Lattice QCD and Baryon Spectroscopy

Huey-Wen Lin

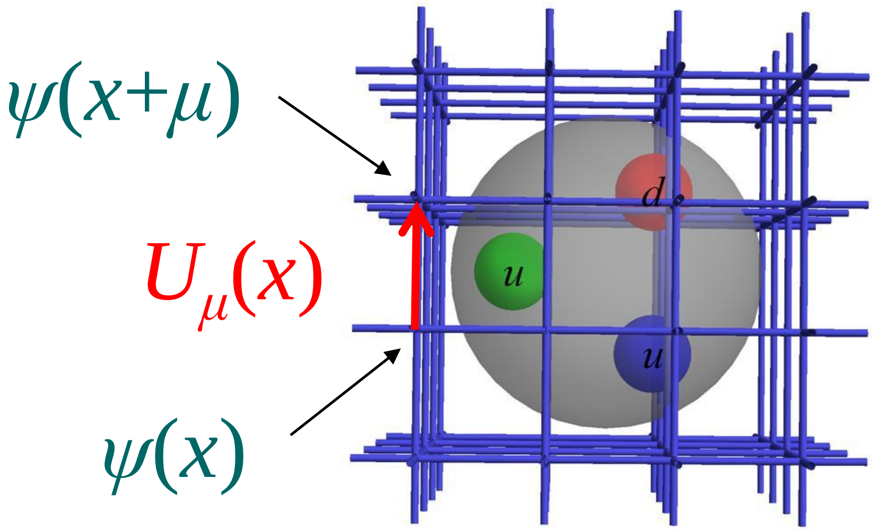
National Taiwan University
2007 Dec. 14

Outline

- Lattice QCD
  - Background, actions, observables, …
- Methodology
  - Group theory and operator design
  - Variational method
  - Ensembles, parameter and analysis
- Numerical Results
  - Octet and decuplet
  - Other ±-parity, spin-1/2 and 3/2 states
  - Roper from full QCD
- Conclusions and Outlook

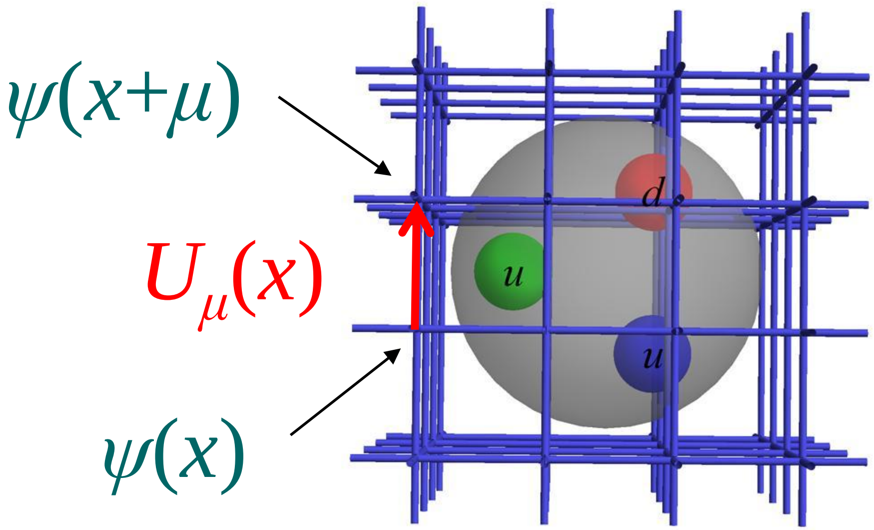
Lattice QCD

- Lattice QCD is a discrete version of continuum QCD theory



Lattice QCD

- Lattice QCD is a discrete version of continuum QCD theory



- Physical observables are calculated from the path integral

$$\langle 0|O(\overline{\psi},\psi,A)|0\rangle = \frac{1}{Z}\int [dA][d\overline{\psi}][d\psi]\, O(\overline{\psi},\psi,A) e^{i\int d^4x\, \mathcal{L}^{\rm QCD}(\overline{\psi},\psi,A)}$$

- Use Monte Carlo integration combined with the "importance sampling" technique to calculate the path integral.
- Take $a \to 0$ and $V \to \infty$ in the continuum limit

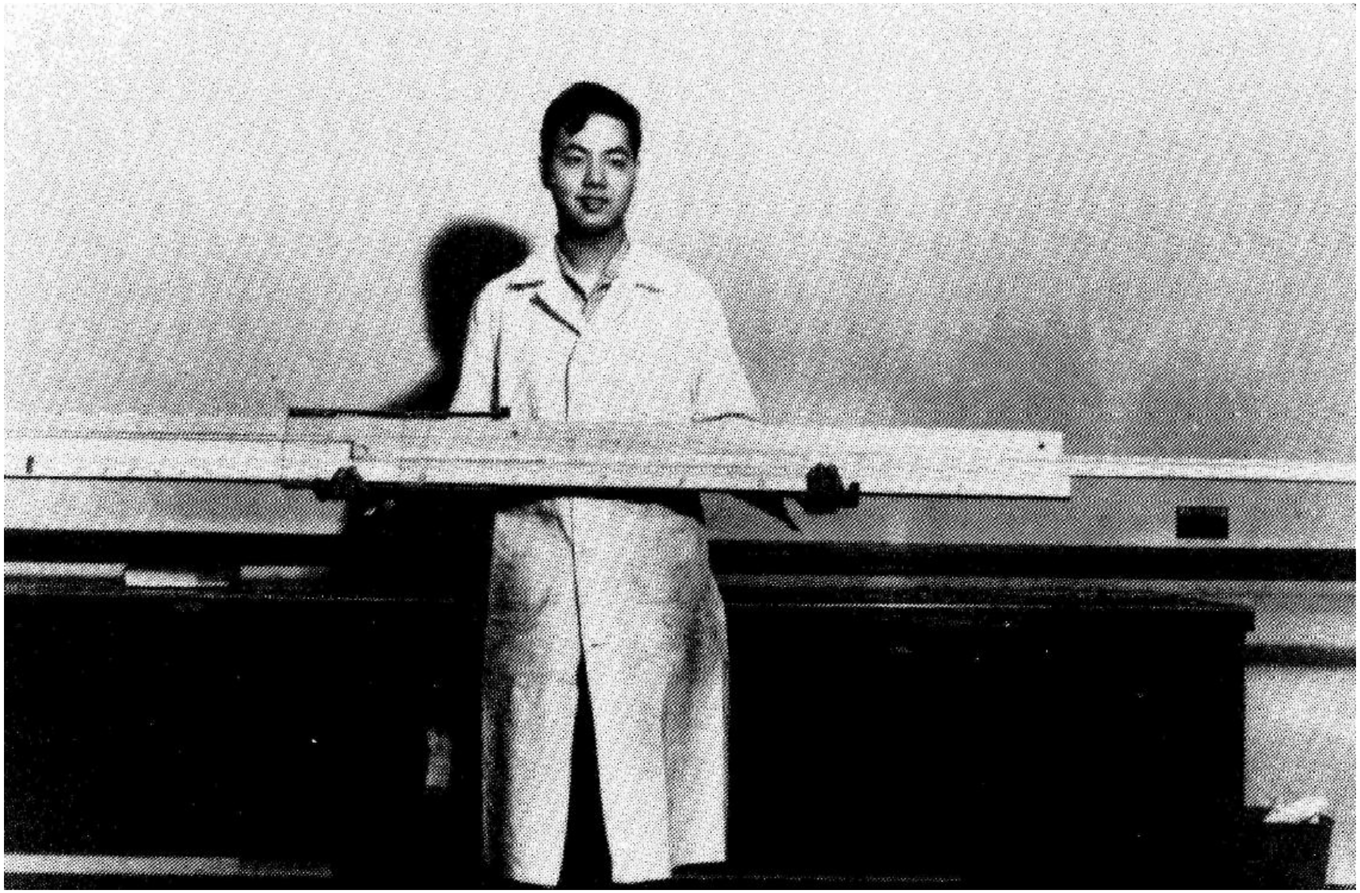
Lattice QCD

- A wide variety of first-principles QCD calculations can be done: In 1970, Wilson wrote down the original lattice action
- Progress is limited by computational resources...

  but assisted by advances in algorithms.

Lattice QCD

T.D. Lee uses an “analog computer” to calculate stellar radiative transfer equations



Lattice QCD

2007: The 13 Tflops cluster at Jefferson Lab

Other joint lattice resources within the US: Fermilab, BNL.

Non-lattice resources open to USQCD: ORNL, LLNL, ANL.

Lattice QCD

- Lattice QCD is computationally intensive

$$\text{Cost} \approx \left(\frac{L}{\text{fm}}\right)^5 L_s \left(\frac{\text{MeV}}{M_\pi}\right)\left(\frac{\text{fm}}{a}\right)^6 \left(C_0 + C_1\left(\frac{\text{fm}}{a}\right)\left(\frac{\text{MeV}}{M_K}\right)^2 + C_2\left(\frac{a}{\text{fm}}\right)^2\left(\frac{\text{MeV}}{M_\pi}\right)^2\right)$$

Norman Christ, LAT2007

- Current major US 2+1-flavor gauge ensemble generation:
  - MILC: staggered, $a \sim 0.06$ fm, $L \sim 3$ fm, $M_\pi \sim 250$ MeV
  - RBC+UKQCD: DWF, $a \sim 0.09$ fm, $L \sim 3$fm, $M_\pi \sim 330$ MeV
- Chiral domain-wall fermions (DWF) at large volume (6 fm) at physical pion mass may be expected in 2011

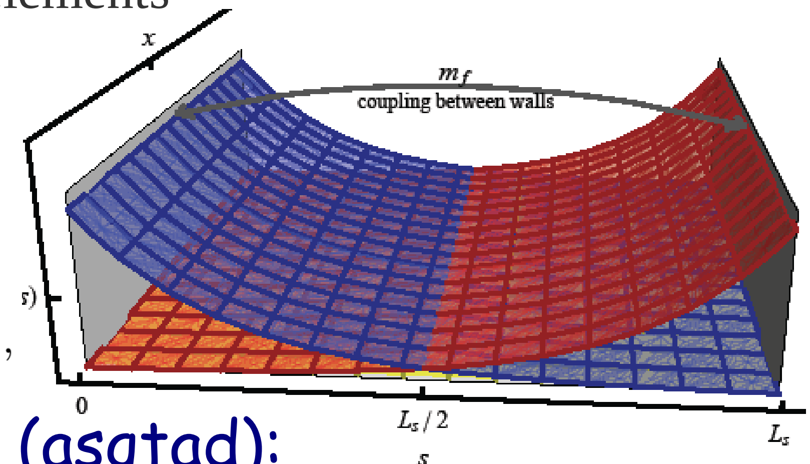
- But for now…
  need a pion mass extrapolation $M_\pi \to (M_\pi)_{\text{phys}}$
  (use chiral perturbation theory, if available)

Lattice Fermion Actions

- Chiral fermions (e.g., **Domain-Wall**/Overlap):
  - Automatically $O(a)$ improved,
    good for spin physics and weak matrix elements
  - Expensive

$$D_{x,s;x',s'} = \delta_{x,x'} D^{\perp}_{s,s'} + \delta_{s,s'} D^{\parallel}_{x,x'}$$

$$\begin{aligned} D^{\perp}_{s,s'} &= \frac{1}{2}[(1-\gamma_5)\delta_{s+1,s'} + (1+\gamma_5)\delta_{s-1,s'} - 2\delta_{s,s'}] \\ &- \frac{m_f}{2}[(1-\gamma_5)\delta_{s,L_s-1}\delta_{0,s'} + (1+\gamma_5)\delta_{s,0}\delta_{L_s-1,s'}], \end{aligned}$$



- (Improved) Staggered fermions (asqtad):
  - Relatively cheap for dynamical fermions (good)
  - Mixing among parities and flavors or "tastes"
  - Baryonic operators a nightmare — not suitable
- Wilson/Clover action:
  - Moderate cost; explicit chiral symmetry breaking
- Twisted Wilson action:
  - Moderate cost; isospin mixing

Mixed Action Parameters

- Mixed action:
  - Staggered sea (cheap) with domain-wall valence (chiral)
  - Match the sea Goldstone pion mass to the DWF pion
  - Only mixes with the "scalar" taste of sea pion
  - Free light quark propagators (LHPC & NPLQCD)
- In this calculation:
  - Pion mass ranges 300–750 MeV
  - Volume fixed at 2.6 fm, box size of $20^3 \times 32$
  - $a \approx 0.125$ fm, $L_s = 16$, $M_5 = 1.7$
  - HYP-smeared gauge fields

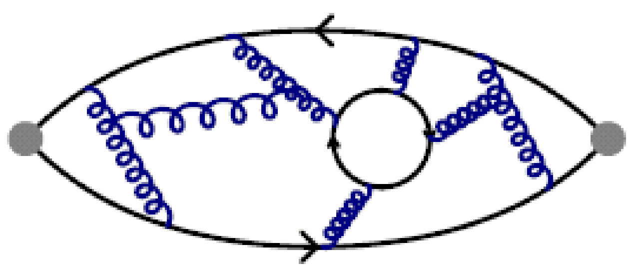
Lattice QCD: Observables

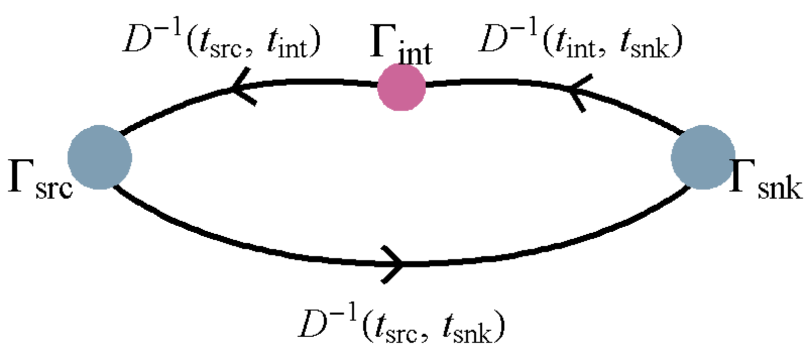
- Two-point Green function

  e.g. spectroscopy

- Three-point Green function

  e.g. form factors, structure functions, …





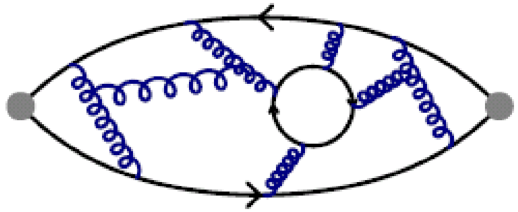
$$\sum_{\alpha,\beta} \Gamma^{\alpha,\beta} \langle J(X_{\rm snk}) J(X_{\rm src}) \rangle_{\alpha,\beta}$$

$$\sum_{\alpha,\beta} \Gamma^{\alpha,\beta} \langle J(X_{\rm snk}) O(X_{\rm int}) J(X_{\rm src}) \rangle_{\alpha,\beta}$$

Lattice QCD: Observables

- Two-point Green function

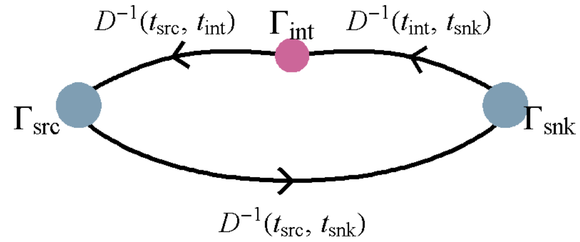
  e.g. spectroscopy



$$\sum_{\alpha,\beta} \Gamma^{\alpha,\beta} \langle J(X_{\text{snk}}) J(X_{\text{src}}) \rangle_{\alpha,\beta}$$

- Three-point Green function

  e.g. form factors, structure functions, …



$$\sum_{\alpha,\beta} \Gamma^{\alpha,\beta} \langle J(X_{\text{snk}}) O(X_{\text{int}}) J(X_{\text{src}}) \rangle_{\alpha,\beta}$$

After taking spin and momentum projection

(ignore the variety of boundary condition choices)

Two-point correlator

$$\sum_n Z_{n,B} e^{-E_n(\vec{P})t}$$

Three-point correlator

$$\sum_n \sum_{n'} Z_{n',B}(p_f) Z_{n,A}(p_i)$$

$$\times \text{FF's} \times e^{-(t_f-t)E'_n(\vec{p}_f)} e^{-(t-t_i)E_n(\vec{p}_i)}$$

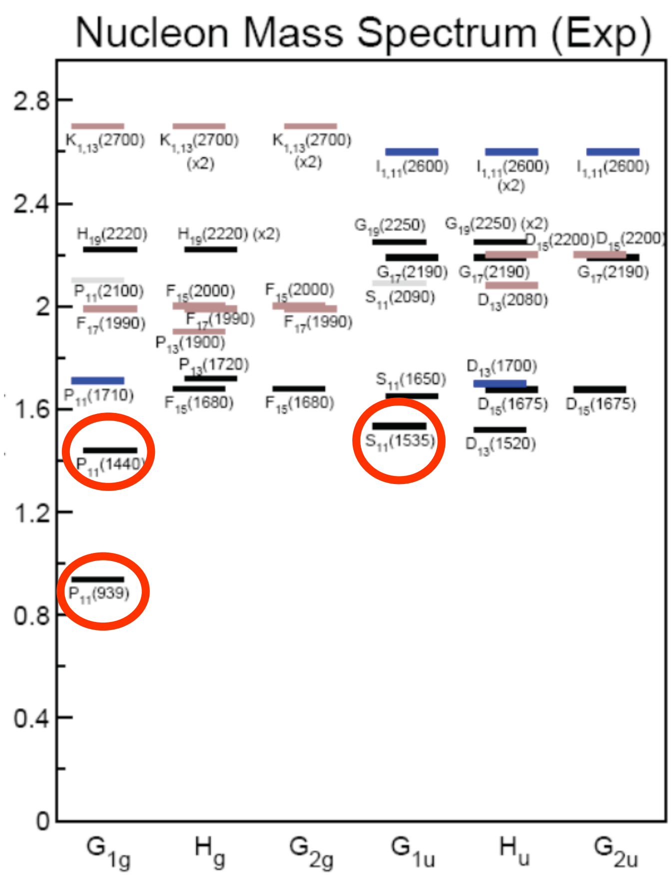
At large enough *t*, the ground-state signal dominates.

Motivations and Methodology

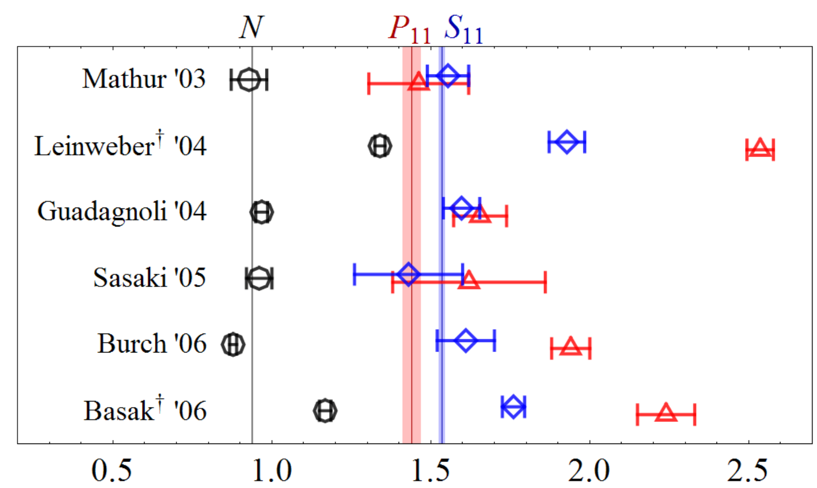
Why Baryons?

Lattice QCD spectrum

- Successfully calculates many ground states (Nature, …)
- Nucleon spectrum, on the other hand… not quite



Example: N, P_{11}, S_{11} spectrum



Strange Baryons

- Strange baryons are of particular interest; challenging even to experiment
- Example from PDG Live:

Ξ BARYONS ($S = -2$, $I = 1/2$)

$\Xi^0 = u\,s\,s$, $\Xi^- = d\,s\,s$

Ξ^0	$1/2(1/2^+)$	****	$\Xi(1820)$ D_{13}	$1/2(3/2^-)$	***	$\Xi(2370)$	$1/2(?^?)$	•**
Ξ^-	$1/2(1/2^+)$	****	$\Xi(1950)$	$1/2(?^?)$	***	$\Xi(2500)$	$1/2(?^?)$	•*
$\Xi(1530)$ P_{13}	$1/2(3/2^+)$	****	$\Xi(2030)$	$1/2(\geq\frac{5}{2}^?)$	***			
$\Xi(1620)$	$1/2(?^?)$	•*	$\Xi(2120)$	$1/2(?^?)$	•*			
$\Xi(1690)$	$1/2(?^?)$	***	$\Xi(2250)$	$1/2(?^?)$	•**			

• — OMITTED FROM SUMMARY TABLE

Ω BARYONS ($S = -3$, $I = 0$)

$\Omega^- = s\,s\,s$

Ω^-	$0(3/2^+)$	****
$\Omega(2250)^-$	$0(?^?)$	***
$\Omega(2380)^-$		•**
$\Omega(2470)^-$		•**

Operator Design

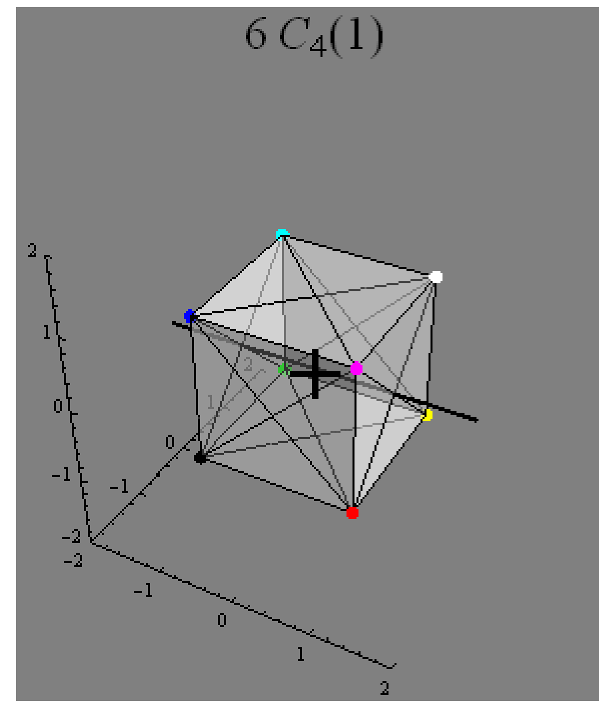
- All baryon spin states wanted: j = 1/2, 3/2, 5/2, …
- Rotation symmetry is reduced due to discretization
  rotation O(3) ⇒ octahedral O_h group

	I	J	$6\,C_4$	$8\,C_6$	$8\,C_3$	$6\,C_8$	$6\,C'_8$	$12\,C'_4$
A_1	1	1	1	1	1	1	1	1
A_2	1	3	−2	1	0	−1	1	0
E	2	1	1	1	−1	−1	−1	0
G_1	2	0	1	−1	1	−2	1	0
G_2	2	−4	0	1	0	0	1	−1
T_1	3	2	0	0	1	1	−1	−1
T_2	3	3	0	−1	−1	1	1	0
H	4	−3	−1	0	0	0	−1	1

Operator Design

- All baryon spin states wanted: j = 1/2, 3/2, 5/2, …
- Rotation symmetry is reduced due to discretization
  rotation O(3) ⇒ octahedral O_h group

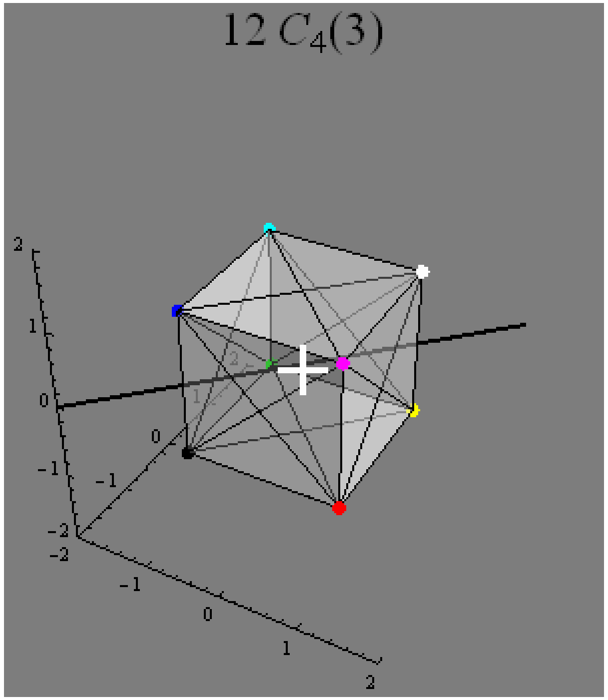
	I	J	6 C_4	8 C_6	8 C_3	6 C_8	6 C'_8	12 C'_4
A_1	1	1	1	1	1	1	1	1
A_2	1	3	-2	1	0	-1	1	0
E	2	1	1	1	-1	-1	-1	0
G_1	2	0	1	-1	1	-2	1	0
G_2	2	-4	0	1	0	0	1	-1
T_1	3	2	0	0	1	1	-1	-1
T_2	3	3	0	-1	-1	1	1	0
H	4	-3	-1	0	0	0	-1	1



Operator Design

- All baryon spin states wanted: j = 1/2, 3/2, 5/2, …
- Rotation symmetry is reduced due to discretization
  rotation O(3) ⇒ octahedral O_h group

	I	J	$6\ C_4$	$8\ C_6$	$8\ C_3$	$6\ C_8$	$6\ C'_8$	$12\ C'_4$
A_1	1	1	1	1	1	1	1	1
A_2	1	3	−2	1	0	−1	1	0
E	2	1	1	1	−1	−1	−1	0
G_1	2	0	1	−1	1	−2	1	0
G_2	2	−4	0	1	0	0	1	−1
T_1	3	2	0	0	1	1	−1	−1
T_2	3	3	0	−1	−1	1	1	0
H	4	−3	−1	0	0	0	−1	1



Operator Design

- All baryon spin states wanted: j = 1/2, 3/2, 5/2, …
- Rotation symmetry is reduced due to discretization

  rotation O(3) ⇒ octahedral O_h

	I	J	$6\,C_4$	$8\,C_6$	$8\,C_3$	$6\,C_8$	$6\,C_8'$	$12\,C_4'$
A_1	1	1	1	1	1	1	1	1
A_2	1	3	−2	1	0	−1	1	0
E	2	1	1	1	−1	−1	−1	0
G_1	2	0	1	−1	1	−2	1	0
G_2	2	−4	0	1	0	0	1	−1
T_1	3	2	0	0	1	1	−1	−1
T_2	3	3	0	−1	−1	1	1	0
H	4	−3	−1	0	0	0	−1	1

Baryons

j	Irreps
$\frac{1}{2}$	G_1
$\frac{3}{2}$	H
$\frac{5}{2}$	$G_2 \oplus H$
$\frac{7}{2}$	$G_1 \oplus G_2 \oplus H$
$\frac{9}{2}$	$G_1 \oplus 2\,H$
$\frac{11}{2}$	$G_1 \oplus G_2 \oplus 2\,H$
$\frac{13}{2}$	$G_1 \oplus 2\,G_2 \oplus 2\,H$
$\frac{15}{2}$	$G_1 \oplus G_2 \oplus 3\,H$
$\frac{17}{2}$	$2\,G_1 \oplus G_2 \oplus 3\,H$
$\frac{19}{2}$	$2\,G_1 \oplus 2\,G_2 \oplus 3\,H$
$\frac{21}{2}$	$G_1 \oplus 2\,G_2 \oplus 4\,H$
$\frac{23}{2}$	$2\,G_1 \oplus 2\,G_2 \oplus 4\,H$

Operator Design

- The basic building blocks

$$\bar{B}^{ABC}_{\alpha\beta\gamma}(x) = \bar{\psi}^{A,i}_{\alpha}\bar{\psi}^{B,j}_{\beta}\bar{\psi}^{C,k}_{\gamma}\epsilon^{ijk}$$

  - *A*, *B*, *C*: quark flavor
  - *i*, *j*, *k*: color
  - α, β, γ: Dirac indices
- Project onto irreducible representations (irreps)

$$\bar{B}^{\Lambda,n}_{\lambda}(x) = \Gamma^{\Lambda,n}_{\lambda}(\alpha,\beta,\gamma)\bar{B}_{\alpha,\beta,\gamma}(x)$$

  - Λ: irrep
  - $\lambda \in [1, \dim(\Lambda)]$
  - *n*: element of interoperating op

Flavor	$G_{1g/u}(2)$	$H_{g/u}(4)$
N	3	1
Δ	1	2
Λ	4	1
Σ	4	3
Ξ	4	3
Ω	1	2

- Correlator matrix

$$C^{m,n}_{\Lambda}(t) = \sum_{\vec{x}}\sum_{\lambda}\langle 0 \mid B^{\Lambda,m}_{\lambda}(\vec{x},t)\bar{B}^{\Lambda,n}_{\lambda}(0) \mid 0\rangle$$

- For more details and extended-link operators:

S. Basak et al., Phys. Rev. D72, 094506 (2005)

Variational Method

C. Michael, Nucl. Phys. B 259, 58 (1985)
M. Lüscher and U. Wolff, Nucl. Phys. B 339, 222 (1990)

- Construct the matrix

$$C_{i\ j}(t) = \langle 0 \mid \mathcal{O}_i(t)^\dagger \mathcal{O}_j(0) \mid 0 \rangle$$

- The O_i could be different choices of operator or smearing parameters
- Solve for the generalized eigensystem of

$$C(t_0)^{-1/2} C(t) C(t_0)^{-1/2} v = \lambda(t, t_0) v$$

with eigenvalues

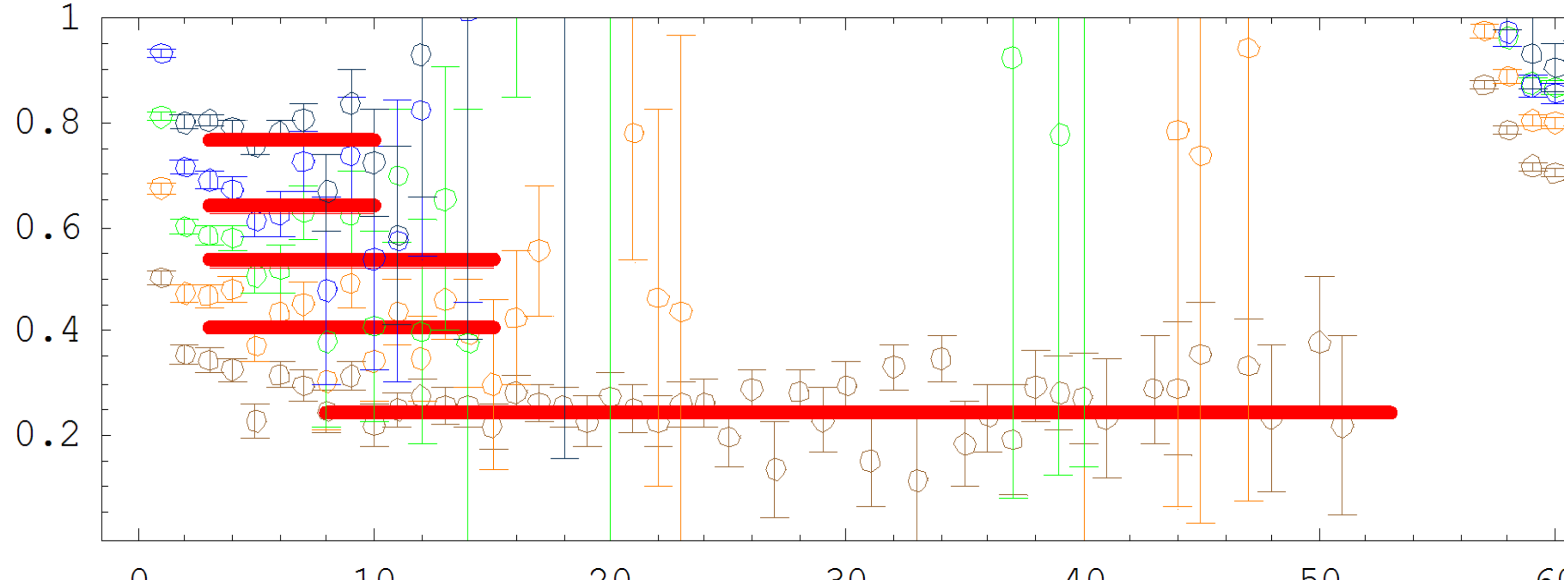
$$\lambda_n(t, t_0) = e^{-(t-t_0)E_n}\left(1 + \mathcal{O}(e^{-|\delta E|(t-t_0)})\right)$$

- At large t, the signal of the desired state dominates.

Variational Method

Quenched Anisotropic ($a_t^{-1} \sim 6$ GeV)

- Clover action, 680 MeV pion
- Example: 5×5 smeared-smeared correlator matrices
- Fit them individually with exponential form (red bars)
- Plotted along with effective masses

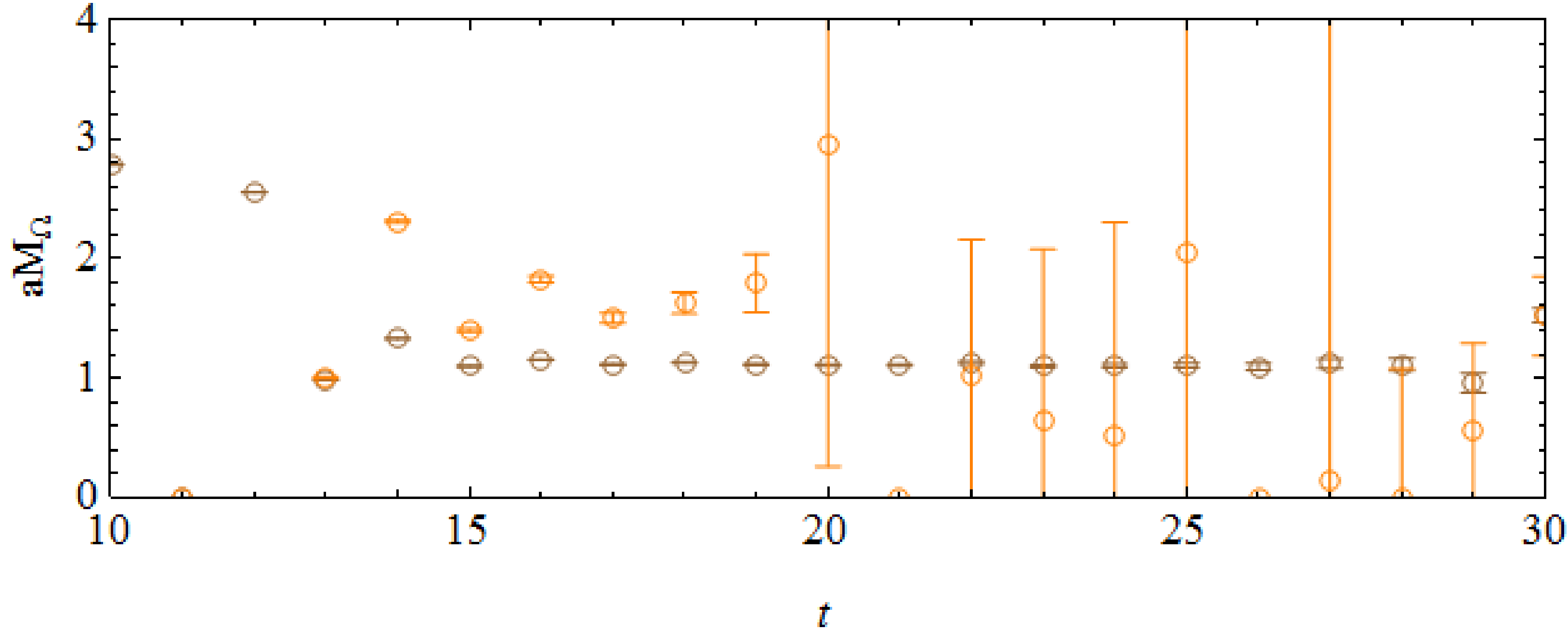


Variational Method

Mixed Action ($a_t^{-1} \sim 1.6$ GeV)

- Example: (~350 MeV pion)

  Omega 2×2 smeared-smeared operator correlator matrices

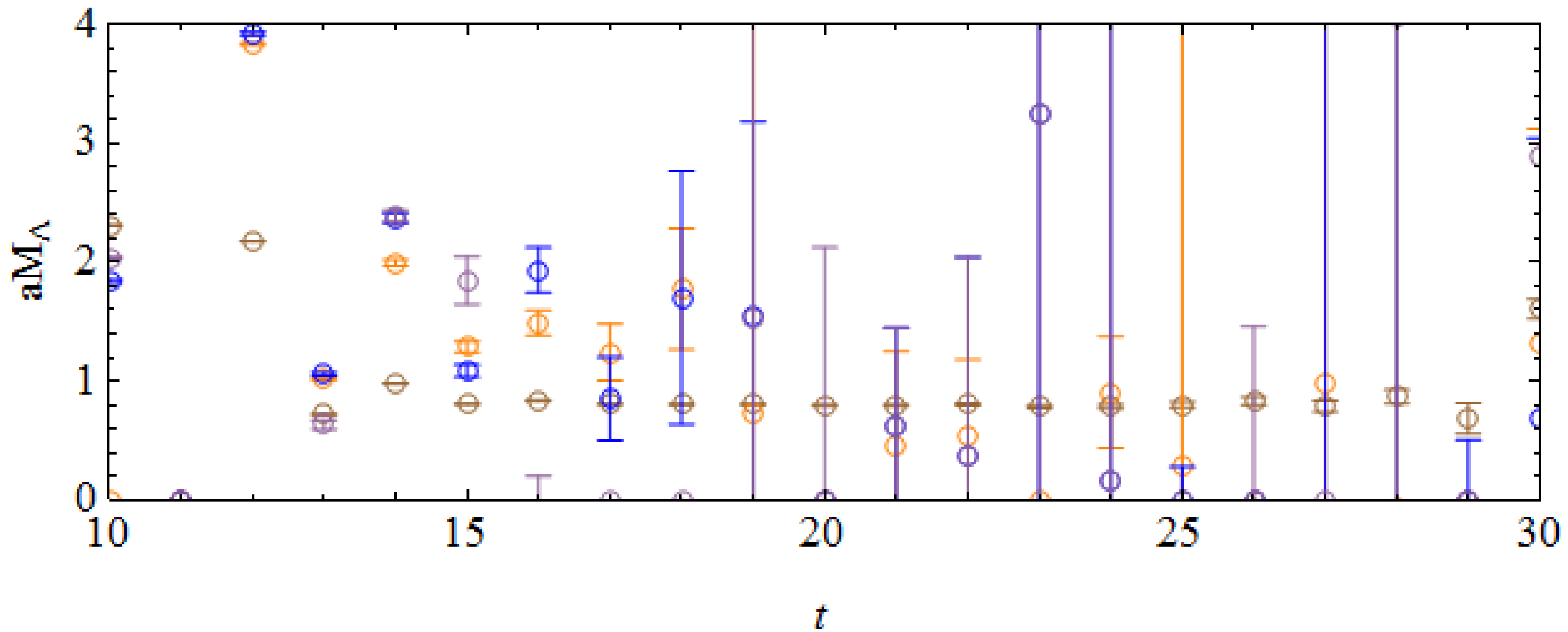


Variational Method

Mixed Action (a_t^{-1} ~ 1.6 GeV)

- Example: (~350 MeV pion)

  Lambda 4×4 smeared-smeared operator correlator matrices

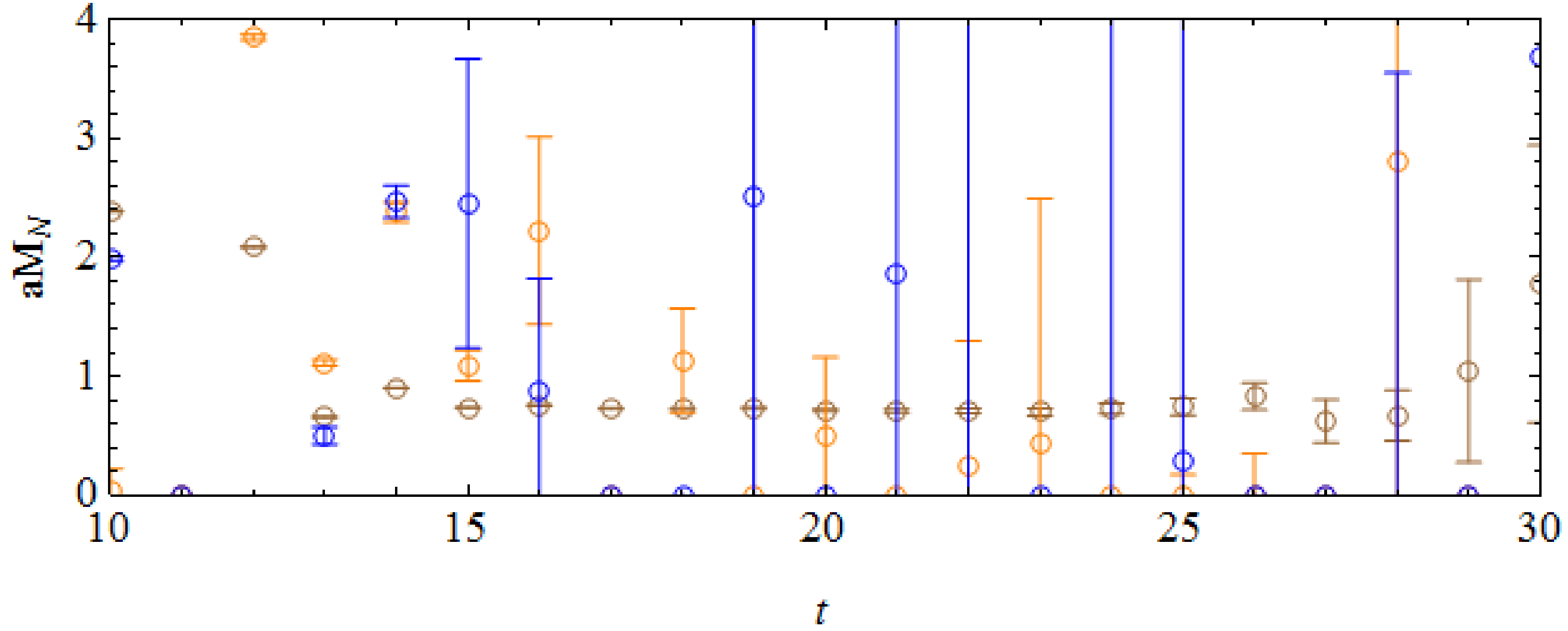


Variational Method

Mixed Action ($a_t^{-1} \sim 1.6$ GeV)

- Example: (~350 MeV pion)

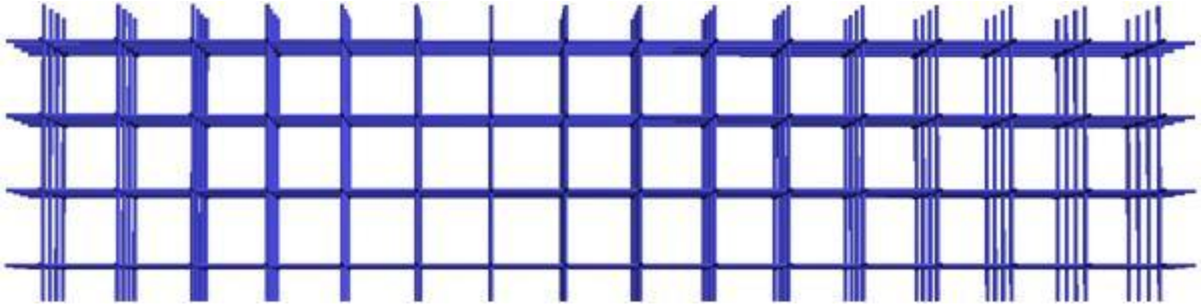
Nucleon 3×3 smeared-smeared operator correlator matrices



- Unfortunately, we cannot see a clear radial excited state

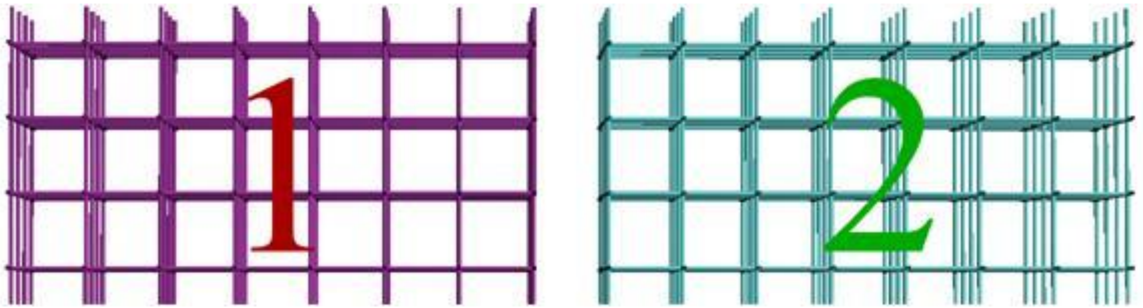
Ensembles and Parameters

- Mixed action: DWF on staggered sea
- Pion mass ranges 300–750 MeV
- $a \approx 0.125$ fm, $L_s = 16$, $M_5 = 1.7$
- Volume fixed at 2.6 fm, box size of $20^3 \times 32$ chopped



Ensembles and Parameters

- Mixed action: DWF on staggered sea
- Pion mass ranges 300–750 MeV
- $a \approx 0.125$ fm, $L_s = 16$, $M_5 = 1.7$
- Volume fixed at 2.6 fm, box size of $20^3 \times 32$ chopped



Ensembles and Parameters

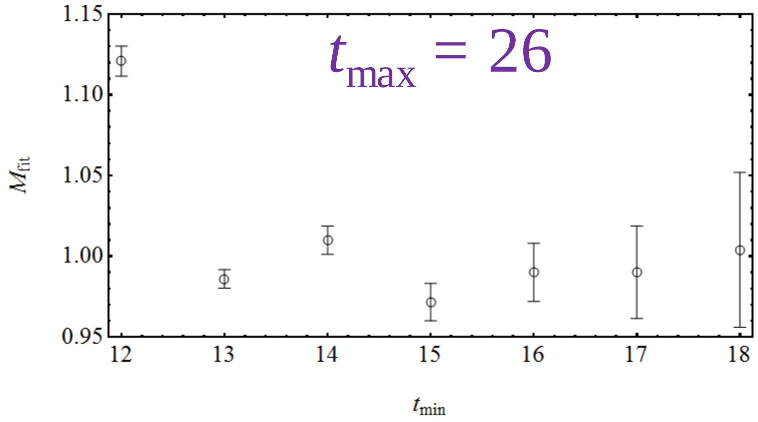
- Mixed action: DWF on staggered sea
- Pion mass ranges 300–750 MeV
- $a \approx 0.125$ fm, $L_s = 16$, $M_5 = 1.7$
- Volume fixed at 2.6 fm, box size of $20^3\times32$ chopped

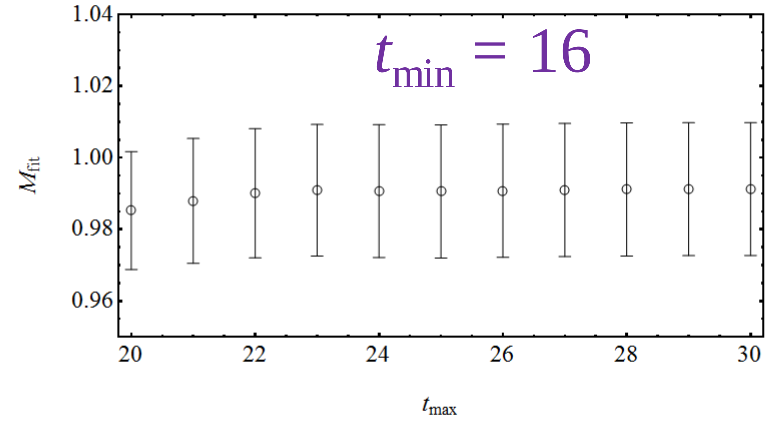
ensem	m007	m010	m020	m030	m040	m050
Conf.	3489	3693	1455	700	324	425

- HYP-smeared gauge fields, Gaussian operator smearing

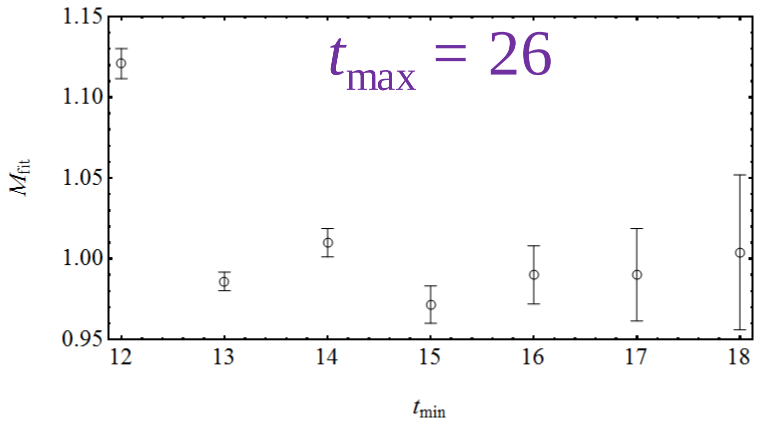
Consistent Analyses

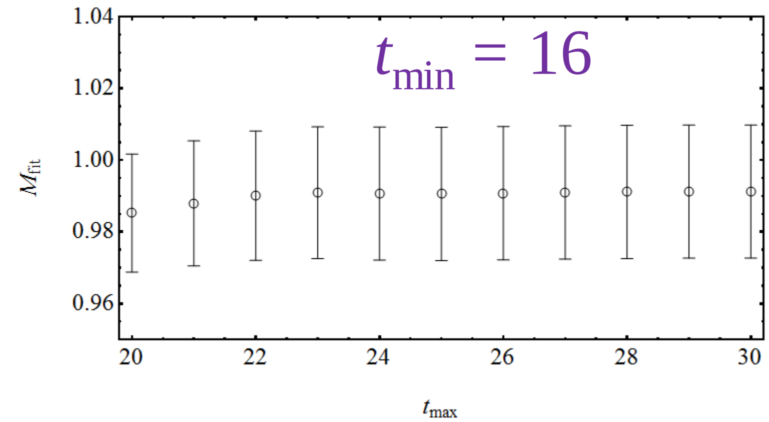
- Systematic error due to fit range
- Example: Nucleon @ 350 MeV





- Example: Delta @ 350 MeV





Consistent Analyses

- Oscillating effective mass is related to transfer matrix with 5th-dimensional mass term

  → treat as a lattice artifact

- Solution: oscillating term + one excited state

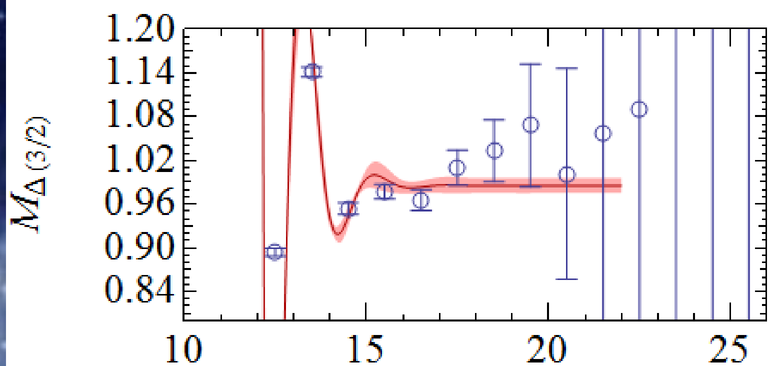
$$C(t)=\sum_{n=0}^{1} A_n exp[-M_n \times (t-t_{\rm src})] + A_{\rm osc}(-1)^t exp[-M_{\rm osc} \times (t-t_{\rm src})].$$

J. Negele et al. LAT2007, 078

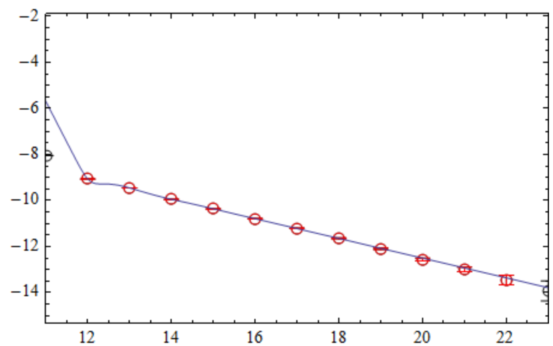
- Example: Delta @ 350 MeV pion ensemble

  $\chi^2/\text{dof} = 0.69$

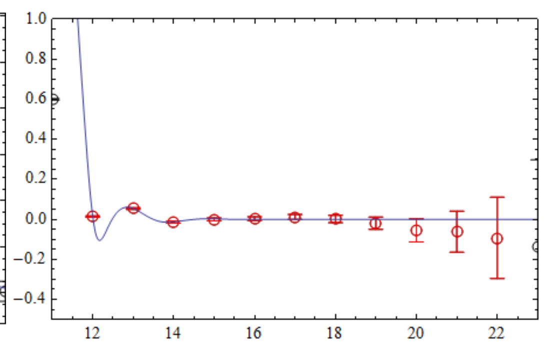
"effective" mass



log corr.



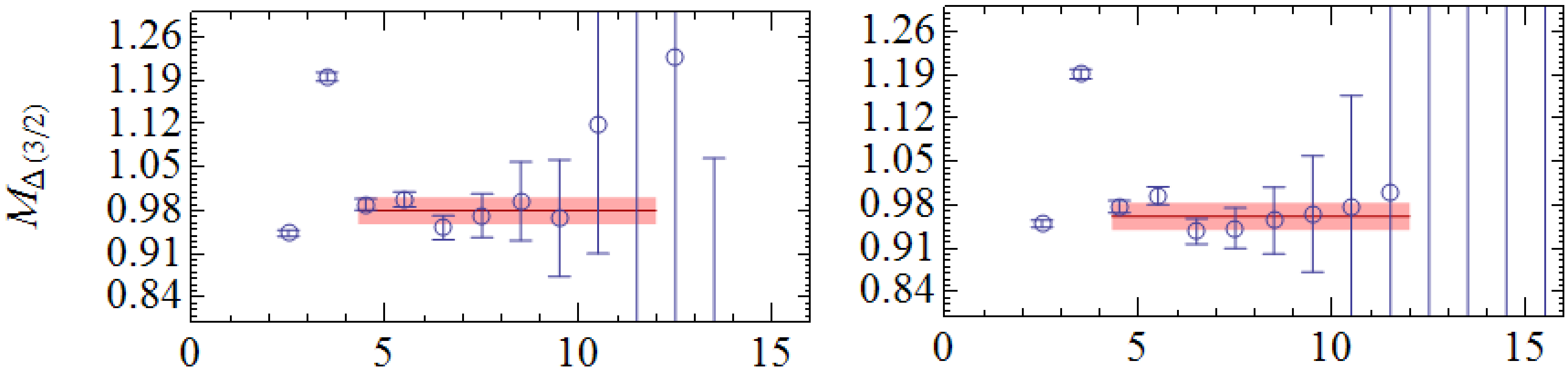
(ground-state removed)



Consistent Analyses

- Chopped lattice? Example: Delta @350 MeV
  - Set A: $20^3 \times 64$, 4 sources, 224 confs.
  - Set B: chopped $20^3 \times 32$, 620 confs.

Set-A Set-B



- Consistent results in both cases

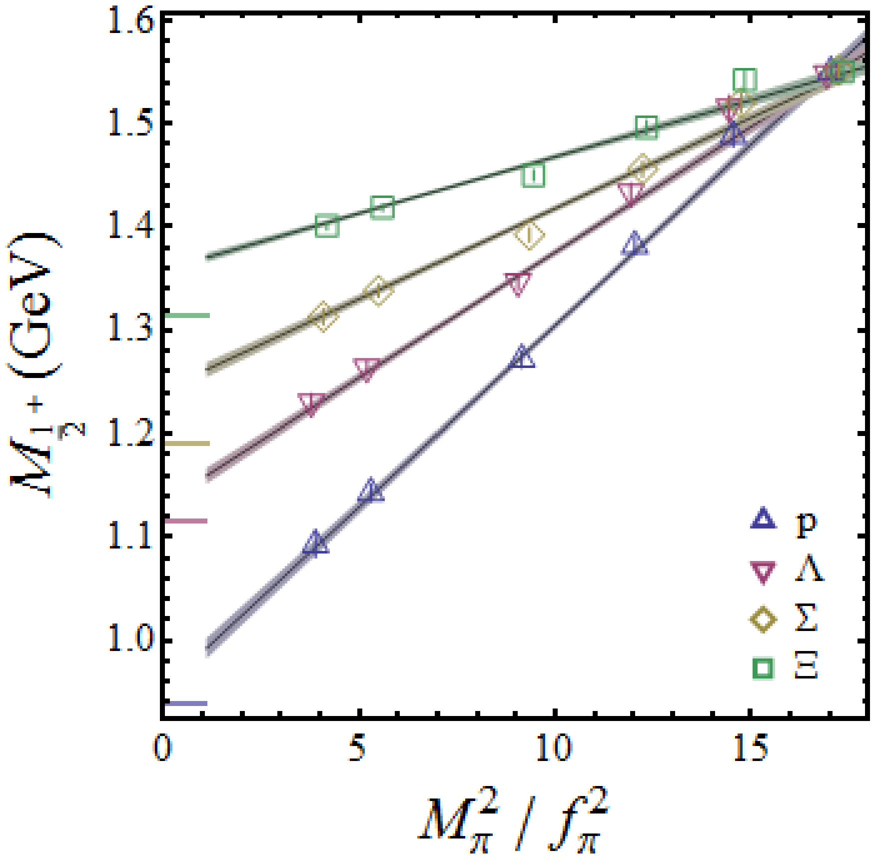
Ground-State Results

work with

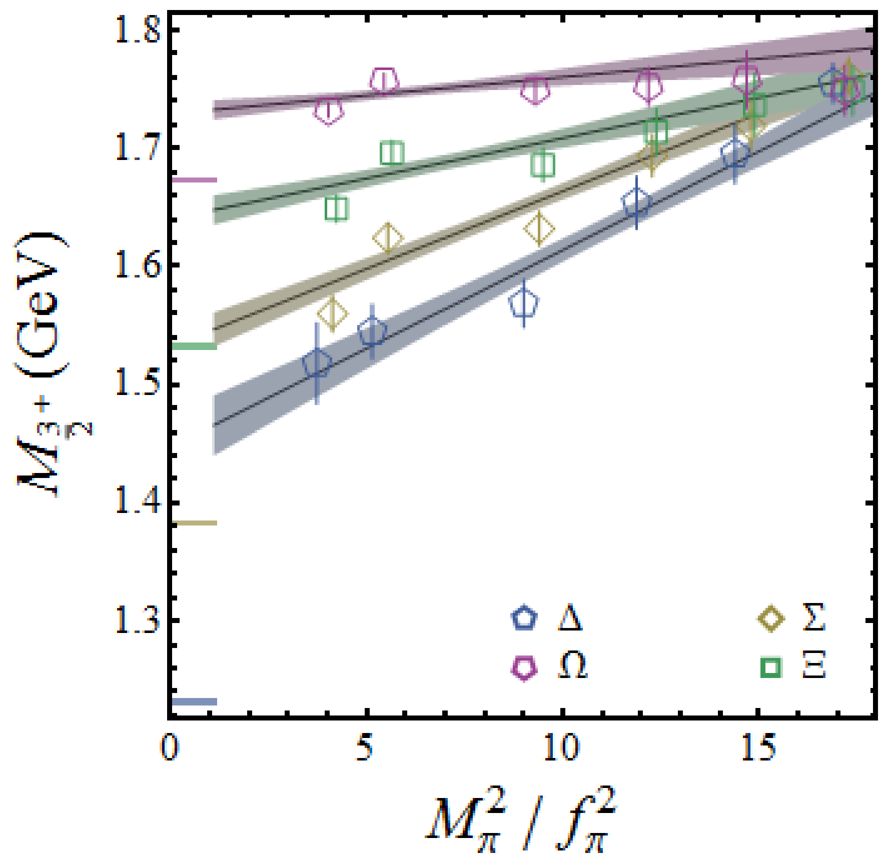
Lattice Hadron Physics Collaboration (LHPC)

Octets and Decuplets

- **Spin-1/2**



- **Spin-3/2**



Multiplet Mass Relations

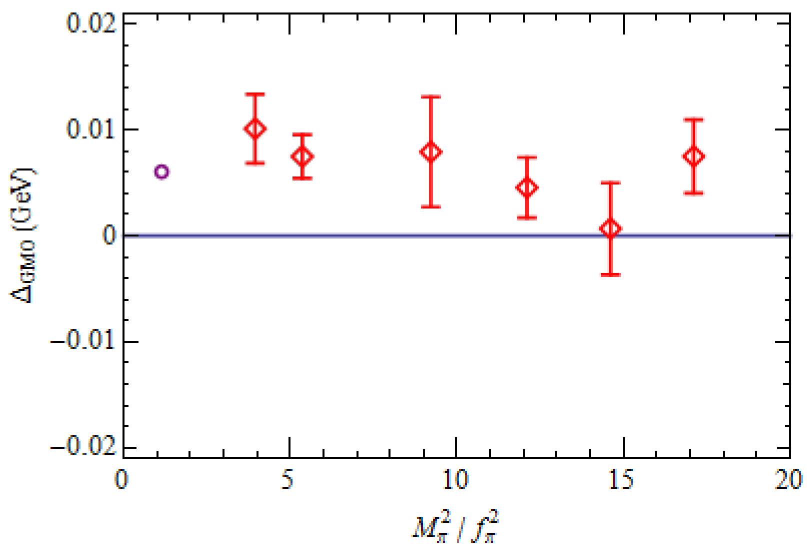
- SU(3) flavor symmetry breaking
  - Gell-Mann-Okubo relation

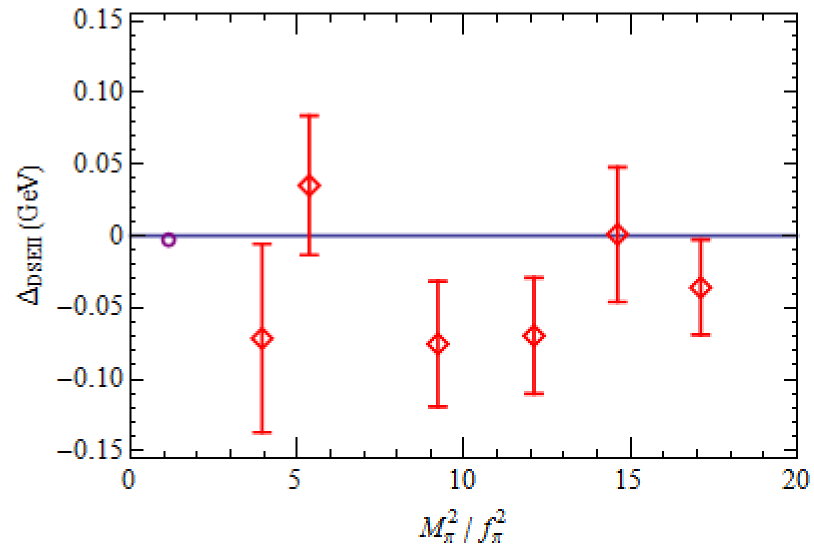
$$\Delta_{GMO} = \frac{3}{4}M_\Lambda + \frac{1}{4}M_\Sigma - \frac{1}{2}M_N - \frac{1}{2}M_\Xi$$

  - Decuplet Equal Spacing relation

$$\Delta_{DESII} = \frac{1}{2}(M_{\Sigma^*} - M_\Delta) + \frac{1}{2}(M_\Omega - M_{\Xi^*}) - M_{\Xi^*} + M_{\Sigma^*}$$

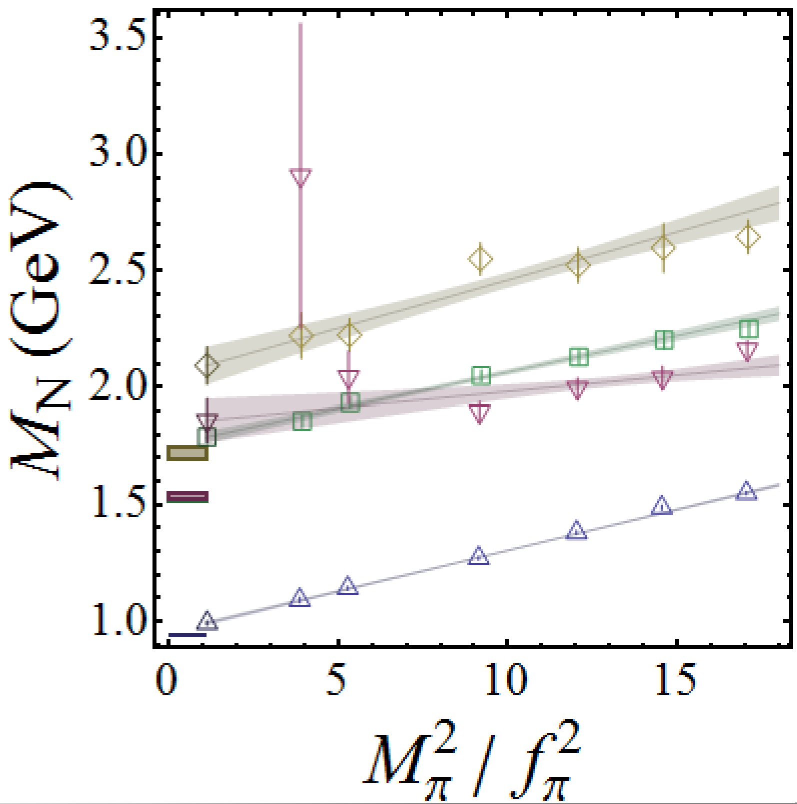
- Mass differences are close to experimental numbers





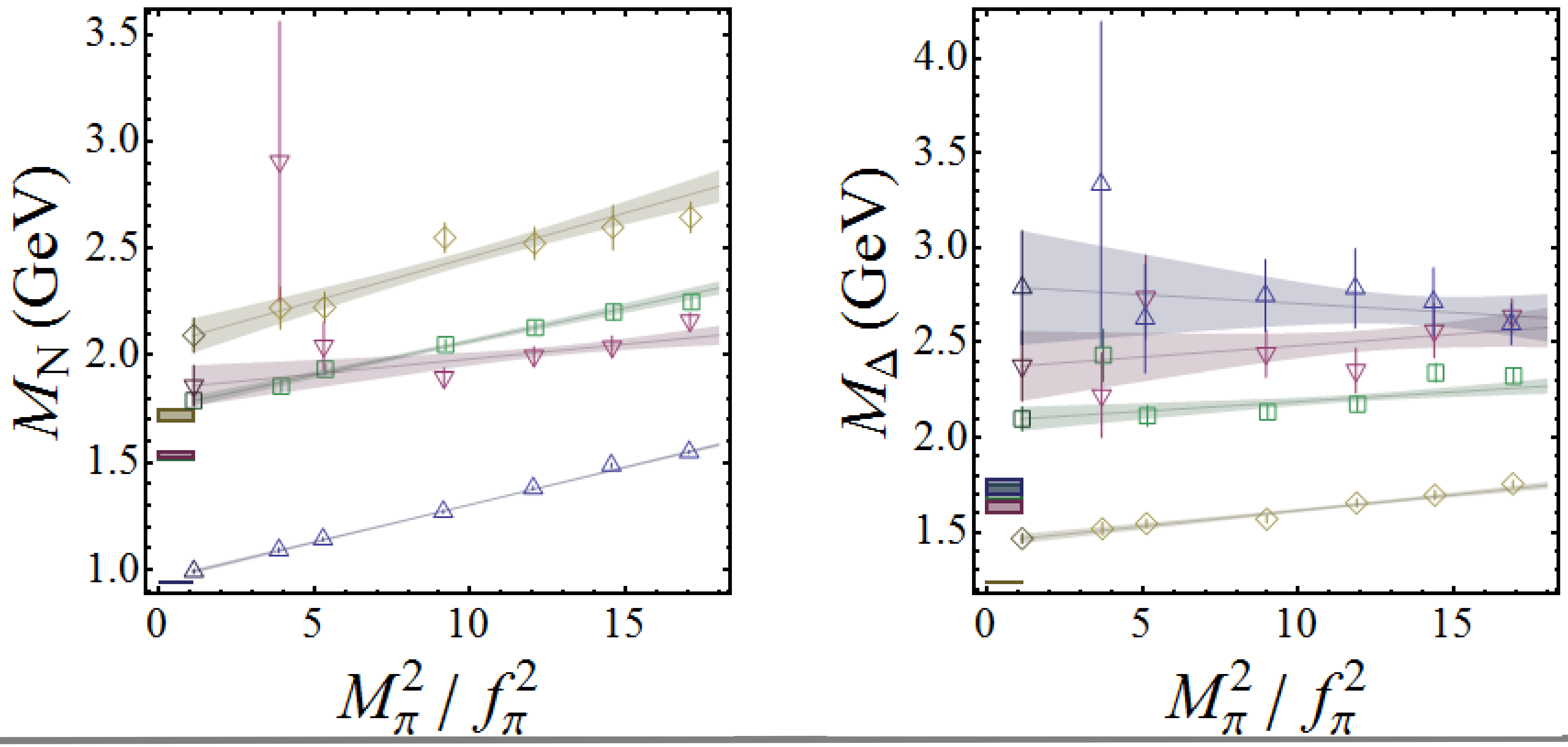
General Spectroscopy

- The non-strange baryons (*N*)
- Symbols: J^P = $1/2^+$ △, $1/2^-$ ▽ , $3/2^+$ ◇, $3/2^-$ □
  - *N* *N*(1535) *N*(1720) *N*(1520)



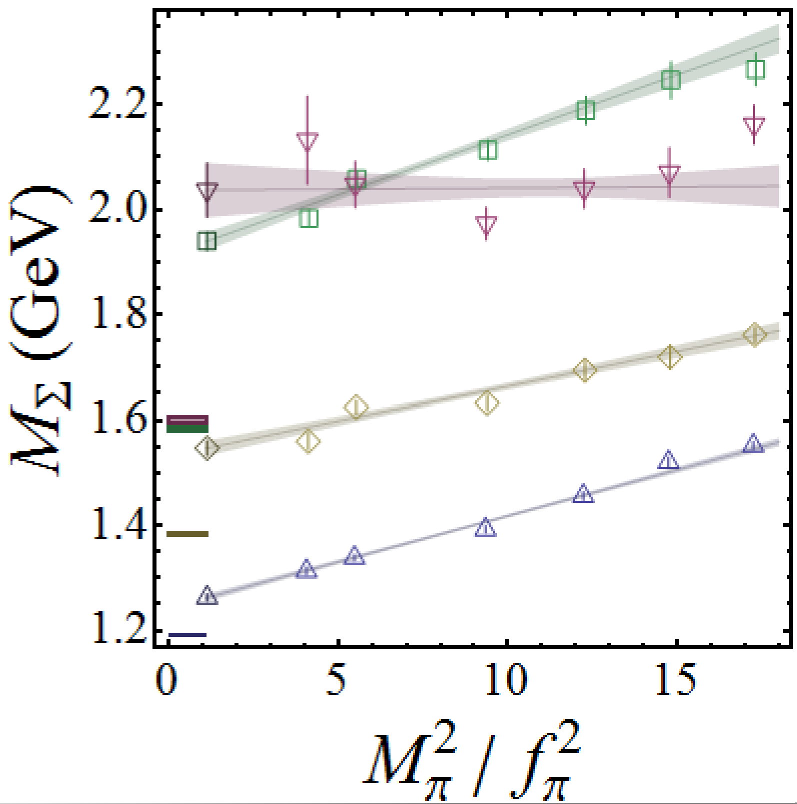
General Spectroscopy

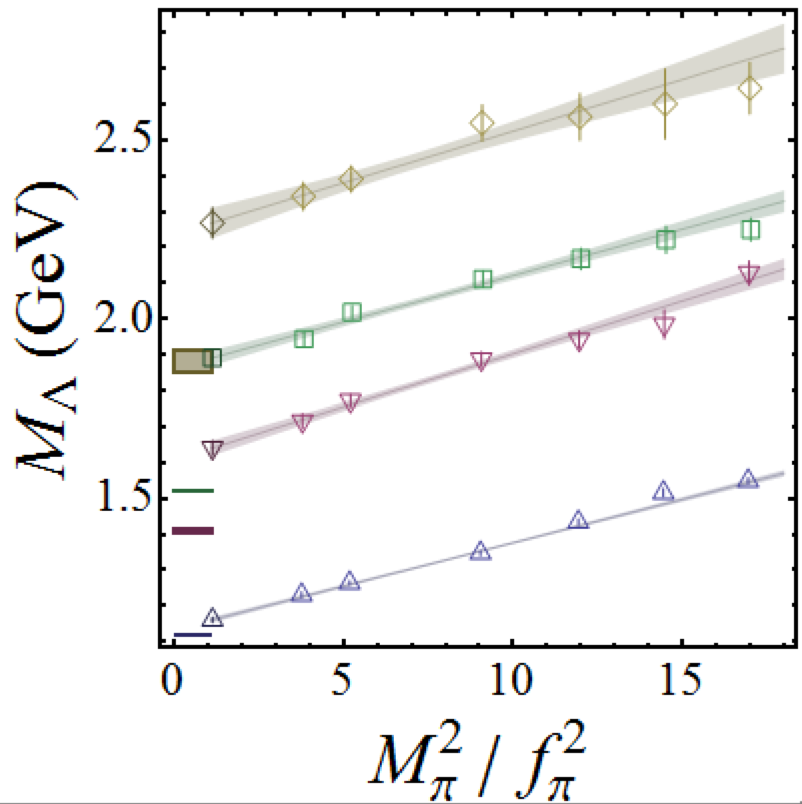
- The non-strange baryons (N and Δ)
- Symbols: J^P = 1/2⁺ △, 1/2⁻ ▽, 3/2⁺ ◇, 3/2⁻ □
  - N $N(1535)$ $N(1720)$ $N(1520)$
  - $\Delta(1620)$ Δ $\Delta(1700)$



General Spectroscopy

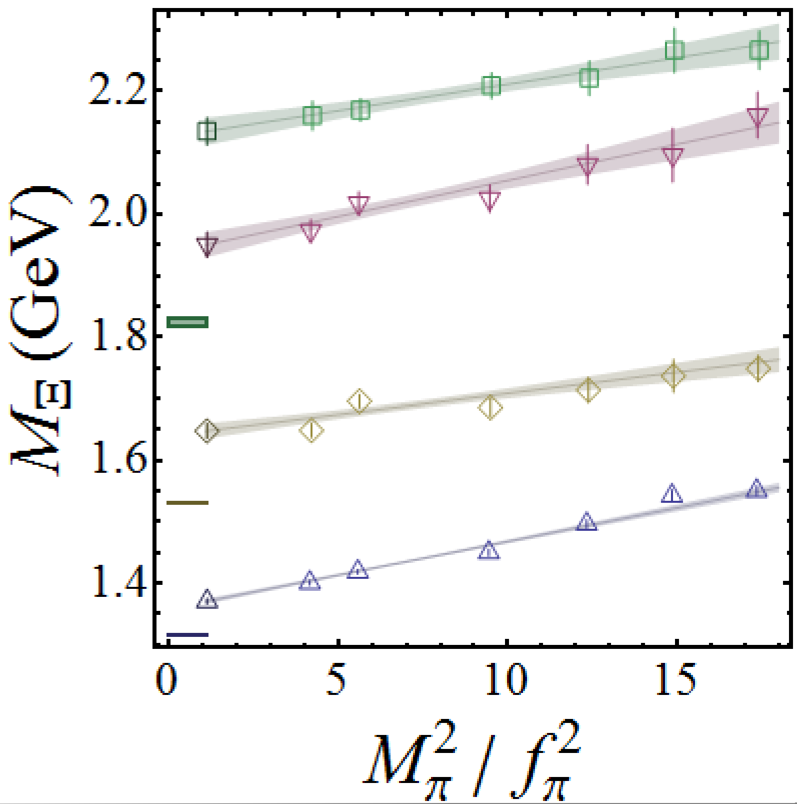
- The singly strange baryons: (Σ and Λ)
- Symbols: J^P = 1/2⁺ △, 1/2⁻ ▽, 3/2⁺ ◇, 3/2⁻ □
  - Σ Σ(1620) Σ* Σ(1580)
  - Λ Λ(1405) Λ(1890) Λ(1520)





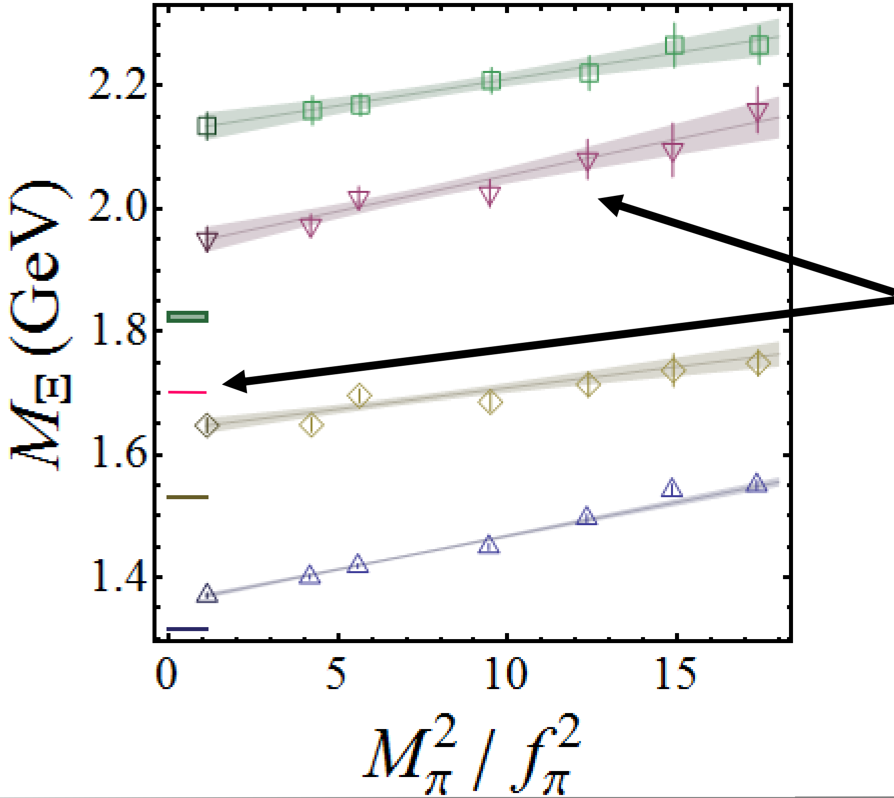
General Spectroscopy

- The less known baryons (Ξ)
- Symbols: J^P = 1/2⁺ △, 1/2⁻ ▽ , 3/2⁺ ◇, 3/2⁻ □
    - Ξ Ξ(1690)? Ξ(1530) Ξ(1820)



General Spectroscopy

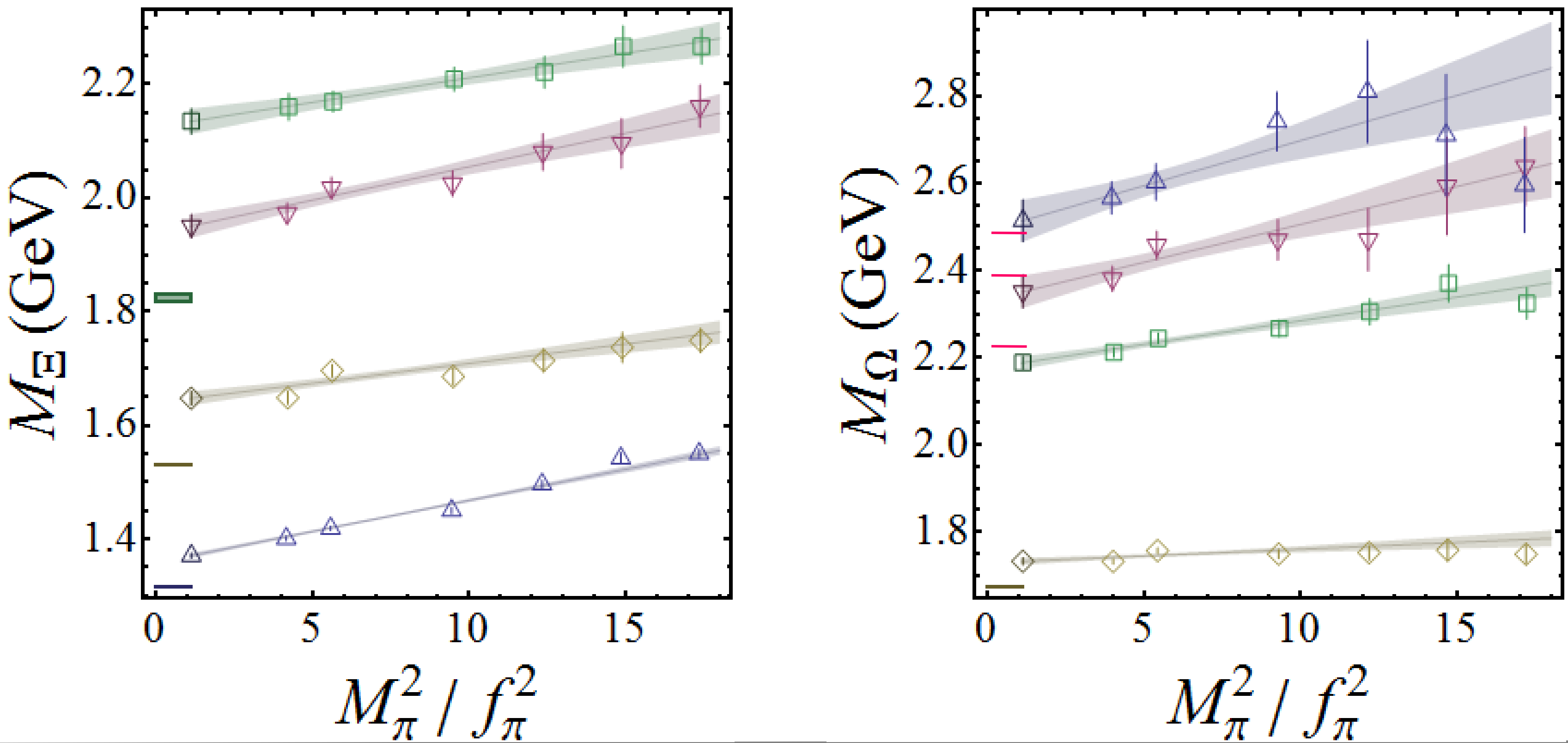
- The less known baryons (Ξ)
- Symbols: J^P = 1/2⁺ △, 1/2⁻ ▽ , 3/2⁺ ◇, 3/2⁻ □
  - Ξ Ξ(1690)? Ξ(1530) Ξ(1820)



- Babar at MENU 2007: $\Xi(1690)^0$ negative parity −1/2

General Spectroscopy

- The less known baryons (Ξ and Ω)
- Symbols: J^P = 1/2⁺ △, 1/2⁻ ▽, 3/2⁺ ◇, 3/2⁻ □
  - Ξ Ξ(1690)? Ξ(1530) Ξ(1820)
  - Could they be Ω(2250), Ω(2380), Ω(2470)?



Excited-State Results

Roper Puzzles

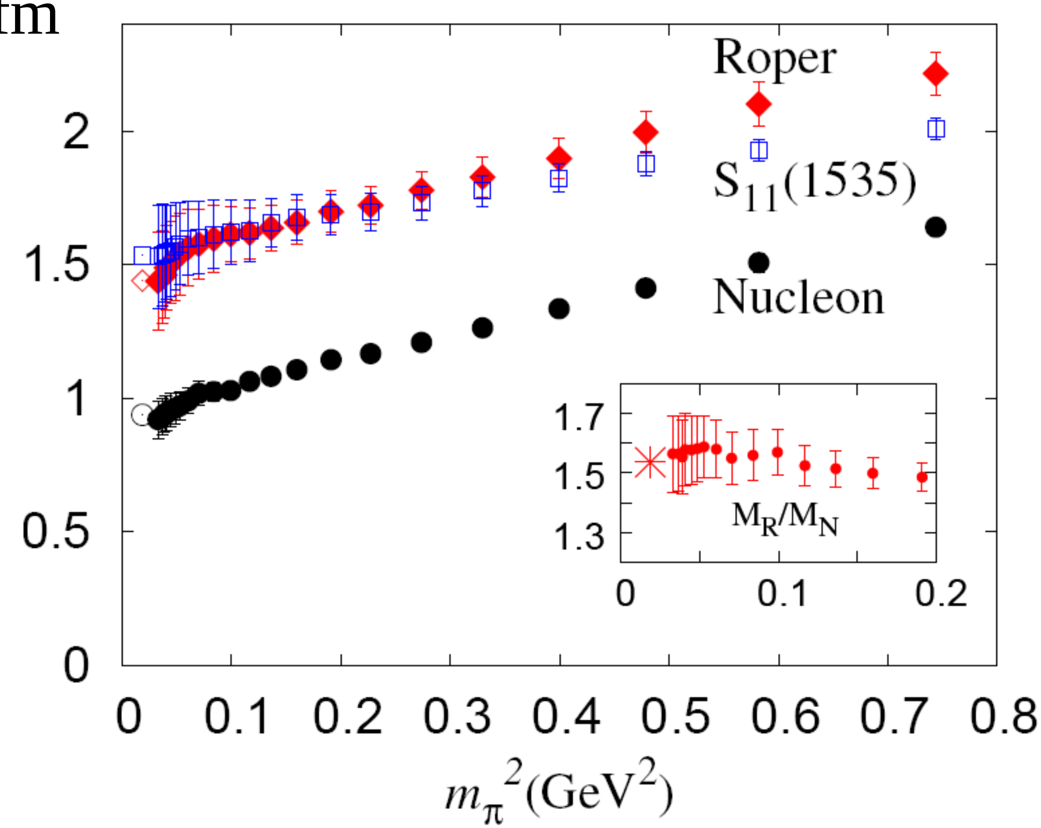
What is the Roper?

- First positive-parity excited state of the nucleon
- Unusual feature: 1st excited state is lower than its negative-parity partner!
- Long-standing puzzle
  - Quark-gluonic (hybrid) state [C. Carlson et al. (1991)]
  - Five-quark (meson-baryon) state [O. Krehl et al. (1999)]
  - Constituent quark models (many different specific approaches)
  - and many other models…

- Lattice gauge theory
  - Many early quenched calculations failed to extract the correct Roper mass
  - Kentucky group (with lightest pion mass = 180 MeV) got $M_{\text{Roper}} = 1462(157)$ MeV

More on the Roper

- Kentucky's calculation
  - Quenched Iwasaki, overlap
    (ghost contributions are included in analysis)
  - Volume: 2.4 and 3.6 fm
  - Large range of m_π



- Conclusive?
  - Many groups fail
  to see similar behaviour

  - War still going on...

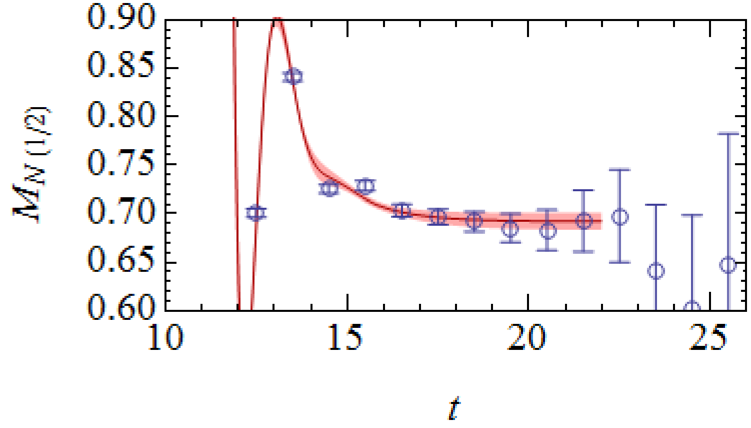
Roper in Full QCD

- Attempt to extract Roper mass from our current data
- Analysis: oscillating term is necessary for small *t*

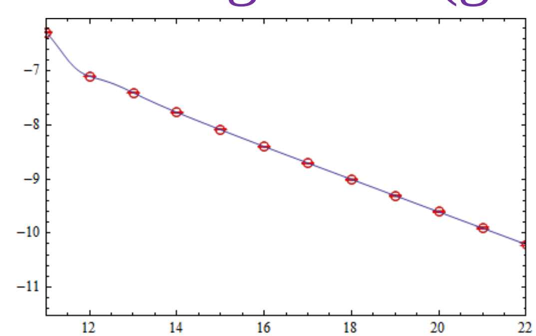
$$C(t) = \sum_{n=0}^{1} A_n exp[-M_n \times (t - t_{\rm src})] + A_{\rm osc}(-1)^t exp[-M_{\rm osc} \times (t - t_{\rm src})].$$

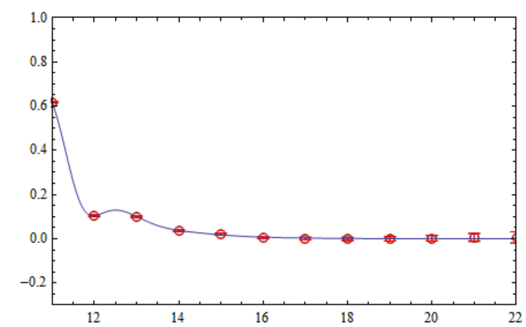
- Example plot (300 MeV ensemble)

"effective" mass



log corr. (ground-state removed)

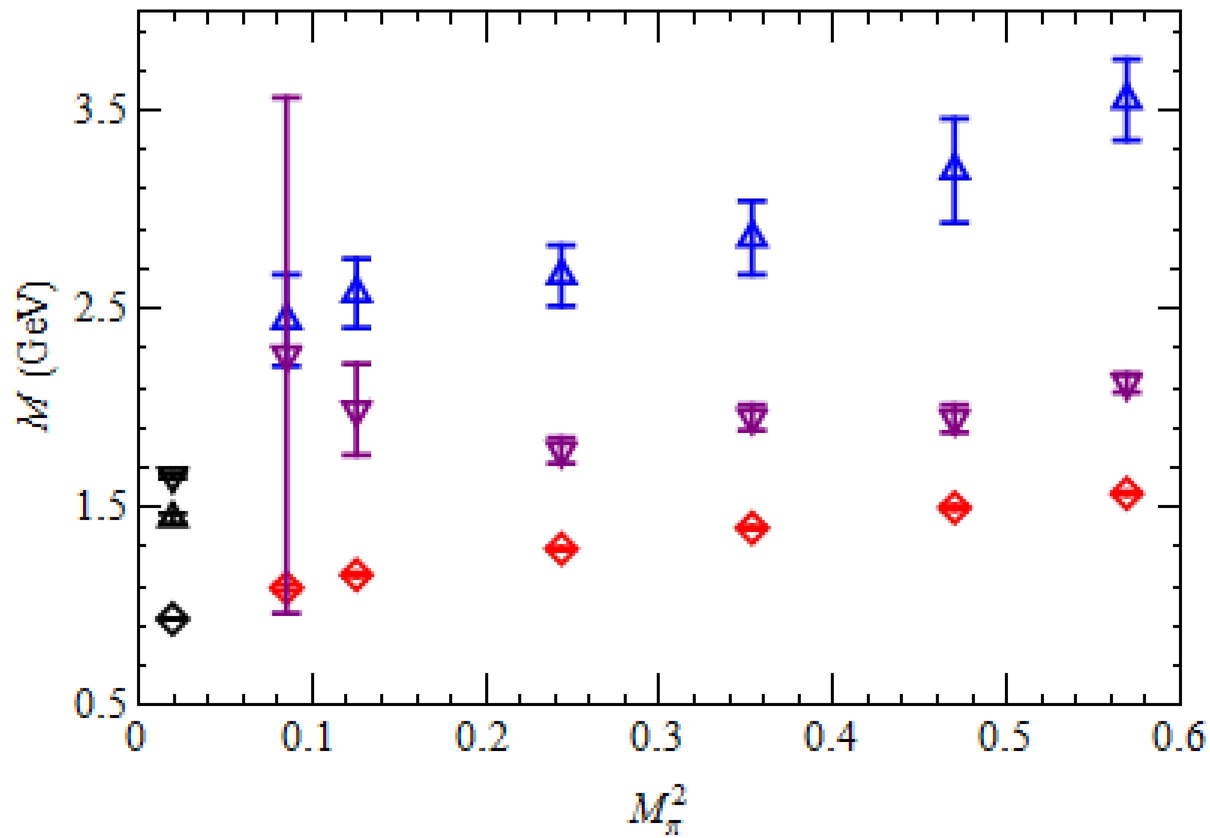




- Reasonable χ^2/dof < 0.6
- Systematic error due to lack of 2nd-excited state in the fit?

Roper in Full QCD

- Results from mixed action
- Symbols: J^P
  - $1/2^+$ ◇
  - $1/2^-$ ▽
  - $1/2^+$ △



- No sign of crossover occurs here
- Finite-volume effects starting at 350 MeV pion?

Summary/Outlook — I

- What we have done:
  - 2+1-flavor calculations with volume around 2.6 fm
  - Preliminary study with lightest pion mass 300 MeV
  - Ground states of $G_{1g/u}$ and $H_{g/u}$ for each flavor
  - Roper state calculated;
    correct mass-ordering pattern is not yet seen
- Currently in progress:
  - Mixed action chiral extrapolation for octet and decuplet
  - Open-minded for extrapolation to physical pion mass for other states
- In the future:
  - Lower pion masses to confirm chiral logarithm drops

The Future

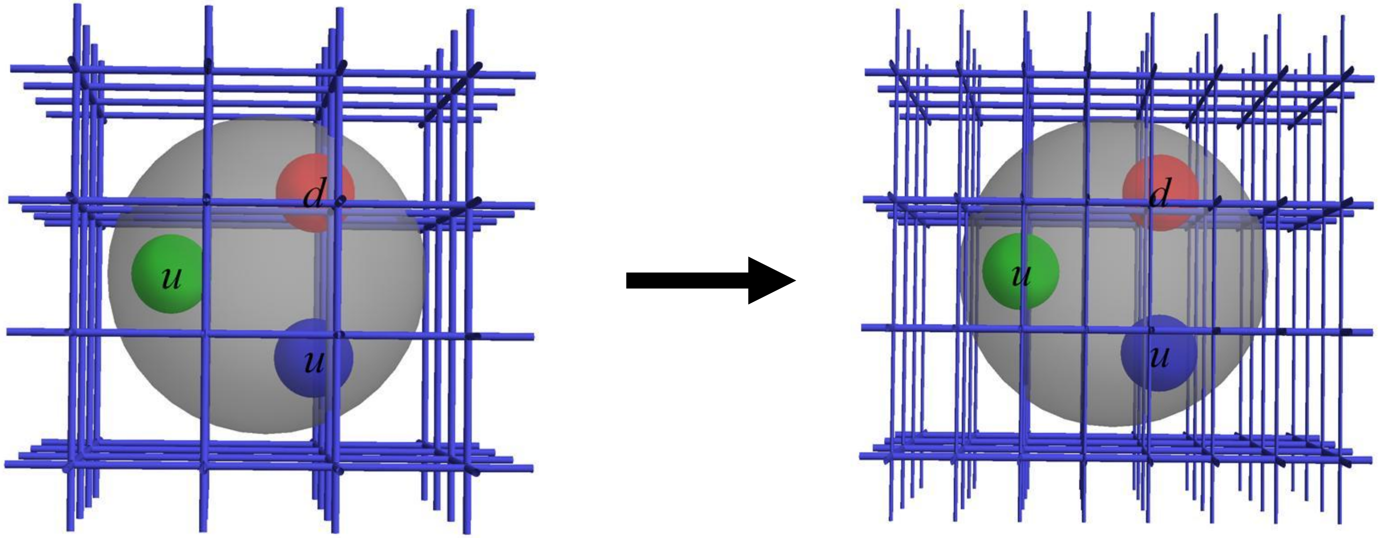
Physical-Pion Era

- Physical pion mass ensembles are near
- Chiral perturbation theory will no longer be a guide, but can be judged against predictions of QCD

- Three major gauge-generation projects within US community
  - Chiral fermions:
    DWF on Iwasaki gauge, 0.093 fm (RBC+LHPC+UKQCD)
    Designing next generation with IBM (RBC+UKQCD)
  - Staggered fermions: MILC
    Considering HISQ instead of the asqtad
  - Anisotropic clover lattices: LHPC
    2+1-flavor dynamical runs

Anisotropic Clover Fermions

- Solution: increase resolution



- Excited-state resonances and form factors
- Glueballs, hybrids, etc.
- Nucleon scattering, four-point Green functions
- Roadmap:
  - 2012: physical pion at $a \sim 0.10$ fm (72^3)
  - 2014: physical pion at $a \sim 0.08$ fm (96^3)