International Conference on Baryons
Osaka, Japan
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Quark-hadron duality in structure functions

– *recent developments* –

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Outline

- Historical perspective

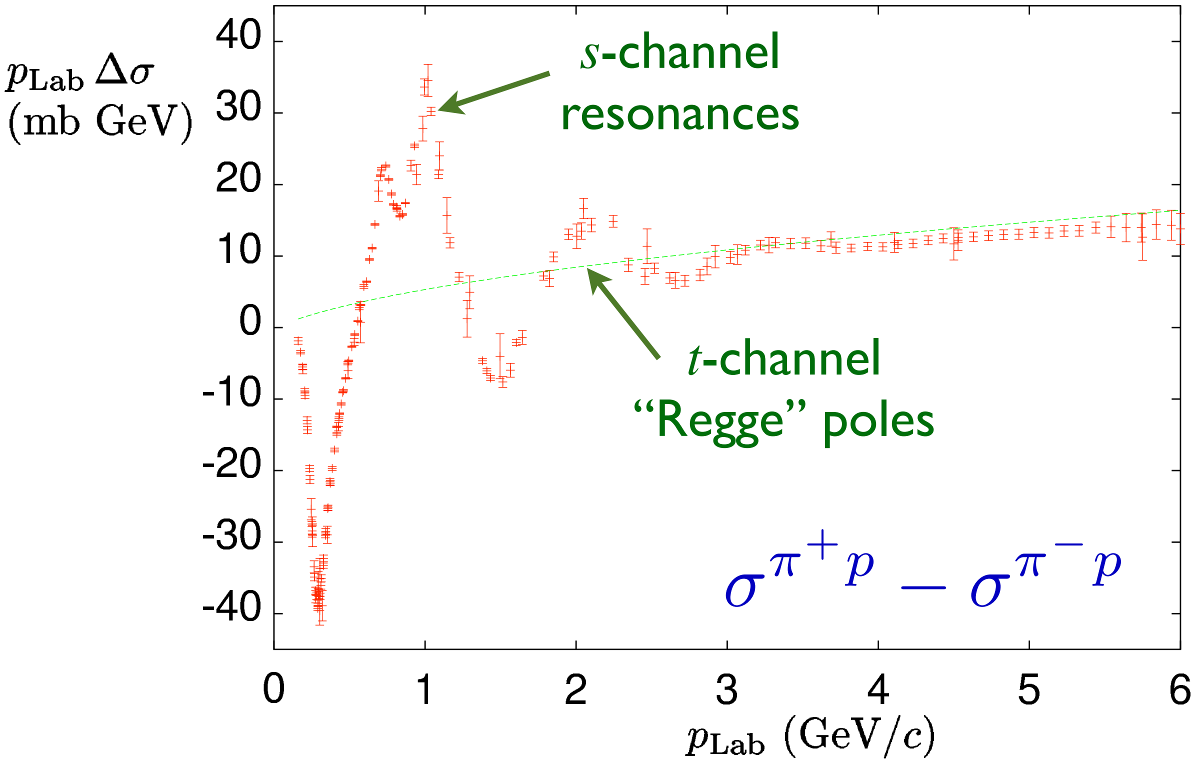
- Modern perspective
  - → twists and truncated moments
  - → insights from models

- Implications for global PDF analysis

- Outlook

Historical perspective

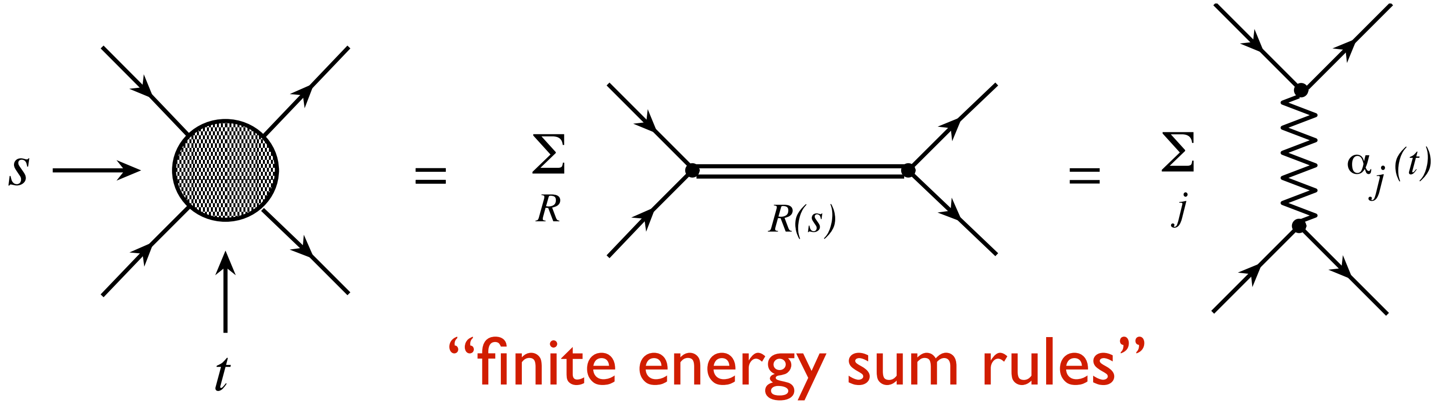
Duality in hadron-hadron scattering



s-channel resonances

t-channel “Regge” poles

$\sigma^{\pi^+p} - \sigma^{\pi^-p}$

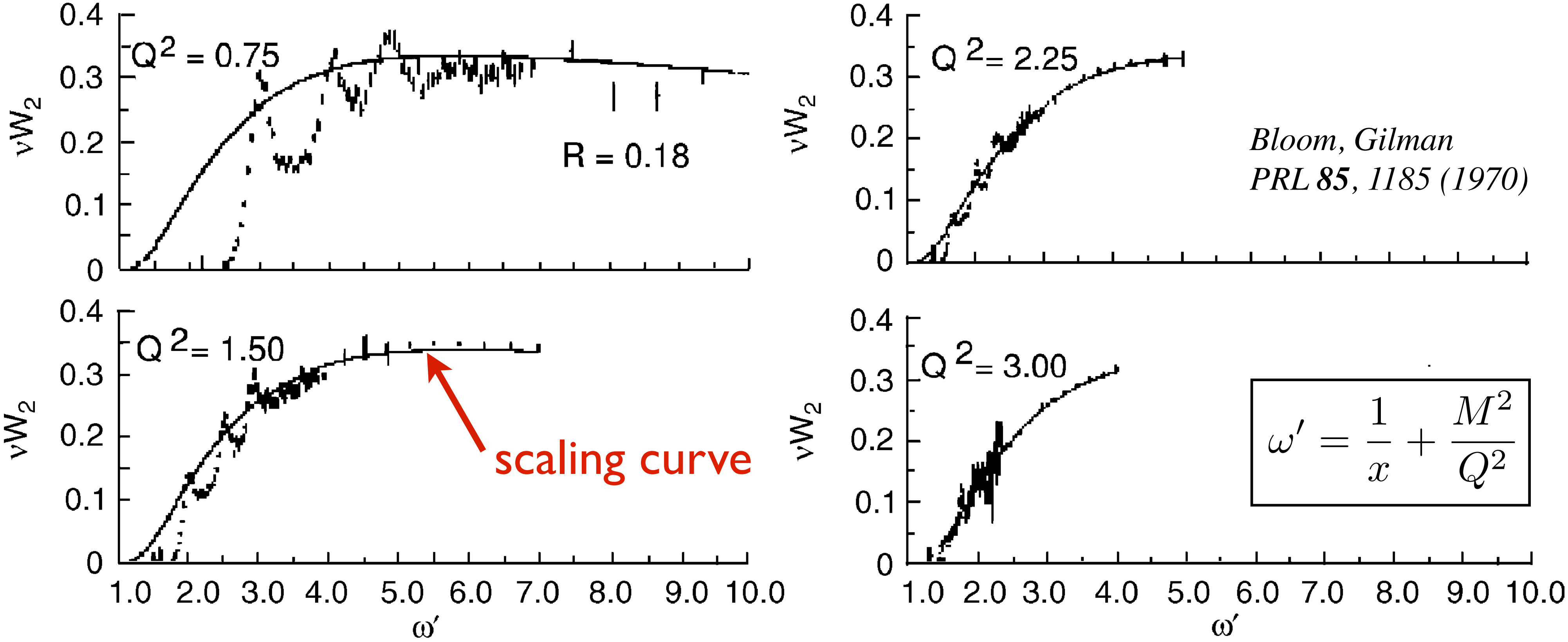


“finite energy sum rules”

Igi (1962), Dolen, Horn, Schmidt (1968)

Duality in electron-hadron scattering

“Bloom-Gilman duality”



scaling curve

Bloom, Gilman
PRL ***85****, 1185 (1970)*

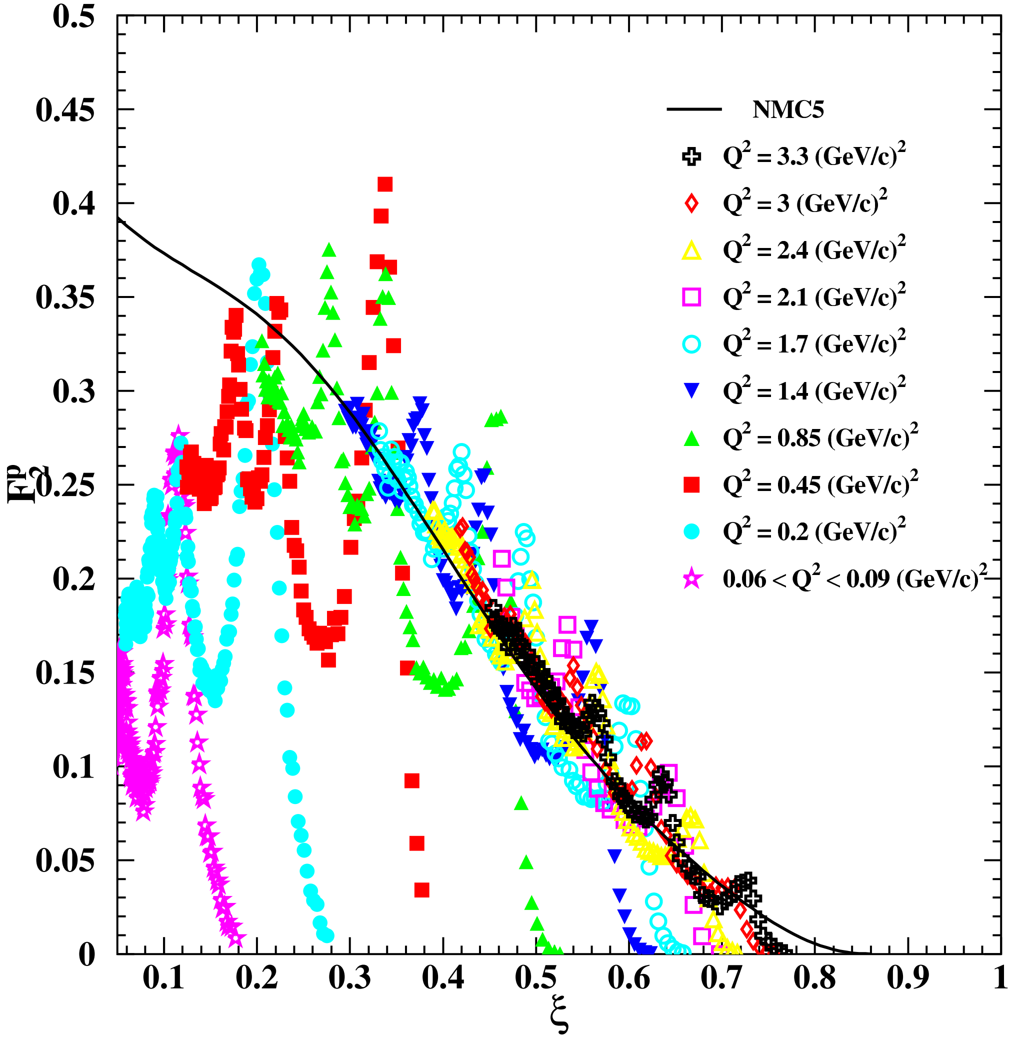
$$\boxed{\omega' = \frac{1}{x} + \frac{M^2}{Q^2}}$$

⟶ finite energy sum rule for eN scattering

$$\frac{2M}{Q^2}\int_0^{\nu_m} d\nu\ \nu W_2(\nu, Q^2) = \int_1^{\omega'_m} d\omega'\ \nu W_2(\omega')$$

“hadrons” “quarks”

Duality in electron-hadron scattering



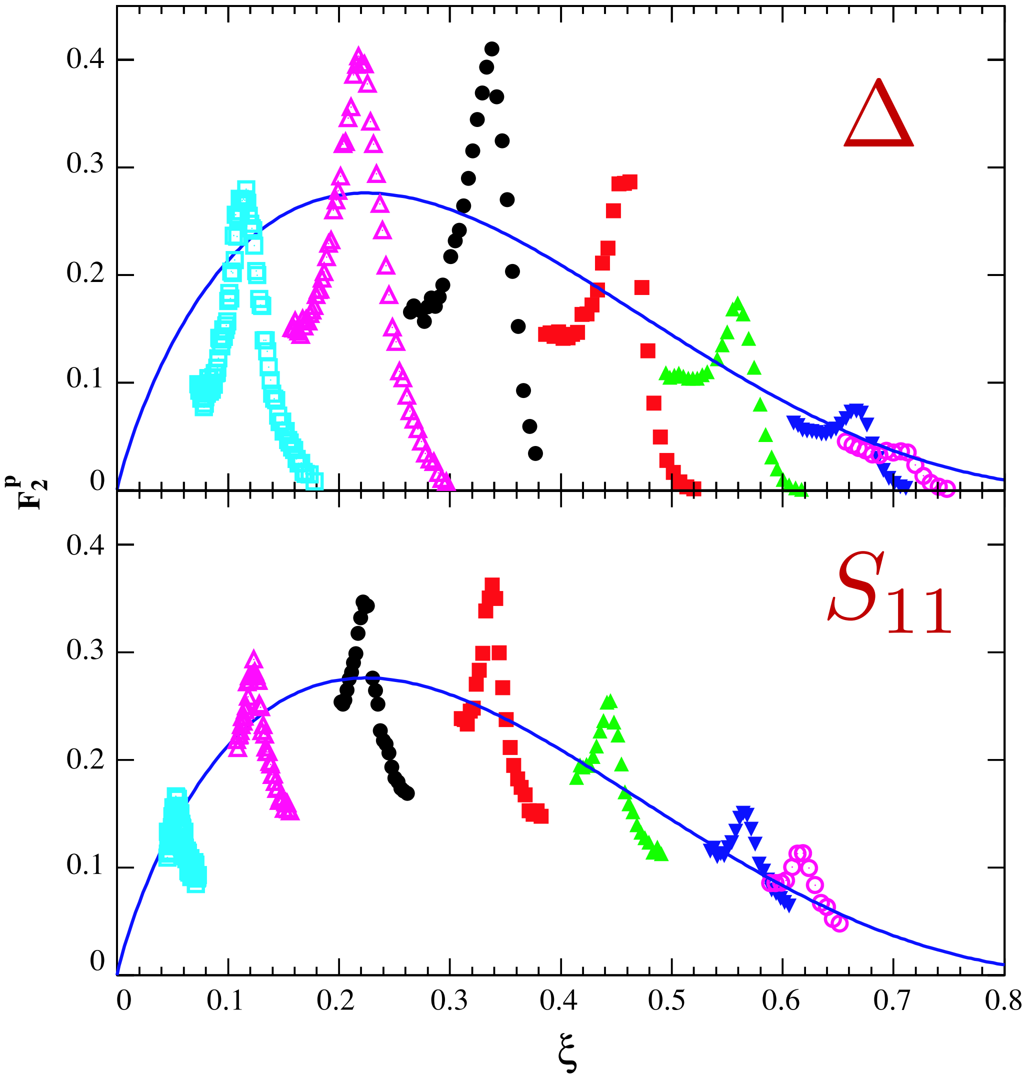
Niculescu et al., PRL ***85****, 1182 (2000)*

average over
(strongly Q^2 dependent)
resonances
$\approx Q^2$ independent
scaling function

"Nachtmann" scaling variable

$$\xi = \frac{2x}{1+\sqrt{1+4M^2x^2/Q^2}}$$

Duality in electron-hadron scattering



→ also exists *locally* in individual resonance regions

Duality in QCD era

- Operator product expansion

  → expand *moments* of structure functions in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx\ x^{n-2}\ F_2(x, Q^2)$$

$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

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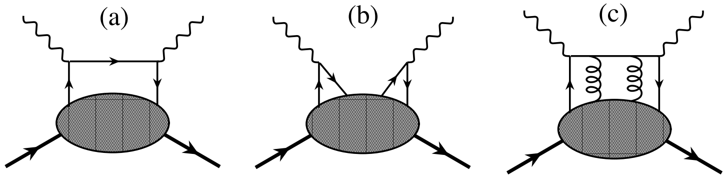
de Rujula, Georgi, Politzer
Ann. Phys. ***103****, 315 (1975)*

matrix elements of operators with specific "twist" τ

$$\tau = \text{dimension} - \text{spin}$$



$\tau = 2$ $\tau > 2$

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Ann. Phys. ***103****, 315 (1975)*

- If moment $\approx$ independent of Q^2

  → higher twist terms $A_n^{(\tau>2)}$ small

- Duality ⟷ suppression of higher twists

Modern perspective:
truncated moments

Truncated moments

- Seldom have sufficient data to form complete moments

→ usually require $x \to 0$ and $x \to 1$ extrapolations

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  ⟶ usually require $x \to 0$ and $x \to 1$ extrapolations

- *Truncated* moments allow study of restricted regions in x (or W) within pQCD in well-defined, systematic way

$$\overline{M}_n(\Delta x, Q^2) = \int_{\Delta x} dx\ x^{n-2}\ F_2(x, Q^2)$$

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$$\overline{M}_n(\Delta x, Q^2) = \int_{\Delta x} dx\ x^{n-2}\ F_2(x, Q^2)$$

- Obey DGLAP-like evolution equations, similar to PDFs

$$\frac{d\overline{M}_n(\Delta x, Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi}\left(P'_{(n)} \otimes \overline{M}_n\right)(\Delta x, Q^2)$$

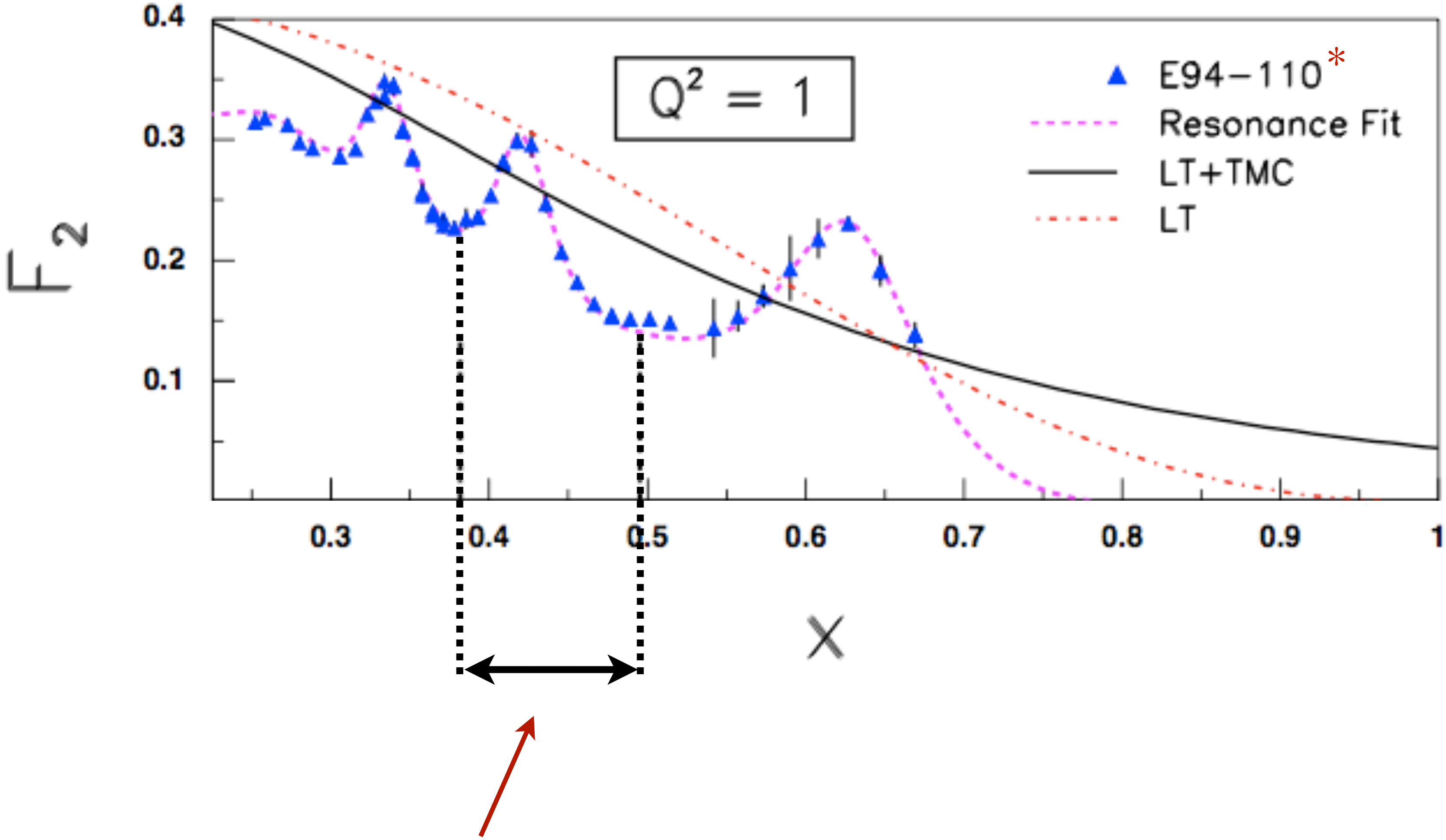
$$P'_{(n)}(z, \alpha_s) = z^n\ P_{NS,S}(z, \alpha_s)$$

truncated splitting function

Forte, Magnea, PLB ***448****, 295 (1999)*
Kotlorz, Kotlorz, PLB ***644****, 284 (2007)*

Truncated moments

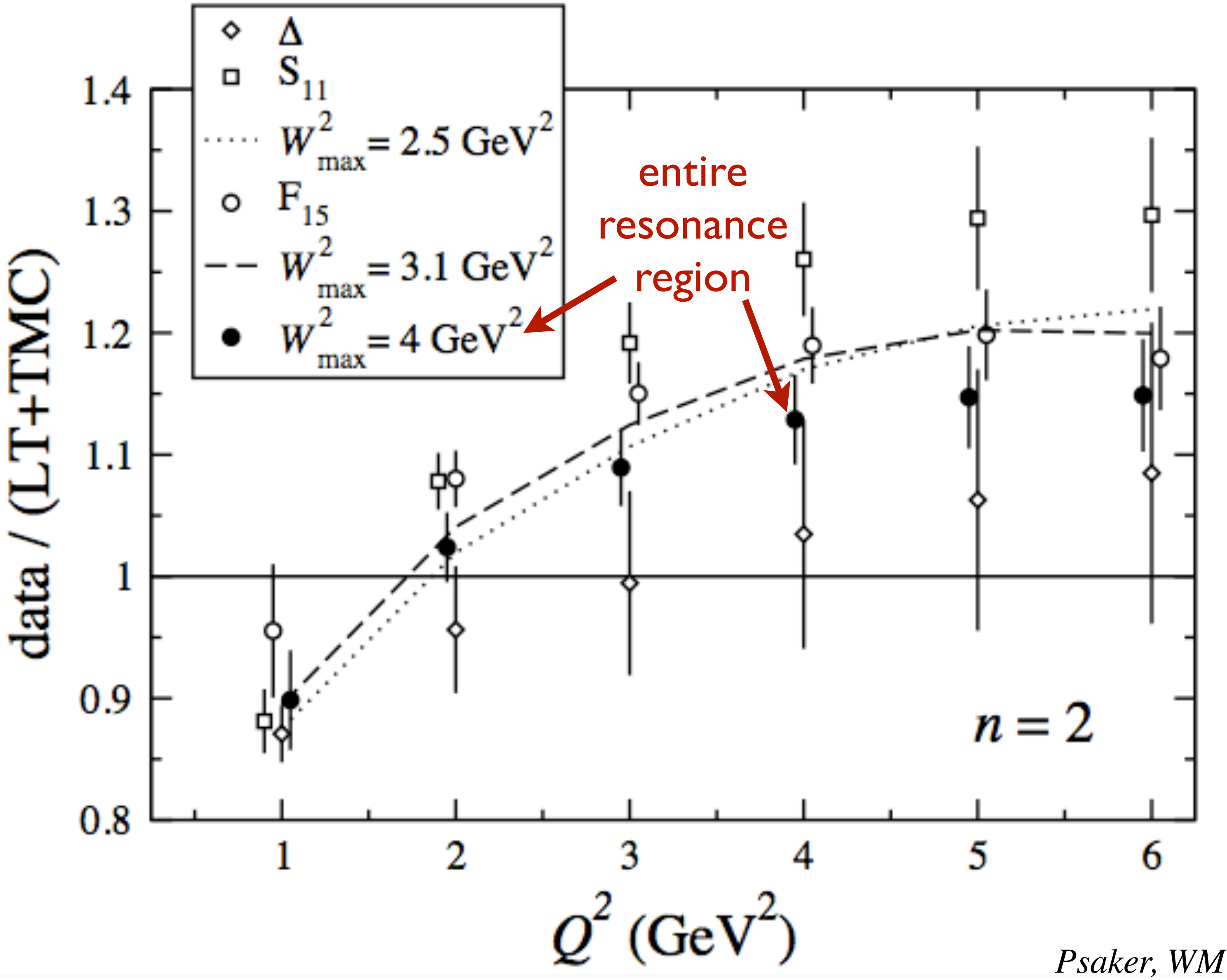
- Follow evolution of *specific resonance (region)* with Q^2 in pQCD framework



* JLab Hall C

how much of this region is *leading twist* ?

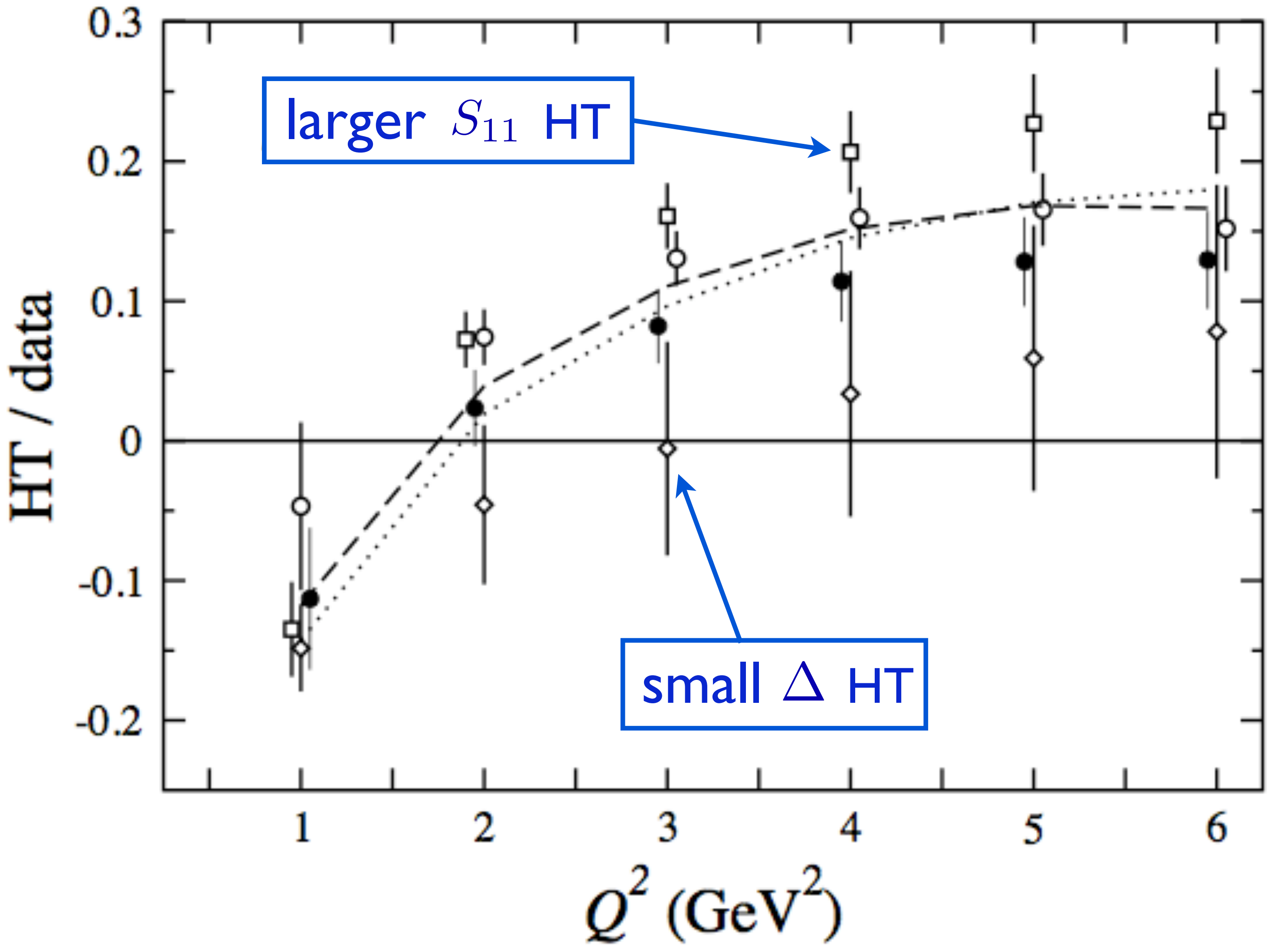
Analysis of JLab F_2^p resonance region data



entire resonance region

Psaker, WM, Christy, Keppel
PRC ***78****, 025206 (2008)*

■ Analysis of JLab F_2^p resonance region data



larger S_{11} HT

small Δ HT

→ higher twists $< 10\text{–}15\%$ for $Q^2 > 1$ GeV^2

Resonances & twists

- Total higher twist "*small*" at scales $Q^2 \sim \mathcal{O}(1\ \mathrm{GeV}^2)$
- On average, nonperturbative interactions between quarks and gluons not dominant (at these scales)

  → nontrivial interference between resonances

Resonances & twists

- Total higher twist "*small*" at scales $Q^2 \sim \mathcal{O}(1\ \mathrm{GeV}^2)$
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  → nontrivial interference between resonances

- Can we understand this dynamically, at quark level?

  → is duality an accident?

- Can we use resonance region data to learn about *leading twist* structure functions?

  → expanded data set has potentially significant implications for global PDF studies

Insights from dynamical models

- Consider simple quark model with spin-flavor symmetric wave function

form factors

⟶ *coherent* scattering from quarks $d\sigma \sim \left(\sum_i e_i\right)^2$

structure functions

⟶ *incoherent* scattering from quarks $d\sigma \sim \sum_i e_i^2$

- Consider simple quark model with spin-flavor symmetric wave function

<u>form factors</u>

→ *coherent* scattering from quarks $d\sigma \sim \left(\sum_i e_i\right)^2$

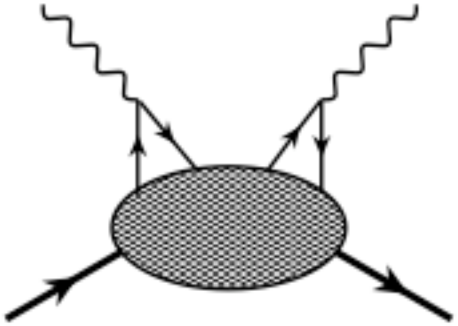
<u>structure functions</u>

→ *incoherent* scattering from quarks $d\sigma \sim \sum_i e_i^2$

- For duality to work, these must be equal

→ how can <u>*square of a sum*</u> become <u>*sum of squares*</u>?

Accidental cancellations of charges?



cat's ears diagram (4-fermion higher twist $\sim 1/Q^2$)

$$\propto \sum_{i \neq j} e_i \, e_j \sim \Big(\sum_i e_i \Big)^2 - \sum_i e_i^2$$

↑ *coherent* ↑ *incoherent*

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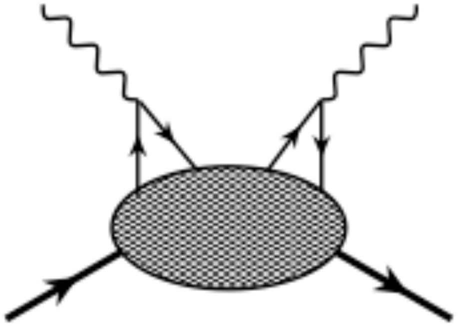

cat’s ears diagram (4-fermion higher twist $\sim 1/Q^2$)

$$\propto \sum_{i \neq j} e_i \, e_j \sim \Big(\underset{\substack{\uparrow \\ \textit{coherent}}}{\sum_i e_i}\Big)^2 - \underset{\substack{\uparrow \\ \textit{incoherent}}}{\sum_i e_i^2}$$

proton $\mathrm{HT} \sim 1 - \left(2 \times \frac{4}{9} + \frac{1}{9}\right) = 0\ !$

neutron $\mathrm{HT} \sim 0 - \left(\frac{4}{9} + 2 \times \frac{1}{9}\right) \neq 0$

Brodsky
hep-ph/0006310

→ duality in proton a *coincidence!*

→ should *not* hold for neutron

- Dynamical cancellations?

→ *e.g.* for toy model of two quarks bound in a harmonic oscillator potential, structure function given by

$$F(\nu,\mathbf{q}^2) \;\sim\; \sum_n \left|G_{0,n}(\mathbf{q}^2)\right|^2 \;\delta(E_n - E_0 - \nu)$$

→ charge operator $\Sigma_i \; e_i \exp(i\mathbf{q}\cdot\mathbf{r}_i)$ excites
even partial waves with strength $\propto (e_1 + e_2)^2$
odd partial waves with strength $\propto (e_1 - e_2)^2$

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→ charge operator $\Sigma_i \ e_i \exp(i\mathbf{q}\cdot\mathbf{r}_i)$ excites *even* partial waves with strength $\propto (e_1+e_2)^2$ *odd* partial waves with strength $\propto (e_1-e_2)^2$

→ resulting structure function

$$F(\nu,\mathbf{q}^2) \ \sim \ \sum_n \left\{(e_1+e_2)^2 \ G^2_{0,2n} + (e_1-e_2)^2 \ G^2_{0,2n+1}\right\}$$

→ if states degenerate, *cross terms* $(\sim e_1 e_2)$ *cancel* when averaged over nearby *even and odd parity* states

- Dynamical cancellations?

→ duality is realized by summing over at least one complete set of *even* and *odd* parity resonances *

Close, Isgur, PLB ***509****, 81 (2001)*

→ in NR Quark Model, even & odd parity states generalize to **56** (L=0) and **70** (L=1) multiplets of spin-flavor SU(6)

- assume magnetic coupling of photon to quarks
  (better approximation at high Q^2)
- in this limit Callan-Gross relation valid $F_2 = 2xF_1$
- structure function given by squared sum of transition FFs

$$F_1(\nu,\vec{q}^{\,2}) \sim \sum_R |F_{N\to R}(\vec{q}^{\,2})|^2 \delta(E_R - E_N - \nu)$$

* realized in many models: ‘t Hooft model, large N_c, RQM, ... see *WM et al., Phys. Rep.* ***406****, 127 (2005)*

- Dynamical cancellations?

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representation	$^2\mathbf{8}[\mathbf{56}^+]$	$^4\mathbf{10}[\mathbf{56}^+]$	$^2\mathbf{8}[\mathbf{70}^-]$	$^4\mathbf{8}[\mathbf{70}^-]$	$^2\mathbf{10}[\mathbf{70}^-]$	Total
F_1^p	$9\rho^2$	$8\lambda^2$	$9\rho^2$	0	λ^2	$18\rho^2+9\lambda^2$
F_1^n	$(3\rho+\lambda)^2/4$	$8\lambda^2$	$(3\rho-\lambda)^2/4$	$4\lambda^2$	λ^2	$(9\rho^2+27\lambda^2)/2$

$\lambda\ (\rho) =$ (anti) symmetric component of ground state wfn.

Close, WM, PRC ***68****, 035210 (2003)*

- SU(6) limit ➡ $\lambda = \rho$

  ⟶ relative strengths of $N \rightarrow N^*$ transitions:

$SU(6):$	$[\mathbf{56}, 0^+]^2\mathbf{8}$	$[\mathbf{56}, 0^+]^4\mathbf{10}$	$[\mathbf{70}, 1^-]^2\mathbf{8}$	$[\mathbf{70}, 1^-]^4\mathbf{8}$	$[\mathbf{70}, 1^-]^2\mathbf{10}$	*total*
F_1^p	9	8	9	0	1	27
F_1^n	4	8	1	4	1	18

- summing over all resonances in $\mathbf{56}^+$ and $\mathbf{70}^-$ multiplets

  ⟶ $\frac{F_1^n}{F_1^p} = \frac{2}{3}$ as in quark-parton model (for u=$2d$) !

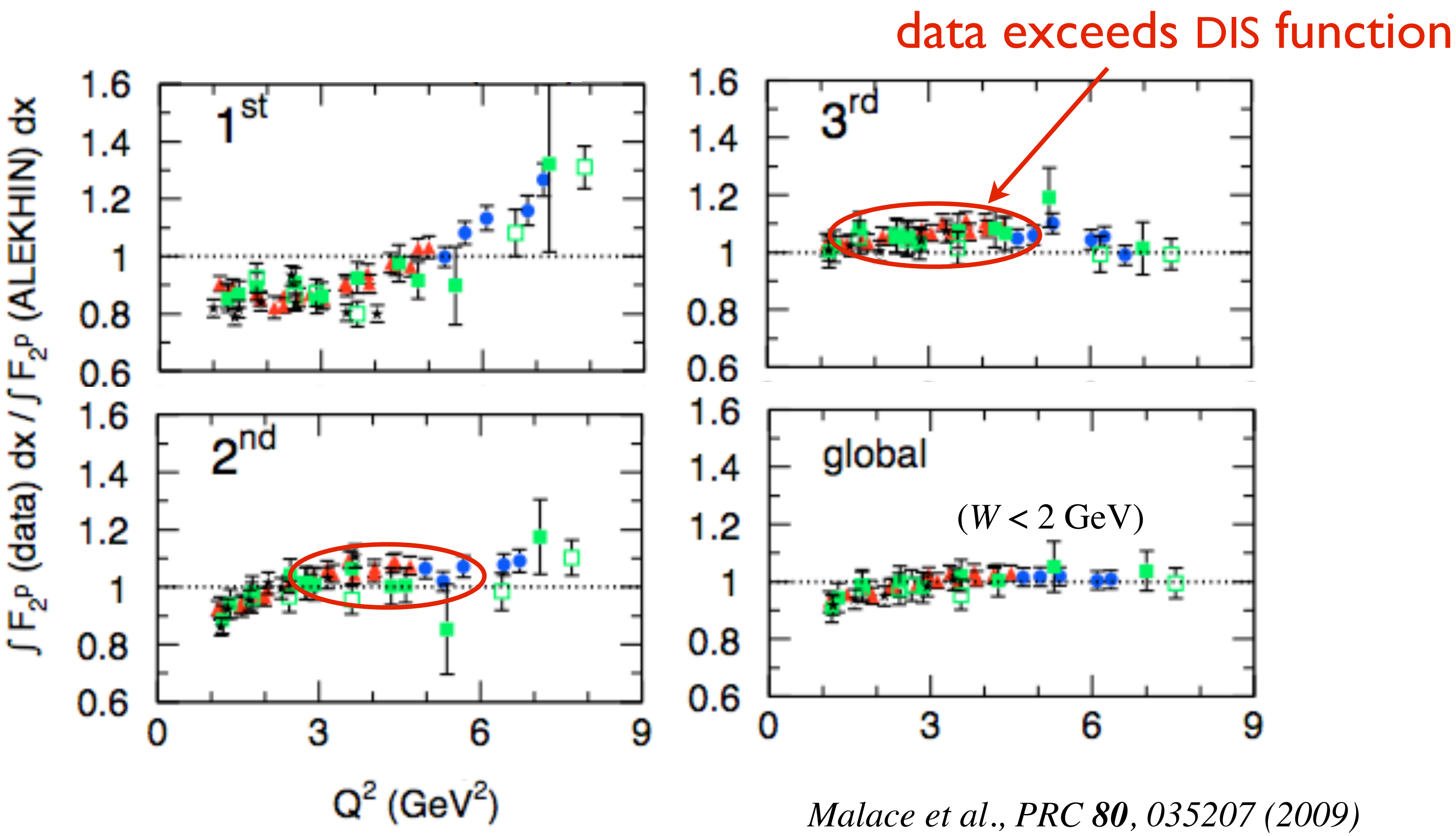
- proton sum saturated by lower-lying resonances

  ⟶ expect duality to appear *earlier* for p than n

Close, WM, PRC ***68****, 035210 (2003)*

Comparison with data

- Proton data expected to *overestimate* DIS function in 2nd and 3rd resonance regions (odd parity states)



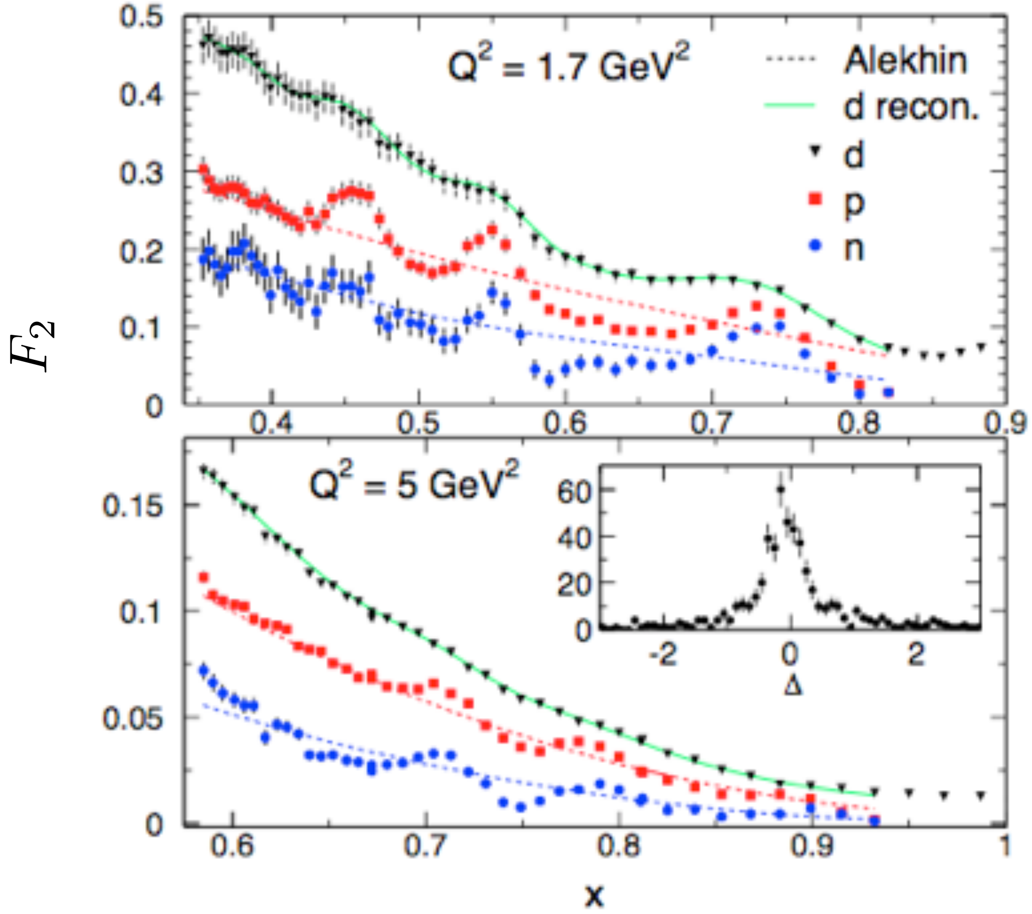
Malace et al., PRC ***80****, 035207 (2009)*

→ duality violation for proton $\lesssim 10\%$, integrated over x

Comparison with data

- Duality in neutron not tested because of absence of free neutron targets
- New extraction method (using iterative procedure for solving integral convolution equations) has allowed first determination of F_2^n in resonance region & test of neutron duality

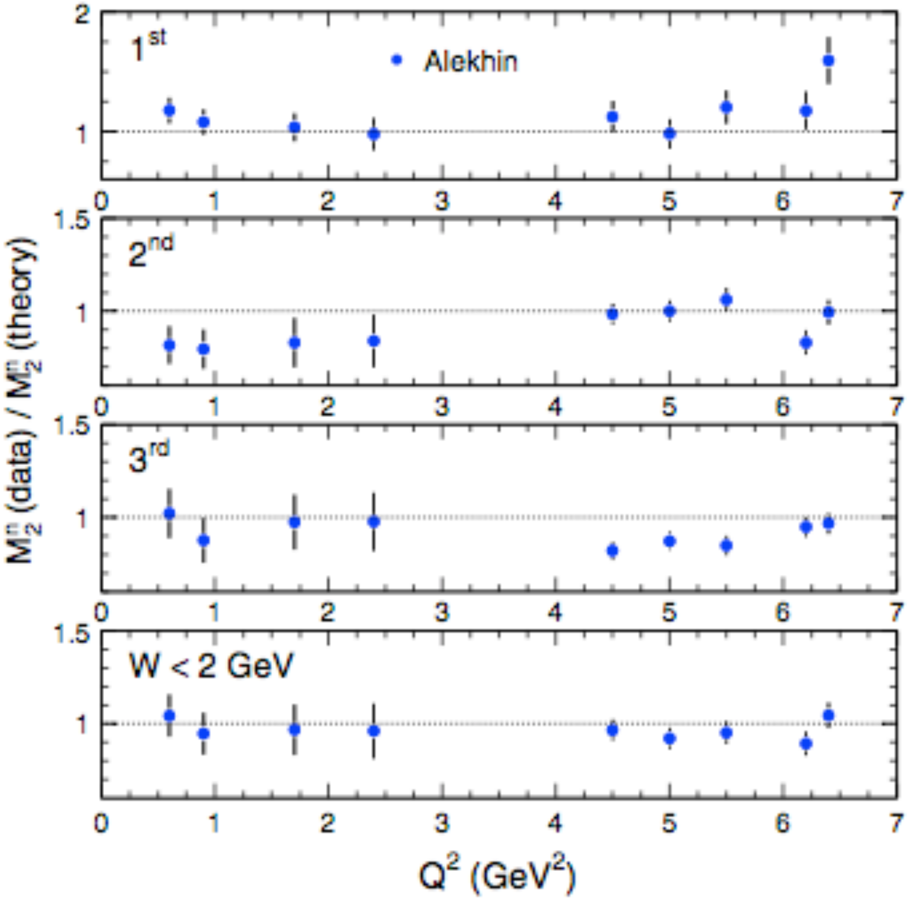
Kahn, WM, Kulagin
PRC ***79****, 035205 (2009)*



Malace, Kahn, WM, Keppel
PRL ***104****, 102001 (2010)*

Comparison with data

- Neutron data expected to lie *below* DIS function in 2nd region



→ "theory": fit to $W > 2$ GeV data

Alekhin et al., 0908.2762 [hep-ph]

→ *locally*, violations of duality in resonance regions < 15−20% (largest in Δ region)

→ *globally*, violations < 10%

Malace, Kahn, WM, Keppel
PRL ***104****, 102001 (2010)*

⇨ duality is *not accidental*, but a general feature of resonance−scaling transition!

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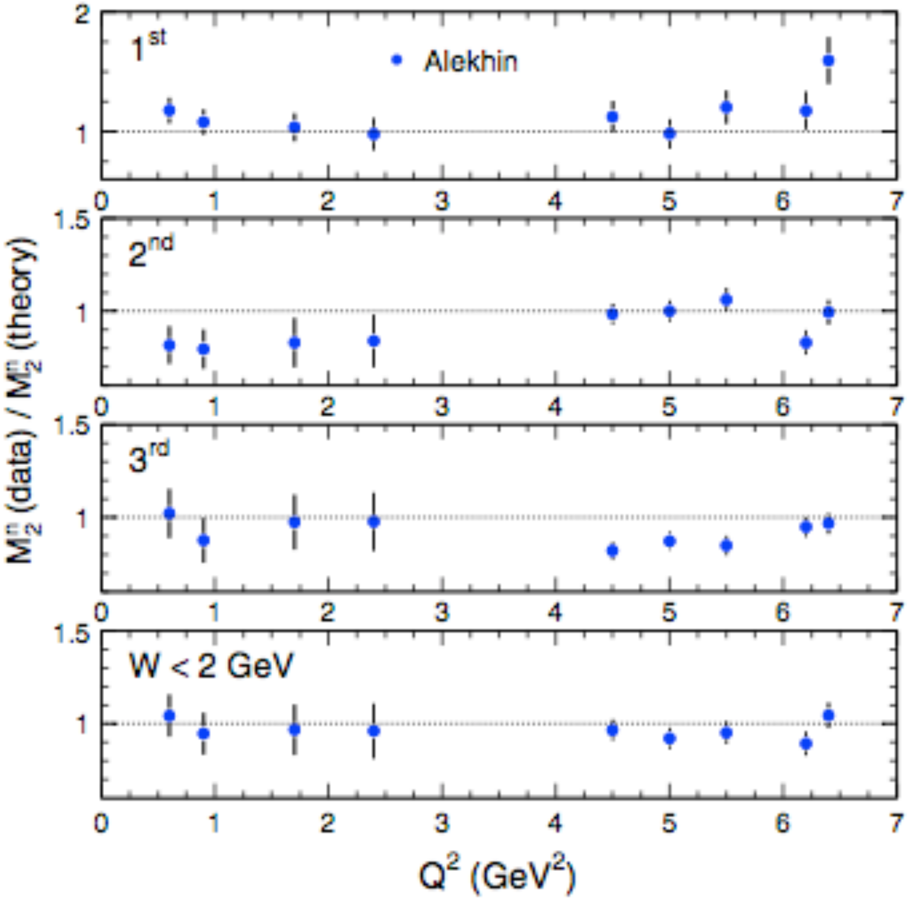

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⇒ use resonance region data to learn about *leading twist* structure functions?

Duality in practice: global PDF analysis

CTEQ6X global PDF analysis

- New global QCD (next-to-leading order) analysis of expanded set of p and d data, including large-x, low-Q^2 region

  → joint JLab-CTEQ theory/experiment collaboration
  (with Hampton, FSU, FNAL, Duke)

- Systematically study effects of Q^2 & W cuts

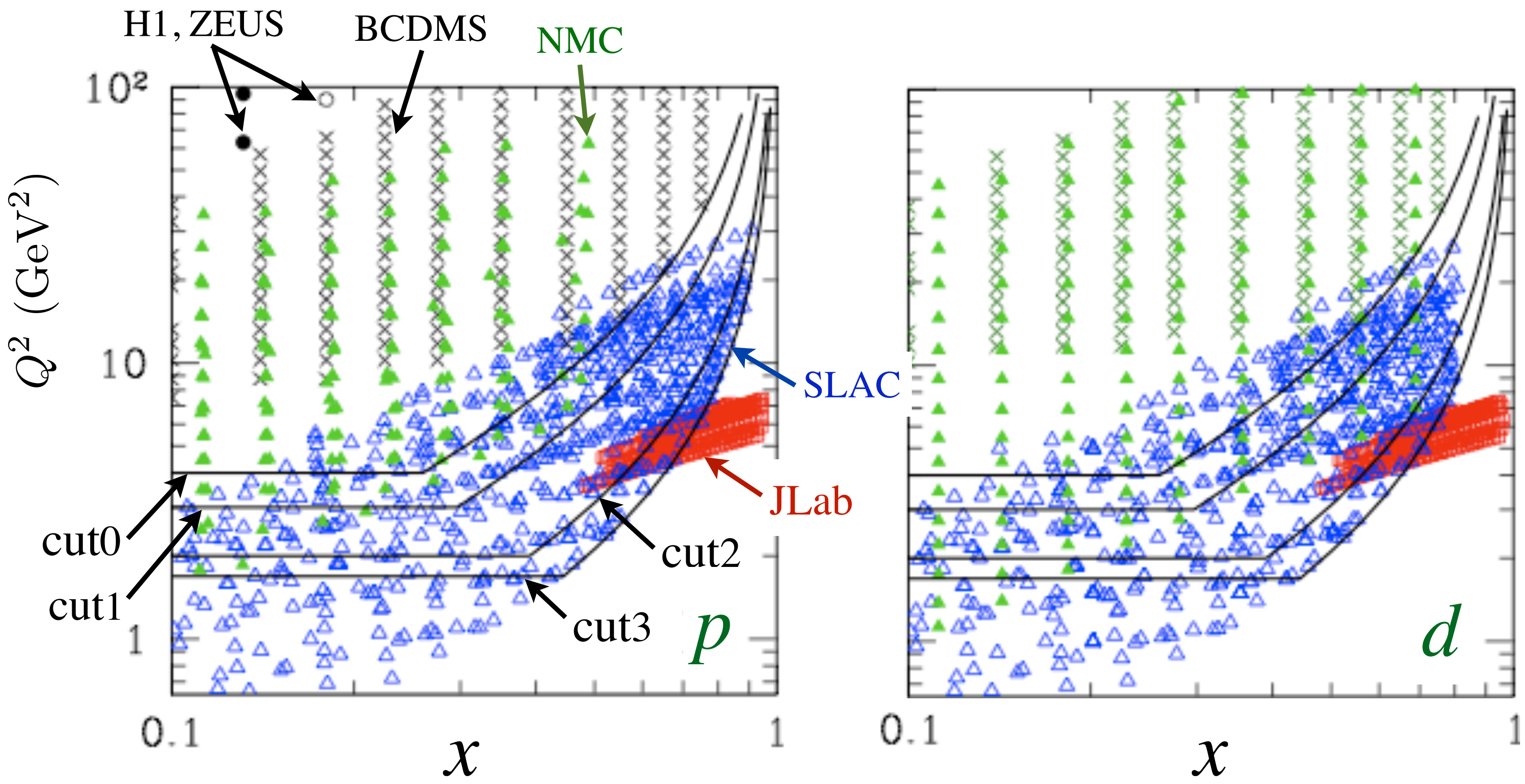
  → as low as $Q \sim m_c$ and $W \sim 1.7\ \mathrm{GeV}$

- Include large-x corrections

  → TMCs & higher twists $F_2(x,Q^2) = F_2^{\mathrm{LT}}(x,Q^2)(1 + C(x)/Q^2)$

  → realistic nuclear effects in deuteron (binding + off-shell)
  (most analyses assume no nuclear corrections)

CTEQ6X – kinematic cuts



cut0: $Q^2 > 4\ \mathrm{GeV}^2, \quad W^2 > 12.25\ \mathrm{GeV}^2$

cut1: $Q^2 > 3\ \mathrm{GeV}^2, \quad W^2 > 8\ \mathrm{GeV}^2$

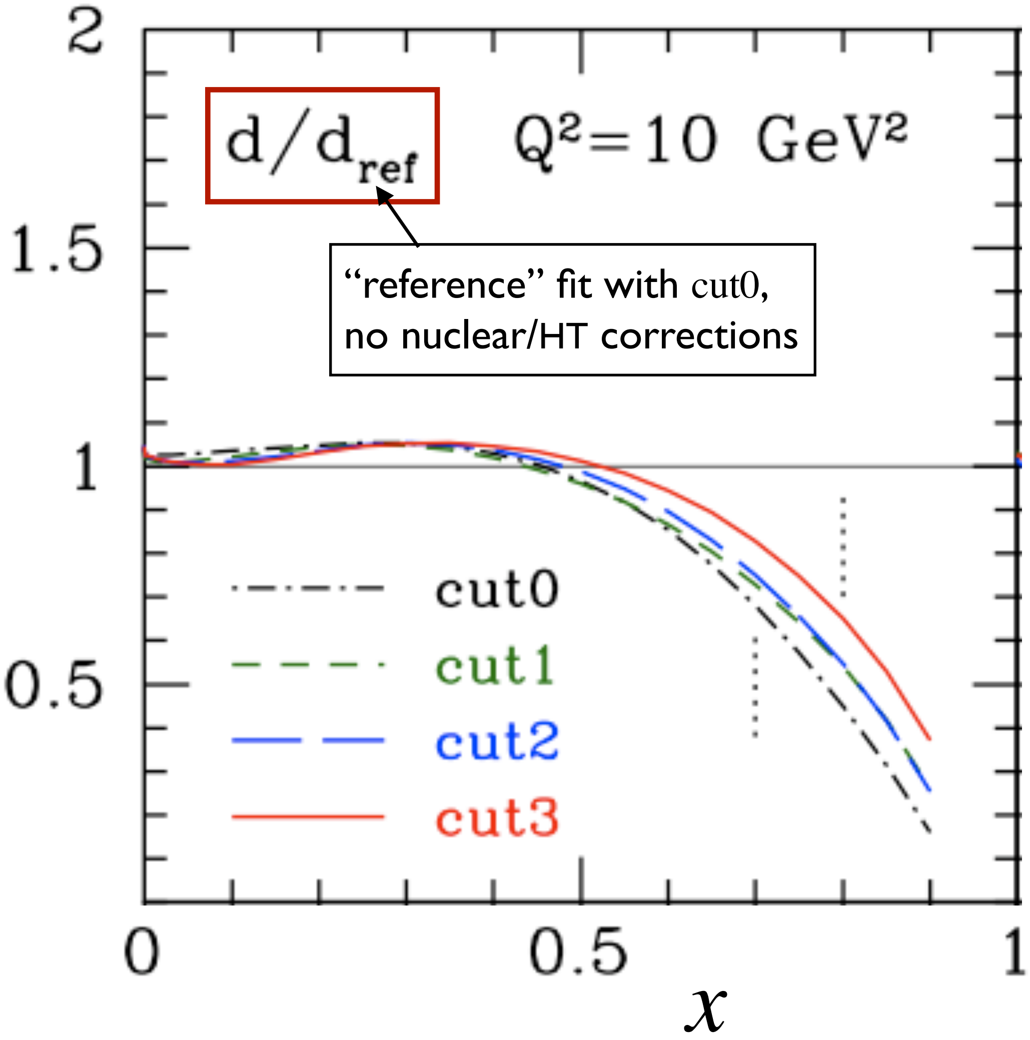
cut2: $Q^2 > 2\ \mathrm{GeV}^2, \quad W^2 > 4\ \mathrm{GeV}^2$

cut3: $Q^2 > m_c^2\ , \quad W^2 > 3\ \mathrm{GeV}^2$

factor 2 increase
in DIS data from
cut0 → cut3

CTEQ6X – kinematic cuts

- Systematically reduce Q^2 and W cuts, including TMC, HT & nuclear corrections



"reference" fit with cut0, no nuclear/HT corrections

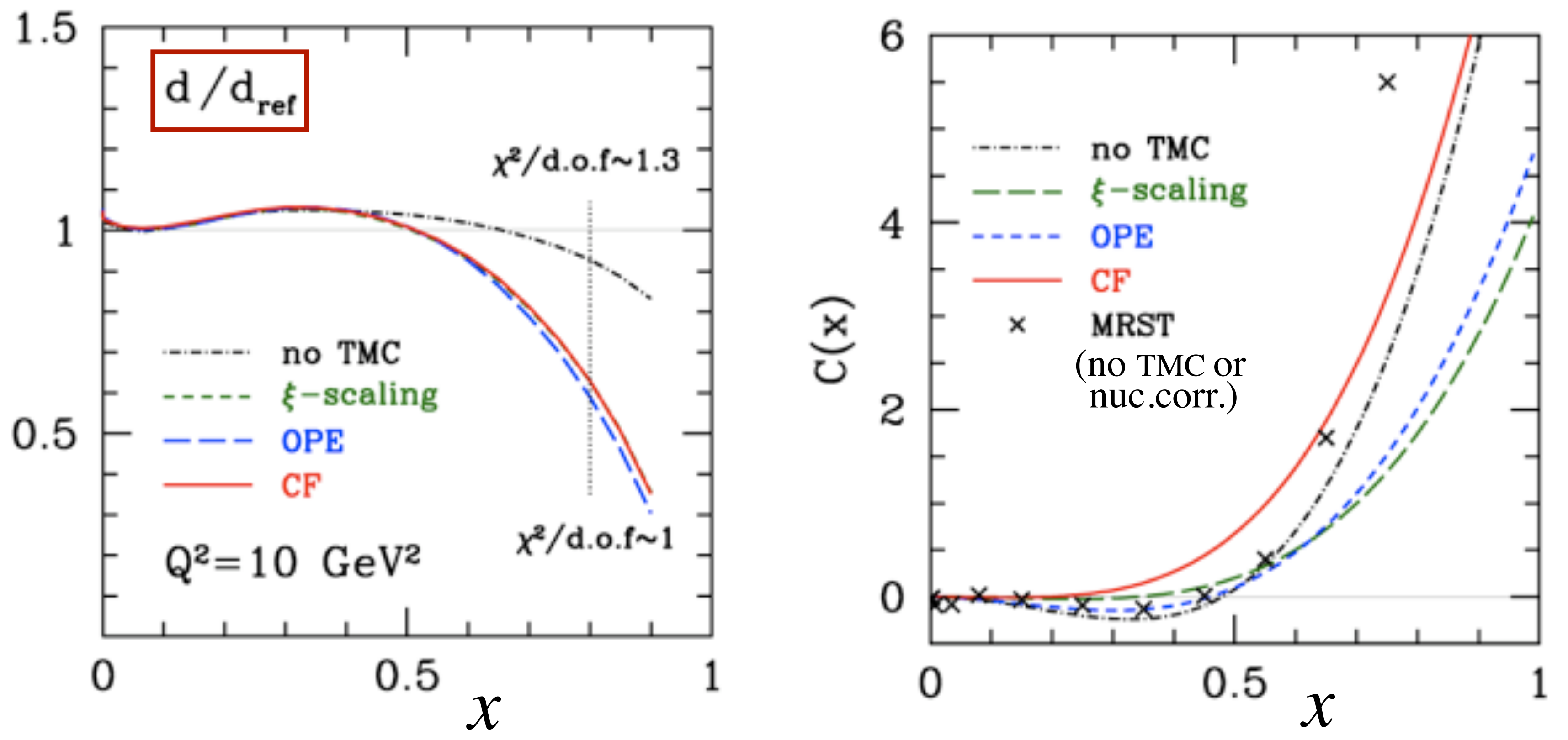
→ *stable* with respect to cut reduction

→ d quark suppressed by ~ 50% for $x > 0.5$

(driven by nuclear corrections)

Accardi et al., Phys. Rev. D ***81****, 034016 (2010)*

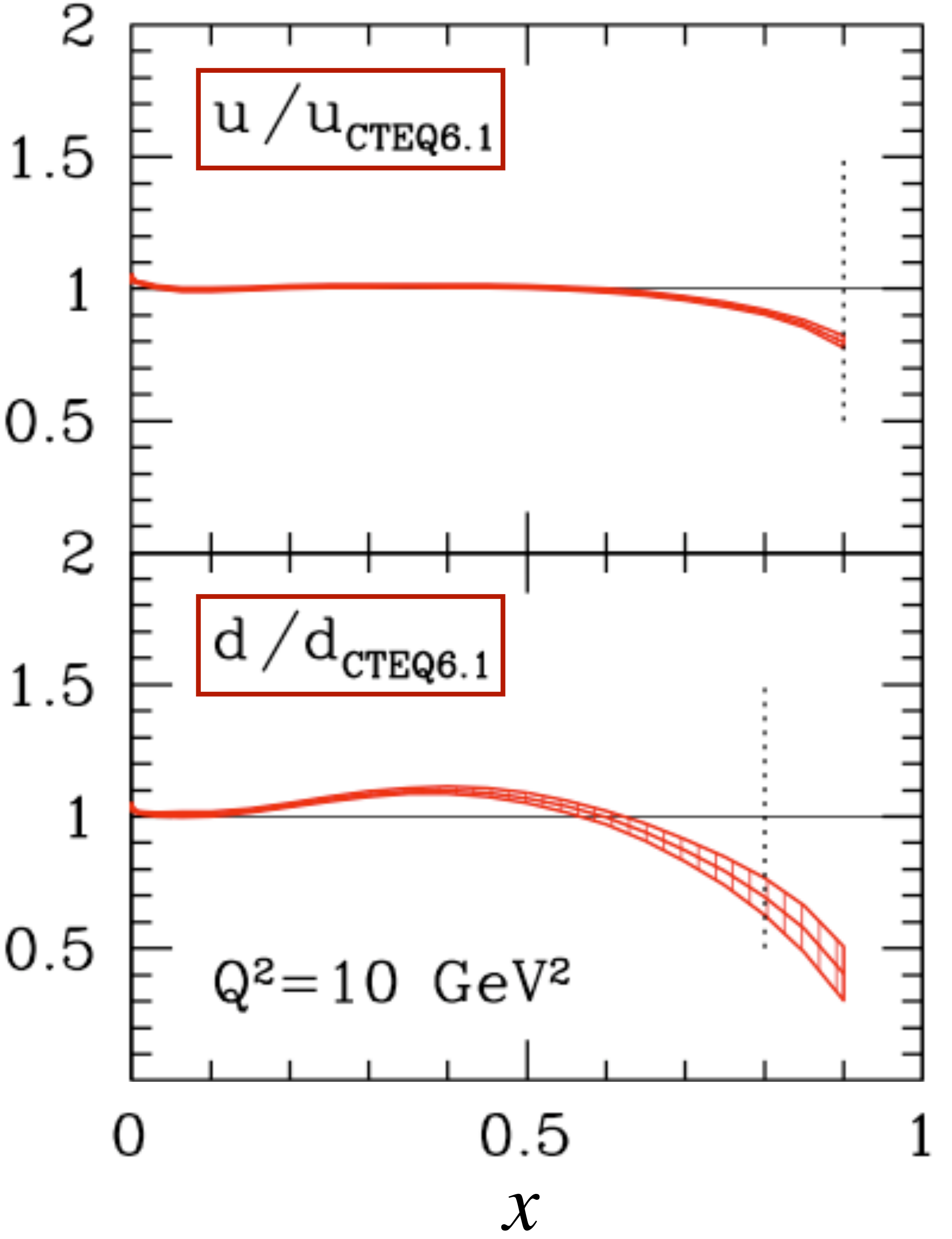
CTEQ6X – $1/Q^2$ corrections



→ important interplay between TMCs and higher twist: HT alone *cannot* accommodate full Q^2 dependence

→ stable leading twist when *both* TMCs and HTs included

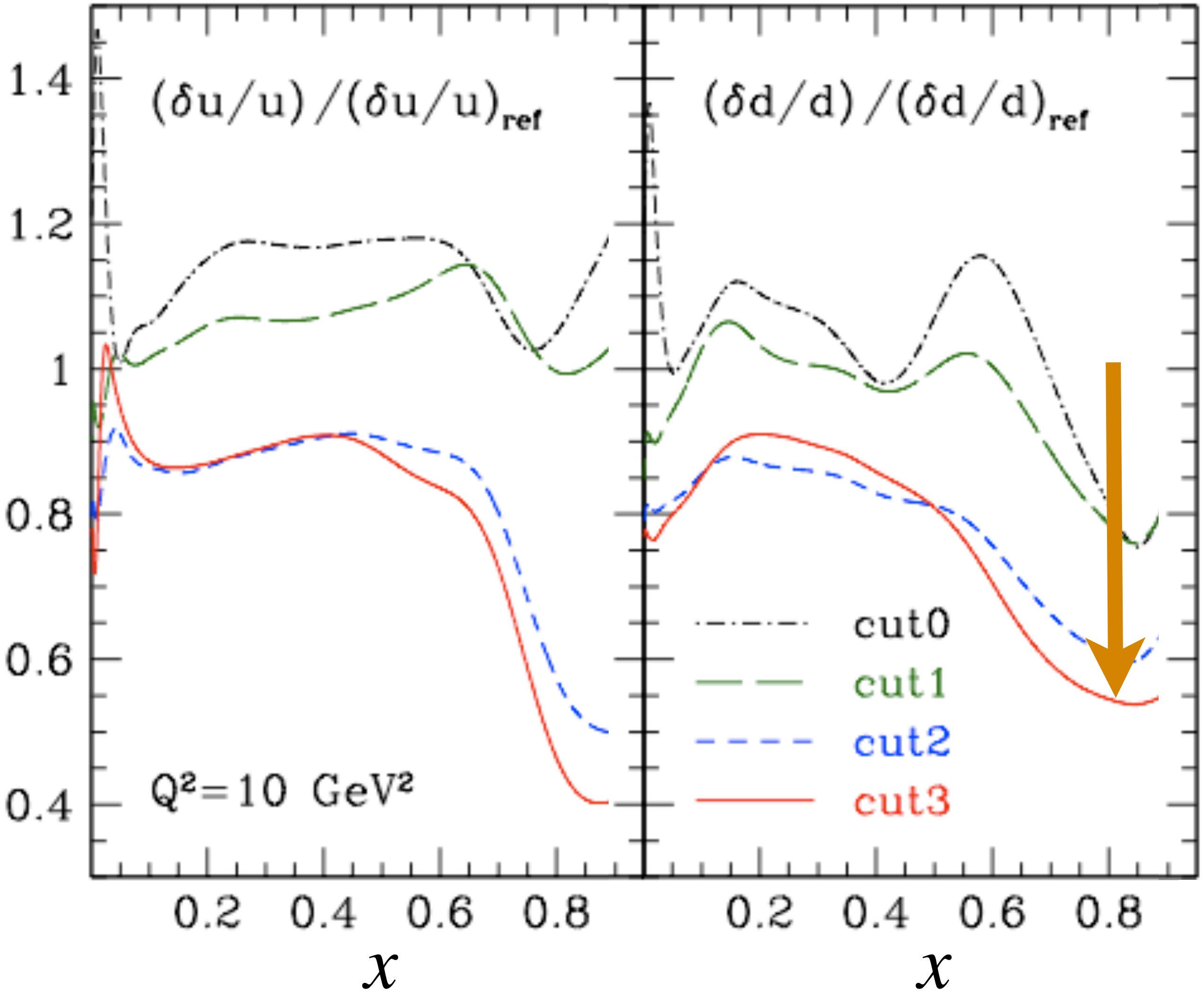
CTEQ6X – final PDF results



→ full fits favors *smaller d/u* ratio

(CTEQ6.1 had no nuclear or TMC/HT corrections)

CTEQ6X – final PDF results



Accardi et al., Phys. Rev. D ***81****, 034016 (2010)*

→ full fits favors *smaller d/u* ratio

(CTEQ6.1 had no nuclear or TMC/HT corrections)

→ up to 40-60% *reduced errors* with weaker cuts extending into resonance region

Summary

- Remarkable confirmation of quark-hadron duality in *proton* and *neutron* structure functions
  - → duality violating higher twists ~ 10–15% in few-GeV range
- Confirmation of duality in *neutron* suggests origin in dynamical cancellations of higher twists
  - → duality *not* due to accidental cancellations of quark charges
- Practical application of duality
  - → use resonance region data to constrain *leading twist* PDFs
  - → stable fits at low Q^2 and large x with significantly reduced uncertainties

The End