Quark-Hadron Duality and the Transition to pQCD
Laboratori Nazionali di Frascati, June 6-8, 2005

Quark Models of Duality

in e and ν scattering

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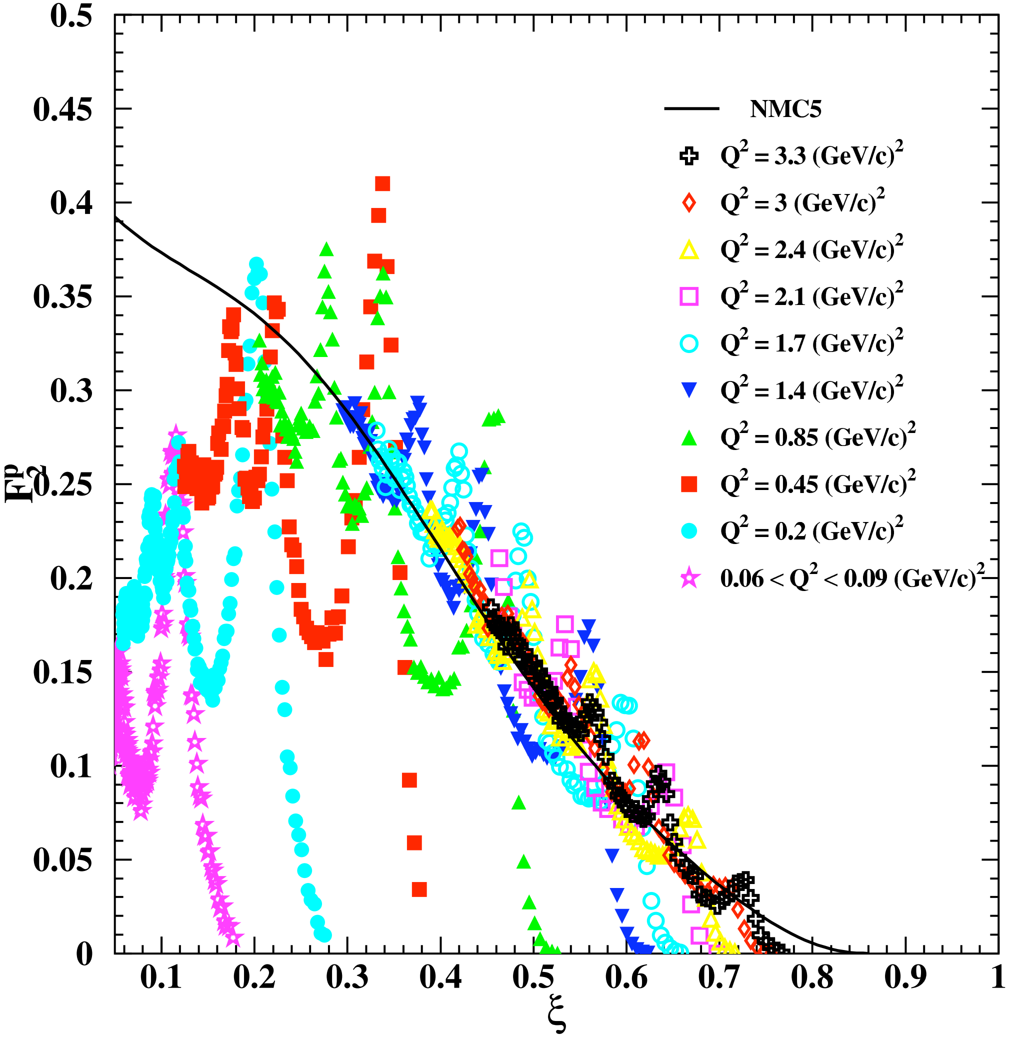
+ F. Close (Oxford), E. Paschos (Dortmund)

Bloom-Gilman duality

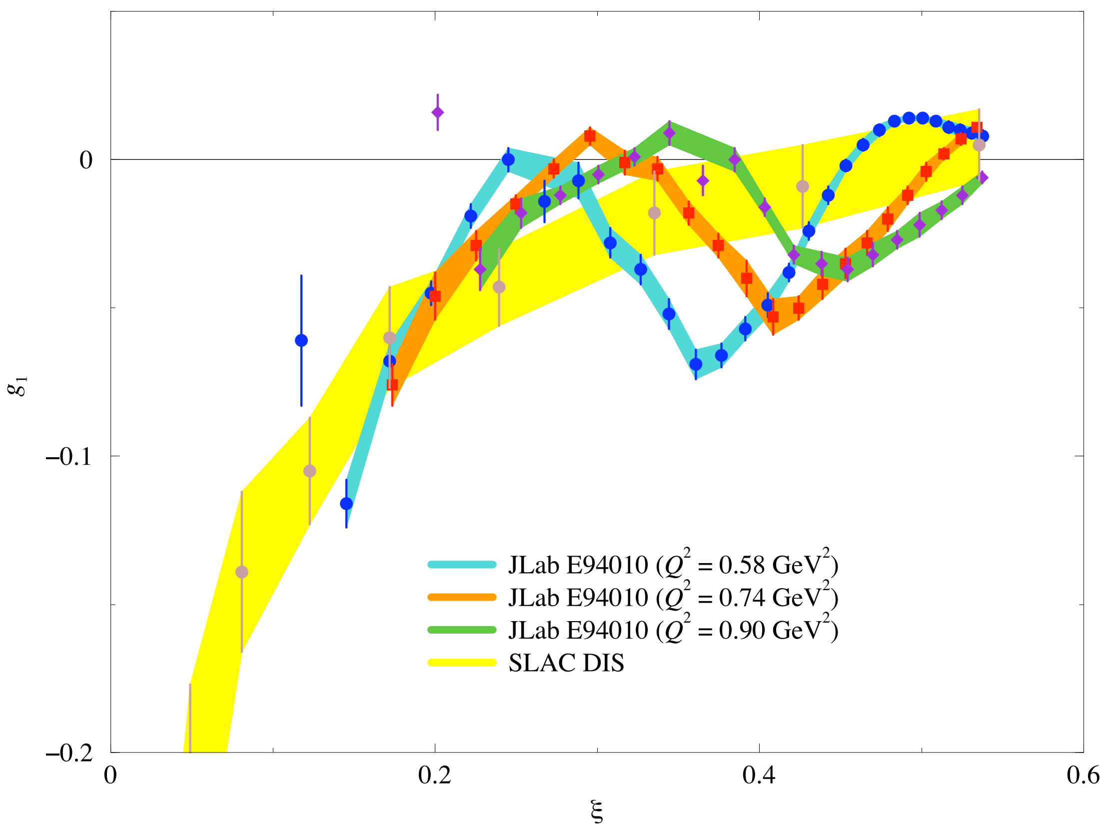


Niculescu et al., Phys. Rev. Lett. 85 (2000) 1182

Average over
(strongly Q^2 dependent)
resonances
$\approx Q^2$ independent
scaling function

... also for spin-dependent...

Neutron (^{3}He) g_1 structure function



Liyanage et al. (JLab Hall A)

Duality in QCD

Operator product expansion

→ expand moments of structure functions in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx\; x^{n-2}\; F_2(x, Q^2)$$

$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

Duality in QCD

Operator product expansion

⇒ expand moments of structure functions in powers of $1/Q^2$

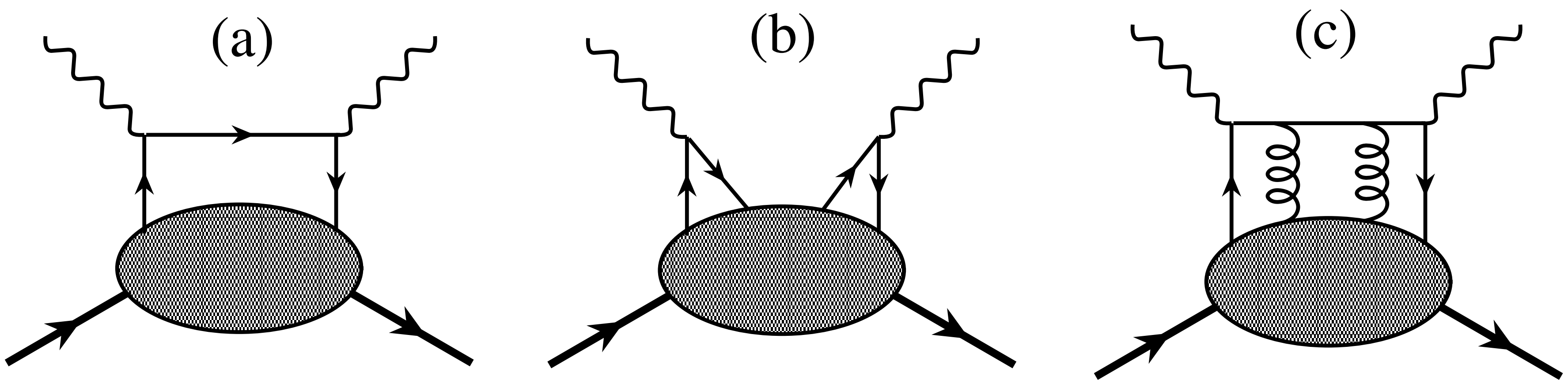
$$M_n(Q^2) = \int_0^1 dx\ x^{n-2}\ F_2(x, Q^2)$$

$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

matrix elements of operators with specific "twist" τ

$$\tau = \text{dimension} - \text{spin}$$

Higher twists



$\tau = 2$

single quark
scattering

$\tau > 2$

qq and qg
correlations

Duality in QCD

Operator product expansion

⟹ expand moments of structure functions in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx \; x^{n-2} \; F_2(x, Q^2)$$

$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

If moment $\approx$ independent of Q^2

⟹ higher twist terms $A_n^{(\tau>2)}$ small

Duality in QCD

Operator product expansion

→ expand moments of structure functions in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx\; x^{n-2}\; F_2(x, Q^2)$$

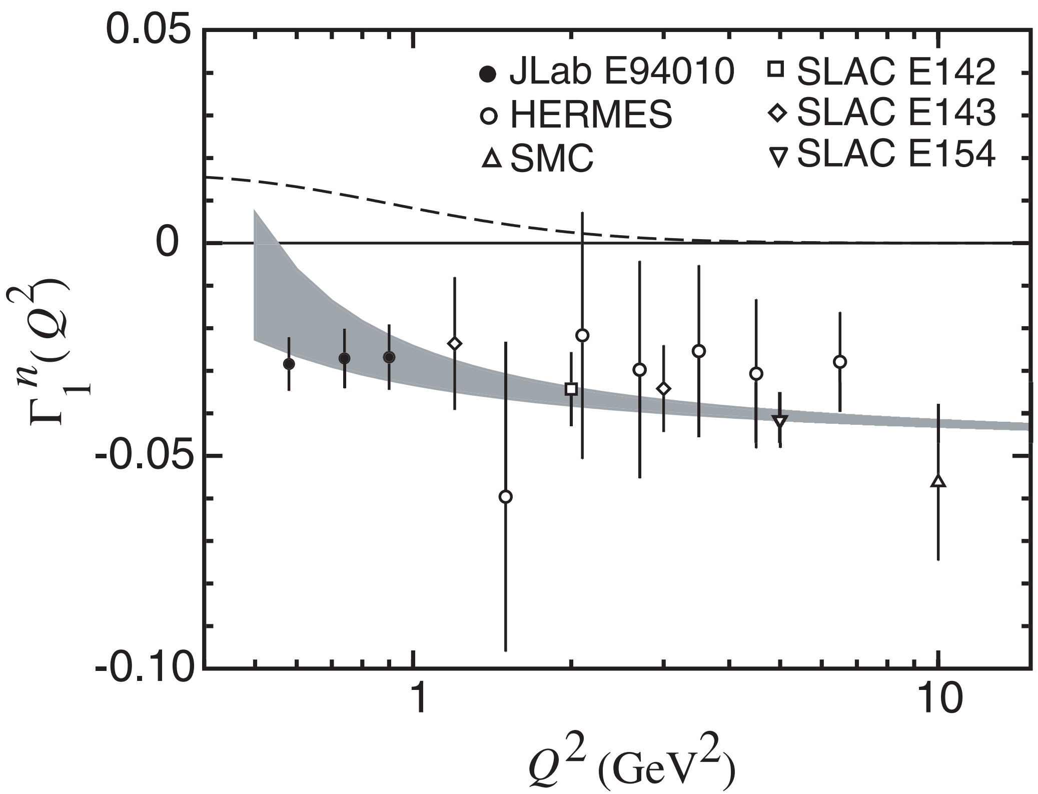
$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

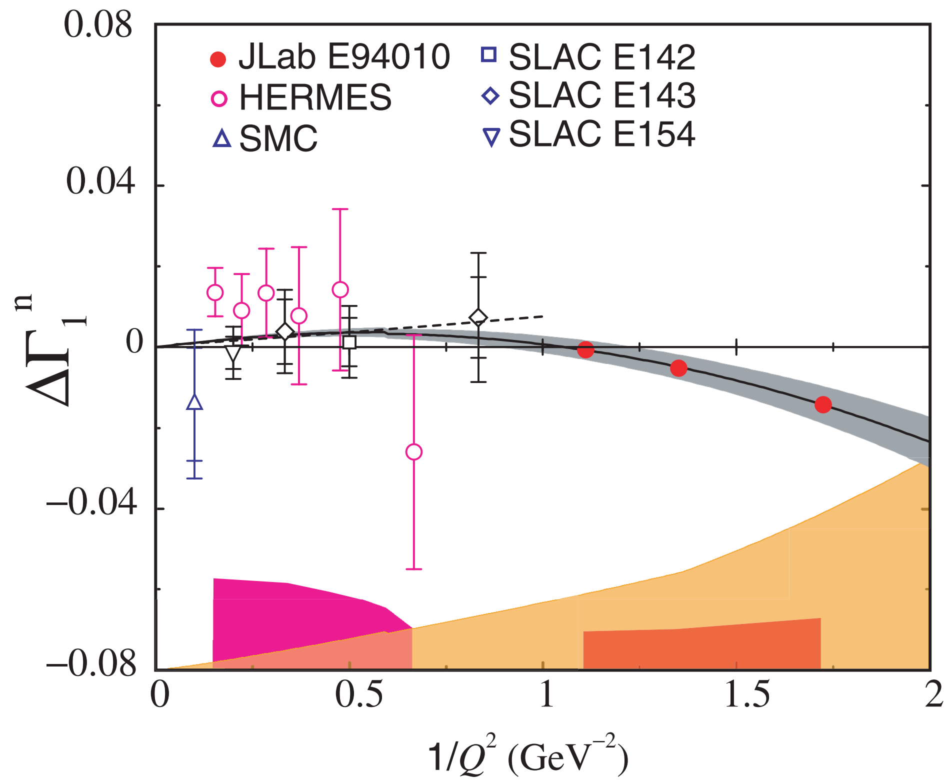
Duality $\Longleftrightarrow$ suppression of higher twists

de Rujula, Georgi, Politzer,
Ann. Phys. 103 (1975) 315

Moment of neutron g_1 structure function

Meziani, WM, et al., Phys. Lett. B613, 148 (2005)



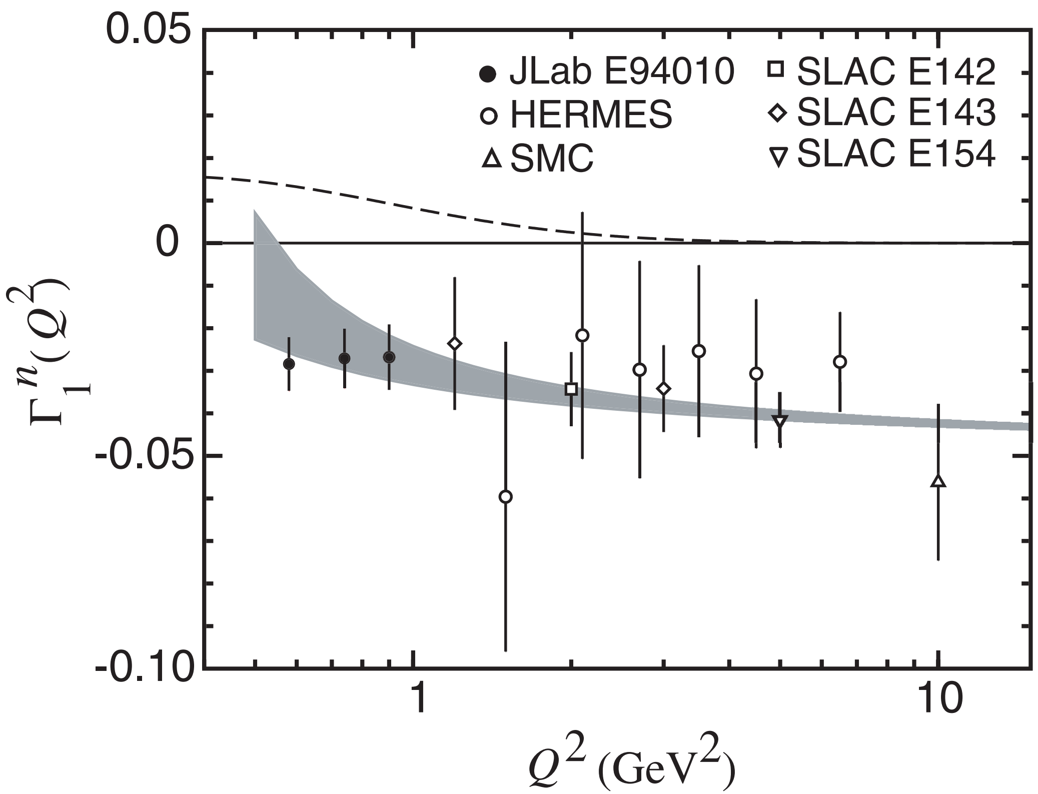


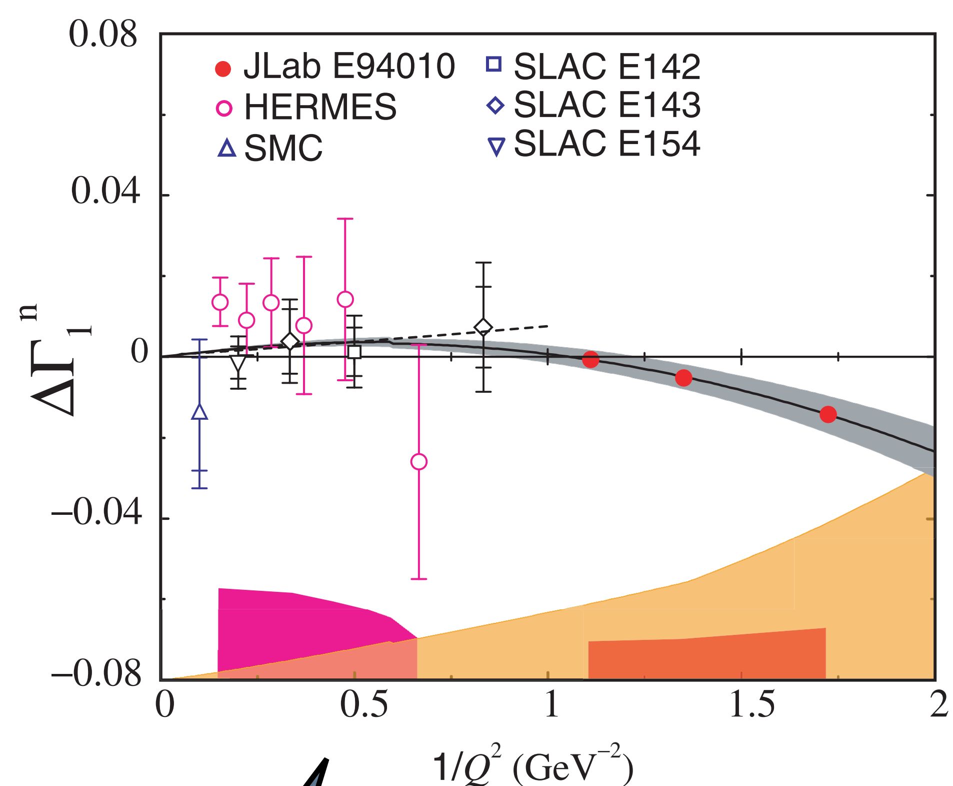
$$\Gamma_1(Q^2) = \int_0^1 dx\; g_1(x, Q^2)$$

$$= \Gamma_1^{(\tau=2)}(Q^2) \;+\; \Delta\Gamma_1(Q^2)$$

Moment of neutron g_1 structure function

Meziani, WM, et al., Phys. Lett. B613, 148 (2005)





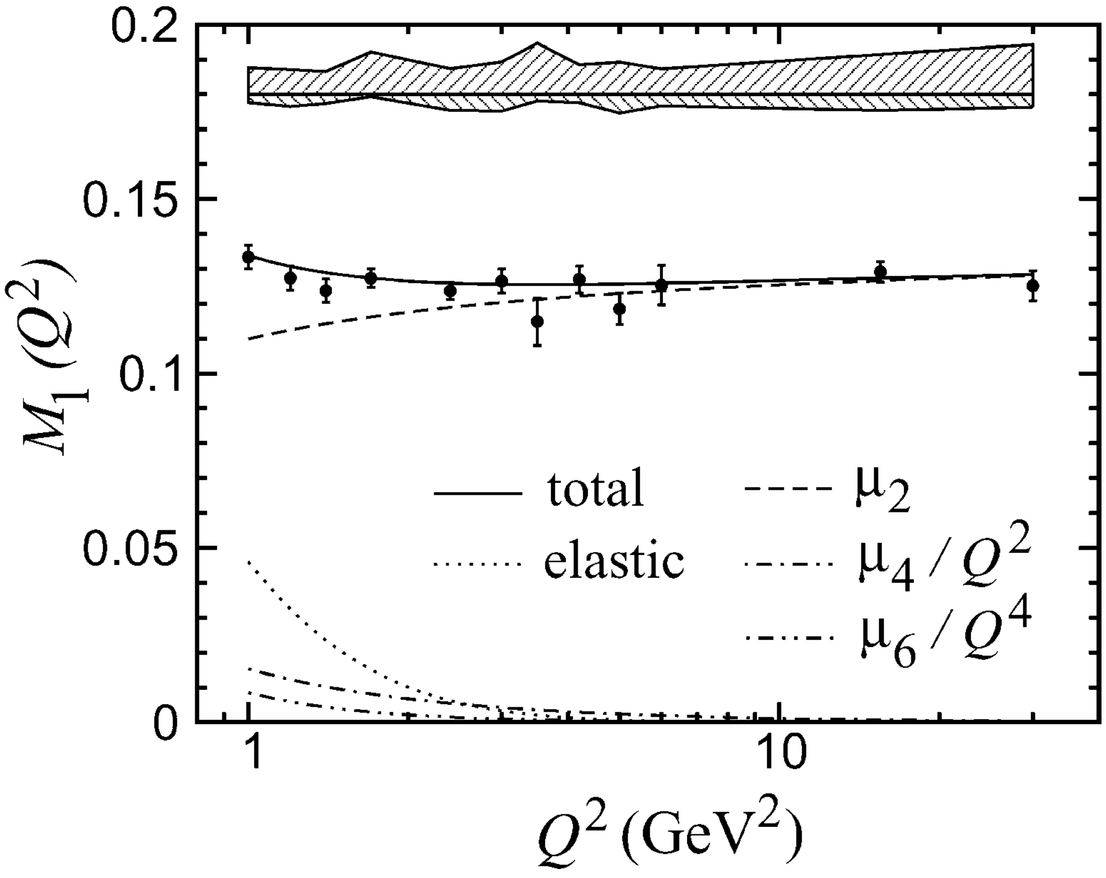
$$\Gamma_1(Q^2) = \int_0^1 dx\ g_1(x, Q^2)$$

$$= \Gamma_1^{(\tau=2)}(Q^2) + \Delta\Gamma_1(Q^2)$$

higher twist small down to $Q^2 \sim 1$ GeV^2

Moment of proton g_1 structure function

Osipenko, WM et al., Phys. Lett. B609, 259 (2005)



⟹ higher twist small down to $Q^2 \sim 2$ GeV^2

Total higher twist $\sim$ *zero* at $Q^2 \sim 1-2\ \mathrm{GeV}^2$

⟹ nonperturbative interactions between quarks and gluons not dominant at these scales

⟹ suggests *strong cancellations* between resonances, resulting in dominance of *leading twist*

⟹ OPE does not tell us *why* higher twists are small !

Can we understand this behavior dynamically?

How do cancellations between *coherent* resonances produce *incoherent* scaling function?

Dynamical quark models

Coherence vs. incoherence

Exclusive form factors

⟹ *coherent* scattering from quarks

$$d\sigma \sim \left(\sum_i e_i\right)^2$$

Inclusive structure functions

⟹ *incoherent* scattering from quarks

$$d\sigma \sim \sum_i e_i^2$$

⟹ How can the *square of a sum* become the *sum of squares*?

Pedagogical model

Two quarks bound in a harmonic oscillator potential

⟹ exactly solvable spectrum

Structure function given by sum of squares of transition form factors

$$F(\nu, \mathbf{q}^2) \ \sim \ \sum_n \left| G_{0,n}(\mathbf{q}^2) \right|^2 \ \delta(E_n - E_0 - \nu)$$

Charge operator $\Sigma_i \ e_i \exp(i\mathbf{q} \cdot \mathbf{r}_i)$ excites

even partial waves with strength $\propto (e_1 + e_2)^2$

odd partial waves with strength $\propto (e_1 - e_2)^2$

Pedagogical model

Resulting structure function

$$F(\nu, \mathbf{q}^2) \;\sim\; \sum_n \left\{ (e_1 + e_2)^2 \; G^2_{0,2n} + (e_1 - e_2)^2 \; G^2_{0,2n+1} \right\}$$

If states degenerate, cross terms $(\sim e_1 e_2)$ cancel when averaged over nearby even and odd parity states

Minimum condition for duality:

⟹ *at least one complete set of <u>even</u> and <u>odd</u> parity resonances must be summed over*

Close, Isgur, Phys. Lett. B509 (2001) 81

Quark model

Even and odd parity states generalize to $\mathbf{56}^+$ (L=0) and $\mathbf{70}^-$ (L=1) multiplets of spin-flavor SU(6)

⟹ scaling occurs if contributions from $\mathbf{56}^+$ and $\mathbf{70}^-$ have equal overall strengths

Simplified case: magnetic coupling of γ^* to quark

⟹ expect dominance over electric at large Q^2

Quark model

Even and odd parity states generalize to **56**$^+$ (L=0) and **70**$^-$ (L=1) multiplets of spin-flavor SU(6)

⟹ scaling occurs if contributions from **56**$^+$ and **70**$^-$ have equal overall strengths

representation	$^2\mathbf{8}[\mathbf{56}^+]$	$^4\mathbf{10}[\mathbf{56}^+]$	$^2\mathbf{8}[\mathbf{70}^-]$	$^4\mathbf{8}[\mathbf{70}^-]$	$^2\mathbf{10}[\mathbf{70}^-]$	Total
F_1^p	$9\rho^2$	$8\lambda^2$	$9\rho^2$	0	λ^2	$18\rho^2+9\lambda^2$
F_1^n	$(3\rho+\lambda)^2/4$	$8\lambda^2$	$(3\rho-\lambda)^2/4$	$4\lambda^2$	λ^2	$(9\rho^2+27\lambda^2)/2$
g_1^p	$9\rho^2$	$-4\lambda^2$	$9\rho^2$	0	λ^2	$18\rho^2-3\lambda^2$
g_1^n	$(3\rho+\lambda)^2/4$	$-4\lambda^2$	$(3\rho-\lambda)^2/4$	$-2\lambda^2$	λ^2	$(9\rho^2-9\lambda^2)/2$

$\lambda\ (\rho) =$ (anti) symmetric component of ground state wfn.

$$|N\rangle = \lambda\ |\varphi \otimes \chi\rangle_{\rm sym} + \rho\ |\varphi \otimes \chi\rangle_{\rm antisym}$$

Quark model

Even and odd parity states generalize to $\mathbf{56}^+$ $(L=0)$ and $\mathbf{70}^-$ $(L=1)$ multiplets of spin-flavor SU(6)

⟹ scaling occurs if contributions from $\mathbf{56}^+$ and $\mathbf{70}^-$ have equal overall strengths

Similarly for neutrinos …

Quark model

Even and odd parity states generalize to $\mathbf{56}^+$ ($L=0$) and $\mathbf{70}^-$ ($L=1$) multiplets of spin-flavor SU(6)

⟹ scaling occurs if contributions from $\mathbf{56}^+$ and $\mathbf{70}^-$ have equal overall strengths

representation	$^2\mathbf{8}[\mathbf{56}^+]$	$^4\mathbf{10}[\mathbf{56}^+]$	$^2\mathbf{8}[\mathbf{70}^-]$	$^4\mathbf{8}[\mathbf{70}^-]$	$^2\mathbf{10}[\mathbf{70}^-]$	Total
$F_1^{\nu p}$	0	$24\lambda^2$	0	0	$3\lambda^2$	$27\lambda^2$
$F_1^{\nu n}$	$(9\rho+\lambda)^2/4$	$8\lambda^2$	$(9\rho-\lambda)^2/4$	$4\lambda^2$	λ^2	$(81\rho^2+27\lambda^2)/2$
$g_1^{\nu p}$	0	$-12\lambda^2$	0	0	$3\lambda^2$	$-9\lambda^2$
$g_1^{\nu n}$	$(9\rho+\lambda)^2/4$	$-4\lambda^2$	$(9\rho-\lambda)^2/4$	$-2\lambda^2$	λ^2	$(81\rho^2-9\lambda^2)/2$

λ (ρ) = (anti) symmetric component of ground state wfn.

Close, WM, Phys. Rev. C68 (2003) 035210

Quark model

SU(6) limit ⟹ $\lambda = \rho$

$SU(6):$	$[\mathbf{56},\mathbf{0}^+]^\mathbf{2}\mathbf{8}$	$[\mathbf{56},\mathbf{0}^+]^\mathbf{4}\mathbf{10}$	$[\mathbf{70},\mathbf{1}^-]^\mathbf{2}\mathbf{8}$	$[\mathbf{70},\mathbf{1}^-]^\mathbf{4}\mathbf{8}$	$[\mathbf{70},\mathbf{1}^-]^\mathbf{2}\mathbf{10}$	*total*
F_1^p	9	8	9	0	1	27
F_1^n	4	8	1	4	1	18
g_1^p	9	-4	9	0	1	15
g_1^n	4	-4	1	-2	1	0

Summing over all resonances in $\mathbf{56}^+$ and $\mathbf{70}^-$ multiplets

⟹ $R^{np} = \frac{F_1^n}{F_1^p} = \frac{2}{3}$ $\quad A_1^p = \frac{g_1^p}{F_1^p} = \frac{5}{9}$ $\quad A_1^n = \frac{g_1^n}{F_1^n} = 0$

⟹ as in quark-parton model !

Quark model

SU(6) limit ⟹ $\lambda = \rho$

$SU(6):$	$[\mathbf{56},0^+]^{\mathbf{2}}\mathbf{8}$	$[\mathbf{56},0^+]^{\mathbf{4}}\mathbf{10}$	$[\mathbf{70},1^-]^{\mathbf{2}}\mathbf{8}$	$[\mathbf{70},1^-]^{\mathbf{4}}\mathbf{8}$	$[\mathbf{70},1^-]^{\mathbf{2}}\mathbf{10}$	*total*
F_1^p	9	8	9	0	1	27
F_1^n	4	8	1	4	1	18
g_1^p	9	-4	9	0	1	15
g_1^n	4	-4	1	-2	1	0

⟹ expect duality to appear earlier for F_1^p than F_1^n

⟹ earlier onset for g_1^n than g_1^p

⟹ cancellations *within* multiplets for g_1^n

Quark model

Similarly for neutrinos ... SU(6) limit $(\lambda = \rho)$

$SU(6):$	$[\mathbf{56},\mathbf{0}^+]^{\mathbf{2}}\mathbf{8}$	$[\mathbf{56},\mathbf{0}^+]^{\mathbf{4}}\mathbf{10}$	$[\mathbf{70},\mathbf{1}^-]^{\mathbf{2}}\mathbf{8}$	$[\mathbf{70},\mathbf{1}^-]^{\mathbf{4}}\mathbf{8}$	$[\mathbf{70},\mathbf{1}^-]^{\mathbf{2}}\mathbf{10}$	$total$
$F_1^{\nu p}$	0	24	0	0	3	27
$F_1^{\nu n}$	25	8	16	4	1	54
$g_1^{\nu p}$	0	−12	0	0	3	−9
$g_1^{\nu n}$	25	−4	16	−2	1	36

Quark model

Similarly for neutrinos ... SU(6) limit $(\lambda = \rho)$

$SU(6):$	$[\mathbf{56},\mathbf{0}^+]^2\mathbf{8}$	$[\mathbf{56},\mathbf{0}^+]^4\mathbf{10}$	$[\mathbf{70},\mathbf{1}^-]^2\mathbf{8}$	$[\mathbf{70},\mathbf{1}^-]^4\mathbf{8}$	$[\mathbf{70},\mathbf{1}^-]^2\mathbf{10}$	*total*
$F_1^{\nu p}$	0	24	0	0	3	27
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$g_1^{\nu p}$	0	-12	0	0	3	-9
$g_1^{\nu n}$	25	-4	16	-2	1	36

Summing over all resonances in $\mathbf{56}^+$ and $\mathbf{70}^-$ multiplets

$$\Longrightarrow \quad R^\nu = \frac{F_1^{\nu p}}{F_1^{\nu n}} = \frac{1}{2} \left(= \frac{d}{u} \right)$$

$$A_1^{\nu p} = -\frac{1}{3} \left(= \frac{\Delta d}{d} \right)$$

$$A_1^{\nu n} = \frac{2}{3} \left(= \frac{\Delta u}{u} \right)$$

$\Longrightarrow$ as in parton model !

Quark model

SU(6) may be $\approx$ valid at $x \sim 1/3$

But significant deviations at large x

→ which combinations of resonances reproduce behavior of structure functions at large x?

Model	SU(6)	No $^4\mathbf{10}$	No $^2\mathbf{10}$, $^4\mathbf{10}$	No $S_{3/2}$	No $\sigma_{3/2}$	No ψ_λ
R^{np}	2/3	10/19	1/2	6/19	3/7	1/4
A_1^p	5/9	1	1	1	1	1
A_1^n	0	2/5	1/3	1	1	1

gives $\Delta u/u > 1$ → *inconsistent with duality*

Quark model

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R^{np}	2/3	10/19	1/2	6/19	3/7	1/4
A_1^p	5/9	1	1	1	1	1
A_1^n	0	2/5	1/3	1	1	1

$^4\mathbf{10}\ [\mathbf{56}^+]$ and $^4\mathbf{8}\ [\mathbf{70}^-]$ suppressed

Quark model

SU(6) may be ≈ valid at $x \sim 1/3$

But significant deviations at large x

→ which combinations of resonances reproduce behavior of structure functions at large x?

Model	SU(6)	No $^4\mathbf{10}$	No $^2\mathbf{10}$, $^4\mathbf{10}$	No $S_{3/2}$	No $\sigma_{3/2}$	No ψ_λ
R^{np}	2/3	10/19	1/2	6/19	3/7	1/4
A_1^p	5/9	1	1	1	1	1
A_1^n	0	2/5	1/3	1	1	1

helicity 3/2 suppression

$N \to N^*$ transitions for helicity-1/2 dominance

SU(6) representation	$^2\mathbf{8}[\mathbf{56}^+]$	$^4\mathbf{10}[\mathbf{56}^+]$	$^2\mathbf{8}[\mathbf{70}^-]$	$^4\mathbf{8}[\mathbf{70}^-]$	$^2\mathbf{10}[\mathbf{70}^-]$	Total
$F_1^p = g_1^p$	9	2	9	0	1	21
$F_1^n = g_1^n$	4	2	1	1	1	9

polarization asymmetries $A_1^N \to 1$

⟹ cf. pQCD "counting rules"

⟹ hard gluon exchange between quarks

neutron to proton ratio $F_2^n / F_2^p \to 3/7$

⟹ cf. "helicity retention" model

Farrar, Jackson, Phys. Rev. Lett. 35 (1975) 1416

Quark model

SU(6) may be $\approx$ valid at $x \sim 1/3$

But significant deviations at large x

$\Longrightarrow$ which combinations of resonances reproduce behavior of structure functions at large x?

Model	SU(6)	No $^4\mathbf{10}$	No $^2\mathbf{10}$, $^4\mathbf{10}$	No $S_{3/2}$	No $\sigma_{3/2}$	No ψ_λ
R^{np}	2/3	10/19	1/2	6/19	3/7	1/4
A_1^p	5/9	1	1	1	1	1
A_1^n	0	2/5	1/3	1	1	1

e.g. through $\vec{S}_i \cdot \vec{S}_j$ interaction between quarks $\Longleftarrow$ suppression of symmetric part of spin-flavor wfn.

Quark model

SU(6) may be $\approx$ valid at $x \sim 1/3$

But significant deviations at large x

→ which combinations of resonances reproduce behavior of structure functions at large x?

Model	SU(6)	No $^4\mathbf{10}$	No $^2\mathbf{10}$, $^4\mathbf{10}$	No $S_{3/2}$	No $\sigma_{3/2}$	No ψ_λ
R^ν	1/2	3/46	0	1/14	1/5	0
$A_1^{\nu p}$	−1/3	1		1		−1/3
$A_1^{\nu n}$	2/3	20/23	13/15	1	1	1

gives d/u, $\Delta u/u$, $\Delta d/d$ inconsistent with e scattering

Quark model

SU(6) may be $\approx$ valid at $x \sim 1/3$

But significant deviations at large x

→ which combinations of resonances reproduce behavior of structure functions at large x?

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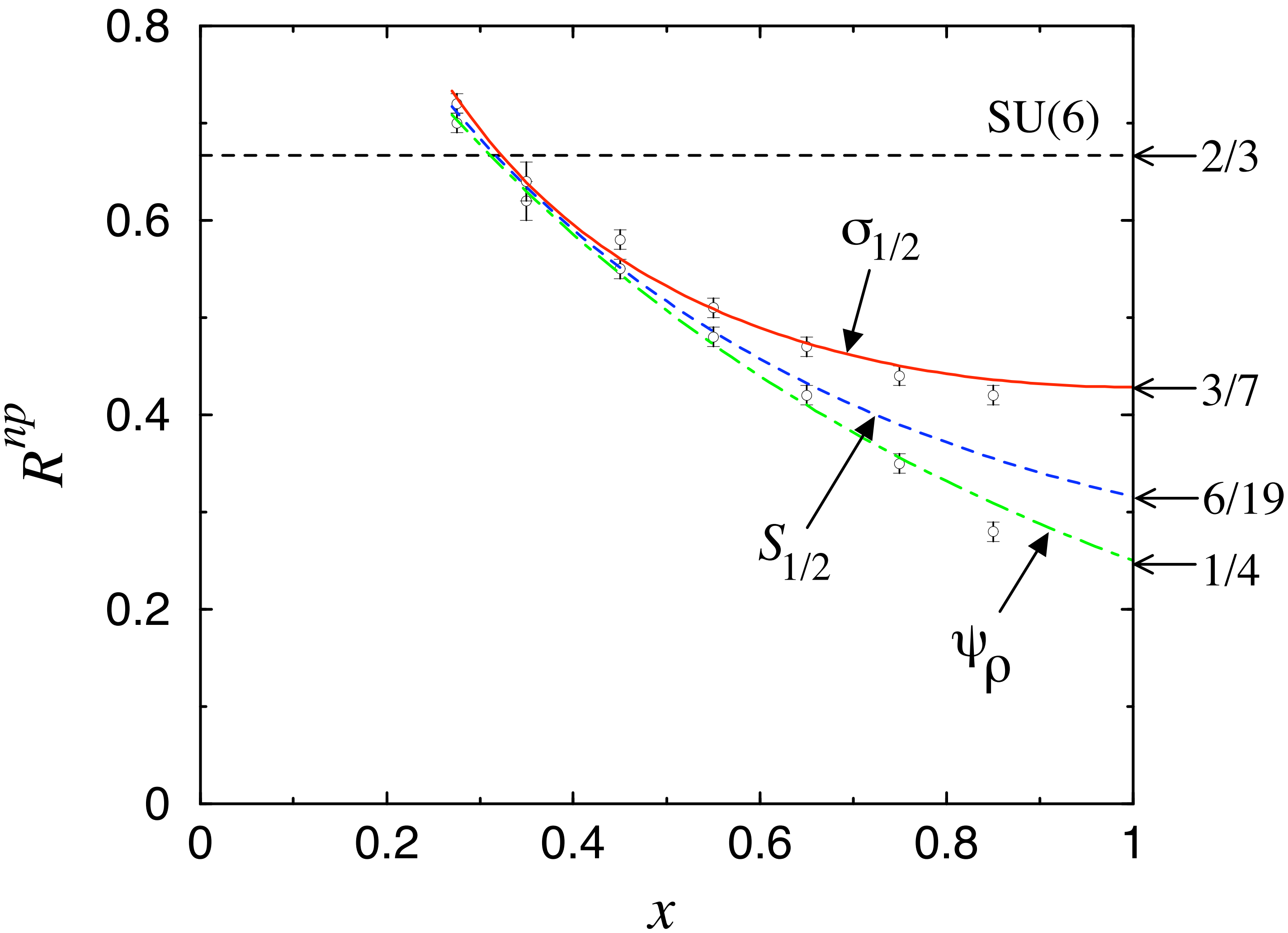
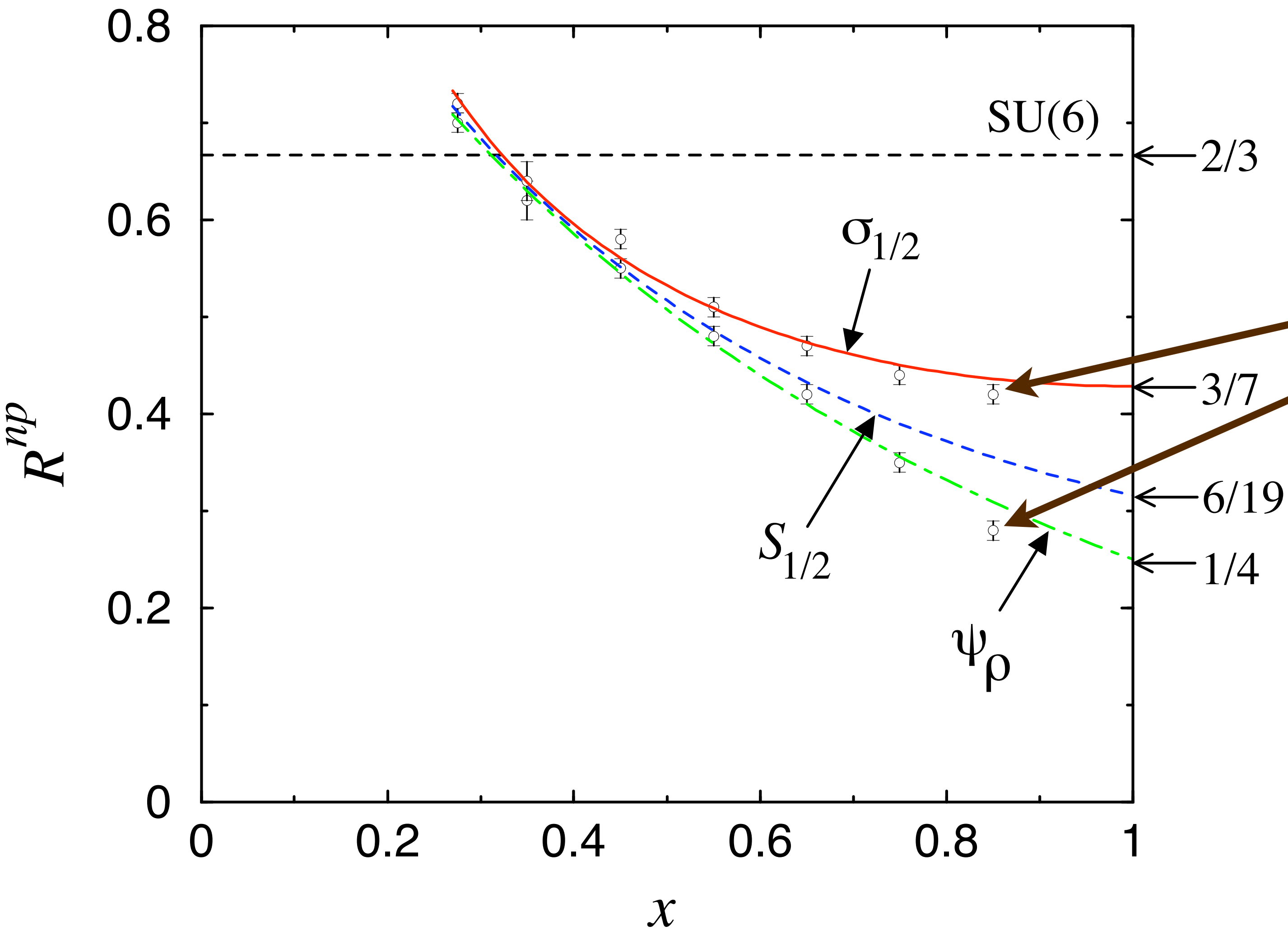
$A_1^{\nu p}$	−1/3	1		1		−1/3
$A_1^{\nu n}$	2/3	20/23	13/15	1	1	1

consistent with duality in e scattering

Fit to { SU(6) symmetry at $x \sim 1/3$; SU(6) breaking at $x \sim 1$ }

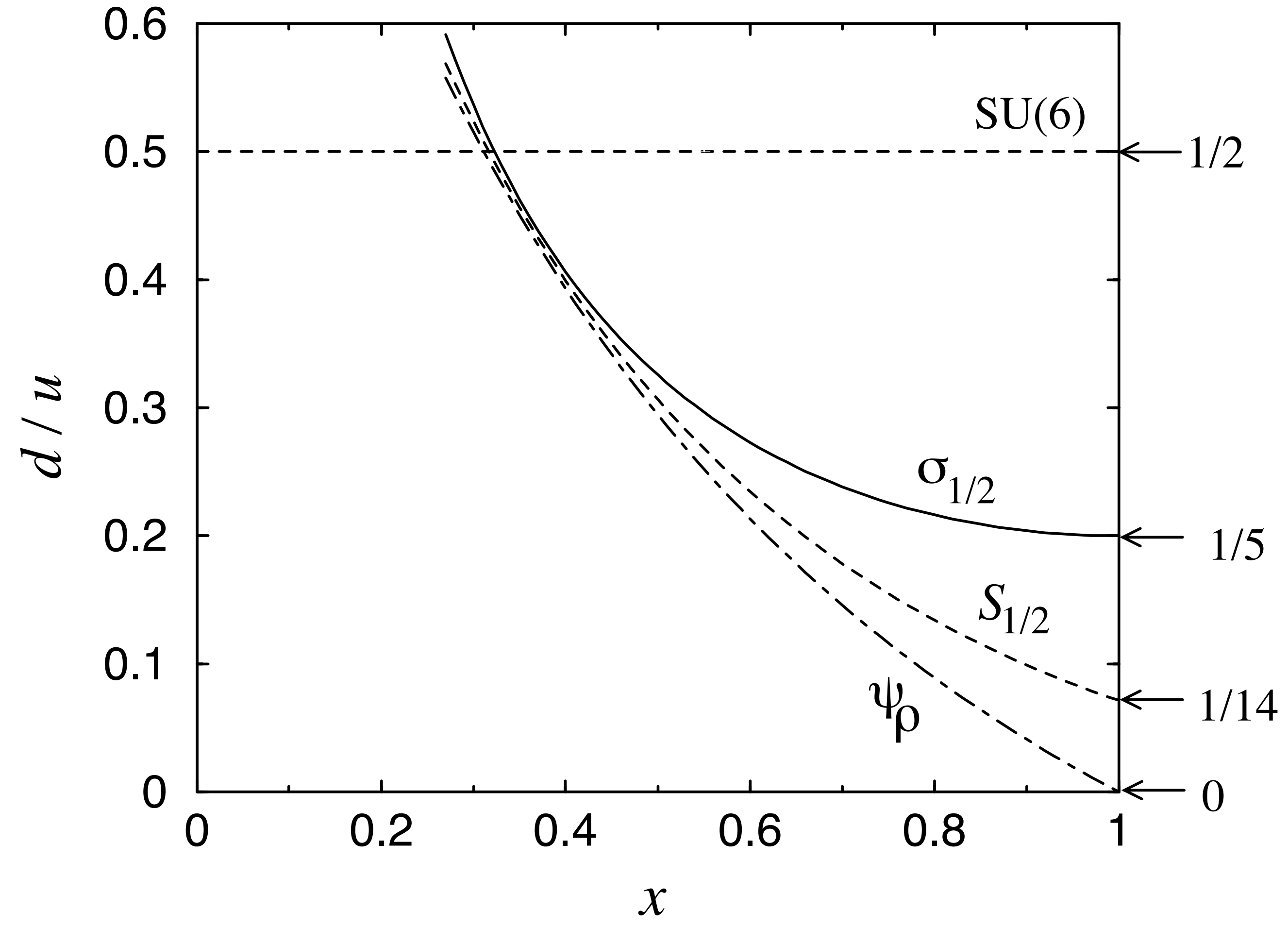


Fit to { SU(6) symmetry at $x \sim 1/3$
SU(6) breaking at $x \sim 1$

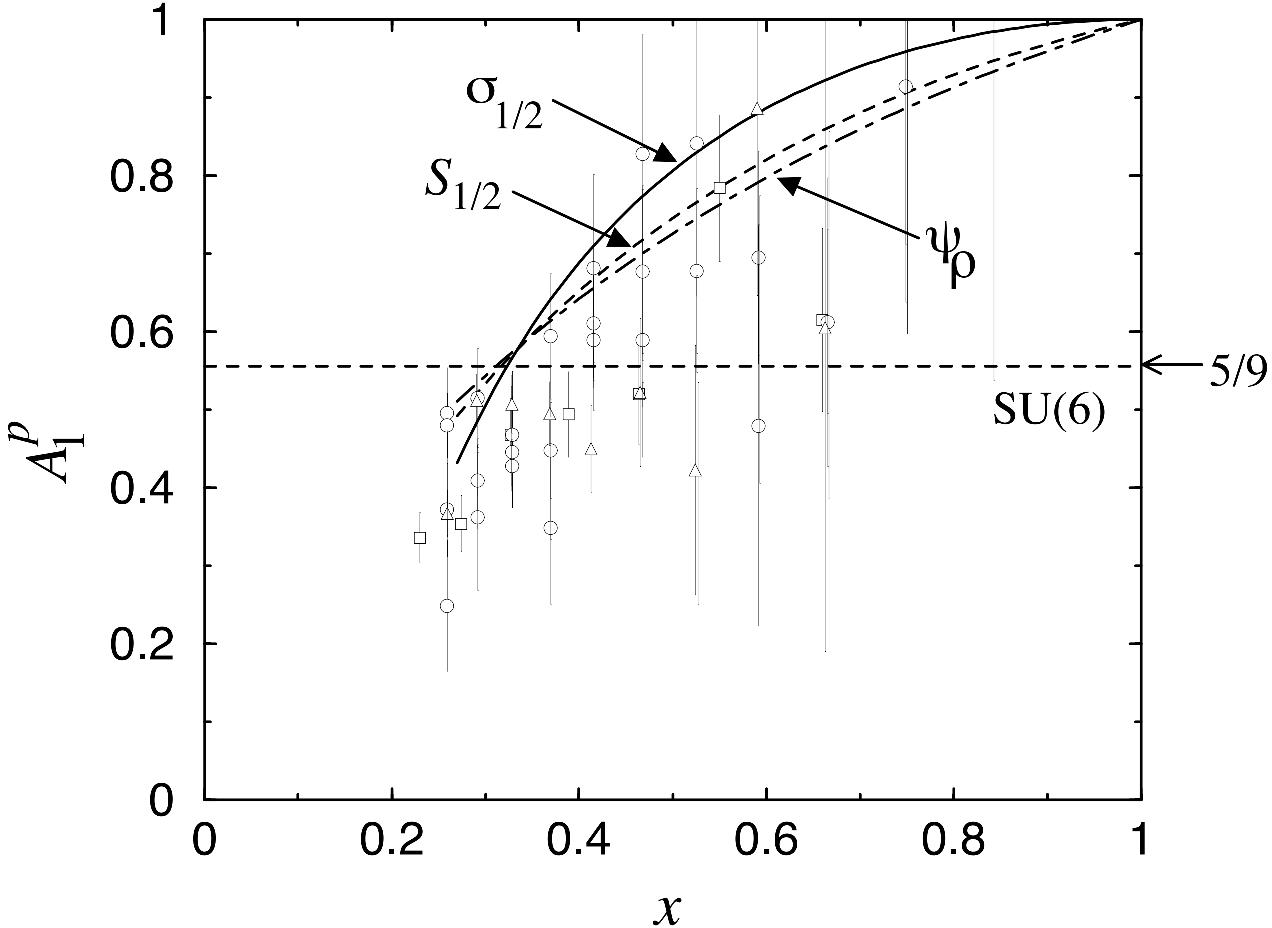


uncertainty in F_2^n due to nuclear effects in deuteron

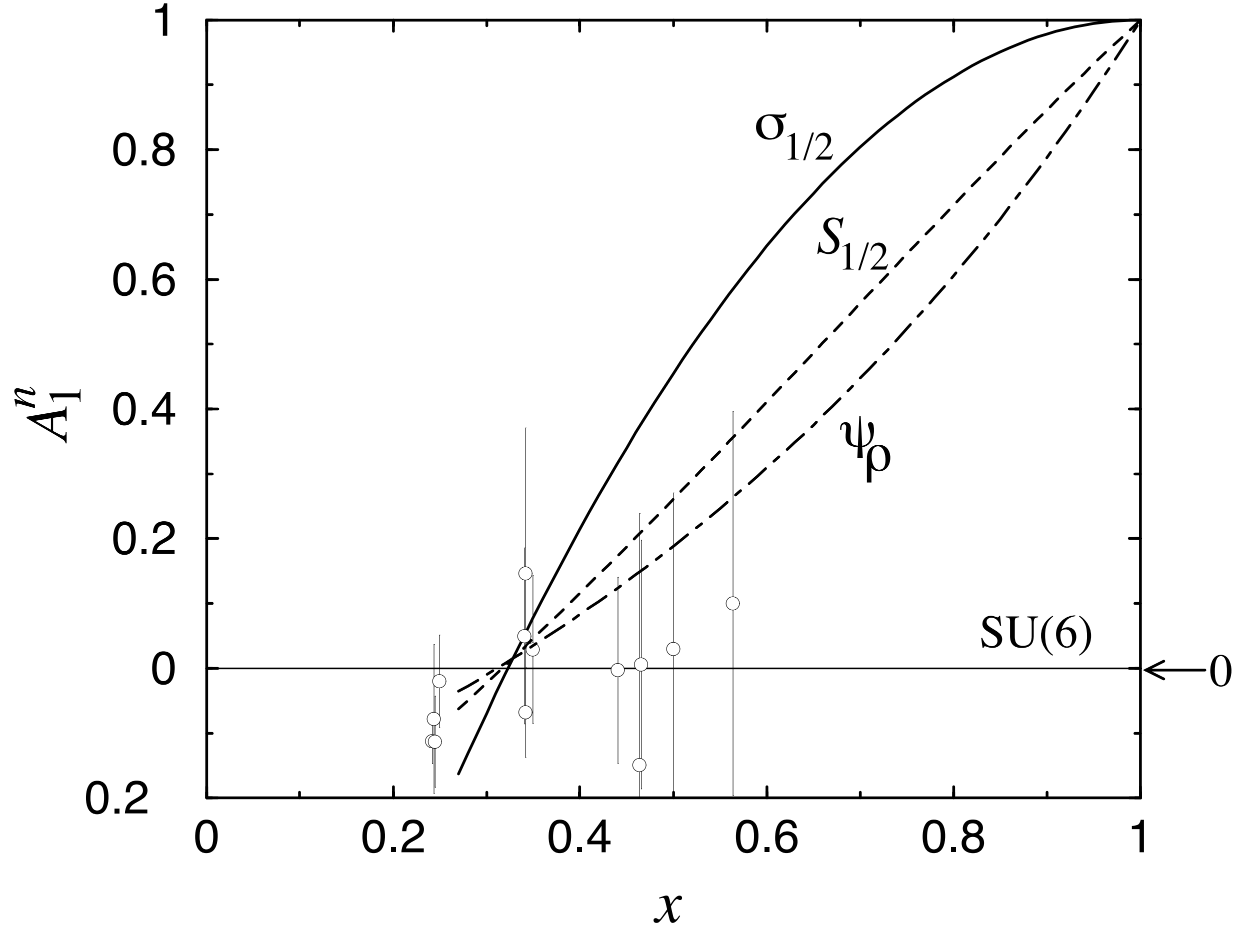
R^ν $(= d/u)$



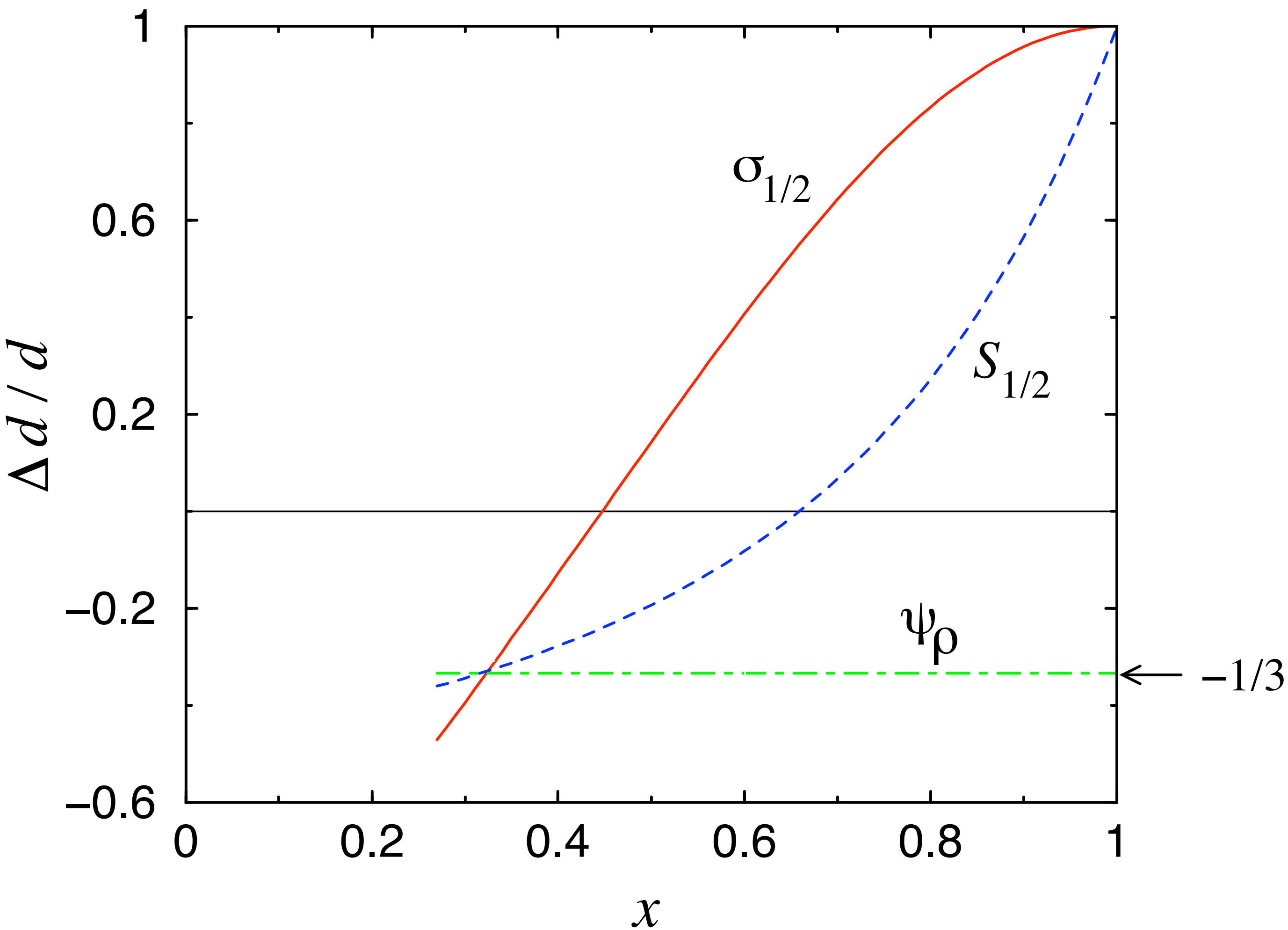
Polarization asymmetry A_1^p



Polarization asymmetry A_1^n

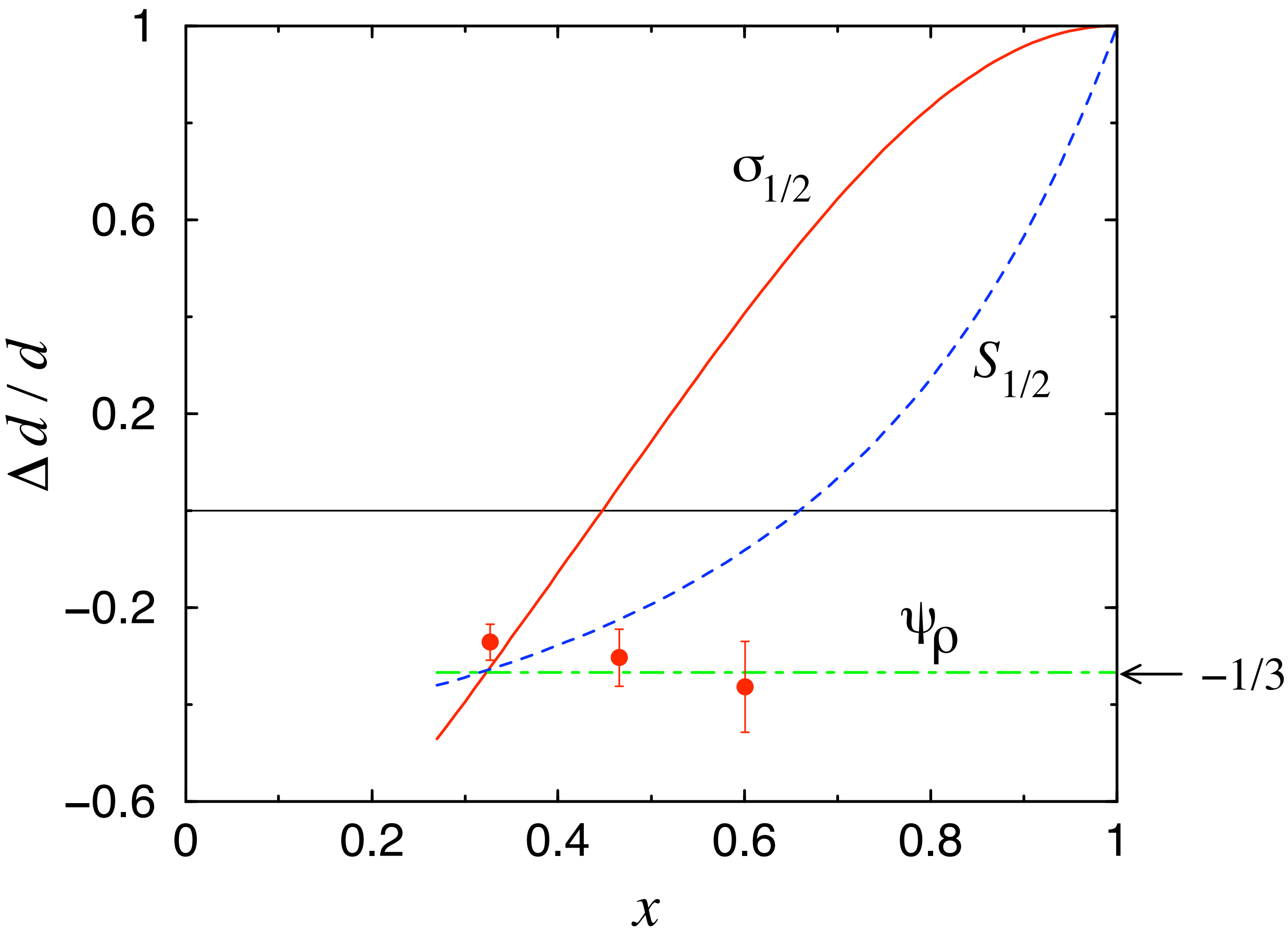


$$\frac{\Delta d}{d} = \frac{4}{15} A_1^n \left(4 + \frac{u}{d}\right) - \frac{1}{15} A_1^p \left(1 + 4\frac{u}{d}\right)$$



$$\frac{u}{d} = \frac{4 - R^{np}}{4R^{np} - 1}$$

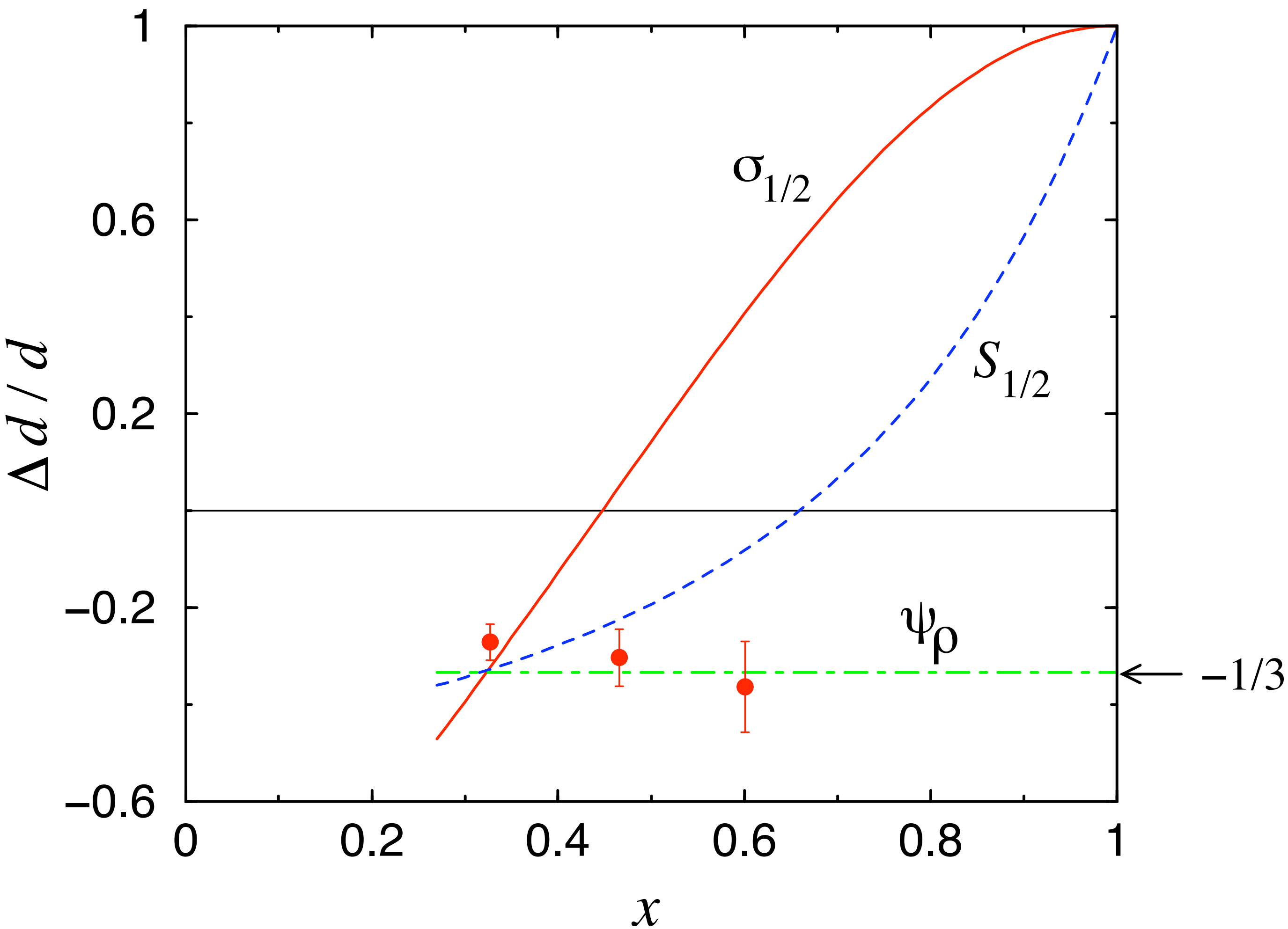
$$\frac{\Delta d}{d} = \frac{4}{15} A_1^n \left(4 + \frac{u}{d}\right) - \frac{1}{15} A_1^p \left(1 + 4\frac{u}{d}\right)$$



$$\frac{u}{d} = \frac{4 - R^{np}}{4R^{np} - 1}$$

Zheng et al. [JLab Hall A], Phys. Rev. Lett. (2004) 012004

$$\frac{\Delta d}{d} = \frac{4}{15} A_1^n \left(4 + \frac{u}{d}\right) - \frac{1}{15} A_1^p \left(1 + 4\frac{u}{d}\right) \quad (= A_1^{\nu p})$$



$$\frac{u}{d} = \frac{4 - R^{np}}{4R^{np} - 1}$$

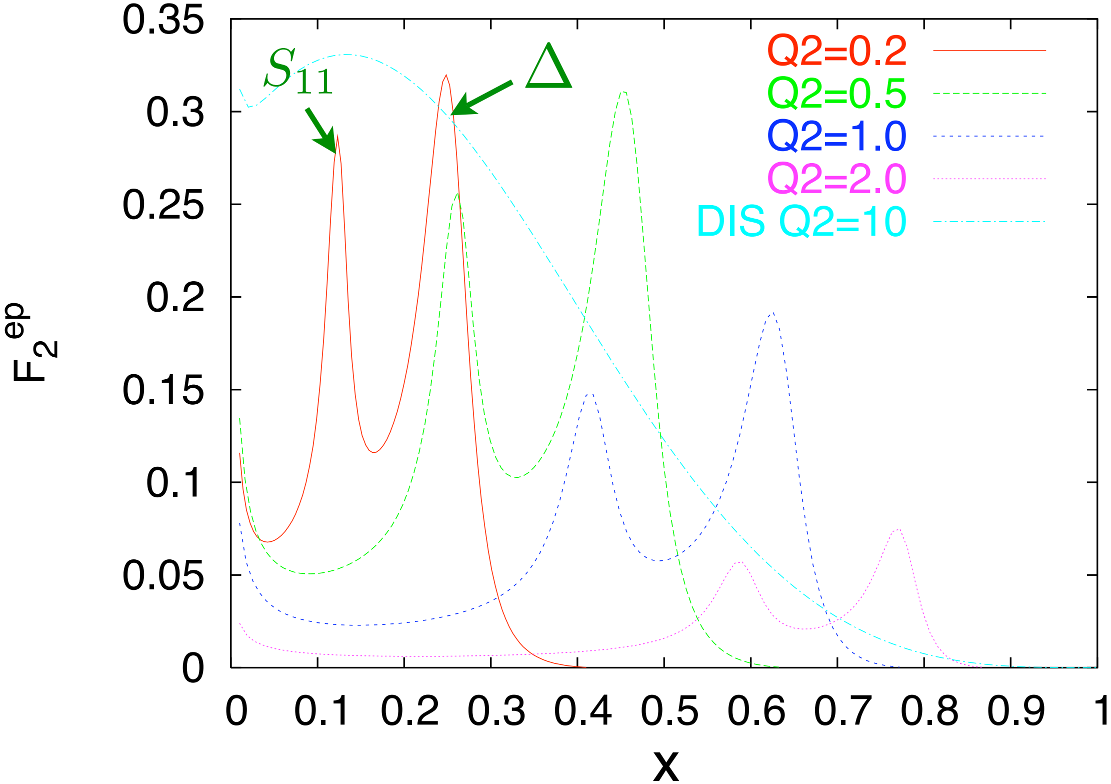
$$(= 1/R^{\nu})$$

Zheng et al. [JLab Hall A], Phys. Rev. Lett. (2004) 012004

Phenomenological models

Phenomenological model

Construct structure function from phenomenological $N \rightarrow N^*$ transition form factors



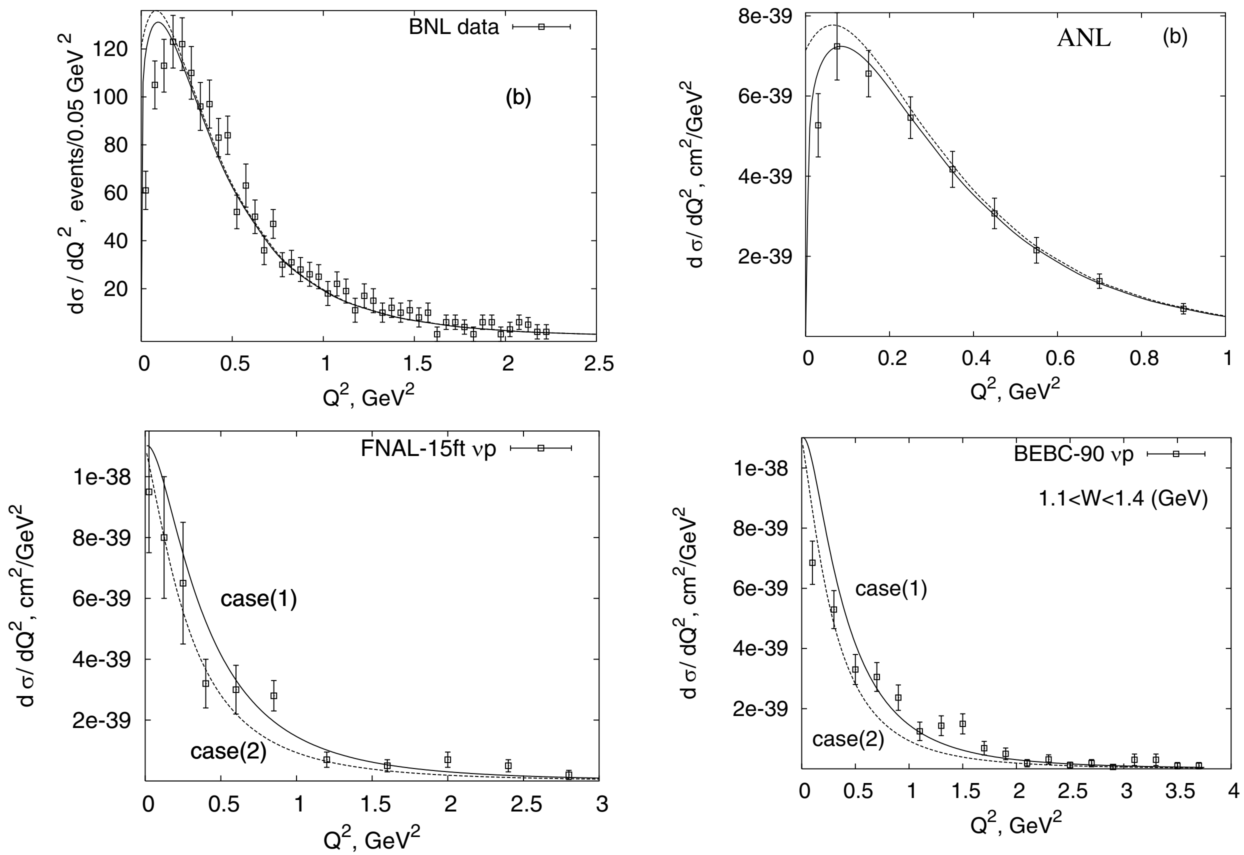
Lalakulich,WM, Paschos (2005)

Resonance widths

$$\delta(W^2 - M_R^2) \longrightarrow \frac{M_R \Gamma_R}{\pi} \frac{1}{(W^2 - M_R^2)^2 + M_R^2 \Gamma_R^2}$$

Neutrino structure functions

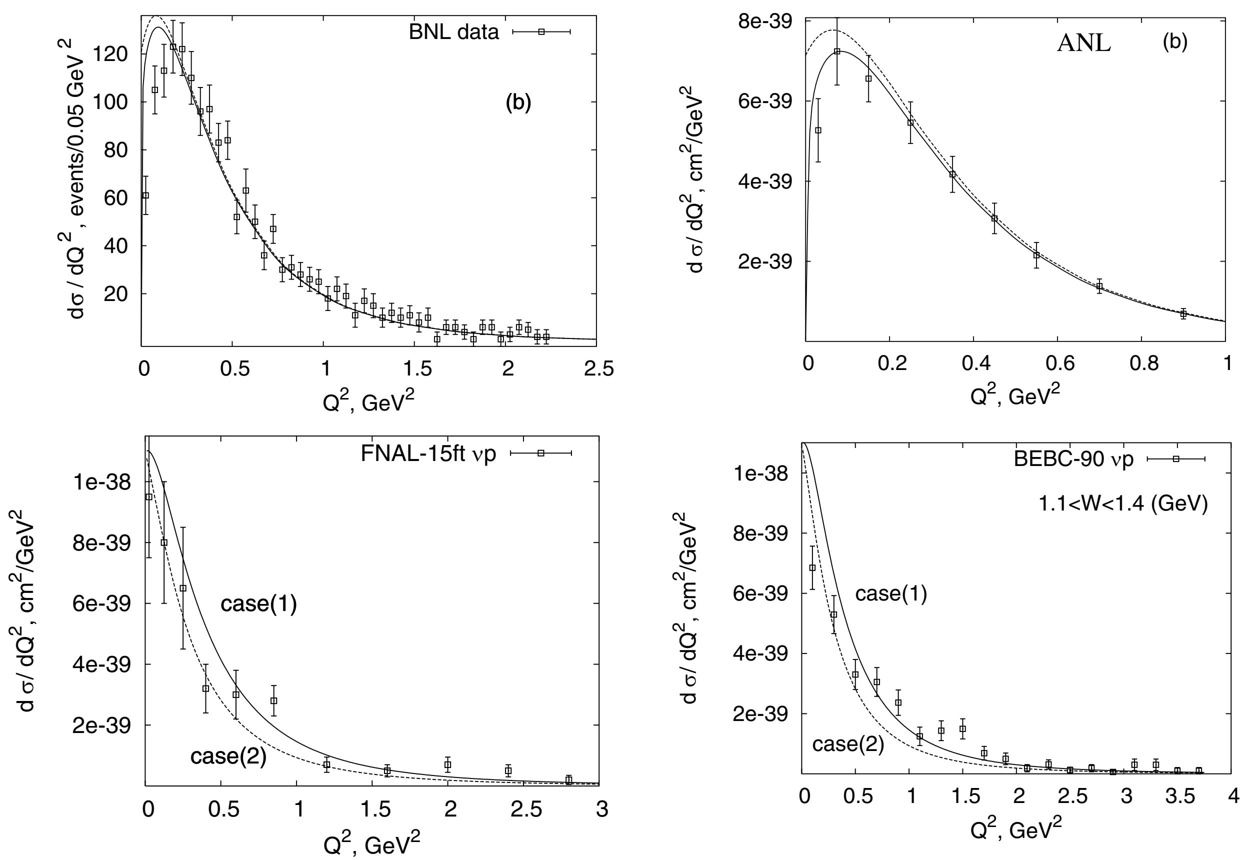
Neutrino form factors fitted to neutrino cross section data from BNL, ANL, BEBC, FNAL ... more to come with MINERνA



Lalakulich, Paschos,
Phys. Rev. D71 (2005) 074003

Neutrino structure functions

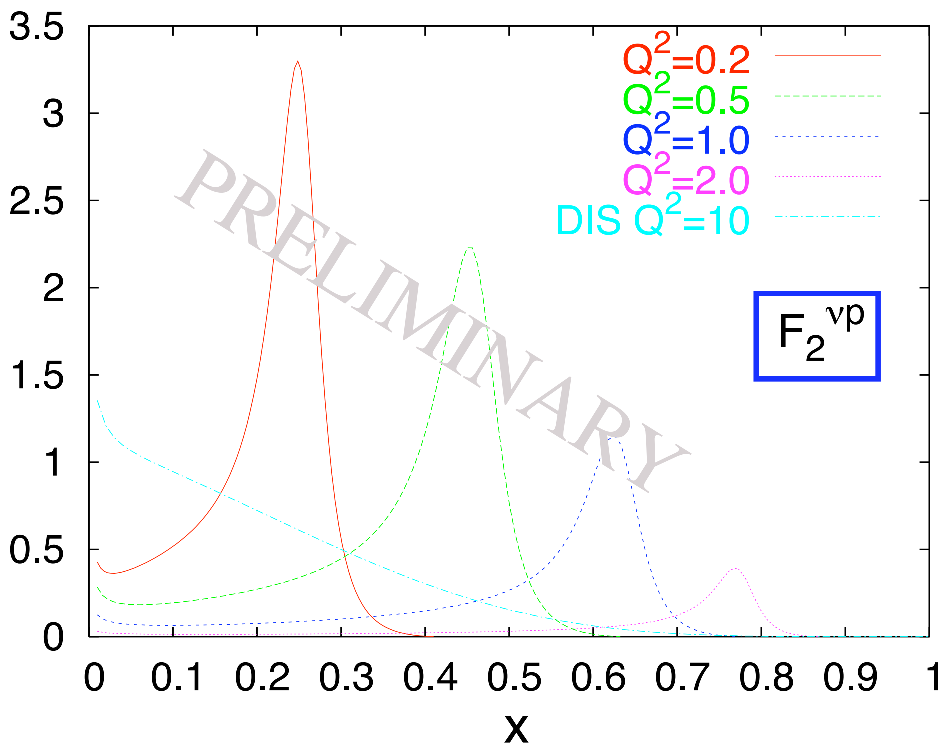
Neutrino form factors fitted to neutrino cross section data from BNL, ANL, BEBC, FNAL ... more to come with MINERνA

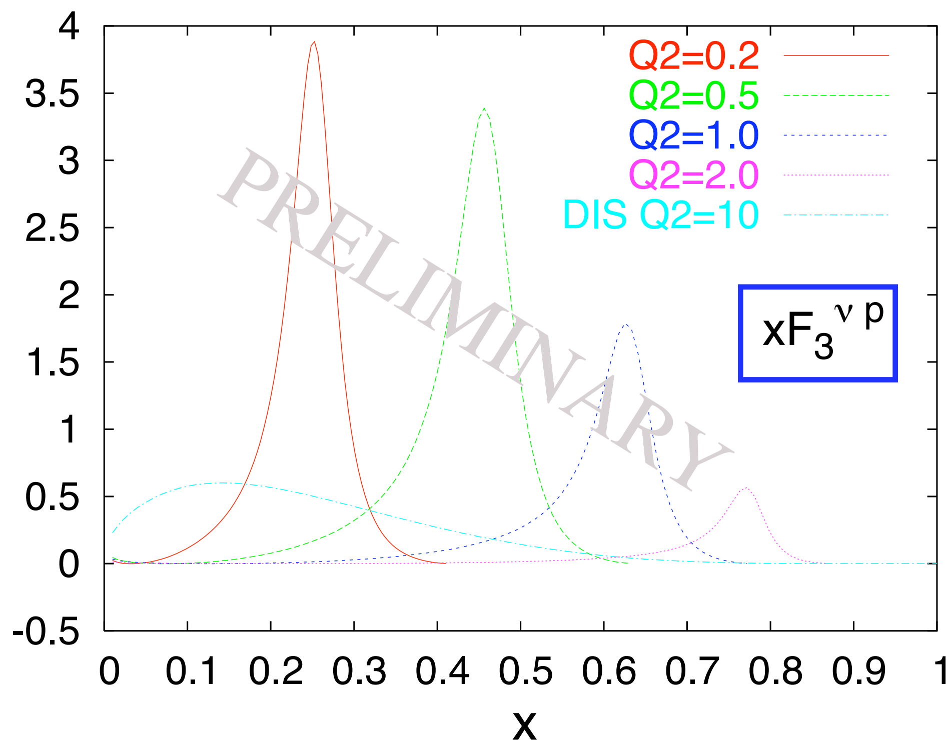


Lalakulich, Paschos,
Phys. Rev. D71 (2005) 074003

→ important for neutrino oscillation experiments

Neutrino structure functions





Lalakulich,WM, Paschos (in progress)

→ Important to understand systematics of duality in ν scattering cf. e scattering

Summary

- Remarkable confirmation of quark-hadron duality in structure functions
  $\longrightarrow$ higher twists "small" down to low $Q^2 (\sim 1 \text{ GeV}^2)$
- Quark models provide clues to origin of resonance cancellations $\longrightarrow$ local duality
- Practical applications
  $\longrightarrow$ broaden kinematic region for studying (leading and higher twist) quark-gluon structure of nucleon
  $\longrightarrow$ understanding duality in ν scattering important for interpretation of oscillation experiments