Outline

Lecture 2

- Quark-hadron duality

- "Bloom-Gilman" duality in structure functions

- Duality in QCD

- Resonances & local quark-hadron duality

  $\longrightarrow$ "truncated" moments in QCD

- Duality in the neutron

  $\longrightarrow$ extraction of neutron resonance structure from nuclear data

Quark-hadron duality

Quark-hadron duality

Complementarity between *quark* and *hadron* descriptions of observables

$$\sum_{hadrons} = \sum_{quarks}$$

Can use either set of complete basis states to describe all physical phenomena

Quark-hadron duality

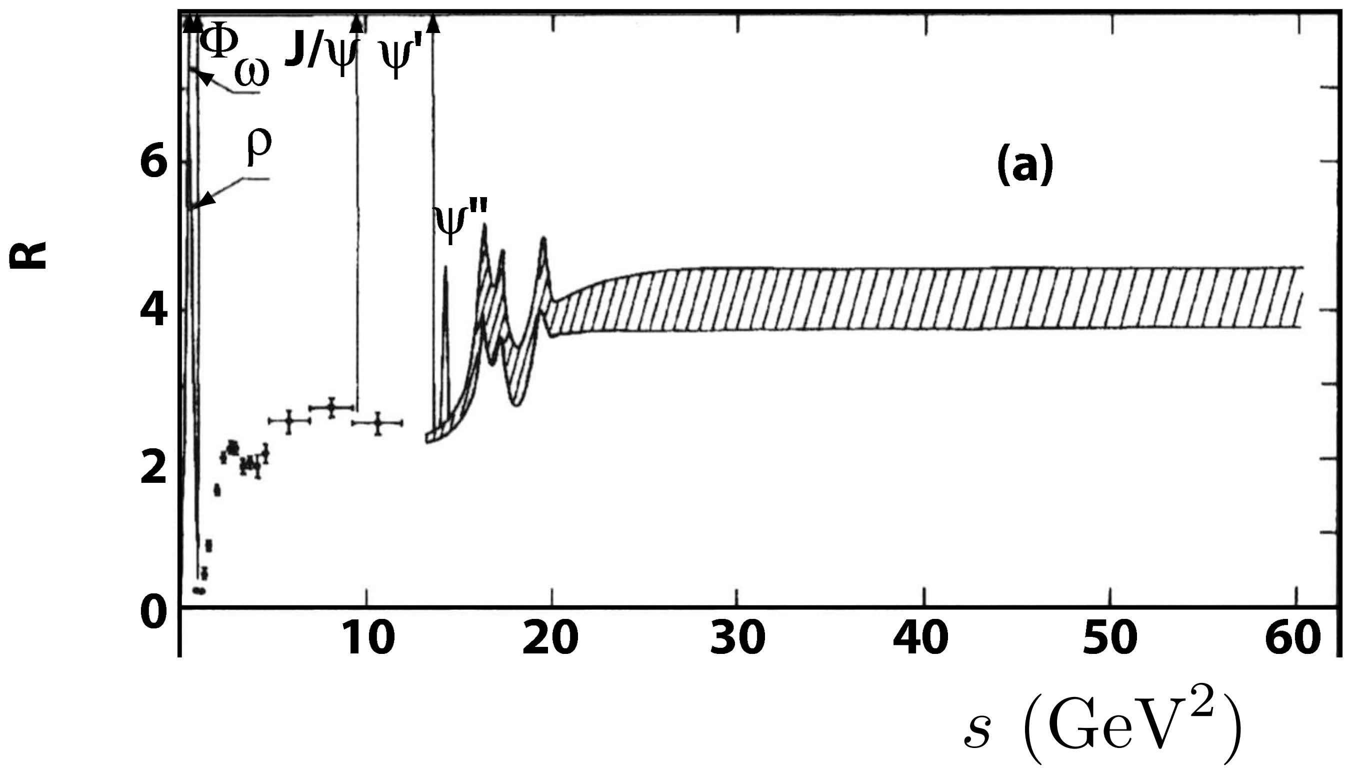
- Duality between quarks *(high energy)* and hadrons *(low energy)* manifests itself in many processes

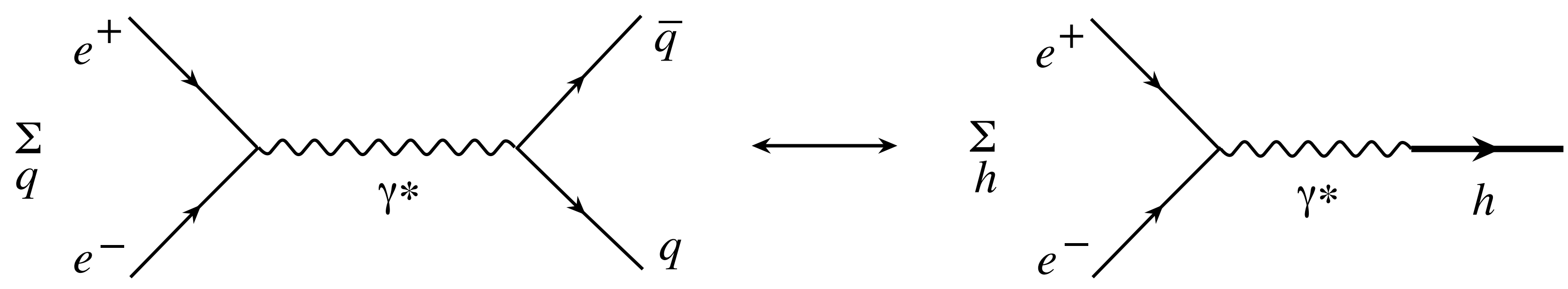
Quark-hadron duality

- Duality between quarks *(high energy)* and hadrons *(low energy)* manifests itself in many processes

- *e*+ *e*- annihilation
  - *total hadronic cross section at high energy averages resonance cross section*

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



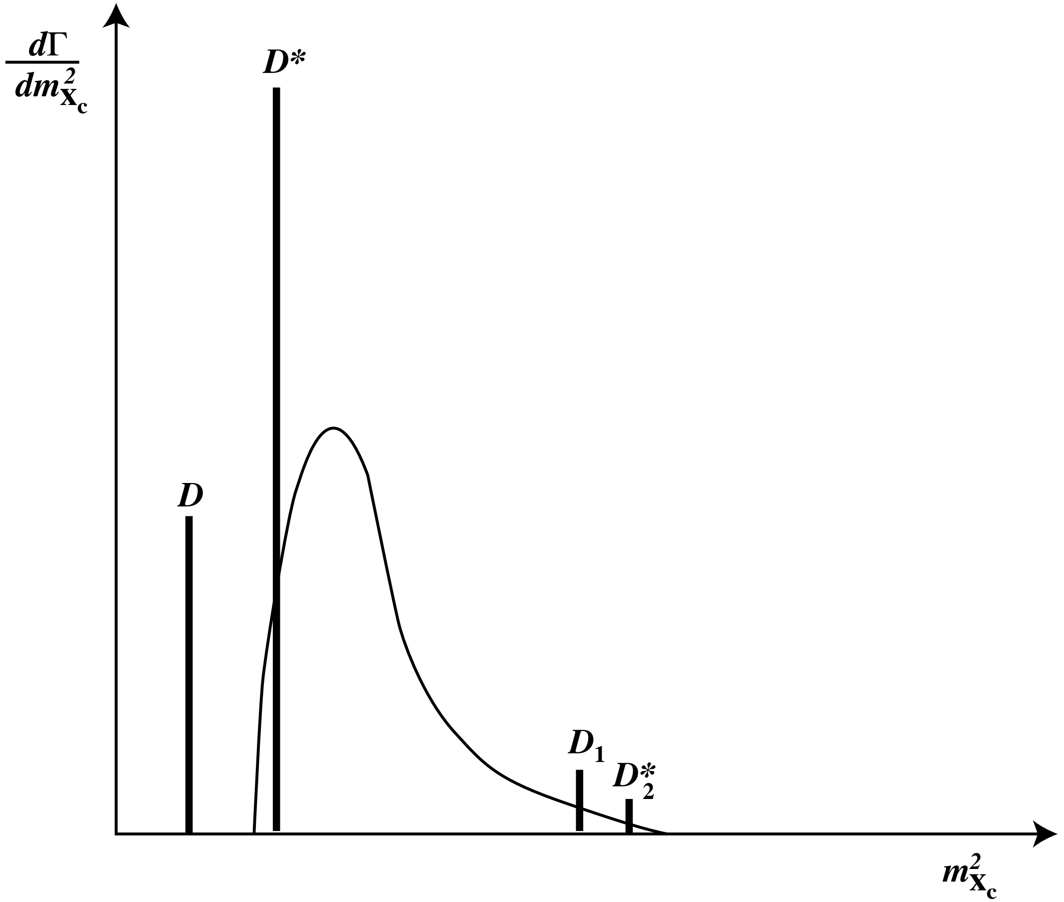


Quark-hadron duality

- Duality between quarks *(high energy)* and hadrons *(low energy)* manifests itself in many processes

- e+ e- annihilation
  - *total hadronic cross section at high energy averages resonance cross section*

- Heavy meson decays
  - *duality between hadronic & quark descriptions of decays in* $m_Q \to \infty$ *limit*



$$B \to D\, l\, \nu$$

$$\Updownarrow$$

$$b \to c\, l\, \nu$$

Voloshin, Shifman, Sov. J. Nucl. Phys. 41 (1985) 120
Isgur, Phys. Lett. B448 (1999) 111

Quark-hadron duality

- Duality between quarks *(high energy)* and hadrons *(low energy)* manifests itself in many processes

- e+ e- annihilation
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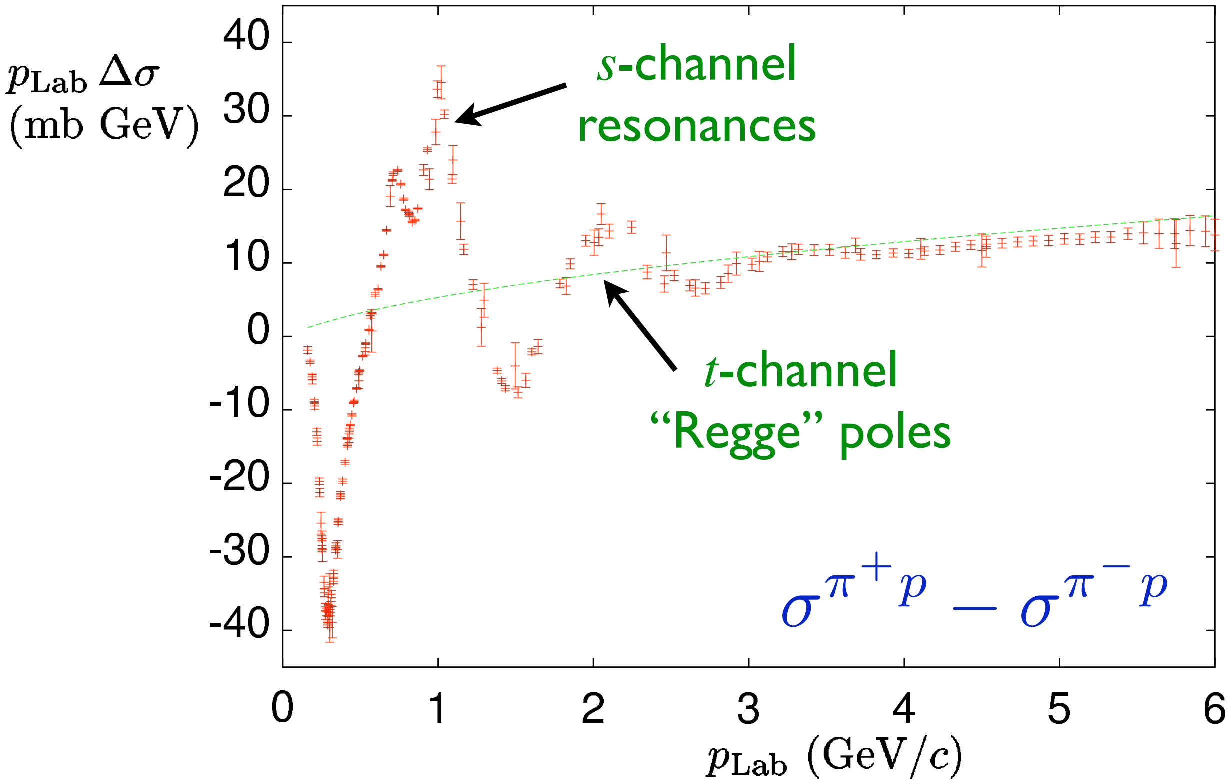
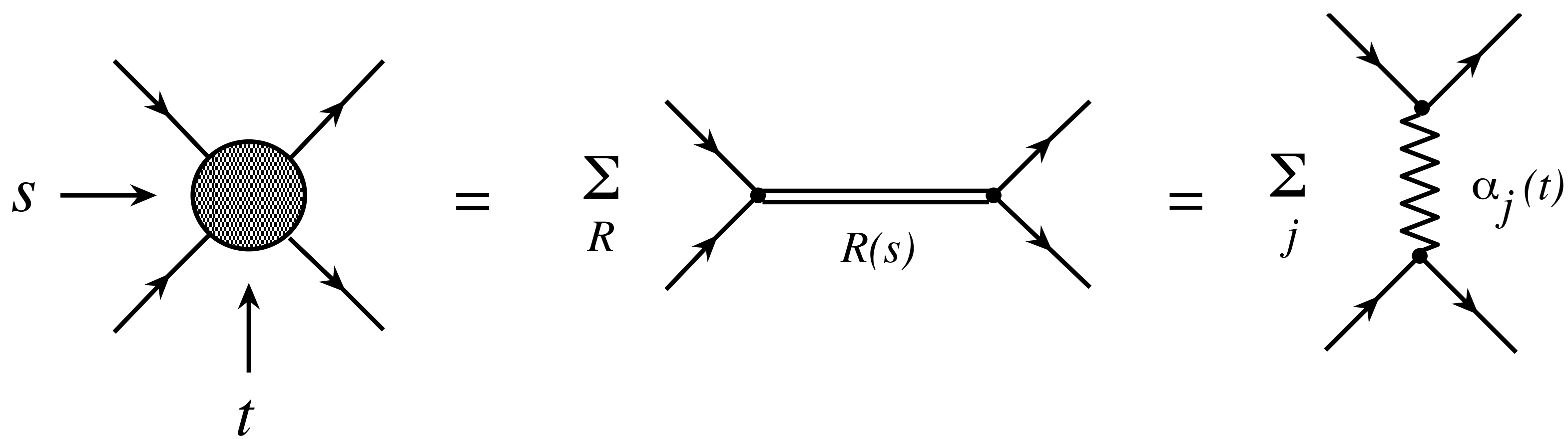
- Heavy meson decays
  - *duality between hadronic & quark descriptions of decays in* $m_Q \to \infty$ *limit*

- Duality between s-channel resonances and t-channel (Regge) poles in hadronic reactions





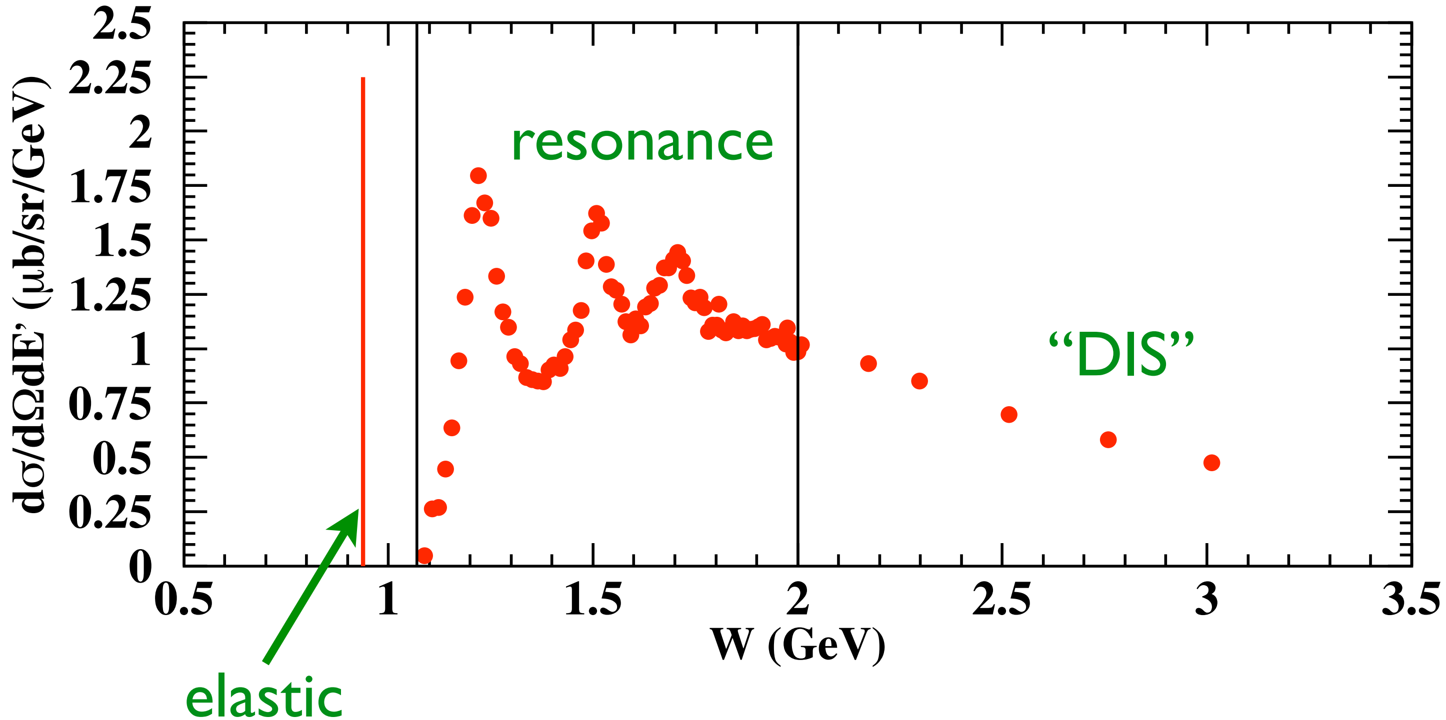
“Finite energy sum rules”

Igi (1962), Dolen, Horn, Schmidt (1968)

Quark-hadron duality in structure functions

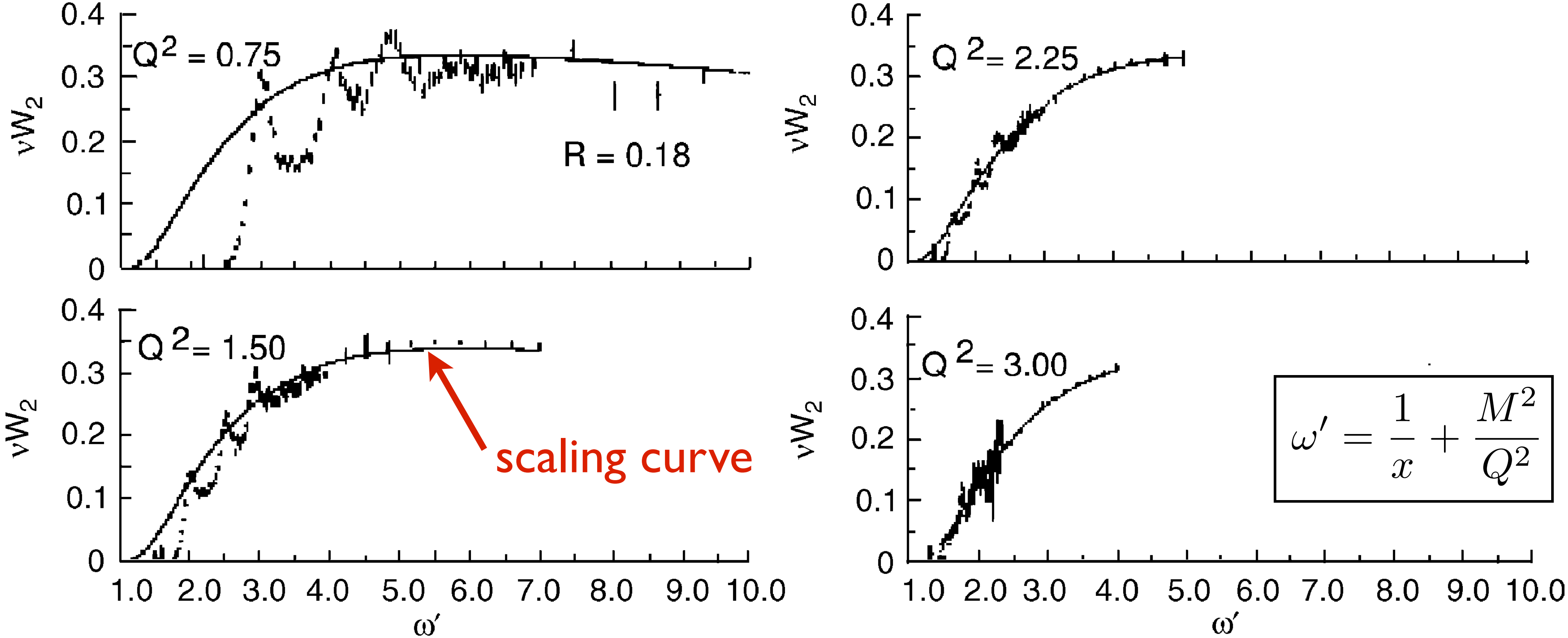
Structure functions in the resonance region

As W decreases, DIS region gives way to region dominated by nucleon resonances



$$x = \frac{Q^2}{W^2 - M^2 + Q^2}$$

"Bloom-Gilman" duality



Bloom, Gilman, Phys. Rev. Lett. 85 (1970) 1185

resonance – scaling duality in proton $\nu W_2 = F_2$ structure function

"Bloom-Gilman" duality

Average over (strongly Q^2 dependent) resonances
$\approx Q^2$ independent scaling function

"Finite energy sum rule" for eN scattering

$$\frac{2M}{Q^2}\int_0^{\nu_m} d\nu\ \nu W_2(\nu, Q^2) = \int_1^{\omega'_m} d\omega'\ \nu W_2(\omega')$$

measured structure function
(function of ν and Q^2)

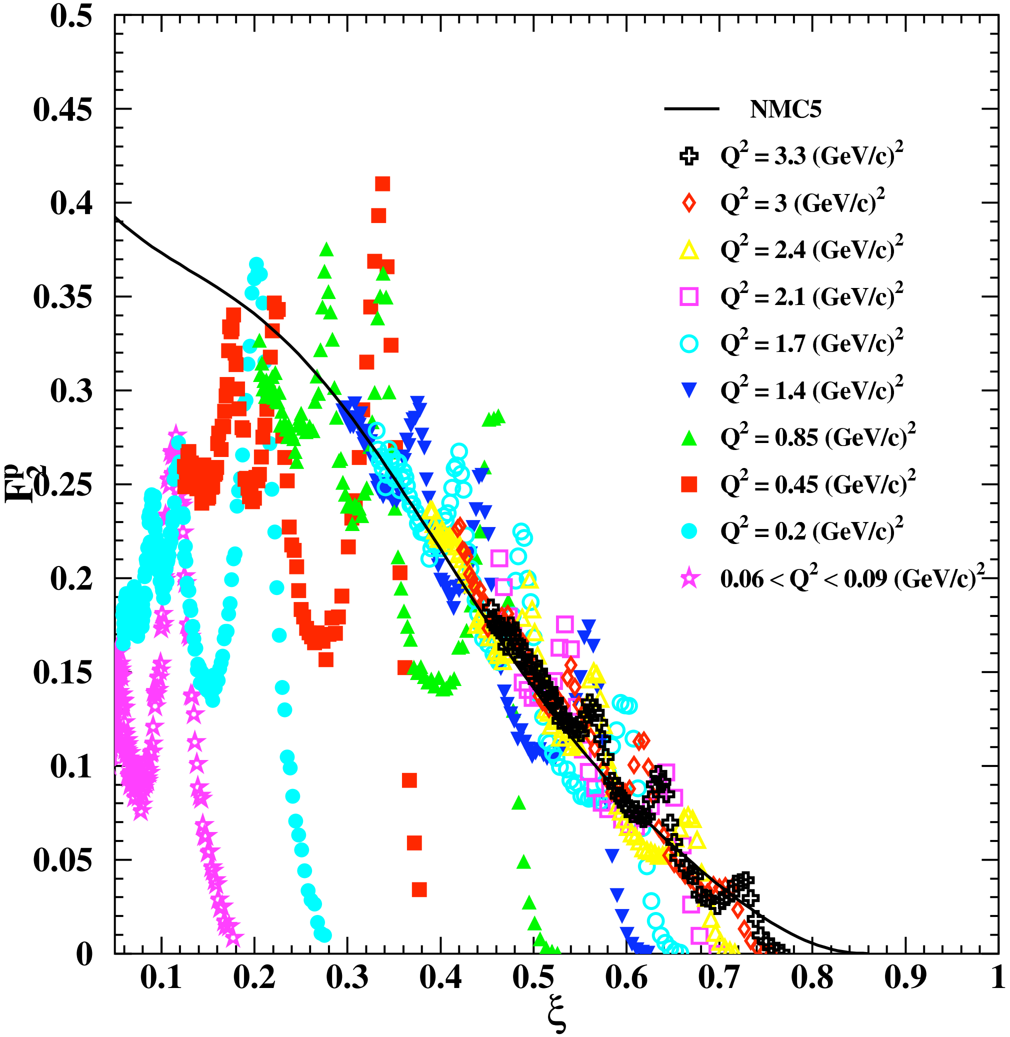
"hadrons"

$$\omega' = \frac{1}{x} + \frac{M^2}{Q^2}$$

scaling function
(function of ω' only)

"quarks"

"Bloom-Gilman" duality



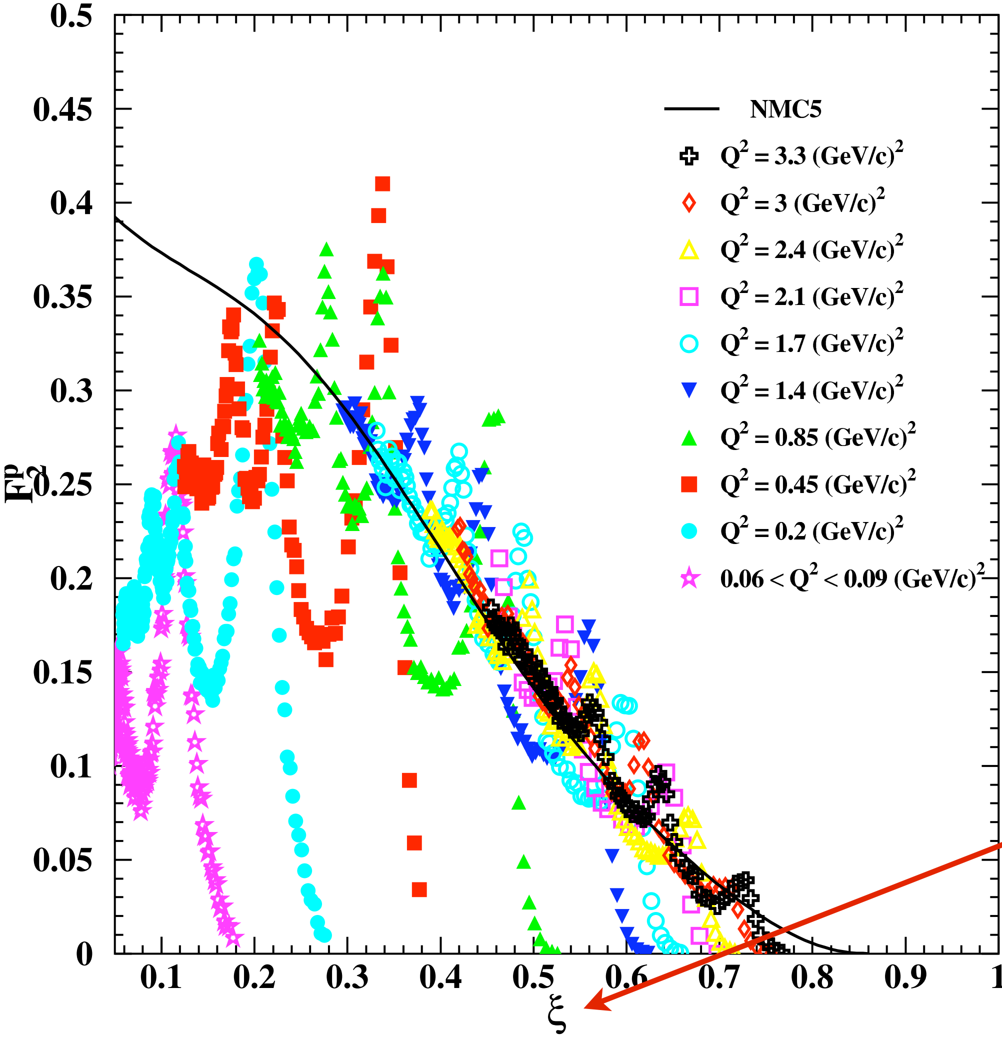
Jefferson Lab (Hall C)

Niculescu et al., Phys. Rev. Lett. 85 (2000) 1182

Average over (strongly Q^2 dependent) resonances

$\approx Q^2$ independent scaling function

“Bloom-Gilman” duality



Jefferson Lab (Hall C)

Niculescu et al., Phys. Rev. Lett. 85 (2000) 1182

Average over (strongly Q^2 dependent) resonances $\approx Q^2$ independent scaling function

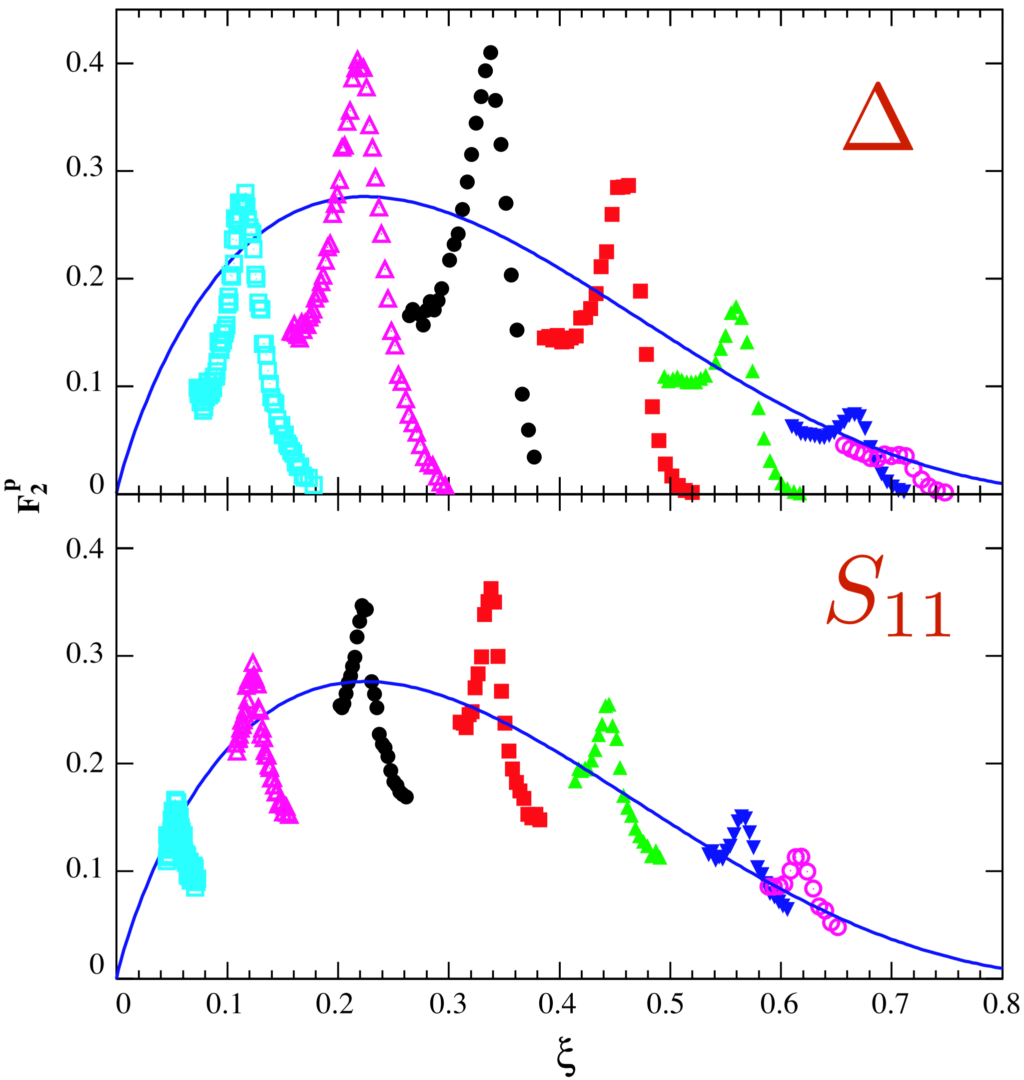
“Nachtmann scaling variable”

$$\xi = \frac{2x}{1 + \sqrt{1 + 4M^2x^2/Q^2}}$$

see also:

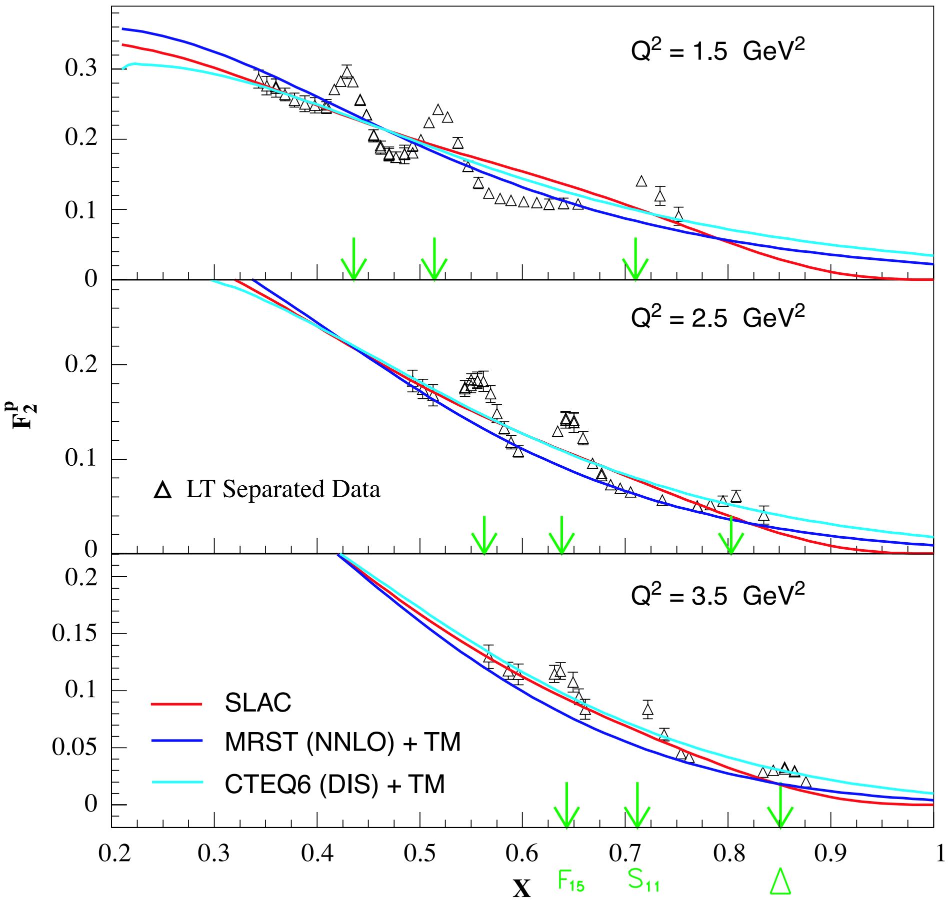
Fritzsch, Proc. of the Coral Gables Conf. (1971)
Greenberg & Bhaumik, PRD4 (1971) 2048

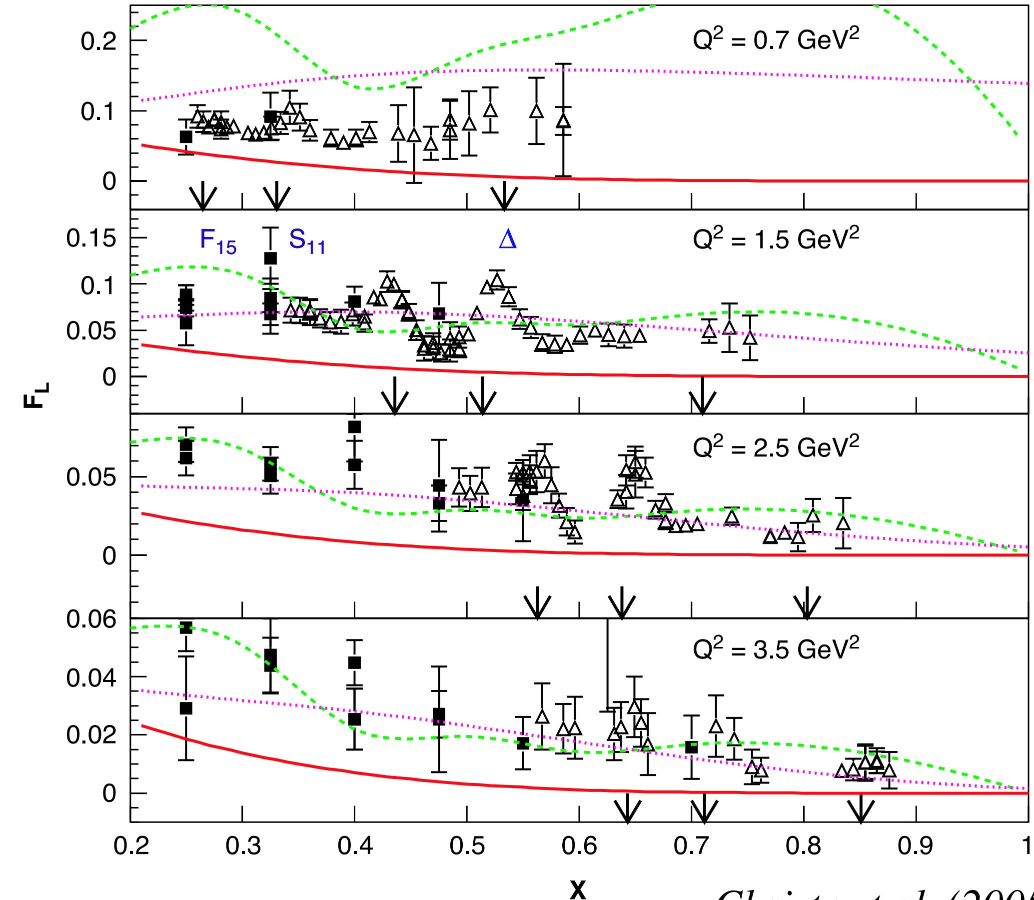
■ Duality exists also in local regions, around individual resonances



→ *"local"* Bloom-Gilman duality

Local Bloom-Gilman duality





Christy et al. (2005)

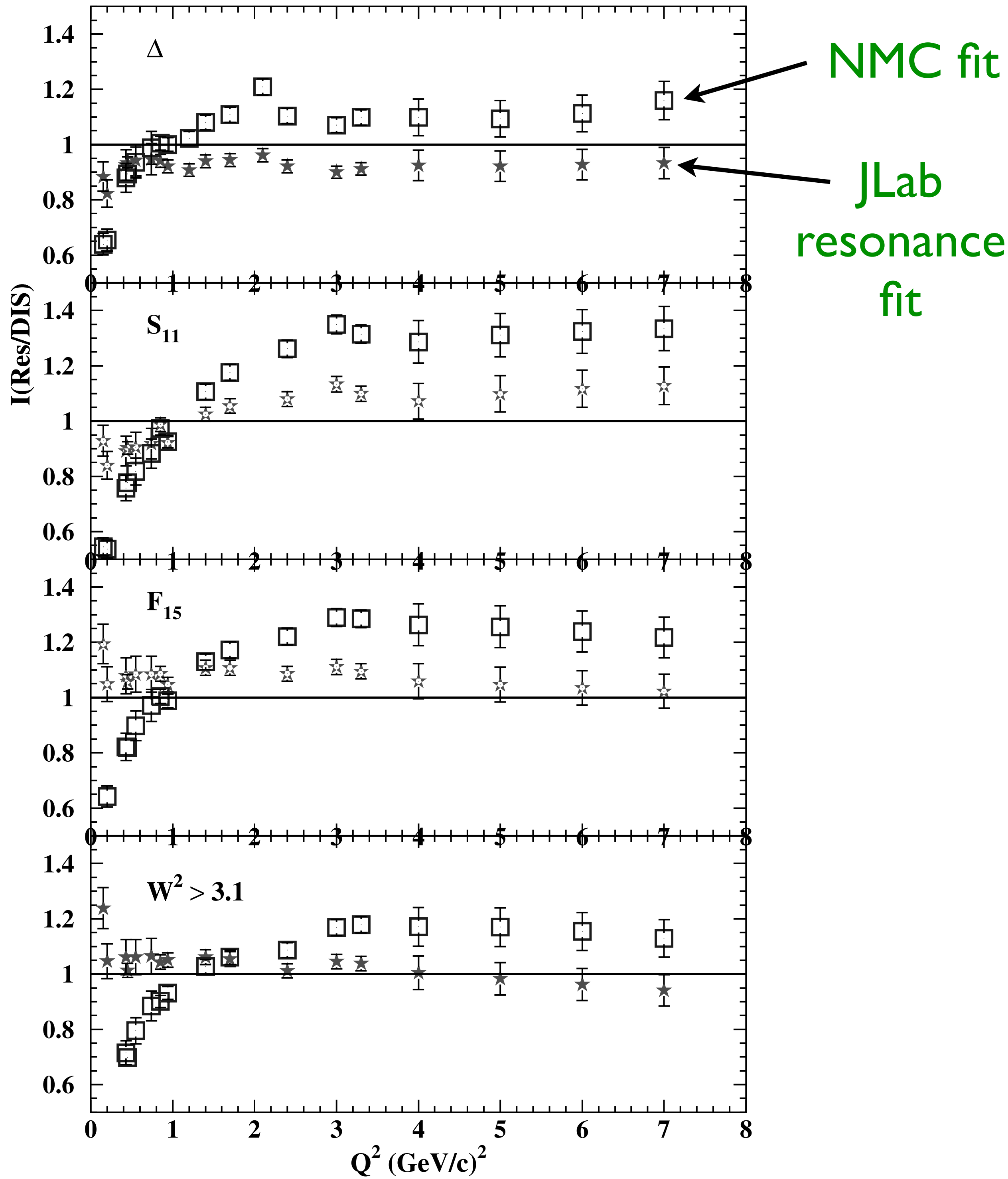
duality in F_2 and F_L structure functions

(from longitudinal-transverse separation)

⇒ importance of target mass corrections

*Steffens, WM, PRC **73** (2006) 055202*

Integrated strength

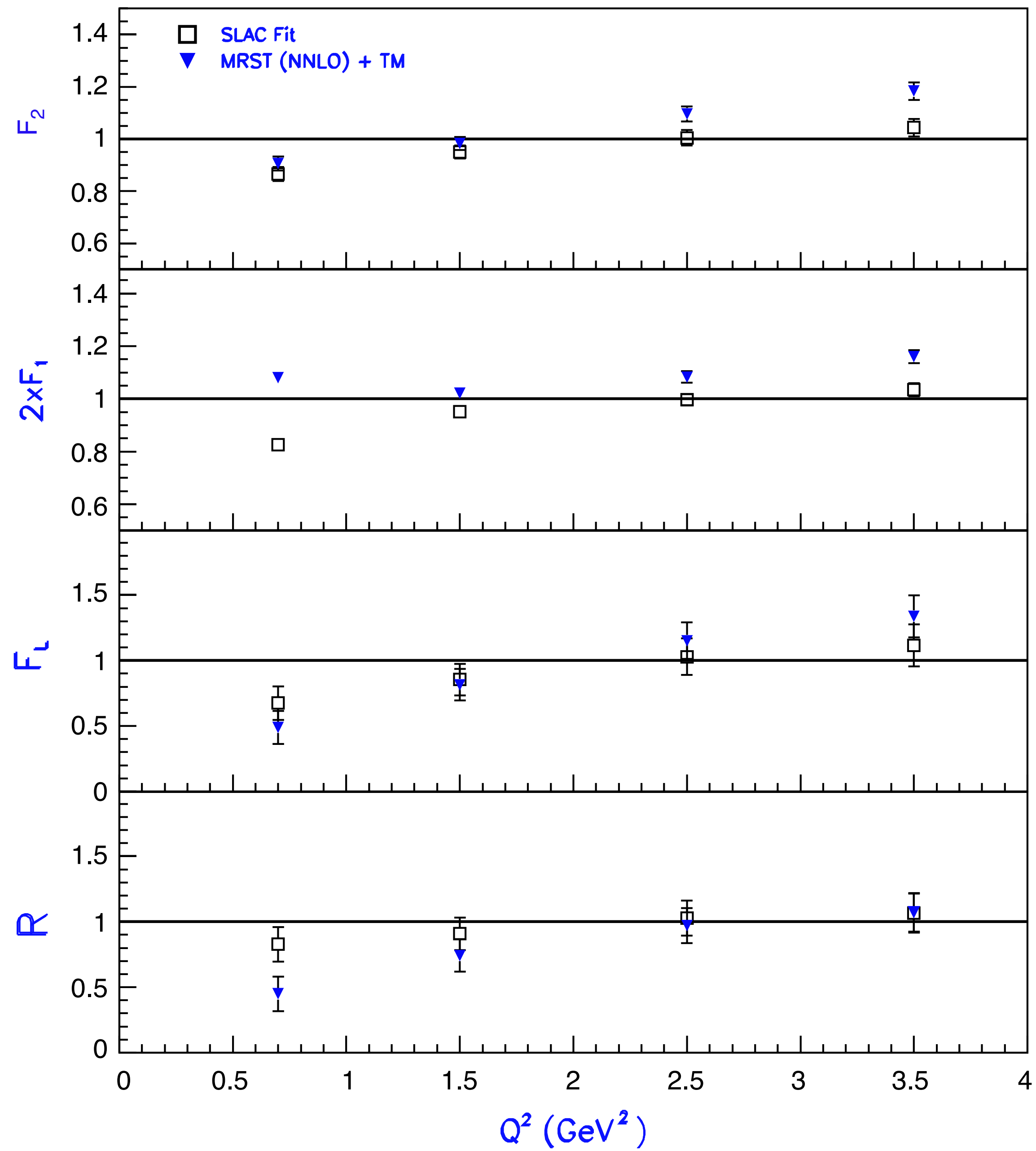


~ 10% agreement for $Q^2 > 1$ GeV^2

Niculescu et al, Phys. Rev. Lett. 85 (2000) 1186

Moments

data from longitudinal-transverse separation !



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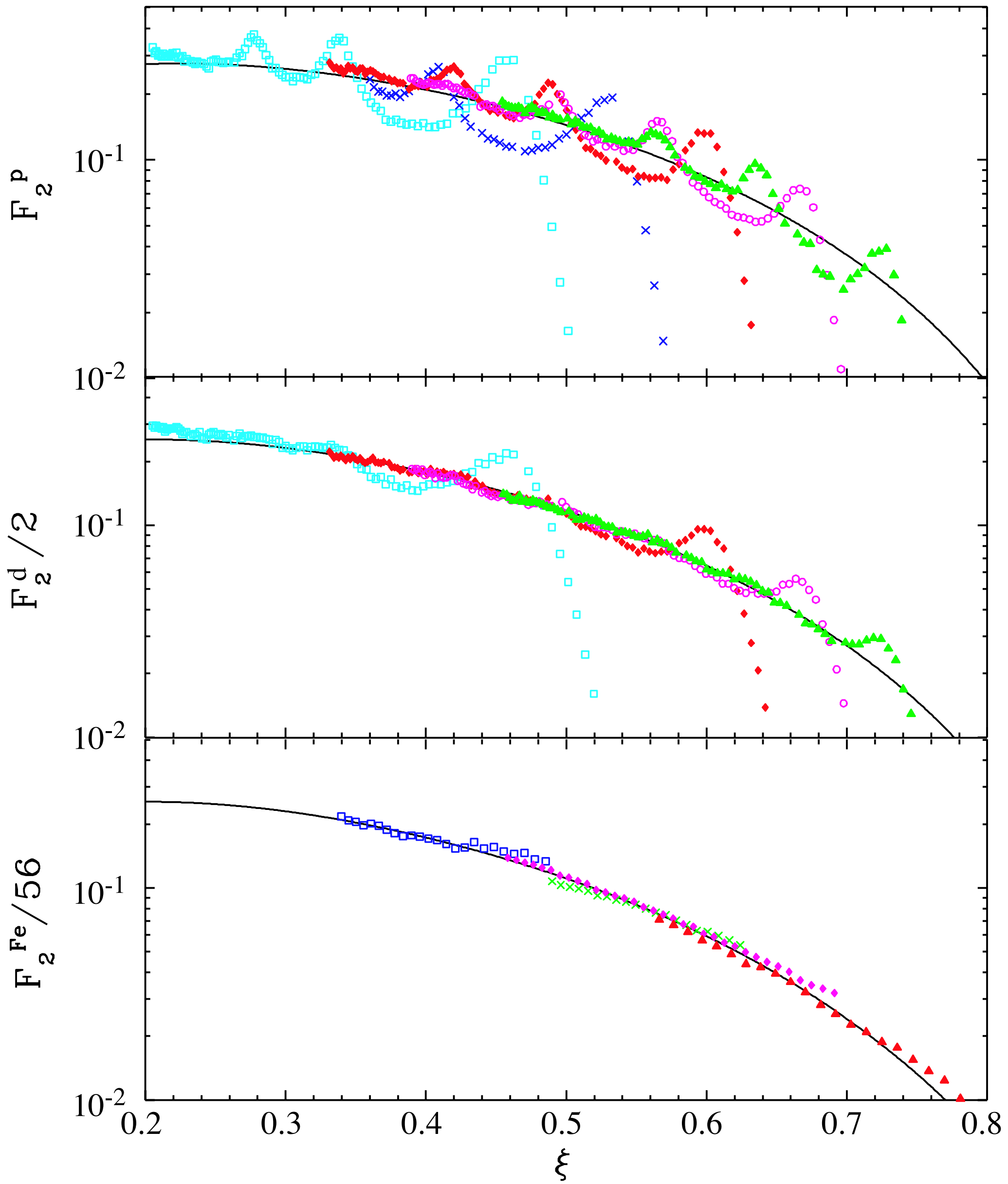
Nuclear structure functions



for larger nuclei, Fermi motion does resonance averaging automatically !

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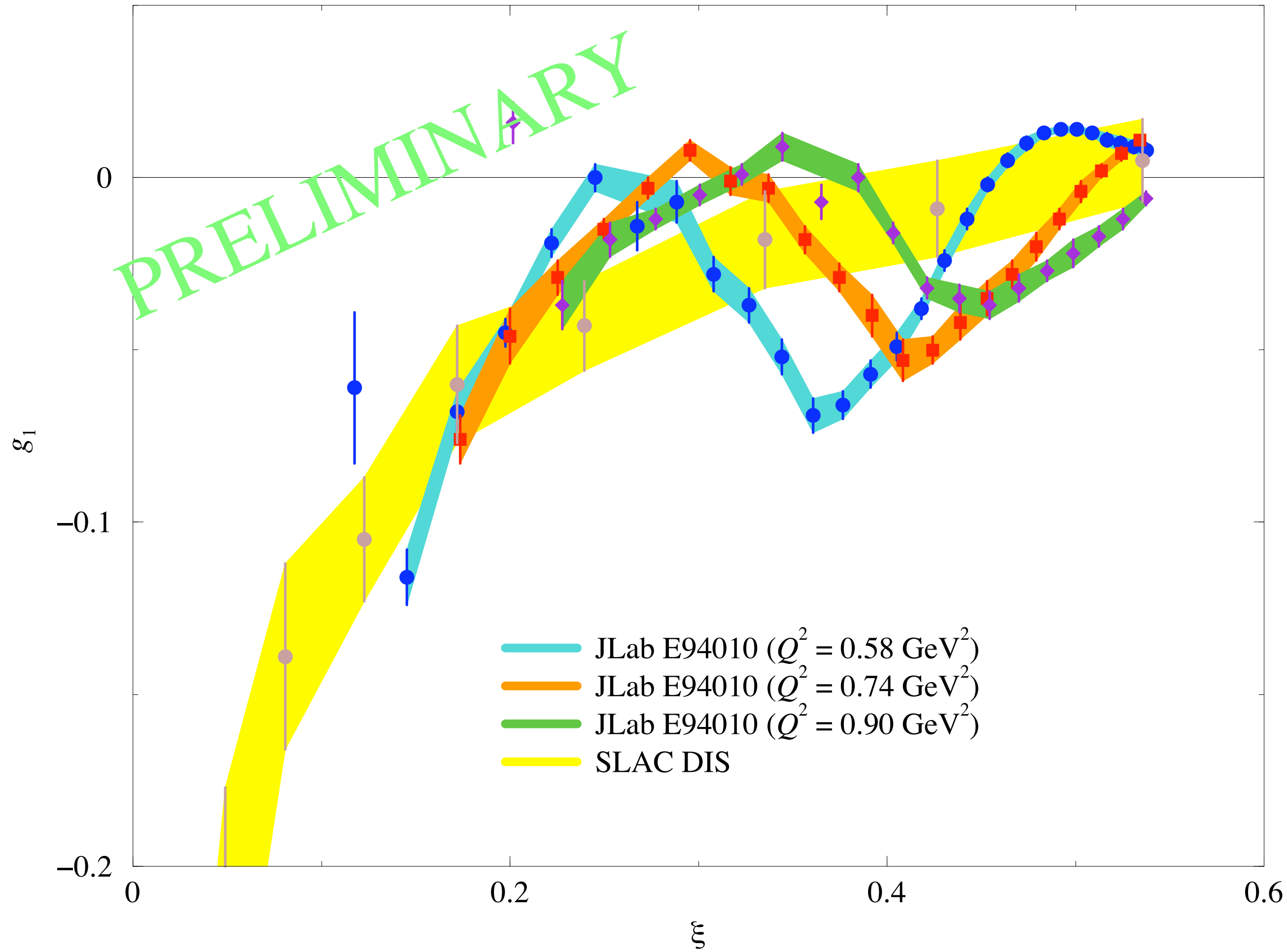
Spin-dependent structure functions



neutron (^{3}He) g_1 structure function

Solvignon et al., PRL 101, 182502 (2008)

Duality in QCD

Duality in QCD

- Operator product expansion

  ⟹ expand *moments* of structure functions in powers of $1/Q^2$

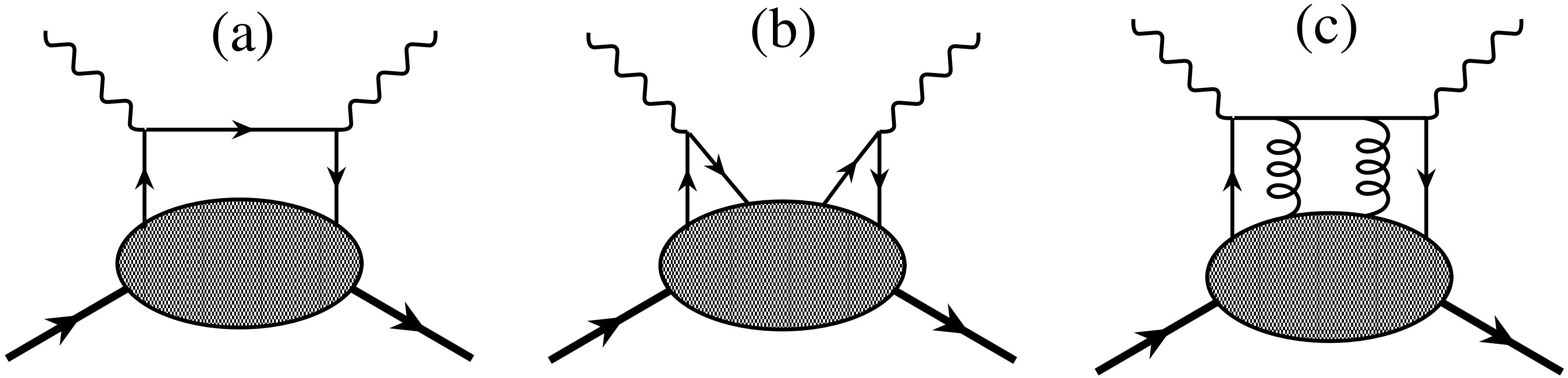
$$M_n(Q^2) = \int_0^1 dx\ x^{n-2}\ F_2(x,Q^2)$$

$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

matrix elements of operators with specific "twist" τ

$$\tau = \text{dimension} - \text{spin}$$

Duality in QCD



$\tau = 2$

single quark scattering

e.g. $\bar{\psi}\ \gamma_\mu\ \psi$

$\tau > 2$

qq and qg correlations

e.g. $\bar{\psi}\ \gamma_\mu\ \psi\ \bar{\psi}\ \gamma_\nu\ \psi$

or $\bar{\psi}\ \tilde{G}_{\mu\nu}\gamma^\nu\ \psi$

Duality in QCD

- Operator product expansion

  ⟹ expand *moments* of structure functions in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx\ x^{n-2}\ F_2(x,Q^2)$$
$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

If moment $\approx$ independent of Q^2

⟹ higher twist terms $A_n^{(\tau>2)}$ small

Duality in QCD

- Operator product expansion

  → expand *moments* of structure functions in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx\ x^{n-2}\ F_2(x, Q^2)$$

$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

Duality $\Longleftrightarrow$ suppression of higher twists

de Rujula, Georgi, Politzer, Ann. Phys. 103 (1975) 315

Duality in QCD

- Much of recent new data is in *resonance* region, $W < 2\ \mathrm{GeV}$

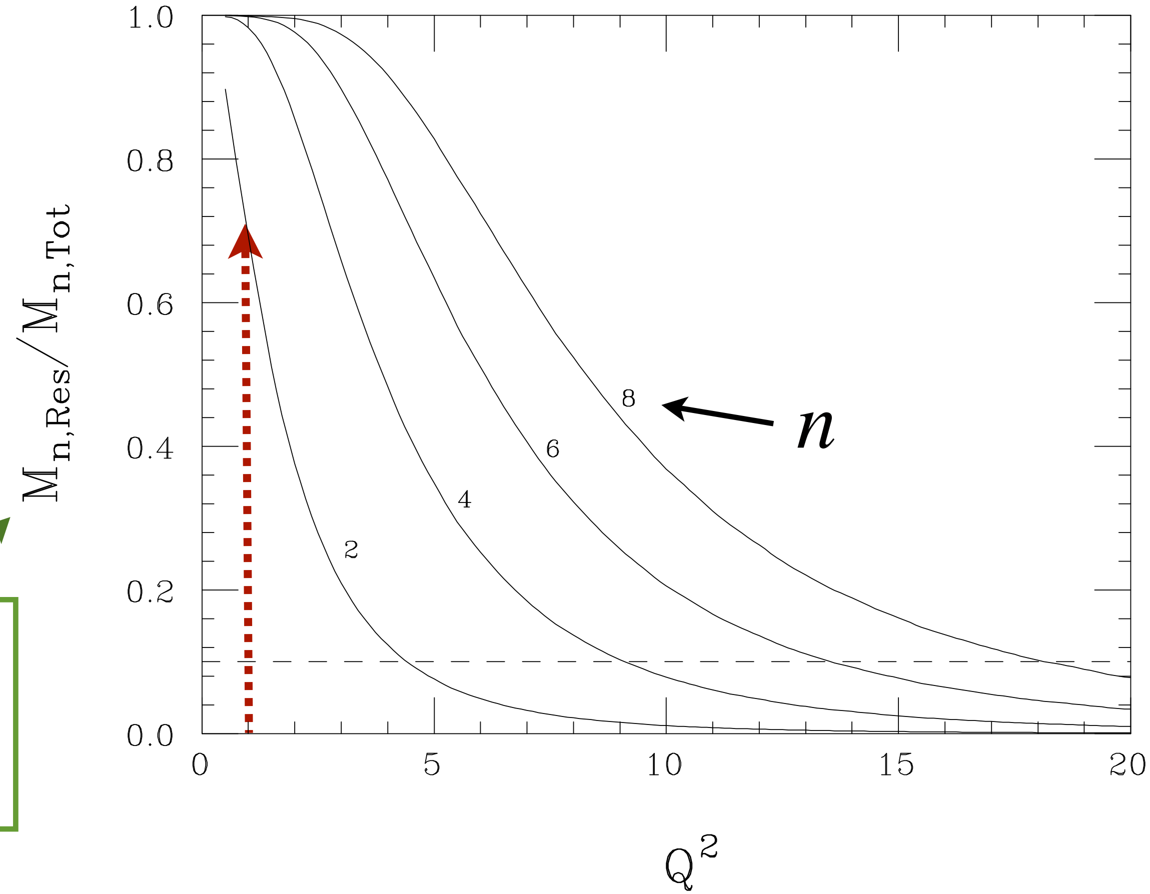
→ *common wisdom:* pQCD analysis not valid in resonance region

→ *in fact:* partonic interpretation of moments *does* include resonance region

- Resonances are an *integral part* of deep inelastic structure functions!

→ implicit role of quark-hadron duality

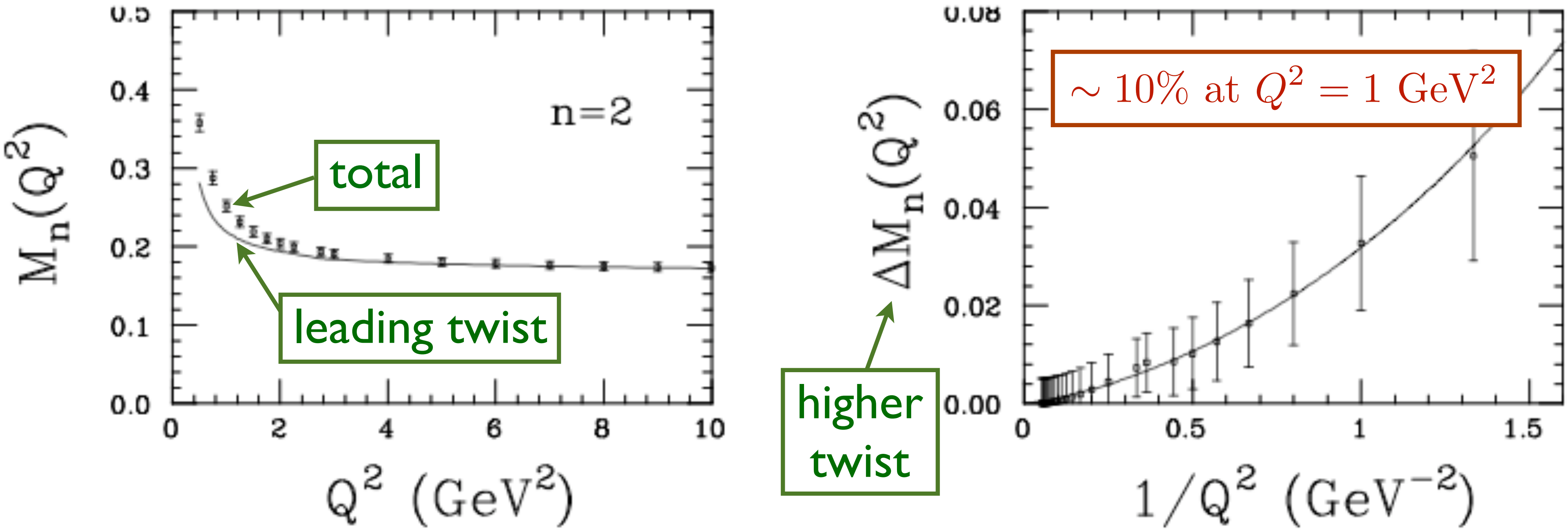
Proton moments



relative contribution of resonance region to n-th moment

At $Q^2 = 1\ \mathrm{GeV}^2$, $\sim$ 70% of lowest moment of F_2^p comes from $W < 2\ \mathrm{GeV}$

Proton moments



➡ BUT resonances and DIS continuum conspire to produce only $\sim$ 10% *higher twist* contribution!

Ji, Unrau,
Phys. Rev. D 52 (1995) 72

→ total higher twist *small* at $Q^2 \sim 1 - 2\ \mathrm{GeV}^2$

- on average, nonperturbative interactions between quarks and gluons not dominant at these scales

- suggests *strong cancellations* between resonances, resulting in dominance of *leading twist*

- OPE does not tell us *why* higher twists are small

  → need more detailed information (*e.g.* about individual resonances) to understand behavior dynamically

Duality & Truncated Moments

Truncated moments

- complete moments can be studied in pQCD via twist expansion

  → Bloom-Gilman duality has a precise meaning

  (*i.e.*, duality violation = higher twists)

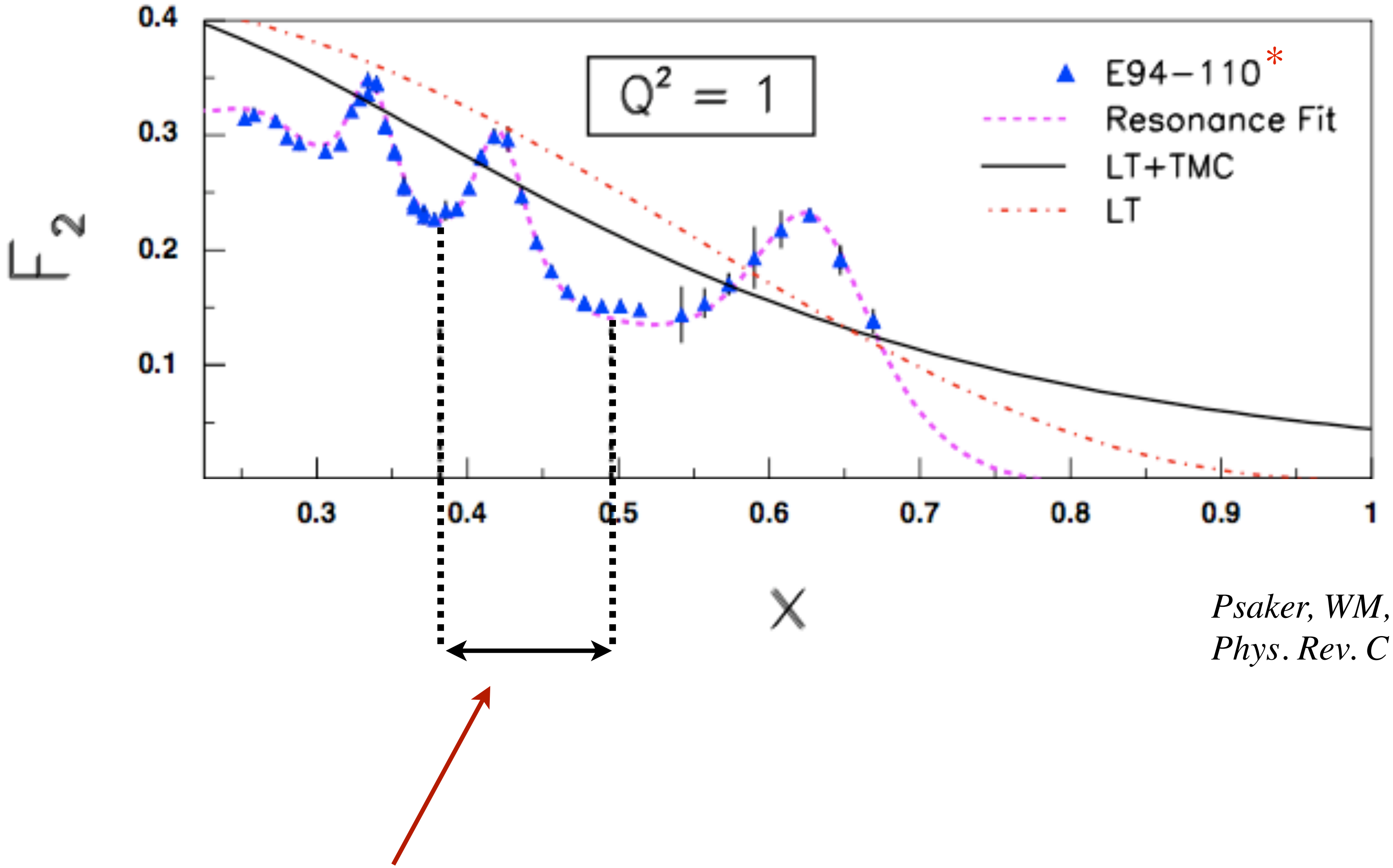
- for local duality, difficult to make rigorous connection with QCD

  → *e.g.* need prescription for how to average over resonances

- *truncated* moments allow study of restricted regions in x (or W) within pQCD in well-defined, systematic way

$$\overline{M}_n(\Delta x, Q^2) = \int_{\Delta x} dx\ x^{n-2}\ F_2(x, Q^2)$$

F_2^p resonance spectrum



* JLab Hall C

Psaker, WM, et al., Phys. Rev. C 78 (2008) 025206

how much of this region is *leading twist* ?

Truncated moments

- truncated moments obey DGLAP-like evolution equations, similar to PDFs

$$\frac{d\overline{M}_n(\Delta x, Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi}\left(P'_{(n)} \otimes \overline{M}_n\right)(\Delta x, Q^2)$$

where modified splitting function is

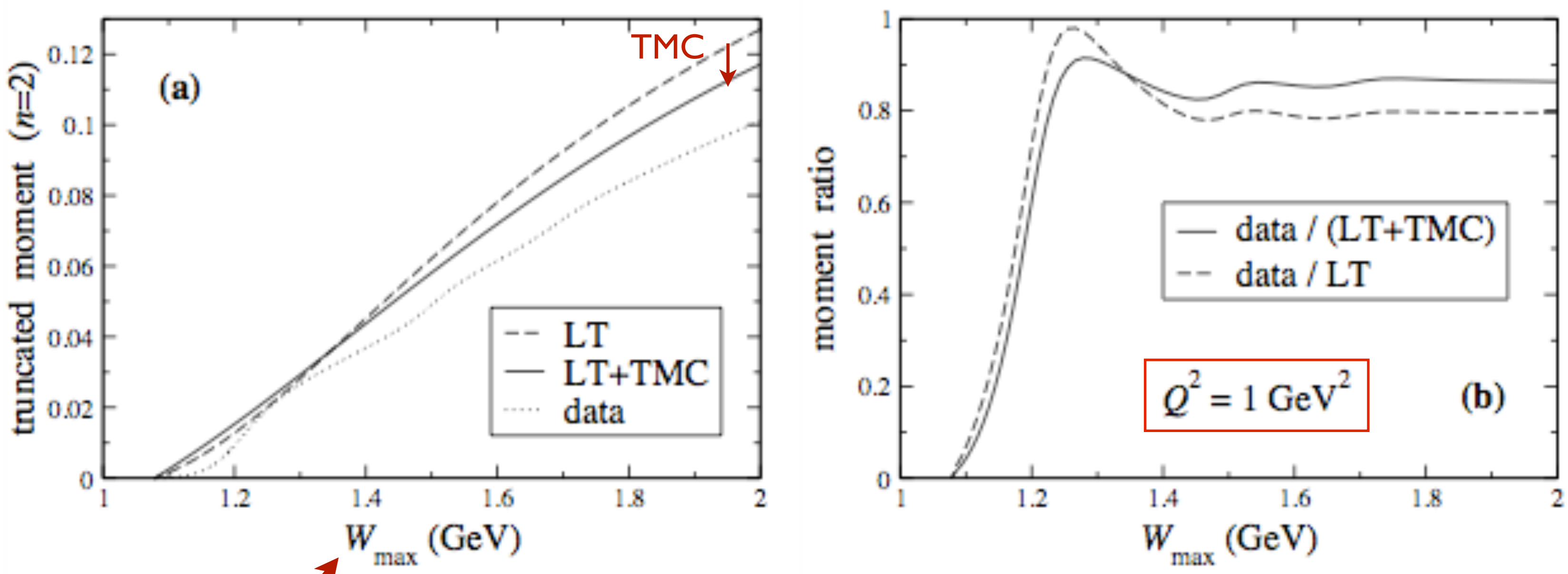
$$P'_{(n)}(z, \alpha_s) = z^n \; P_{NS,S}(z, \alpha_s)$$

→ can follow evolution of specific resonance (region) with Q^2 in pQCD framework!

→ suitable when complete moments not available

Data analysis

- assume data at large enough Q^2 are entirely leading twist
- evolve fit to data at large Q^2 down to lower Q^2
- apply target mass corrections (TMC) and compare with low-Q^2 data



$$W^2 = M^2 + \frac{Q^2}{x}(1-x)$$

Psaker, WM, et al.,
Phys. Rev. C 78 (2008) 025206

- consider individual resonance regions:

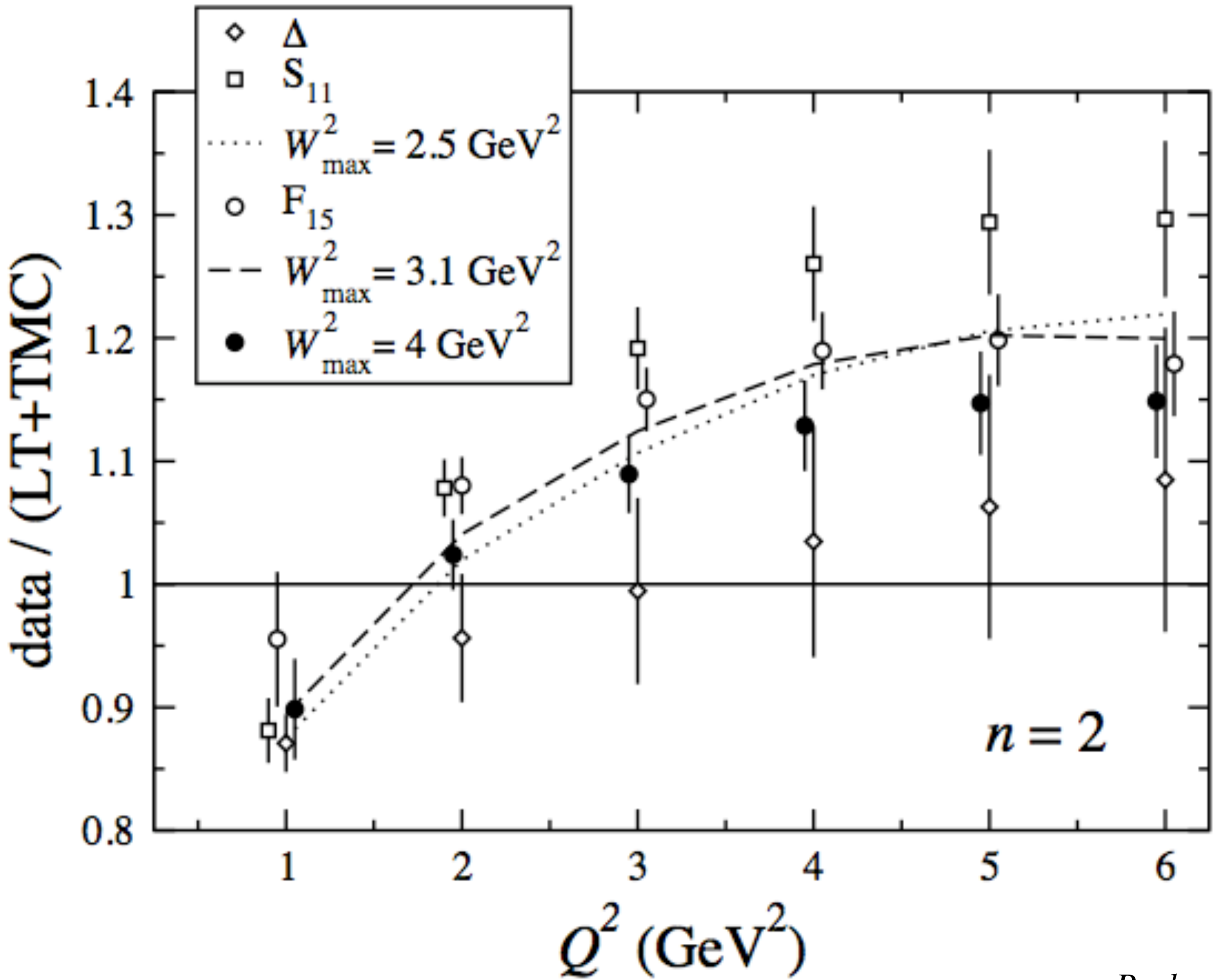
→ $W_{\rm thr}^2 < W^2 < 1.9\ {\rm GeV}^2$ "$\Delta(1232)$"

→ $1.9 < W^2 < 2.5\ {\rm GeV}^2$ "$S_{11}(1535)$"

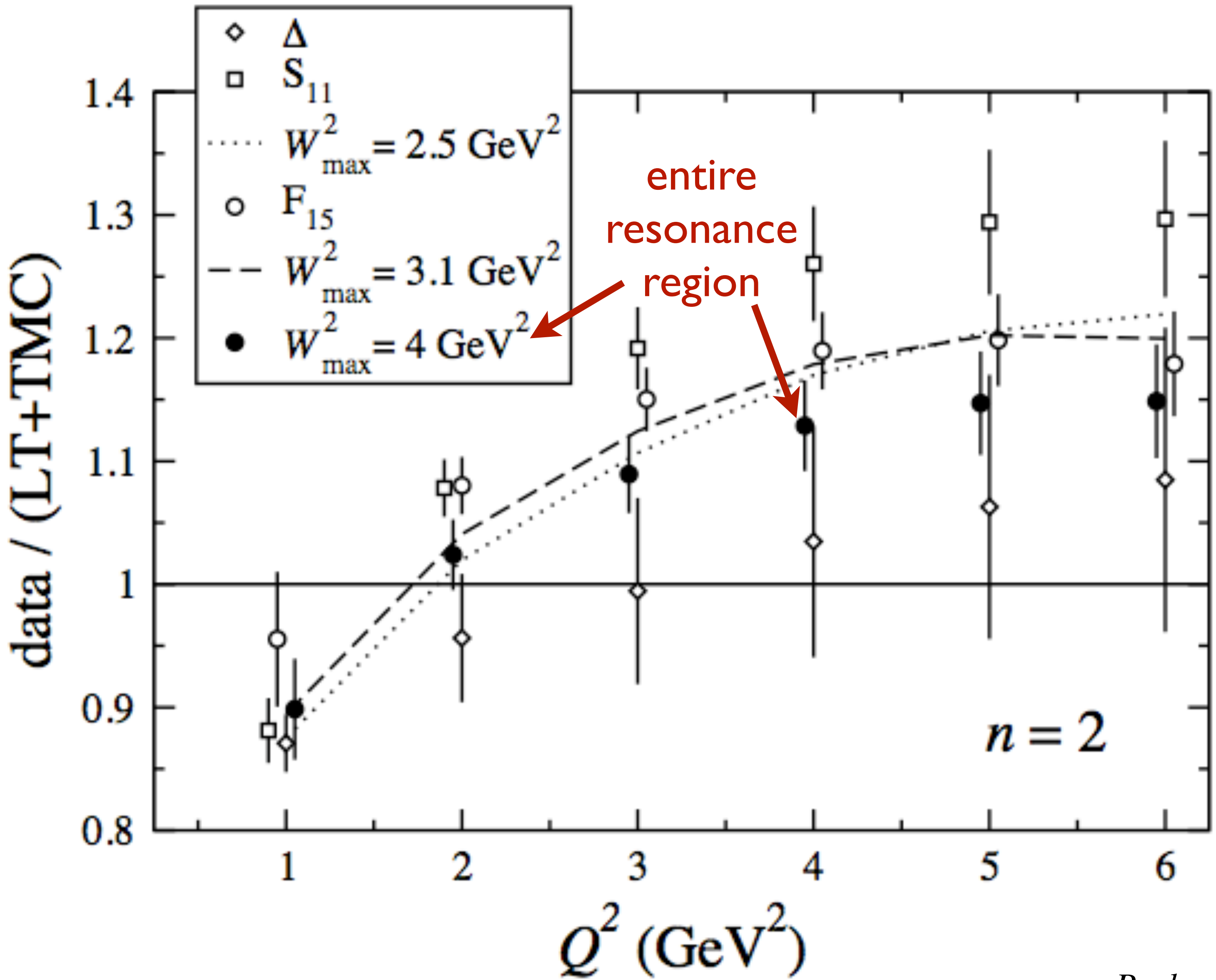
→ $2.5 < W^2 < 3.1\ {\rm GeV}^2$ "$F_{15}(1680)$"

as well as total resonance region:

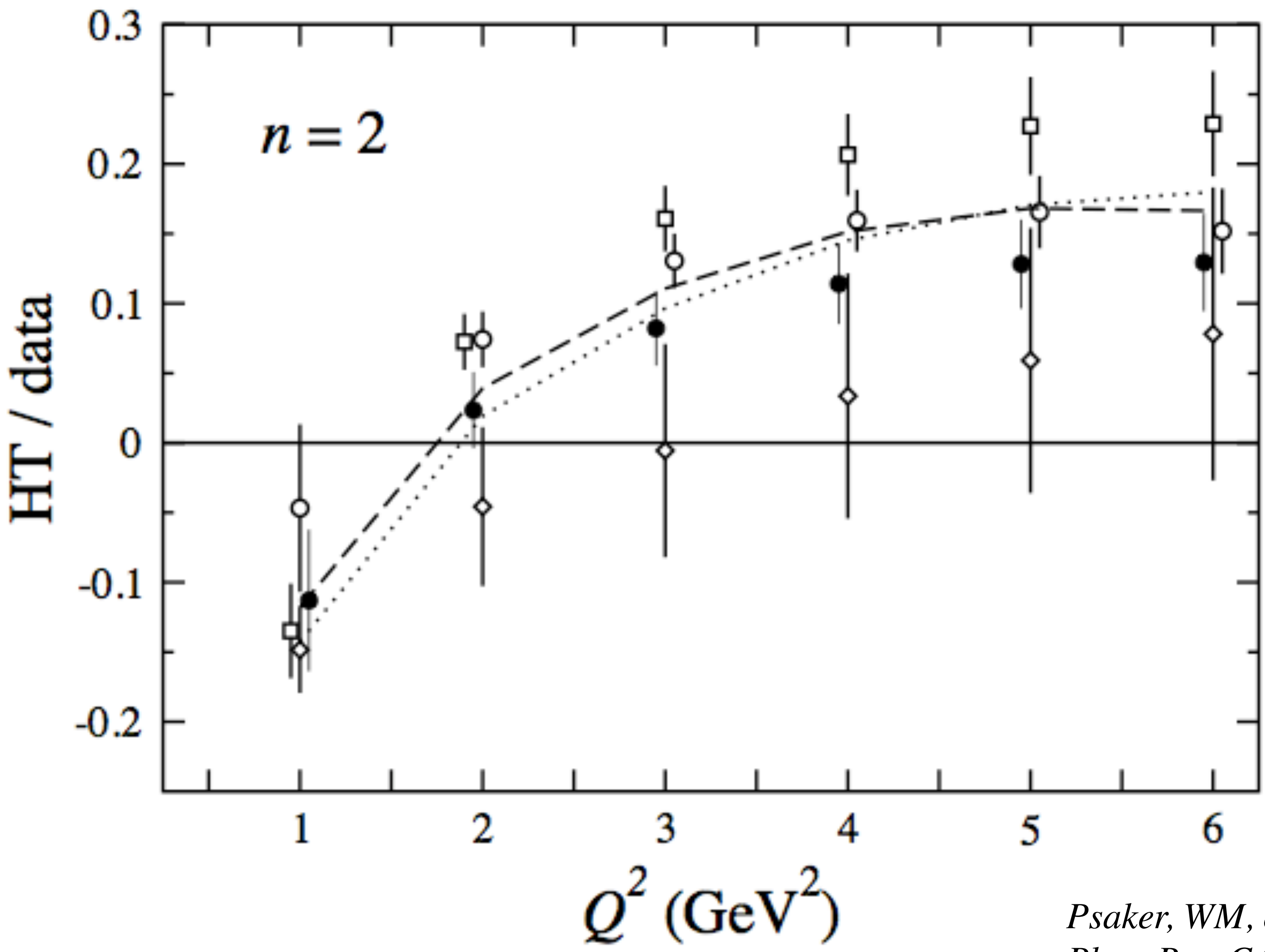
→ $W^2 < 4\ {\rm GeV}^2$



Psaker, WM, et al.,
Phys. Rev. C 78 (2008) 025206

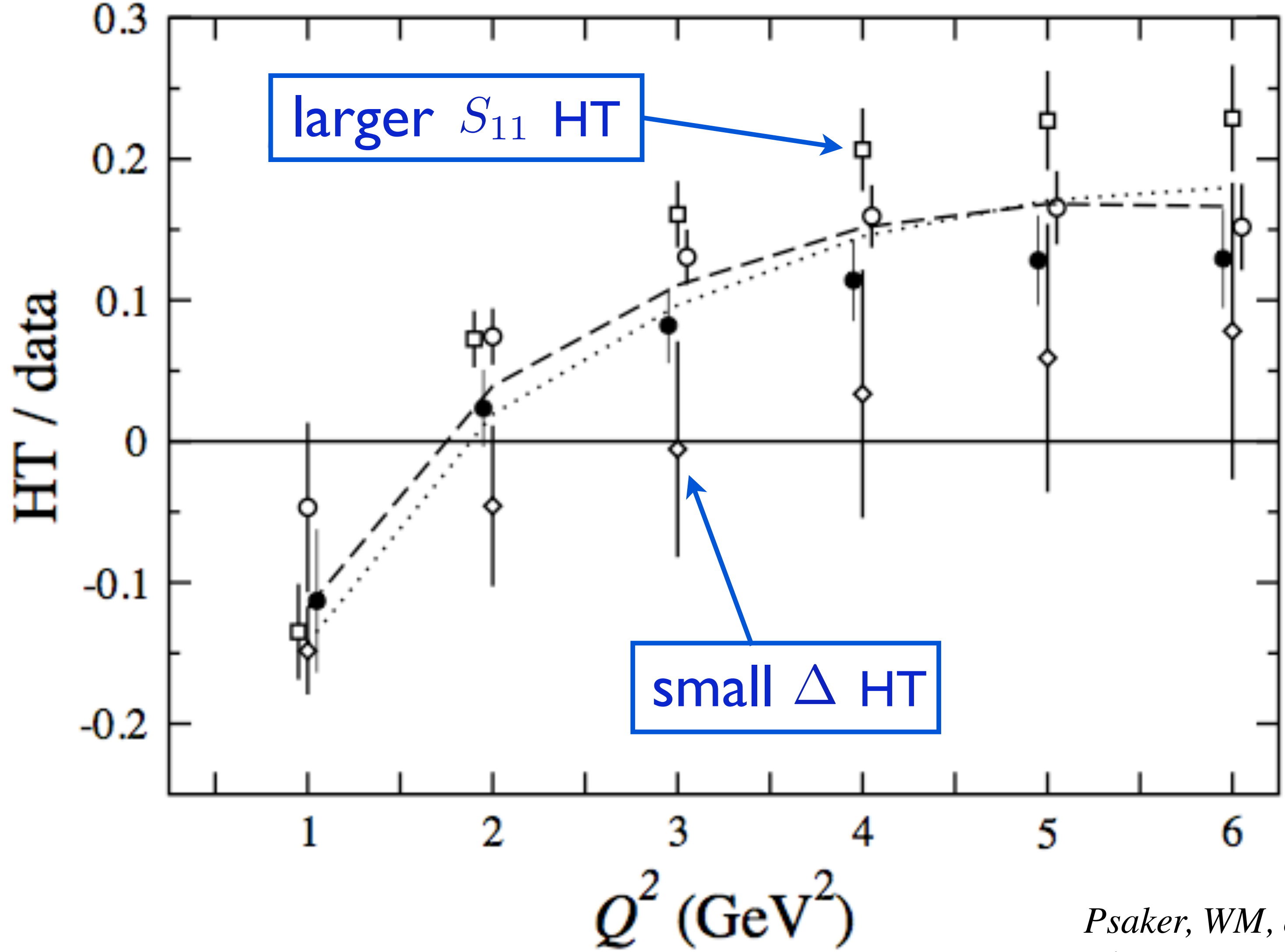


Psaker, WM, et al.,
Phys. Rev. C 78 (2008) 025206



Psaker, WM, et al.,
Phys. Rev. C 78 (2008) 025206

→ higher twists $< 10–15\%$ for $Q^2 > 1\ \mathrm{GeV}^2$



Psaker, WM, et al.,
Phys. Rev. C 78 (2008) 025206

→ higher twists $< 10\text{–}15\%$ for $Q^2 > 1\ \mathrm{GeV}^2$

Can we understand this behavior dynamically?

Can we understand this behavior dynamically?

How do cancellations between *coherent* resonances produce *incoherent* scaling function?

Coherence vs. incoherence

Exclusive form factors

→ *coherent* scattering from quarks

$$d\sigma \sim \left(\sum_i e_i \right)^2$$

Coherence vs. incoherence

Exclusive form factors

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Inclusive structure functions

→ *incoherent* scattering from quarks

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Coherence vs. incoherence

Exclusive form factors

⟹ *coherent* scattering from quarks

$$d\sigma \sim \left(\sum_i e_i \right)^2$$

Inclusive structure functions

⟹ *incoherent* scattering from quarks

$$d\sigma \sim \sum_i e_i^2$$

⟹ How can the *square of a sum* become the *sum of squares*?

Pedagogical model

Two quarks bound in a harmonic oscillator potential

$\Longrightarrow$ exactly solvable spectrum

Pedagogical model

Two quarks bound in a harmonic oscillator potential

$\Longrightarrow$ exactly solvable spectrum

Structure function given by sum of squares of transition form factors

$$F(\nu, \mathbf{q}^2) \;\sim\; \sum_n \left|G_{0,n}(\mathbf{q}^2)\right|^2 \;\delta(E_n - E_0 - \nu)$$

Pedagogical model

Two quarks bound in a harmonic oscillator potential

⟹ exactly solvable spectrum

Structure function given by sum of squares of transition form factors

$$F(\nu, \mathbf{q}^2) \;\sim\; \sum_n \left|G_{0,n}(\mathbf{q}^2)\right|^2 \;\delta(E_n - E_0 - \nu)$$

Charge operator $\Sigma_i \; e_i \exp(i\mathbf{q}\cdot\mathbf{r}_i)$ excites

even partial waves with strength $\propto (e_1 + e_2)^2$

odd partial waves with strength $\propto (e_1 - e_2)^2$

Pedagogical model

Resulting structure function

$$F(\nu, \mathbf{q}^2) \;\sim\; \sum_n \left\{ (e_1 + e_2)^2 \; G^2_{0,2n} + (e_1 - e_2)^2 \; G^2_{0,2n+1} \right\}$$

Pedagogical model

Resulting structure function

$$F(\nu, \mathbf{q}^2) \;\sim\; \sum_n \left\{ (e_1 + e_2)^2 \; G^2_{0,2n} + (e_1 - e_2)^2 \; G^2_{0,2n+1} \right\}$$

If states degenerate, cross terms $(\sim e_1 e_2)$ cancel when averaged over nearby even and odd parity states

Pedagogical model

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If states degenerate, cross terms $(\sim e_1 e_2)$ cancel when averaged over nearby even and odd parity states

Minimum condition for duality:

⟹ *at least one complete set of <u>even</u> and <u>odd</u> parity resonances must be summed over*

Close, Isgur, Phys. Lett. B509 (2001) 81

Quark model

- In NR Quark Model, even and odd parity states generalize to **56** (L=0) and **70** (L=1) multiplets of spin-flavor SU(6)

→ scaling occurs if contributions from **56** and **70** have equal overall strengths

Close, Nucl. Phys. B80, 269 (1974)
Isgur, Karl, Phys. Rev. D 18, 4187 (1978)

SU(6) symmetric proton wave function

$$
\begin{aligned}
p^{\uparrow} \; &= -\frac{1}{3} d^{\uparrow}(uu)_1 - \frac{\sqrt{2}}{3} d^{\downarrow}(uu)_1 \\
&\quad + \frac{\sqrt{2}}{6} u^{\uparrow}(ud)_1 - \frac{1}{3} u^{\downarrow}(ud)_1 + \frac{1}{\sqrt{2}} u^{\uparrow}(ud)_0
\end{aligned}
$$

Quark model

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Close, Nucl. Phys. B80, 269 (1974)
Isgur, Karl, Phys. Rev. D 18, 4187 (1978)

- Simplified case: magnetic coupling of γ^* to quark

  ⟹ expect dominance over electric at large Q^2

Quark model

- In NR Quark Model, even and odd parity states generalize to **56** (L=0) and **70** (L=1) multiplets of spin-flavor SU(6)

→ scaling occurs if contributions from **56** and **70** have equal overall strengths

representation	$^2\mathbf{8}[\mathbf{56}^+]$	$^4\mathbf{10}[\mathbf{56}^+]$	$^2\mathbf{8}[\mathbf{70}^-]$	$^4\mathbf{8}[\mathbf{70}^-]$	$^2\mathbf{10}[\mathbf{70}^-]$	Total
F_1^p	$9\rho^2$	$8\lambda^2$	$9\rho^2$	0	λ^2	$18\rho^2+9\lambda^2$
F_1^n	$(3\rho+\lambda)^2/4$	$8\lambda^2$	$(3\rho-\lambda)^2/4$	$4\lambda^2$	λ^2	$(9\rho^2+27\lambda^2)/2$

λ (ρ) $=$ (anti) symmetric component of ground state wfn.

Close, WM, Phys. Rev. C68 (2003) 035210

- SU(6) limit $\Rightarrow$ $\lambda = \rho$

$\longrightarrow$ relative strengths of $N \rightarrow N^*$ transitions:

$SU(6):$	$[\mathbf{56},0^+]^2\mathbf{8}$	$[\mathbf{56},0^+]^4\mathbf{10}$	$[\mathbf{70},1^-]^2\mathbf{8}$	$[\mathbf{70},1^-]^4\mathbf{8}$	$[\mathbf{70},1^-]^2\mathbf{10}$	*total*
F_1^p	9	8	9	0	1	27
F_1^n	4	8	1	4	1	18

- summing over all resonances in **56**$^+$ and **70**$^-$ multiplets

$\longrightarrow$ $\dfrac{F_1^n}{F_1^p} = \dfrac{2}{3}$ as in quark-parton model (for u=$2d$) !

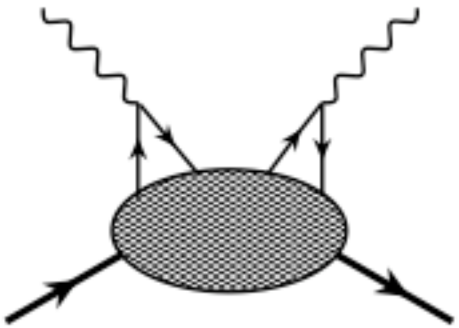
- proton sum saturated by lower-lying resonances

$\longrightarrow$ expect duality to appear *earlier* for p than n

Close, WM, Phys. Rev. C68 (2003) 035210

Is duality in the proton a coincidence?

$\longrightarrow$ consider symmetric nucleon wave function



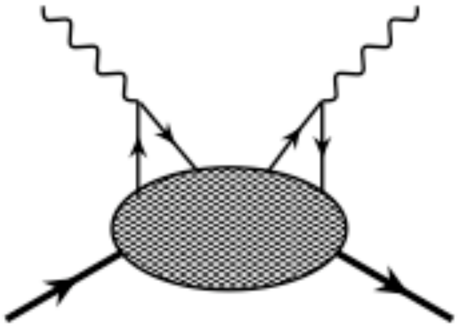
cat's ears diagram (4-fermion higher twist $\sim 1/Q^2$)

$$\propto \sum_{i \neq j} e_i \, e_j \sim \Big(\sum_i e_i\Big)^2 - \sum_i e_i^2$$

coherent incoherent

Is duality in the proton a coincidence?

$\longrightarrow$ consider symmetric nucleon wave function



cat's ears diagram (4-fermion higher twist $\sim 1/Q^2$)

$$\propto \sum_{i \neq j} e_i \, e_j \sim \Big(\sum_i e_i\Big)^2 - \sum_i e_i^2$$

coherent — incoherent

- *proton* $\quad \mathrm{HT} \sim 1 - \left(2 \times \frac{4}{9} + \frac{1}{9}\right) = 0\ !$
- *neutron* $\quad \mathrm{HT} \sim 0 - \left(\frac{4}{9} + 2 \times \frac{1}{9}\right) \neq 0$

$\Rightarrow$ need to test duality in the neutron!

Extraction of Neutron Structure Function

- Problem: no free neutron targets!

  (neutron half-life ~ 12 mins)

  $\longrightarrow$ use deuteron as "effective neutron target"

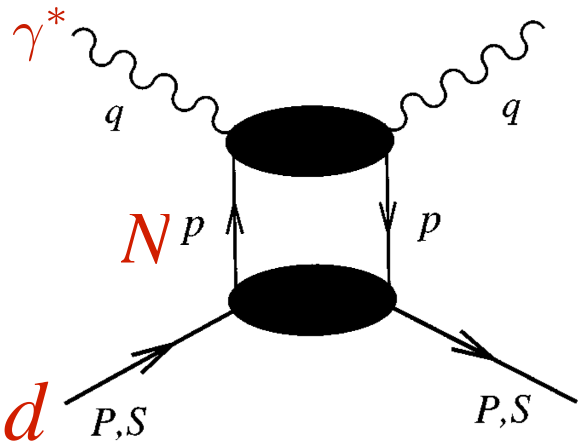
- But: deuteron is a nucleus, and $F_2^d \neq F_2^p + F_2^n$

  $\longrightarrow$ nuclear effects (nuclear binding, Fermi motion, shadowing) *obscure neutron structure* information

  $\longrightarrow$ need to correct for "nuclear EMC effect"

- nuclear "impulse approximation"

→ incoherent scattering from individual nucleons in d
(good approx. at $x >> 0$)



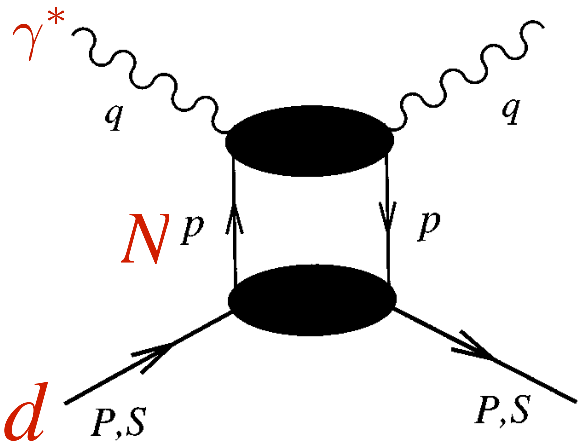
$\boxed{N=p+n}$

$$F_2^d(x,Q^2) = \int_x dy\ f(y,\gamma)\ F_2^N(x/y,Q^2)$$
$$+\ \delta^{(\mathrm{off})} F_2^d$$

nuclear "impulse approximation"

→ incoherent scattering from individual nucleons in d

(good approx. at $x >> 0$)



$N=p+n$

$$F_2^d(x,Q^2) = \int_x dy\ f(y,\gamma)\ F_2^N(x/y,Q^2)$$

$$+\ \delta^{(\mathrm{off})} F_2^d$$

nucleon momentum distribution in d ("smearing function")

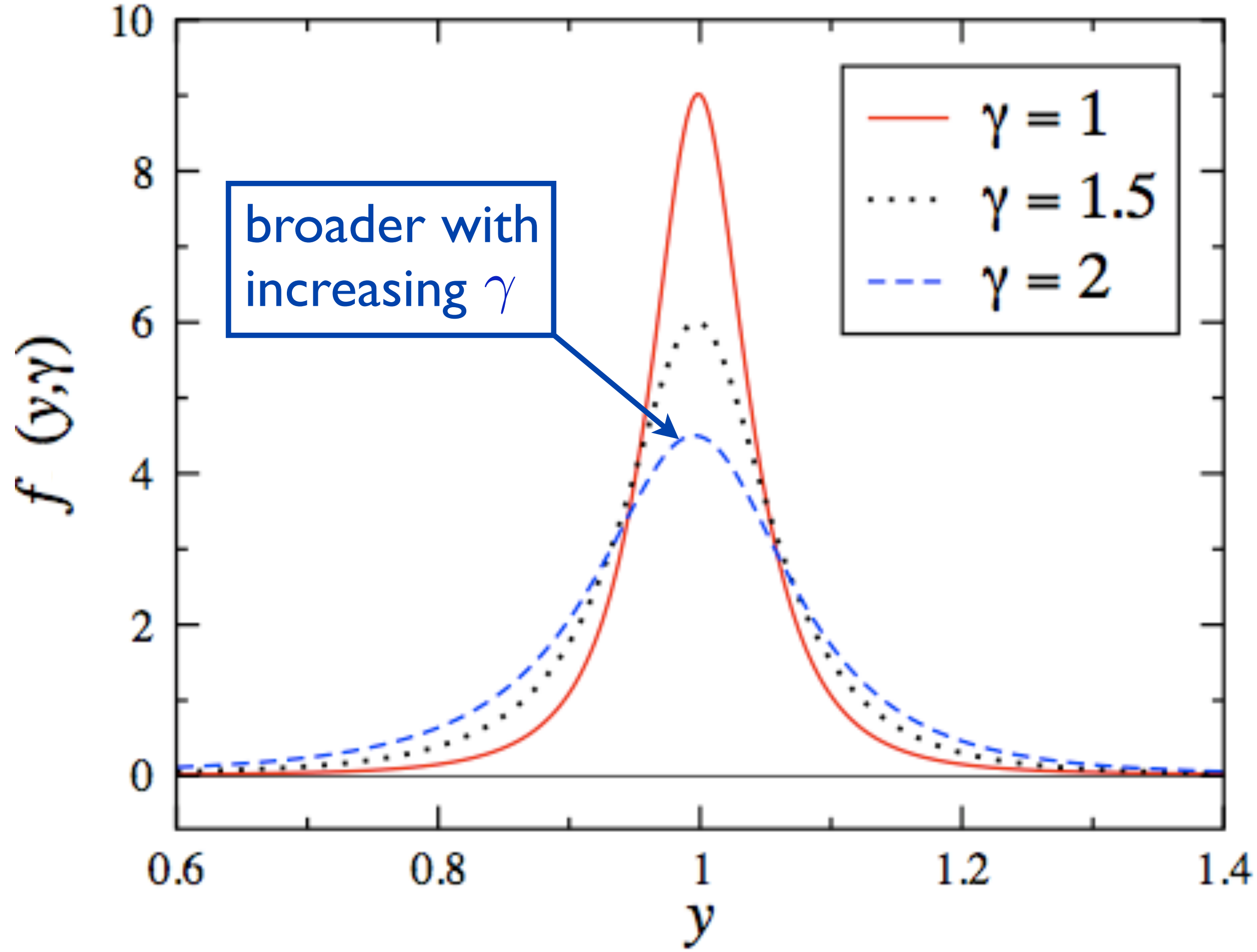
off-shell correction (~1%)

→ at finite Q^2, smearing function depends also on parameter

$$\gamma = |\mathbf{q}|/q_0 = \sqrt{1+4M^2x^2/Q^2}$$

Kulagin, WM, Phys. Rev. C ***77****, 015210 (2008)*

N momentum distributions in d



$\longrightarrow$ for most kinematics $\gamma \lesssim 2$

Unsmearing – additive method

- calculated F_2^d depends on input F_2^n

  $\longrightarrow$ extracted n depends on input n ... cyclic argument

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Solution: <u>iteration procedure</u>

0. subtract $\delta^{(\mathrm{off})} F_2^d$ from d data: $F_2^d \to F_2^d - \delta^{(\mathrm{off})} F_2^d$

1. define difference between smeared and free SFs

$$\boxed{F_2^d - \widetilde{F}_2^p = \widetilde{F}_2^n \equiv f \otimes F_2^n \equiv F_2^n + \Delta}$$

2. first guess for $F_2^{n(0)}$ $\longrightarrow$ $\Delta^{(0)} = \widetilde{F}_2^{n(0)} - F_2^n$

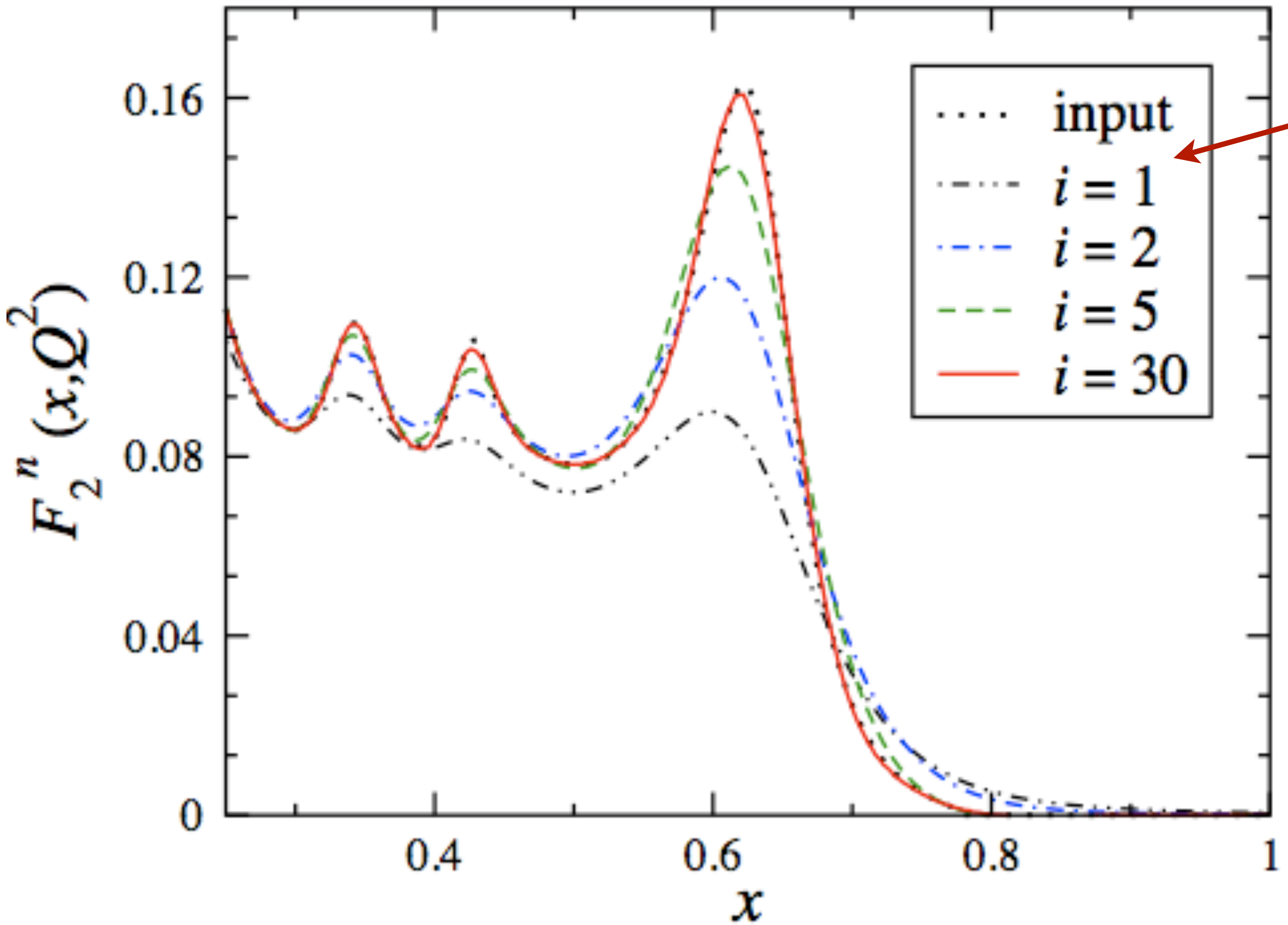
3. after one iteration, gives

$$F_2^{n(1)} = F_2^{n(0)} + (\widetilde{F}_2^n - \widetilde{F}_2^{n(0)})$$

4. repeat until convergence

Unsmearing – test of convergence

- F_2^d constructed from known F_2^p and F_2^n inputs
  (using MAID resonance parameterization)



initial guess
$F_2^{n(0)} = 0$ *

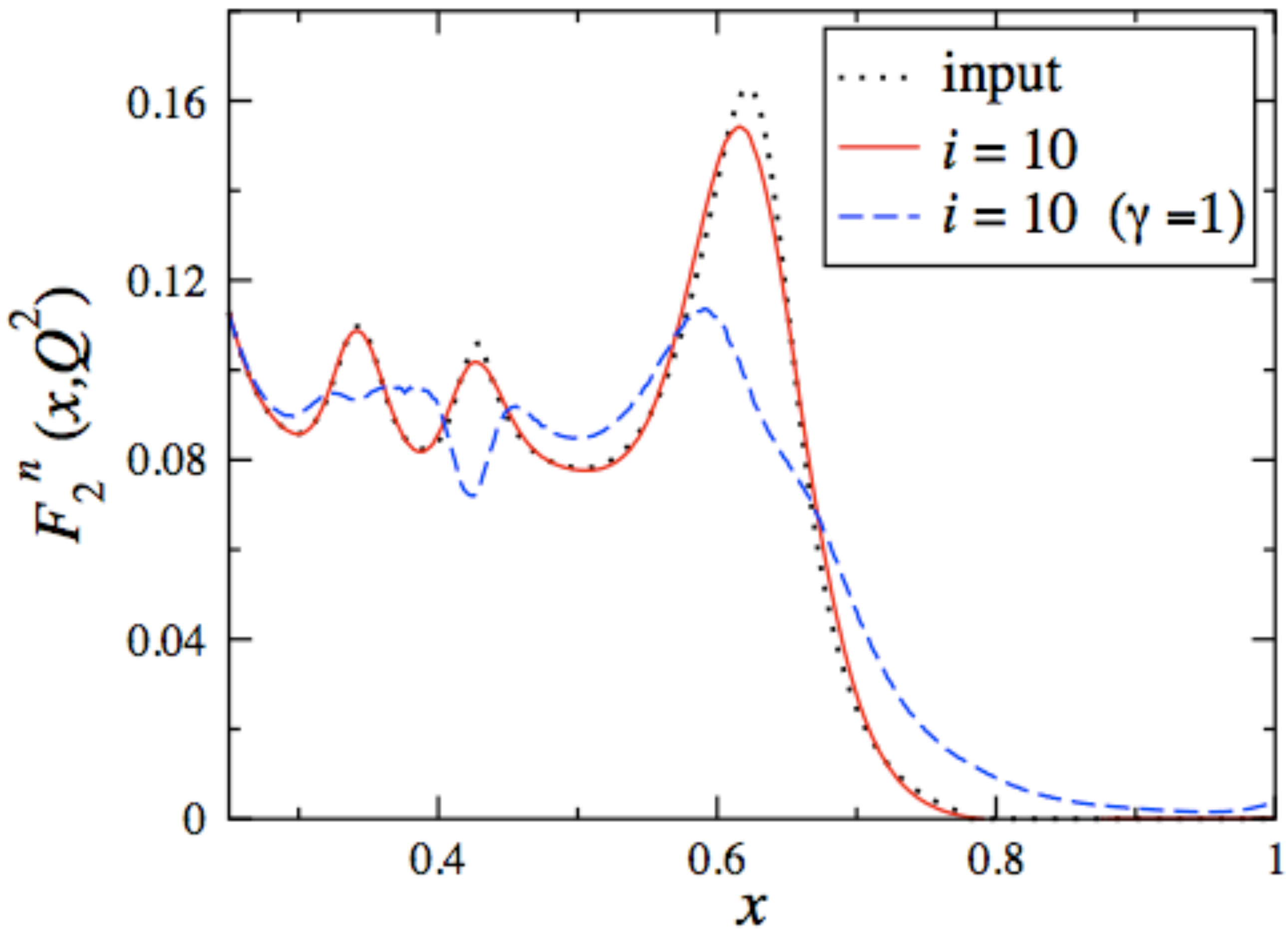
* even faster convergence if choose $F_2^{n(0)} = F_2^p$

Kahn, WM (2008)

→ can reconstruct almost arbitrary shape

Unsmearing – Q^2 dependence

- important to use correct γ dependence in extraction



Kahn, WM (2008)

→ important also in DIS region
(do not have resonance “benchmarks”)

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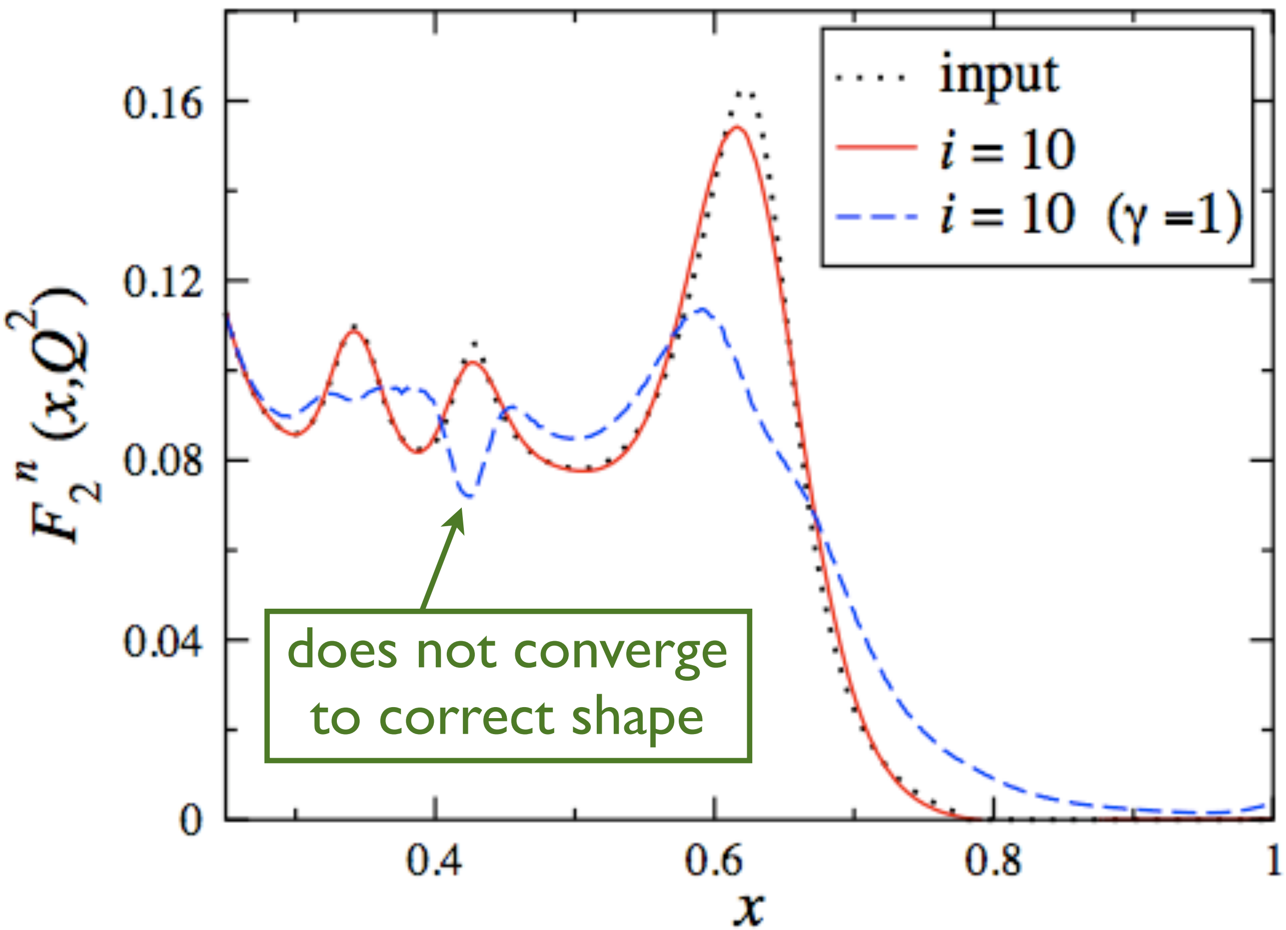

Kahn, WM (2008)

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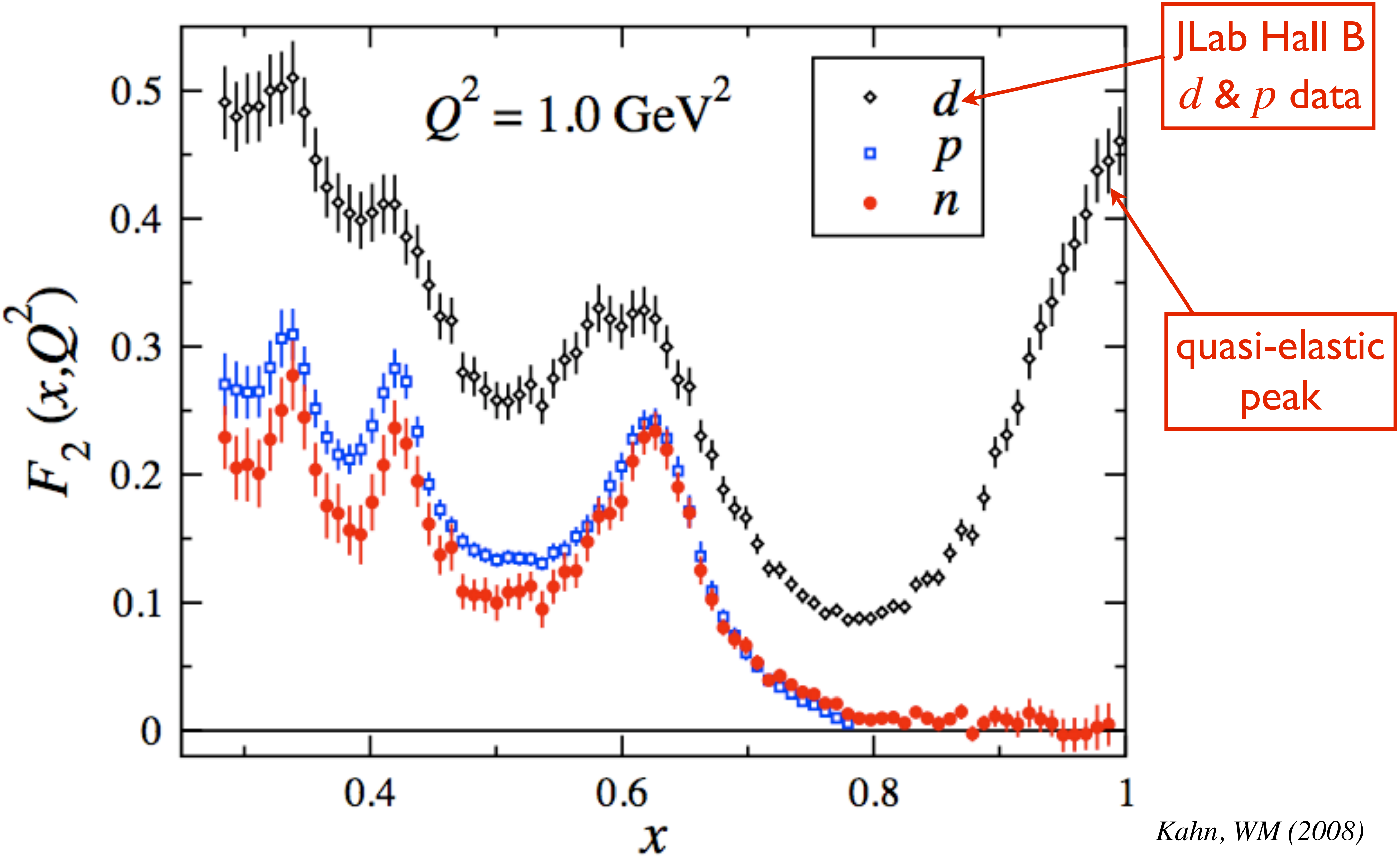
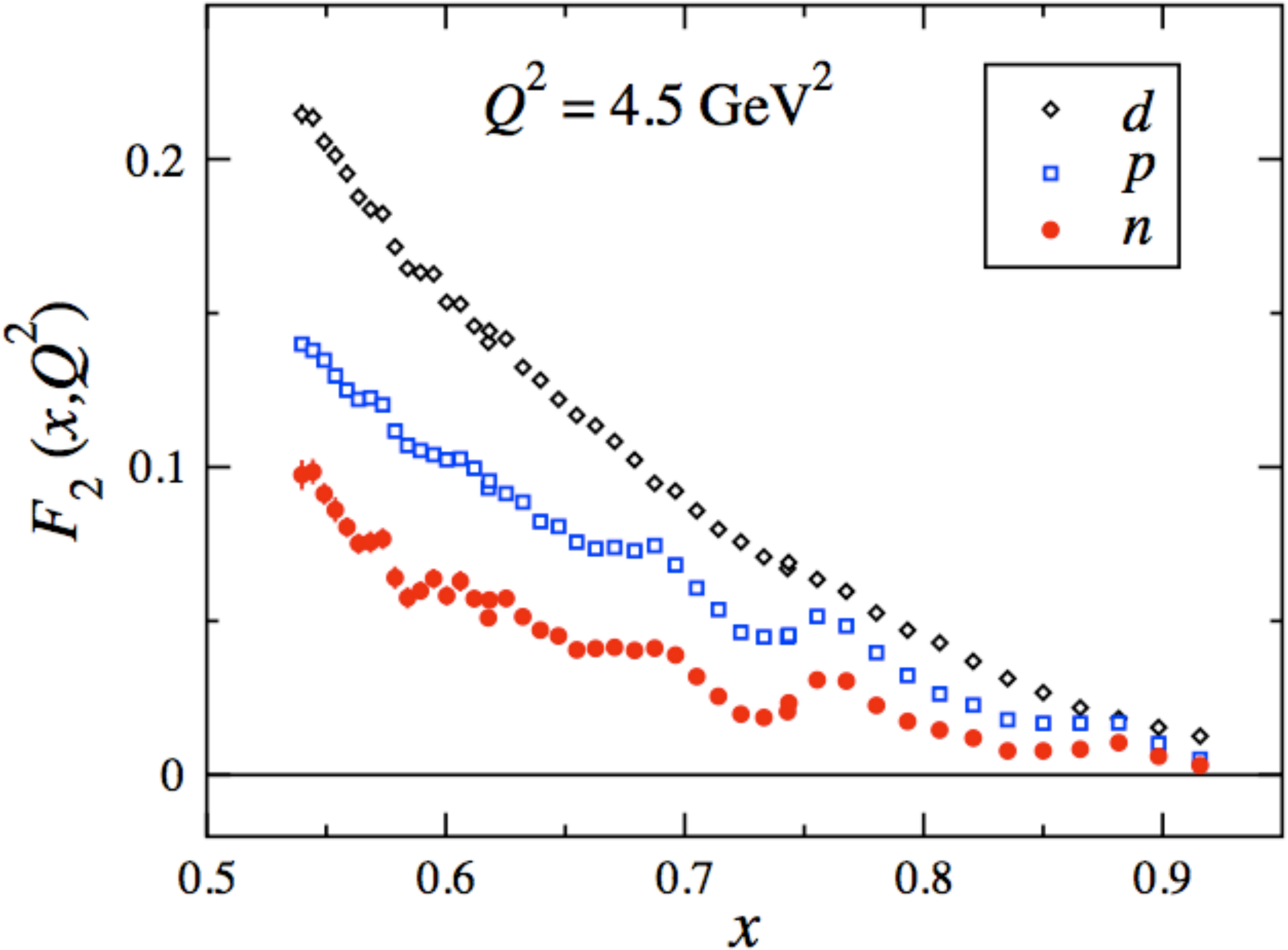


Kahn, WM (2008)

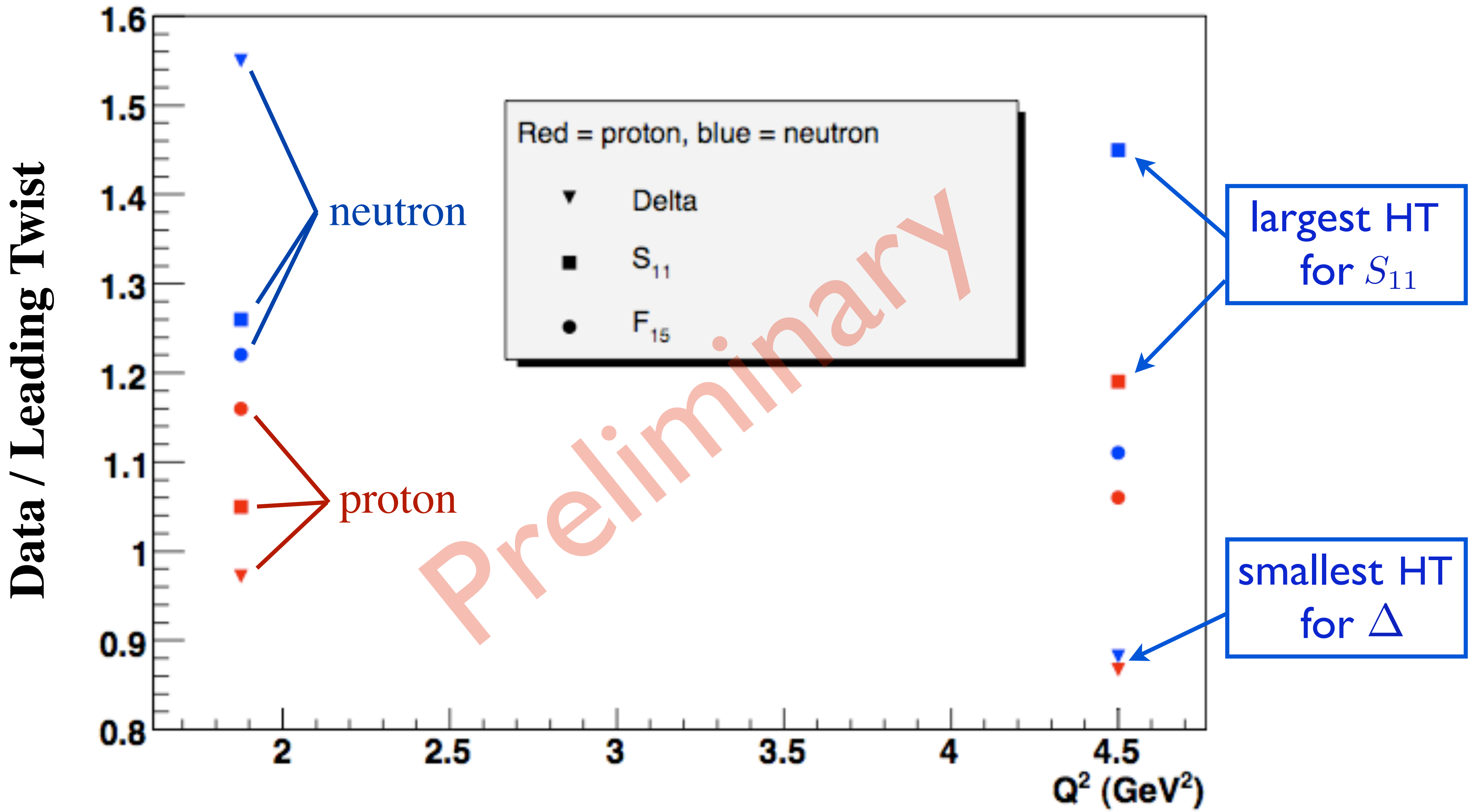
→ first extraction of F_2^n in resonance region



Kahn, WM (2008)

→ works also in DIS region

Structure function integral ratios



Kahn, WM (2008)

→ neutron HT indeed larger than proton!

Summary

- Remarkable confirmation of quark-hadron duality in *proton* structure functions

  → duality violating higher twists $\sim 10\%$ in few-GeV range

- Truncated moments

  → firm foundation for study of *local* duality in QCD

  → HTs largest in S_{11} region, smallest in Δ region

- Duality in the *neutron*

  → extraction of neutron structure function from deuteron data

  → neutron HTs *larger* than proton HTs
  (as expected from quark models)

Future

- Complete analysis of neutron structure function extraction

  → quantify isospin dependence of HTs

- Application to spin-dependent structure functions

  → extraction method works also for functions with zeros

- Cross-check with neutron extracted from ^{3}He data