*

→ WW and ZZ box diagrams dominated by short distances, evaluated perturbatively

→ γZ box diagram sensitive to long distance physics, has two contributions $\Box_{\gamma Z} = \Box_{\gamma Z}^{A} + \Box_{\gamma Z}^{V}$

vector e – axial h
(finite at E=0)

axial e – vector h
(vanishes at E=0)

Axial h correction

- axial h correction $\Box^A_{\gamma Z}$ dominant γZ correction in atomic parity violation at very low (zero) energy

→ computed by Marciano & Sirlin as sum of two parts:

- ★ low-energy part approximated by *Born* contribution (elastic intermediate state)
- ★ high-energy part (above scale $\Lambda \sim 1$ GeV) computed in terms of scattering from *free quarks*

$$\Box^A_{\gamma Z} = \frac{5\alpha}{2\pi}(1 - 4\sin^2\theta_W)\left[\underbrace{\ln\frac{M_Z^2}{\Lambda^2}}_{\text{short−distance}} + \underbrace{C_{\gamma Z}(\Lambda)}_{\text{long−distance}}\right]$$

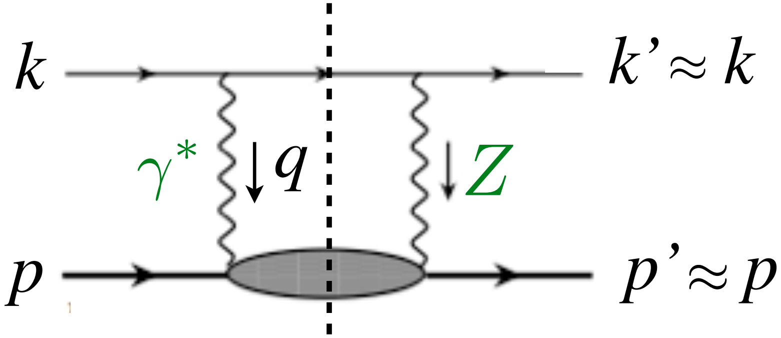
$$\approx \; 0.0028$$

*Marciano, Sirlin, PRD **29**, 75 (1984)*
*Erler et al., PRD **68**, 016006 (2003)*

Axial h correction

- axial h correction $\Box^A_{\gamma Z}$ dominant γZ correction in atomic parity violation at very low (zero) energy

  → repeat calculation using forward dispersion relations with realistic (structure function) input



- ★ axial h contribution *anti*symmetric under $E' \leftrightarrow -E'$:

$$\Re e \, \Box^A_{\gamma Z}(E) = \frac{2}{\pi} \int_0^\infty dE' \, \frac{E'}{E'^2 - E^2} \; \Im m \, \Box^A_{\gamma Z}(E')$$

- ★ imaginary part can only grow as $\log E' / E'$

Axial h correction

→ imaginary part given by interference $F_3^{\gamma Z}$ structure function

$$\Im m\, \square^{A}_{\gamma Z}(E) = \frac{\alpha}{(s-M^2)^2} \int_{W_\pi^2}^{s} dW^2 \int_0^{Q^2_{\max}} \frac{dQ^2}{1+Q^2/M_Z^2} \times \frac{g_V^e}{2g_A^e}\left(\frac{4ME}{W^2-M^2+Q^2}-1\right) F_3^{\gamma Z}$$

with $g_A^e = -\frac{1}{2}, \quad g_V^e = -\frac{1}{2}(1-4\sin^2\theta_W)$

→ $F_3^{\gamma Z}$ structure function

- ★ elastic part given by $G_M^p\, G_A^Z$
- ★ resonance part from parametrization of ν scattering data (Lalakulich-Paschos)
- ★ DIS part dominated by leading twist PDFs at small x (MSTW, CTEQ, Alekhin)

Axial h correction

→ energy dependence is weak

→ correction at $E = 0$

$$\Re e\,\Box^{A}_{\gamma Z}(0) \;=\; \underset{\substack{\uparrow\\ \text{elastic}}}{0.0006} \;+\; \underset{\substack{\uparrow\\ \text{resonance}}}{0.0002} \;+\; \underset{\substack{\uparrow\\ \text{DIS}}}{0.0025} \;=\; 0.0033$$

→ *cf.* MS value 0.0028 (or 0.7% increase)

→ resulting shift in weak charge

$$\boxed{Q^p_W = 0.0713 \to 0.0718}$$

Blunden, WM, Thomas (2010)

Vector h correction

- vector h correction $\Box^V_{\gamma Z}$ vanishes at $E = 0$, but experiment has $E \sim 1\ \mathrm{GeV}$ – what is energy dependence?

→ forward dispersion relation

- ★ $\Re e\, \Box^V_{\gamma Z}(E) = \frac{2E}{\pi} \int_0^\infty dE' \frac{1}{E'^2 - E^2}\, \Im m\, \Box^V_{\gamma Z}(E')$
- ★ integration over $E' < 0$ corresponds to crossed-box, vector h contribution symmetric under $E' \leftrightarrow -E'$

→ imaginary part given by

$$\Im m\, \Box^V_{\gamma Z}(E) = \frac{\alpha}{(s - M^2)^2} \int_{W_\pi^2}^{s} dW^2 \int_0^{Q^2_{\max}} \frac{dQ^2}{1 + Q^2/M_Z^2}$$

$$\times \left(F_1^{\gamma Z} + F_2^{\gamma Z} \frac{s\,(Q^2_{\max} - Q^2)}{Q^2(W^2 - M^2 + Q^2)} \right)$$

Gorchtein, Horowitz, PRL ***102****, 091806 (2009)*

Vector h correction

→ $F_{1,2}^{\gamma Z}$ structure functions

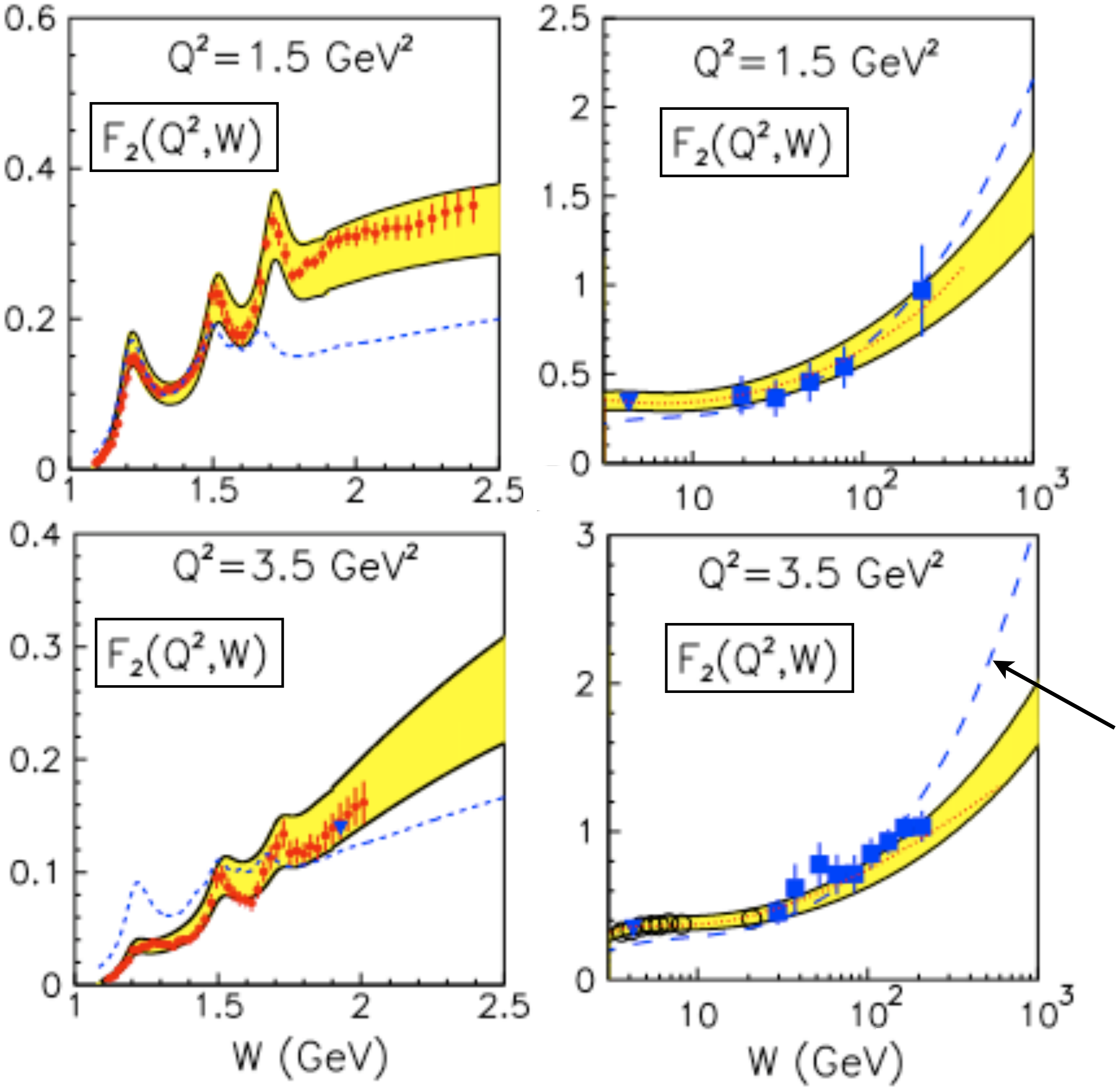
- ★ parton model for DIS region $F_2^{\gamma Z} = 2x \sum_q e_q\, g_V^q\, (q+\bar{q}) = 2x F_1^{\gamma Z}$
  - → $F_2^{\gamma Z} \approx F_2^{\gamma}$ good approximation at *low* x
  - → provides upper limit at *large* x $(F_2^{\gamma Z} \lesssim F_2^{\gamma})$

- ★ in resonance region use phenomenological input for F_2, empirical (SLAC) fit for R
  - → for transitions to $I = 3/2$ states ($e.g.$ Δ), CVC and isospin symmetry give $F_i^{\gamma Z} = (1 + Q_W^p) F_i^{\gamma}$
  - → for transitions to $I = 1/2$ states, SU(6) wave functions predict Z & γ transition couplings equal to a few %

Vector *h* correction

→ compare structure function input with data

low *W*



high *W*

GVMD model
(used as input by
Gorchtein & Horowitz)

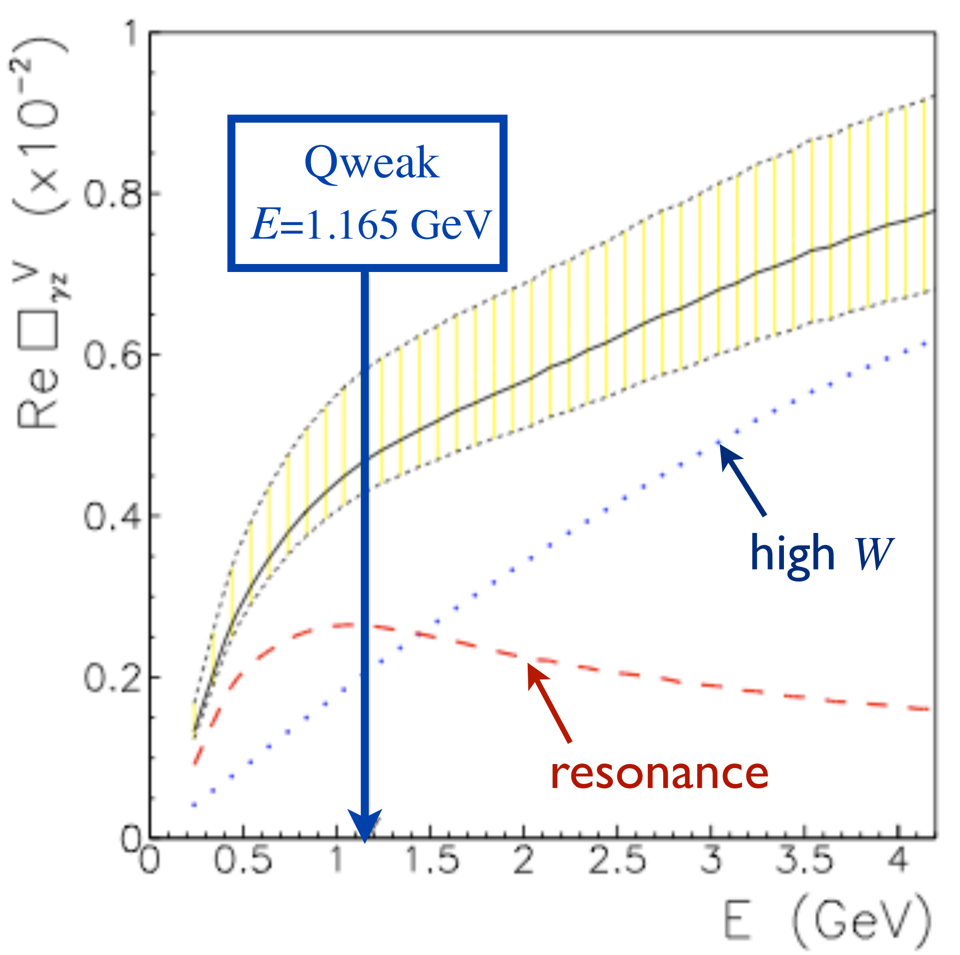
Vector h correction

→ total $\Box^V_{\gamma Z}$ correction:

$$\Re e\, \Box^V_{\gamma Z} = 0.0047^{+0.0011}_{-0.0004}$$

or $6.6^{+1.5}_{-0.6}\,\%$ of uncorrected Q^p_W

$$\boxed{Q^p_W = 0.0713 \rightarrow 0.0760}$$

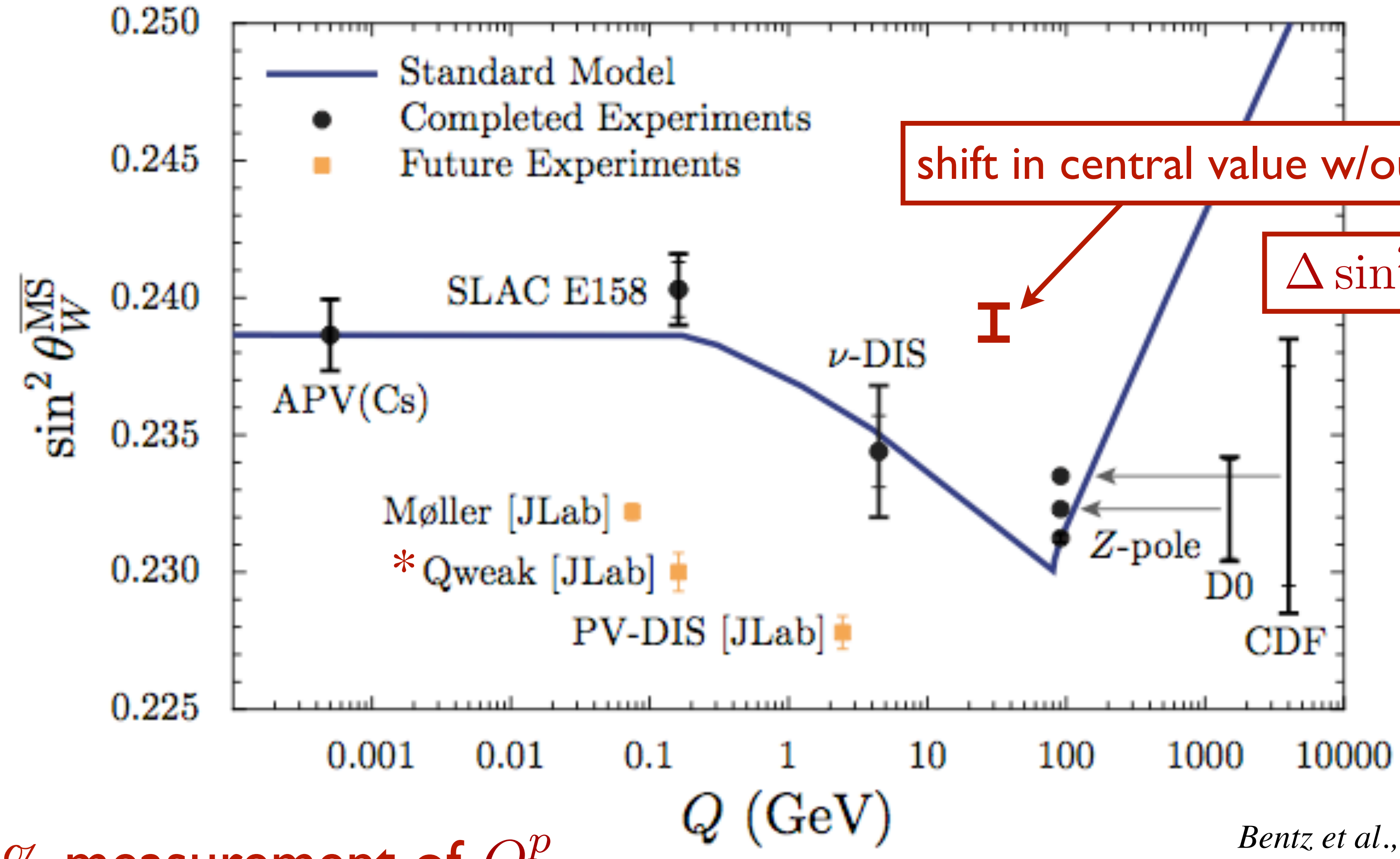


Sibirtsev, Blunden, WM, Thomas, PRD ***82****, 013011 (2010)*

Combined vector and axial h correction

$$\boxed{Q_W^p = 0.0713(8) \rightarrow 0.0765^{+0.0014}_{-0.0009}}$$

→ significant shift in central value, errors within projected experimental uncertainty $\Delta Q_W^p = \pm 0.003$



shift in central value w/out correction

$\Delta \sin^2\theta_W \approx 0.0013$

Bentz et al., PLB ***693****, 462 (2010)*

* 4% measurement of Q_W^p

Summary

- Two-photon exchange corrections resolve most of Rosenbluth / polarization transfer G_E^p/G_M^p discrepancy

  → striking demonstration of limitation of one-photon exchange approximation in ep scattering

  → direct tests from e^+/e^- comparison; polarization observables

- Dramatic effect of $\gamma(Z\gamma)$ corrections at forward angles on proton weak charge, $\Delta Q_W^p \sim 7\%$, *cf*. PDG

  → would significantly shift extracted weak angle

  → will be better constrained by direct measurement of $F_{1,2,3}^{\gamma Z}$ (*e.g*. in PVDIS at JLab)

The End