QCD and the Strong Interactions
CSSM, Adelaide
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Quark-Hadron Duality
in Electron-Nucleon Scattering

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Outline

1. Bloom-Gilman duality
2. Duality in QCD
3. Local duality
   - *quark models*
   - *phenomenological models*
4. Target mass corrections

Quark-hadron duality

Complementarity between *quark* and *hadron* descriptions of observables

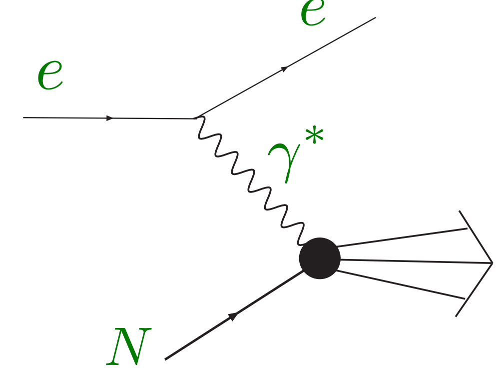
$$\sum_{hadrons} = \sum_{quarks}$$

Can use either set of complete basis states to describe all physical phenomena

1.
Bloom-Gilman duality

Electron scattering

Inclusive cross section for $eN \to eX$



leptonic tensor

$$\frac{d^2\sigma}{d\Omega dE'} \sim L^{\mu\nu} W_{\mu\nu}$$

Hadronic tensor

$$W_{\mu\nu} = \sum_X \langle X|J_\mu(z)|N\rangle \langle N|J_\nu(0)|X\rangle \, \delta^4(p+q-p_X)$$

$$= \int d^4z \, e^{iq\cdot z} \langle N|J_\mu(z)J_\nu(0)|N\rangle$$

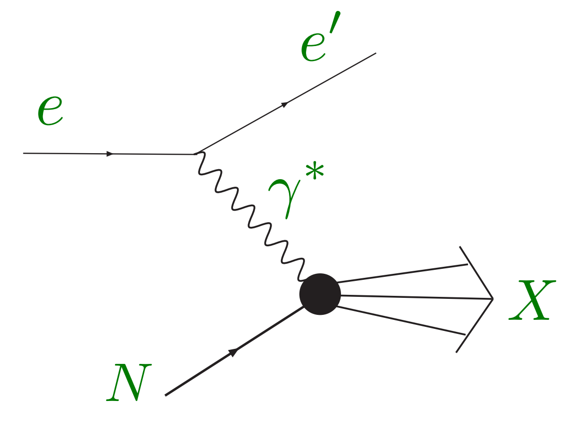
using completeness (sum over ALL states X)

$$\sum_X |X\rangle\langle X| = 1$$

"hadrons" "duality" *"quarks"*

Electron scattering

Inclusive cross section for $eN \to eX$



$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2 \cos^2\frac{\theta}{2}}{Q^4}\left(2\tan^2\frac{\theta}{2}\frac{F_1}{M} + \frac{F_2}{\nu}\right)$$

$$\left.\begin{aligned} \nu &= E - E' \\ Q^2 &= \vec{q}^{\,2} - \nu^2 = 4EE'\sin^2\frac{\theta}{2} \end{aligned}\right\} \quad x = \frac{Q^2}{2M\nu}$$

Bjorken scaling variable

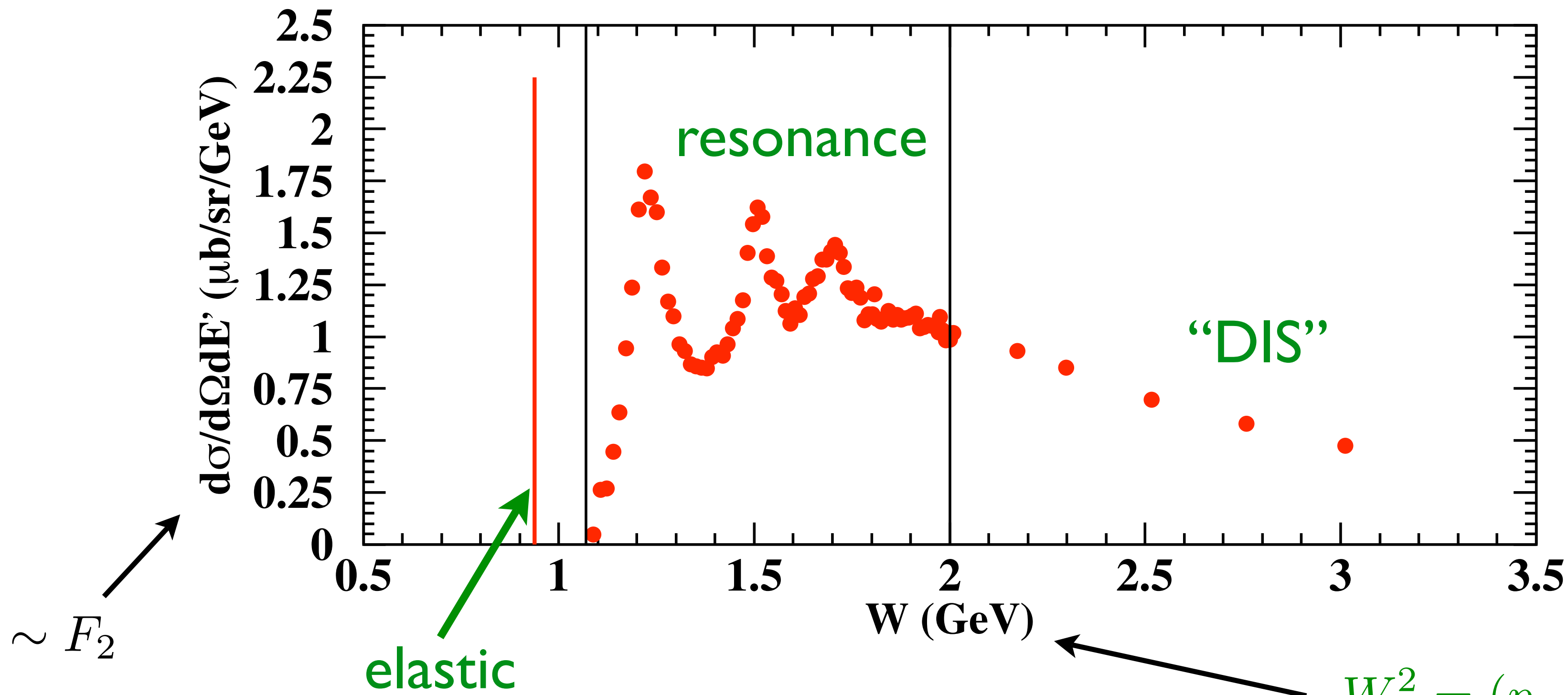
$F_1\ , F_2$ "structure functions"

⟶ contain all information about structure of nucleon

⟶ functions of $x,\ Q^2$ in general

Electron scattering

As W decreases, DIS region gives way to region dominated by nucleon resonances

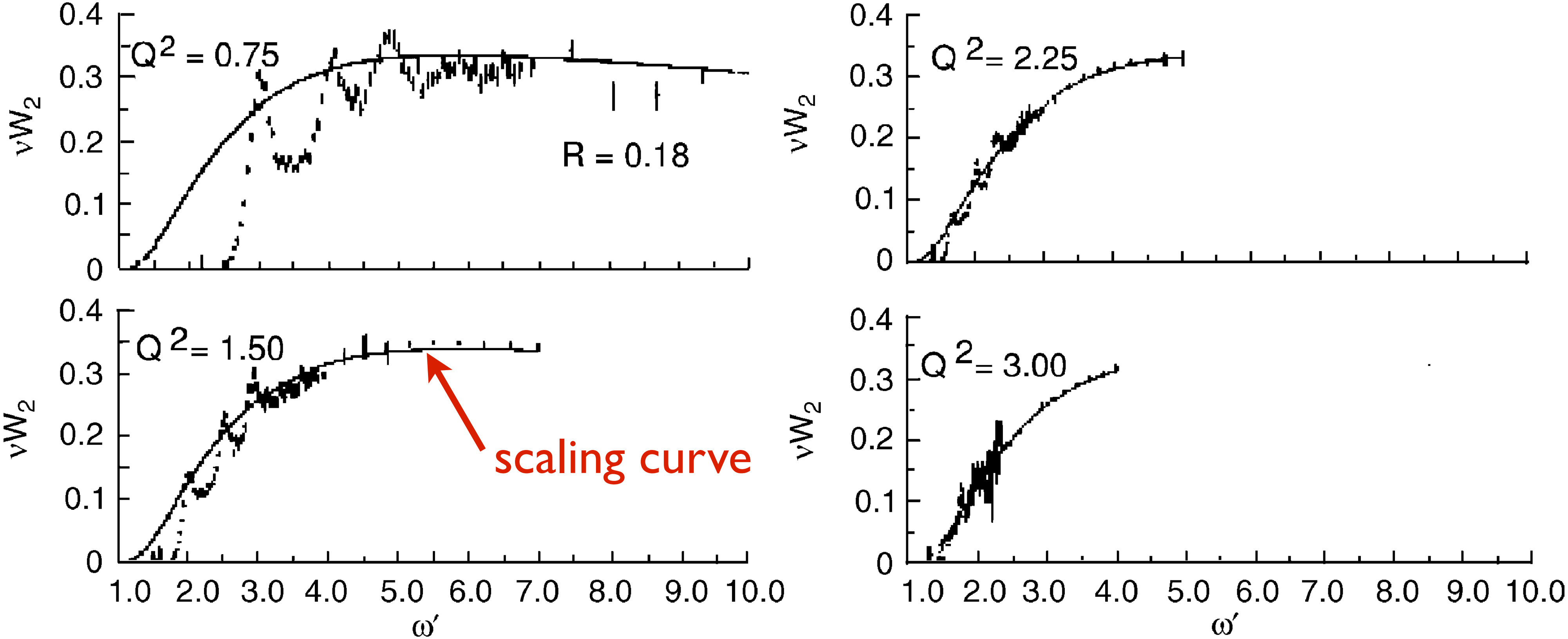


$$W^2 = (p_N + q)^2$$
$$= M^2 + 2M\nu - Q^2$$

hadronic final state mass

$$x = \frac{Q^2}{W^2 - M^2 + Q^2}$$

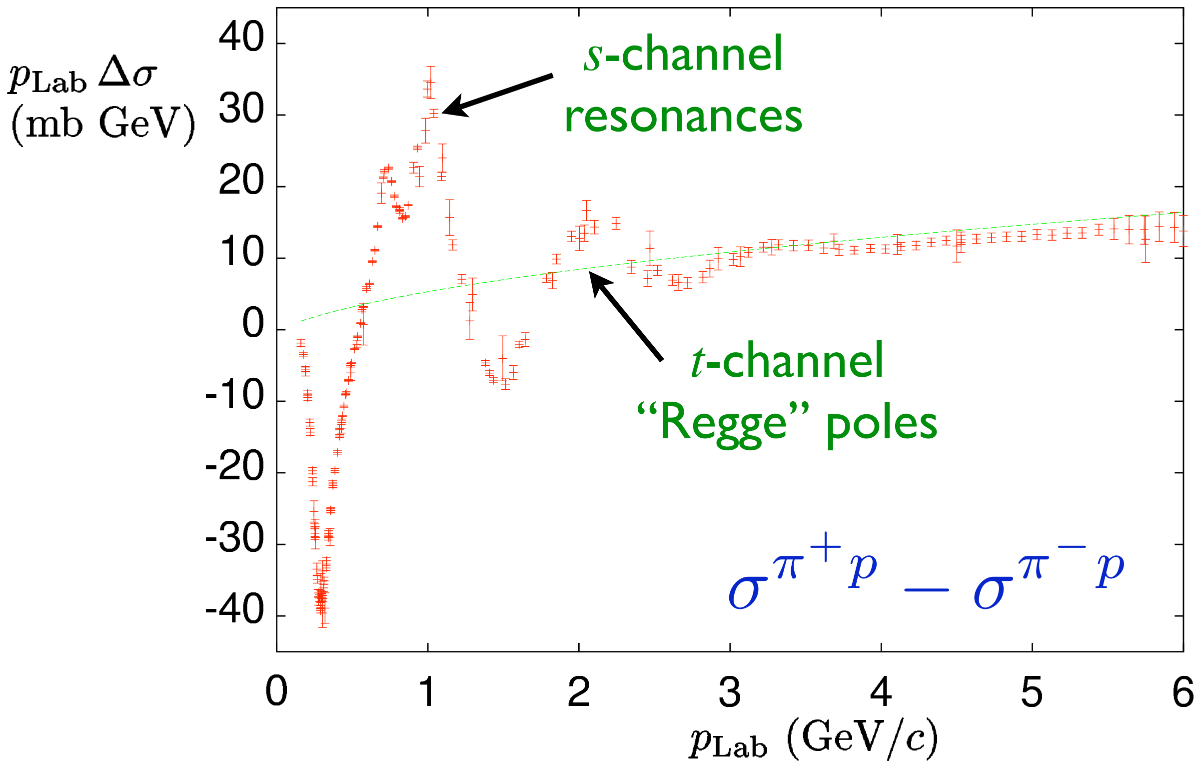
Electron scattering

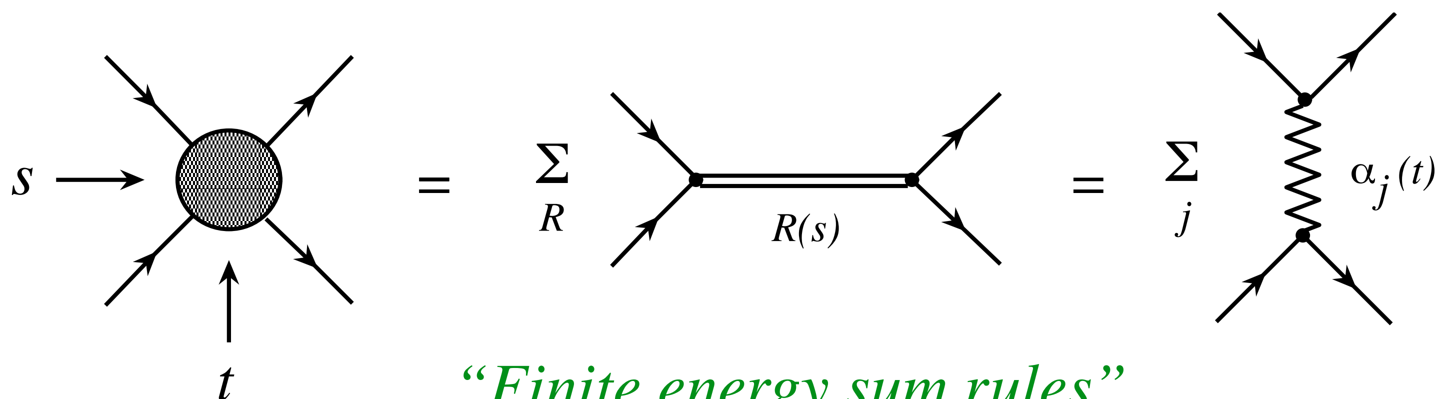


Bloom, Gilman, Phys. Rev. Lett. 85 (1970) 1185

resonance – scaling duality in proton $\nu W_2 = F_2$ structure function

cf. hadron-hadron scattering





"Finite energy sum rules"

Igi (1962), Dolen, Horn, Schmidt (1968)

Bloom-Gilman duality

Average over (strongly Q^2 dependent) resonances
$\approx Q^2$ independent scaling function

Finite energy sum rule for eN scattering

$$\frac{2M}{Q^2}\int_0^{\nu_m} d\nu\ \nu W_2(\nu, Q^2) = \int_1^{\omega'_m} d\omega'\ \nu W_2(\omega')$$

measured structure function
(function of ν and Q^2)

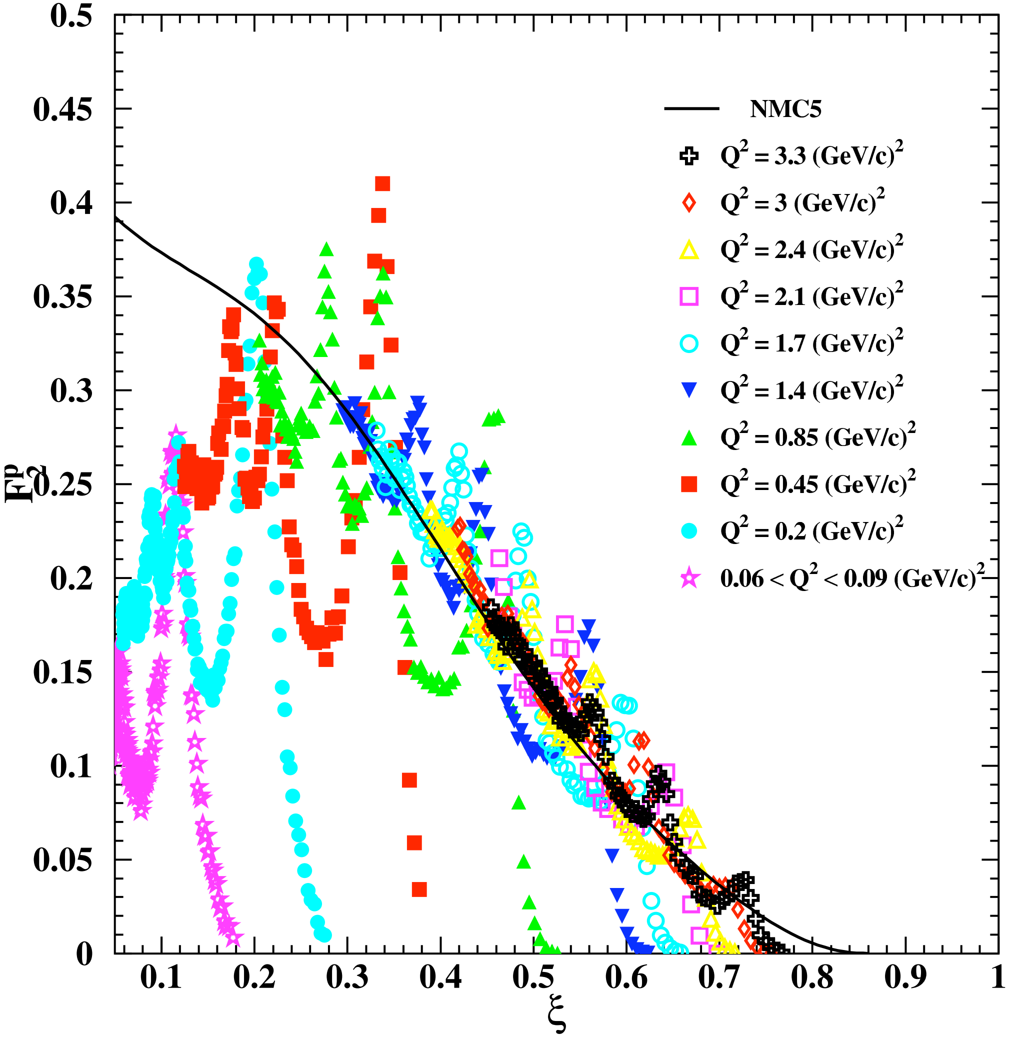
"hadrons"

scaling function
(function of ω' only)

"quarks"

$$\omega' = \frac{1}{x} + \frac{M^2}{Q^2}$$

Bloom-Gilman duality



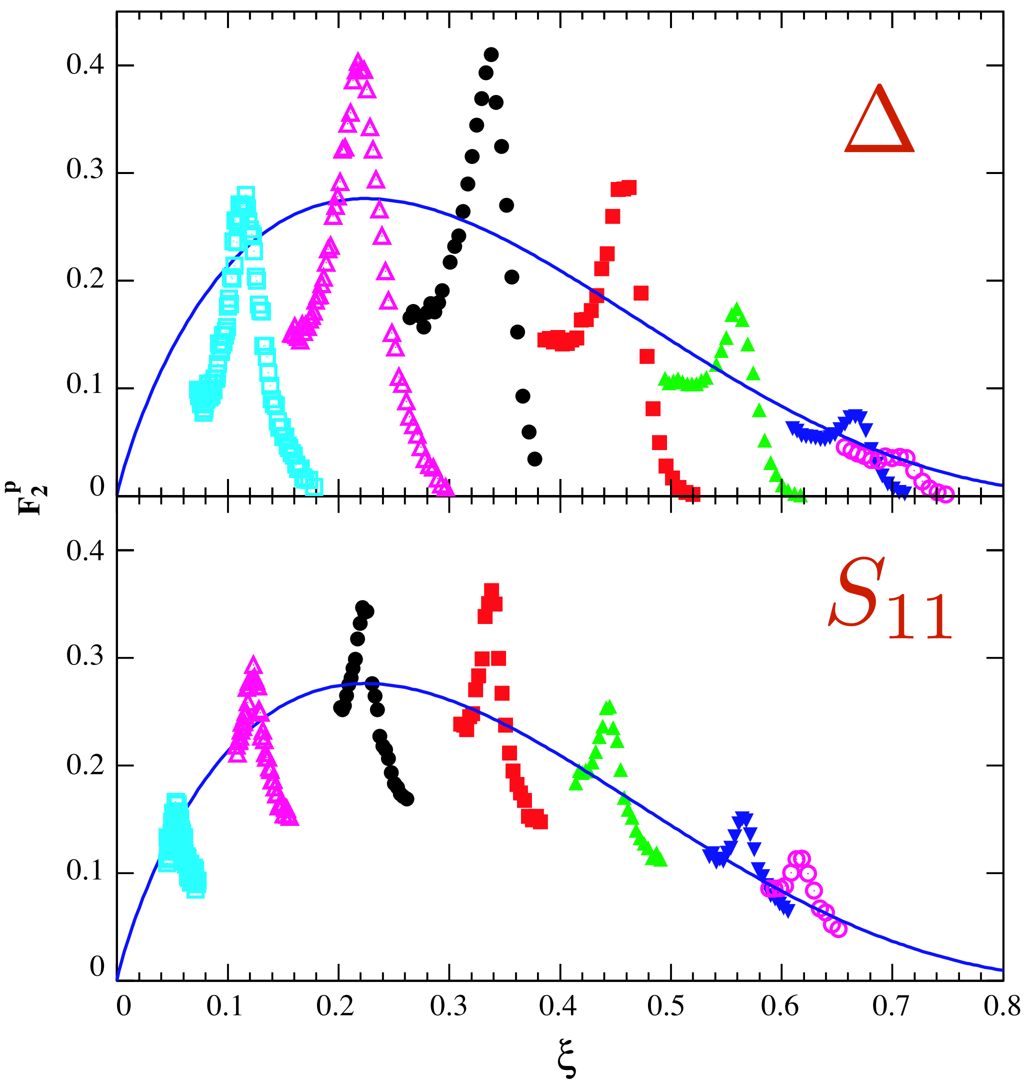
Jefferson Lab (Hall C)

Niculescu et al., Phys. Rev. Lett. 85 (2000) 1182

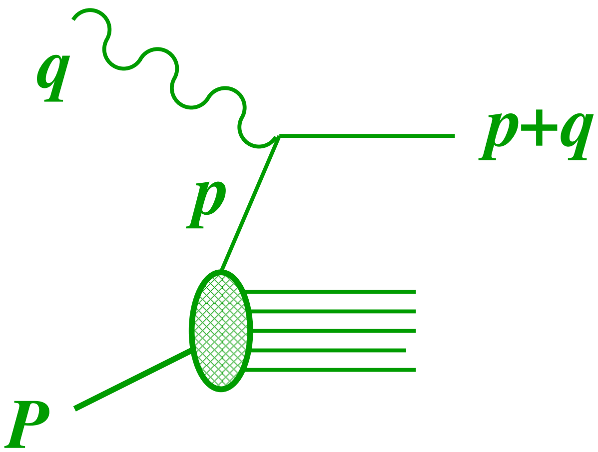
Average over (strongly Q^2 dependent) resonances

$\approx$ Q^2 independent scaling function

(Local) Bloom-Gilman duality



Scaling variables



$$(p+q)^2 = m_q^2 \quad \left\{ \begin{array}{l} m_q = 0 \\ p_T = 0 \end{array} \right.$$

light-cone fraction of target's momentum carried by parton

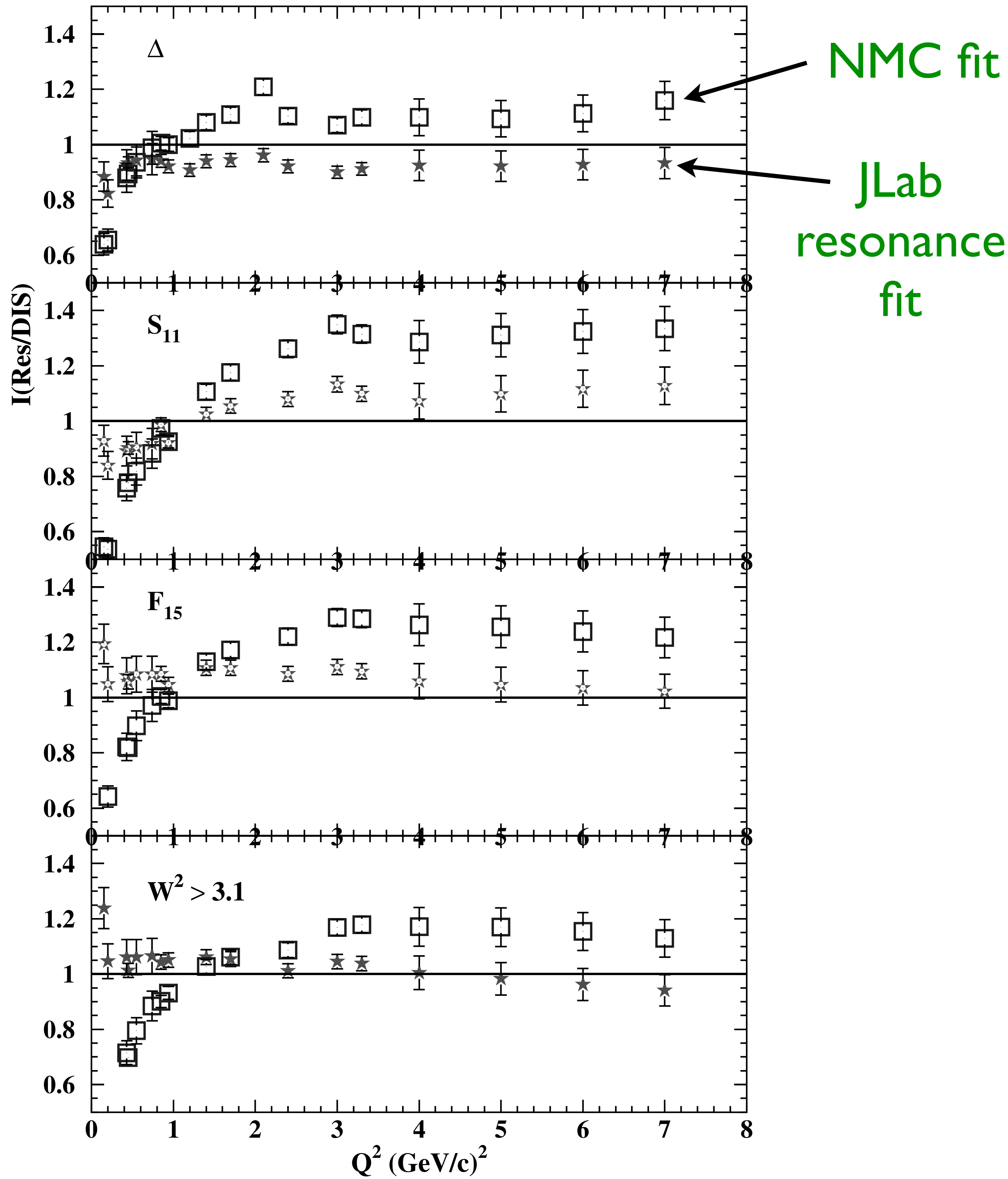
$$\xi = \frac{p^+}{P^+} = \frac{p^0 + p^z}{M}$$

$$\Rightarrow \quad \xi = \frac{2x}{1 + \sqrt{1 + 4x^2 M^2 / Q^2}} \quad \rightarrow \quad x \quad \text{as} \quad Q^2 \rightarrow \infty$$

Nachtmann scaling variable

Integrated strength

~ 10% agreement for $Q^2 > 1$ GeV^2



Niculescu et al., Phys. Rev. Lett. 85 (2000) 1186

2.
Duality in QCD

Duality in QCD

Operator product expansion

⟹ expand moments of structure functions in powers of $1/Q^2$

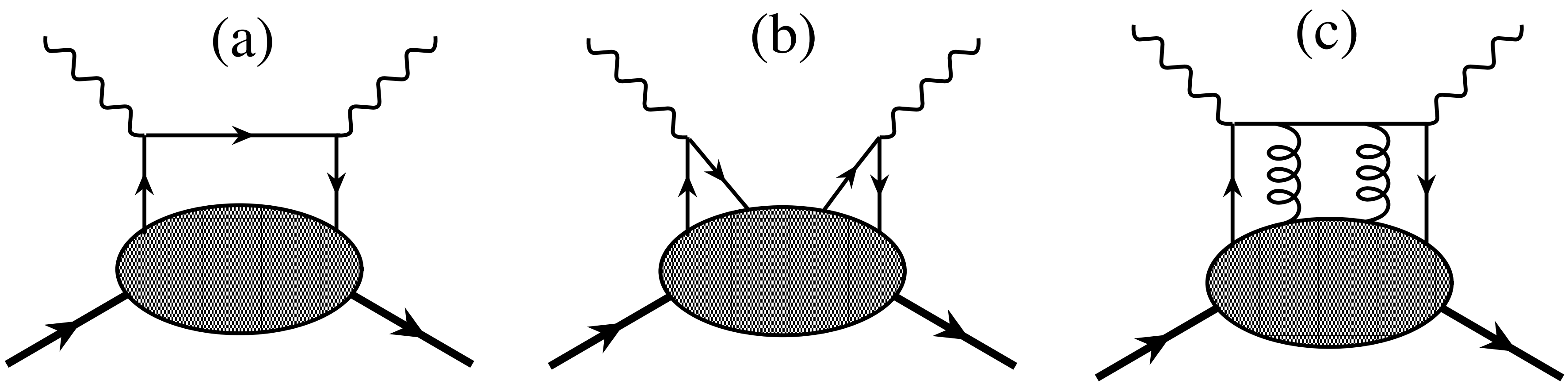
$$M_n(Q^2) = \int_0^1 dx\ x^{n-2}\ F_2(x,Q^2)$$

$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

matrix elements of operators with specific "twist" τ

$$\tau = \text{dimension} - \text{spin}$$

Higher twists



$\tau = 2$

single quark scattering

e.g. $\bar{\psi}\ \gamma_\mu\ \psi$

$\tau > 2$

qq and qg correlations

e.g. $\bar{\psi}\ \gamma_\mu\ \psi\ \bar{\psi}\ \gamma_\nu\ \psi$

or $\bar{\psi}\ \tilde{G}_{\mu\nu}\gamma^\nu\ \psi$

Duality in QCD

Operator product expansion

⟹ expand moments of structure functions in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx\ x^{n-2}\ F_2(x,Q^2)$$
$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

If moment $\approx$ independent of Q^2

⟹ higher twist terms $A_n^{(\tau>2)}$ small

Duality in QCD

Operator product expansion

⟹ expand moments of structure functions in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx\ x^{n-2}\ F_2(x, Q^2)$$

$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

Duality $\Longleftrightarrow$ suppression of higher twists

de Rujula, Georgi, Politzer,
Ann. Phys. 103 (1975) 315

Duality in QCD

- Much of recent new data is in *resonance* region, $W < 2$ GeV

→ *common wisdom:* pQCD analysis not valid in resonance region

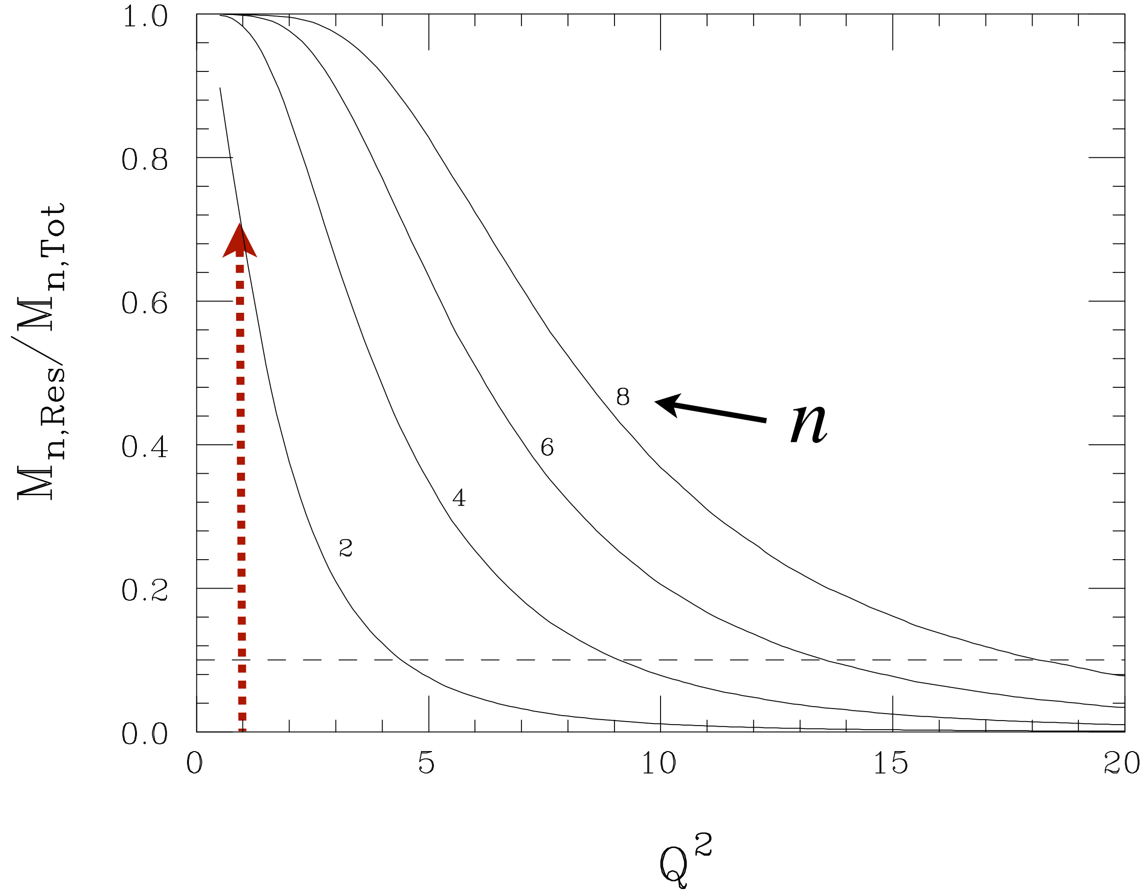
→ *in fact:* partonic interpretation of moments *does* include resonance region

- Resonances are an *integral part* of deep inelastic structure functions!

→ implicit role of quark-hadron duality

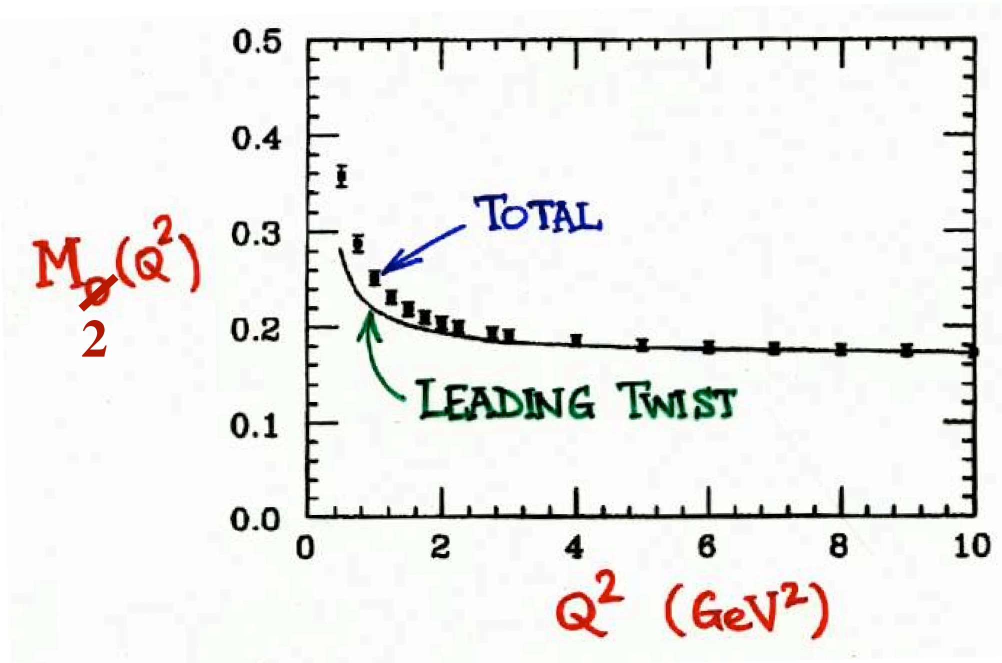
Proton F_2 moments

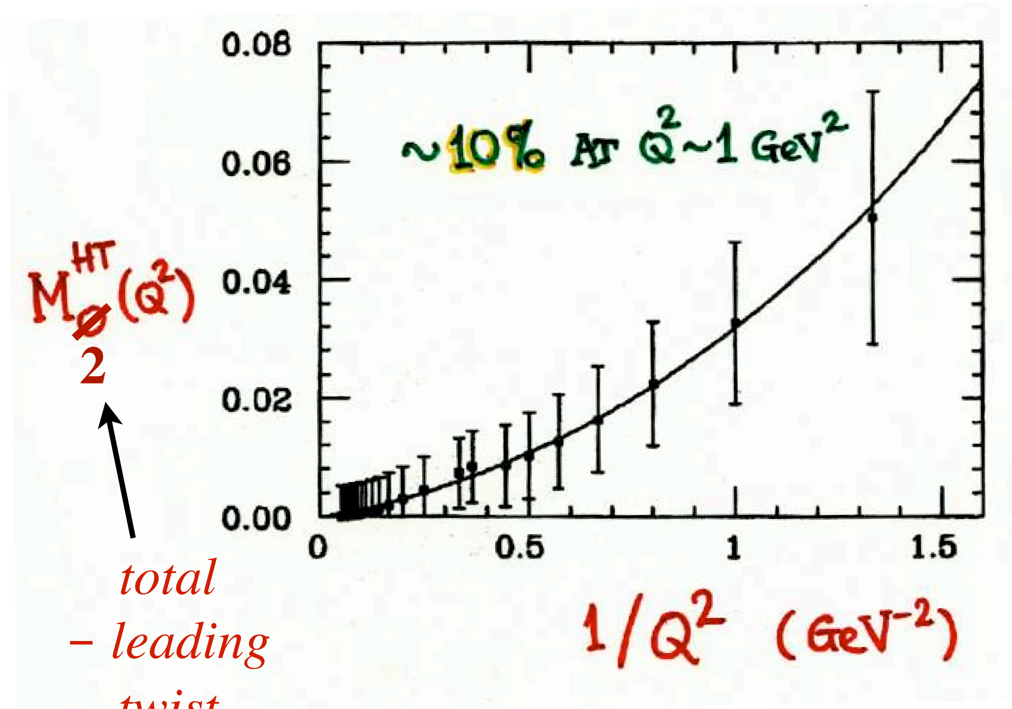
relative contribution of resonance region to n-th moment



At $Q^2 = 1\ \mathrm{GeV}^2$, $\sim$ 70% of lowest moment of F_2^p comes from $W < 2\ \mathrm{GeV}$

Proton F_2 moments



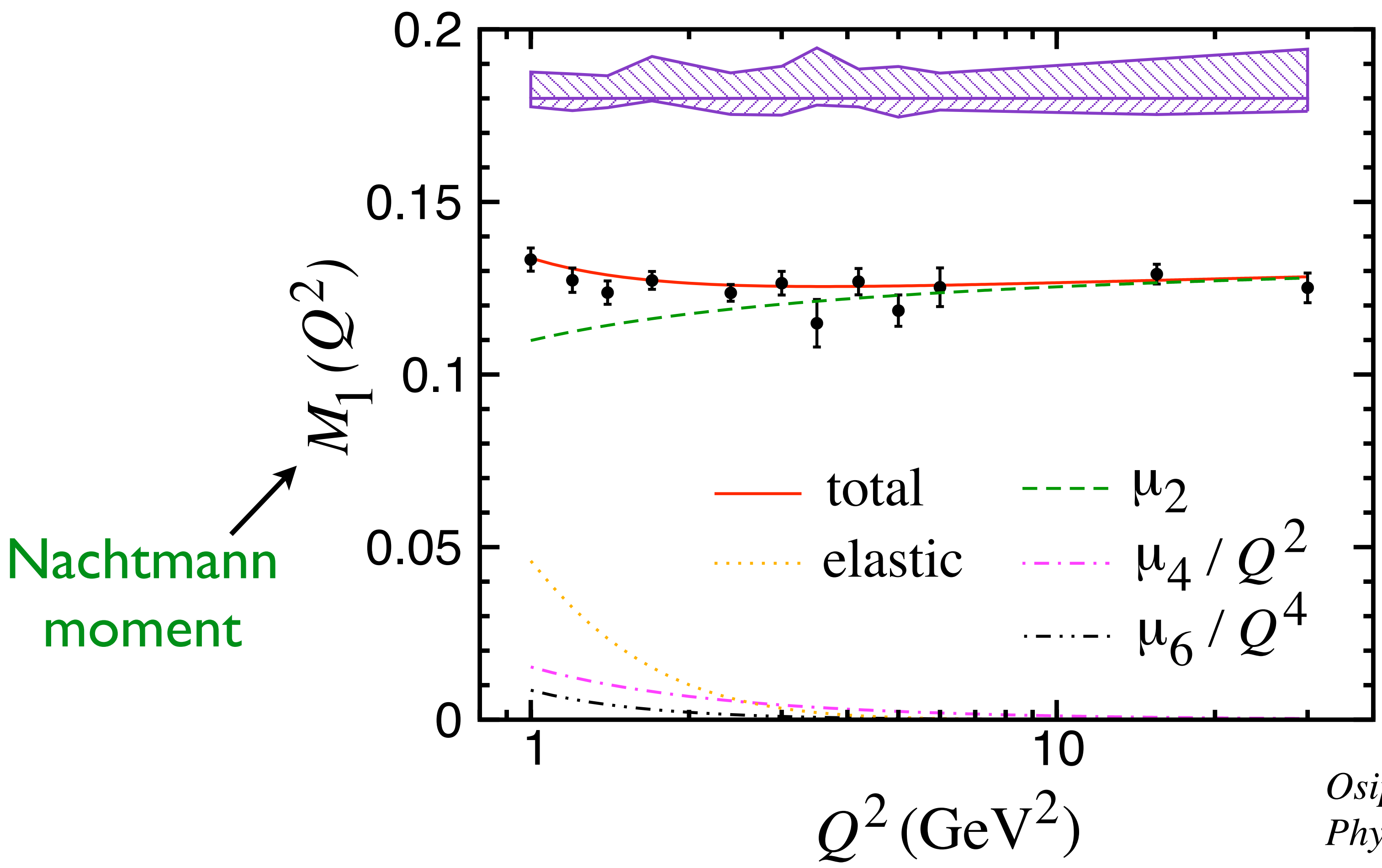


total
– leading
twist

BUT resonances and DIS continuum conspire to produce only ~ 10% *higher twist* contribution!

Ji, Unrau,
Phys. Rev. D 52 (1995) 72

Proton g_1 moment

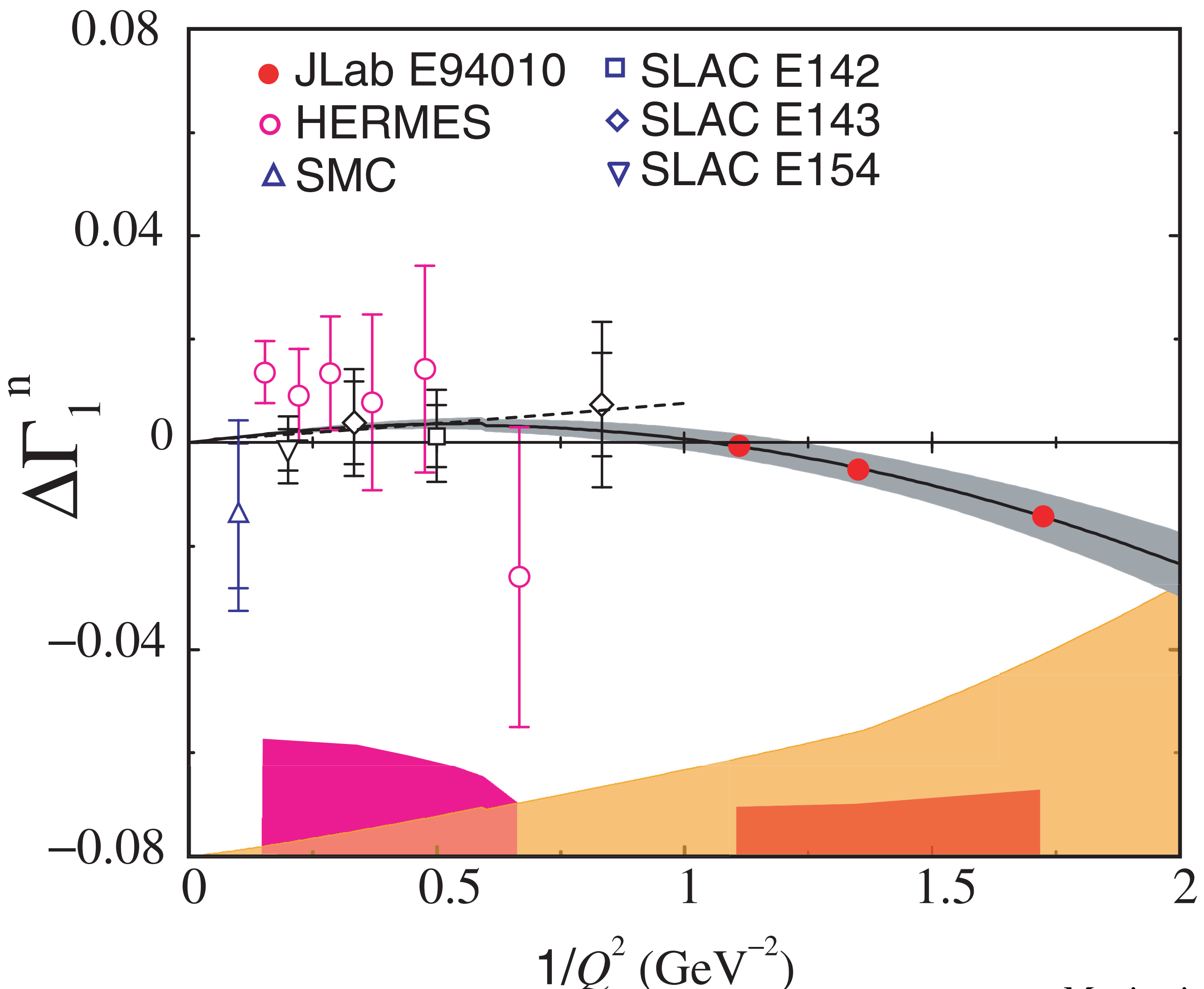


Osipenko, WM et al,
Phys. Lett. B 609 (2005) 259

$$M_1 = \int_0^1 dx \frac{\xi^2}{x^2} \left[g_1 \left(\frac{x}{\xi} - \frac{M^2 x \xi}{9Q^2} \right) - g_2 \frac{4M^2 x^2}{3Q^2} \right] \;=\; \mu_2 + \frac{4M^2}{9Q^2} f_2 + \cdots$$

Neutron g_1 moment

→ higher twist contribution



Meziani, WM et al.,
Phys. Lett. B613 (2005) 148

Total higher twist *small* at $Q^2 \sim 1 - 2\ \mathrm{GeV}^2$

→ nonperturbative interactions between quarks and gluons not dominant at these scales

→ suggests *strong cancellations* between resonances, resulting in dominance of *leading twist*

→ OPE does not tell us *why* higher twists are small !

Can we understand this behavior dynamically?

How do cancellations between *coherent* resonances produce *incoherent* scaling function?

3.
Local duality

- quark models

Coherence vs. incoherence

Exclusive form factors

$\Longrightarrow$ *coherent* scattering from quarks

$$d\sigma \sim \left(\sum_i e_i\right)^2$$

Inclusive structure functions

$\Longrightarrow$ *incoherent* scattering from quarks

$$d\sigma \sim \sum_i e_i^2$$

$\Longrightarrow$ How can *square of a sum* $\approx$ *sum of squares* ?

Pedagogical model

Two quarks bound in a harmonic oscillator potential

⟹ exactly solvable spectrum

Structure function given by sum of squares of transition form factors

$$F(\nu, \mathbf{q}^2) \sim \sum_n \left|G_{0,n}(\mathbf{q}^2)\right|^2 \delta(E_n - E_0 - \nu)$$

Charge operator $\Sigma_i \; e_i \exp(i\mathbf{q}\cdot\mathbf{r}_i)$ excites

even partial waves with strength $\propto (e_1 + e_2)^2$

odd partial waves with strength $\propto (e_1 - e_2)^2$

Pedagogical model

Resulting structure function

$$F(\nu, \mathbf{q}^2) \;\sim\; \sum_n \left\{ (e_1 + e_2)^2 \; G^2_{0,2n} + (e_1 - e_2)^2 \; G^2_{0,2n+1} \right\}$$

If states degenerate, cross terms $(\sim e_1 e_2)$ cancel when averaged over nearby even and odd parity states

Minimum condition for duality:

⟹ *at least one complete set of <u>even</u> and <u>odd</u> parity resonances must be summed over*

Close, Isgur, Phys. Lett. B509 (2001) 81

Quark model

Even and odd parity states generalize to $\mathbf{56}^{+}$ $(L=0)$ and $\mathbf{70}^{-}$ $(L=1)$ multiplets of spin-flavor SU(6)

⟹ scaling occurs if contributions from $\mathbf{56}^{+}$ and $\mathbf{70}^{-}$ have equal overall strengths

representation	$^{2}\mathbf{8}[\mathbf{56}^{+}]$	$^{4}\mathbf{10}[\mathbf{56}^{+}]$	$^{2}\mathbf{8}[\mathbf{70}^{-}]$	$^{4}\mathbf{8}[\mathbf{70}^{-}]$	$^{2}\mathbf{10}[\mathbf{70}^{-}]$	Total
F_1^p	$9\rho^2$	$8\lambda^2$	$9\rho^2$	0	λ^2	$18\rho^2+9\lambda^2$
F_1^n	$(3\rho+\lambda)^2/4$	$8\lambda^2$	$(3\rho-\lambda)^2/4$	$4\lambda^2$	λ^2	$(9\rho^2+27\lambda^2)/2$
g_1^p	$9\rho^2$	$-4\lambda^2$	$9\rho^2$	0	λ^2	$18\rho^2-3\lambda^2$
g_1^n	$(3\rho+\lambda)^2/4$	$-4\lambda^2$	$(3\rho-\lambda)^2/4$	$-2\lambda^2$	λ^2	$(9\rho^2-9\lambda^2)/2$

λ (ρ) $=$ (anti) symmetric component of ground state wfn.

Close, WM
Phys. Rev. C 68 (2003) 035210

Quark model

SU(6) limit $\Longrightarrow$ $\lambda = \rho$

$SU(6):$	$[\mathbf{56},\mathbf{0}^+]^\mathbf{2}\mathbf{8}$	$[\mathbf{56},\mathbf{0}^+]^\mathbf{4}\mathbf{10}$	$[\mathbf{70},\mathbf{1}^-]^\mathbf{2}\mathbf{8}$	$[\mathbf{70},\mathbf{1}^-]^\mathbf{4}\mathbf{8}$	$[\mathbf{70},\mathbf{1}^-]^\mathbf{2}\mathbf{10}$	*total*
F_1^p	9	8	9	0	1	27
F_1^n	4	8	1	4	1	18
g_1^p	9	-4	9	0	1	15
g_1^n	4	-4	1	-2	1	0

Summing over all resonances in $\mathbf{56}^+$ and $\mathbf{70}^-$ multiplets

$$\longrightarrow \quad R^{np} = \frac{F_1^n}{F_1^p} = \frac{2}{3} \qquad A_1^p = \frac{g_1^p}{F_1^p} = \frac{5}{9} \qquad A_1^n = \frac{g_1^n}{F_1^n} = 0$$

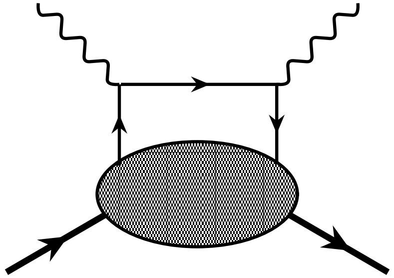
$\longrightarrow$ as in quark-parton model !

Quark model

- proton wave function

$$p^{\uparrow} = -\frac{1}{3}d^{\uparrow}(uu)_1 - \frac{\sqrt{2}}{3}d^{\downarrow}(uu)_1$$

$$+ \frac{\sqrt{2}}{6}u^{\uparrow}(ud)_1 - \frac{1}{3}u^{\downarrow}(ud)_1 + \frac{1}{\sqrt{2}}u^{\uparrow}(ud)_0$$



interacting quark

spectator diquark

diquark spin

$\longrightarrow$ $u(x) = 2\ d(x)$ for all x $\longrightarrow$ $\frac{F_2^n}{F_2^p} = \frac{4u+d}{u+4d} = \frac{2}{3}$

Quark model

SU(6) may be $\approx$ valid at $x \sim 1/3$

But significant deviations at large x

$\Longrightarrow$ which combinations of resonances reproduce behavior of structure functions at large x?

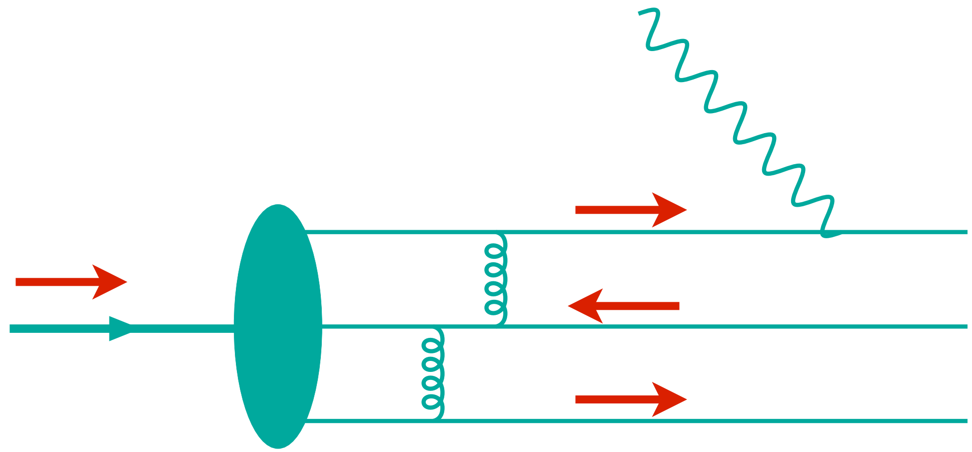
Model	SU(6)	No $^4\mathbf{10}$	No $^2\mathbf{10}$, $^4\mathbf{10}$	No $S_{3/2}$	No $\sigma_{3/2}$	No ψ_λ
R^{np}	2/3	10/19	1/2	6/19	3/7	1/4
A_1^p	5/9	1	1	1	1	1
A_1^n	0	2/5	1/3	1	1	1

$^4\mathbf{10}\ [56^+]$ and $^4\mathbf{8}\ [70^-]$ suppressed

helicity 3/2 suppression

hard gluon exchange

at large x, helicity of struck quark = helicity of hadron



$$q^{\uparrow} \gg q^{\downarrow}$$

$\Longrightarrow$ helicity-zero diquark dominant in $x \to 1$ limit

→ $$\frac{d}{u} \to \frac{1}{5}$$

→ $$\frac{F_2^n}{F_2^p} \to \frac{3}{7}$$

Farrar, Jackson 1975

Quark model

SU(6) may be $\approx$ valid at $x \sim 1/3$

But significant deviations at large x

⟹ which combinations of resonances reproduce behavior of structure functions at large x?

Model	SU(6)	No $^4\mathbf{10}$	No $^2\mathbf{10}$, $^4\mathbf{10}$	No $S_{3/2}$	No $\sigma_{3/2}$	No ψ_λ
R^{np}	2/3	10/19	1/2	6/19	3/7	1/4
A_1^p	5/9	1	1	1	1	1
A_1^n	0	2/5	1/3	1	1	1

suppression of symmetric part of spin-flavor wfn.

e.g. $\vec{S}_i \cdot \vec{S}_j$ interaction

scalar diquark dominance

$M_\Delta > M_N \quad \Longrightarrow \quad (qq)_1$ has larger energy than $(qq)_0$

$\Longrightarrow$ scalar diquark dominant in $x \to 1$ limit

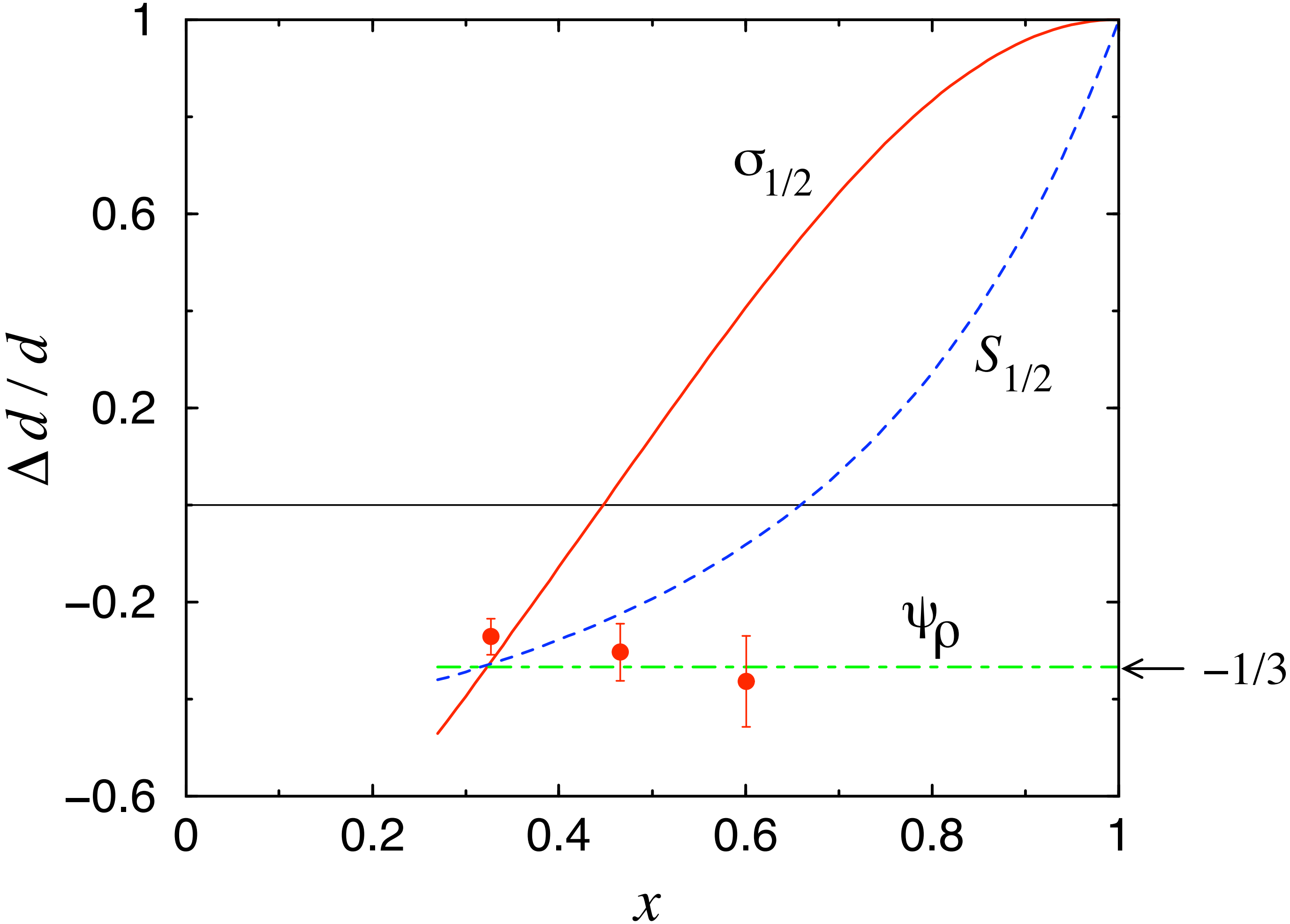
since only u quarks couple to scalar diquarks

$$\longrightarrow \quad \frac{d}{u} \to 0$$

$$\longrightarrow \quad \frac{F_2^n}{F_2^p} \to \frac{1}{4}$$

Feynman 1972, Close 1973, Close/Thomas 1988

$$\frac{\Delta d}{d} = \frac{4}{15} A_1^n \left(4 + \frac{u}{d}\right) - \frac{1}{15} A_1^p \left(1 + 4\frac{u}{d}\right)$$



$$\frac{u}{d} = \frac{4 - R^{np}}{4R^{np} - 1}$$

Zheng et al. (JLab Hall A)
PRL (2004) 012004

Close, WM
Phys. Rev. C 68 (2003) 035210

→ nonperturbative physics still dominant at $x \sim 0.6$!

λ suppression model $\Rightarrow$ identical production rates in $\mathbf{56}^+$ and $\mathbf{70}^-$ channels

	representation	${}^2\mathbf{8}[\mathbf{56}^+]$	${}^4\mathbf{10}[\mathbf{56}^+]$	${}^2\mathbf{8}[\mathbf{70}^-]$	${}^4\mathbf{8}[\mathbf{70}^-]$	${}^2\mathbf{10}[\mathbf{70}^-]$	Total
γ^*	F_1^p	$9\rho^2$	$8\lambda^2$	$9\rho^2$	0	λ^2	$18\rho^2+9\lambda^2$
	F_1^n	$(3\rho+\lambda)^2/4$	$8\lambda^2$	$(3\rho-\lambda)^2/4$	$4\lambda^2$	λ^2	$(9\rho^2+27\lambda^2)/2$
	g_1^p	$9\rho^2$	$-4\lambda^2$	$9\rho^2$	0	λ^2	$18\rho^2-3\lambda^2$
	g_1^n	$(3\rho+\lambda)^2/4$	$-4\lambda^2$	$(3\rho-\lambda)^2/4$	$-2\lambda^2$	λ^2	$(9\rho^2-9\lambda^2)/2$

	representation	${}^2\mathbf{8}[\mathbf{56}^+]$	${}^4\mathbf{10}[\mathbf{56}^+]$	${}^2\mathbf{8}[\mathbf{70}^-]$	${}^4\mathbf{8}[\mathbf{70}^-]$	${}^2\mathbf{10}[\mathbf{70}^-]$	Total
ν	$F_1^{\nu p}$	0	$24\lambda^2$	0	0	$3\lambda^2$	$27\lambda^2$
	$F_1^{\nu n}$	$(9\rho+\lambda)^2/4$	$8\lambda^2$	$(9\rho-\lambda)^2/4$	$4\lambda^2$	λ^2	$(81\rho^2+27\lambda^2)/2$
	$g_1^{\nu p}$	0	$-12\lambda^2$	0	0	$3\lambda^2$	$-9\lambda^2$
	$g_1^{\nu n}$	$(9\rho+\lambda)^2/4$	$-4\lambda^2$	$(9\rho-\lambda)^2/4$	$-2\lambda^2$	λ^2	$(81\rho^2-9\lambda^2)/2$

λ suppression model ⇨ identical production rates in $\mathbf{56}^+$ and $\mathbf{70}^-$ channels

	representation	$^2\mathbf{8}[\mathbf{56}^+]$	$^4\mathbf{10}[\mathbf{56}^+]$	$^2\mathbf{8}[\mathbf{70}^-]$	$^4\mathbf{8}[\mathbf{70}^-]$	$^2\mathbf{10}[\mathbf{70}^-]$	Total
γ^*	F_1^p	$9\rho^2$	~~$8\lambda^2$~~	$9\rho^2$	0	~~λ^2~~	$18\rho^2+9\lambda^2$
	F_1^n	$(3\rho+\lambda)^2/4$	~~$8\lambda^2$~~	$(3\rho-\lambda)^2/4$	~~$4\lambda^2$~~	~~λ^2~~	$(9\rho^2+27\lambda^2)/2$
	g_1^p	$9\rho^2$	~~$-4\lambda^2$~~	$9\rho^2$	0	~~λ^2~~	$18\rho^2-3\lambda^2$
	g_1^n	$(3\rho+\lambda)^2/4$	~~$-4\lambda^2$~~	$(3\rho-\lambda)^2/4$	~~$-2\lambda^2$~~	~~λ^2~~	$(9\rho^2-9\lambda^2)/2$

	representation	$^2\mathbf{8}[\mathbf{56}^+]$	$^4\mathbf{10}[\mathbf{56}^+]$	$^2\mathbf{8}[\mathbf{70}^-]$	$^4\mathbf{8}[\mathbf{70}^-]$	$^2\mathbf{10}[\mathbf{70}^-]$	Total
ν	$F_1^{\nu p}$	0	~~$24\lambda^2$~~	0	0	~~$3\lambda^2$~~	$27\lambda^2$
	$F_1^{\nu n}$	$(9\rho+\lambda)^2/4$	~~$8\lambda^2$~~	$(9\rho-\lambda)^2/4$	~~$4\lambda^2$~~	~~λ^2~~	$(81\rho^2+27\lambda^2)/2$
	$g_1^{\nu p}$	0	~~$-12\lambda^2$~~	0	0	~~$3\lambda^2$~~	$-9\lambda^2$
	$g_1^{\nu n}$	$(9\rho+\lambda)^2/4$	~~$-4\lambda^2$~~	$(9\rho-\lambda)^2/4$	~~$-2\lambda^2$~~	~~λ^2~~	$(81\rho^2-9\lambda^2)/2$

⇨ important test for future experiments

3.
Local duality

- phenomenological models

Phenomenological model

- Extract $N \to N^*$ form factors from exclusive data (for $Q^2 \leq 2\ \mathrm{GeV}^2$)

  $\longrightarrow$ consider both γ and ν scattering

- Calculate structure function from J=1/2 and 3/2 resonance form factors $\longrightarrow$ $P_{33}(1232), D_{13}(1520), P_{11}(1440), S_{11}(1535)$

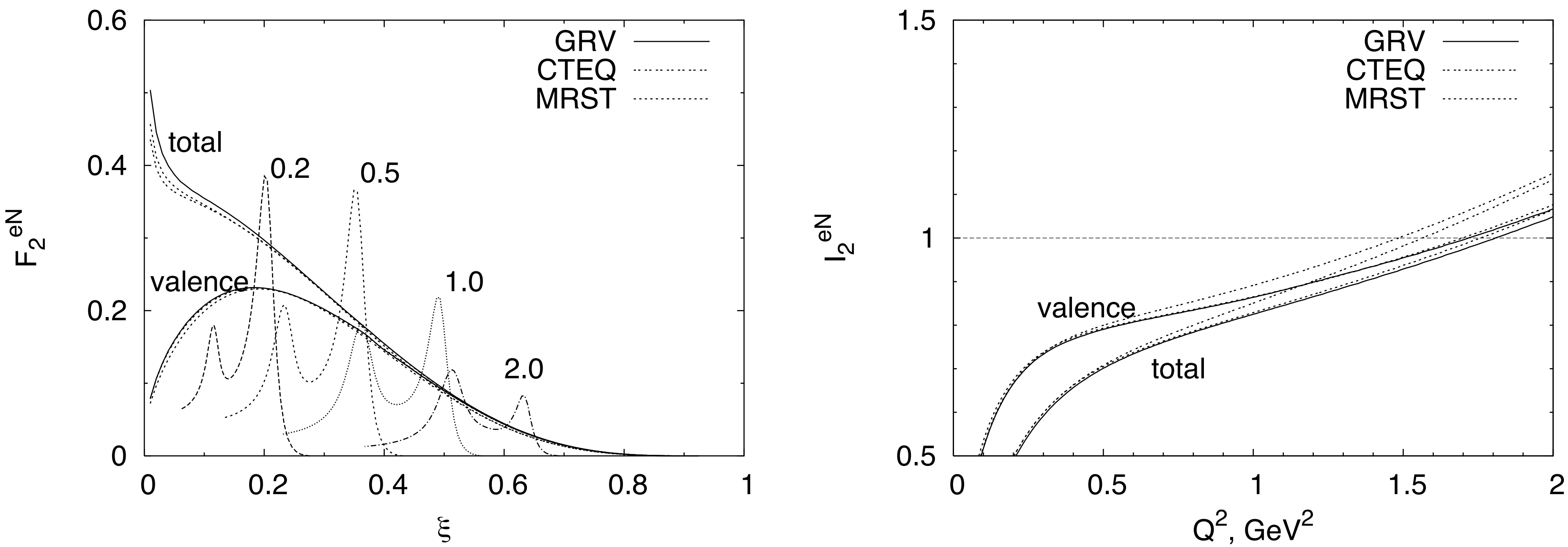
$$F_2(\nu, Q^2) = \frac{1}{M} V_2\ \delta(W^2 - M_R^2)$$

vector and axial form factors

$$\frac{V_2}{3} = (C_3^V)^2 \frac{2}{3M_R^2} Q^2[q \cdot p + m_N^2 + M_R^2] + \frac{(C_4^V)^2}{m_N^2} \frac{2}{3} Q^2[q \cdot p + m_N^2 - m_N M_R]$$

$$+ \frac{C_3^V C_4^V}{m_N} \frac{2}{3M_R} Q^2[q \cdot p + (M_R - m_N)^2] + \frac{2}{3}\left[(C_5^A)^2 \frac{m_N^2}{M_R^2} + \frac{(C_4^A)^2}{m_N^2} Q^2\right][q \cdot p + m_N^2 + m_N M_R]$$

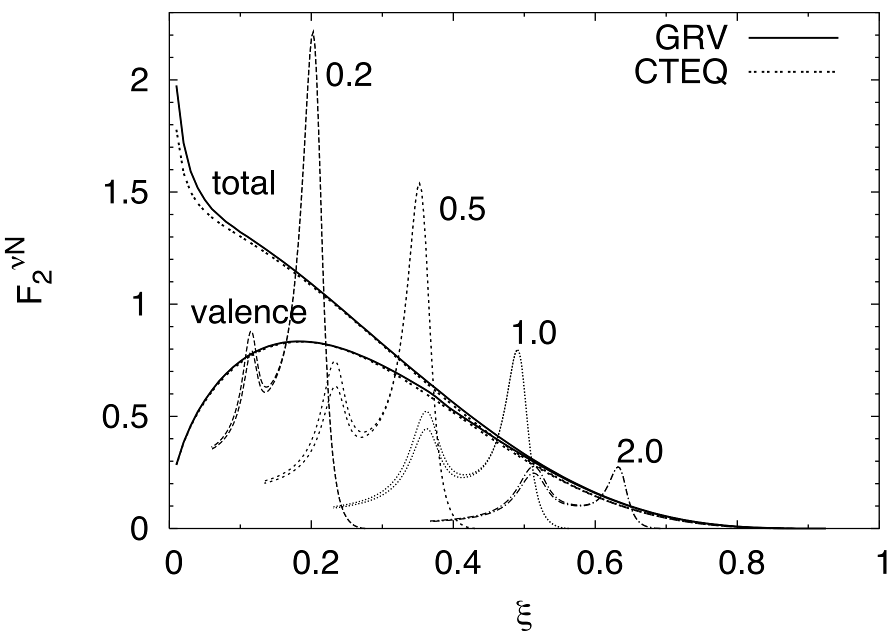
Phenomenological model

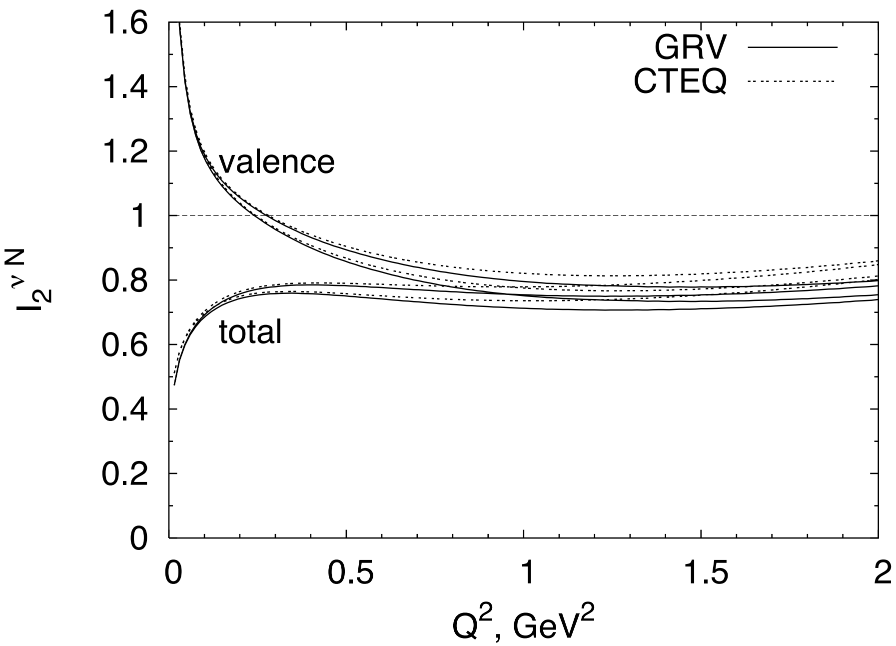


$\sim$ 10 - 20% agreement for $1 < Q^2 < 2$ GeV^2

Lalakulich, WM, Paschos
PRC, hep-ph/0608058

Phenomenological model



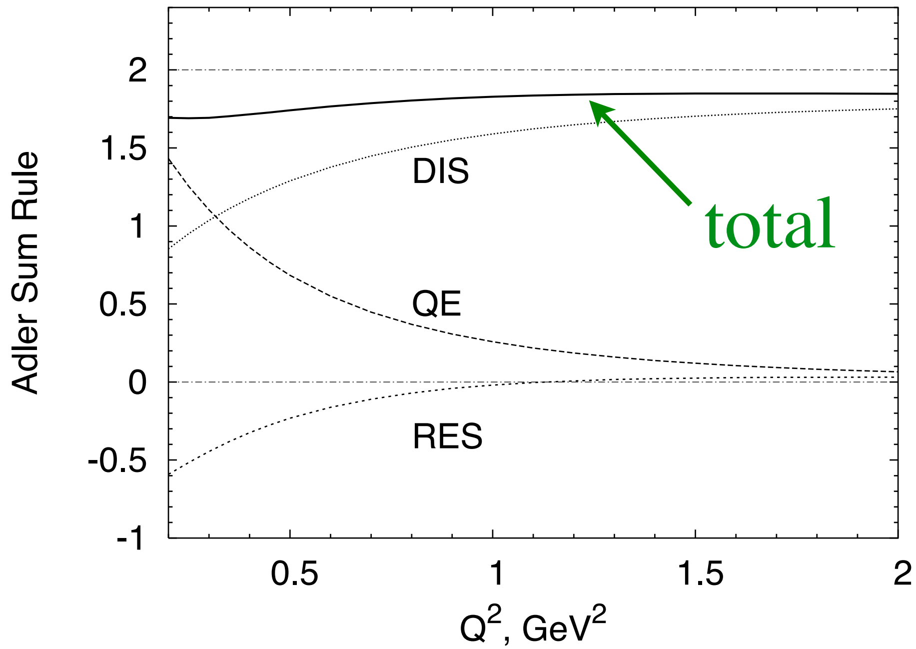


- $\sim 20\%$ agreement for $0.5 < Q^2 < 2\ \mathrm{GeV}^2$
- need to average over proton and neutron

Phenomenological model

Adler sum rule (valid for *all* Q^2)

$$\left[g_{1V}^{(QE)}(Q^2)\right]^2 + \left[g_{1A}^{(QE)}(Q^2)\right]^2 + \left[g_{2V}^{(QE)}(Q^2)\right]^2 \frac{Q^2}{4M^2} + \int d\nu \left[W_2^{\nu n}(Q^2,\nu) - W_2^{\nu p}(Q^2,\nu)\right] = 2$$



Lalakulich, WM, Paschos
PRC, hep-ph/0608058

- saturated at $\sim 90\%$ level $0.5 < Q^2 < 2\ \mathrm{GeV}^2$
- remainder likely indicates need for more resonances or better determined transition form factors

4.
Target mass corrections

Target mass corrections

Operator Product Expansion

$$\int d^4x\; e^{iq\cdot x}\langle N|T(J^\mu(x)J^\nu(0))|N\rangle$$

$$= \sum_k \left(-g^{\mu\nu}q^{\mu_1}q^{\mu_2} + g^{\mu\mu_1}q^\nu q^{\mu_2} + q^\mu q^{\mu_1}g^{\nu\mu_2} + g^{\mu\mu_1}g^{\nu\mu_2}Q^2\right)$$

$$\times q^{\mu_3}\cdots q^{\mu_{2k}}\frac{2^{2k}}{Q^{4k}}\underbrace{A_{2k}\Pi_{\mu_1\cdots\mu_{2k}}}_{\langle N|\mathcal{O}_{\mu_1\cdots\mu_{2k}}|N\rangle}$$

Georgi, Politzer (1976)

$$\Pi_{\mu_1\cdots\mu_{2k}} = p_{\mu_1}\cdots p_{\mu_{2k}} - (g_{\mu_i\mu_j}\ \text{terms})$$

$$= \sum_{j=0}^{k}(-1)^j\frac{(2k-j)!}{2^j(2k)^j}g\cdots g\; p\cdots p$$

traceless, symmetric rank-$2k$ tensor

Target mass corrections

- n-th moment of F_2 structure function

$$M_2^n(Q^2) = \int dx \; x^{n-2} \; F_2(x, Q^2)$$

$$= \sum_{j=0}^{\infty} \left(\frac{M^2}{Q^2} \right)^j \frac{(n+j)!}{j!(n-2)!} \frac{A_{n+2j}}{(n+2j)(n+2j-1)}$$

$$\Rightarrow \quad A_n = \int_0^1 dy \; y^n \; F(y)$$

"quark distribution function"

$$F(y) \equiv \frac{F_2(y)}{y^2}$$

Target mass corrections

- inverse Mellin transform (+ tedious manipulations)

$$F_2^{\mathrm{GP}}(x,Q^2) = \frac{x^2}{r^3} F(\xi) \; + \; 6 \frac{M^2}{Q^2} \frac{x^3}{r^4} \int_\xi^1 d\xi' \; F(\xi')$$

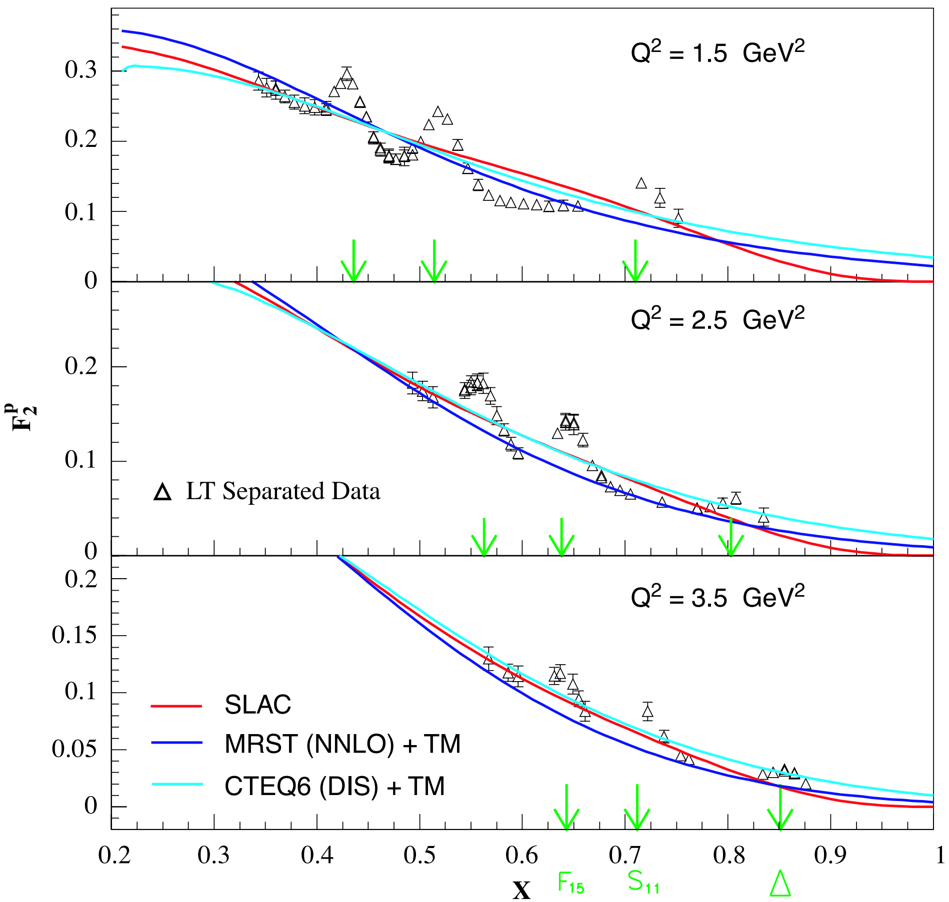
$$+ \; 12 \frac{M^4}{Q^4} \frac{x^4}{r^5} \int_\xi^1 d\xi' \; \int_{\xi'}^1 d\xi'' \; F(\xi'')$$

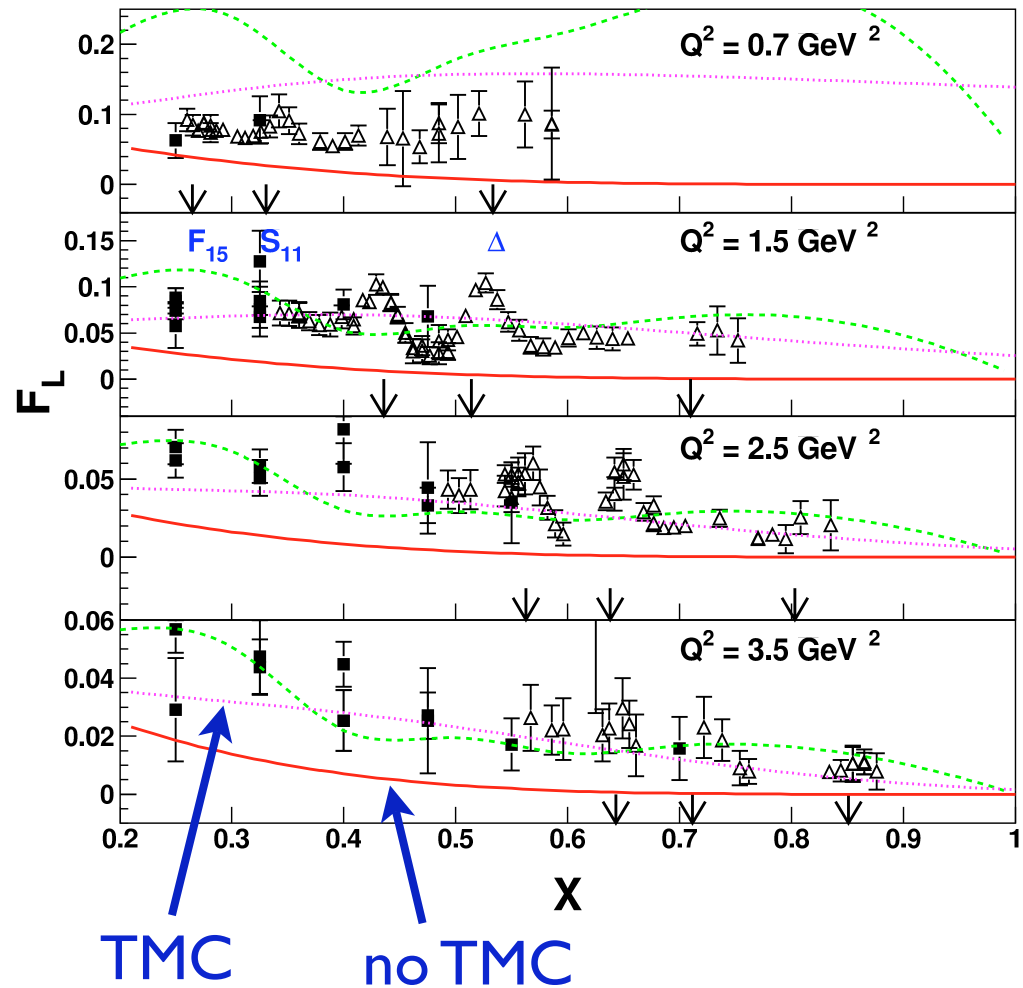
$$\xi = \frac{2x}{1+r} \qquad\qquad r = \sqrt{1 + 4x^2 M^2/Q^2}$$

... similarly for other structure functions F_1, F_L

Target mass corrections

Christy et al. (2005)





TMCs significant at large x^2/Q^2, especially for F_L

Threshold problem

- if $F(y) \sim (1-y)^\beta$ at large y

  then since $\xi_0 \equiv \xi(x=1) < 1$

  ⇒ $F(\xi_0) > 0$

  ⇒ $F_i^{\mathrm{TMC}}(x=1, Q^2) > 0$

  is this physical?

  ⇒ problem with GP formulation?

Possible solution

- work with ξ_0 dependent PDFs

→ n-th moment A_n of distribution function

$$A_n = \int_0^{\xi_{\rm max}} d\xi\ \xi^n\ F(\xi)$$

→ what is $\xi_{\rm max}$?

- GP use $\xi_{\rm max} = 1,\ \ \xi_0 < \xi < 1$ unphysical
- strictly, should use $\xi_{\rm max} = \xi_0$

Steffens, WM
PRC 73 (2006) 055202

Possible solution

- what is effect on phenomenology?

  $\longrightarrow$ try several "toy distributions"

standard TMC ("sTMC")

$$q(\xi) = \mathcal{N}\ \xi^{-1/2}\ (1-\xi)^3\ , \qquad \xi_{\rm max} = 1$$

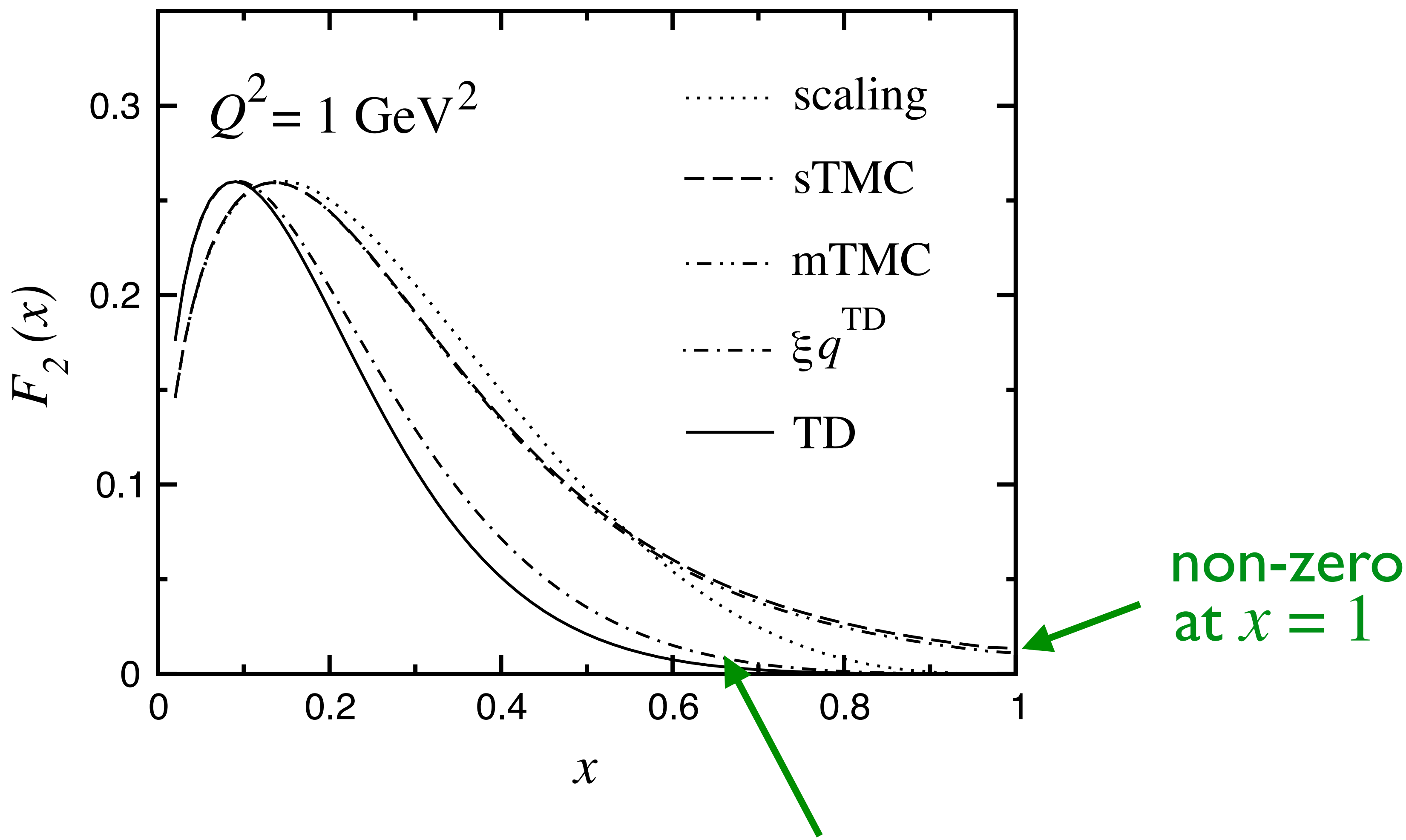
modified TMC ("mTMC")

$$q(\xi) = \mathcal{N}\ \xi^{-1/2}\ (1-\xi)^3\ \Theta(\xi - \xi_0), \qquad \xi_{\rm max} = \xi_0$$

threshold dependent ("TD")

$$q^{\rm TD}(\xi) = \mathcal{N}\ \xi^{-1/2}\ (\xi_0 - \xi)^3\ , \qquad \xi_{\rm max} = \xi_0$$

TMCs in F_2

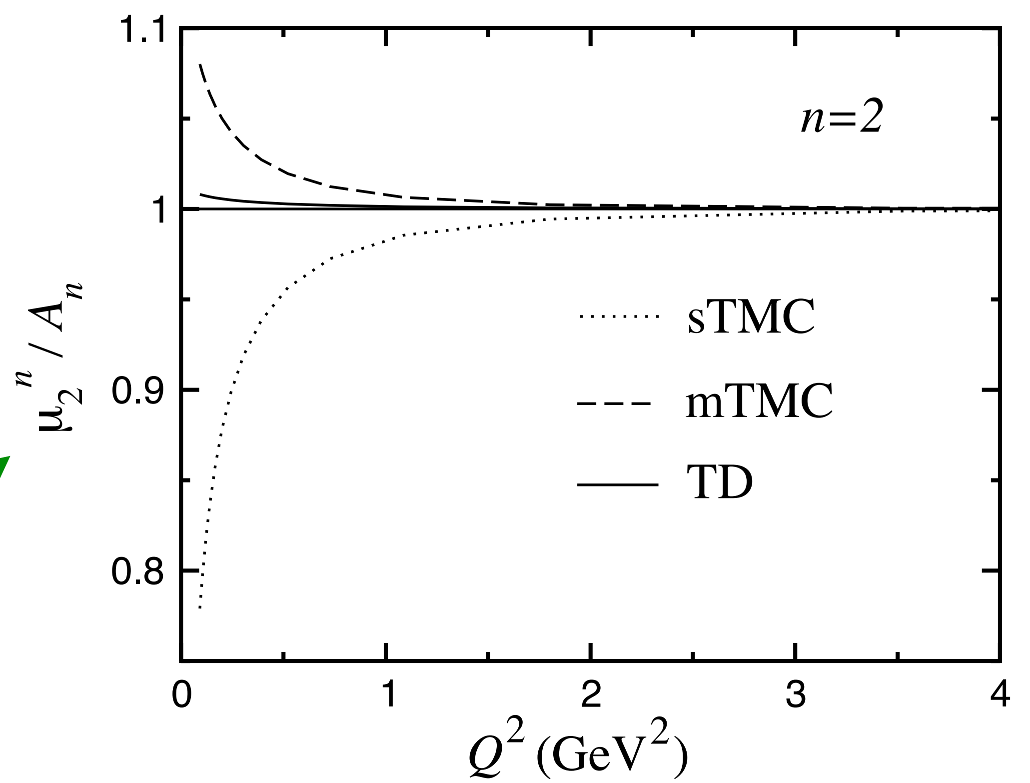


→ correct threshold behavior for "TD" correction

Nachtmann F_2 moments

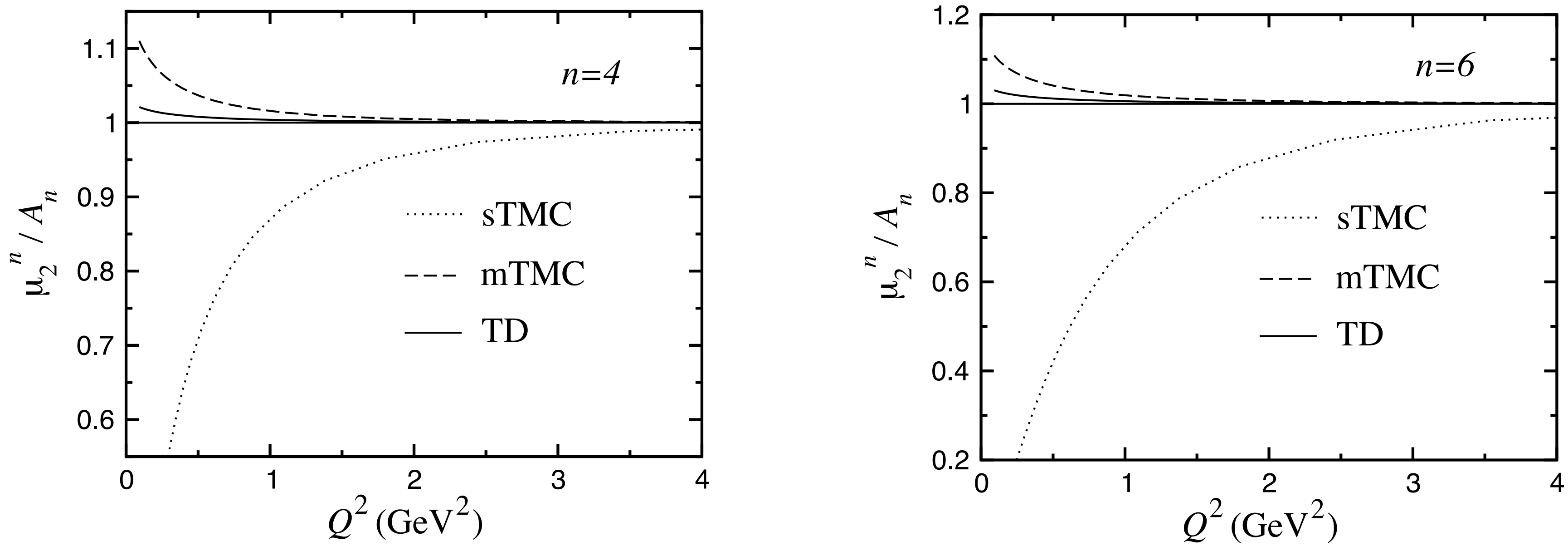
- designed to remove target mass effects explicitly from structure function moment

Nachtmann moment



→ moment of structure function agrees with moment of PDF to 1% down to very low Q^2

Nachtmann F_2 moments



→ $$\frac{\mu_2^n(\text{finite } Q^2)}{A_n(\text{finite } Q^2)} = \frac{\mu_2^n(Q^2 \to \infty)}{A_n(Q^2 \to \infty)}$$

→ extract PDFs from structure function data at lower Q^2

Summary

- Remarkable confirmation of quark-hadron duality in structure functions
  → higher twists "small" down to low Q^2 (~ 1 GeV^2)

- OPE "organizes" duality violations in terms of higher twists *but* need quark models to understand origin of resonance cancellations
  → phenomenological models for local duality
  → need higher-Q^2 transition form factor data
  → quantify role of background vs. resonances

- Importance of target mass corrections at low Q^2
  → avoid unphysical "threshold problem" by using threshold-dependent PDFs

Summary

- References: WM, Ent, Keppel: *Phys. Rept.* 406 (2005) 127

  Steffens, WM: *Phys. Rev.* C 73 (2006) 055202

  Olness, WM, Steffens et al. (CTEQ):
  review on TMCs, in preparation

  Lalakulich, WM, Paschos: *hep-ph/0608058*

  Close, WM: *Phys. Rev.* C 68 (2003) 035210