JLab Science & Technology Review
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Recent Developments in Radiative Corrections

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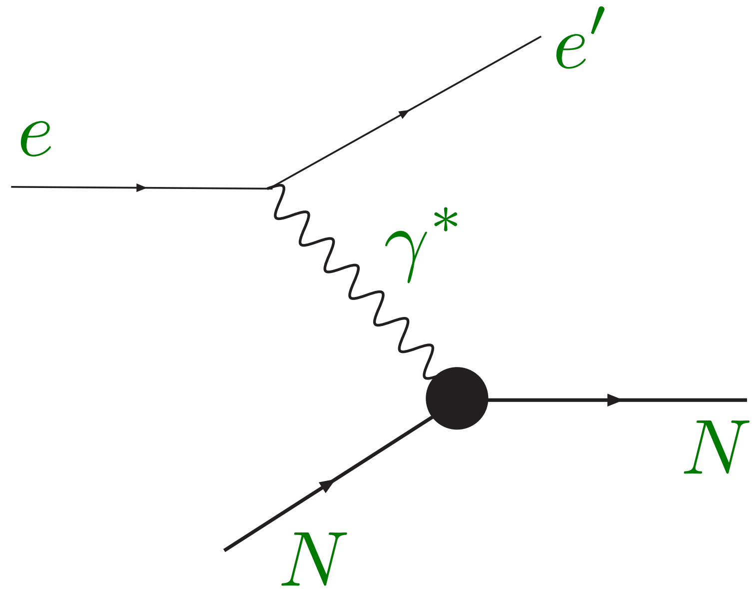
Outline

- Elastic *ep* scattering
- Two-photon exchange
  - ➡ Rosenbluth separation vs. polarization transfer
- Global analysis of form factors
- Parity-violating electron scattering
  - ➡ photon-Z interference & strangeness in the proton
- Summary

Elastic eN scattering

Elastic cross section



$$\frac{d\sigma}{d\Omega} = \sigma_{\text{Mott}} \; \frac{\tau}{\varepsilon\;(1+\tau)} \; \sigma_R$$

$$\tau = Q^2/4M^2$$

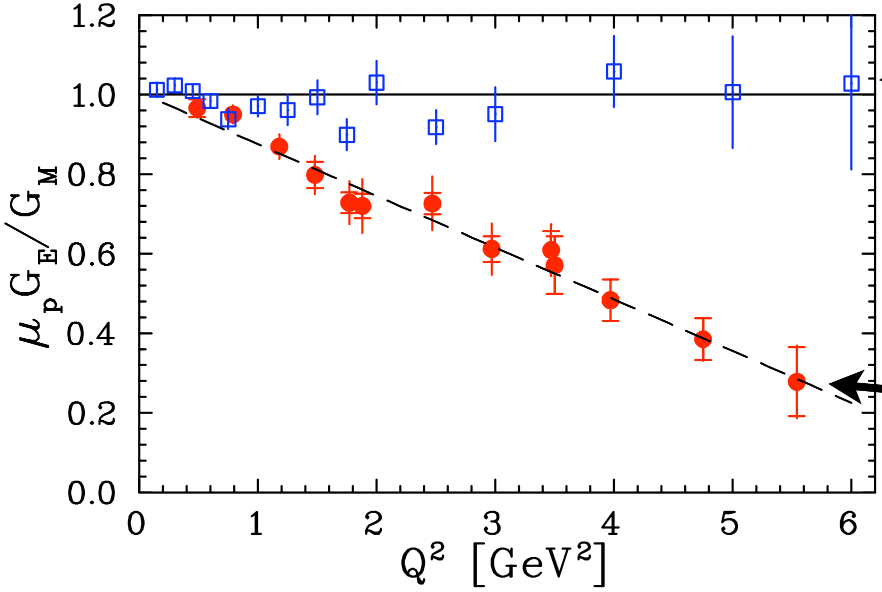
$$\varepsilon = \left(1 + 2(1+\tau)\tan^2(\theta/2)\right)^{-1}$$

$$\sigma_{\text{Mott}} = \frac{\alpha^2 E' \cos^2\frac{\theta}{2}}{4E^3 \sin^4\frac{\theta}{2}}$$ ← cross section for scattering from point particle

$$\sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$ ← reduced cross section

$G_E \; , \; G_M$ ← Sachs electric and magnetic form factors

Proton G_E/G_M Ratio



Rosenbluth (Longitudinal-Transverse) Separation

Polarization Transfer

LT method

$$\sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

→ G_E from slope in ε plot

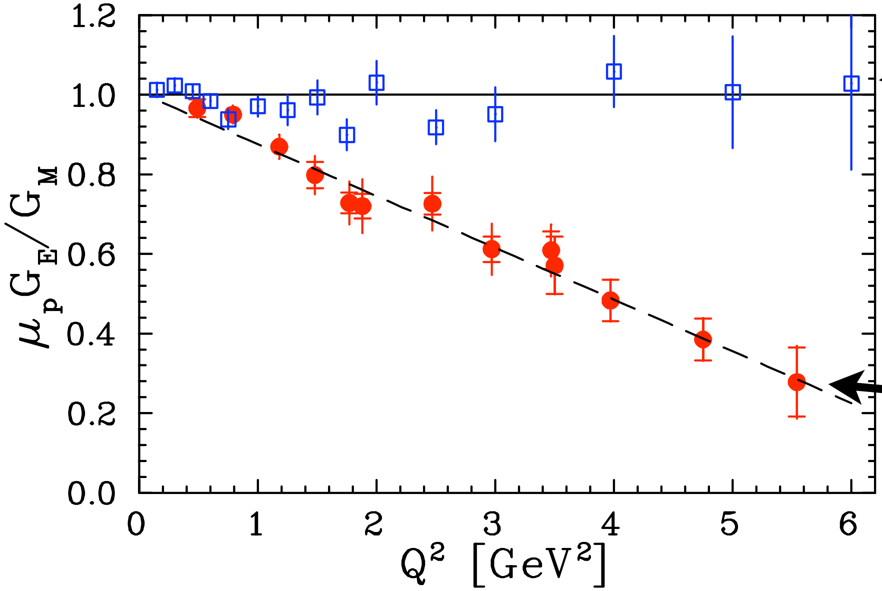
→ suppressed at large Q^2

PT method

$$\frac{G_E}{G_M} = -\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}} \frac{P_T}{P_L}$$

→ $P_{T,L}$ recoil proton polarization in $\vec{e}\, p \to e\, \vec{p}$

Proton G_E/G_M Ratio



LT method

$$\sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

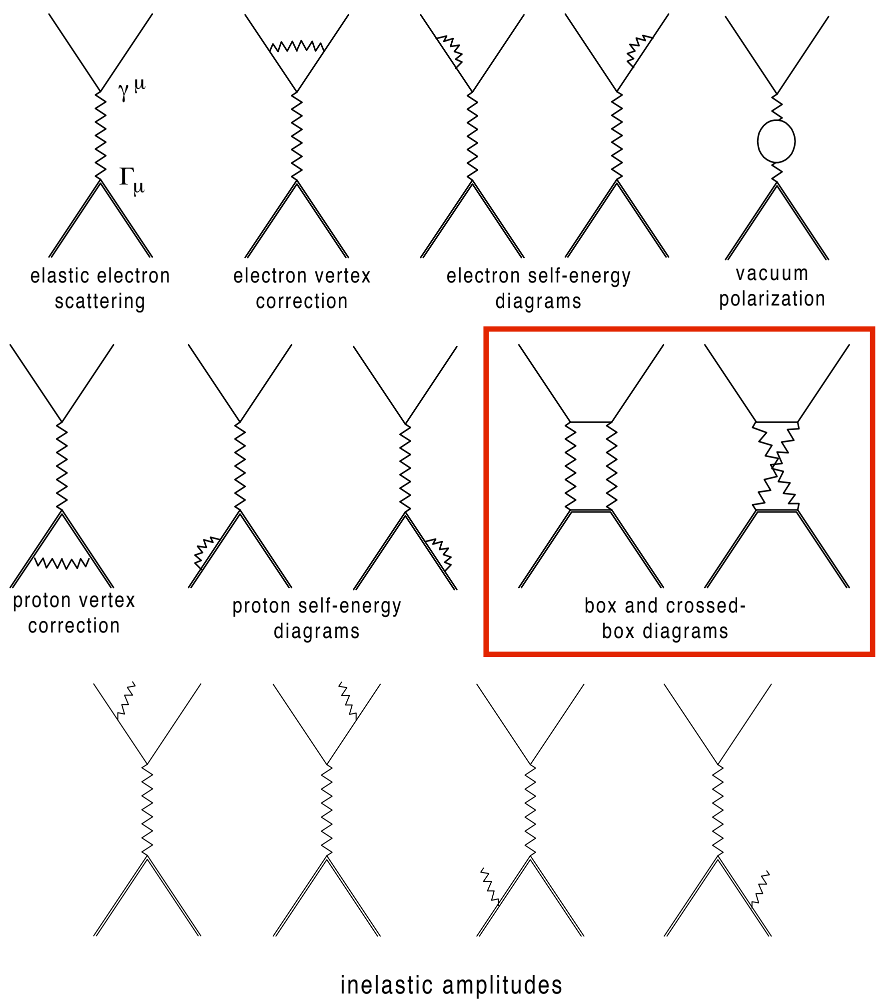
PT method

$$\frac{G_E}{G_M} = -\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}} \frac{P_T}{P_L}$$

Are the G_E^p/G_M^p data consistent?

QED Radiative Corrections

- cross section modified by 1γ loop effects



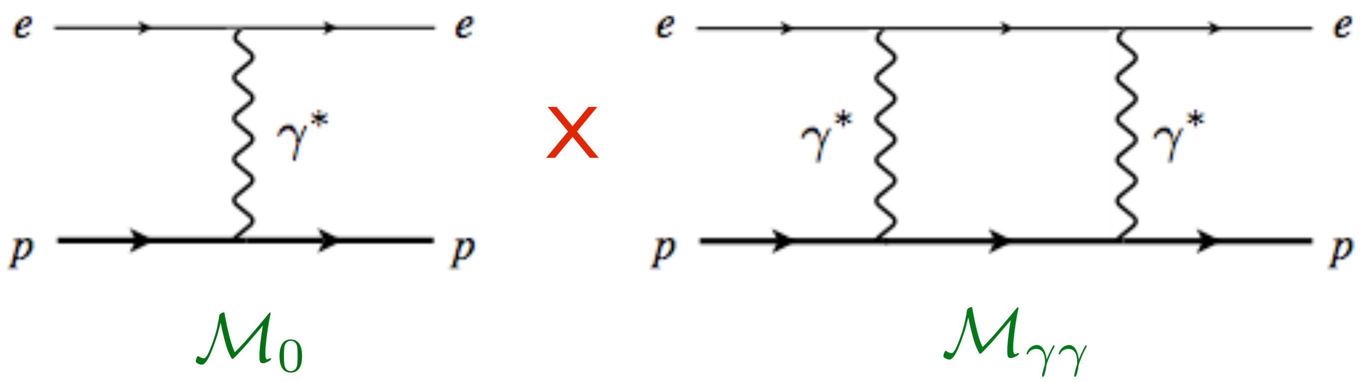
$$d\sigma = d\sigma_0\,(1+\delta)$$

δ contains additional ε dependence, mostly from box diagrams

(most difficult to calculate)

Two-photon exchange

- interference between Born and two-photon exchange amplitudes



- contribution to cross section:

$$\delta^{(2\gamma)} = \frac{2\mathcal{R}e\left\{\mathcal{M}_0^\dagger\ \mathcal{M}_{\gamma\gamma}\right\}}{|\mathcal{M}_0|^2}$$

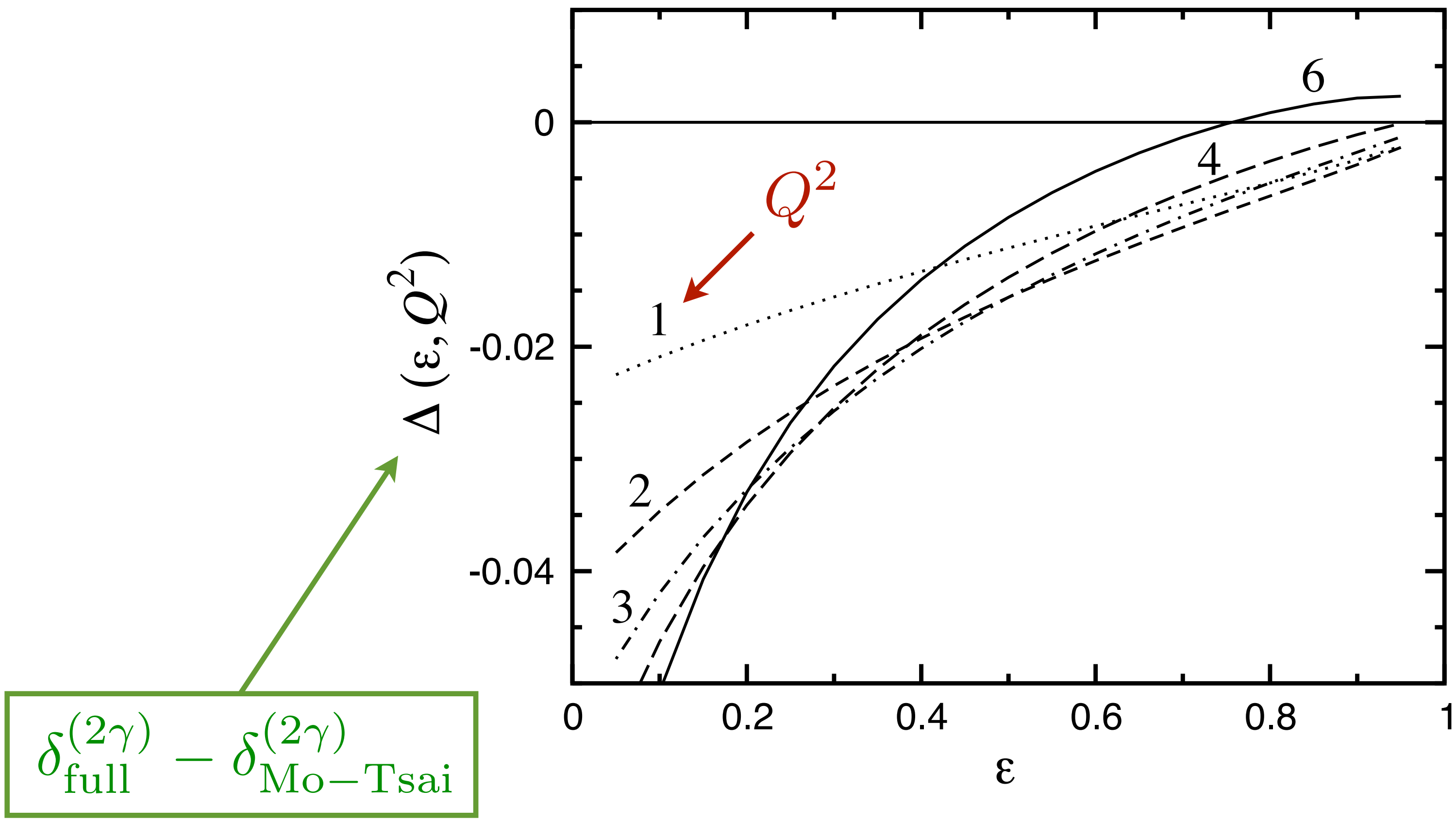
- standard "soft photon approximation" (used in most data analyses)

  $\longrightarrow$ approximate integrand in $\mathcal{M}_{\gamma\gamma}$ by values at γ^* poles

  $\longrightarrow$ neglect nucleon structure (no form factors) *Mo, Tsai (1969)*

Two-photon exchange

- "exact" calculation of loop diagram (including $\gamma^* NN$ form factors)



Blunden, Melnitchouk, Tjon
*PRL **91** (2003) 142304;*
*PRC **72** (2005) 034612*

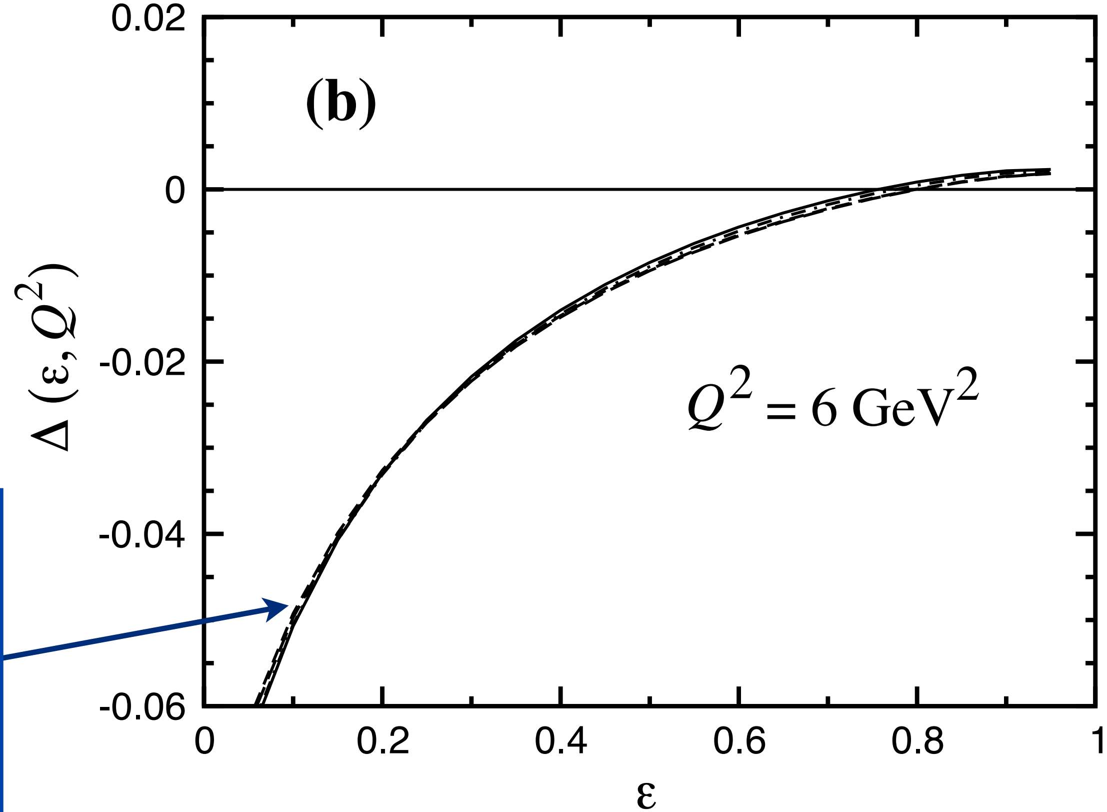
⟹ few % magnitude

⟹ <u>positive</u> slope

⟹ non-linearity in ε

Two-photon exchange

- "exact" calculation of loop diagram (including $\gamma^* NN$ form factors)

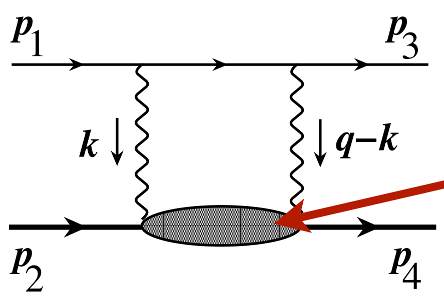


form factors:
Mergell et al. (1996)
Brash et al. (2002)
Arrington LT (2004)
Arrington PT (2004)

Blunden, Melnitchouk, Tjon
PRL ***91*** *(2003) 142304;*
PRC ***72*** *(2005) 034612*

⟹ results essentially independent of form factor input

What about higher-mass intermediate states?



$N,\ \Delta,\ P_{11},\ S_{11},\ S_{31},\ldots$

- Lowest mass excitation is P_{33} $\Delta(1232)$ resonance

➡ relativistic $\gamma^* N\Delta$ vertex

form factor $\frac{\Lambda_\Delta^4}{(\Lambda_\Delta^2 - q^2)^2}$

$$\Gamma^{\nu\alpha}_{\gamma\Delta\to N}(p,q) \equiv iV^{\nu\alpha}_{\Delta in}(p,q) = i\frac{eF_\Delta(q^2)}{2M_\Delta^2}\Big\{g_1\left[\,g^{\nu\alpha}\not{p}\not{q} - p^\nu\gamma^\alpha\not{q} - \gamma^\nu\gamma^\alpha p\cdot q + \gamma^\nu\not{p}q^\alpha\,\right]$$

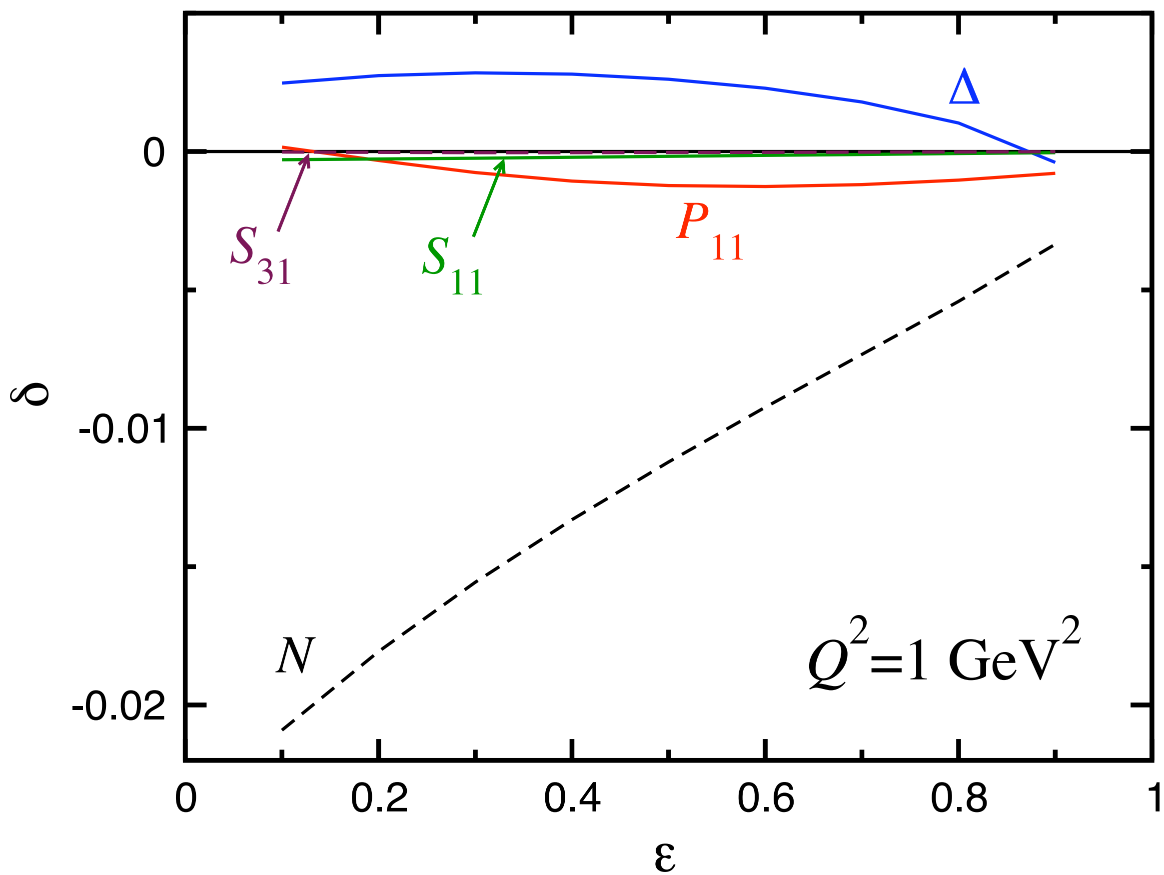
$$+g_2\left[\,p^\nu q^\alpha - g^{\nu\alpha}p\cdot q\,\right] + (g_3/M_\Delta)\left[\,q^2(p^\nu\gamma^\alpha - g^{\nu\alpha}\not{p}) + q^\nu(q^\alpha\not{p} - \gamma^\alpha p\cdot q)\,\right]\Big\}\gamma_5\, T_3$$

➡ coupling constants

g_1	magnetic	➡	7
$g_2 - g_1$	electric	➡	9
g_3	Coulomb	➡	-2 ... 0

- Higher-mass intermediate states have also been calculated

→ more model dependent, since couplings & form factors not well known (especially at high Q^2)



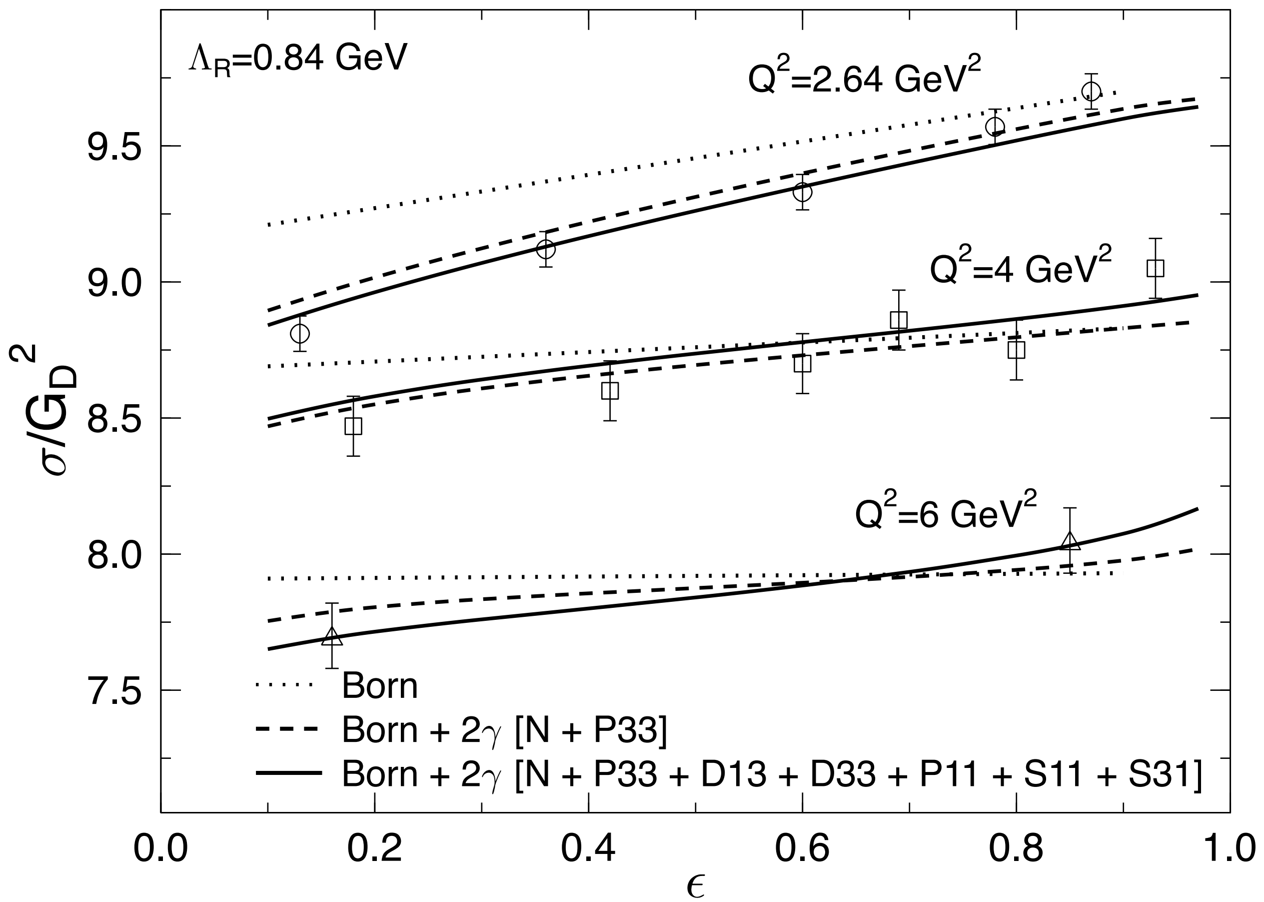
Kondratyuk, Blunden, Melnitchouk, Tjon Phys. Rev. Lett ***95*** *(2005) 172503*

Kondratyuk, Blunden Phys. Rev. C ***75*** *(2007) 038201*

⇒ dominant contribution from N

⇒ Δ partially cancels N contribution

Higher-mass intermediate states have also been calculated



Kondratyuk, Blunden
Phys. Rev. C ***75*** *(2007) 038201*

➡ higher mass resonance contributions small

➡ much better fit to data including TPE

Global analysis

- reanalyze *all* elastic *ep* data (Rosenbluth, PT), including TPE corrections consistently from the beginning

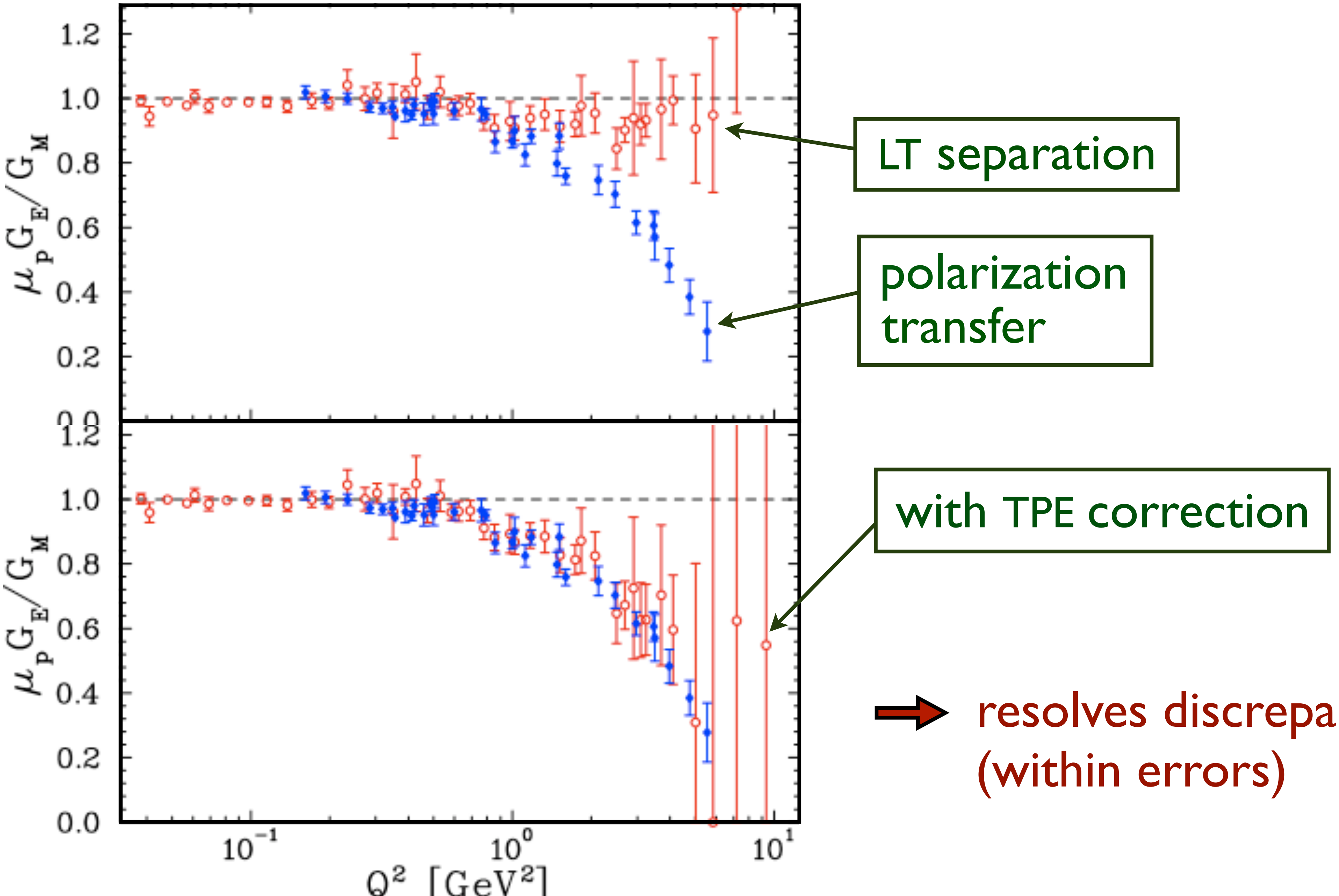
- use explicit calculation of N elastic contribution

- approximate higher mass contributions by phenomenological form, based on N^* calculations:

$$\delta^{(2\gamma)}_{\mathrm{high\ \ mass}} = -0.01\ (1-\varepsilon)\ \log Q^2 / \log 2.2$$

for $Q^2 > 1\ \mathrm{GeV}^2$, with $\pm 100\%$ uncertainty

⟹ decreases $\varepsilon = 0$ cross section by 1% (2%) at $Q^2 = 2.2\ (4.8)\ \mathrm{GeV}^2$



resolves discrepancy (within errors)

Arrington, Melnitchouk, Tjon
Phys. Rev. C ***76*** *(2007) 035205*

Non-linearity in ε

- unique feature of TPE correction to cross section
- observation of non-linearity in ε would provide direct evidence of TPE in elastic scattering
- fit reduced cross section as:

$$\sigma_R = P_0 \left[1 + P_1 \, (\varepsilon - \frac{1}{2}) + P_2 \, (\varepsilon - \frac{1}{2})^2 \right]$$

- current data give average non-linearity parameter:

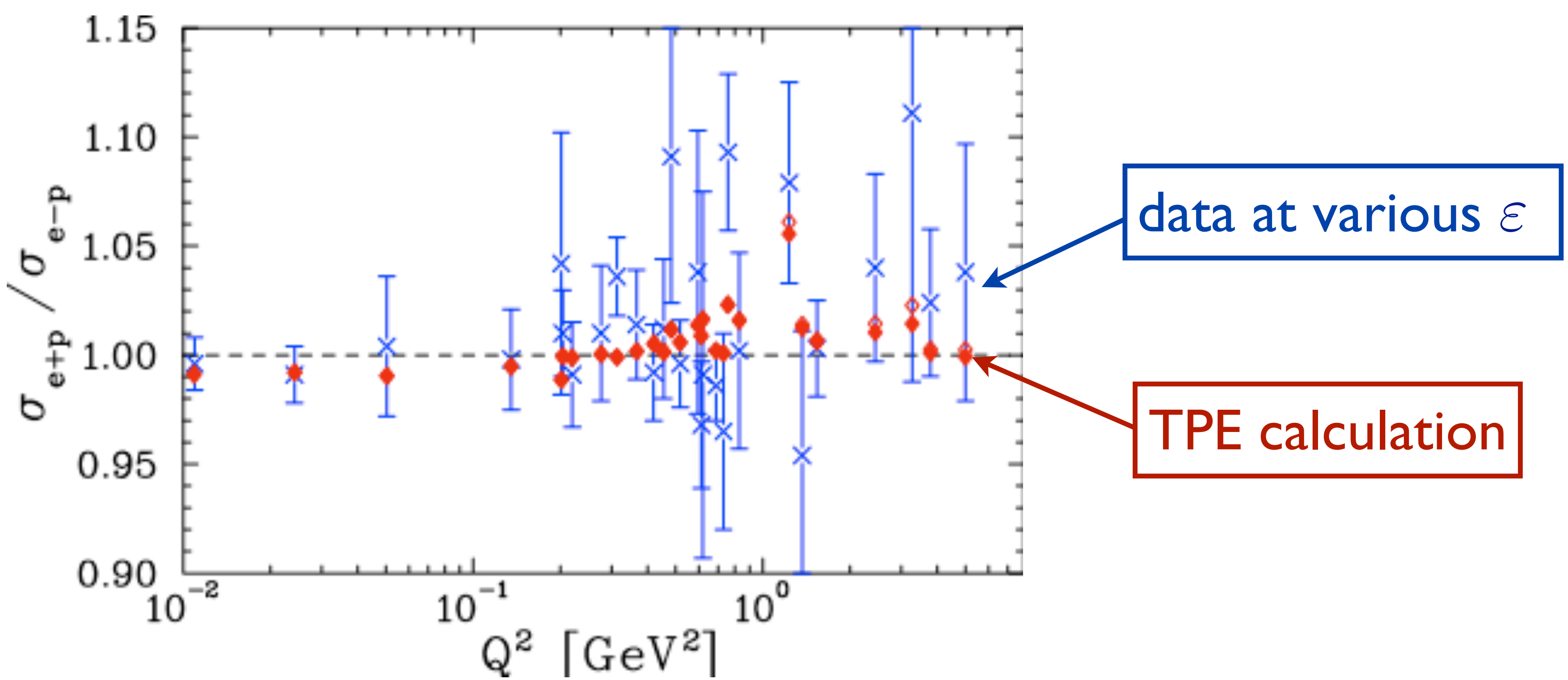
$$\langle P_2 \rangle = 4.3 \pm 2.8\%$$

- Hall C experiment E-05-017 will provide accurate measurement of ε dependence

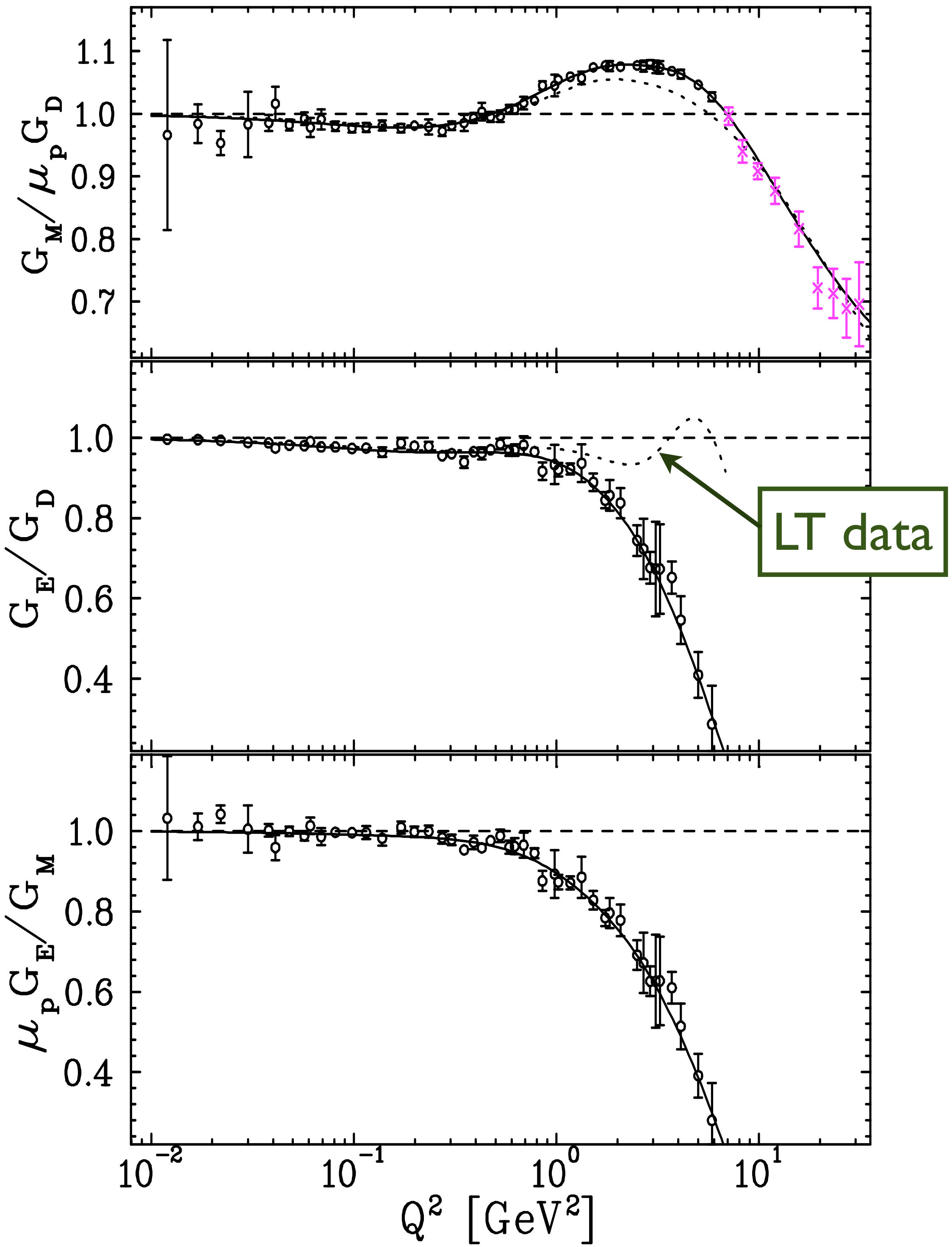
e^+/e^- comparison

- 1γ (2γ) exchange changes sign (invariant) under $e^+ \leftrightarrow e^-$
- ratio of $e^+p\,/\,e^-p$ elastic cross sections sensitive to $\Delta(\varepsilon, Q^2)$:

$$\sigma_{e^+p}/\sigma_{e^-p} \approx 1 - 2\Delta$$



⇒ simultaneous e^-p/e^+p measurement using tertiary e^+/e^- beam to Q^2 ~ 1-2 GeV^2 (Hall B expt. E-04-116)



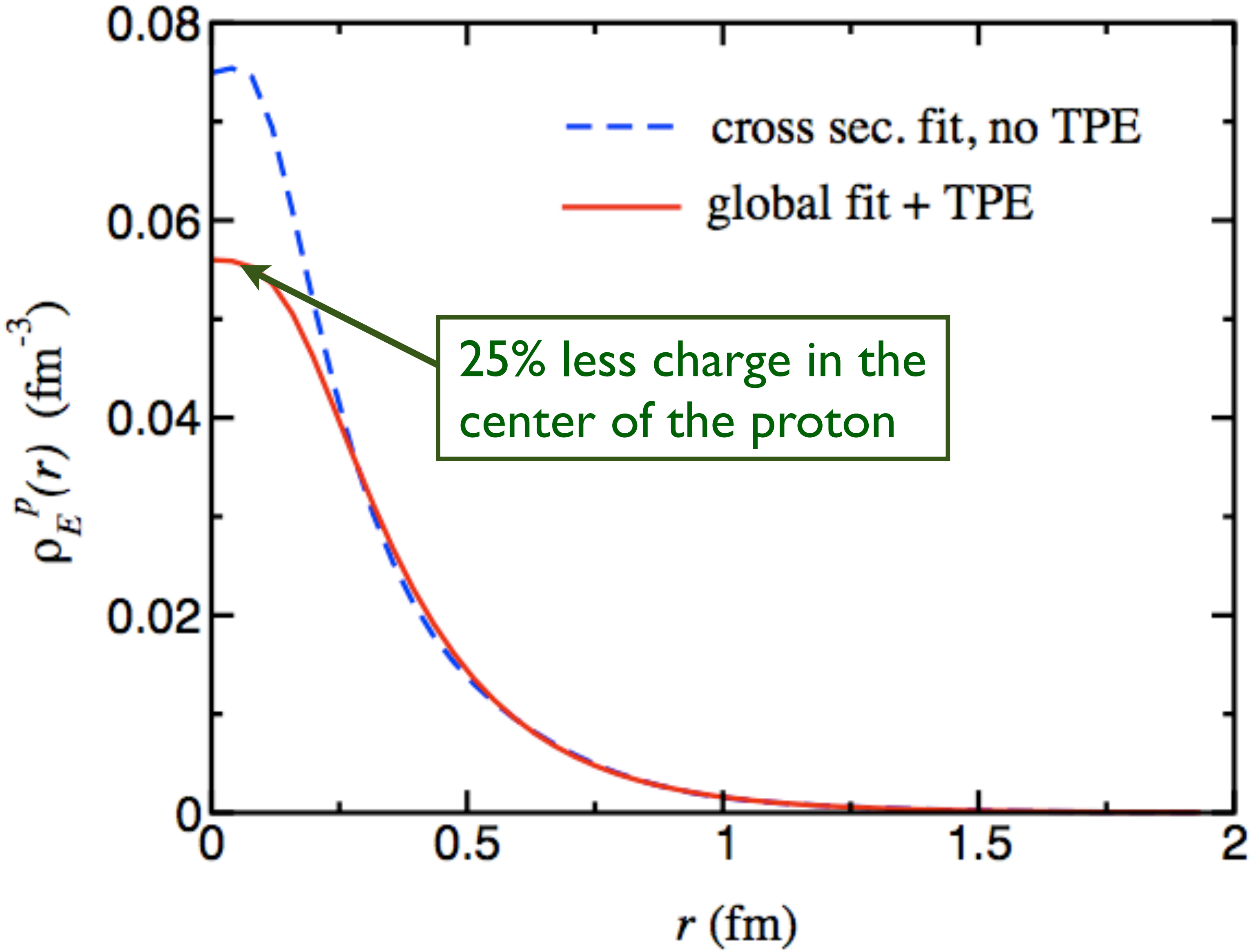
final form factor results from global analysis *including TPE corrections*

$$\left\{ G_E,\ \frac{G_M}{\mu_p} \right\} = \frac{1 + \sum_{i=1}^{n} a_i \tau^i}{1 + \sum_{i=1}^{n+2} b_i \tau^i}$$

Parameter	G_M/μ_p	G_E
a_1	−1.465	3.439
a_2	1.260	−1.602
a_3	0.262	0.068
b_1	9.627	15.055
b_2	0.000	48.061
b_3	0.000	99.304
b_4	11.179	0.012
b_5	13.245	8.650

Arrington, Melnitchouk, Tjon
Phys. Rev. C ***76*** *(2007) 035205*

Charge density

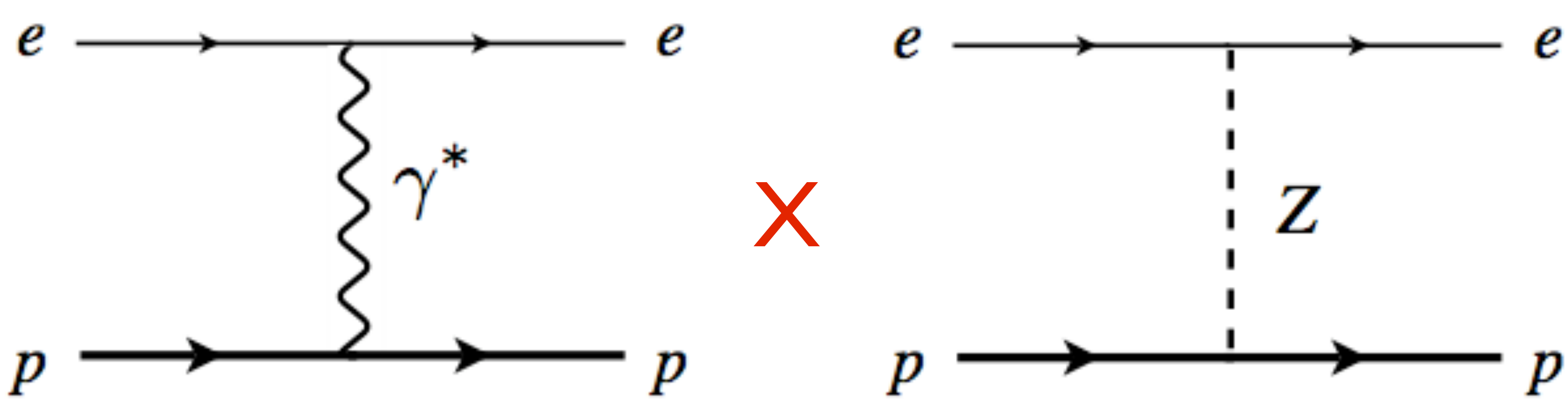


Parity-violating e scattering

- Left-right polarization asymmetry in $\vec{e}\,p \to e\,p$ scattering

$$A_{\rm PV} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\left(\frac{G_F Q^2}{4\sqrt{2}\alpha}\right)(A_V + A_A + A_s)$$

→ measure interference between e.m. and weak currents



Born (tree) level

Parity-violating e scattering

- Left-right polarization asymmetry in $\vec{e}\,p \to e\,p$ scattering

$$A_{\mathrm{PV}} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\left(\frac{G_F Q^2}{4\sqrt{2}\alpha}\right)(A_V + A_A + A_s)$$

→ measure interference between e.m. and weak currents

$$A_V = g_A^e \rho \left[(1 - 4\kappa \sin^2\theta_W) - (\varepsilon G_E^{\gamma p} G_E^{\gamma n} + \tau G_M^{\gamma p} G_M^{\gamma n})/\sigma^{\gamma p}\right]$$

radiative corrections, including TBE

using relations between weak and e.m. form factors

$$G_{E,M}^{Zp} = (1 - 4\sin^2\theta_W) G_{E,M}^{\gamma p} - G_{E,M}^{\gamma n} - G_{E,M}^{s}$$

Parity-violating e scattering

- Left-right polarization asymmetry in $\vec{e}\,p \to e\,p$ scattering

$$A_{\rm PV} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\left(\frac{G_F Q^2}{4\sqrt{2}\alpha}\right)(A_V + A_A + A_s)$$

→ measure interference between e.m. and weak currents

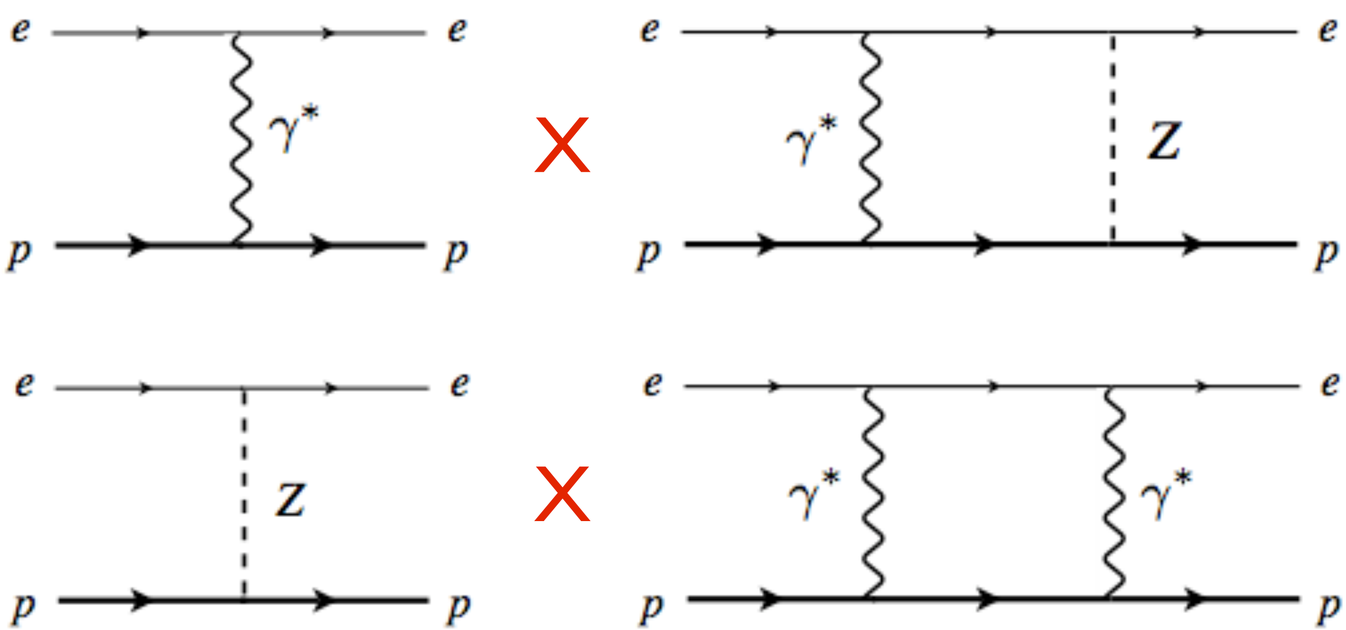
$$A_A = g_V^e \sqrt{\tau(1+\tau)(1-\varepsilon^2)}\;\widetilde{G}_A^{Zp} G_M^{\gamma p} / \sigma^{\gamma p}$$

includes axial RCs + anapole term

$$A_s = -g_A^e \rho \left(\varepsilon G_E^{\gamma p} G_E^s + \tau G_M^{\gamma p} G_M^s\right) / \sigma^{\gamma p}$$

strange electric & magnetic form factors

Two-boson exchange corrections



“$\gamma(Z\gamma)$”

“$Z(\gamma\gamma)$”

- current PDG estimates computed at $Q^2 = 0$

  Marciano, Sirlin (1980)
  Erler, Ramsey-Musolf (2003)

- do not include hadron structure effects
  (parameterized via VNN form factors)

Two-boson exchange corrections

- At tree level, $\rho = \kappa = 1$

- Including TBE corrections,

$$\rho = \rho_0 + \Delta\rho \ , \qquad \kappa = \kappa_0 + \Delta\kappa$$

standard RCs (ρ_0)

Born-TBE interference ($\Delta\rho$)

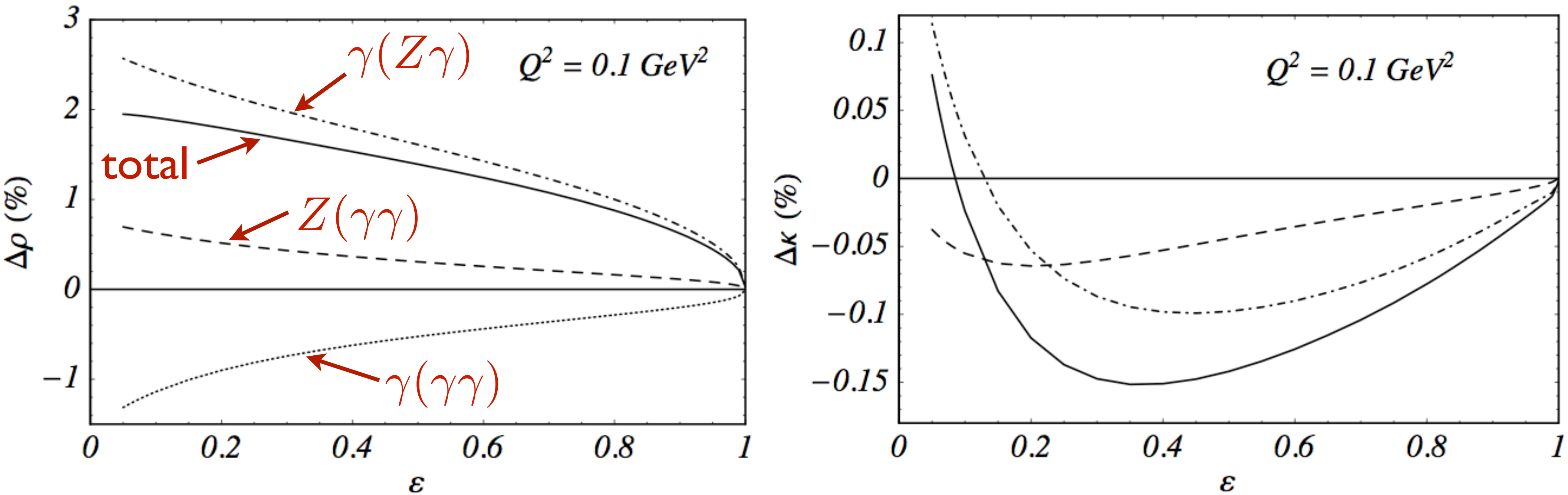
→ from vector part of asymmetry,

$$\Delta\rho = \frac{A_V^p + A_V^n}{A_V^{p,\text{tree}} + A_V^{n,\text{tree}}} - \frac{\Delta\sigma^{\gamma(\gamma\gamma)}}{\sigma^{\gamma p}}$$

$$\Delta\kappa = \frac{A_V^p}{A_V^{p,\text{tree}}} - \frac{A_V^p + A_V^n}{A_V^{p,\text{tree}} + A_V^{n,\text{tree}}}$$

tree level contribution ($A_V^{p,\text{tree}} + A_V^{n,\text{tree}}$)

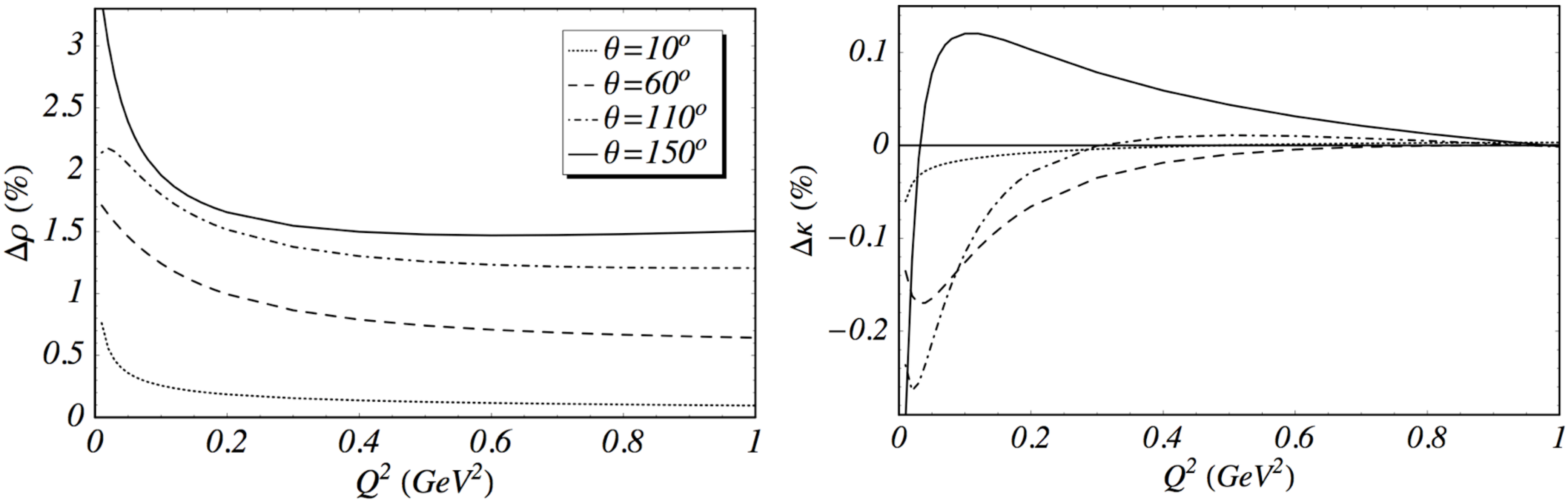
Two-boson exchange corrections



Tjon, Melnitchouk, PRL ***100****, 082003 (2008)*

- some cancellation between $Z(\gamma\gamma)$ and $\gamma(\gamma\gamma)$ corrections in $\Delta\rho$
- no $\gamma(\gamma\gamma)$ contribution to $\Delta\kappa$

Two-boson exchange corrections



Tjon, Melnitchouk, PRL ***100****, 082003 (2008)*

- 2-3% correction at $Q^2 < 0.1$ GeV^2
- strong Q^2 dependence at low Q^2
- *cf.* Marciano-Sirlin ($Q^2 = 0$): $\Delta\rho = -0.37\%$, $\Delta\kappa = -0.53\%$

Two-boson exchange corrections

- dependence on input form factors

$$\delta = A_{\rm PV}^{\rm TBE} \ / \ A_{\rm PV}^{\rm tree}$$

Q^2 (GeV^2)	θ	δ(%) Empirical	Dipole	Monopole	
0.1	144.0°	1.62	1.52	1.72	SAMPLE (97)
0.23	35.31°	0.63	0.58	0.84	PVA4 (04)
0.477	12.3°	0.16	0.15	0.24	HAPPEX (04)
0.997	20.9°	0.22	0.23	0.30	G0 (05)
0.109	6.0°	0.20	0.16	0.32	HAPPEX (07)
0.23	110.0°	1.39	1.33	1.52	G0 } results to come
0.03	8.0°	0.58	0.47	0.86	Qweak } results to come

→ "dipole" results ~ 5-10% smaller than "empirical" [1]

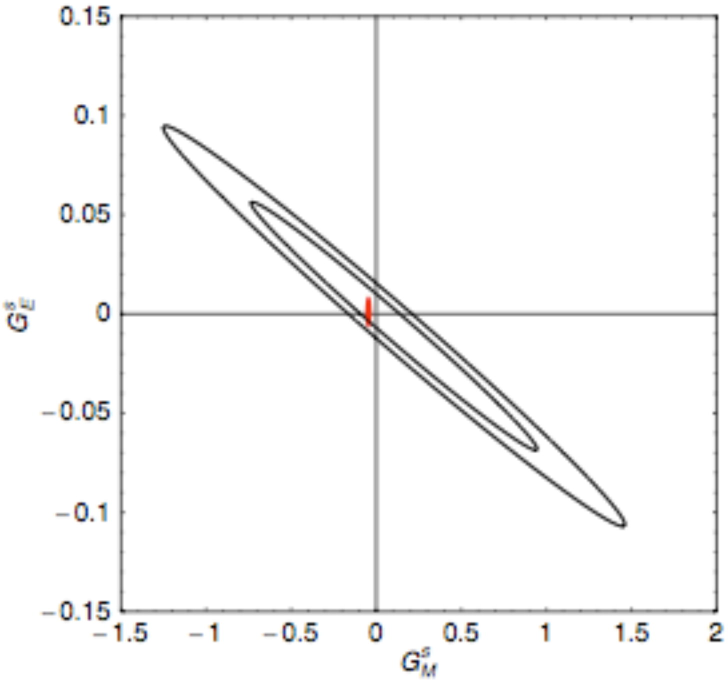
→ "monopole" [2] results ~ 50% larger than "empirical" [1]

[1] *Tjon, Melnitchouk, PRL* ***100****, 082003 (2008)*

[2] *Zhou, Kao, Yang, PRL* ***99****, 262001 (2007)*

Effects on strange form factors

- global analysis of all PVES data at $Q^2 < 0.3\ \mathrm{GeV}^2$



$$G_E^s = 0.0025 \pm 0.0182$$

$$G_M^s = -0.011 \pm 0.254$$

at $Q^2 = 0.1\ \mathrm{GeV}^2$

Young et al., PRL ***97****, 102002 (2006)*

- including TBE corrections:

$$G_E^s = 0.0023 \pm 0.0182$$

$$G_M^s = -0.020 \pm 0.254$$

at $Q^2 = 0.1\ \mathrm{GeV}^2$

fixed mainly by ^{4}He data ...

...TBE for ^{4}He not yet included

Summary

- TPE corrections resolve most of Rosenbluth / PT G_E^p/G_M^p discrepancy

  $\longrightarrow$ excited state contributions $(\Delta, P_{11}(1440), S_{11}(1535), ...)$ small relative to nucleon

- Reanalysis of global data, including TPE from the outset

  $\longrightarrow$ first consistent form factor fit at order α^3

  $\longrightarrow$ "25% less charge" in the center of the proton

- $\gamma(Z\gamma)$ and $Z(\gamma\gamma)$ contributions give $\sim 2\%$ corrections to PVES at small Q^2

  $\longrightarrow$ strong Q^2 dependence at low Q^2

  $\longrightarrow$ affects extraction of strange form factors