MENU 07
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Quark-Hadron Duality in Electron Scattering

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Goal: QCD-based understanding of structure function data at low Q^2 and W

Outline:

- Quark-hadron (*"Bloom-Gilman"*) duality
- Duality in QCD
- Local duality & truncated moments
- Conclusions

Quark-Hadron
(*“Bloom-Gilman”*)
Duality

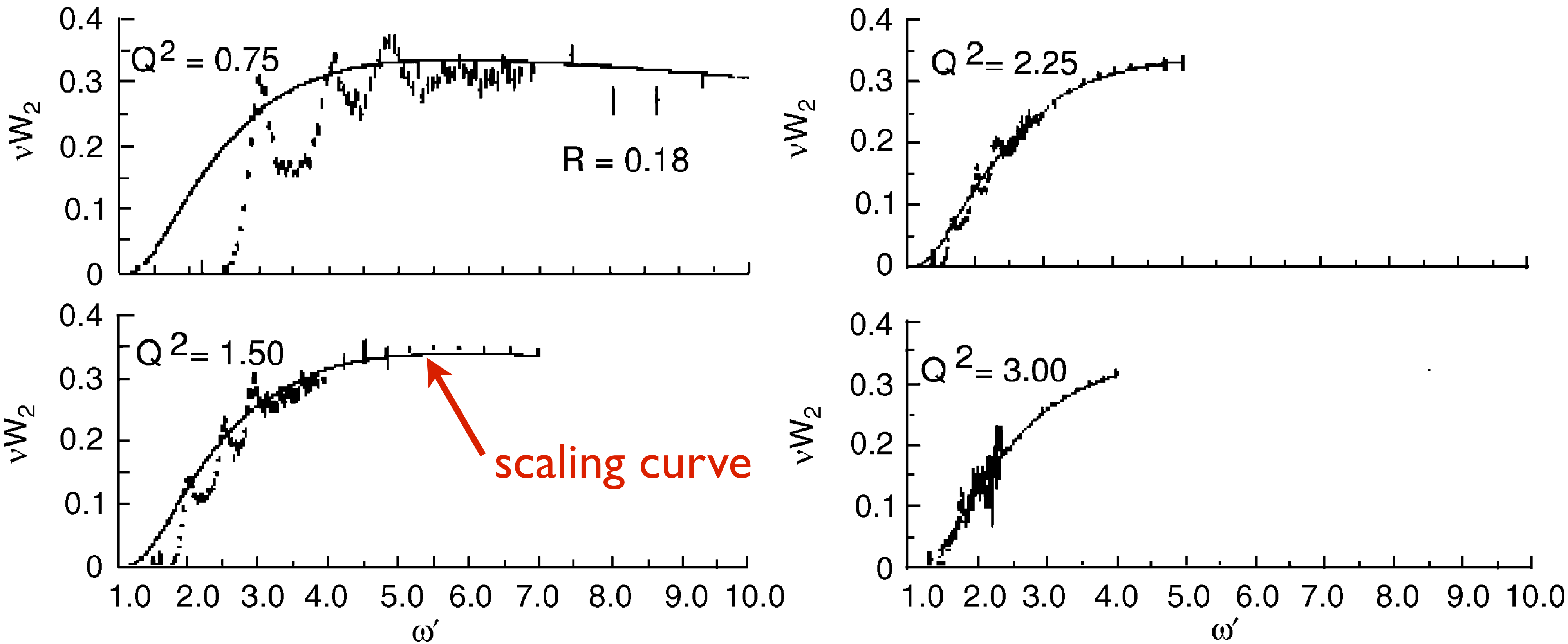
Quark-hadron duality

Complementarity between *quark* and *hadron* descriptions of observables

$$\sum_{hadrons} = \sum_{quarks}$$

Can use <u>either</u> set of complete basis states to describe all physical phenomena

Inclusive electron-proton scattering



Bloom, Gilman, Phys. Rev. Lett. 85 (1970) 1185

➡ resonance – scaling duality in proton $\nu W_2 = F_2$ structure function

Bloom-Gilman duality

Average over (strongly Q^2 dependent) resonances
$\approx Q^2$ independent scaling function

Finite energy sum rule for eN scattering

$$\frac{2M}{Q^2}\int_0^{\nu_m} d\nu\ \nu W_2(\nu, Q^2) = \int_1^{\omega'_m} d\omega'\ \nu W_2(\omega')$$

measured structure function
(function of ν and Q^2)

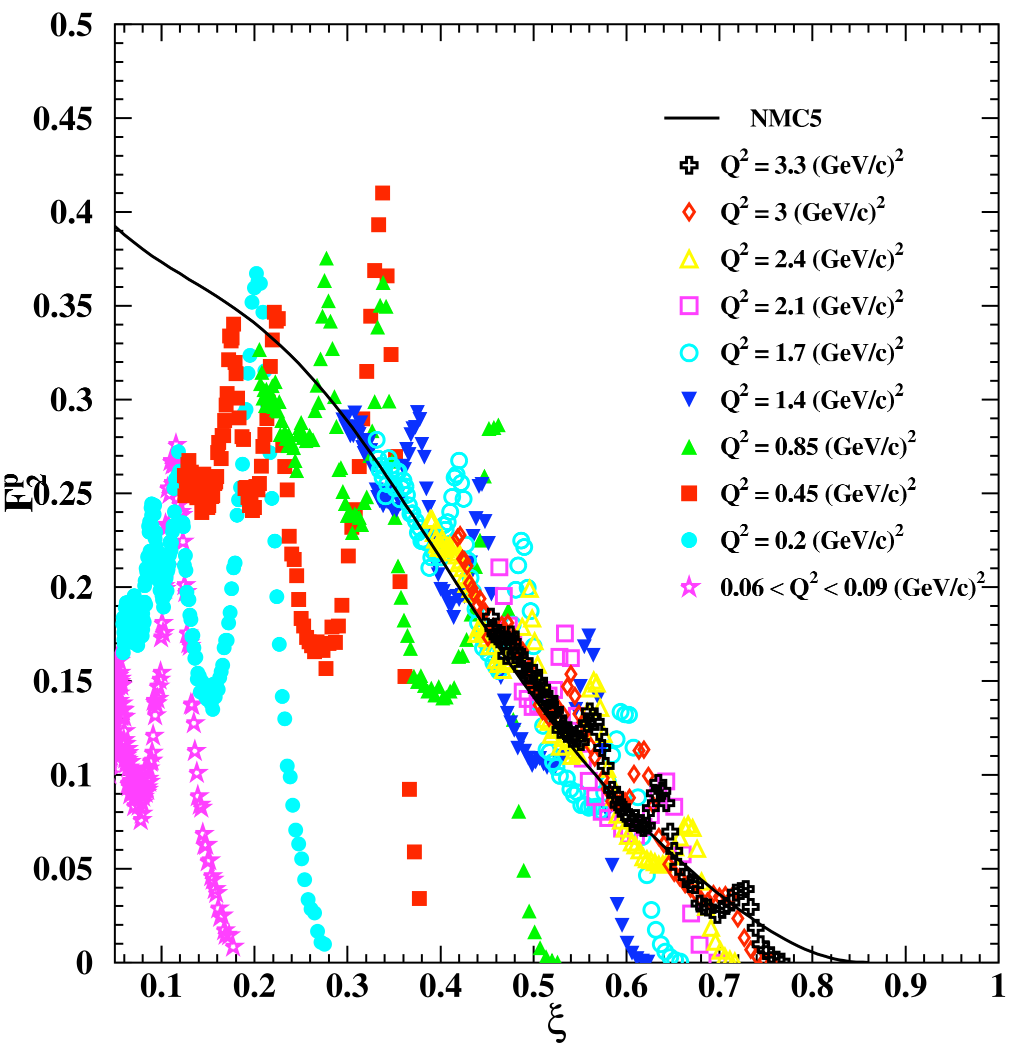
"hadrons"

$$\omega' = \frac{1}{x} + \frac{M^2}{Q^2}$$

scaling function
(function of ω' only)

"quarks"

Bloom-Gilman duality

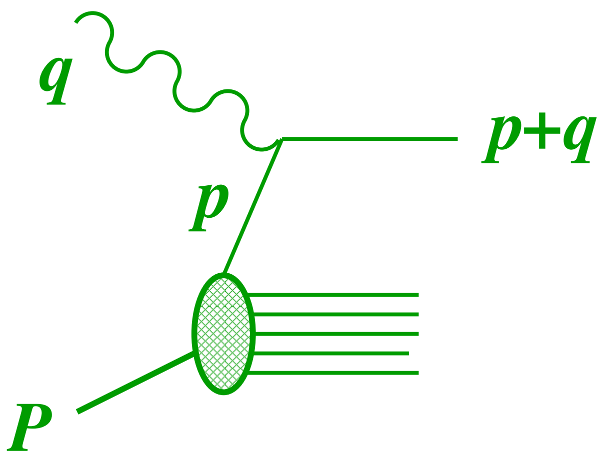


Jefferson Lab (Hall C)

Niculescu et al., Phys. Rev. Lett. 85 (2000) 1182

Average over
(strongly Q^2 dependent)
resonances
$\approx$ Q^2 independent
scaling function

Scaling variables



$$(p+q)^2 = m_q^2 \quad \left\{ \begin{array}{l} m_q = 0 \\ p_T = 0 \end{array} \right.$$

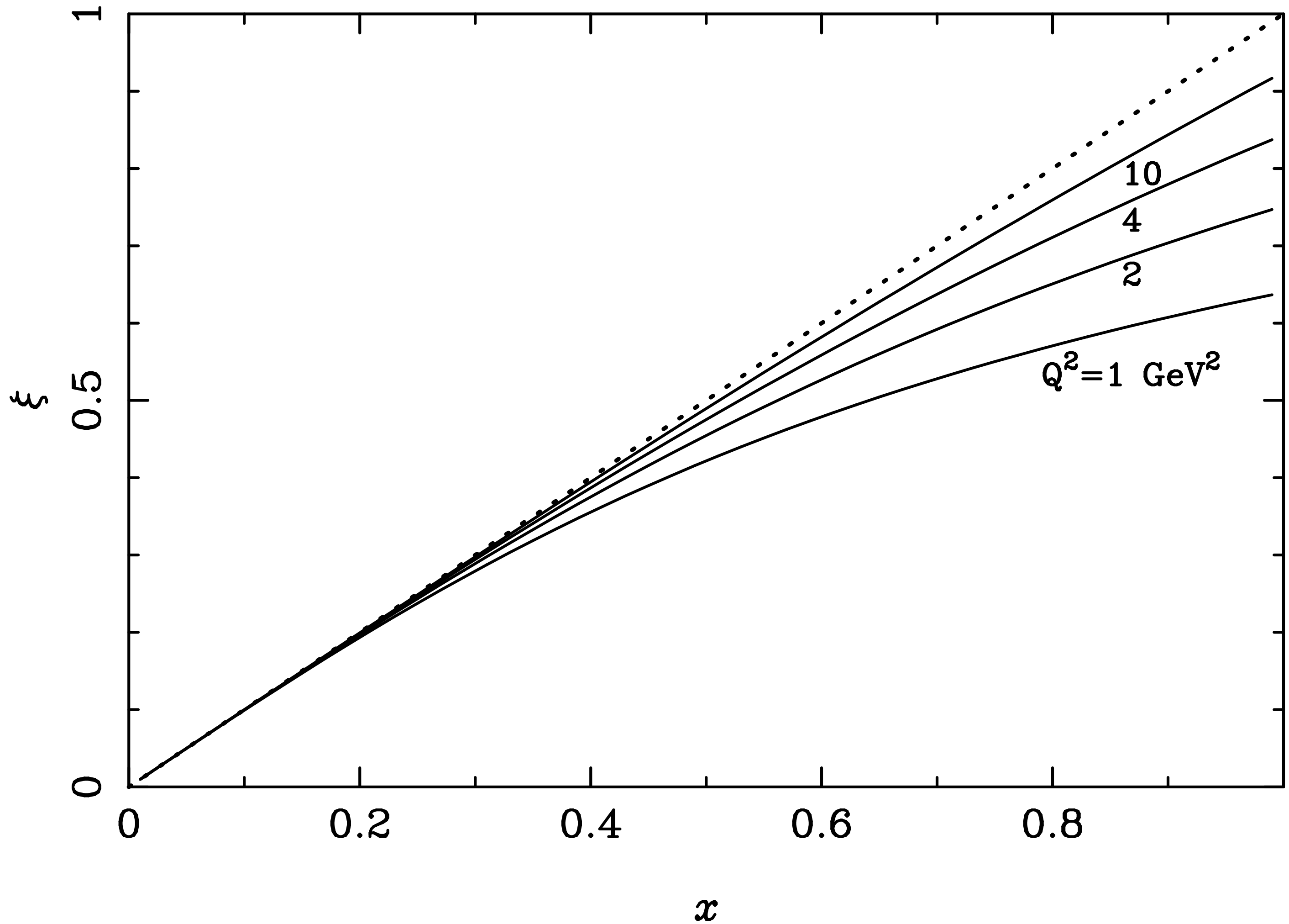
light-cone fraction of target's momentum carried by parton

$$\xi = \frac{p^+}{P^+} = \frac{p^0 + p^z}{M}$$

$$\Longrightarrow \quad \xi = \frac{2x}{1 + \sqrt{1 + 4x^2 M^2 / Q^2}} \quad \rightarrow \quad x \quad \text{as} \quad Q^2 \rightarrow \infty$$

Nachtmann scaling variable

Scaling variables



Duality in QCD

Duality in QCD

Operator product expansion

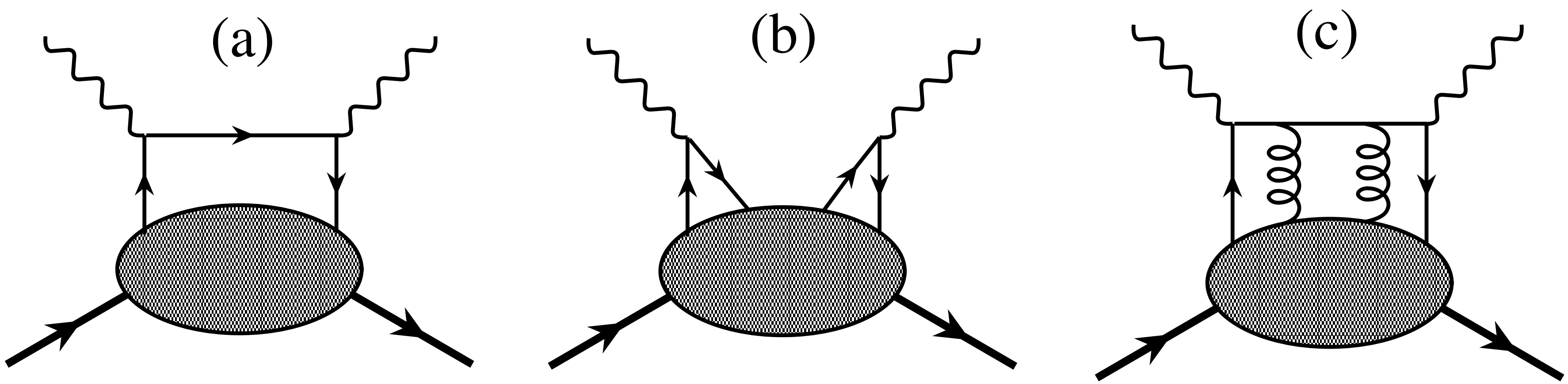
$\Longrightarrow$ expand moments of structure functions in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx\ x^{n-2}\ F_2(x,Q^2)$$
$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

matrix elements of operators with specific "twist" τ

$$\tau = \text{dimension} - \text{spin}$$

Higher twists



$\tau = 2$

$\tau > 2$

single quark scattering

qq and qg correlations

$e.g. \quad \bar{\psi}\ \gamma_\mu\ \psi$

$e.g. \quad \bar{\psi}\ \gamma_\mu\ \psi\ \bar{\psi}\ \gamma_\nu\ \psi$

or $\quad \bar{\psi}\ \tilde{G}_{\mu\nu}\gamma^\nu\ \psi$

Duality in QCD

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If moment $\approx$ independent of Q^2

$\Longrightarrow$ higher twist terms $A_n^{(\tau>2)}$ small

Duality in QCD

Operator product expansion

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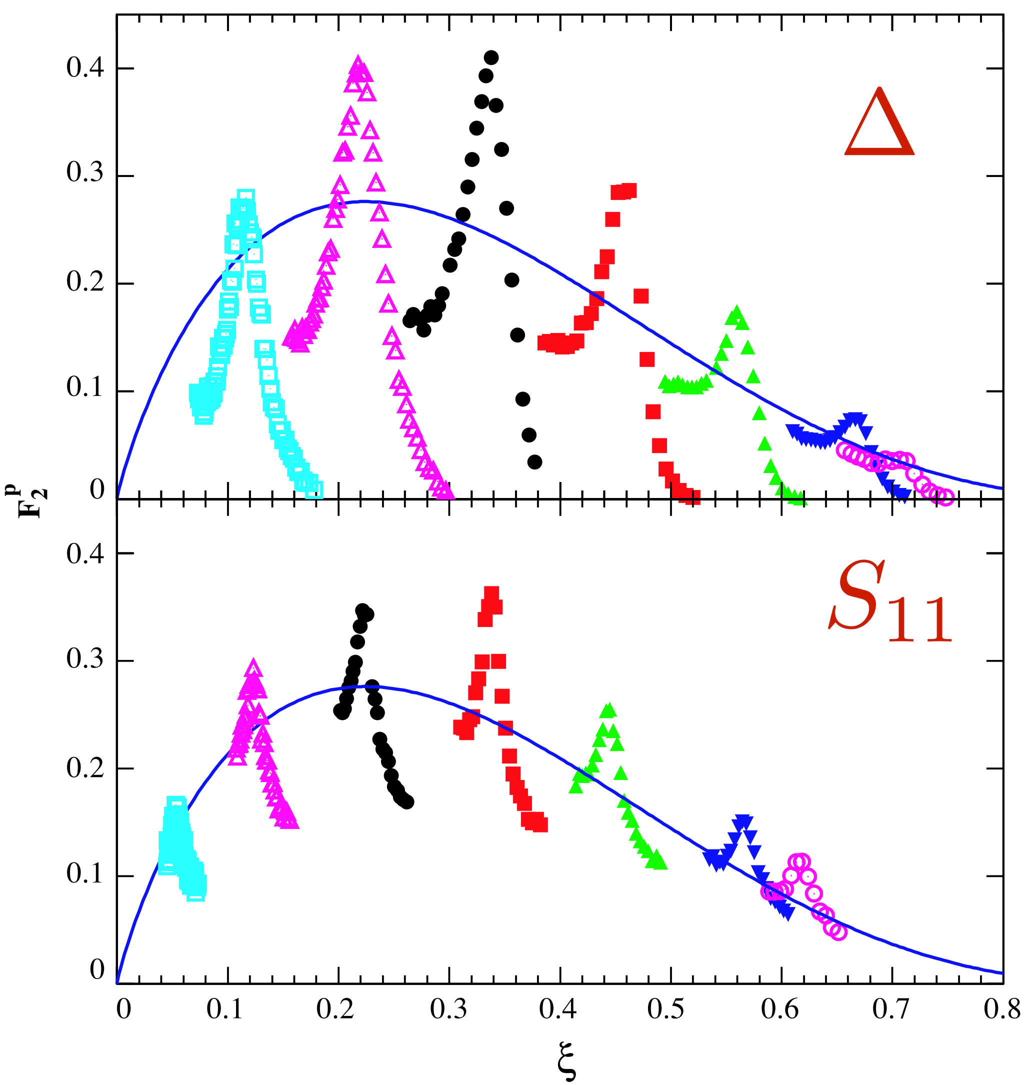
Duality $\Longleftrightarrow$ suppression of higher twists

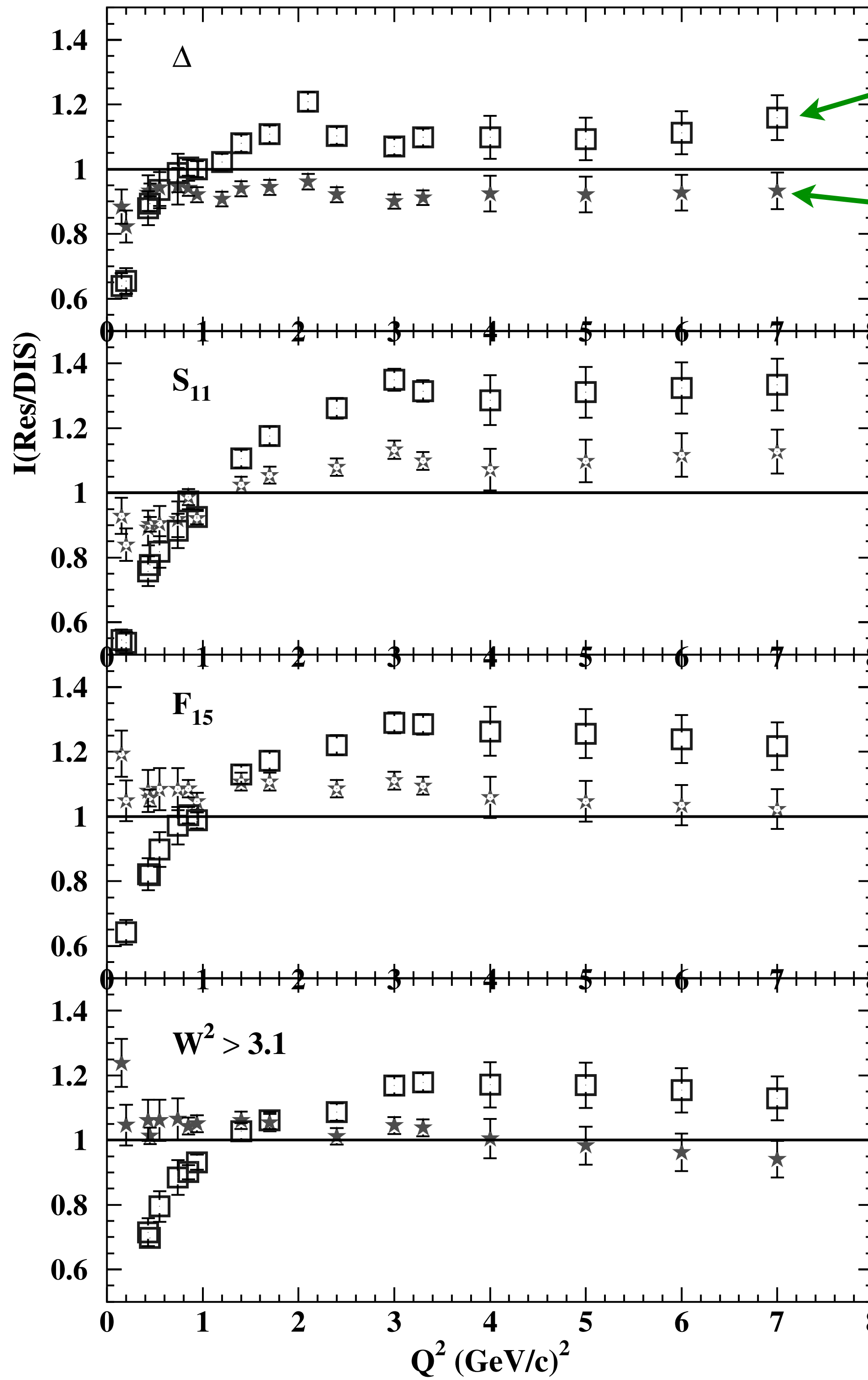
de Rujula, Georgi, Politzer,
Ann. Phys. 103 (1975) 315

Duality exists also in <u>local</u> regions, around individual resonances



⇒ "local Bloom-Gilman duality"

ratio of <u>integrated</u> resonance to DIS contributions



~10% agreement for $Q^2 > 1$ GeV^2

Niculescu et al., Phys. Rev. Lett. 85 (2000) 1186

Truncated Moments

Truncated moments

- complete moments can be studied in QCD via twist expansion

  $\longrightarrow$ Bloom-Gilman duality has a precise meaning

  (*i.e.*, duality violation = higher twists)

- for local duality, difficult to make rigorous connection with QCD

  $\longrightarrow$ *e.g.* need prescription for how to average over resonances

- *truncated* moments allow study of restricted regions in x (or W) within QCD in well-defined, systematic way

$$\overline{M}_n(\Delta x, Q^2) = \int_{\Delta x} dx\ x^{n-2}\ F_2(x, Q^2)$$

Truncated moments

- truncated moments obey DGLAP-like evolution equations, similar to PDFs

$$\frac{d\overline{M}_n(\Delta x, Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi}\left(P'_{(n)} \otimes \overline{M}_n\right)(\Delta x, Q^2)$$

where modified splitting function is

$$P'_{(n)}(z, \alpha_s) = z^n \; P_{NS,S}(z, \alpha_s)$$

→ can follow evolution of <u>specific resonance (region)</u> with Q^2 in pQCD framework!

→ suitable when complete moments not available

Truncated moments

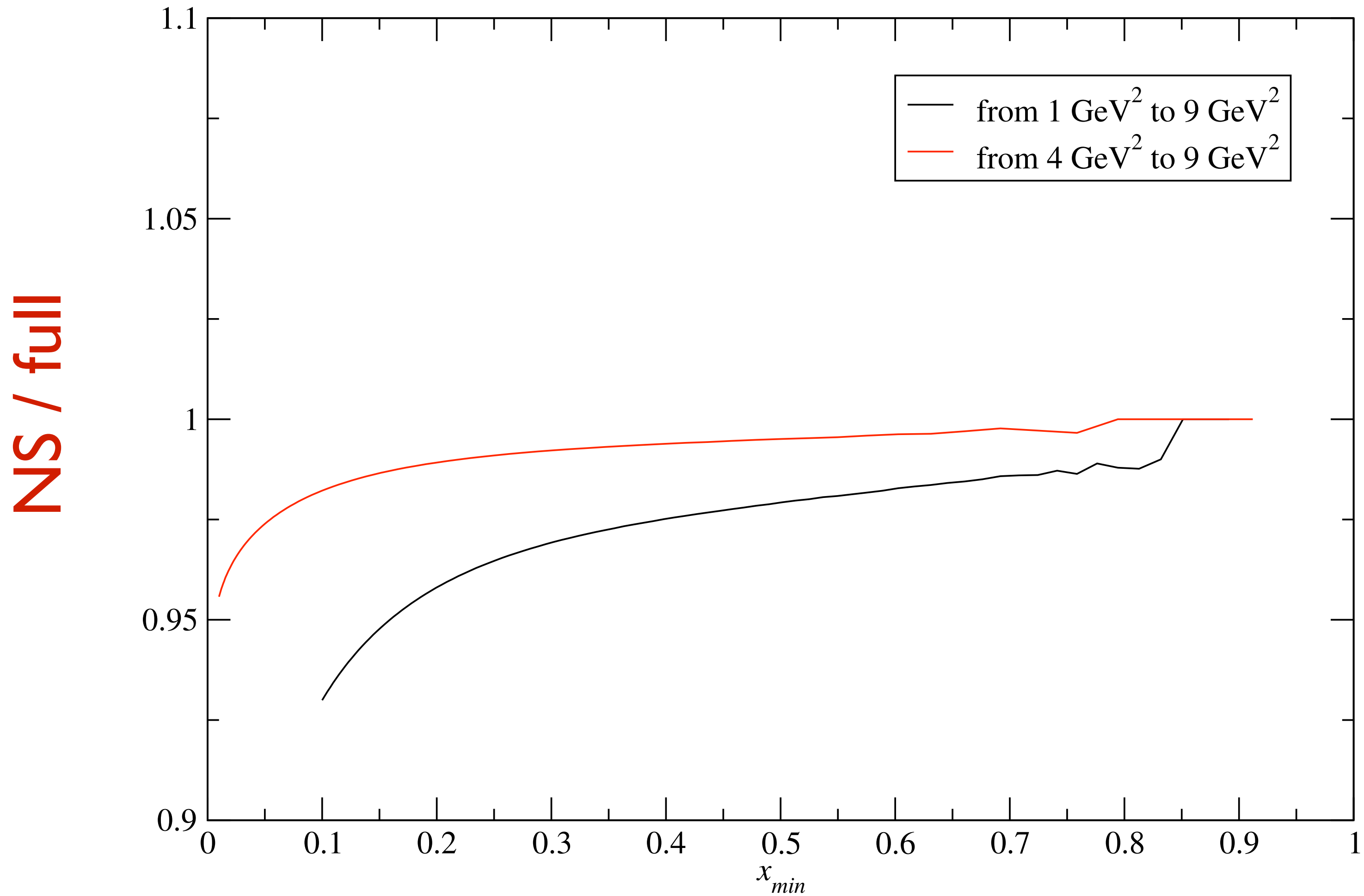
- truncated moment evolution equations exist for singlet (S) and nonsinglet (NS) separately

- for analysis of data, do not know much of experimental structure function is NS and how much is S

  → for lowest ($n=2$) truncated moment, assumption that total $\approx$ NS is good to few % for $x_{\min} > 0.2$

  → for higher moments, small-x region is further suppressed, so that NS is a very good approximation to total

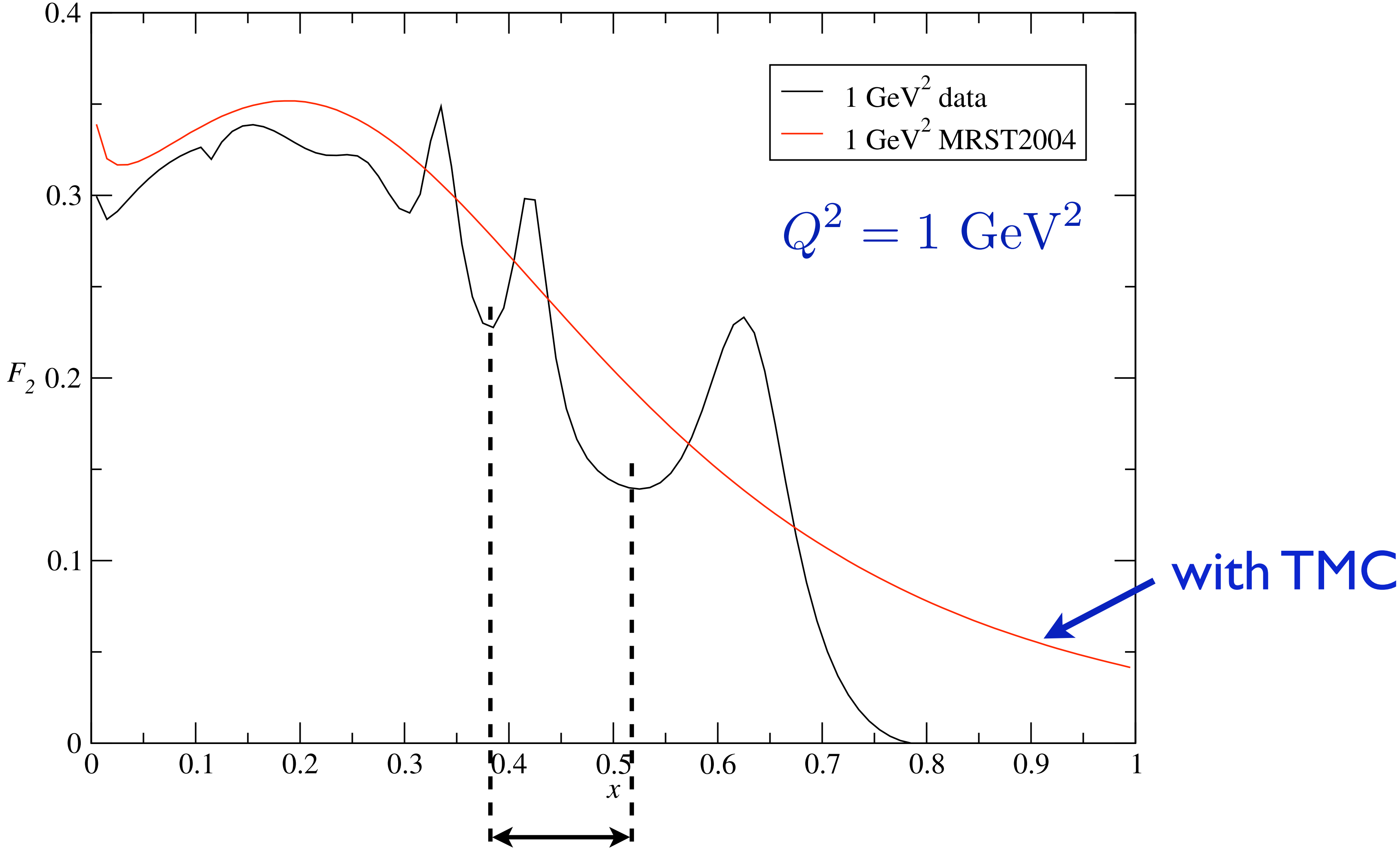
$n = 2$ truncated moment of F_2^p



$$\Delta x = [x_{min}, 1]$$

Psaker, WM, Christy, Keppel (2007)

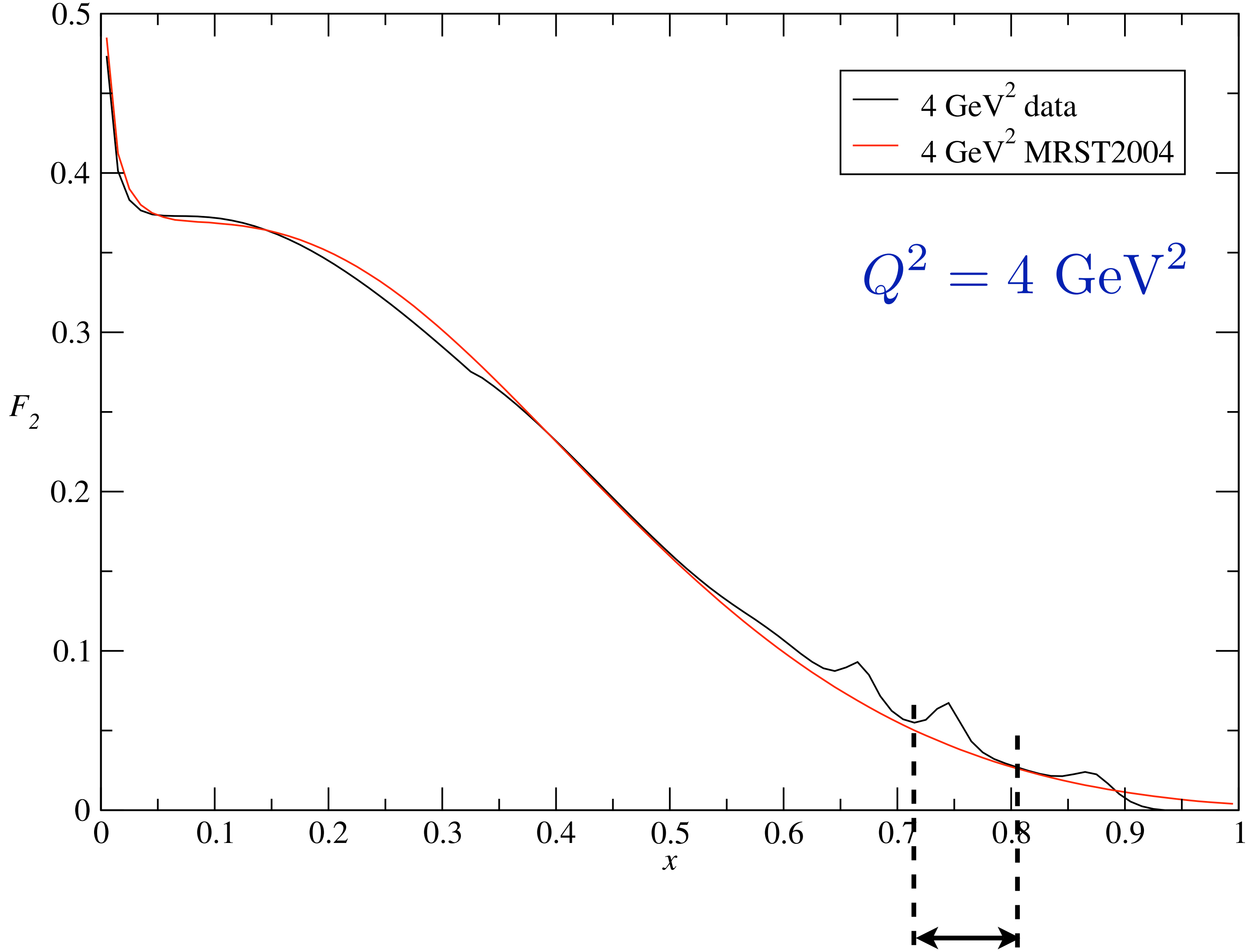
Parameterization of F_2^p data



how much of this region
is <u>leading twist</u> ?

Psaker et al. (2007)

Parameterization of F_2^p data



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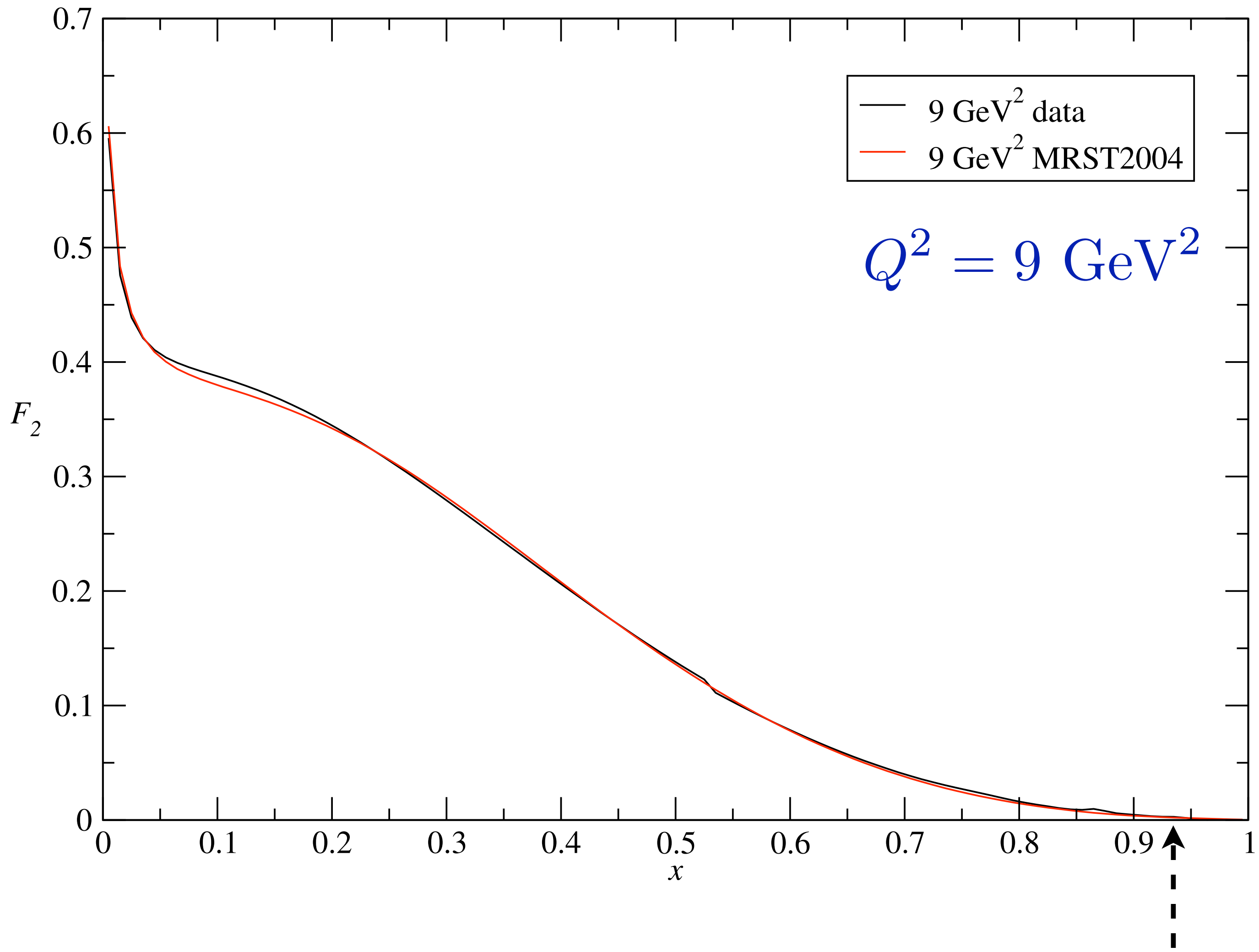
Psaker et al. (2007)

Parameterization of F_2^p data



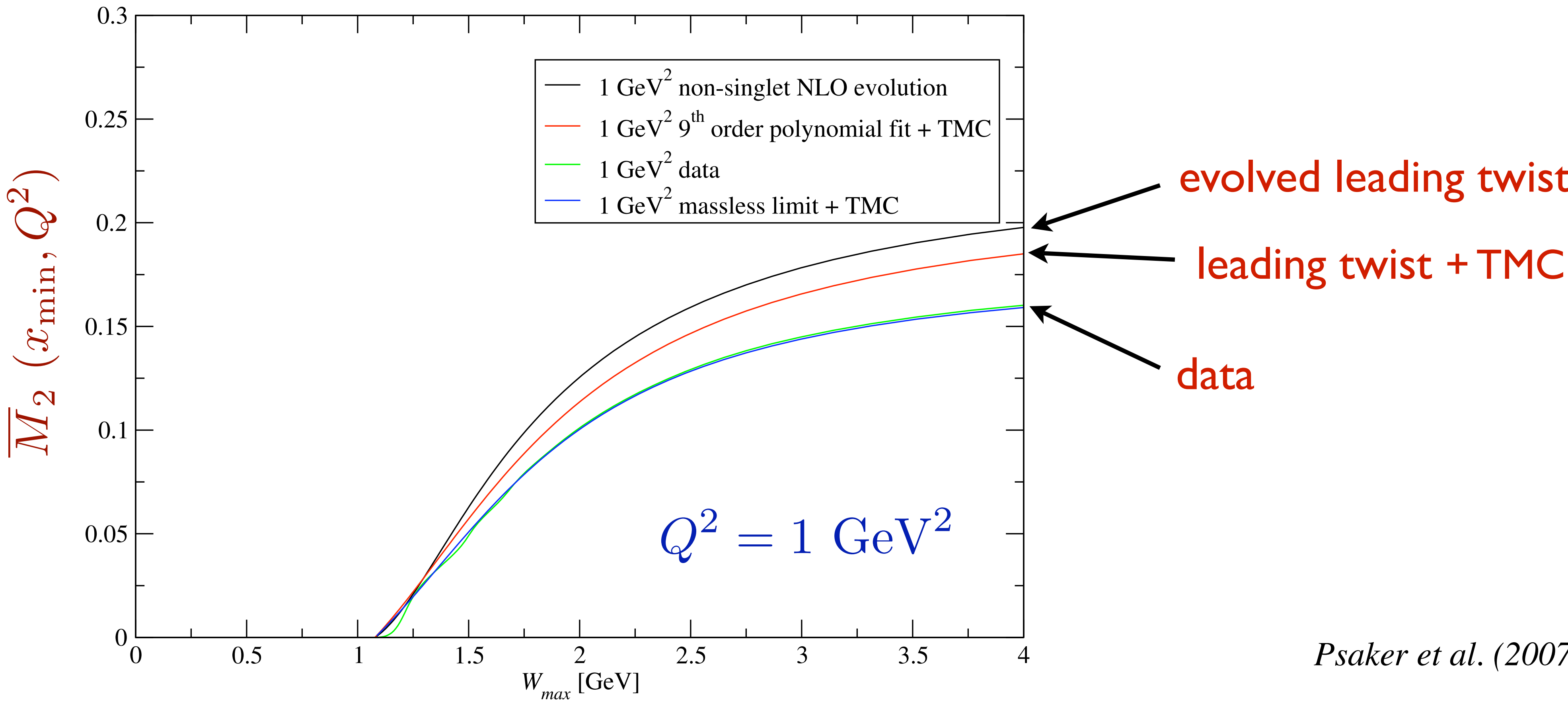
how much of this region
is <u>leading twist</u> ?

Psaker et al. (2007)

Analysis of JLab data

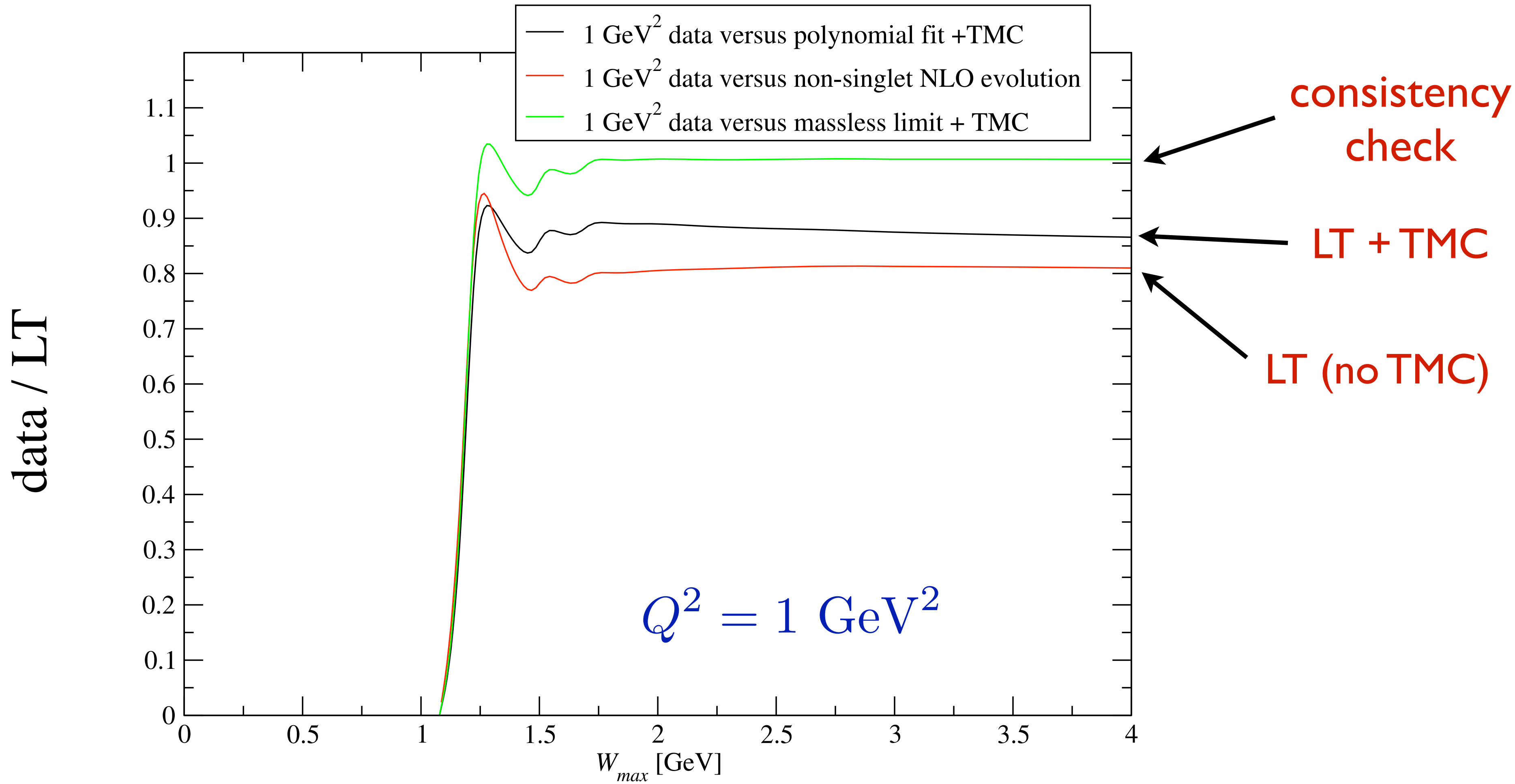
- assume data at highest Q^2 ($Q^2 = 9\ \mathrm{GeV}^2$) is entirely leading twist
- evolve (as NS) fit to data at $Q^2 = 9\ \mathrm{GeV}^2$ down to lower Q^2

$\longrightarrow$ apply TMC, and compare with data at lower Q^2



Psaker et al. (2007)

ratio of data to leading twist



- consider individual resonance regions:

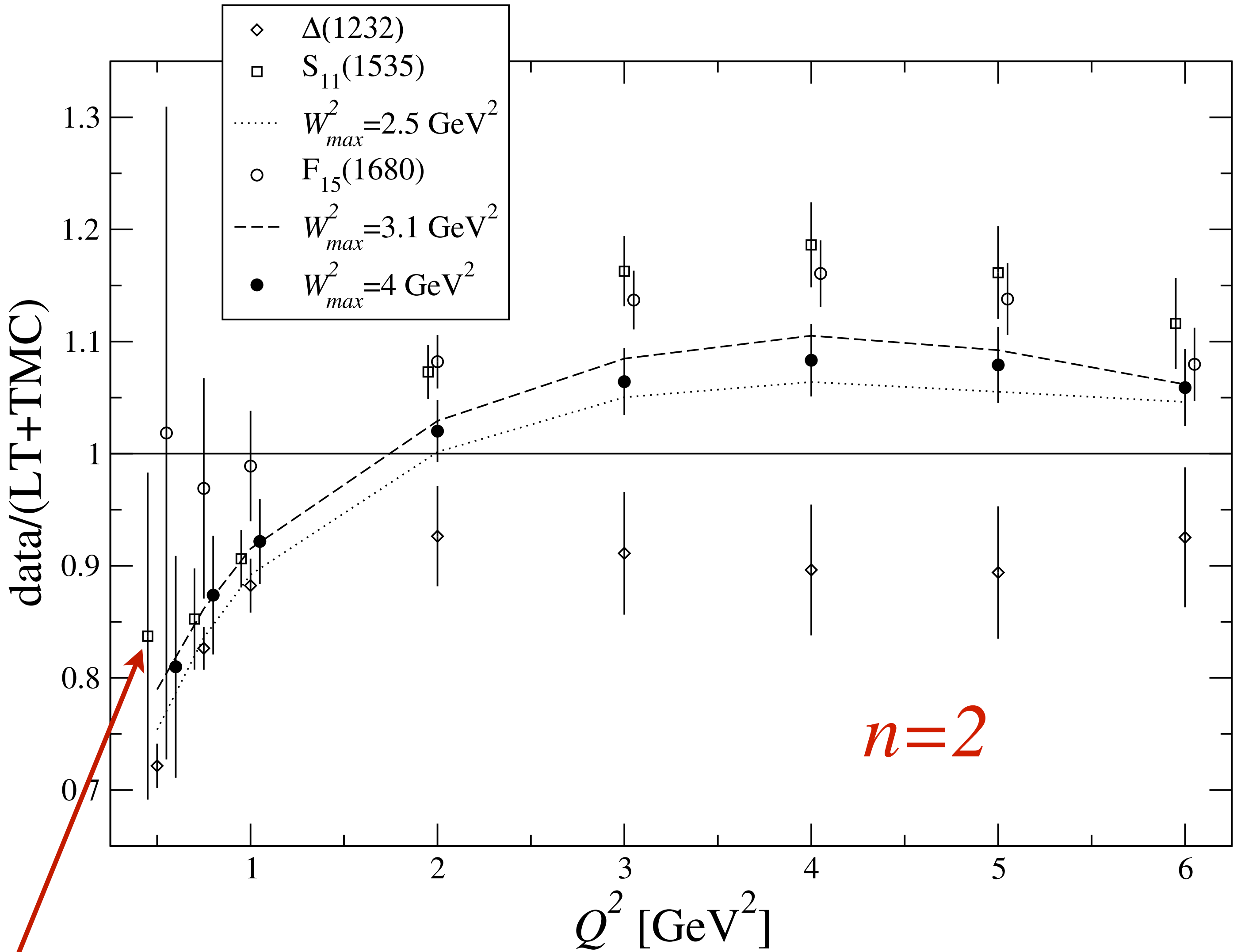
$$W_{\mathrm{thr}}^2 < W^2 < 1.9\ \mathrm{GeV}^2 \qquad \text{“}\Delta(1232)\text{”}$$

$$1.9 < W^2 < 2.5\ \mathrm{GeV}^2 \qquad \text{“}S_{11}(1535)\text{”}$$

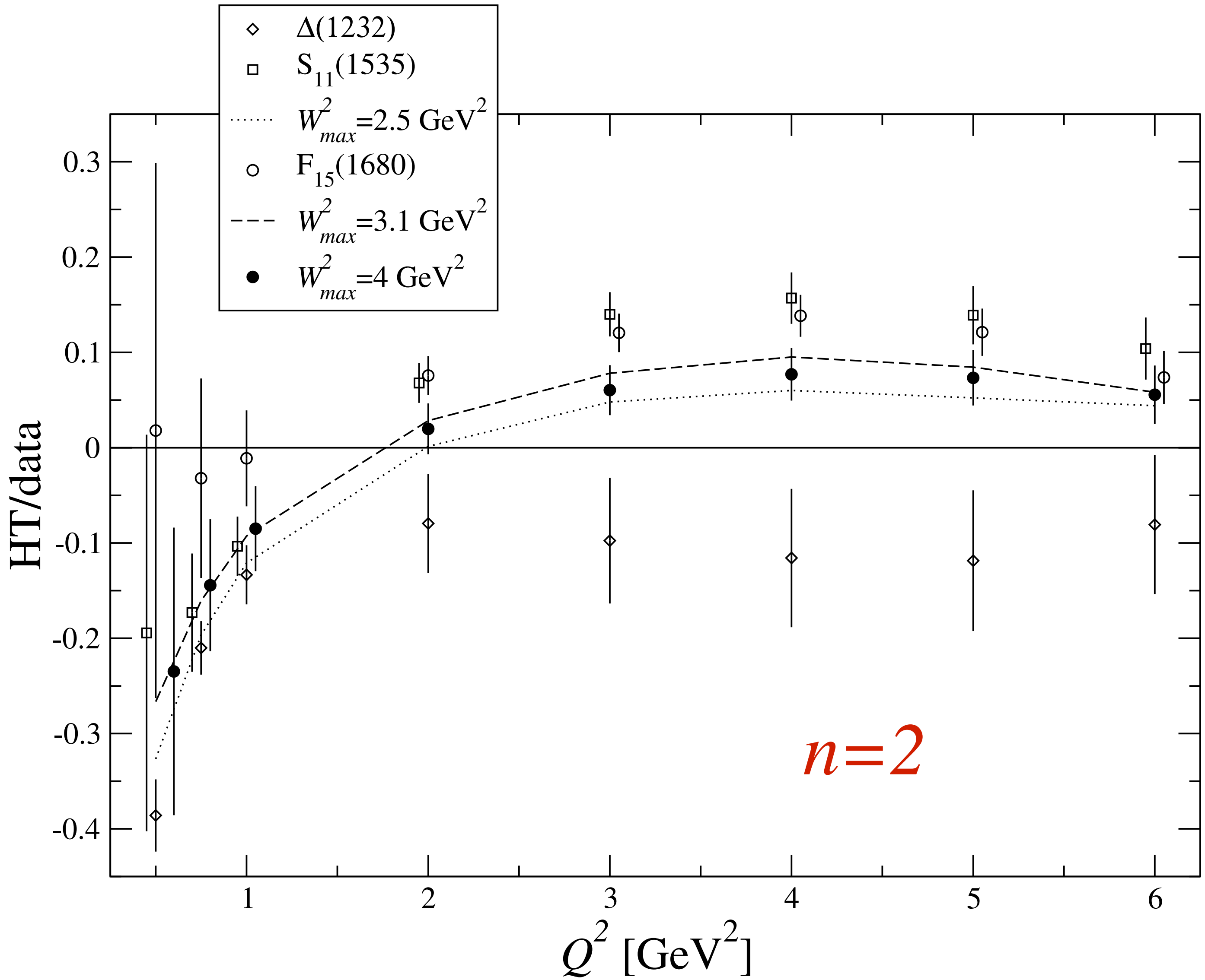
$$2.5 < W^2 < 3.1\ \mathrm{GeV}^2 \qquad \text{“}F_{15}(1680)\text{”}$$

as well as total resonance region:

$$W^2 < 4\ \mathrm{GeV}^2$$

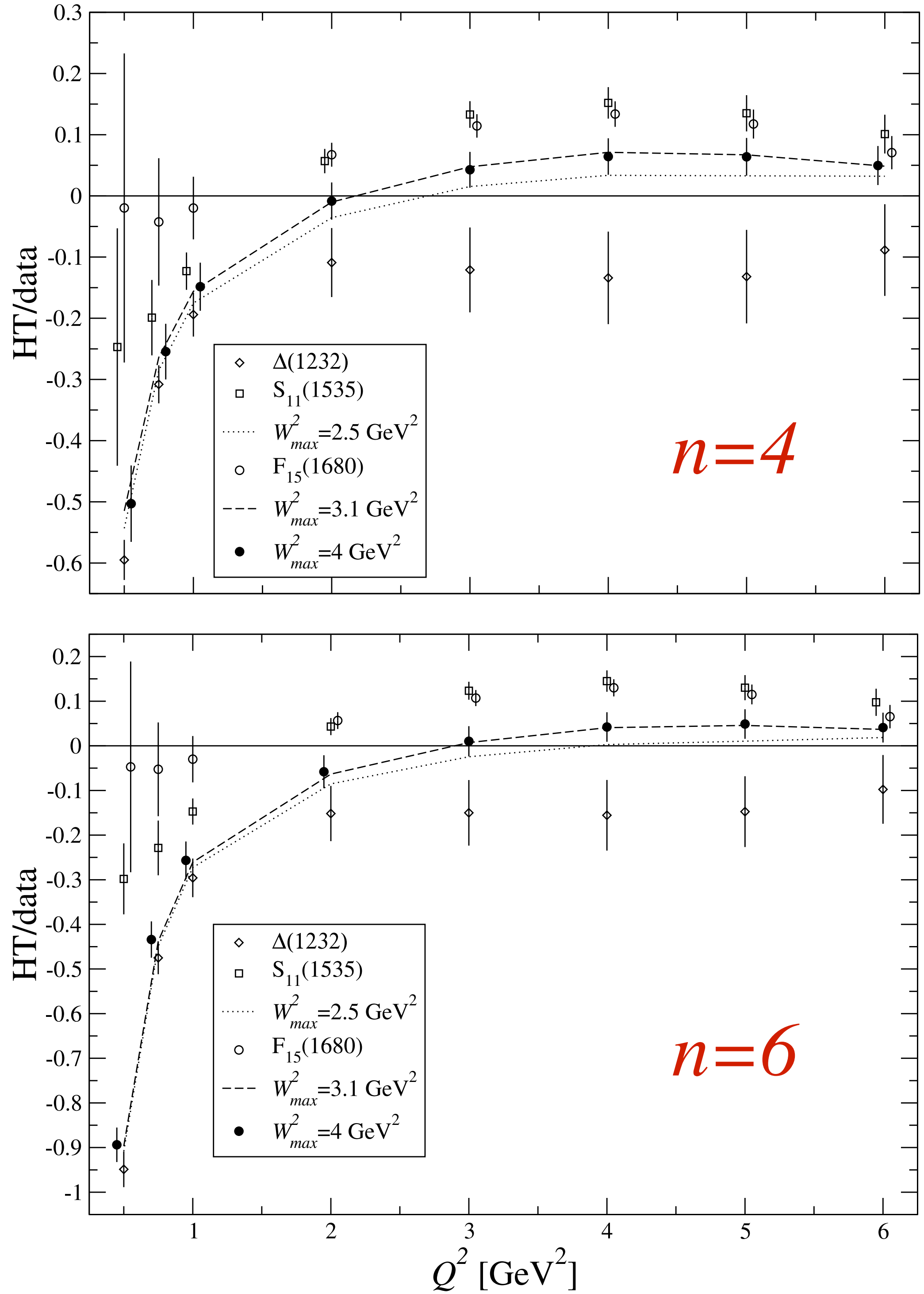


method breaks down for
low x (high W) at low Q^2



→ higher twists < 10% for $Q^2 > 1\ \mathrm{GeV}^2$

■ higher moments



→ higher twists < 5-10%

for Q^2 > 2-3 GeV^2

in resonance region

Summary

- Observation of quark-hadron duality in structure functions
  - → higher twists "small" down to low Q^2 (~ 1 GeV^2)
  - → global duality understood within QCD moments

- Local duality
  - → duality exists in local regions of x (or W)
  - → difficult to understand within QCD

- Truncated moments
  - → firm foundation for study of local duality in QCD
  - → higher twists $< 10\%$ for $Q^2 > 1$ GeV^2 in resonance region