E-Weak Tests with P-Violating e Scattering in the LHC Era
Institute for Nuclear Theory
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Finite-Q^2 corrections to parity-violating deep-inelastic scattering

Wally Melnitchouk
Jefferson Lab

with Tim Hobbs (U. Chicago)

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arXiv:0801.4791 [hep-ph]

- PVDIS as a tool to measure flavor (p target) and isospin (d target) dependence of nucleon PDFs

- Formalism assumes leading twist (parton model) dominance

  $\longrightarrow$ $Q^2 \to \infty$

- Experiments at finite kinematics $(Q^2 \sim 5 - 10\ \mathrm{GeV}^2)$

  $\longrightarrow$ how large are finite-Q^2 corrections?

- Lagrangian for parity-violating lepton-quark interaction

$$\mathcal{L}^{\rm PV} = \frac{G_F}{\sqrt{2}}[\bar{e}\gamma^\mu\gamma_5 e(C_{1u}\bar{u}\gamma_\mu u + C_{1d}\bar{d}\gamma_\mu d) + \bar{e}\gamma^\mu e(C_{2u}\bar{u}\gamma_\mu\gamma_5 u + C_{2d}\bar{d}\gamma_\mu\gamma_5 d)]$$

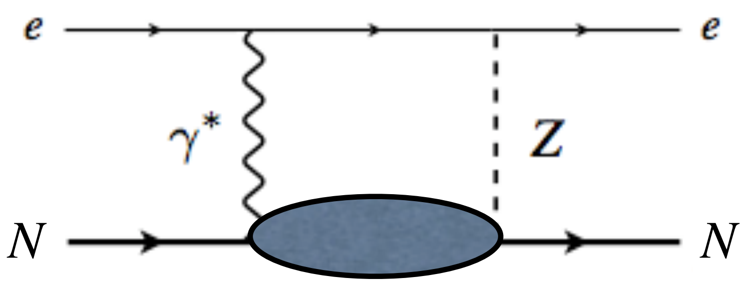
with electroweak couplings (at tree level)

$$\begin{aligned}
C_{1u} &= g_A^e \cdot g_V^u = -\tfrac{1}{2} + \tfrac{4}{3}\sin^2\theta_W,\\
C_{1d} &= g_A^e \cdot g_V^d = \tfrac{1}{2} - \tfrac{2}{3}\sin^2\theta_W,\\
C_{2u} &= g_V^e \cdot g_A^u = -\tfrac{1}{2} + 2\sin^2\theta_W,\\
C_{2d} &= g_V^e \cdot g_A^d = \tfrac{1}{2} - 2\sin^2\theta_W.
\end{aligned}$$

- Asymmetry between left- and right-handed inclusive electron-nucleon cross sections

$$\boxed{A^{\rm PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}}$$

→ for $Q^2 \ll M_Z^2$, numerator sensitive to γ–Z interference only



→ denominator dominated by e.m. component

- In terms of structure functions:

$$A^{\mathrm{PV}} = -\left(\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\right)\left[g_A^e Y_1 \frac{F_1^{\gamma Z}}{F_1^\gamma} + \frac{g_V^e}{2} Y_3 \frac{F_3^{\gamma Z}}{F_1^\gamma}\right]$$

$\rightarrow$ $Y_{1,3}$ parameterize dependence on $y = \nu/E$

$$Y_1 = \frac{1 + (1-y)^2 - y^2(1 - r^2/(1+R^{\gamma Z})) - 2xyM/E}{1 + (1-y)^2 - y^2(1 - r^2/(1+R^{\gamma})) - 2xyM/E}\left(\frac{1+R^{\gamma Z}}{1+R^\gamma}\right)$$

$$Y_3 = \frac{1-(1-y)^2}{1 + (1-y)^2 - y^2(1 - r^2/(1+R^{\gamma})) - 2xyM/E}\left(\frac{r^2}{1+R^\gamma}\right)$$

with

$$r^2 = 1 + \frac{Q^2}{\nu^2} = 1 + \frac{4M^2x^2}{Q^2}$$

$$R^{\gamma(\gamma Z)} \equiv \frac{\sigma_L^{\gamma(\gamma Z)}}{\sigma_T^{\gamma(\gamma Z)}} = r^2 \frac{F_2^{\gamma(\gamma Z)}}{2xF_1^{\gamma(\gamma Z)}} - 1$$

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with

unknown phenomenology

$$r^2 = 1 + \frac{Q^2}{\nu^2} = 1 + \frac{4M^2x^2}{Q^2}$$

$$R^{\gamma(\gamma Z)} \equiv \frac{\sigma_L^{\gamma(\gamma Z)}}{\sigma_T^{\gamma(\gamma Z)}} = r^2 \frac{F_2^{\gamma(\gamma Z)}}{2xF_1^{\gamma(\gamma Z)}} - 1$$

- At leading twist, electroweak structure functions given by PDFs

→ electromagnetic

$$\boxed{\begin{aligned} F_1^\gamma(x) &= \frac{1}{2}\sum_q e_q^2(q(x)+\bar{q}(x)),\\ F_2^\gamma(x) &= 2xF_1^\gamma(x), \end{aligned}}$$

→ interference

$$\boxed{\begin{aligned} F_1^{\gamma Z}(x) &= \sum_q e_q g_V^q(q(x)+\bar{q}(x)),\\ F_2^{\gamma Z}(x) &= 2xF_1^{\gamma Z}(x),\\ F_3^{\gamma Z}(x) &= 2\sum_q e_q g_A^q(q(x)-\bar{q}(x)) \end{aligned}}$$

PV asymmetry in terms of PDFs

$$\boxed{A^{\mathrm{PV}} = -\left(\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\right)(Y_1 a_1 + Y_3 a_3)}$$

<u>(hadronic) *vector* term</u>

$$\boxed{a_1 = \frac{2\sum_q e_q C_{1q}(q+\bar{q})}{\sum_q e_q^2(q+\bar{q})}}$$

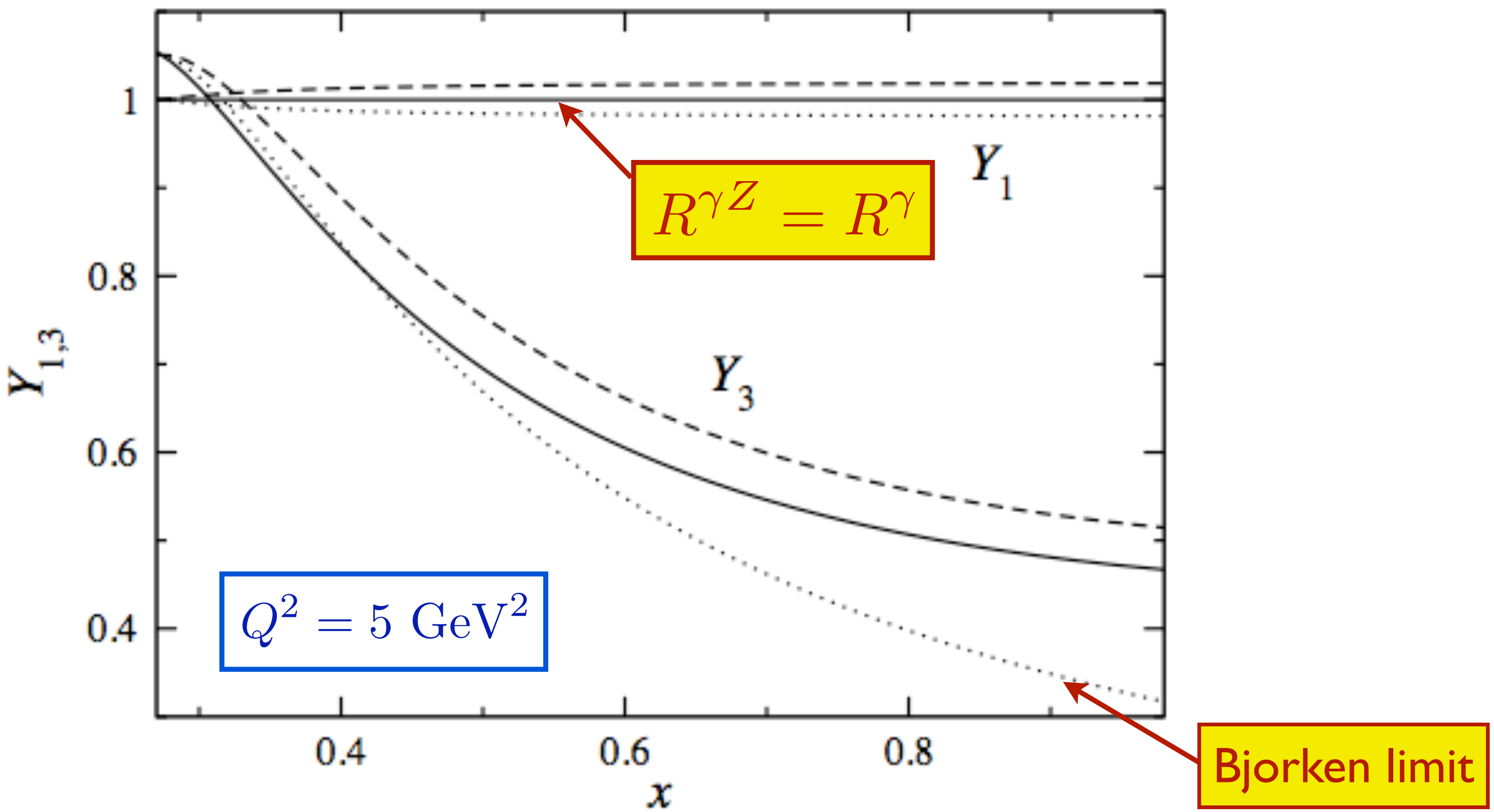
<u>(hadronic) *axial-vector* term</u>

$$\boxed{a_3 = \frac{2\sum_q e_q C_{2q}(q-\bar{q})}{\sum_q e_q^2(q+\bar{q})}}$$

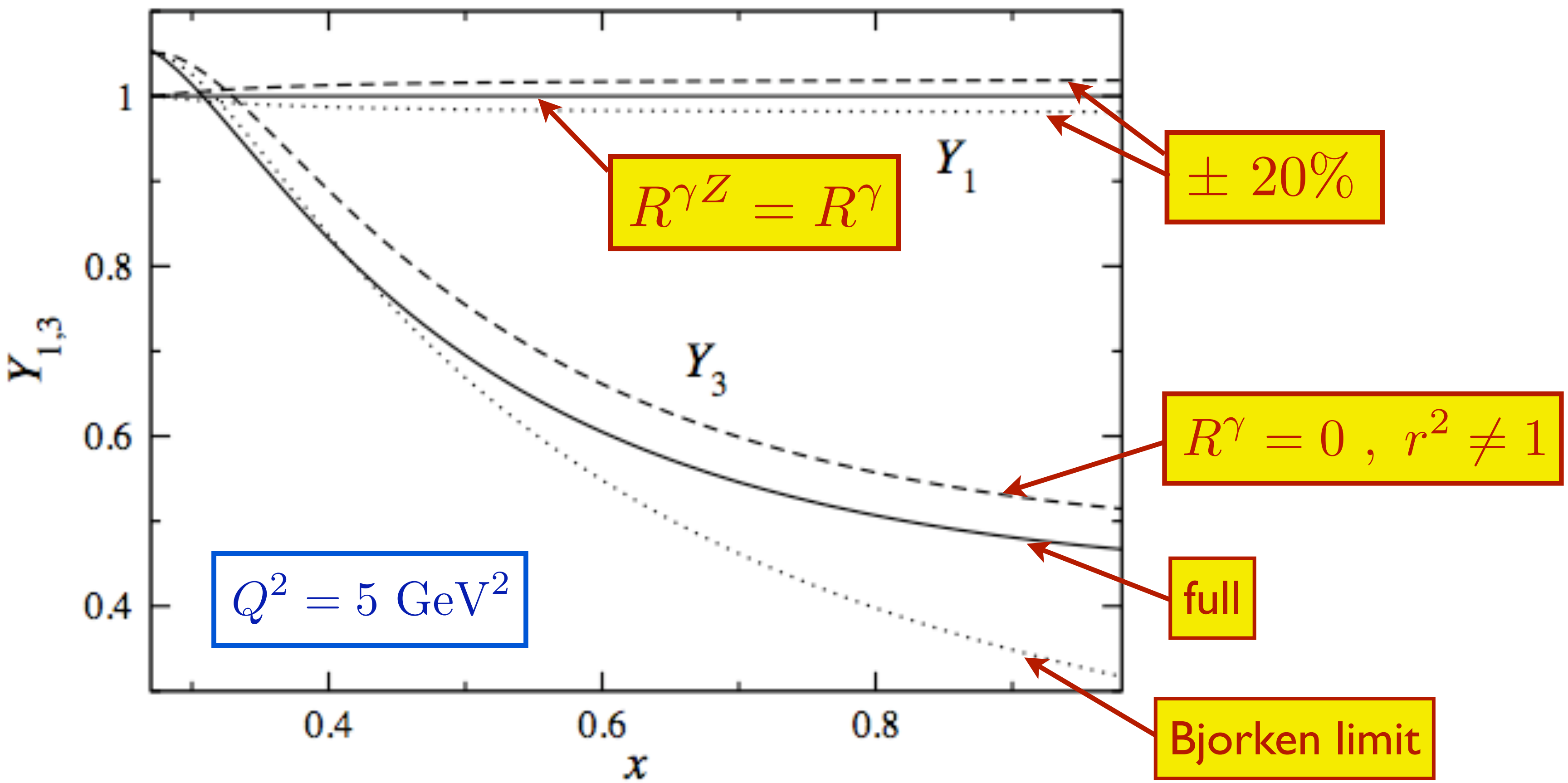
$\longrightarrow$ simplified y dependence

$$\boxed{\begin{aligned} &Y_1 \to 1, \\ &Y_3 \to \frac{1-(1-y)^2}{1+(1-y)^2} \end{aligned}}$$

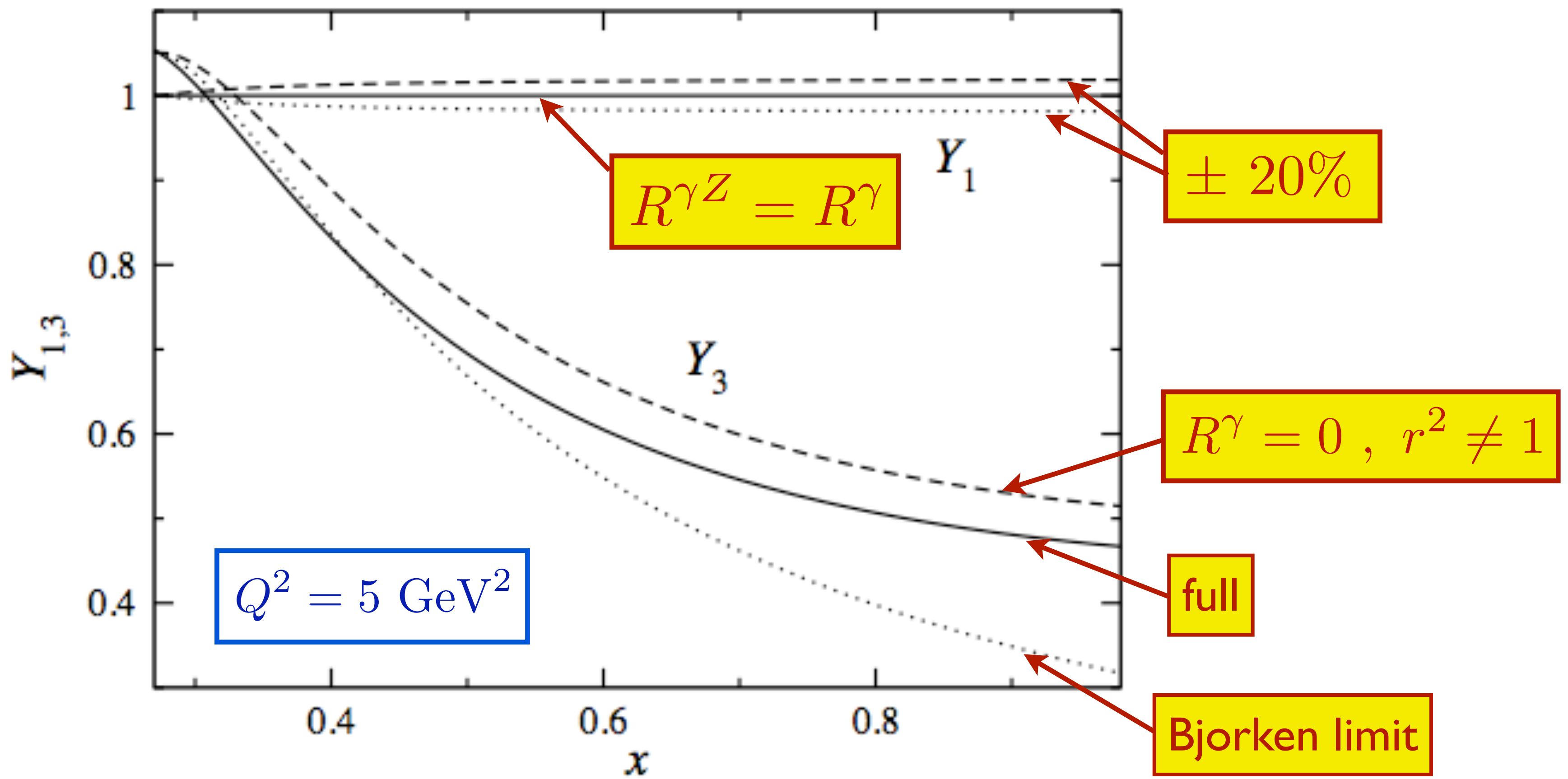
Importance of axial-vector term



Importance of axial-vector term



Importance of axial-vector term



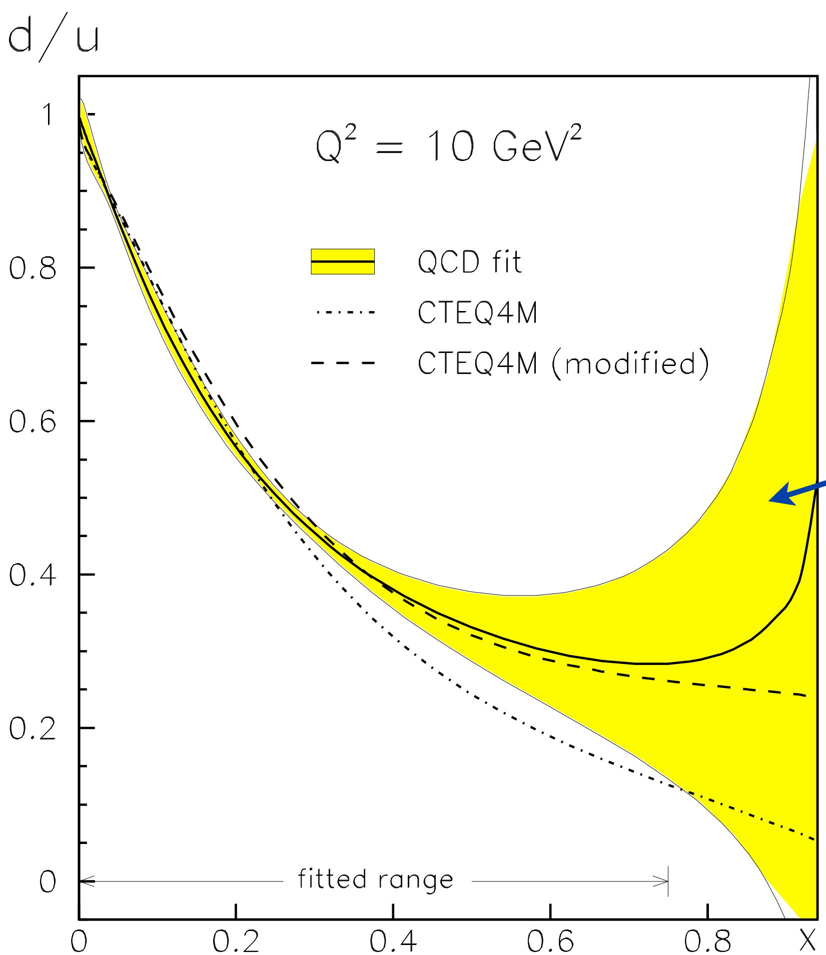
→ hadronic axial-vector term relatively more important at finite Q^2

Proton target

- sensitive to d/u ratio at large x

$$a_1^p = \frac{12C_{1u} - 6C_{1d}d/u}{4 + d/u}$$

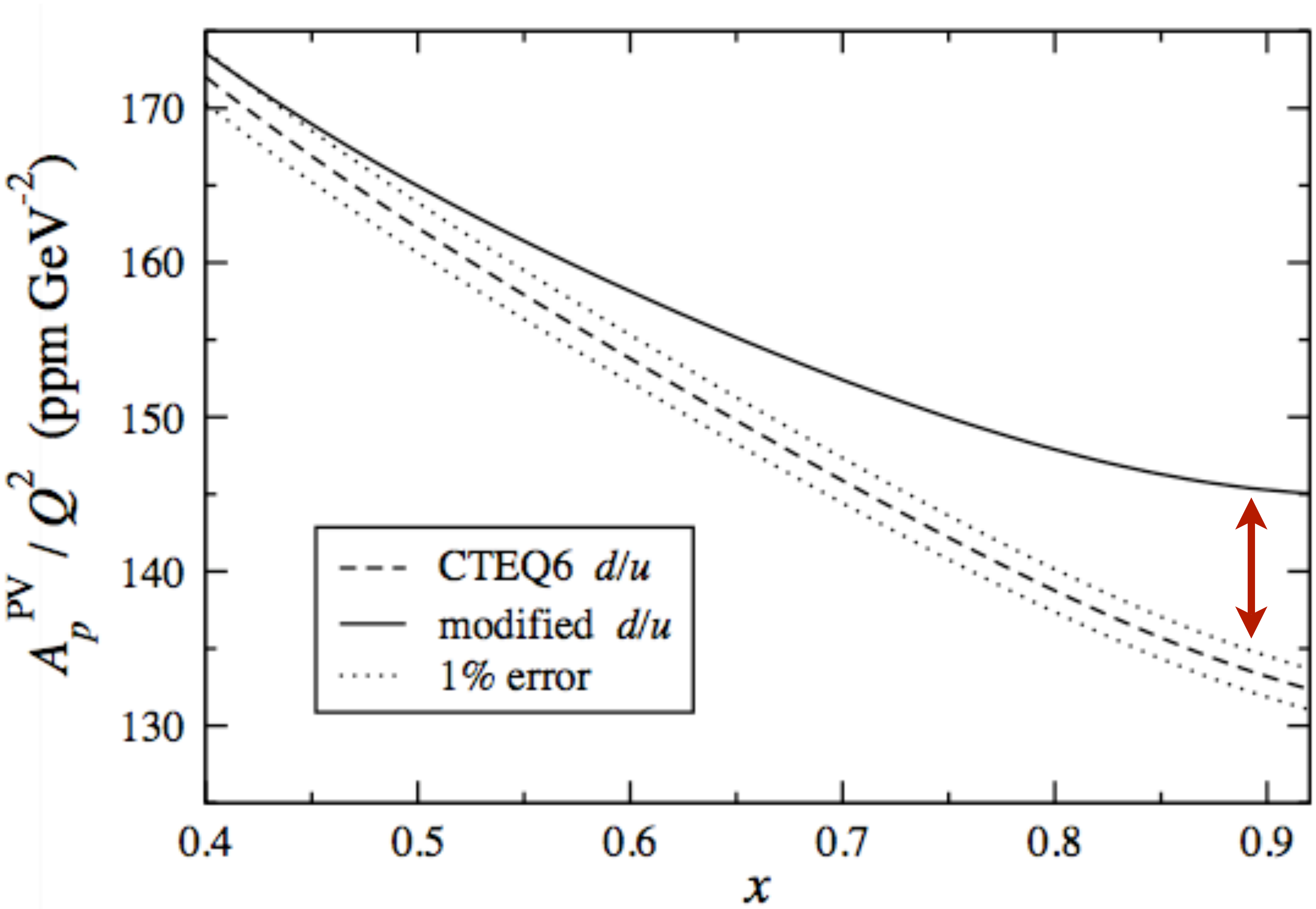
$$a_3^p = \frac{12C_{2u} - 6C_{2d}d/u}{4 + d/u}$$



large uncertainty at high x in d/u ratio

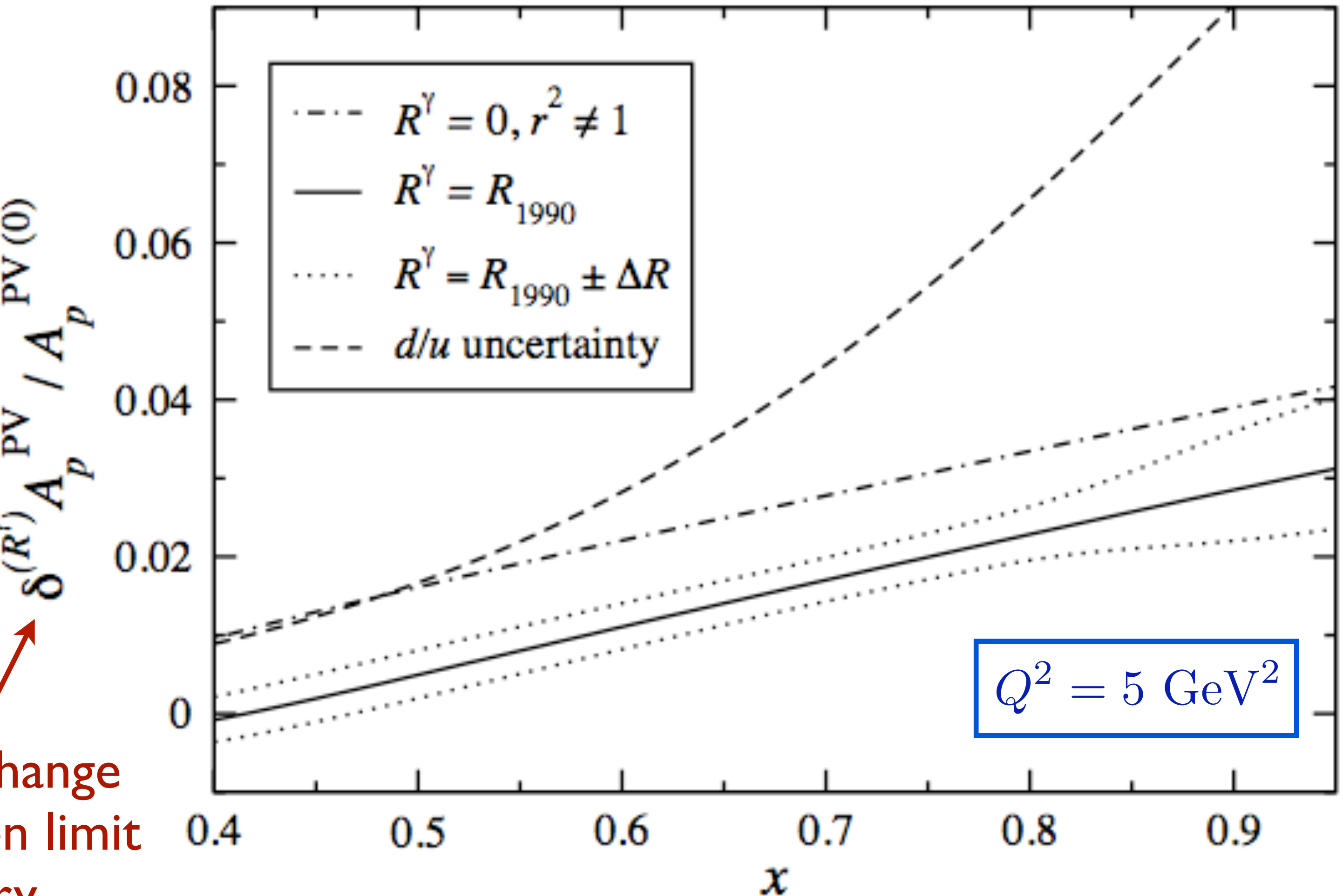
Botje, Eur. Phys. J. C 14 (2000) 285

Sensitivity to d/u



$* \; d/u \to 0.2$ as $x \to 1$

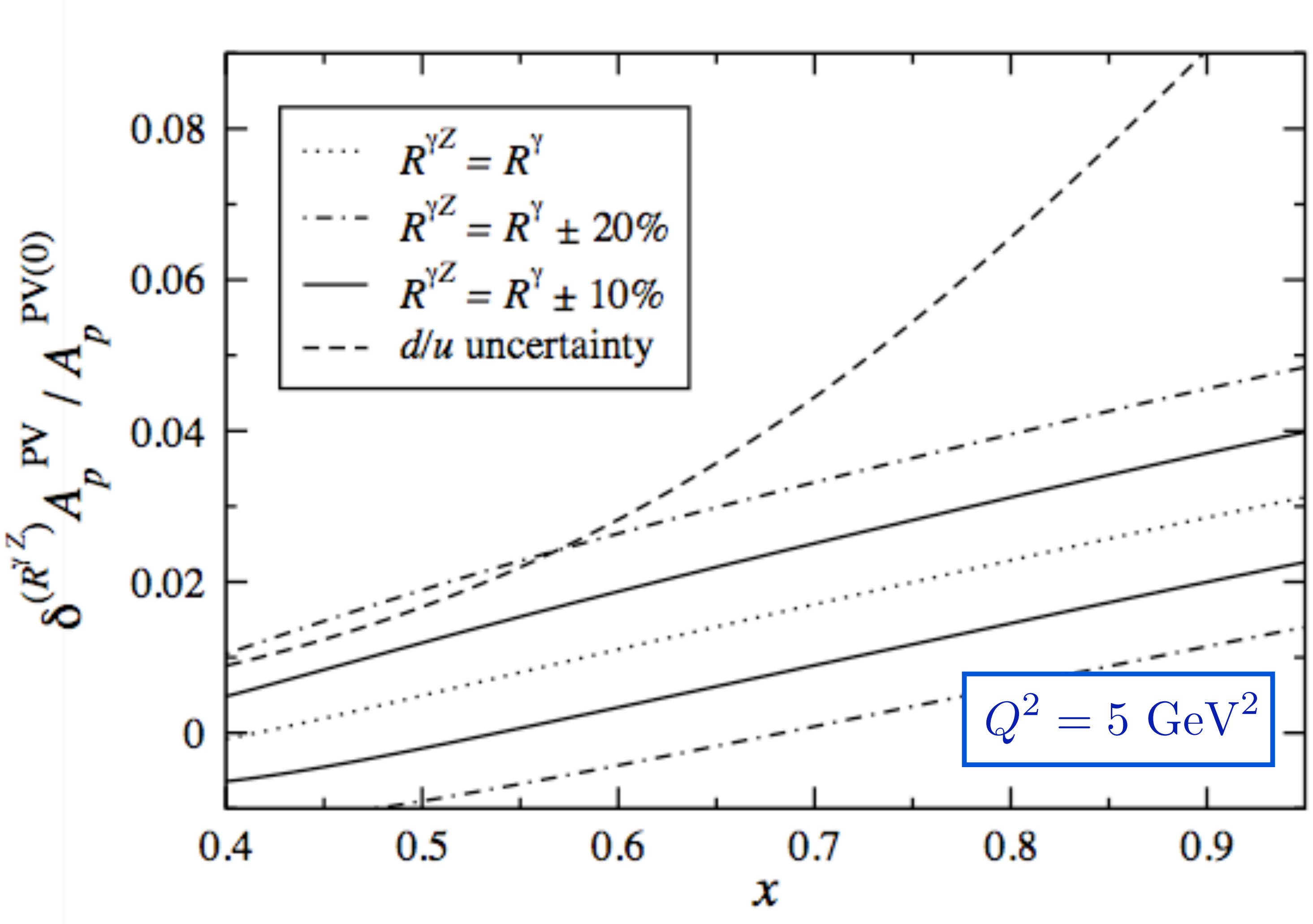
Sensitivity to R^γ



relative change to Bjorken limit asymmetry

→ uncertainty due to R^γ smaller than d/u differences at large x

Sensitivity to $R^{\gamma Z}$



→ correction from $R^{\gamma Z}$ needs further investigation

Deuteron target

- isoscalar target, dependence on PDFs cancels at large Q^2

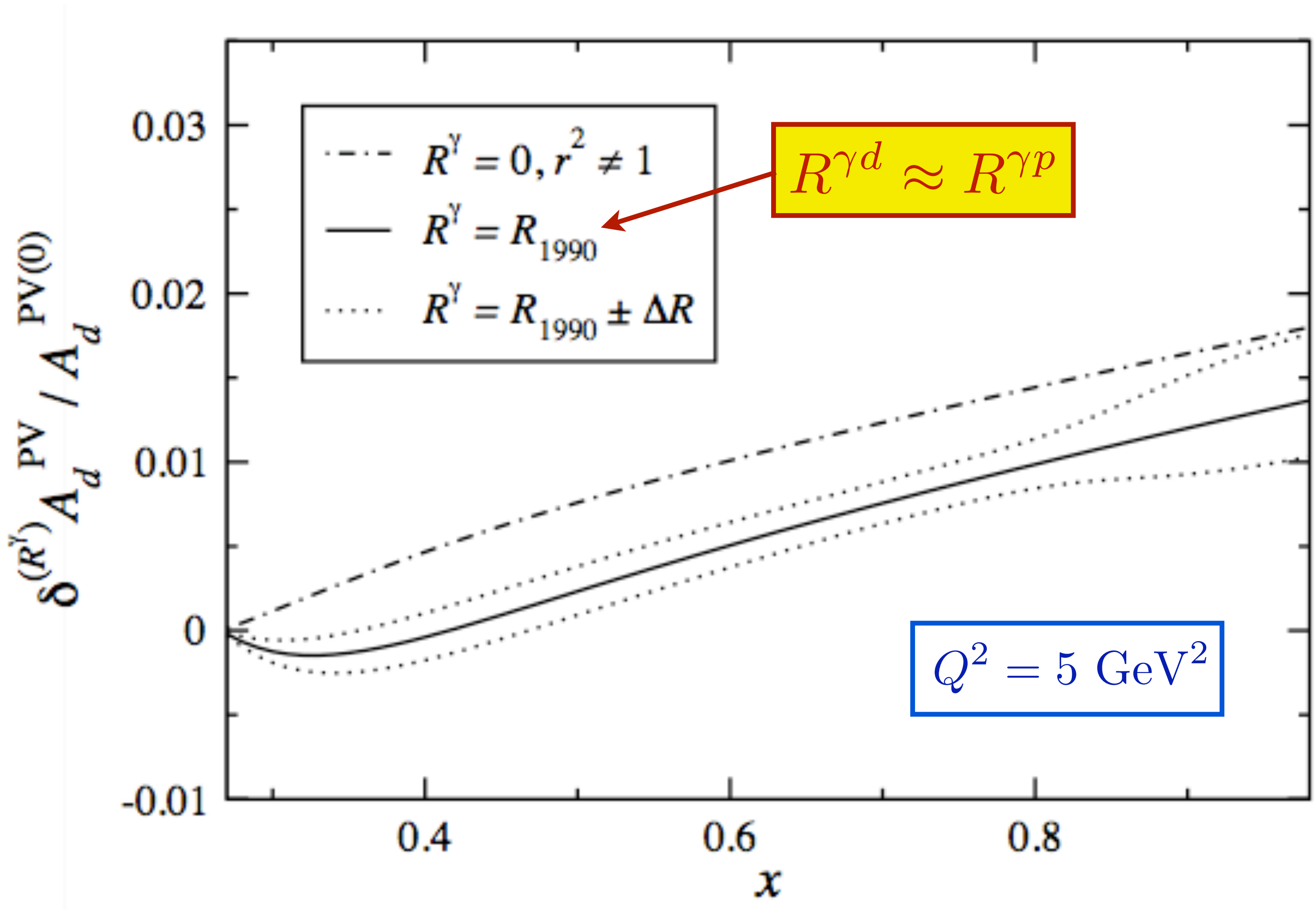
$$\boxed{\begin{aligned} a_1^d &= \tfrac{6}{5}(2C_{1u} - C_{1d}) \\ a_3^d &= \tfrac{6}{5}(2C_{2u} - C_{2d}) \end{aligned}}$$

⟶ PV asymmetry becomes independent of hadronic structure

$$\boxed{A^{\mathrm{PV}} = -\left(\frac{3G_F Q^2}{10\sqrt{2}\pi\alpha}\right)[Y_1(2C_{1u} - C_{1d}) + Y_3(2C_{2u} - C_{2d})]}$$

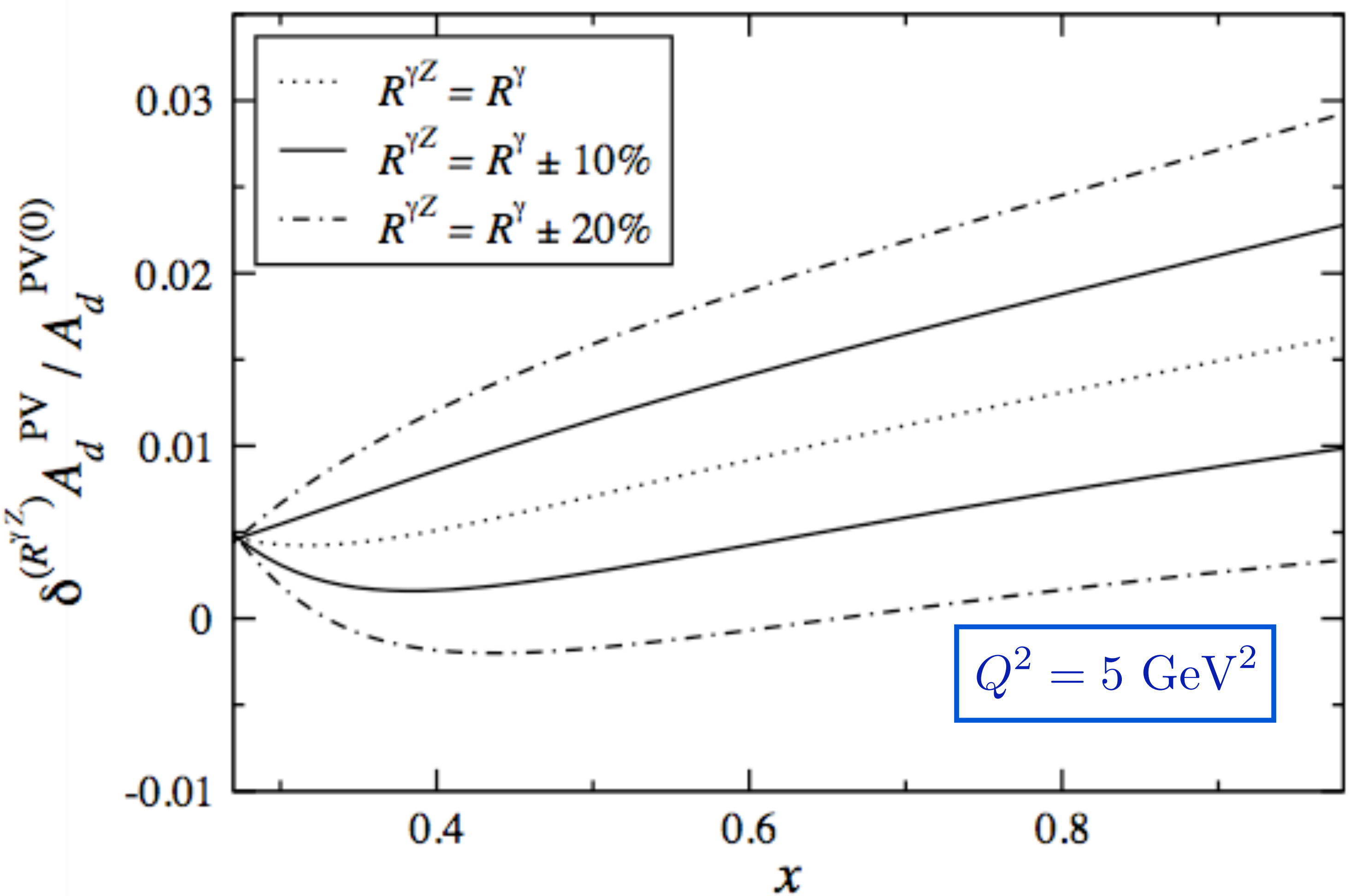
⟶ sensitivity to electroweak couplings an important early test of standard model

Sensitivity to R^γ



→ correction $< 1\%$ for $x < 0.8$

Sensitivity to $R^{\gamma Z}$



→ potentially important uncertainty in asymmetry from $R^{\gamma Z}$

Constraints on $R^{\gamma Z}$

- at large Q^2 , perturbative QCD predicts $R^{\gamma} \approx R^{\gamma Z}$

- in limit $Q^2 \to 0$, conserved vector current requires $R^{\gamma}, R^{\gamma Z} \to 0$

  cf. $R^Z \neq 0$ in $Q^2 \to 0$ limit, since axial current not conserved

  *Kulagin, Petti, PRD **76** (2007) 094023*

- in intermediate Q^2 region, interpolate $R^{\gamma Z}$ between pQCD and (axial) vector meson dominance behaviors

  Kulagin, Hobbs, WM (in progress)

Charge symmetry violation

- define quark distributions in presence of CSV

$$\boxed{\begin{aligned} u &\equiv u^p - \frac{\delta u}{2} = d^n + \frac{\delta u}{2} \\ d &\equiv d^p - \frac{\delta d}{2} = u^n + \frac{\delta d}{2} \end{aligned}}$$

K. Kumar

$$\Rightarrow \boxed{\begin{aligned} \delta u &= u^p - d^n \\ \delta d &= d^p - u^n \end{aligned}}$$

CSV PDFs

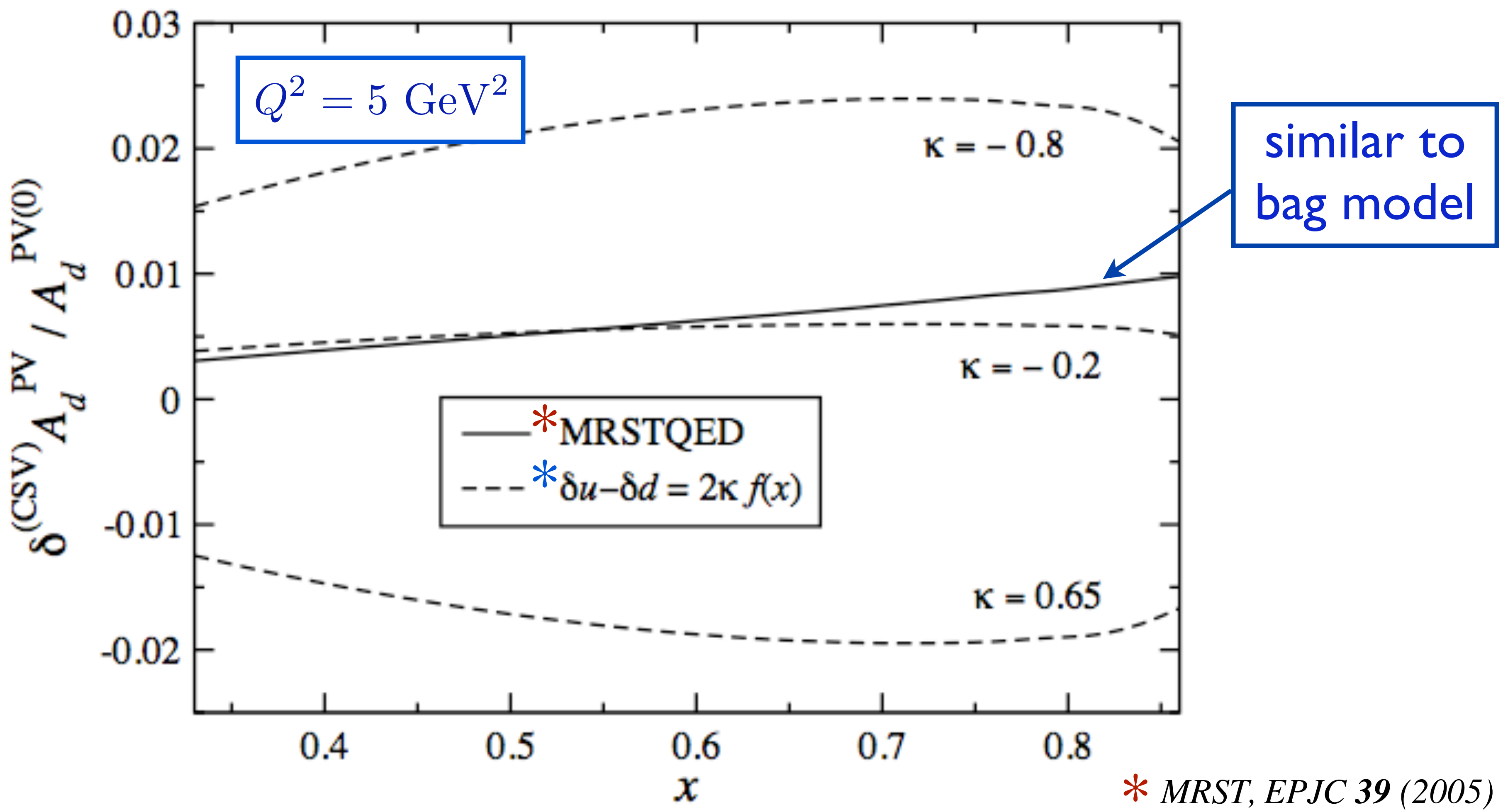
- PV asymmetry then depends on

$$a_1^d = a_1^{d(0)} + \delta^{(\mathrm{CSV})} a_1^d$$
$$a_3^d = a_3^{d(0)} + \delta^{(\mathrm{CSV})} a_3^d$$

fractional CSV correction

$$\boxed{\begin{aligned} \frac{\delta^{(\mathrm{CSV})} a_1^d}{a_1^{d(0)}} &= \left(-\frac{3}{10} + \frac{2C_{1u} + C_{1d}}{2(2C_{1u} - C_{1d})}\right)\left(\frac{\delta u - \delta d}{u + d}\right) \\ \frac{\delta^{(\mathrm{CSV})} a_3^d}{a_3^{d(0)}} &= \left(-\frac{3}{10} + \frac{2C_{2u} + C_{2d}}{2(2C_{2u} - C_{2d})}\right)\left(\frac{\delta u - \delta d}{u + d}\right) \end{aligned}}$$

Sensitivity to CSV



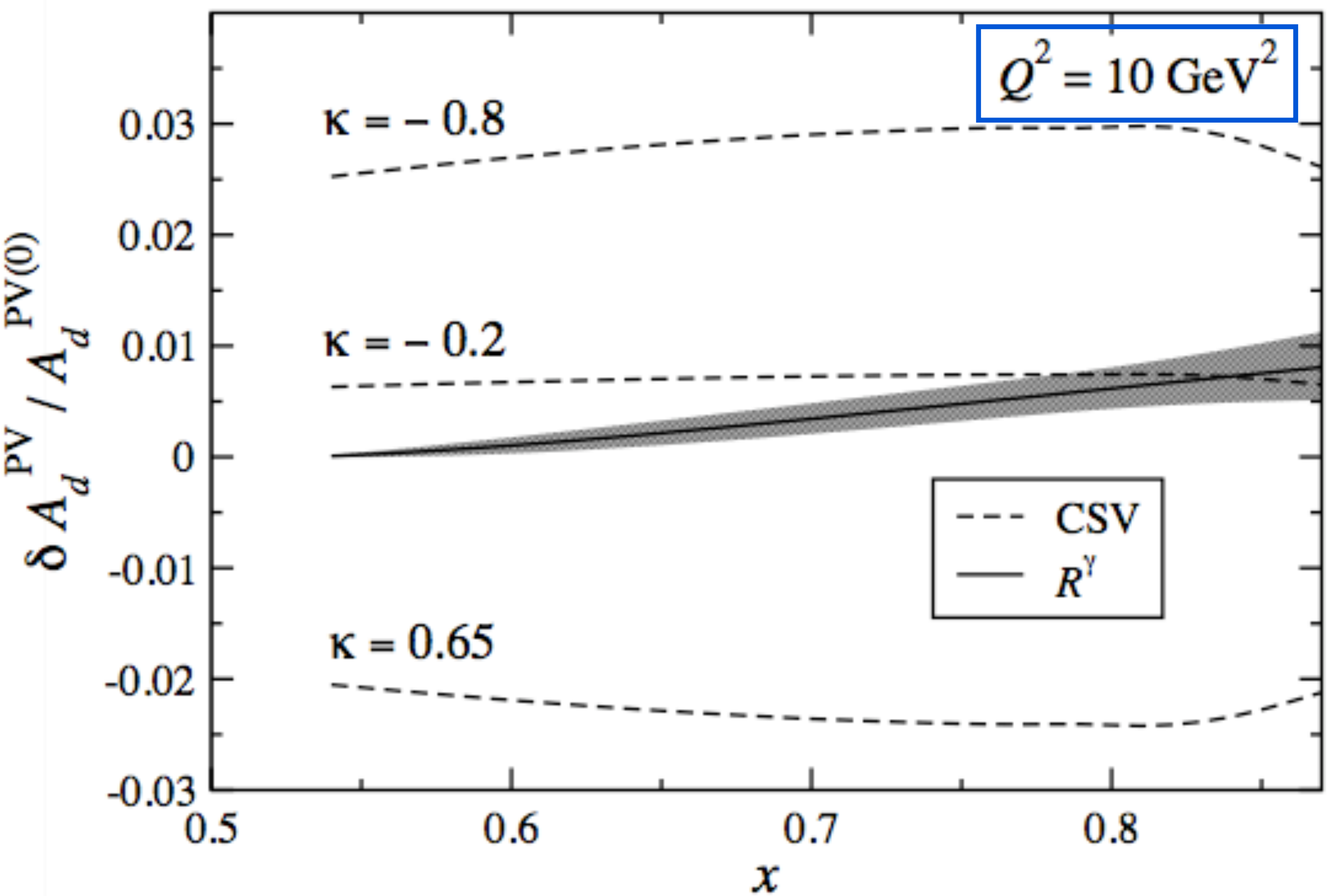
* *MRST, EPJC **39** (2005) 155*

* *MRST, EPJC **35** (2004) 325*

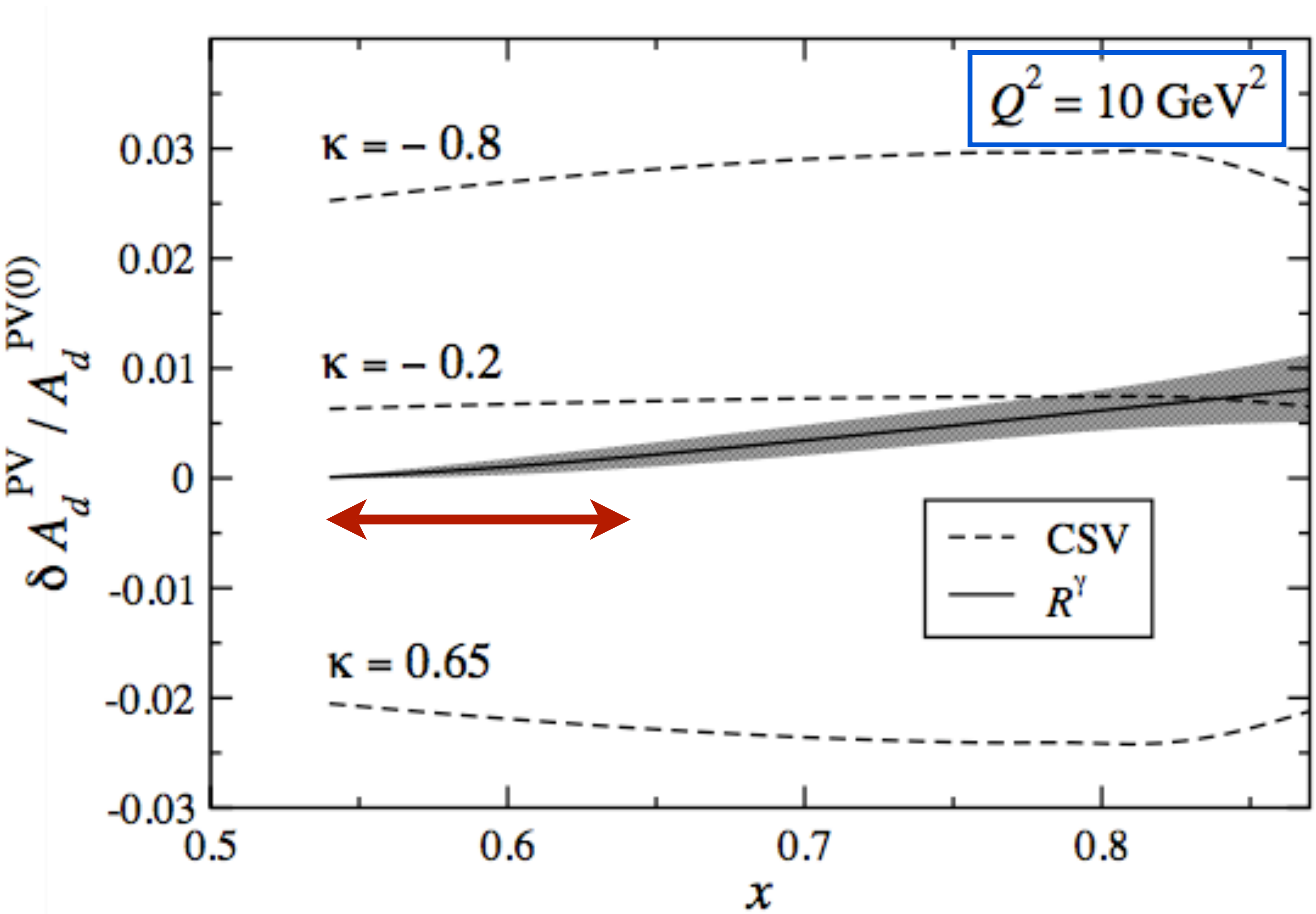
→ "best fit" CSV effect could be smaller than uncertainty in R^γ & $R^{\gamma Z}$

→ need larger Q^2 to reveal (leading twist) CSV

CSV *vs.* finite-Q^2



CSV *vs.* finite-Q^2



→ if CSV $\sim 0.5\%$, optimal value $x \sim 0.6$

→ if CSV larger, could be visible at larger x

Target mass corrections

- Additional corrections from kinematical Q^2/ν^2 effects

  $\longrightarrow$ "target mass corrections" (TMC)

- Important at large x and low Q^2

  $\longrightarrow$ but *not unique* – depend on formalism
  ($e.g.$ OPE, collinear factorization)

  $\longrightarrow$ most implementations exhibit "threshold problem"

$$F(x=1) \neq 0$$

  $\longrightarrow$ uncertainties not overwhelming, except at very large x

  $\longrightarrow$ new scaling variable $\xi = \dfrac{2x}{1+\sqrt{1+4M^2x^2/Q^2}}$

Operator product expansion

$$\int d^4x\, e^{iq\cdot x} \langle N|T(J^\mu(x)J^\nu(0))|N\rangle$$

$$= \sum_k \left(-g^{\mu\nu}q^{\mu_1}q^{\mu_2} + g^{\mu\mu_1}q^\nu q^{\mu_2} + q^\mu q^{\mu_1}g^{\nu\mu_2} + g^{\mu\mu_1}g^{\nu\mu_2}Q^2\right)$$

$$\times q^{\mu_3}\cdots q^{\mu_{2k}} \frac{2^{2k}}{Q^{4k}} \underbrace{A_{2k}\Pi_{\mu_1\cdots\mu_{2k}}}_{\langle N|\mathcal{O}_{\mu_1\cdots\mu_{2k}}|N\rangle}$$

local operators

Georgi, Politzer (1976)

Operator product expansion

$$\int d^4x\ e^{iq\cdot x}\langle N|T(J^\mu(x)J^\nu(0))|N\rangle$$

$$= \sum_k \left(-g^{\mu\nu}q^{\mu_1}q^{\mu_2} + g^{\mu\mu_1}q^\nu q^{\mu_2} + q^\mu q^{\mu_1}g^{\nu\mu_2} + g^{\mu\mu_1}g^{\nu\mu_2}Q^2\right)$$

$$\times q^{\mu_3}\cdots q^{\mu_{2k}}\frac{2^{2k}}{Q^{4k}}\underbrace{A_{2k}\Pi_{\mu_1\cdots\mu_{2k}}}_{\langle N|\mathcal{O}_{\mu_1\cdots\mu_{2k}}|N\rangle}$$

local operators

$$\Pi_{\mu_1\cdots\mu_{2k}} = p_{\mu_1}\cdots p_{\mu_{2k}} - (g_{\mu_i\mu_j}\ \text{terms})$$

$$= \sum_{j=0}^{k}(-1)^j\frac{(2k-j)!}{2^j(2k)^j}g\cdots g\ p\cdots p$$

traceless, symmetric
rank-$2k$ tensor

Georgi, Politzer (1976)

n-th Cornwall-Norton moment of F_2 structure function

$$M_2^n(Q^2) = \int dx \ x^{n-2} \ F_2(x, Q^2)$$

$$= \sum_{j=0}^{\infty} \left(\frac{M^2}{Q^2}\right)^j \frac{(n+j)!}{j!(n-2)!} \frac{A_{n+2j}}{(n+2j)(n+2j-1)}$$

$j = 0 \implies$ parton model

"quark distribution function"

$$\Rightarrow \quad A_n = \int_0^1 dy \ y^n \ F(y) \qquad F(y) \equiv \frac{F_2(y)}{y^2}$$

- take inverse Mellin transform (+ tedious manipulations)

  ⇨ target mass corrected structure function

$$F_2^{\mathrm{GP}}(x,Q^2) = \frac{x^2}{r^3} F(\xi) \; + \; 6\frac{M^2}{Q^2}\frac{x^3}{r^4}\int_\xi^1 d\xi' \; F(\xi')$$
$$+ \; 12\frac{M^4}{Q^4}\frac{x^4}{r^5}\int_\xi^1 d\xi' \; \int_{\xi'}^1 d\xi'' \; F(\xi'')$$

$$\xi = \frac{2x}{1+r} \qquad\qquad r = \sqrt{1+4x^2M^2/Q^2}$$

- take inverse Mellin transform (+ tedious manipulations)

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$$F_2^{\mathrm{GP}}(x,Q^2) = \frac{x^2}{r^3}F(\xi) \; + \; 6\frac{M^2}{Q^2}\frac{x^3}{r^4}\int_\xi^1 d\xi' \; F(\xi')$$

$$+ \; 12\frac{M^4}{Q^4}\frac{x^4}{r^5}\int_\xi^1 d\xi' \; \int_{\xi'}^1 d\xi'' \; F(\xi'')$$

Georgi-Politzer prescription for TMCs

massless limit function

$$\xi = \frac{2x}{1+r} \qquad\qquad r = \sqrt{1+4x^2M^2/Q^2}$$

Threshold problem

- if $F(y) \sim (1-y)^\beta$ at large y

  then since $\xi_0 \equiv \xi(x=1) < 1$

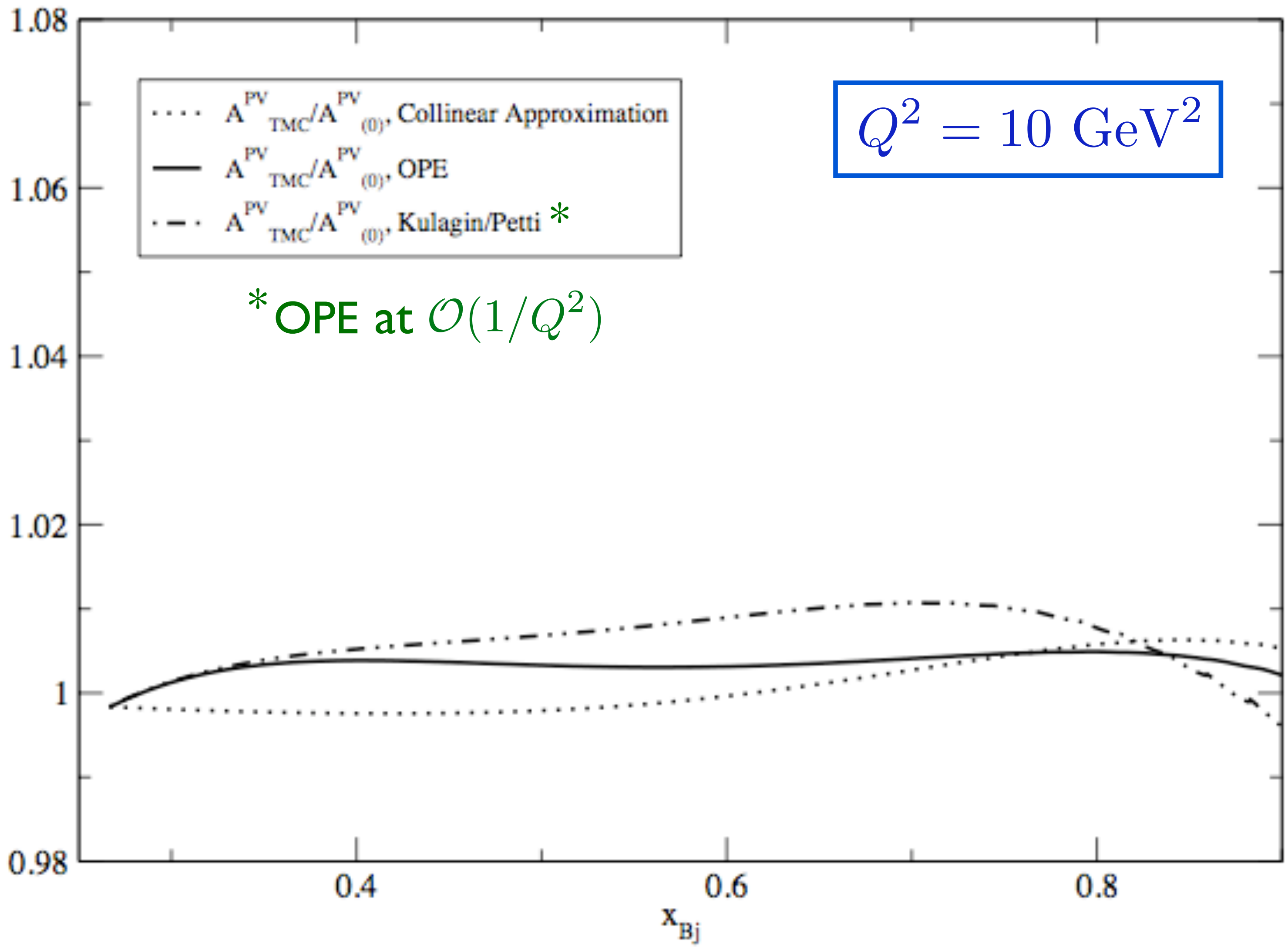
$\Rightarrow$ $F(\xi_0) > 0$

$\Rightarrow$ $F_i^{\rm TMC}(x=1, Q^2) > 0$

is this physical?

$\Rightarrow$ several (partial) attempts to circumvent problem

Target mass corrections



→ additional uncertainty, increases with x^2/Q^2

Summary & outlook

- Sources of Q^2 dependence identified and quantified

  $\longrightarrow$ can mask sought-after signals at finite Q^2

- Largest uncertainty in interference $\mathrm{L/T}$ ratio $R^{\gamma Z}$

  $\longrightarrow$ theoretical study ongoing

  $\longrightarrow$ empirical constraints?

- Target mass corrections

  $\longrightarrow$ corrections scale as x^2/Q^2 , involve some uncertainty

- Higher twist effects

  $\longrightarrow$ studied by several authors, need revisiting

Summary & outlook

Thanks to Krishna Kumar & Paul Sauder
for motivating this study

The End