



Weak charge of proton: *loop corrections to parity-violating electron scattering*

Wally Melnitchouk



with *Peter Blunden* (Manitoba), *Alex Sibirtsev* (Juelich),
Tony Thomas (Adelaide), *John Tjon*[†] (Utrecht)

Why proton weak charge $Q_W^p = 1 - 4 \sin^2 \theta_W$?

electromagnetic

$$G_E^\gamma = \sum_q e_q G_E^q$$

weak

$$G_E^Z = \sum_q g_q^V G_E^q$$

PDG convention
(= 1/2 x nuclear physics convention)

$$g_q^V = I_q^w - 2e_q \sin^2 \theta_W$$

Why proton weak charge $Q_W^p = 1 - 4 \sin^2 \theta_W$?

electromagnetic

$$G_E^\gamma = \sum_q e_q G_E^q$$

weak

$$G_E^Z = \sum_q g_q^V G_E^q$$

at $Q^2 = 0$

$$G_E^{u/p} \equiv G_E^{d/n} = 2, \quad G_E^{d/p} \equiv G_E^{u/n} = 1$$

$$G_E^{\gamma p} = 1$$

$$G_E^{Zp} = \frac{1}{2} Q_W^p$$

$$G_E^{\gamma n} = 0$$

$$G_E^{Zn} = -\frac{1}{2}$$

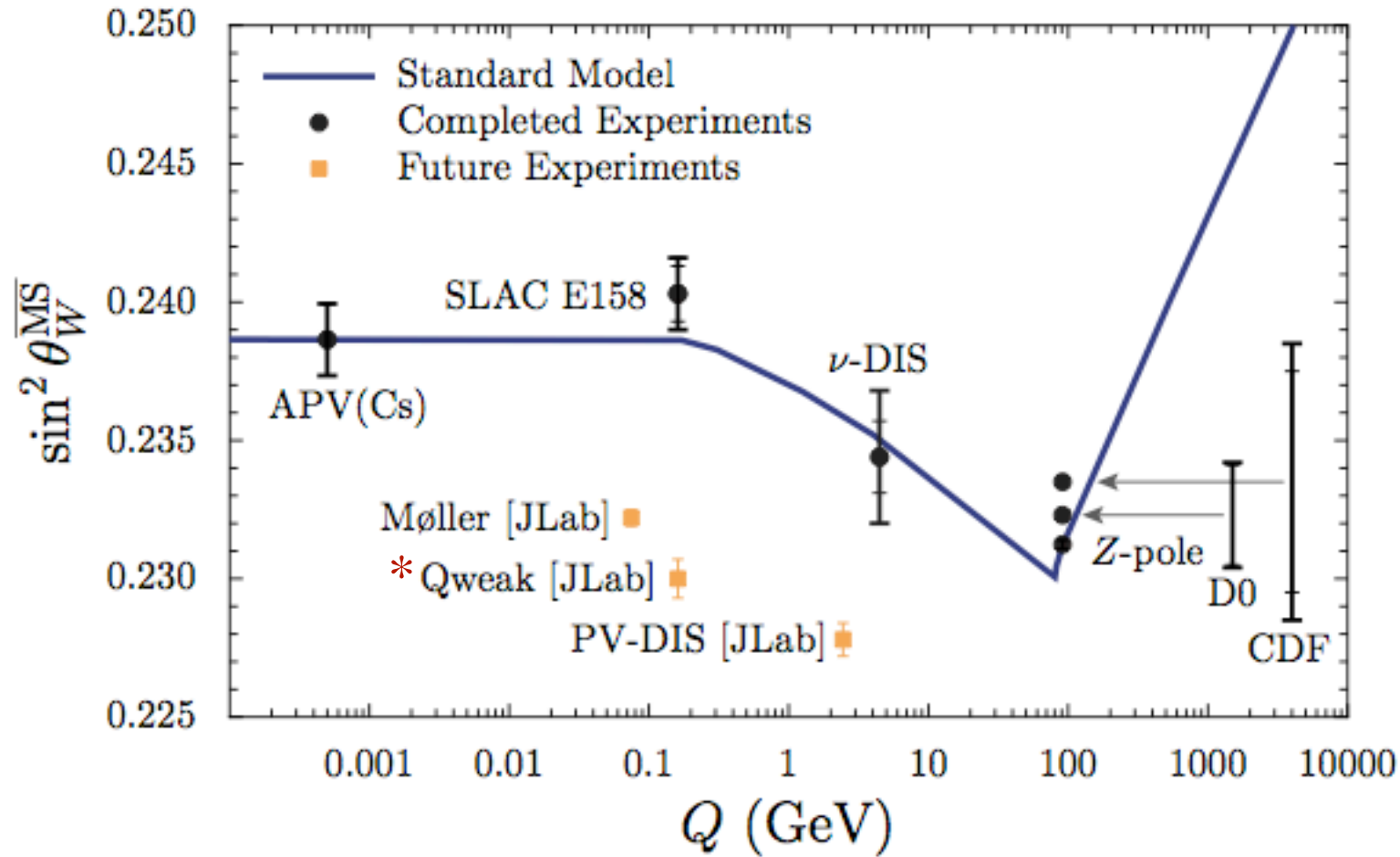
$$G_E^{\gamma n} \ll G_E^{\gamma p}$$

$$|G_E^{Zp}| \ll |G_E^{Zn}|$$

→ G_E^{Zp} small but fundamental quantity!

→ measured in *Qweak* experiment at JLab

Why proton weak charge $Q_W^p = 1 - 4 \sin^2 \theta_W$?



*Bentz, Cloet, Londergan, Thomas
PLB 693, 462 (2010)*

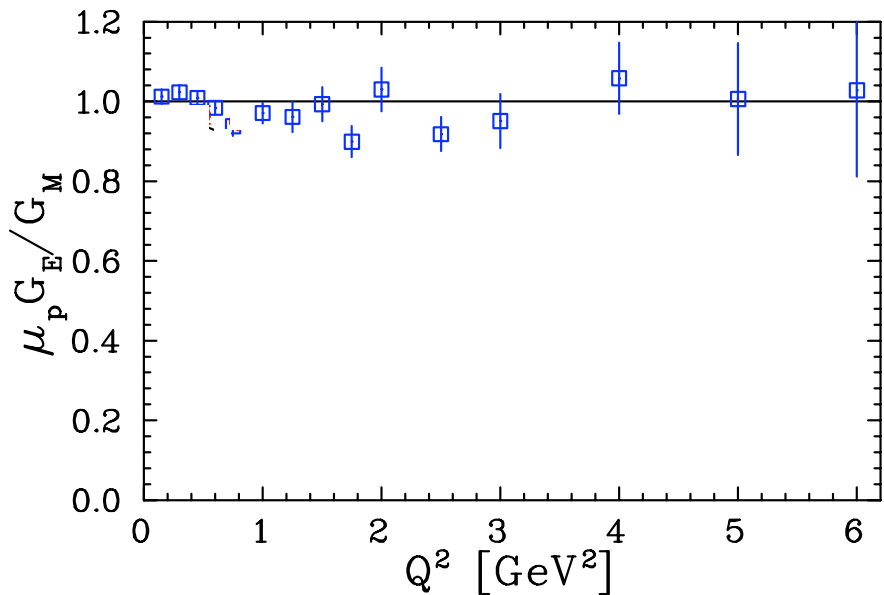
* 4% measurement of Q_W^p

Outline

- *Background: two-photon exchange in elastic ep scattering*
 - electric/magnetic form factor ratio:
Rosenbluth separation *vs.* polarization transfer
- *Parity-violating electron scattering*
 - effect of γZ exchange on strange form factors
 - dispersive corrections to proton's weak charge:
“Qweak” experiment at Jefferson Lab
- *Summary*

Two-photon exchange
in elastic $e-p$ scattering

Proton G_E/G_M ratio



Rosenbluth (Longitudinal-Transverse) Separation

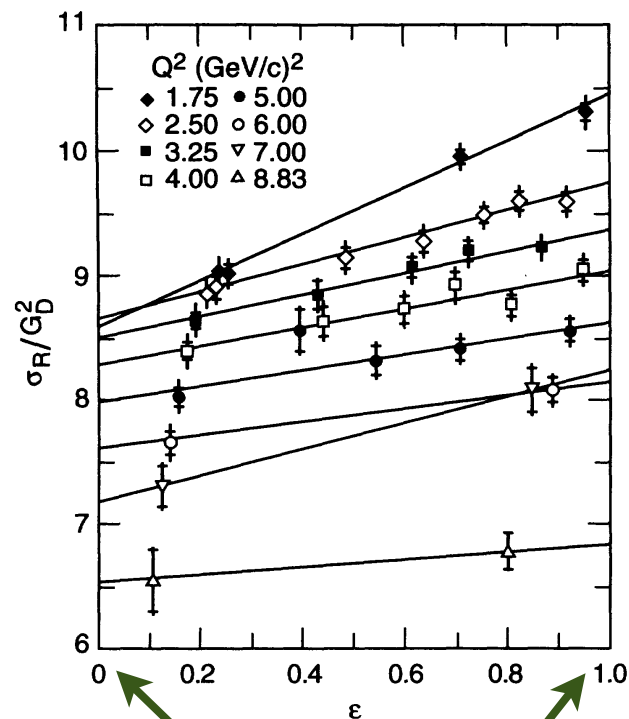
Arrington et al., PRC 68, 034325 (2003)

LT method

$$\sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

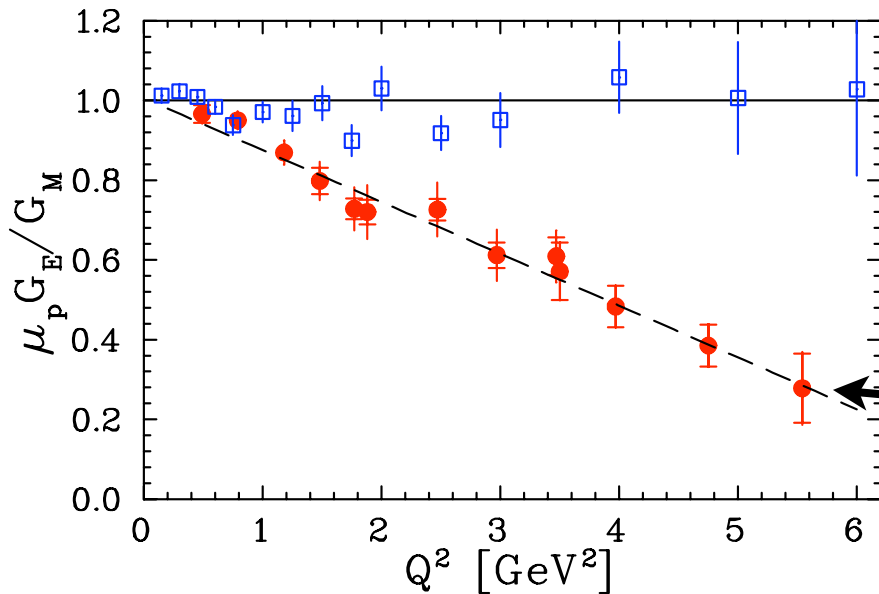
→ G_E from slope in ε plot

→ suppressed at large Q^2



backward scattering forward scattering

Proton G_E/G_M ratio



Rosenbluth (Longitudinal-Transverse)
Separation

Arrington et al., PRC 68, 034325 (2003)

Polarization Transfer

Jones et al., PRL 84, 1398 (2000)

Gayou et al., PRL 88, 092301 (2002)

LT method

$$\sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

→ G_E from slope in ε plot

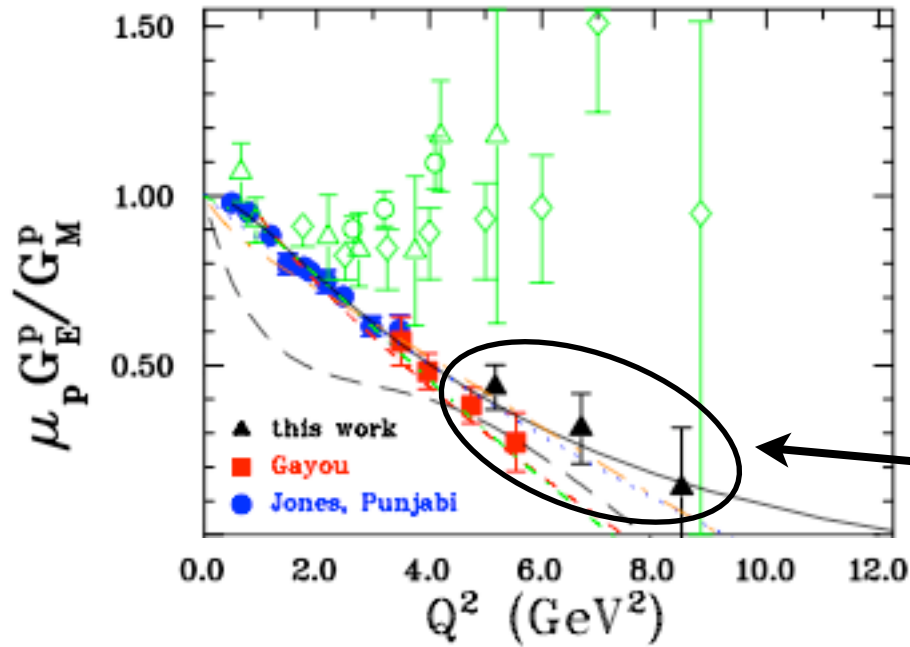
→ suppressed at large Q^2

PT method

$$\frac{G_E}{G_M} = -\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}} \frac{P_T}{P_L}$$

→ $P_{T,L}$ recoil proton
polarization in $\vec{e} p \rightarrow e \vec{p}$

Proton G_E/G_M ratio



Polarization Transfer (latest from JLab)

Puckett et al., PRL 104, 242301 (2010)

LT method

$$\sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

→ G_E from slope in ε plot

→ suppressed at large Q^2

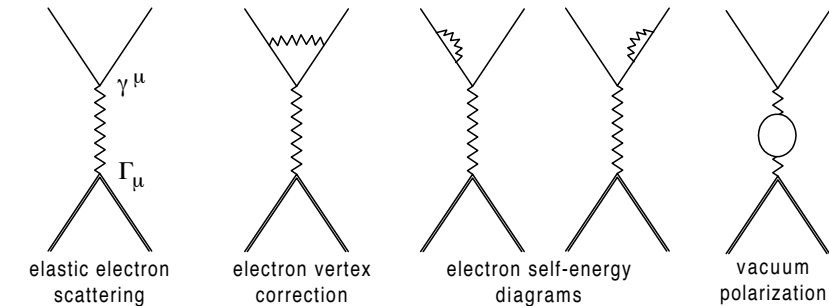
PT method

$$\frac{G_E}{G_M} = -\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}} \frac{P_T}{P_L}$$

→ $P_{T,L}$ recoil proton polarization in $\vec{e} p \rightarrow e \vec{p}$

QED radiative corrections

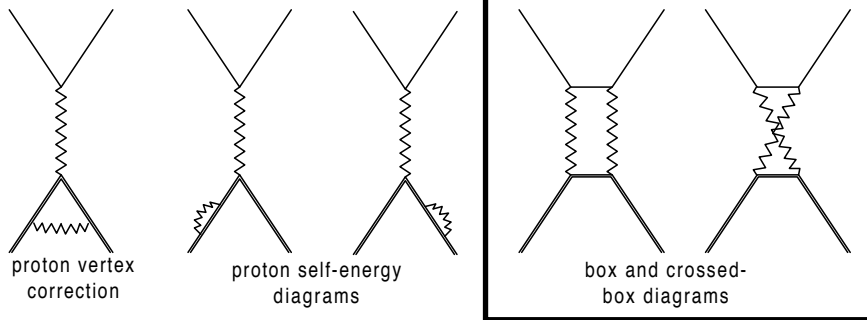
- cross section modified by 1γ loop effects



Born

TPE

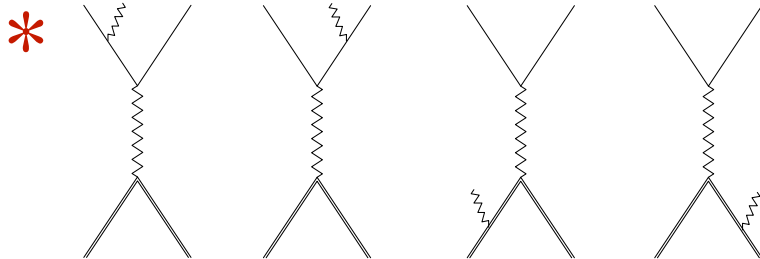
$$d\sigma = d\sigma_0 (1 + \delta)$$



*

δ contains additional ϵ dependence, mostly from box diagrams

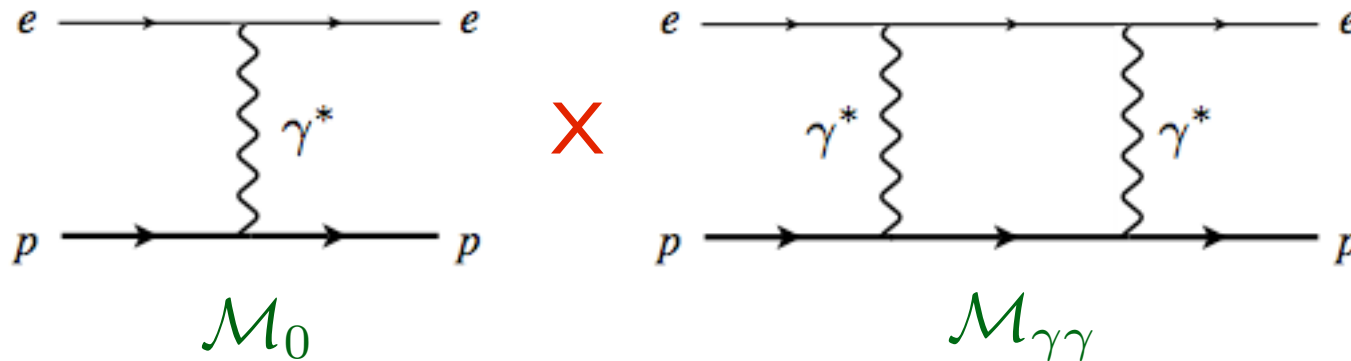
(most difficult to calculate)



* IR divergences cancel

Two-photon exchange

- interference between Born and TPE amplitudes



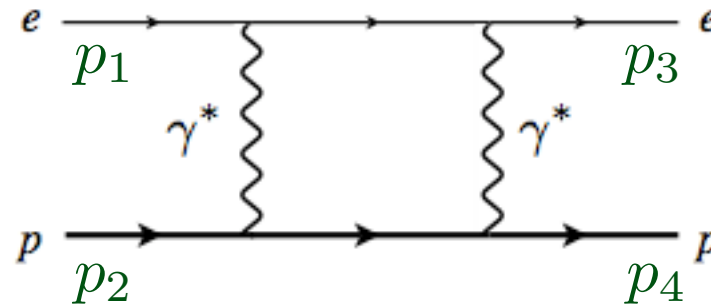
- contribution to cross section:

$$\delta^{(2\gamma)} = \frac{2\text{Re} \left\{ \mathcal{M}_0^\dagger \mathcal{M}_{\gamma\gamma} \right\}}{|\mathcal{M}_0|^2}$$

- “soft photon approximation” (used in all previous data analyses)
 - approximate integrand in $\mathcal{M}_{\gamma\gamma}$ by values at γ^* poles
 - neglect nucleon structure (no form factors)

Mo, Tsai (1969)

Two-photon exchange



$$\mathcal{M}_{\gamma\gamma} = e^4 \int \frac{d^4k}{(2\pi)^4} \frac{N(k)}{D(k)}$$

where

$$N(k) = \bar{u}(p_3) \gamma_\mu (\not{p}_1 - \not{k} + m_e) \gamma_\nu u(p_1) \\ \times \bar{u}(p_4) \Gamma^\mu(q - k) (\not{p}_2 + \not{k} + M) \Gamma^\nu(k) u(p_2)$$

and

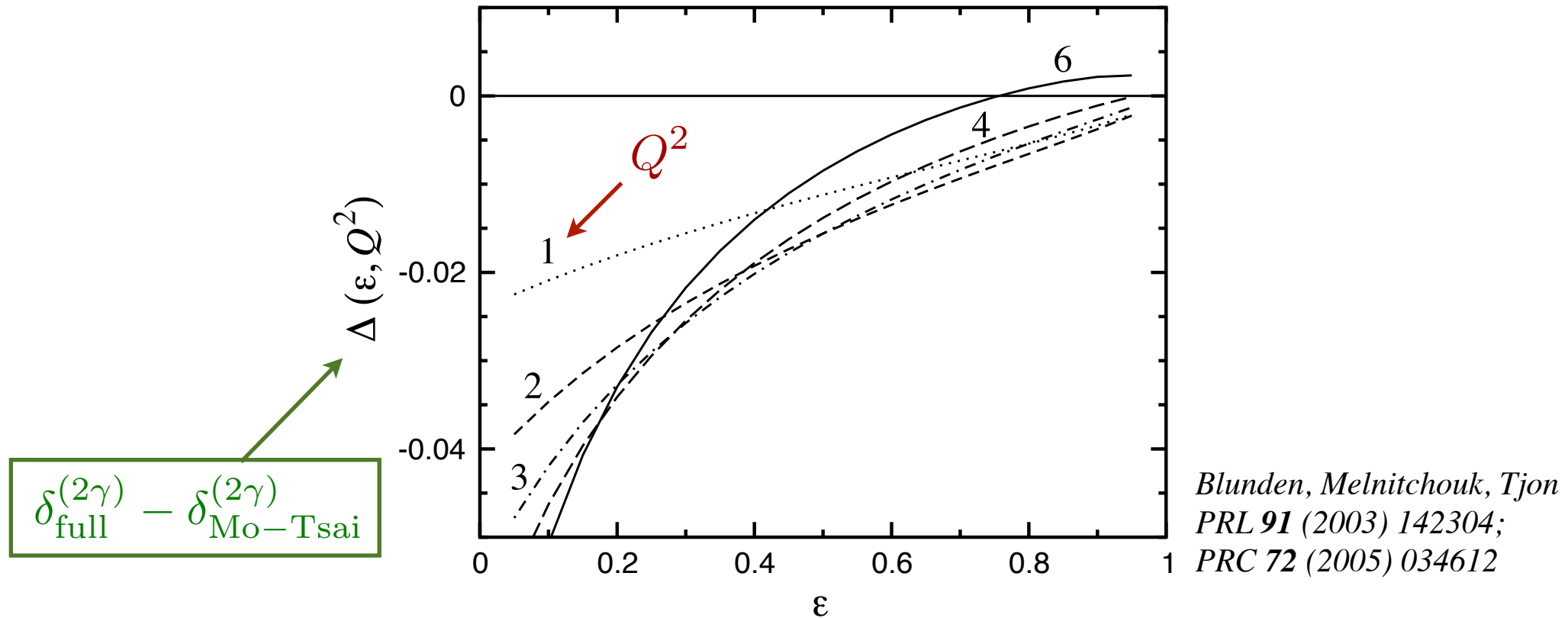
$$D(k) = (k^2 - \lambda^2) ((k - q)^2 - \lambda^2) \\ \times ((p_1 - k)^2 - m^2) ((p_2 + k)^2 - M^2)$$

with λ an IR regulator, and e.m. current is

$$\Gamma^\mu(q) = \gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M} F_2(q^2) \leftarrow \text{on-shell approximation}$$

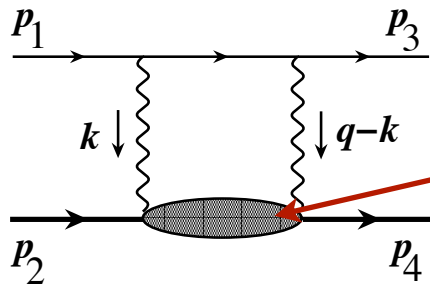
Two-photon exchange

- “exact” calculation of loop diagram (including hadron structure)



- few % magnitude, non-linear in ε , *positive slope*
- *will reduce Rosenbluth ratio*
- does not depend strongly on vertex form factors

Higher-mass intermediate states



$N, \Delta, P_{11}, S_{11}, S_{31}, \dots$

- lowest mass excitation is P_{33} $\Delta(1232)$ resonance

→ relativistic $\gamma^* N \Delta$ vertex

form factor $\frac{\Lambda_\Delta^4}{(\Lambda_\Delta^2 - q^2)^2}$

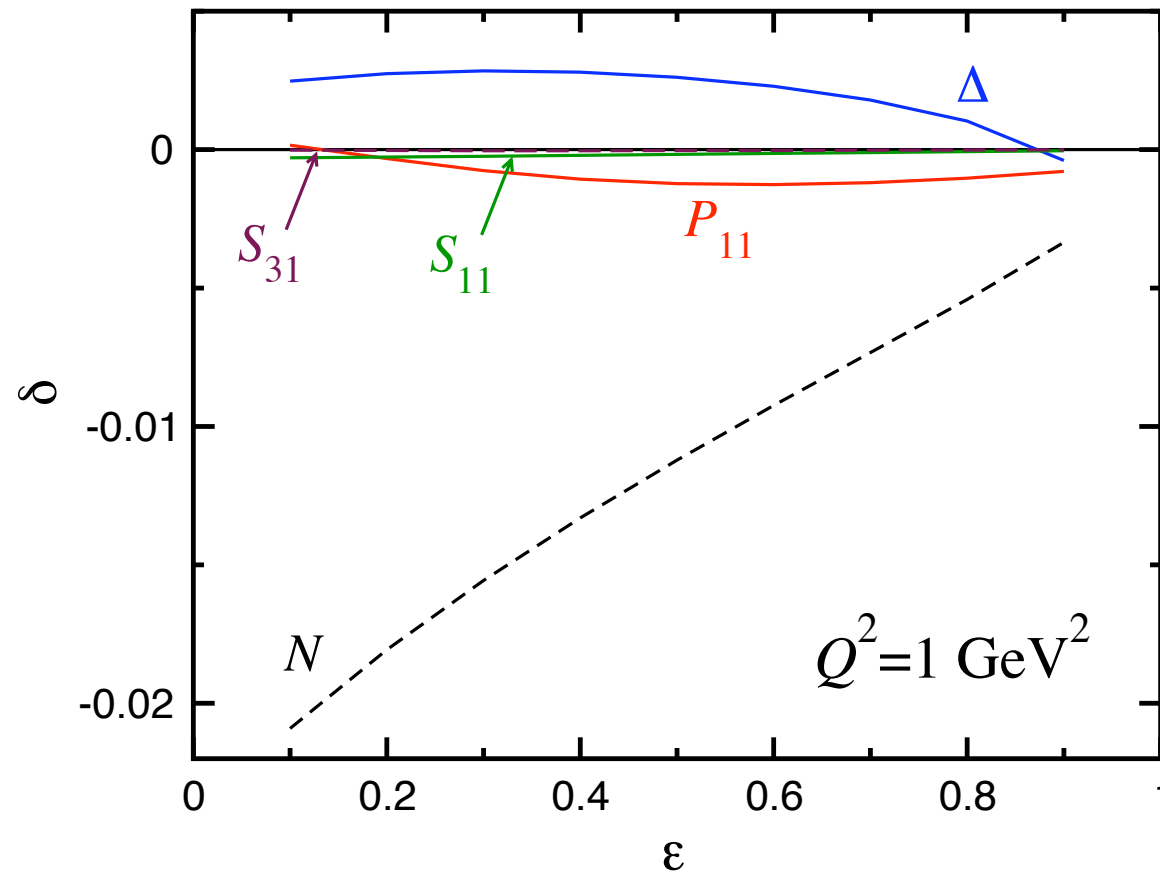
$$\Gamma_{\gamma\Delta \rightarrow N}^{\nu\alpha}(p, q) \equiv iV_{\Delta in}^{\nu\alpha}(p, q) = i \frac{eF_\Delta(q^2)}{2M_\Delta^2} \left\{ g_1 [g^{\nu\alpha} \not{p} \not{q} - p^\nu \gamma^\alpha \not{q} - \gamma^\nu \gamma^\alpha p \cdot q + \gamma^\nu \not{p} q^\alpha] \right. \\ \left. + g_2 [p^\nu q^\alpha - g^{\nu\alpha} p \cdot q] + (g_3/M_\Delta) [q^2 (p^\nu \gamma^\alpha - g^{\nu\alpha} \not{p}) + q^\nu (q^\alpha \not{p} - \gamma^\alpha p \cdot q)] \right\} \gamma_5 T_3$$

→ coupling constants

g_1	magnetic	→	7
$g_2 - g_1$	electric	→	9
g_3	Coulomb	→	-2 ... 0

■ higher-mass intermediate states

→ more model dependent, since couplings & form factors not as well known (especially at high Q^2)



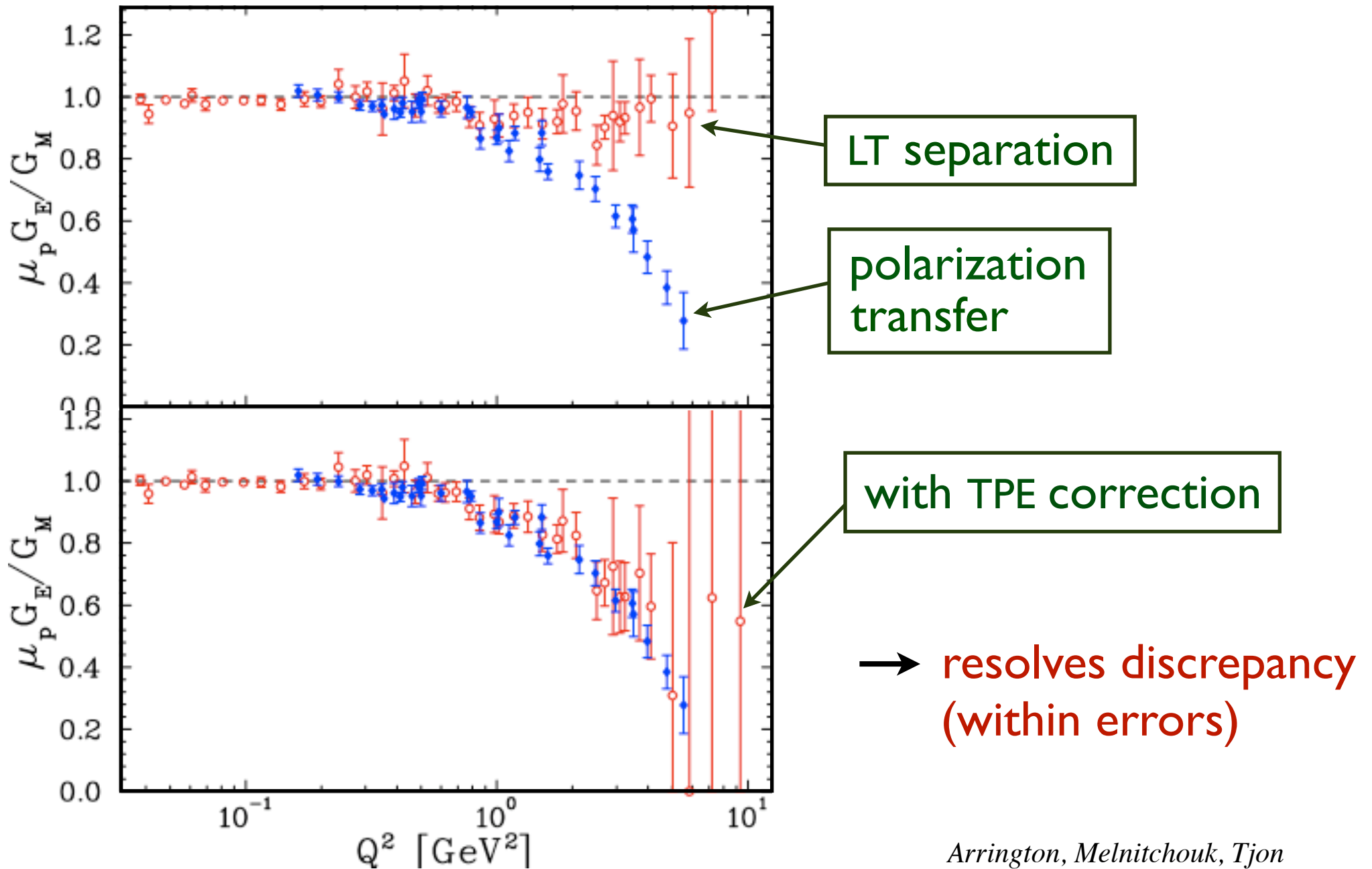
*Kondratyuk, Blunden,
Melnitchouk, Tjon
PRL 95 (2005) 172503*

*Kondratyuk, Blunden
PRC 75 (2007) 038201*

→ dominant contribution from N

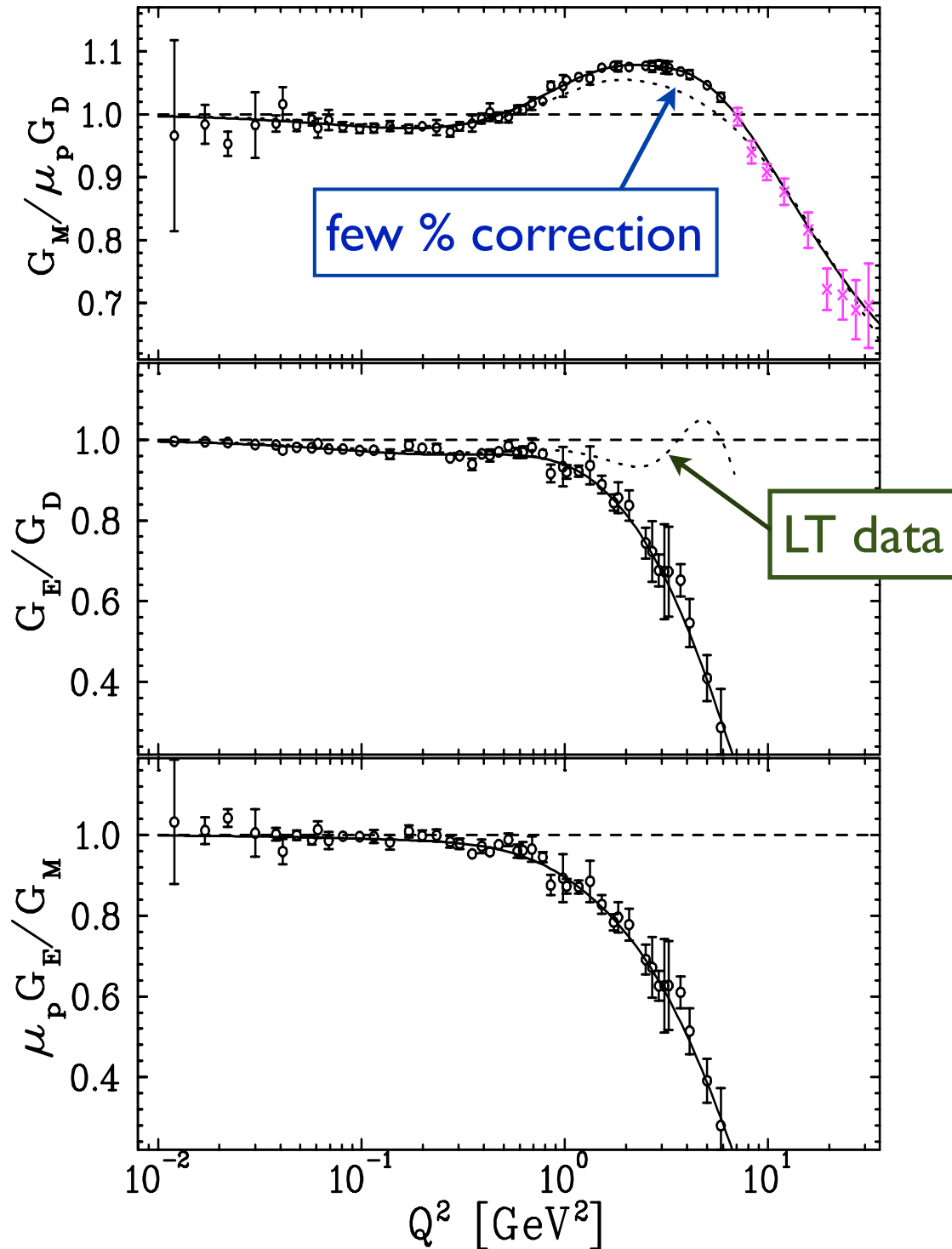
→ Δ partially cancels N contribution

Global analysis



*Arrington, Melnitchouk, Tjon
PRC 76 (2007) 035205*

final form factor results
from global analysis
including TPE corrections



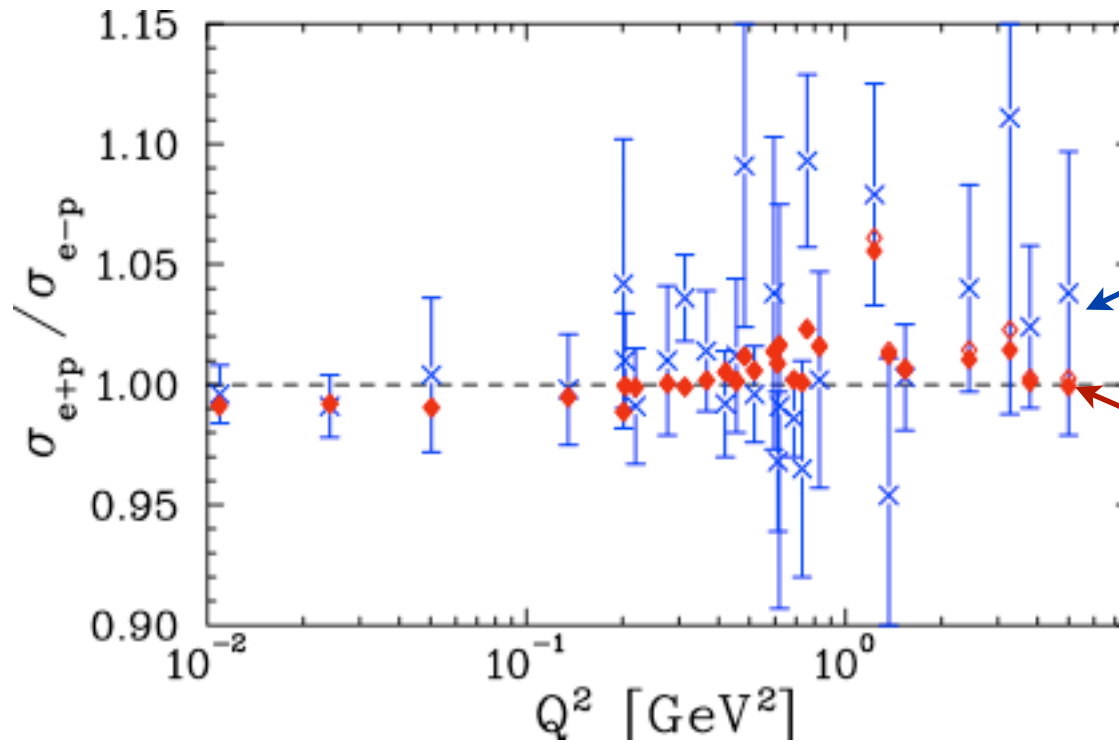
$$\left\{ G_E, \frac{G_M}{\mu_p} \right\} = \frac{1 + \sum_{i=1}^n a_i \tau^i}{1 + \sum_{i=1}^{n+2} b_i \tau^i}$$

Parameter	G_M/μ_p	G_E
a_1	-1.465	3.439
a_2	1.260	-1.602
a_3	0.262	0.068
b_1	9.627	15.055
b_2	0.000	48.061
b_3	0.000	99.304
b_4	11.179	0.012
b_5	13.245	8.650

Arrington, Melnitchouk, Tjon
PRC 76 (2007) 035205

e^+/e^- comparison

- 1γ (2γ) exchange changes sign (invariant) under $e^+ \leftrightarrow e^-$
→ ratio of e^+p/e^-p cross sections sensitive to $\Delta(\varepsilon, Q^2)$



$1 - 2\Delta$

data at various ε

TPE calculation

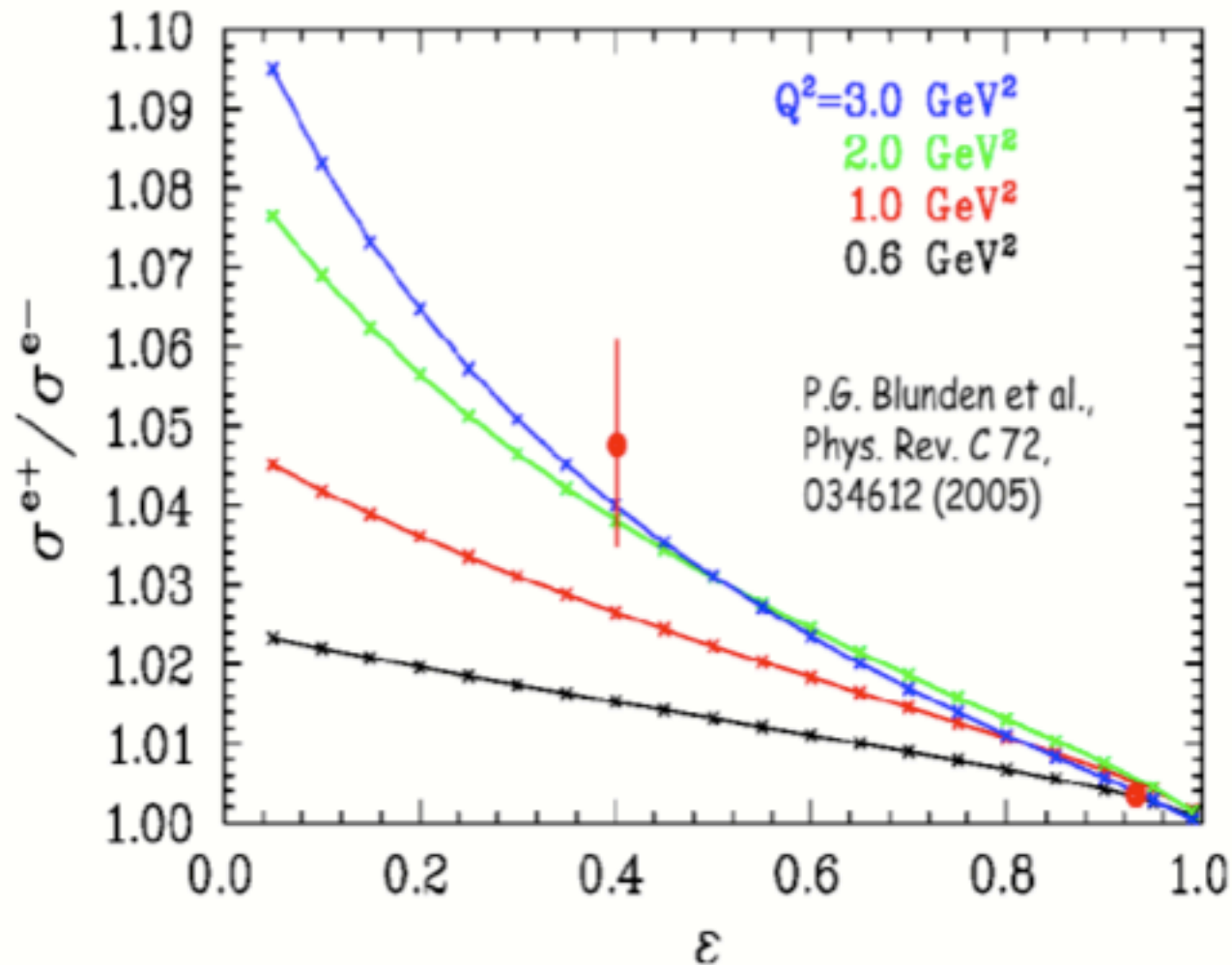
- simultaneous e^+p/e^-p measurement using tertiary e^+/e^- beam to $Q^2 \sim 1-2$ GeV² (Hall B experiment E04-116)

e^+/e^- comparison

- 1γ (2γ) exchange changes sign (invariant) under $e^+ \leftrightarrow e^-$

Very preliminary Novosibirsk data

e^+p/e^-p cross section ratio



Arrington, Holt *et al.* (2010)

e^+/e^- comparison

- 1γ (2γ) exchange changes sign (invariant) under $e^+ \leftrightarrow e^-$
 - strong indication of *inadequacy* of one-photon exchange approximation in ep scattering
 - significant role played by *hadron structure dependent* two-photon exchange corrections

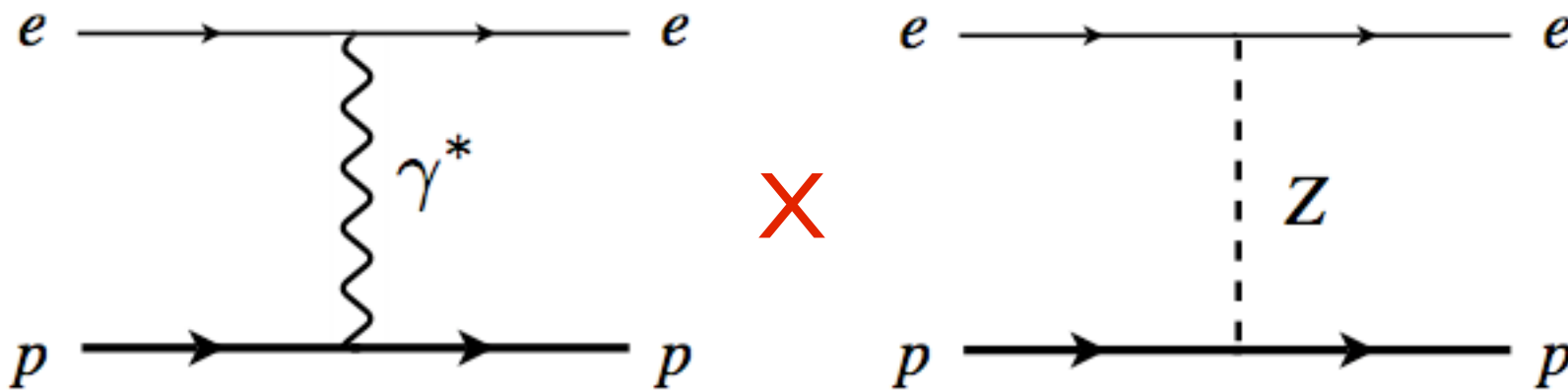
Parity-violating electron scattering

Parity-violating e scattering

- Left-right polarization asymmetry in $\vec{e} p \rightarrow e p$ scattering

$$A_{\text{PV}} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = - \left(\frac{G_F Q^2}{4\sqrt{2}\alpha} \right) (A_V + A_A + A_S)$$

→ measure interference between e.m. and weak currents



Born (tree) level

Parity-violating e scattering

- Left-right polarization asymmetry in $\vec{e} p \rightarrow e p$ scattering

$$A_{\text{PV}} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = - \left(\frac{G_F Q^2}{4\sqrt{2}\alpha} \right) (A_V + A_A + A_s)$$

→ measure interference between e.m. and weak currents

$$A_V = g_A^e \rho \left[(1 - 4\kappa \sin^2 \theta_W) - (\varepsilon G_E^{\gamma p} G_E^{\gamma n} + \tau G_M^{\gamma p} G_M^{\gamma n}) / \sigma^{\gamma p} \right]$$

radiative corrections,
including TBE

→ using relations between weak and e.m. form factors

$$G_{E,M}^{Zp} = (1 - 4 \sin^2 \theta_W) G_{E,M}^{\gamma p} - G_{E,M}^{\gamma n} - G_{E,M}^s$$

Parity-violating e scattering

- Left-right polarization asymmetry in $\vec{e} p \rightarrow e p$ scattering

$$A_{\text{PV}} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = - \left(\frac{G_F Q^2}{4\sqrt{2}\alpha} \right) (A_V + A_A + A_s)$$

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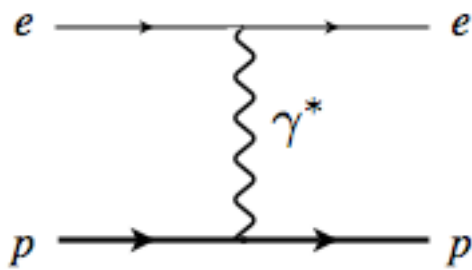
$$A_A = g_V^e \sqrt{\tau(1+\tau)(1-\varepsilon^2)} \tilde{G}_A^{Zp} G_M^{\gamma p} / \sigma^{\gamma p}$$

includes axial RCs + anapole term

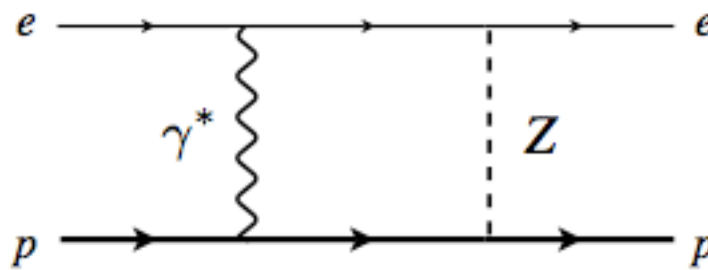
$$A_s = -g_A^e \rho (\varepsilon G_E^{\gamma p} G_E^s + \tau G_M^{\gamma p} G_M^s) / \sigma^{\gamma p}$$

strange electric &
magnetic form factors

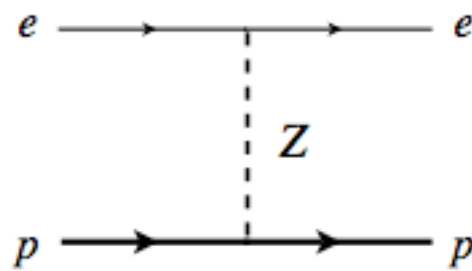
Two-boson exchange corrections



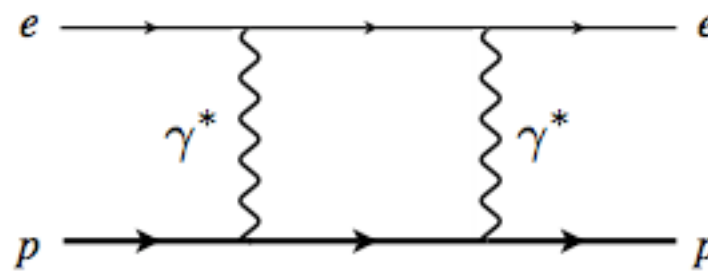
X



“ $\gamma(Z\gamma)$ ”



X



“ $Z(\gamma\gamma)$ ”

- current PDG estimates computed at $Q^2 = 0$

Marciano, Sirlin (1980)

Erlar, Ramsey-Musolf (2003)

- do not include hadron structure effects

Two-boson exchange corrections

- parameterize corrections to asymmetry as

$$A_{PV} = (1 + \delta) A_{PV}^0 \equiv \left(\frac{1 + \delta_{Z(\gamma\gamma)} + \delta_{\gamma(Z\gamma)}}{1 + \delta_{\gamma(\gamma\gamma)}} \right) A_{PV}^0$$

Born asymmetry



$$\delta_{Z(\gamma\gamma)} = \frac{2\Re(\mathcal{M}_Z^* \mathcal{M}_{\gamma\gamma})}{2\Re(\mathcal{M}_Z^* \mathcal{M}_\gamma)}$$

$$\delta_{\gamma(Z\gamma)} = \frac{2\Re(\mathcal{M}_\gamma^* \mathcal{M}_{\gamma Z} + \mathcal{M}_\gamma^* \mathcal{M}_{Z\gamma})}{2\Re(\mathcal{M}_Z^* \mathcal{M}_\gamma)}$$

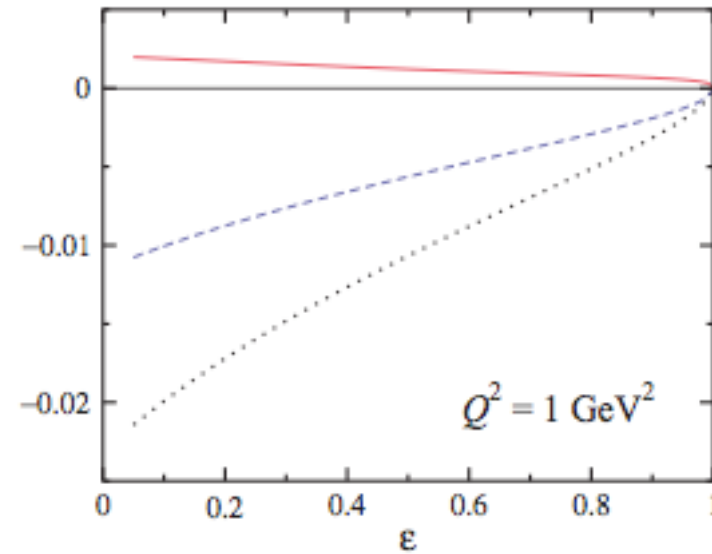
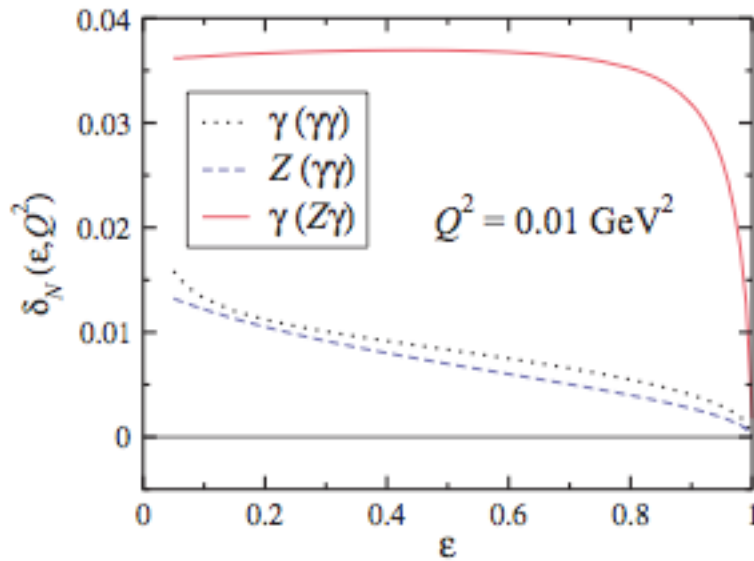
$$\delta_{\gamma(\gamma\gamma)} = \frac{2\Re(\mathcal{M}_\gamma^* \mathcal{M}_{\gamma\gamma})}{|\mathcal{M}_\gamma|^2}$$

→ total TBE correction

$$\delta \approx \delta_{Z(\gamma\gamma)} + \delta_{\gamma(Z\gamma)} - \delta_{\gamma(\gamma\gamma)}$$

Two-boson exchange corrections

■ nucleon intermediate states



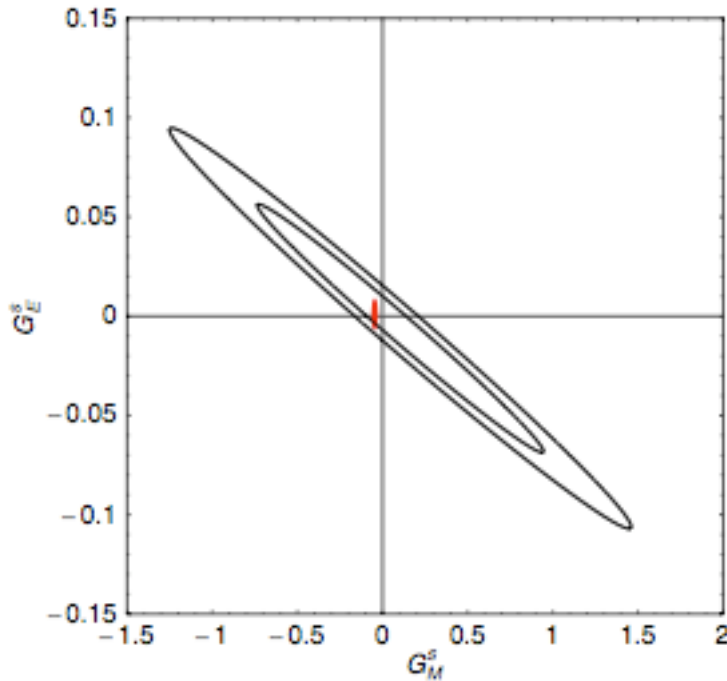
Tjon, Melnitchouk, PRL 100 (2008) 082003

Tjon, Blunden, Melnitchouk, PRC 79 (2009) 055201

- cancellation between $Z(\gamma\gamma)$ and $\gamma(\gamma\gamma)$ corrections, especially at low Q^2
- dominated by $\gamma(Z\gamma)$ contribution

Effects on strange form factors

- global analysis of all PVES data at $Q^2 < 0.3 \text{ GeV}^2$



$$G_E^s = 0.0025 \pm 0.0182$$

$$G_M^s = -0.011 \pm 0.254$$

$$\text{at } Q^2 = 0.1 \text{ GeV}^2$$

Young et al., PRL 97 (2006) 102002

- including TBE corrections:

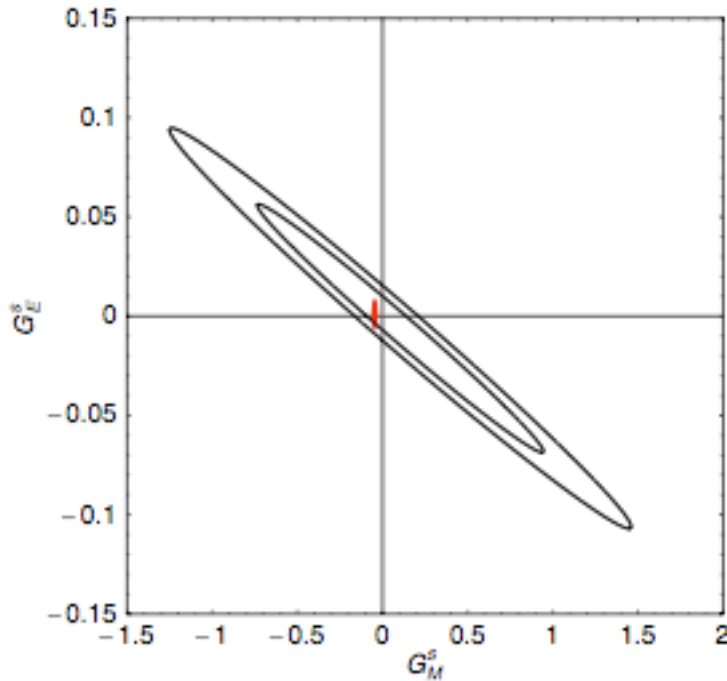
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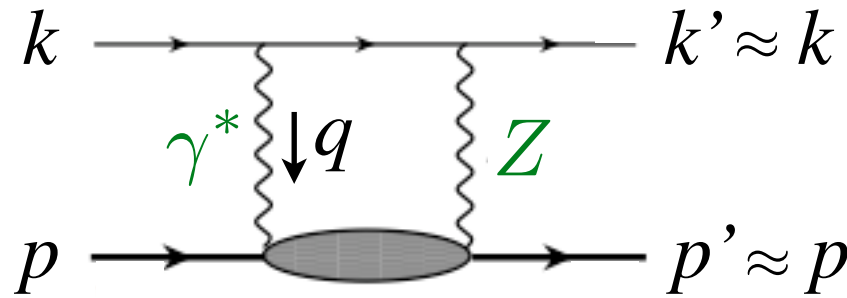
at $Q^2 = 0.1 \text{ GeV}^2$

fixed mainly by ^4He data ...
... TBE for ^4He not yet included

Correction to proton weak charge

- in forward limit A_{PV} measures weak charge of proton Q_W^p

$$A_{PV} \rightarrow \frac{G_F Q_W^p}{4\sqrt{2}\pi\alpha} t$$



forward limit

$$t = (k - k')^2 \rightarrow 0$$


$$s = (k + p)^2 = M(M + 2E)$$

- at tree level Q_W^p gives weak mixing angle

$$Q_W^p = 1 - 4 \sin^2 \theta_W$$

Correction to proton weak charge

- including higher order radiative corrections

$$Q_W^p = (1 + \Delta\rho + \Delta_e)(1 - 4 \sin^2 \theta_W(0) + \Delta'_e)$$


“standard” electroweak vertex & other corrections

Correction to proton weak charge

- including higher order radiative corrections

$$Q_W^p = (1 + \Delta\rho + \Delta_e)(1 - 4 \sin^2 \theta_W(0) + \Delta'_e) + \square_{WW} + \square_{ZZ} + \square_{\gamma Z} \leftarrow \text{box diagrams}$$
$$= 0.0713 \pm 0.0008$$

Erler et al., PRD 72, 073003 (2005)

→ WW and ZZ box diagrams dominated by short distances, evaluated perturbatively

→ γZ box diagram sensitive to long distance physics, has two contributions

$$\square_{\gamma Z} = \square_{\gamma Z}^A + \square_{\gamma Z}^V$$

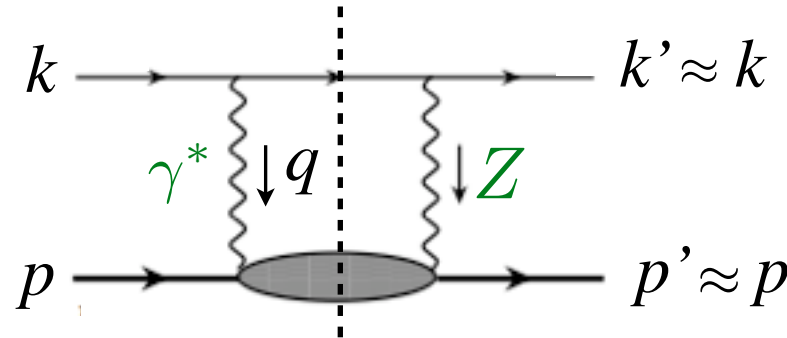
vector e - axial h
(finite at $E=0$)

axial e - vector h
(vanishes at $E=0$)

Vector h correction

- what is energy dependence of vector h correction $\square_{\gamma Z}^V$?

→ computed in forward limit using dispersion relations



Gorchtein, Horowitz
PRL 102, 091806 (2009)

- ★ $\Re \square_{\gamma Z}^V(E) = \frac{2E}{\pi} \int_0^\infty dE' \frac{1}{E'^2 - E^2} \Im \square_{\gamma Z}^V(E')$
- ★ integration over $E' < 0$ corresponds to crossed-box, vector h contribution symmetric under $E' \leftrightarrow -E'$
- ★ vanishes as $E \rightarrow 0$ (e.g. atomic parity violation)
but what about at $\mathcal{O}(1 \text{ GeV})$ of $Qweak$ experiment?

Vector h correction

→ imaginary part given by γZ interference structure functions

$$\Im \square_{\gamma Z}^V(E) = \frac{\alpha}{(s - M^2)^2} \int_{W_\pi^2}^s dW^2 \int_0^{Q_{\max}^2} \frac{dQ^2}{1 + Q^2/M_Z^2} \\ \times \left(F_1^{\gamma Z} + F_2^{\gamma Z} \frac{s(Q_{\max}^2 - Q^2)}{Q^2(W^2 - M^2 + Q^2)} \right)$$

★ little direct data on interference structure functions
(neutral currents at HERA at very small x)

★ in parton model $F_2^{\gamma Z} = 2x \sum_q e_q g_V^q (q + \bar{q}) = 2x F_1^{\gamma Z}$

→ $F_2^{\gamma Z} \approx F_2^\gamma$ good approximation at *low* x

→ provides upper limit at *large* x ($F_2^{\gamma Z} \lesssim F_2^\gamma$)

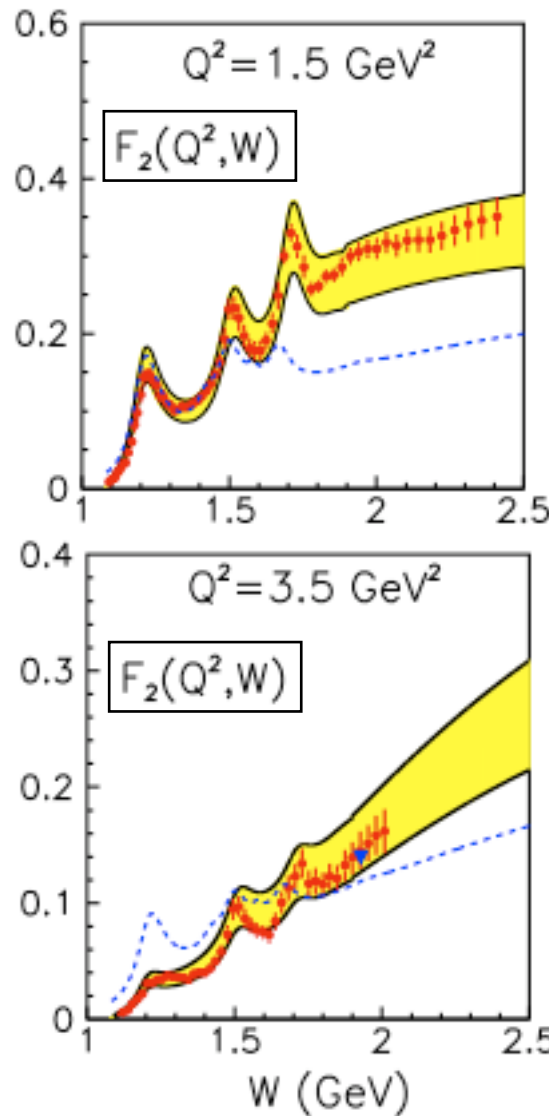
Vector h correction

- in resonance region use phenomenological input for F_2 , empirical SLAC fit for $R = \sigma_L/\sigma_T = (1 + 4M^2x^2/Q^2)F_2/(2xF_1) - 1$
 - for transitions to $I = 3/2$ states (e.g. Δ), CVC and isospin symmetry give $F_i^{\gamma Z} = (1 + Q_W^p) F_i^\gamma$
 - for transitions to $I = 1/2$ states, SU(6) wave functions predict Z & γ transition couplings equal to few percent
 - include contributions from prominent resonances:
 $P_{33}(1232)$, $D_{13}(1520)$, $F_{15}(1680)$, $F_{37}(1950)$

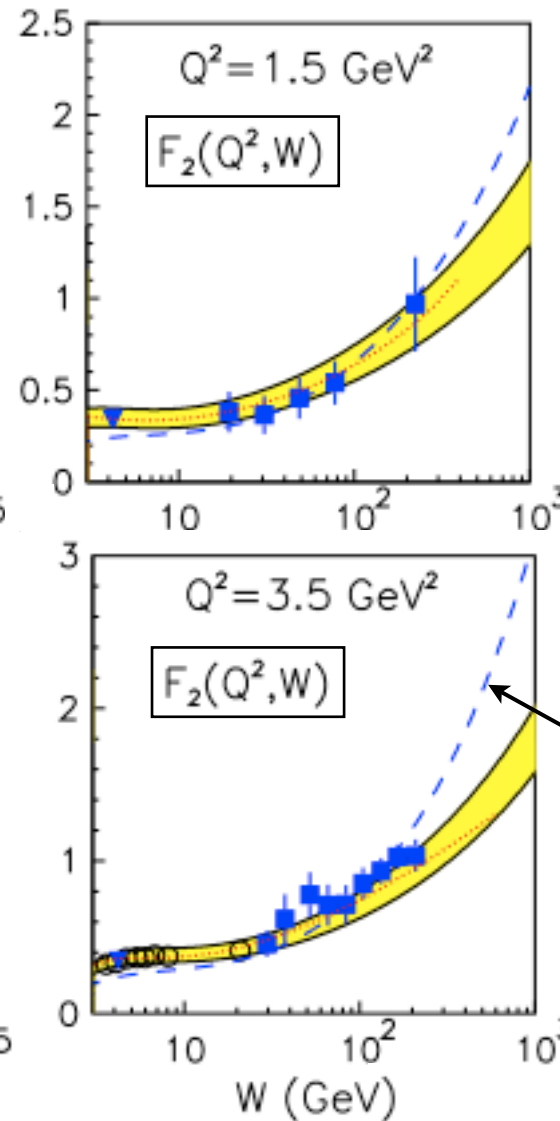
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low W



high W



GVMD model
(used as input by
Gorchtein & Horowitz)

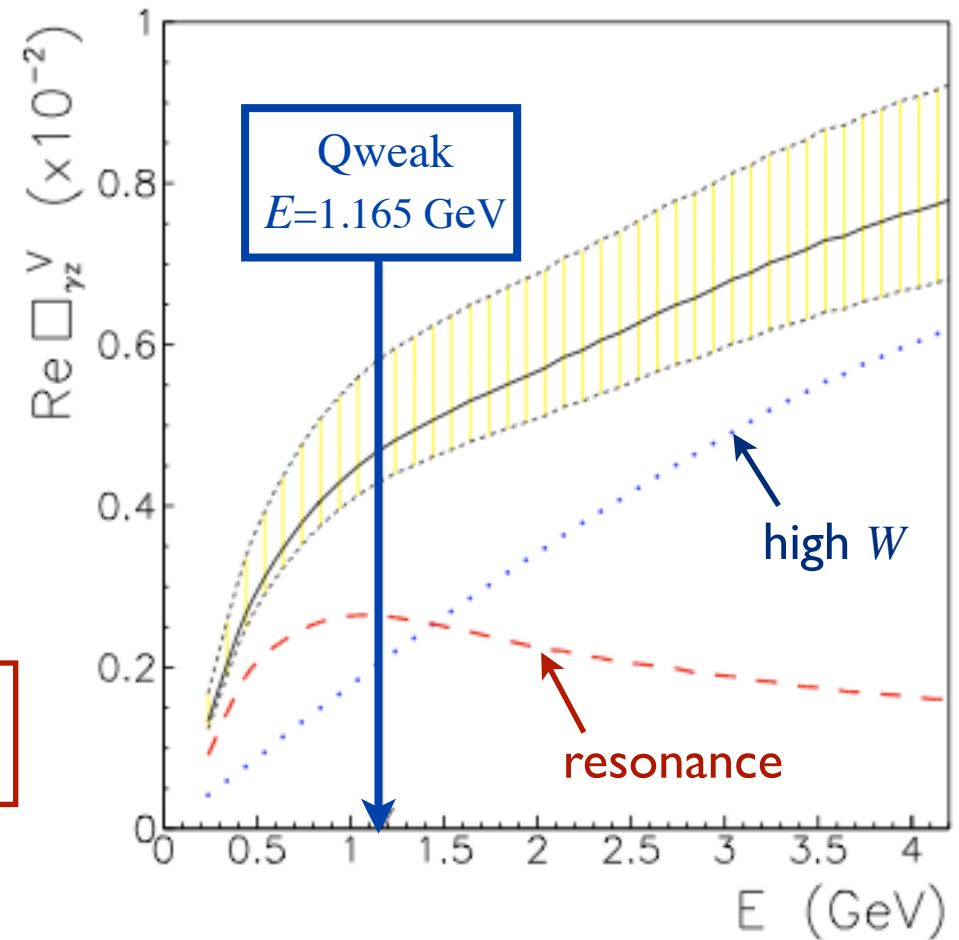
Vector h correction

■ total $\square_{\gamma Z}^V$ correction:

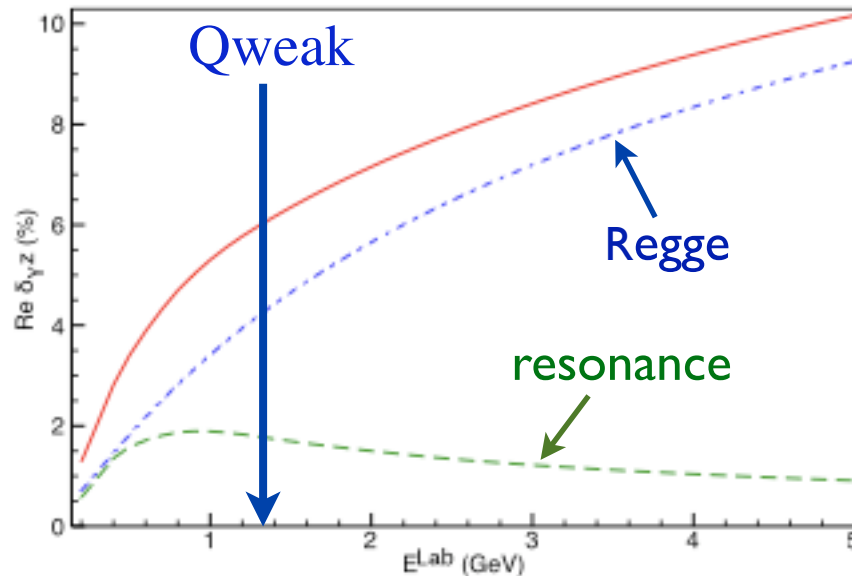
$$\Re \square_{\gamma Z}^V = 0.0047^{+0.0011}_{-0.0004}$$

or $6.6^{+1.5}_{-0.6}$ % of uncorrected Q_W^p

$$Q_W^p = 0.0713(8) \rightarrow 0.0760^{+0.0014}_{-0.0009}$$



→ significant shift in central value, errors within projected experimental uncertainty $\Delta Q_W^p = \pm 0.003$



$$\Re \delta_{\gamma Z} = \Re \square_{\gamma Z}^V / Q_W^p \approx 6\%$$

mostly from high- W
("Regge") contribution

- our formula for $\Im m \square_{\gamma Z}^V$ factor 2 larger
(incorrect definition of parton model structure functions:
"nuclear physics" vs. "particle physics" weak charges!)
- GH omit factor $(1-x)$ in definition of $F_{1,2}$
(spurious $\sim 30\%$ enhancement)
- GH use $Q_W^p \sim 0.05$ cf. ~ 0.07
(spurious $\sim 40\%$ enhancement)
- numerical agreement purely coincidental!

Axial h correction

- axial h correction $\square_{\gamma Z}^A$ dominant γZ correction in atomic parity violation at very low (zero) energy

→ computed by Marciano & Sirlin as sum of two parts:

- ★ low-energy part approximated by *Born* contribution (elastic intermediate state)
- ★ high-energy part (above scale $\Lambda \sim 1$ GeV) computed in terms of scattering from *free quarks*

$$\square_{\gamma Z}^A = \frac{5\alpha}{2\pi} (1 - 4 \sin^2 \theta_W) \left[\ln \frac{M_Z^2}{\Lambda^2} + C_{\gamma Z}(\Lambda) \right]$$

≈ 0.0028

short-distance

long-distance

Marciano, Sirlin, PRD 29, 75 (1984)

Erlar et al., PRD 68, 016006 (2003)

Axial h correction

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- ★ repeat calculation for realistic (structure function) input

Axial h correction

→ imaginary part given by interference $F_3^{\gamma Z}$ structure function

$$\Im \Pi_{\gamma Z}^A(E) = \frac{\alpha}{(s - M^2)^2} \int_{W_\pi^2}^s dW^2 \int_0^{Q_{\max}^2} \frac{dQ^2}{1 + Q^2/M_Z^2} \\ \times \frac{g_V^e}{2g_A^e} \left(\frac{4ME}{W^2 - M^2 + Q^2} - 1 \right) F_3^{\gamma Z}$$

with $g_A^e = -\frac{1}{2}$, $g_V^e = -\frac{1}{2}(1 - 4 \sin^2 \theta_W)$

★ axial h contribution *antisymmetric* under $E' \leftrightarrow -E'$:

$$\Re \Pi_{\gamma Z}^A(E) = \frac{2}{\pi} \int_0^\infty dE' \frac{E'}{E'^2 - E^2} \Im \Pi_{\gamma Z}^A(E')$$

★ imaginary part can only grow as $\log E' / E'$

Axial h correction

■ $F_3^{\gamma Z}$ structure function

- ★ elastic part given by $G_M^p G_A^Z$
- ★ resonance part from parametrization of ν scattering data (Lalakulich-Paschos)
- ★ DIS part dominated by leading twist PDFs at small x (MSTW, CTEQ, Alekhin)

■ real part of $\square_{\gamma Z}^A$ from dispersion relation

$$\Re \square_{\gamma Z}^A(0) = \underset{\substack{\uparrow \\ \text{elastic}}}{0.0006} + \underset{\substack{\uparrow \\ \text{resonance}}}{0.0002} + \underset{\substack{\uparrow \\ \text{DIS}}}{0.0025} = 0.0033$$

→ additional + 0.7% correction

$$Q_W^p = 0.0760 \rightarrow 0.0765$$

Blunden et al. (2010)

Summary

- Two-photon exchange corrections resolve most of Rosenbluth / polarization transfer G_E^p / G_M^p discrepancy
 - striking demonstration of limitation of one-photon exchange approximation in ep scattering
- Dramatic effect of $\gamma(Z\gamma)$ corrections at forward angles on proton weak charge, $\Delta Q_W^p \sim 6-7\%$
 - would shift extracted weak angle by $\Delta \sin^2 \theta_W \approx 0.0013$
 - will be better constrained by direct measurement of $F_{1,2,3}^{\gamma Z}$ (e.g. in PVDIS at JLab)

The End