The Lead Radius Experiment (PREx) and
Neutron Rich Matter in the Heavens and on Earth
August 19, 2008

$\gamma - Z^0$ Contributions to the Parity-Violating Asymmetry

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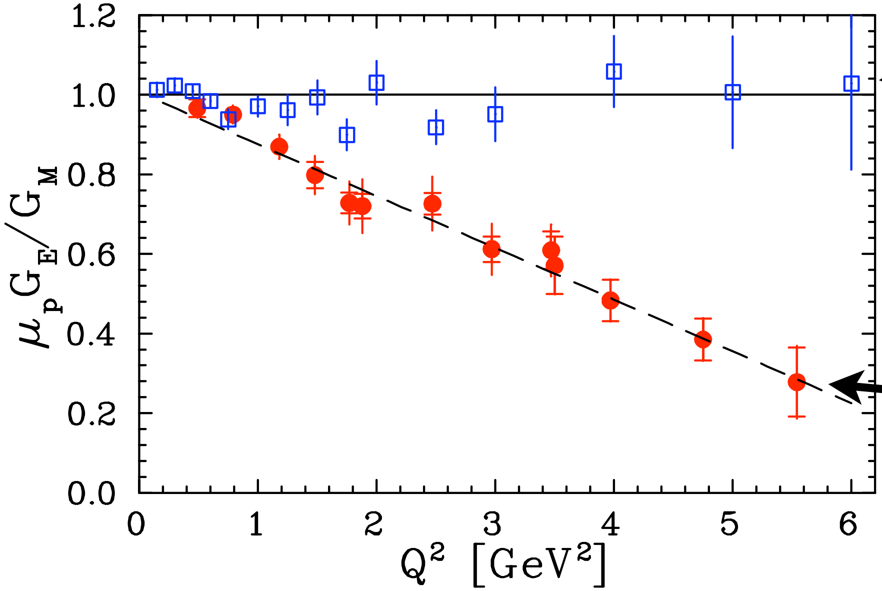
(also Peter Blunden (Manitoba) & John Arrington (Argonne))

Proton G_E/G_M Ratio



Rosenbluth (<u>L</u>ongitudinal-<u>T</u>ransverse) Separation

<u>P</u>olarization <u>T</u>ransfer

<u>LT</u> method

$$\sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

→ G_E from slope in ε plot

→ suppressed at large Q^2

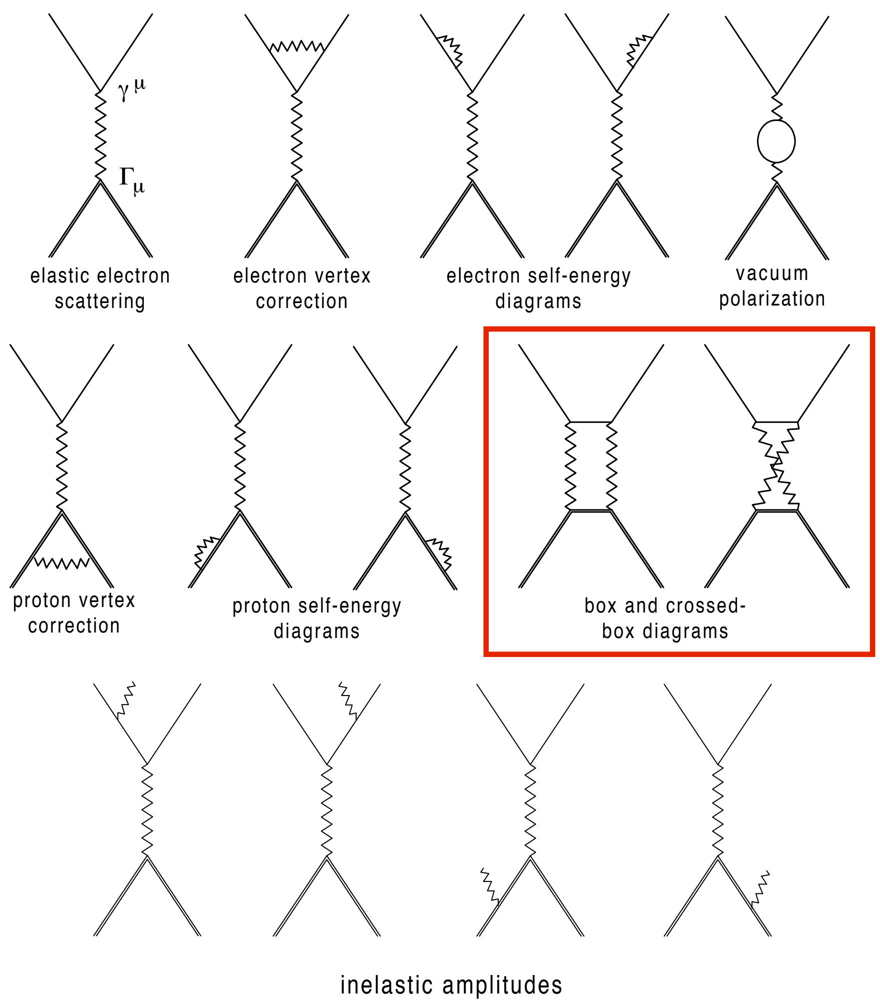
<u>PT</u> method

$$\frac{G_E}{G_M} = -\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}} \frac{P_T}{P_L}$$

→ $P_{T,L}$ recoil proton polarization in $\vec{e}\, p \to e\, \vec{p}$

Possible reason – QED Radiative Corrections

- cross section modified by 1γ loop effects



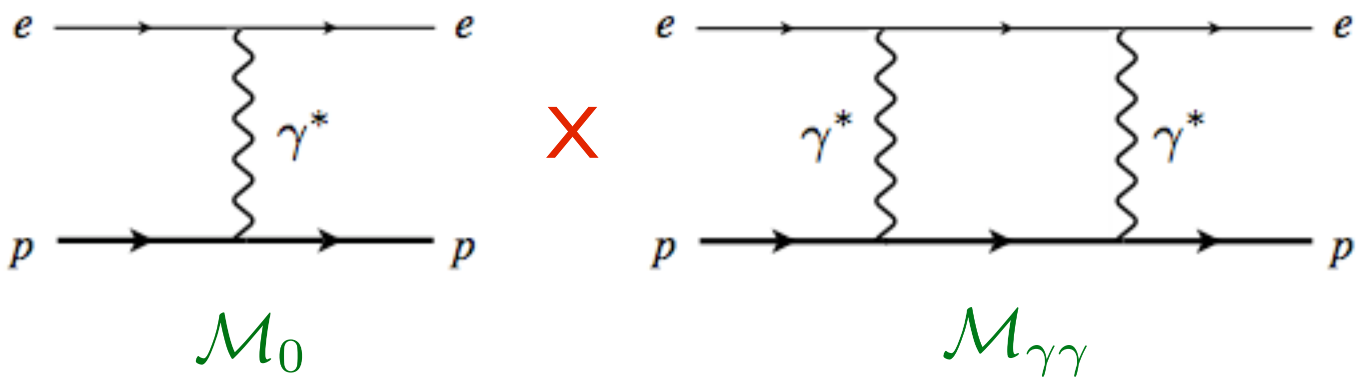
$$d\sigma = d\sigma_0\,(1+\delta)$$

δ contains additional ε dependence, mostly from box diagrams

(most difficult to calculate)

Two-photon exchange

- interference between Born and two-photon exchange amplitudes



- contribution to cross section:

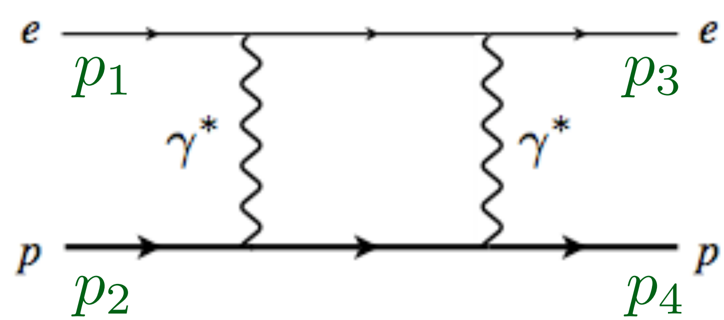
$$\delta^{(2\gamma)} = \frac{2\mathcal{R}e\left\{\mathcal{M}_0^\dagger\ \mathcal{M}_{\gamma\gamma}\right\}}{\left|\mathcal{M}_0\right|^2}$$

- standard "soft photon approximation" (used in most data analyses)

$\longrightarrow$ approximate integrand in $\mathcal{M}_{\gamma\gamma}$ by values at γ^* poles

$\longrightarrow$ neglect nucleon structure (no form factors) *Mo, Tsai (1969)*

Two-photon exchange



$$\mathcal{M}_{\gamma\gamma} = e^4 \int \frac{d^4k}{(2\pi)^4} \frac{N(k)}{D(k)}$$

where

$$\begin{aligned} N(k) &= \bar{u}(p_3)\, \gamma_\mu(\not{p}_1 - \not{k} + m_e)\gamma_\nu\, u(p_1) \\ &\quad \times \bar{u}(p_4)\, \Gamma^\mu(q-k)\, (\not{p}_2 + \not{k} + M)\, \Gamma^\nu(k)\, u(p_2) \end{aligned}$$

and

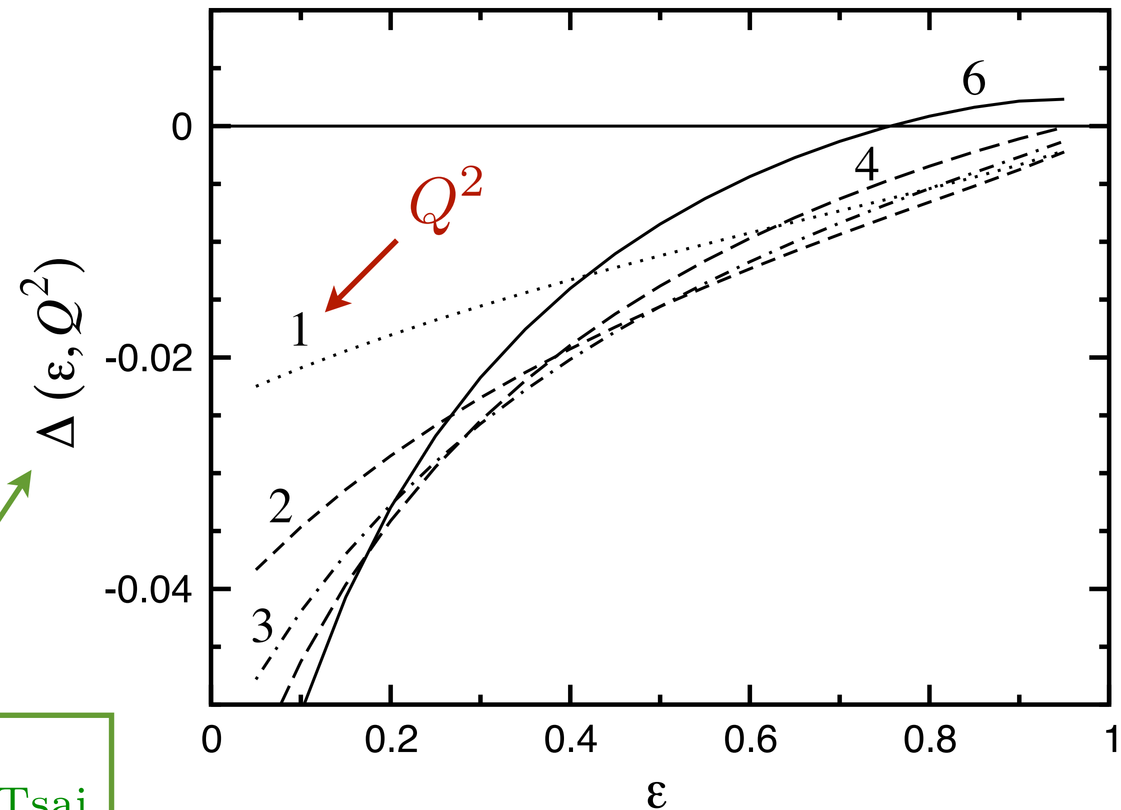
$$\begin{aligned} D(k) &= (k^2 - \lambda^2)\, ((k-q)^2 - \lambda^2) \\ &\quad \times ((p_1 - k)^2 - m^2)\, ((p_2 + k)^2 - M^2) \end{aligned}$$

with λ an IR regulator, and e.m. current is

$$\Gamma^\mu(q) = \gamma^\mu\, F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M}\, F_2(q^2)$$

Two-photon exchange

- "exact" calculation of loop diagram (including $\gamma^* NN$ form factors)



Blunden, Melnitchouk, Tjon
*PRL **91** (2003) 142304;*
*PRC **72** (2005) 034612*

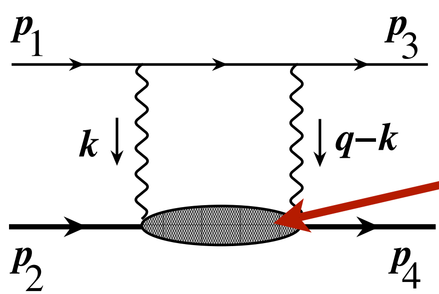
$\delta^{(2\gamma)}_{\rm full} - \delta^{(2\gamma)}_{\rm Mo-Tsai}$

⟹ few % magnitude

⟹ positive slope

⟹ non-linearity in ε

What about higher-mass intermediate states?



$N,\ \Delta,\ P_{11},\ S_{11},\ S_{31},\ldots$

- Lowest mass excitation is P_{33} $\Delta(1232)$ resonance
  - ➡ relativistic $\gamma^* N \Delta$ vertex

form factor $\dfrac{\Lambda_\Delta^4}{\left(\Lambda_\Delta^2 - q^2\right)^2}$

$$\Gamma^{\nu\alpha}_{\gamma\Delta\to N}(p,q) \equiv iV^{\nu\alpha}_{\Delta in}(p,q) = i\frac{eF_\Delta(q^2)}{2M_\Delta^2}\Big\{ g_1\left[\, g^{\nu\alpha}\not{p}\not{q} - p^\nu\gamma^\alpha\not{q} - \gamma^\nu\gamma^\alpha p\cdot q + \gamma^\nu\not{p}q^\alpha \,\right]$$

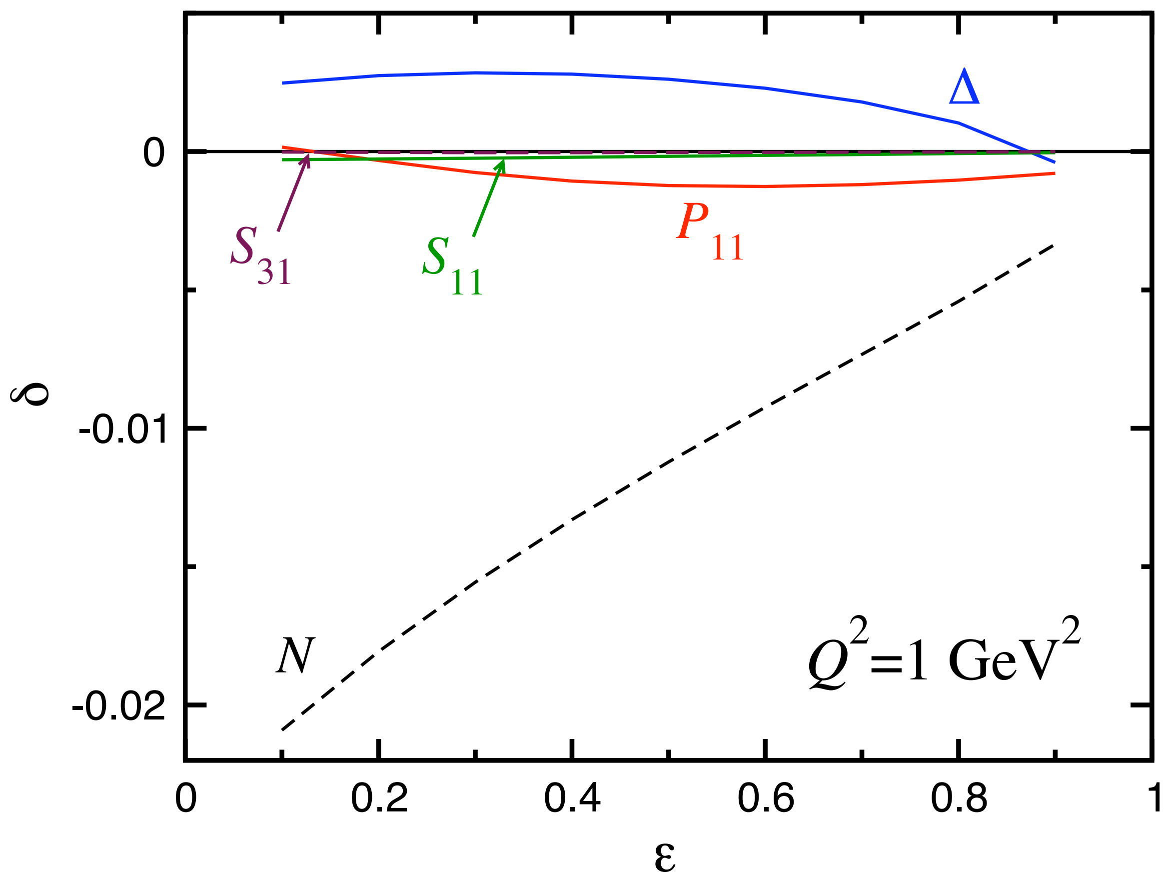
$$+g_2\left[\, p^\nu q^\alpha - g^{\nu\alpha}p\cdot q \,\right] + (g_3/M_\Delta)\left[\, q^2(p^\nu\gamma^\alpha - g^{\nu\alpha}\not{p}) + q^\nu(q^\alpha\not{p} - \gamma^\alpha p\cdot q) \,\right]\Big\}\gamma_5\, T_3$$

  - ➡ coupling constants

g_1	magnetic	➡	7
$g_2 - g_1$	electric	➡	9
g_3	Coulomb	➡	-2 ... 0

Higher-mass intermediate states have also been calculated

→ more model dependent, since couplings & form factors not well known (especially at high Q^2)

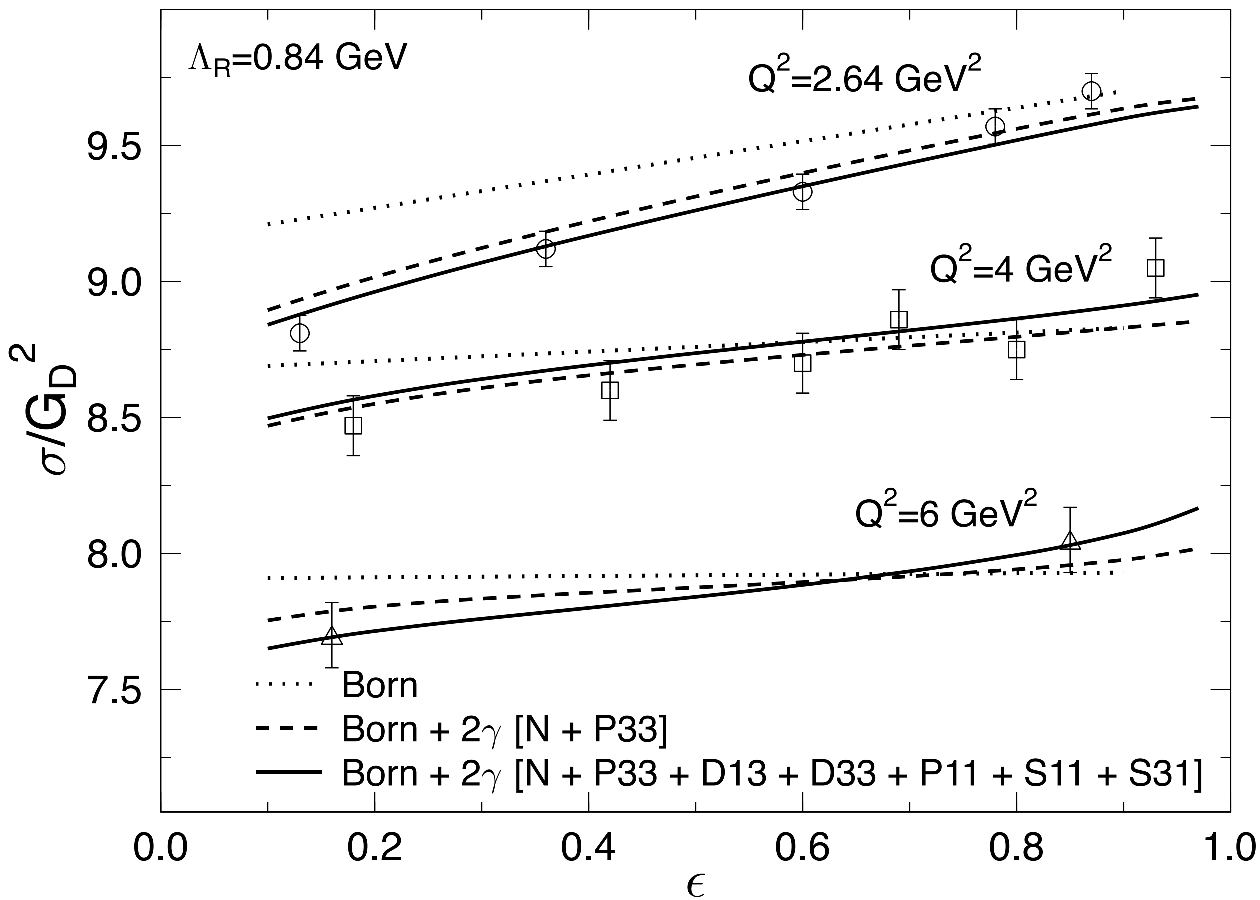


Kondratyuk, Blunden, Melnitchouk, Tjon
Phys. Rev. Lett ***95*** *(2005) 172503*

Kondratyuk, Blunden
Phys. Rev. C ***75*** *(2007) 038201*

- dominant contribution from N
- Δ partially cancels N contribution

Higher-mass intermediate states have also been calculated



Kondratyuk, Blunden
*Phys. Rev. C **75** (2007) 038201*

- higher mass resonance contributions small
- much better fit to data including TPE

Global analysis

- reanalyze *all* elastic *ep* data (Rosenbluth, PT), including TPE corrections consistently *from the beginning*

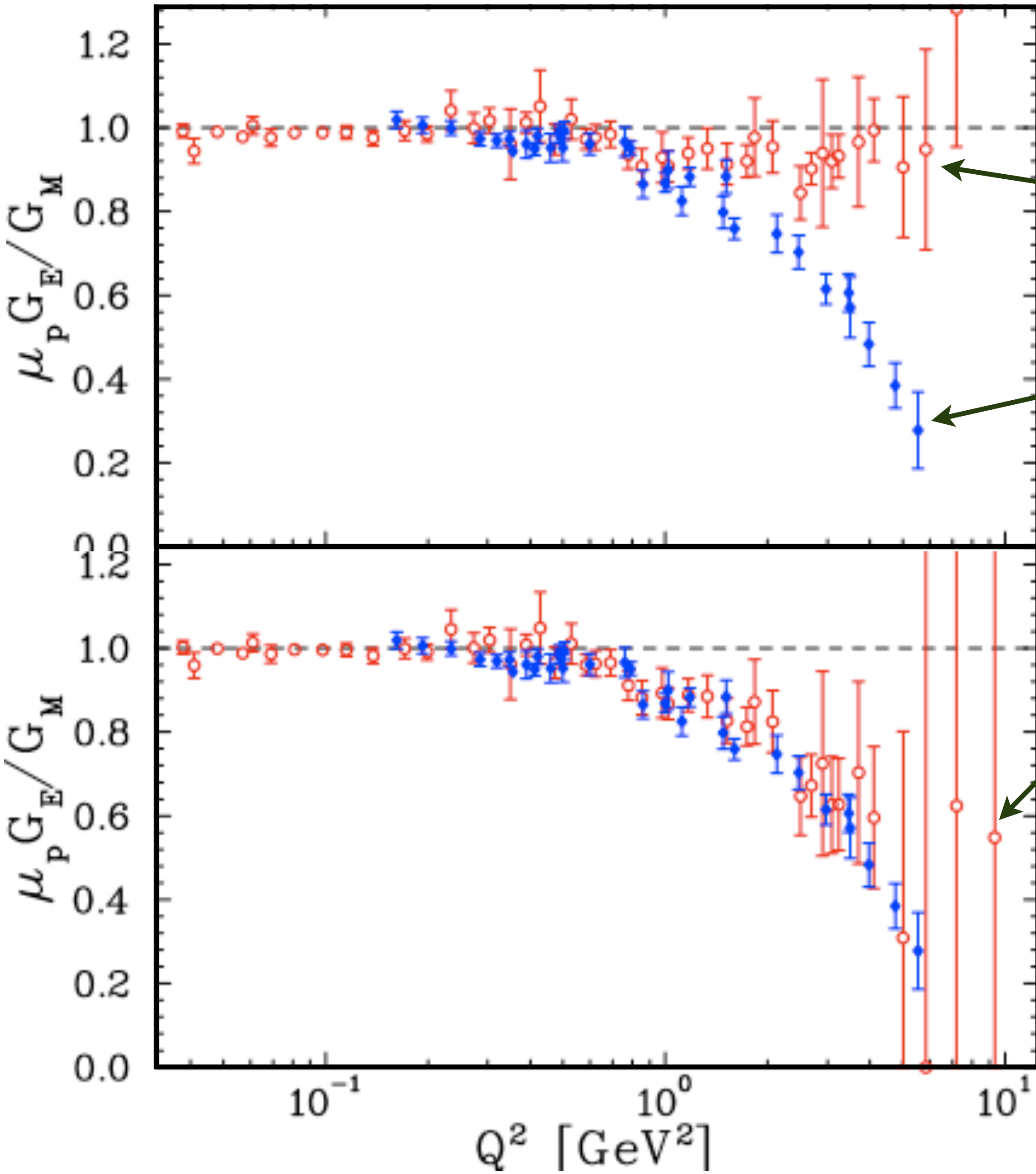
- use explicit calculation of N elastic contribution

- approximate higher mass contributions by phenomenological form, based on N^* calculations:

$$\delta^{(2\gamma)}_{\text{high mass}} = -0.01\ (1-\varepsilon)\ \log Q^2 / \log 2.2$$

for $Q^2 > 1\ \mathrm{GeV}^2$, with $\pm 100\%$ uncertainty

→ decreases $\varepsilon = 0$ cross section by 1% (2%) at $Q^2 = 2.2\ (4.8)\ \mathrm{GeV}^2$



LT separation

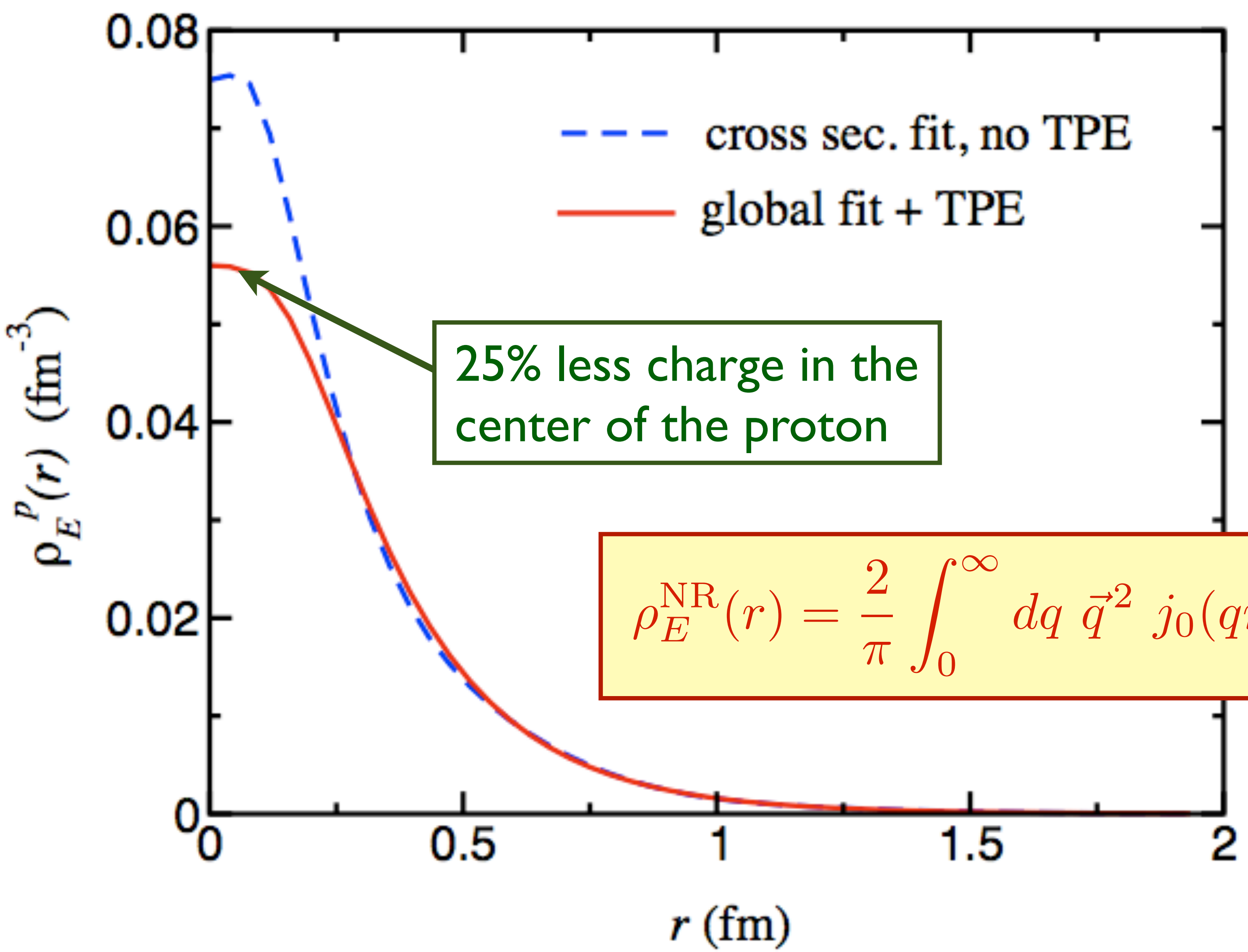
polarization transfer

with TPE correction

➡ resolves discrepancy (within errors)

Arrington, Melnitchouk, Tjon
Phys. Rev. C ***76*** *(2007) 035205*

Charge density



$$\rho_E^{\mathrm{NR}}(r) = \frac{2}{\pi} \int_0^\infty dq\ \vec{q}^{\,2}\ j_0(qr)\ G_E(\vec{q}^{\,2})$$

Parity-violating e scattering

- Left-right polarization asymmetry in $\vec{e}\, p \to e\, p$ scattering

$$A_{\mathrm{PV}} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\left(\frac{G_F Q^2}{4\sqrt{2}\alpha}\right)(A_V + A_A + A_s)$$

$\longrightarrow$ measure interference between e.m. and weak currents

$$A_V = g_A^e \rho \left[(1 - 4\kappa \sin^2\theta_W) - (\varepsilon G_E^{\gamma p} G_E^{\gamma n} + \tau G_M^{\gamma p} G_M^{\gamma n})/\sigma^{\gamma p}\right]$$

radiative corrections, including TBE

using relations between weak and e.m. form factors

$$G_{E,M}^{Zp} = (1 - 4\sin^2\theta_W) G_{E,M}^{\gamma p} - G_{E,M}^{\gamma n} - G_{E,M}^{s}$$

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$\longrightarrow$ measure interference between e.m. and weak currents

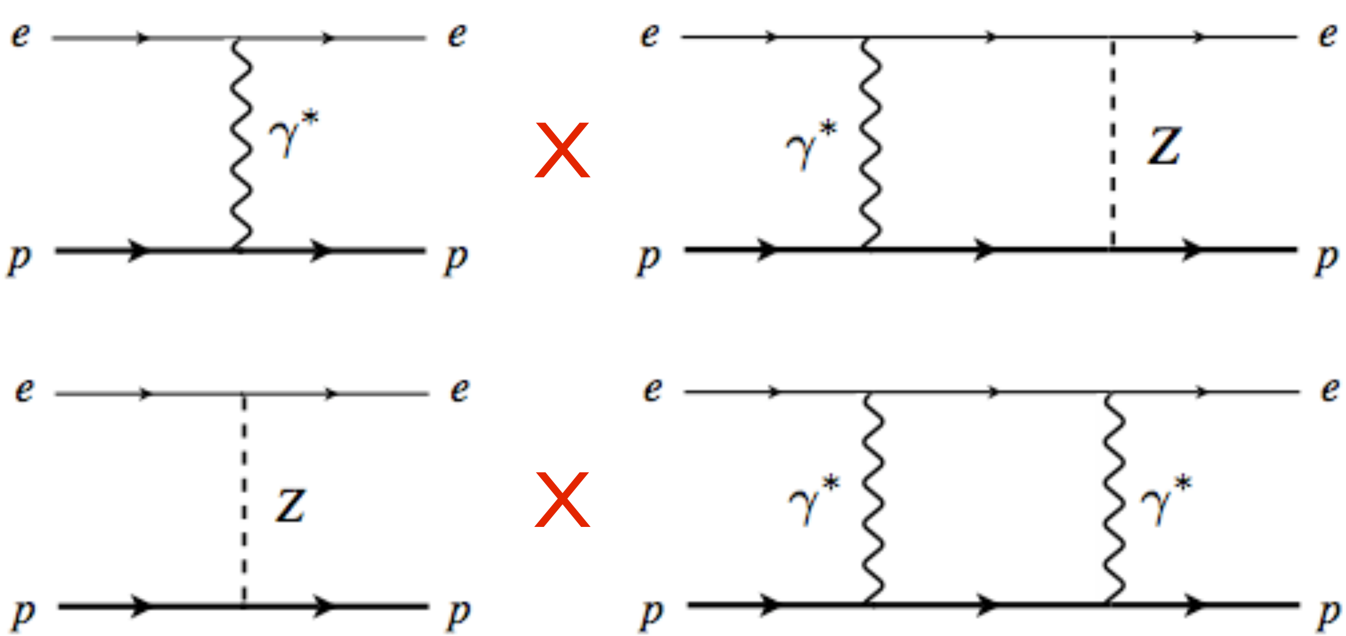
$$A_A = g_V^e \sqrt{\tau(1+\tau)(1-\varepsilon^2)}\ \widetilde{G}_A^{Zp} G_M^{\gamma p} / \sigma^{\gamma p}$$

includes axial RCs + anapole term

$$A_s = -g_A^e \rho \left(\varepsilon G_E^{\gamma p} G_E^s + \tau G_M^{\gamma p} G_M^s\right) / \sigma^{\gamma p}$$

strange electric &
magnetic form factors

Two-boson exchange corrections



"$\gamma(Z\gamma)$"

"$Z(\gamma\gamma)$"

- current PDG estimates (of "$\gamma(Z\gamma)$") computed at $Q^2 = 0$

Marciano, Sirlin (1980)

Erler, Ramsey-Musolf (2003)

- do not include hadron structure effects (parameterized via VNN form factors)

Two-boson exchange corrections

- At tree level, $\rho = \kappa = 1$

- Including TBE corrections,

$$\rho = \rho_0 + \Delta\rho \ , \qquad \kappa = \kappa_0 + \Delta\kappa$$

standard RCs

Born-TBE interference

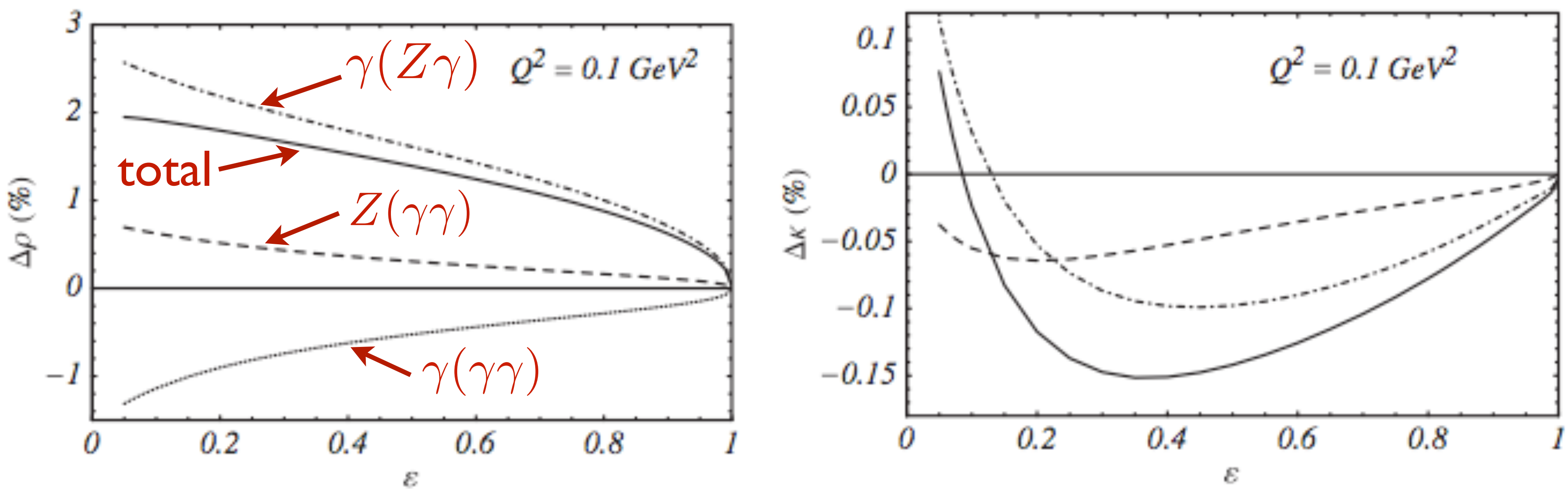
→ from vector part of asymmetry,

$$\Delta\rho = \frac{A_V^p + A_V^n}{A_V^{p,\mathrm{tree}} + A_V^{n,\mathrm{tree}}} - \frac{\Delta\sigma^{\gamma(\gamma\gamma)}}{\sigma^{\gamma p}}$$

$$\Delta\kappa = \frac{A_V^p}{A_V^{p,\mathrm{tree}}} - \frac{A_V^p + A_V^n}{A_V^{p,\mathrm{tree}} + A_V^{n,\mathrm{tree}}}$$

tree level contribution

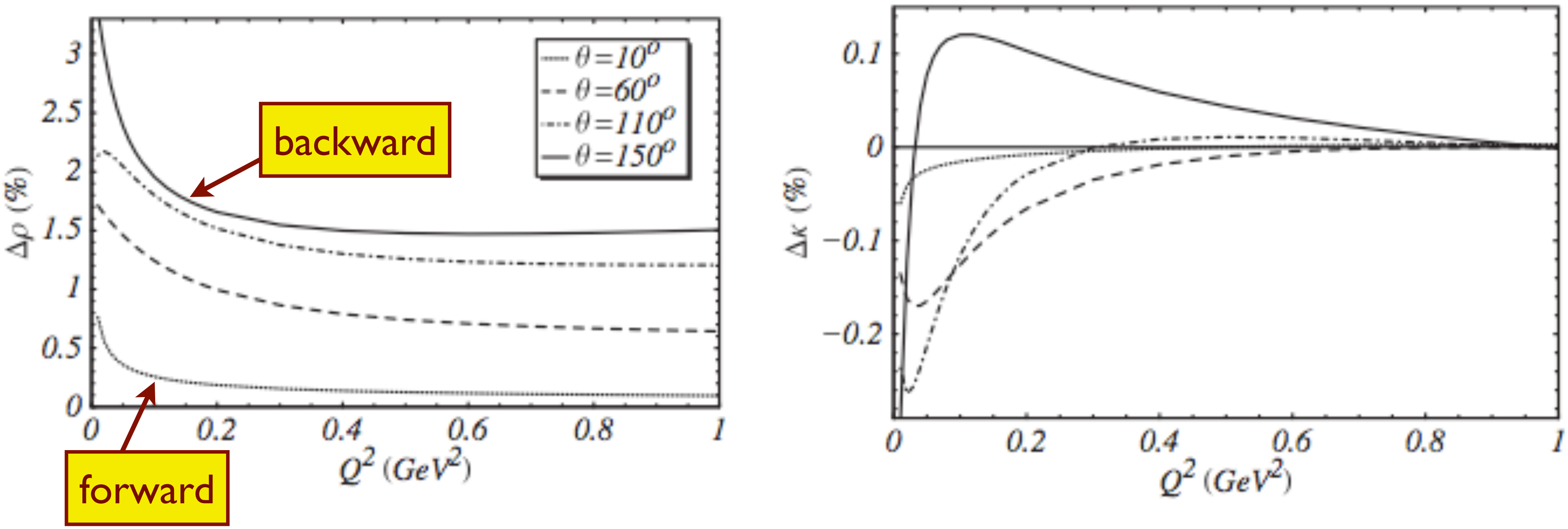
Two-boson exchange corrections



Tjon, Melnitchouk, PRL ***100****, 082003 (2008)*

- some cancellation between $Z(\gamma\gamma)$ and $\gamma(\gamma\gamma)$ corrections in $\Delta\rho$
- no $\gamma(\gamma\gamma)$ contribution to $\Delta\kappa$

Two-boson exchange corrections



*Tjon, Melnitchouk, PRL **100**, 082003 (2008)*

- 2-3% correction at $Q^2 < 0.1$ GeV^2
- strong Q^2 dependence at low Q^2
- *cf.* Marciano-Sirlin ($Q^2 = 0$): $\Delta\rho = -0.37\%$, $\Delta\kappa = -0.53\%$

Two-boson exchange corrections

- dependence on input form factors

$$\delta = A_{\rm PV}^{\rm TBE} \; / \; A_{\rm PV}^{\rm tree}$$

Q^2 (GeV^2)	θ	$\delta(\%)$ Empirical	Dipole	Monopole	
0.1	144.0°	1.62	1.52	1.72	SAMPLE (97)
0.23	35.31°	0.63	0.58	0.84	PVA4 (04)
0.477	12.3°	0.16	0.15	0.24	HAPPEX (04)
0.997	20.9°	0.22	0.23	0.30	G0 (05)
0.109	6.0°	0.20	0.16	0.32	HAPPEX (07)
0.23	110.0°	1.39	1.33	1.52	G0 } results to come
0.03	8.0°	0.58	0.47	0.86	Qweak } results to come

→ "dipole" results ~ 5-10% smaller than "empirical"[1]

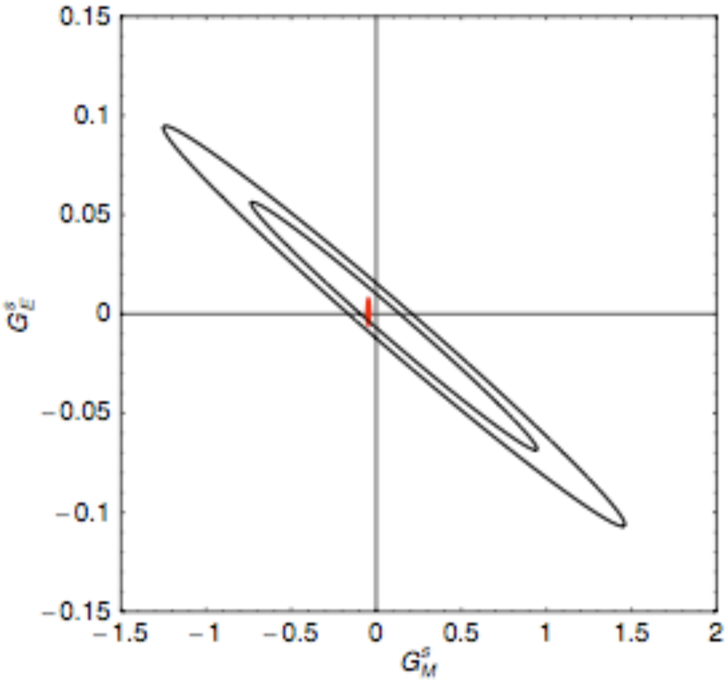
→ "monopole"[2] results ~ 50% larger than "empirical"[1]

[1] *Tjon, Melnitchouk, PRL **100**, 082003 (2008)*

[2] *Zhou, Kao, Yang, PRL **99**, 262001 (2007)*

Effects on strange form factors

- global analysis of all PVES data at $Q^2 < 0.3\ \mathrm{GeV}^2$



$$G_E^s = 0.0025 \pm 0.0182$$

$$G_M^s = -0.011 \pm 0.254$$

$$\text{at } Q^2 = 0.1\ \mathrm{GeV}^2$$

*Young et al., PRL **97**, 102002 (2006)*

- including TBE corrections:

$$\boxed{\begin{aligned} G_E^s &= 0.0023 \pm 0.0182\ ^* \\ G_M^s &= -0.020 \pm 0.254 \end{aligned}}$$

$$\text{at } Q^2 = 0.1\ \mathrm{GeV}^2$$

* fixed mainly by ^{4}He data

TBE in nuclei

- scatter from individual nucleons (*quasi-elastic*), or whole nuclei?

- assume nucleus is Z protons and $(A-Z)$ neutrons

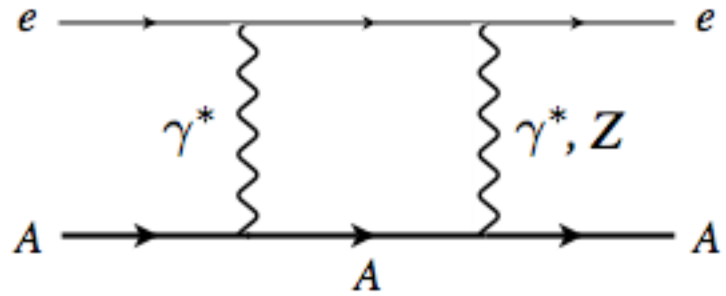
  ($i.e.$ nuclear corrections in $A_{\mathrm{PV}}^{A} \to A_{\mathrm{PV}}^{N}$ have already been removed)

	$\Delta\rho$ (%)	$\Delta\kappa$ (%)
$\gamma(\gamma\gamma)$	–0.11	
$Z(\gamma\gamma)$	0.05	0.00
$\gamma(Z\gamma)$	0.61	–0.04
total	0.56	–0.04

Tjon, Melnitchouk (2008)

TBE in nuclei

- at the nuclear level, consider TBE with elastic intermediate state



- assume dipole form factor with cut-off $\Lambda_{\rm Pb} = \sqrt{12/\langle r^2 \rangle} \approx 0.12$ GeV

$\delta_{\gamma(\gamma\gamma)}$	0.052
$\delta_{Z(\gamma\gamma)}$	−0.026
$\delta_{\gamma(Z\gamma)}$	0.018
$\dfrac{A_{\rm PV}}{A_{\rm PV}^{(0)}}$	0.944

$$\boxed{1 + \delta_{\gamma(Z\gamma)} + \delta_{Z(\gamma\gamma)} - \delta_{\gamma(\gamma\gamma)}}$$

Tjon, Melnitchouk (2008)

Summary

- TPE corrections resolve most of Rosenbluth *vs.* PT G_E^p/G_M^p discrepancy

  $\longrightarrow$ "25% less charge" in the center of the proton

  $\longrightarrow$ first consistent form factor fit at order α^3

- $\gamma(Z\gamma)$ and $Z(\gamma\gamma)$ contributions give ~ 2% corrections to PVES at small Q^2

  $\longrightarrow$ strong Q^2 dependence at low Q^2

  $\longrightarrow$ affects extraction of strange form factors

- First results on TBE in nuclei (^{208}Pb)

  $\longrightarrow$ at nucleon level, correction < 1% $(\Delta\rho)$

  $\longrightarrow$ larger effect at nuclear level (elastic intermediate state only)

The End