# Outline

# Lecture 3

- Elastic *ep* scattering
- Two-photon exchange
  - $\rightarrow$  Rosenbluth separation *vs.* polarization transfer
- Global analysis of form factors
- Parity-violating electron scattering
  - $\rightarrow$  strangeness in the proton
  - → constraints on "new" physics

Elastic scattering

# Elastic *eN* scattering



# Elastic *eN* scattering





cf. classical (Non-Relativistic) current density

$$J^{\rm NR} = \left( e \ \rho_E^{\rm NR} \ , \ \mu \ \vec{\sigma} \times \vec{\nabla} \rho_M^{\rm NR} \right)$$

$$\rho_E^{\rm NR}(r) = \frac{2}{\pi} \int_0^\infty dq \ \vec{q}^2 \ j_0(qr) \ G_E(\vec{q}^2) \quad \leftarrow \text{ charge density}$$
$$\mu \ \rho_M^{\rm NR}(r) = \frac{2}{\pi} \int_0^\infty dq \ \vec{q}^2 \ j_0(qr) \ G_M(\vec{q}^2) \quad \leftarrow \text{ magnetization density}$$

#### neutron





# **Proton densities**



# Neutron densities



# Neutron densities





 $\underline{\text{LT}} \text{ method}$  $\sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$ 

- $\rightarrow G_E$  from slope in  $\varepsilon$  plot
- $\rightarrow$  suppressed at large  $Q^2$



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- $\rightarrow$  suppressed at large  $Q^2$

Rosenbluth (<u>L</u>ongitudinal-<u>T</u>ransverse) Separation

**pQCD:**  $G_E^p/G_M^p \to \text{constant as } Q^2 \to \infty$ 





 $\underline{\text{LT}} \text{ method}$  $\sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$ 

- $\rightarrow$   $G_E$  from slope in  $\varepsilon$  plot
- $\rightarrow$  suppressed at large  $Q^2$

 $\frac{PT}{G_E} = -\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}} \frac{P_T}{P_L}$ 



Are the  $G_E^p/G_M^p$  data consistent?

# Two-photon exchange

### **QED** Radiative Corrections

#### $\blacksquare$ cross section modified by $1\gamma$ loop effects



 $d\sigma = d\sigma_0 (1+\delta)$ 

### **QED** Radiative Corrections

#### $\blacksquare$ cross section modified by $1\gamma$ loop effects





### **QED** Radiative Corrections

### $\blacksquare$ cross section modified by $1\gamma$ loop effects





(most difficult to calculate)

# Two-photon exchange

interference between Born and two-photon exchange amplitudes



contribution to cross section:

$$\delta^{(2\gamma)} = \frac{2\mathcal{R}e\left\{\mathcal{M}_{0}^{\dagger} \ \mathcal{M}_{\gamma\gamma}\right\}}{\left|\mathcal{M}_{0}\right|^{2}}$$

standard "soft photon approximation" (used in most data analyses)

- $\rightarrow$  approximate integrand in  $\mathcal{M}_{\gamma\gamma}$  by values at  $\gamma^*$  poles
- $\rightarrow$  neglect nucleon structure (no form factors) *Mo*, *Tsai* (1969)



$$\mathcal{M}_{\gamma\gamma} = e^4 \int \frac{d^4k}{(2\pi)^4} \frac{N(k)}{D(k)}$$

where

$$N(k) = \bar{u}(p_3) \gamma_{\mu}(\not p_1 - \not k + m_e) \gamma_{\nu} u(p_1) \\ \times \bar{u}(p_4) \Gamma^{\mu}(q-k) (\not p_2 + \not k + M) \Gamma^{\nu}(k) u(p_2)$$

and

$$D(k) = (k^2 - \lambda^2) ((k - q)^2 - \lambda^2) \times ((p_1 - k)^2 - m^2) ((p_2 + k)^2 - M^2)$$

with  $\lambda$  an IR regulator, and e.m. current is

$$\Gamma^{\mu}(q) = \gamma^{\mu} F_1(q^2) + \frac{i\sigma^{\mu\nu}q_{\nu}}{2M} F_2(q^2)$$

- Mo-Tsai: soft  $\gamma$  approximation
  - $\longrightarrow$  integrand most singular when k=0 and k=q
  - $\longrightarrow$  replace  $\gamma$  propagator which is not at pole by  $1/q^2$
  - $\longrightarrow$  approximate numerator  $N(k) \approx N(0)$
  - $\longrightarrow$  neglect all structure effects
- <u>Maximon-Tjon</u>: improved loop calculation
  - $\longrightarrow$  exact treatment of propagators
  - $\longrightarrow$  still evaluate N(k) at k = 0
  - $\longrightarrow$  first study of form factor effects
  - $\longrightarrow$  additional  $\varepsilon$  dependence
- <u>Blunden-WM-Tjon</u>: exact loop calculation
  - $\longrightarrow$  no approximation in N(k) or D(k)
  - $\longrightarrow$  include form factors

## Two-photon exchange

I "exact" calculation of loop diagram (including  $\gamma^*NN$  form factors)



→ few % magnitude
→ positive slope
→ non-linearity in ε

# Two-photon exchange

"exact" calculation of loop diagram (including  $\gamma^*NN$  form factors)



results essentially independent of form factor input

### Effect on cross section



Born cross section with PT form factors
including TPE effects

\* Super-Rosenbluth Qattan et al.,

PRL 94 (2005) 142301

- estimate effect of TPE on  $\varepsilon$  dependence
- approximate correction by linear function of  $\varepsilon$

 $1 + \Delta \approx a + b\varepsilon$ 

- **estimate effect of TPE on**  $\varepsilon$  dependence
  - approximate correction by linear function of  $\varepsilon$

 $1 + \Delta \approx a + b\varepsilon$ 

reduced cross section is then

$$\sigma_R \approx a \ G_M^2 \left[ 1 + \frac{\varepsilon}{\mu^2 \tau} \left( R^2 (1 + \varepsilon \ b/a) + \mu^2 \tau \ b/a \right) \right]$$

where "true" ratio is









 $^{25}_{25}$ 

→ resolves much of the form factor discrepancy

how does TPE affect polarization transfer ratio?

$$\implies \widetilde{R} = R\left(\frac{1+\Delta_T}{1+\Delta_L}\right)$$

where  $\Delta_{L,T} = \delta_{L,T}^{\text{full}} - \delta_{\text{IR}}^{\text{Mo-Tsai}}$  is finite part of  $2\gamma$  contribution relative to IR part of Mo-Tsai

experimentally measure ratio of polarized to unpolarized cross sections

$$\rightarrow \frac{P_{L,T}^{1\gamma+2\gamma}}{P_{L,T}^{1\gamma}} = \frac{1 + \Delta_{L,T}}{1 + \Delta}$$



 $\gamma$ 



→ large  $Q^2$  data typically at large  $\varepsilon$ → < 3% suppression at large  $Q^2$ 

# Excited intermediate states

# What about higher-mass intermediate states?

 $N, \Delta, P_{11}, S_{11}, S_{31}, \ldots$  $\left\{ \int q - k \right\}$ k .  $p_{\gamma}$ Amplitude for box diagram (cross-box is simil r) Lowest mass excitation is  $\mathcal{P}_{33}$   $\overline{\Delta}(1232) \frac{d^4k}{(232)} \frac{N(k)}{(232)}$  $\rightarrow$  relativistic  $\gamma^* N \Delta$  vertex  $\Delta \text{ vertex} \qquad \qquad \text{form factor } \frac{\Lambda_{\Delta}^{4}}{\left(\Lambda_{\Delta}^{2} - q^{2}\right)^{2}} \\ N(k) = \overline{u}(p_{3}) \gamma_{\mu}(p_{1} - k + m_{e}) \gamma_{\nu} u(p_{1})$  $\Gamma^{\nu\alpha}_{\gamma\Delta\to N}(p,q) \equiv i V^{\nu\alpha}_{\Delta in}(p,q) = \overline{u} \left( \frac{eF_{\Delta}(q^2)}{p_{\Delta}} \left\{ g_1 \left[ \frac{d\nu}{p} \right]^{\alpha} p \left( \frac{d\nu}{p} - \frac{M}{p} \right)^{\alpha} \gamma^{\alpha} p \left( \frac{d\nu}{p} - \frac{M}{p} \right)^{\alpha} \right\} \right)$  $+g_{2}\left[p^{\nu}q^{\alpha}-g^{\nu\alpha}p\cdot q\right]+\left(g_{3}/M_{\Delta}\right)\left[q^{2}\left(p^{\nu}\gamma^{\alpha}-g^{\nu\alpha}\not{p}\right)+q^{\nu}\left(q^{\alpha}\not{p}-\gamma^{\alpha}p\cdot q\right)\right]\right\}\gamma_{5}T_{3}$  $D(k) = (k^{2}-\lambda^{2})\left((k-q)^{2}-\lambda^{2}\right)$ coupling constants  $\times ((p_{a_1} - k)^2 - m_{a_2}^2) ((p_{a_2} + k)^2 - M^2)$ with  $\lambda \approx IR regulator and model current is$ 

#### Two-photon exchange amplitude with $\Delta$ intermediate state



$$\mathcal{M}^{\gamma\gamma}_{\Delta} = -e^4 \int \frac{d^4k}{(2\pi)^4} \frac{N^{\Delta}_{box}(k)}{D^{\Delta}_{box}(k)} - e^4 \int \frac{d^4k}{(2\pi)^4} \frac{N^{\Delta}_{x-box}(k)}{D^{\Delta}_{x-box}(k)}$$

#### numerators

$$N_{box}^{\Delta}(k) = \overline{U}(p_4) V_{\Delta in}^{\mu\alpha}(p_2 + k, q - k) \left[ \not p_2 + \not k + M_{\Delta} \right] \mathcal{P}_{\alpha\beta}^{3/2}(p_2 + k) V_{\Delta out}^{\beta\nu}(p_2 + k, k) U(p_2) \\ \times \overline{u}(p_3) \gamma_{\mu} \left[ \not p_1 - \not k + m_e \right] \gamma_{\nu} u(p_1)$$

 $N_{x-box}^{\Delta}(k) = \overline{U}(p_4) V_{\Delta in}^{\mu\alpha}(p_2 + k, q - k) \left[ \not p_2 + \not k + M_{\Delta} \right] \mathcal{P}_{\alpha\beta}^{3/2}(p_2 + k) V_{\Delta out}^{\beta\nu}(p_2 + k, k) U(p_2) \\ \times \overline{u}(p_3) \gamma_{\nu} \left[ \not p_3 + \not k + m_e \right] \gamma_{\mu} u(p_1)$   $spin-3/2 \text{ projection operator} \\ \mathcal{P}_{\alpha\beta}^{3/2}(p) = g_{\alpha\beta} - \frac{1}{3} \gamma_{\alpha} \gamma_{\beta} - \frac{1}{3p^2} \left( \not p \gamma_{\alpha} p_{\beta} + p_{\alpha} \gamma_{\beta} \not p \right)$ 



Kondratyuk, Blunden, WM, Tjon PRL **95** (2005) 172503

- $\rightarrow \Delta$  has <u>opposite</u> slope to N
  - $\blacktriangleright$  cancels some of TPE correction from N

### Higher-mass intermediate states have also been calculated

 $\rightarrow$  more model dependent, since couplings & form factors not well known (especially at high  $Q^2$ )



Kondratyuk, Blunden, WM, Tjon PRL 95 (2005) 172503

> Kondratyuk, Blunden PRC **75** (2007) 038201

dominant contribution from N

 $\longrightarrow \Delta$  partially cancels N contribution

#### Higher-mass intermediate states have also been calculated



Kondratyuk, Blunden PRC 75 (2007) 038201

higher mass resonance contributions small
much better fit to data including TPE

Global analysis
## Global analysis

- reanalyze  $\underline{all}$  elastic ep data (Rosenbluth, PT), including TPE corrections consistently from the beginning
- use explicit calculation of N elastic contribution
- approximate higher mass contributions by phenomenological form, based on  $N^*$  calculations:

 $\delta_{\text{high mass}}^{(2\gamma)} = -0.01 \ (1-\varepsilon) \ \log Q^2 / \log 2.2$ 

for  $Q^2 > 1 \ {
m GeV}^2$ , with  $\pm 100\%$  uncertainty

→ decreases  $\varepsilon = 0$  cross section by 1% (2%) at  $Q^2 = 2.2$  (4.8) GeV<sup>2</sup>



*PRC* **76** (2007) 035205

## Non-linearity in $\varepsilon$

- unique feature of TPE correction to cross section
- observation of non-linearity in *ɛ* would provide direct evidence of TPE in elastic scattering
- fit reduced cross section as:

$$\sigma_R = P_0 \left[ 1 + P_1 \ (\varepsilon - \frac{1}{2}) + P_2 \ (\varepsilon - \frac{1}{2})^2 \right]$$

current data give average non-linearity parameter:

$$\langle P_2 \rangle = 4.3 \pm 2.8\%$$

■ Hall C experiment E-05-017 will provide accurate measurement of  $\varepsilon$  dependence

# $e^+/e^-$ comparison

- 1γ (2γ) exchange changes sign (invariant) under  $e^+ \leftrightarrow e^-$
- ratio of  $e^+p / e^-p$  elastic cross sections sensitive to  $\Delta(\varepsilon, Q^2)$ :

$$\sigma_{e^+p}/\sigma_{e^-p}\approx 1-2\Delta$$



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$$\sigma_{e^+p}/\sigma_{e^-p} \approx 1 - 2\Delta$$



► simultaneous  $e^-p/e^+p$  measurement using tertiary  $e^+/e^$ beam to  $Q^2 \sim 1-2 \text{ GeV}^2$  (Hall B expt. E-04-116)



final form factor results from global analysis including TPE corrections

$$\left\{G_E, \ \frac{G_M}{\mu_p}\right\} = \frac{1 + \sum_{i=1}^n a_i \tau^i}{1 + \sum_{i=1}^{n+2} b_i \tau^i}$$

Parameter	$G_M/\mu_p$	$G_E$
$a_1$	-1.465	3.439
$a_2$	1.260	-1.602
$a_3$	0.262	0.068
$b_1$	9.627	15.055
$b_2$	0.000	48.061
$b_3$	0.000	99.304
$b_4$	11.179	0.012
$b_5$	13.245	8.650

Arrington, WM, Tjon PRC **76** (2007) 035205

## Charge density



Strange quarks in the nucleon

## How strange is the proton?

- Suggestions for major role of strange quarks in the nucleon
  - $\rightarrow$  nucleon "sigma"-term (~100 MeV contribution to N mass?)
  - $\rightarrow$  proton "spin crisis" (s quarks carry large fraction of p spin)
  - $\rightarrow$  how large is contribution to *N* magnetic moment?
- Proton and neutron electromagnetic form factors give two combinations of 3 unknowns:

$$G_{E,M}^{p} = \frac{2}{3}G_{E,M}^{u} - \frac{1}{3}G_{E,M}^{d} - \frac{1}{3}G_{E,M}^{s}$$
$$G_{E,M}^{n} = \frac{2}{3}G_{E,M}^{d} - \frac{1}{3}G_{E,M}^{u} - \frac{1}{3}G_{E,M}^{s}$$

- $\rightarrow$  need third observable to extract  $G_{E,M}^s$
- $\rightarrow$  parity-violating *e* scattering (interference of  $\gamma$  and  $Z^0$  exchange)

• Left-right polarization asymmetry in  $\vec{e} \ p \to e \ p$  scattering  $A_{\rm PV} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\left(\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\right) (A_V + A_A + A_s)$ 

measure interference between e.m. and weak currents



Born (tree) level

**Left-right polarization asymmetry in**  $\vec{e} \ p \rightarrow e \ p$  scattering

$$A_{\rm PV} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\left(\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\right) \left(A_V + A_A + A_s\right)$$

measure interference between e.m. and weak currents

$$A_V = g_A^e \rho \left[ (1 - 4\kappa \sin^2 \theta_W) - (\varepsilon G_E^{\gamma p} G_E^{\gamma n} + \tau G_M^{\gamma p} G_M^{\gamma n}) / \sigma^{\gamma p} \right]$$

using relations between weak and e.m. form factors

$$G_{E,M}^{Zp} = (1 - 4\sin^2\theta_W)G_{E,M}^{\gamma p} - G_{E,M}^{\gamma n} - G_{E,M}^{s}$$

 $\blacksquare$  Left-right polarization asymmetry in  $\vec{e} \ p \rightarrow e \ p$  scattering

$$A_{\rm PV} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\left(\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\right) \left(A_V + A_A + A_s\right)$$

measure interference between e.m. and weak currents

$$A_{V} = g_{A}^{e} \rho \left[ (1 - 4\kappa \sin^{2} \theta_{W}) - (\varepsilon G_{E}^{\gamma p} G_{E}^{\gamma n} + \tau G_{M}^{\gamma p} G_{M}^{\gamma n}) / \sigma^{\gamma p} \right]$$
  
+1 radiative corrections, including TBE

using relations between weak and e.m. form factors

$$G_{E,M}^{Zp} = (1 - 4\sin^2\theta_W)G_{E,M}^{\gamma p} - G_{E,M}^{\gamma n} - G_{E,M}^{s}$$

• Left-right polarization asymmetry in  $\vec{e} \ p \to e \ p$  scattering  $A_{\rm PV} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\left(\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\right) (A_V + A_A + A_s)$ 

measure interference between e.m. and weak currents

$$A_{s} = -g_{A}^{e}\rho\left(\varepsilon G_{E}^{\gamma p}G_{E}^{s} + \tau G_{M}^{\gamma p}G_{M}^{s}\right)/\sigma^{\gamma p}$$
strange electric &
magnetic form factors

# **G0 Experiment at Jefferson Lab**



#### Extracted strange form factors



### Extracted strange form factors



 $\implies$  intriguing  $Q^2$  dependence !

 $\Rightarrow$  trend to positive values at larger  $Q^2$ 

**global analysis of <u>all</u> PVES data at Q^2 < 0.3 \text{ GeV}^2** 



 $G_E^s = 0.0025 \pm 0.0182$   $G_M^s = -0.011 \pm 0.254$  at  $Q^2 = 0.1 \text{ GeV}^2$ 

Young et al., PRL 97 (2006) 102002

Two-boson exchange corrections

## Two-boson exchange corrections



I current PDG estimates computed at  $Q^2=0$ 

Marciano, Sirlin (1980) Erler, Ramsey-Musolf (2003)

do not include hadron structure effects (parameterized via ZNN form factors)

#### Including TBE corrections,



→ some cancellation between  $Z(\gamma\gamma)$  and  $\gamma(\gamma\gamma)$  corrections in  $\Delta\rho$ → effect driven by  $\gamma(Z\gamma)$ 

## Two-boson exchange corrections



Tjon, WM, PRL 100 (2008) 082003

2-3% correction at Q<sup>2</sup> < 0.1 GeV<sup>2</sup>
 strong Q<sup>2</sup> dependence at low Q<sup>2</sup>

# Effects on strange form factors global analysis of PVES data for $Q^2 < 0.3 \text{ GeV}^2$



$$G_E^s = 0.0025 \pm 0.0182$$
  
 $G_M^s = -0.011 \pm 0.254$   
at  $Q^2 = 0.1 \text{ GeV}^2$ 

Young et al., PRL 97 (2006) 102002

#### including TBE corrections:

$$G_E^s = 0.0023 \pm 0.0182$$
  
 $G_M^s = -0.020 \pm 0.254$ 

at  $Q^2 = 0.1 \text{ GeV}^2$ 

qualitative result does not change

even more recent data, from HAPPEX experiment at JLab (H and <sup>4</sup>He targets)



#### combining new HAPPEX results with global data



#### combining new HAPPEX results with global data



Young et al., PRL 99 (2008) 122003

#### combining new HAPPEX results with global data



Young et al., PRL 99 (2008) 122003

strangeness content of nucleon very small

electromagnetic structure is valence quark dominated

Constraints on "new physics" expand asymmetry in powers of  $Q^2$  at low  $Q^2$ 



Constraints on "new physics" expand asymmetry in powers of  $Q^2$  at low  $Q^2$ 



Constraints on "new physics" expand asymmetry in powers of  $Q^2$  at low  $Q^2$ 



proton weak charge

 $Q_{W}^{p} = G_{E}^{Zp}(0)$  $= -2 (2C_{1u} + C_{1d})$  $= 1 - 4 \sin^{2} \theta_{W}$ 

Constraints on "new physics" expand asymmetry in powers of  $Q^2$  at low  $Q^2$ 



 $C_{1d} = g_A^e \ g_V^d = +\frac{1}{2} - \frac{2}{3}\sin^2\theta_W$ 











constraints complementary to LHC potential
## Summary

- **TPE corrections resolve most of Rosenbluth / PT**  $G_E^p/G_M^p$  discrepancy
  - → excited state contributions  $(\Delta, P_{11}(1440), S_{11}(1535), ...)$ small relative to nucleon
- Reanalysis of global data, including TPE from the outset
  - $\rightarrow$  first consistent form factor fit at order  $\alpha^3$
  - → "25% less charge" in the center of the proton
- Precise measurement of strange form factor
  - → very small (consistent with zero!)
  - $\rightarrow$  photon-Z exchange gives ~2% corrections
  - $\rightarrow$  constrains "new physics" to above ~ 1 TeV

The End

### Research opportunities at JLab

Ph. D. studies

www.jlab.org

- → through nearby universities (William & Mary, Old Dominion, etc.)
- $\rightarrow$  "sandwich" doctorate from Brazil (~ 1 year at JLab)
- Undergraduate summer\* internships
  - $\rightarrow$  ~ 3 months research experience at JLab (June-August)
- HUGS (Hampton University Graduate Studies) summer\* school
  - $\rightarrow$  annual 3 week school at JLab for graduate students
- Contact <u>wmelnitc@jlab.org</u> for more information

\* northern summer

# Obrigado!

## Obrigado!

#### Boa sorte!