Strange Baryon Physics in Full Lattice QCD

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INT Seminar
INT & University of Washington
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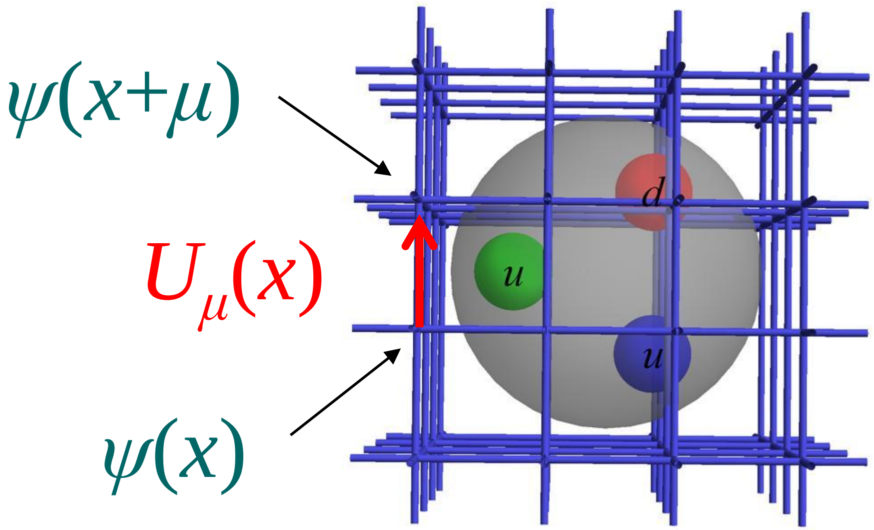
Outline

- Lattice QCD
  - Background, actions, observables, …
- Two-point Green functions
  - Group theory, operator design, spectroscopy results
- Three-point Green functions
  - Hyperon axial coupling constants
  - Strangeness in nucleon magnetic and electric moments
  - Hyperon semi-leptonic decays

Lattice QCD

- Lattice QCD is a discrete version of continuum QCD theory



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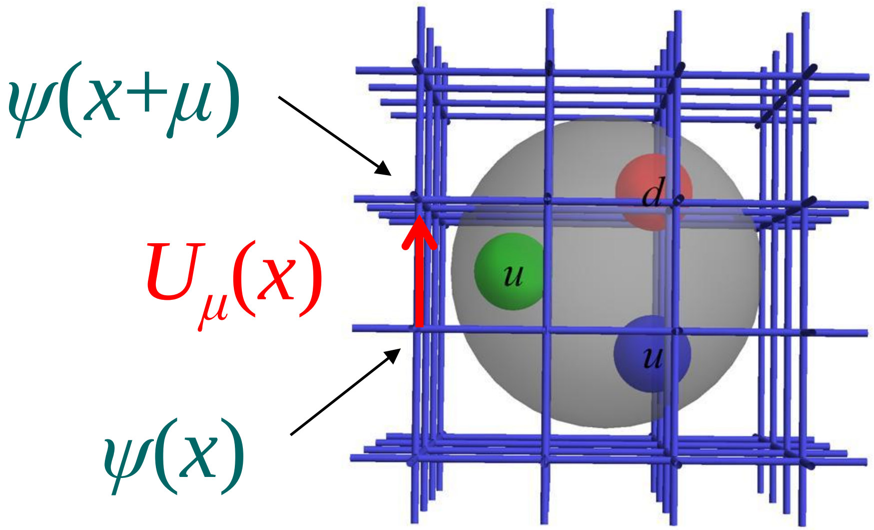


- Physical observables are calculated from the path integral

$$\langle 0|O(\overline{\psi},\psi,A)|0\rangle = \frac{1}{Z}\int [dA][d\overline{\psi}][d\psi]\, O(\overline{\psi},\psi,A)e^{i\int d^4x\, \mathcal{L}^{\mathrm{QCD}}(\overline{\psi},\psi,A)}$$

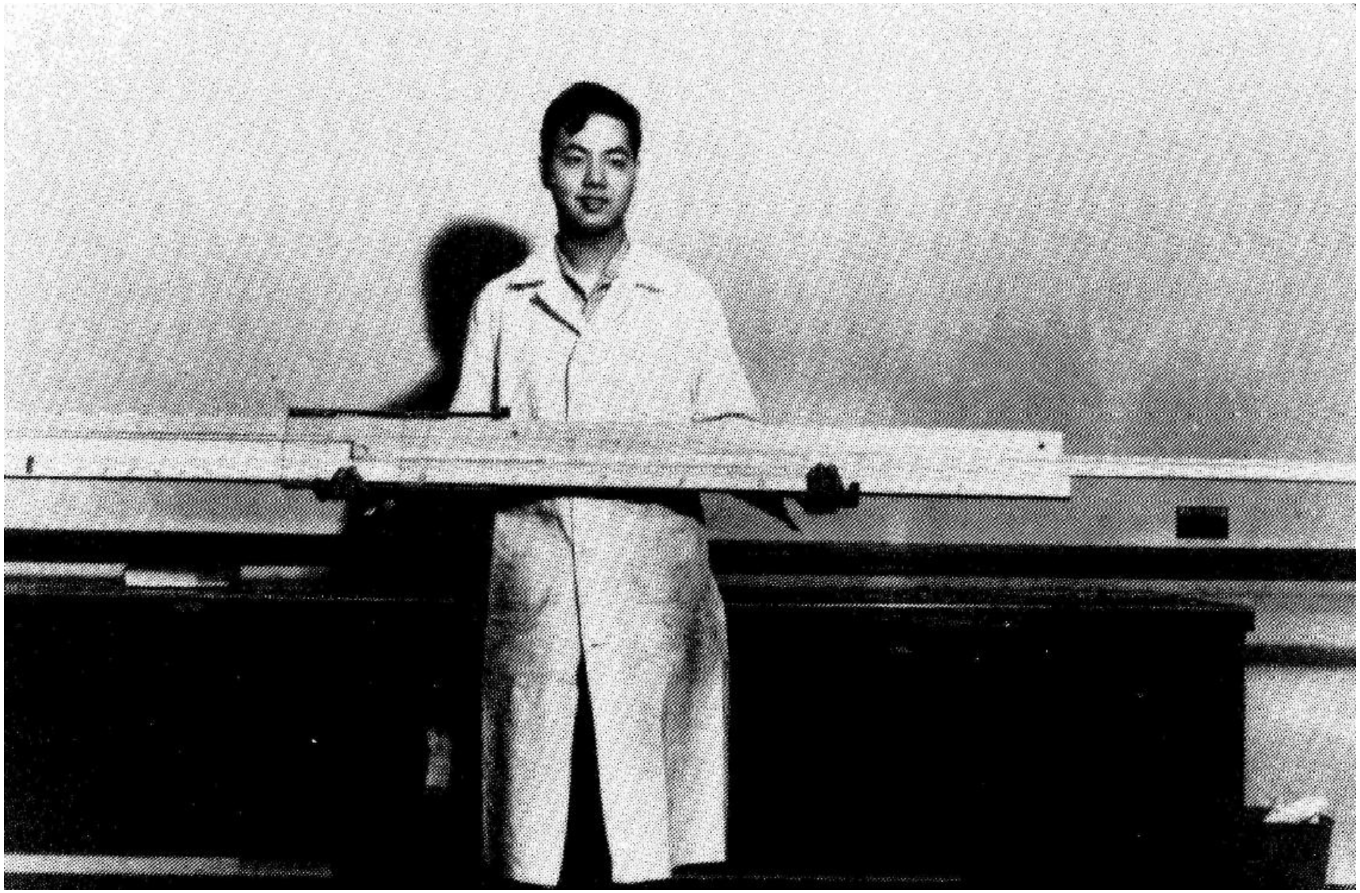
- Use Monte Carlo integration combined with the "importance sampling" technique to calculate the path integral.
- Take $a \to 0$ and $V \to \infty$ in the continuum limit

Lattice QCD

- A wide variety of first-principles QCD calculations can be done: In1970, Wilson started off by writing down the first actions
- Progress is limited by computational resources
  - But assisted by advances in algorithms

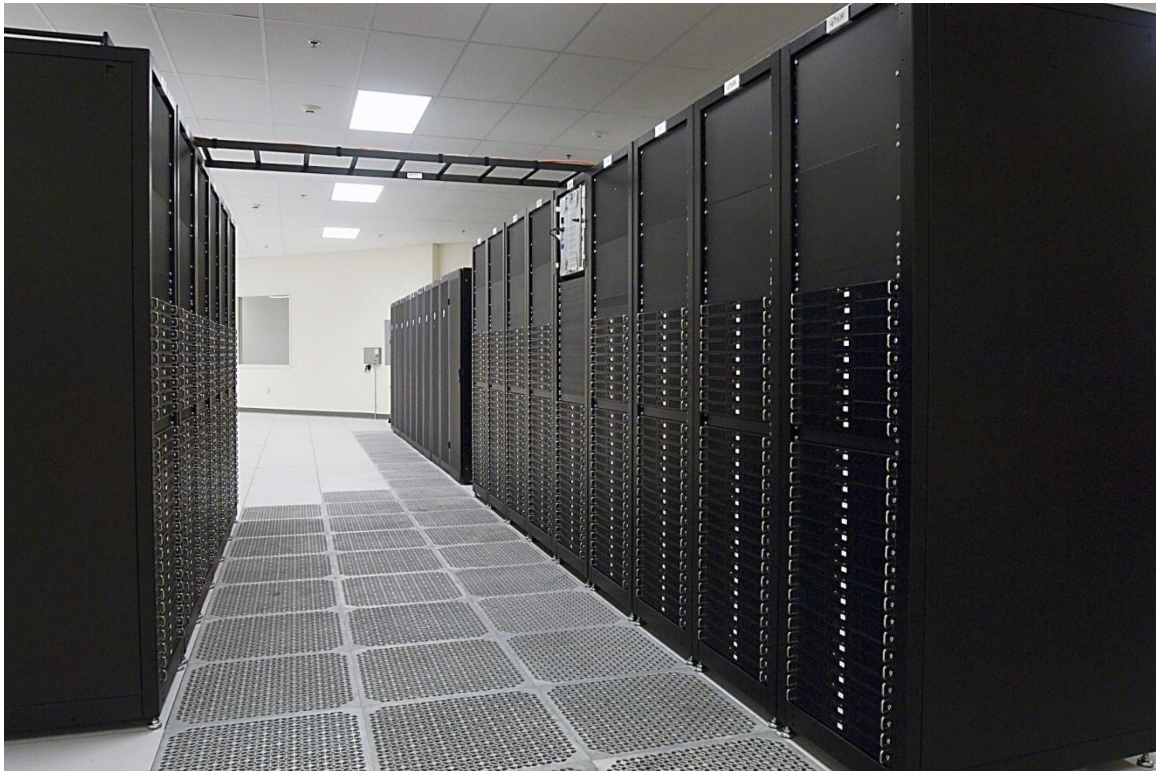
Lattice QCD

T.D. Lee uses an "analog computer" to calculate stellar radiative transfer equations



Lattice QCD

2007: The 13 Tflops cluster at Jefferson Lab



Other joint lattice resources within the US: Fermilab, BNL.

Non-lattice resources open to USQCD: ORNL, LLNL, ANL.

Lattice QCD

- Lattice QCD is computationally intensive

$$\text{Cost} \approx \left(\frac{L}{\text{fm}}\right)^5 L_s \left(\frac{\text{MeV}}{M_\pi}\right)\left(\frac{\text{fm}}{a}\right)^6 \left(C_0 + C_1\left(\frac{\text{fm}}{a}\right)\left(\frac{\text{MeV}}{M_K}\right)^2 + C_2\left(\frac{a}{\text{fm}}\right)^2\left(\frac{\text{MeV}}{M_\pi}\right)^2\right)$$

Norman Christ, LAT2007

- Current major US 2+1-flavor gauge ensemble generation:
  - MILC: staggered, $a \sim 0.06$ fm, $L \sim 3$ fm, $M_\pi \sim 250$ MeV
  - RBC+UKQCD: DWF, $a \sim 0.09$ fm, $L \sim 3$fm, $M_\pi \sim 330$ MeV
- Chiral domain-wall fermions (DWF) at large volume (6 fm) at physical pion mass may be expected in 2011

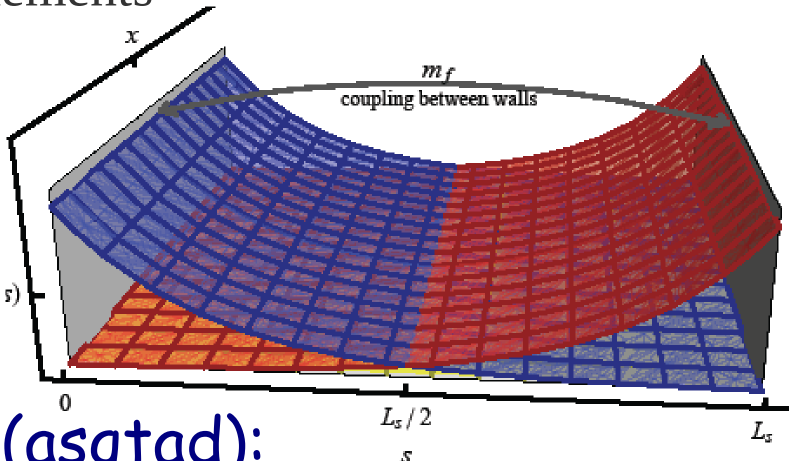
- But for now….
  need a pion mass extrapolation $M_\pi \to (M_\pi)_{\text{phys}}$
  (use chiral perturbation theory, if available)

Lattice Fermion Actions

- **Chiral fermions (e.g., Domain-Wall/Overlap):**
  - Automatically $O(a)$ improved,
    good for spin physics and weak matrix elements
  - Expensive

$$D_{x,s;x',s'} = \delta_{x,x'} D^{\perp}_{s,s'} + \delta_{s,s'} D^{\parallel}_{x,x'}$$

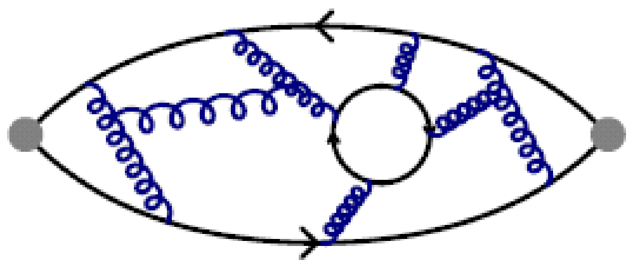
$$\begin{aligned} D^{\perp}_{s,s'} &= \frac{1}{2}[(1-\gamma_5)\delta_{s+1,s'} + (1+\gamma_5)\delta_{s-1,s'} - 2\delta_{s,s'}] \\ &- \frac{m_f}{2}[(1-\gamma_5)\delta_{s,L_s-1}\delta_{0,s'} + (1+\gamma_5)\delta_{s,0}\delta_{L_s-1,s'}], \end{aligned}$$

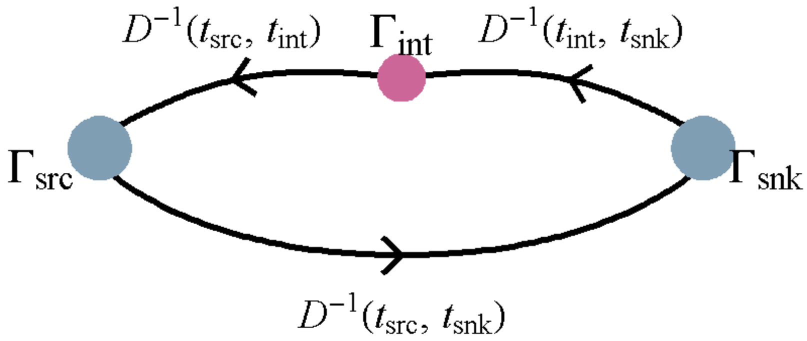


- **(Improved) Staggered fermions (asqtad):**
  - Relatively cheap for dynamical fermions (good)
  - Mixing among parities and flavors or "tastes"
  - Baryonic operators a nightmare — not suitable
- **Mixed action:**
  - Staggered sea (cheap) with domain-wall valence (chiral)
  - Match the sea Goldstone pion mass to the DWF pion
  - Only mixes with the "scalar" taste of sea pion
  - Free light quark propagators (LHPC & NPLQCD)

Lattice QCD: Observables

- Two-point Green function
  e.g. spectroscopy
- Three-point Green function
  e.g. form factors, structure functions, …





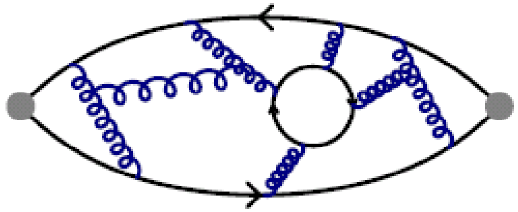
$$\sum_{\alpha,\beta} \Gamma^{\alpha,\beta} \langle J(X_{\rm snk}) J(X_{\rm src}) \rangle_{\alpha,\beta}$$

$$\sum_{\alpha,\beta} \Gamma^{\alpha,\beta} \langle J(X_{\rm snk}) O(X_{\rm int}) J(X_{\rm src}) \rangle_{\alpha,\beta}$$

Lattice QCD: Observables

- Two-point Green function

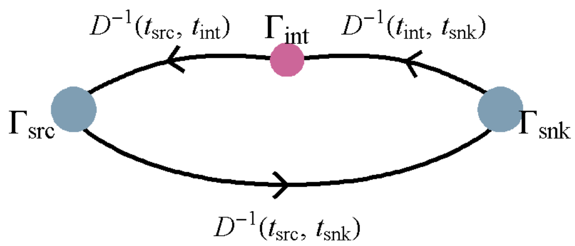
  e.g. spectroscopy



$$\sum_{\alpha,\beta}\Gamma^{\alpha,\beta}\langle J(X_{\rm snk})J(X_{\rm src})\rangle_{\alpha,\beta}$$

- Three-point Green function

  e.g. form factors. structure functions, …



$$\sum_{\alpha,\beta}\Gamma^{\alpha,\beta}\langle J(X_{\rm snk})O(X_{\rm int})J(X_{\rm src})\rangle_{\alpha,\beta}$$

After taking spin and momentum projection

(ignore the variety of boundary condition choices)

Two-point correlator

$$\sum_n Z_{n,B}\, e^{-E_n(\vec{P})t}$$

Three-point correlator

$$\sum_n\sum_{n'} Z_{n',B}(p_f)Z_{n,A}(p_i)$$

$$\times\text{FF's}\times e^{-(t_f-t)E'_n(\vec{p}_f)}e^{-(t-t_i)E_n(\vec{p}_i)}$$

At large enough *t*, the ground-state signal dominates

Two-Point Green Functions

work with

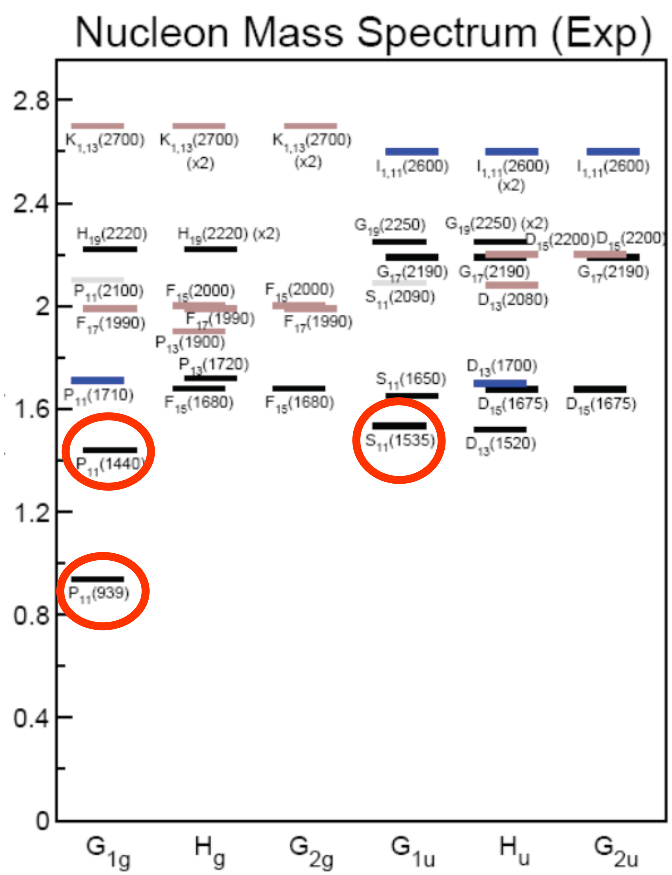
Lattice Hadron Physics Collaboration (LHPC)

Strange quark propagators from NPLQCD

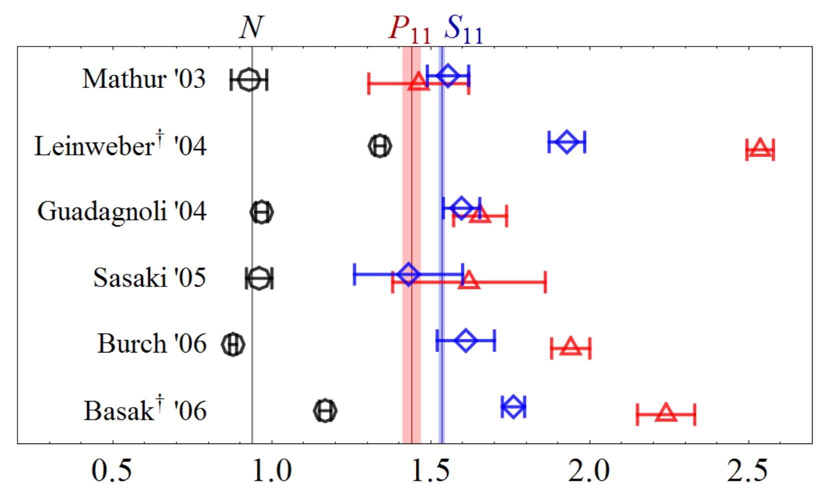
Why Baryons?

Lattice QCD spectrum

- Successfully calculates many ground states (Nature, …)
- Nucleon spectrum, on the other hand… not quite



Example: N, P_{11}, S_{11} spectrum



Strange Baryons

- Strange baryons are of special interest; challenging even to experiment
- Example from PDG Live:

Ξ BARYONS ($S = -2$, $I = 1/2$)

$\Xi^0 = uss$, $\Xi^- = dss$

Ξ^0	$1/2(1/2^+)$	****	$\Xi(1820)$ D_{13}	$1/2(3/2^-)$	***	$\Xi(2370)$	$1/2(?^?)$	•**
Ξ^-	$1/2(1/2^+)$	****	$\Xi(1950)$	$1/2(?^?)$	***	$\Xi(2500)$	$1/2(?^?)$	•*
$\Xi(1530)$ P_{13}	$1/2(3/2^+)$	****	$\Xi(2030)$	$1/2(\geq \frac{5}{2}^?)$	***			
$\Xi(1620)$	$1/2(?^?)$	•*	$\Xi(2120)$	$1/2(?^?)$	•*			
$\Xi(1690)$	$1/2(?^?)$	***	$\Xi(2250)$	$1/2(?^?)$	•**			

• — OMITTED FROM SUMMARY TABLE

Ω BARYONS ($S = -3$, $I = 0$)

$\Omega^- = sss$

Ω^-	$0(3/2^+)$	****
$\Omega(2250)^-$	$0(?^?)$	***
$\Omega(2380)^-$		•**
$\Omega(2470)^-$		•**

Operator Design

- All baryon spin states wanted: j = 1/2, 3/2, 5/2, …
- Rotation symmetry is reduced due to discretization

  rotation O(3) ⇒ octahedral O_h group

	I	J	$6\ C_4$	$8\ C_6$	$8\ C_3$	$6\ C_8$	$6\ C'_8$	$12\ C'_4$
A_1	1	1	1	1	1	1	1	1
A_2	1	3	−2	1	0	−1	1	0
E	2	1	1	1	−1	−1	−1	0
G_1	2	0	1	−1	1	−2	1	0
G_2	2	−4	0	1	0	0	1	−1
T_1	3	2	0	0	1	1	−1	−1
T_2	3	3	0	−1	−1	1	1	0
H	4	−3	−1	0	0	0	−1	1

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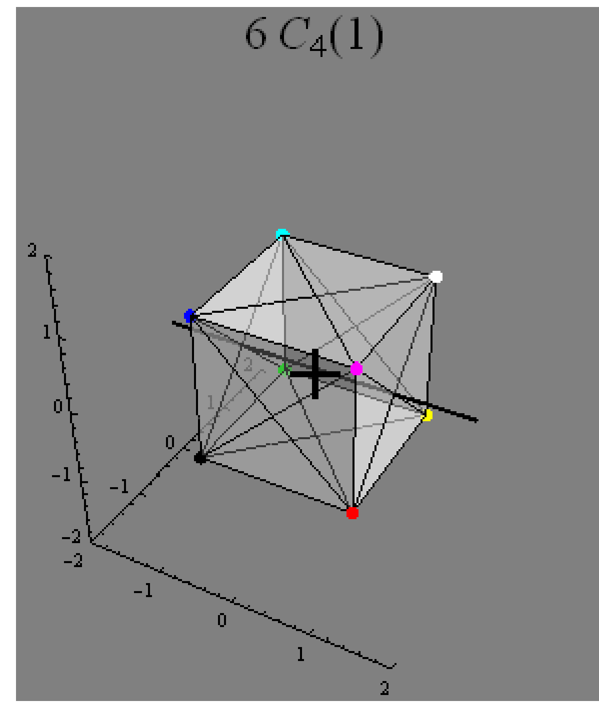
A_1	1	1	1	1	1	1	1	1
A_2	1	3	-2	1	0	-1	1	0
E	2	1	1	1	-1	-1	-1	0
G_1	2	0	1	-1	1	-2	1	0
G_2	2	-4	0	1	0	0	1	-1
T_1	3	2	0	0	1	1	-1	-1
T_2	3	3	0	-1	-1	1	1	0
H	4	-3	-1	0	0	0	-1	1



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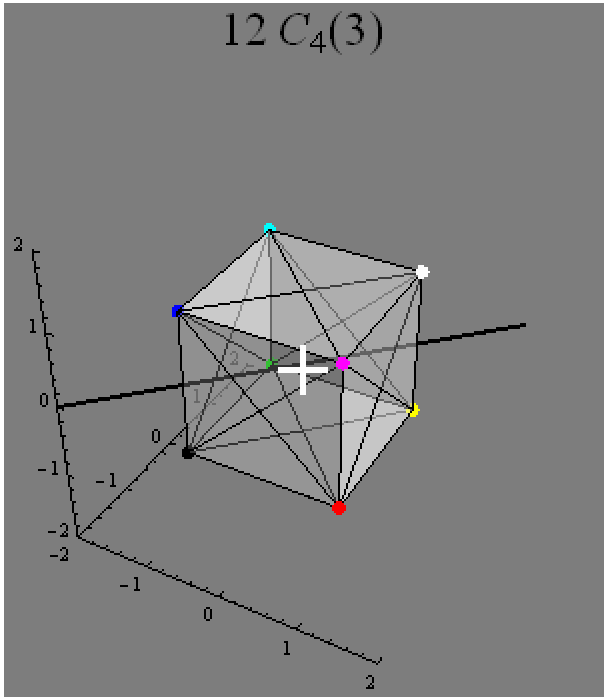
E	2	1	1	1	-1	-1	-1	0
G_1	2	0	1	-1	1	-2	1	0
G_2	2	-4	0	1	0	0	1	-1
T_1	3	2	0	0	1	1	-1	-1
T_2	3	3	0	-1	-1	1	1	0
H	4	-3	-1	0	0	0	-1	1



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G_1	2	0	1	−1	1	−2	1	0
G_2	2	−4	0	1	0	0	1	−1
T_1	3	2	0	0	1	1	−1	−1
T_2	3	3	0	−1	−1	1	1	0
H	4	−3	−1	0	0	0	−1	1

Baryons

j	Irreps
$\frac{1}{2}$	G_1
$\frac{3}{2}$	H
$\frac{5}{2}$	$G_2 \oplus H$
$\frac{7}{2}$	$G_1 \oplus G_2 \oplus H$
$\frac{9}{2}$	$G_1 \oplus 2\,H$
$\frac{11}{2}$	$G_1 \oplus G_2 \oplus 2\,H$
$\frac{13}{2}$	$G_1 \oplus 2\,G_2 \oplus 2\,H$
$\frac{15}{2}$	$G_1 \oplus G_2 \oplus 3\,H$
$\frac{17}{2}$	$2\,G_1 \oplus G_2 \oplus 3\,H$
$\frac{19}{2}$	$2\,G_1 \oplus 2\,G_2 \oplus 3\,H$
$\frac{21}{2}$	$G_1 \oplus 2\,G_2 \oplus 4\,H$
$\frac{23}{2}$	$2\,G_1 \oplus 2\,G_2 \oplus 4\,H$

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j	Irreps
$\frac{1}{2}$	G_1
$\frac{3}{2}$	H
[illegible]	[illegible]
$\frac{19}{2}$	$2\,G_1 \oplus 2\,G_2 \oplus 3\,H$
$\frac{21}{2}$	$G_1 \oplus 2\,G_2 \oplus 4\,H$
$\frac{23}{2}$	$2\,G_1 \oplus 2\,G_2 \oplus 4\,H$

	I	J	6 C_4	8 C_6	8 C_3	6 C_8	6 C_8'	12 C_4'
A_1	1	1	1	1	1	1	1	1
A_2	1	3	-2	1	0	-1	1	0
E	2	1	1	1	-1	-1	-1	0
G_1	2	0	1	-1	1	-2	1	0
G_2	2	-4	0	1	0	0	1	-1
T_1	3	2	0	0	1	1	-1	-1
T_2	3	3	0	-1	-1	1	1	0
H	4	-3	-1	0	0	0	-1	1

This calculation:

Three quarks in a baryon located at a single site

Baryons

- For more details and extended-link operators:

  S. Basak et al., Phys. Rev. D72, 094506 (2005)

Variational Method

- Construct the correlator matrix

$$C_\Lambda^{m,n}(t) = \sum_{\vec{x}} \sum_{\lambda} \langle 0 \mid B_\lambda^{\Lambda,m}(\vec{x},t)\bar{B}_\lambda^{\Lambda,n}(0) \mid 0 \rangle$$

Flavor	$G_{1g/u}(2)$	$H_{g/u}(4)$
N	3	1
Δ	1	2
Λ	4	1
Σ	4	3
Ξ	4	3
Ω	1	2

- Construct the matrix

$$C_{i\ j}(t) = \langle 0 \mid \mathcal{O}_i(t)^\dagger \mathcal{O}_j(0) \mid 0 \rangle$$

- Solve for the generalized eigensystem of

$$C(t)\psi = \lambda(t,t_0)C(t_0)\psi$$

with eigenvalues

$$\lambda_n(t,t_0) = e^{-(t-t_0)E_n}(1 + \mathcal{O}(e^{-|\delta E|(t-t_0)}))$$

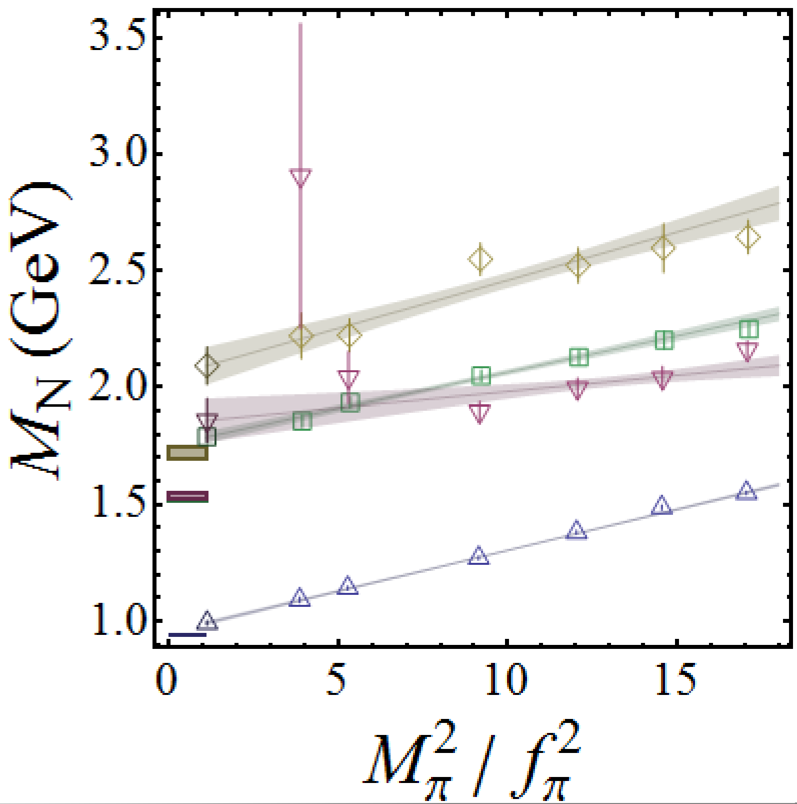
C. Michael, Nucl. Phys. B 259, 58 (1985)

M. Lüscher and U. Wolff, Nucl. Phys. B 339, 222 (1990)

- At large *t*, the signal of the desired state dominates.
- Unfortunately, we cannot see a clear radial excited state with the smeared propagators we got for free.

General Spectroscopy

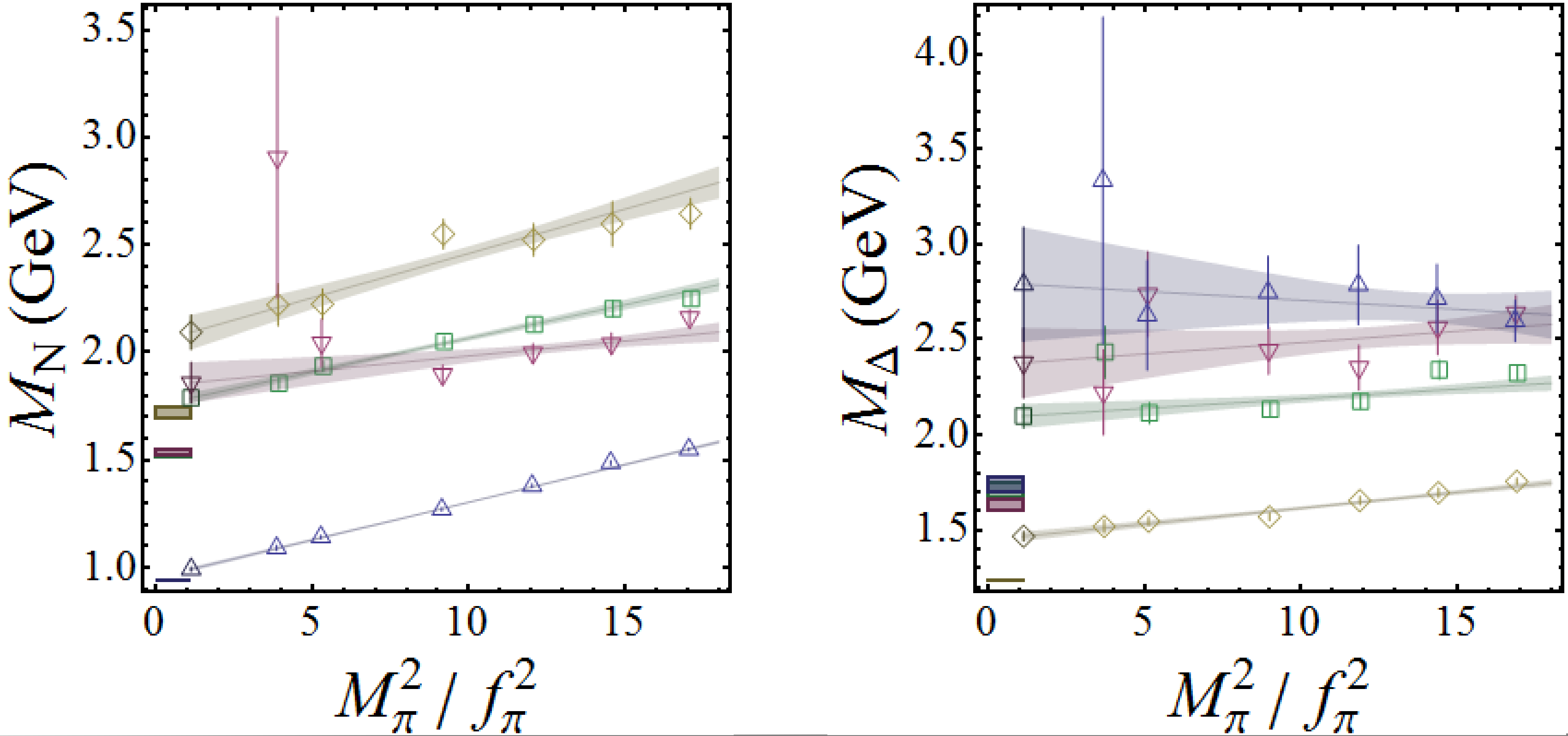
- The non-strange baryons (N)
- Symbols: J^P = 1/2⁺ △ , 1/2⁻ ▽ , 3/2⁺ ◇ , 3/2⁻ □
  N $N(1535)$ $N(1720)$ $N(1520)$



General Spectroscopy

- The non-strange baryons (*N* and Δ)
- Symbols: J^P = 1/2⁺ △, 1/2⁻ ▽ , 3/2⁺ ◇, 3/2⁻ □

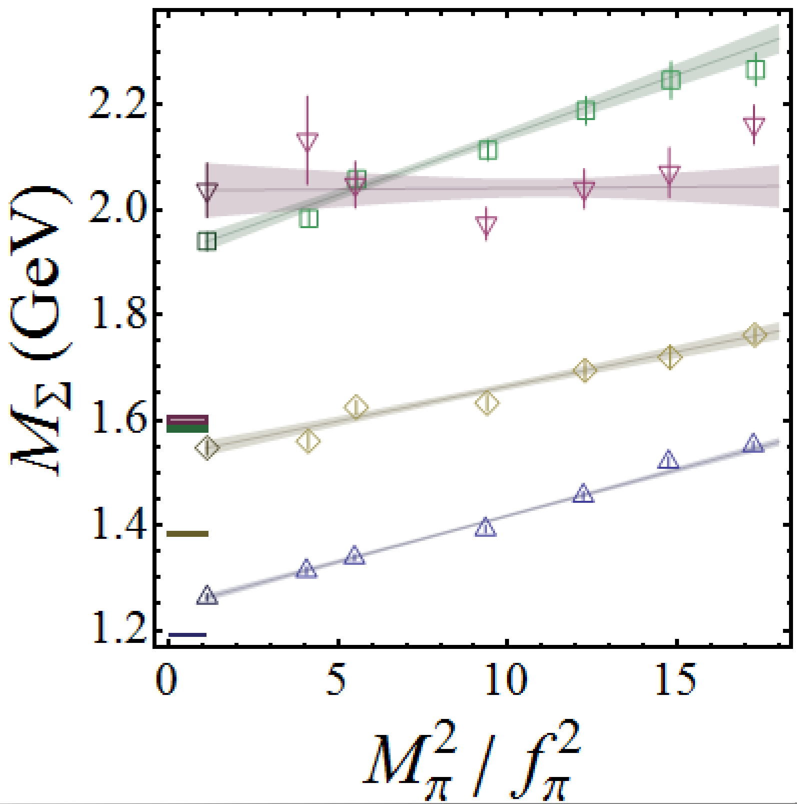
1/2⁺	1/2⁻	3/2⁺	3/2⁻
N	*N*(1535)	*N*(1720)	*N*(1520)
	Δ(1620)	Δ	Δ(1700)

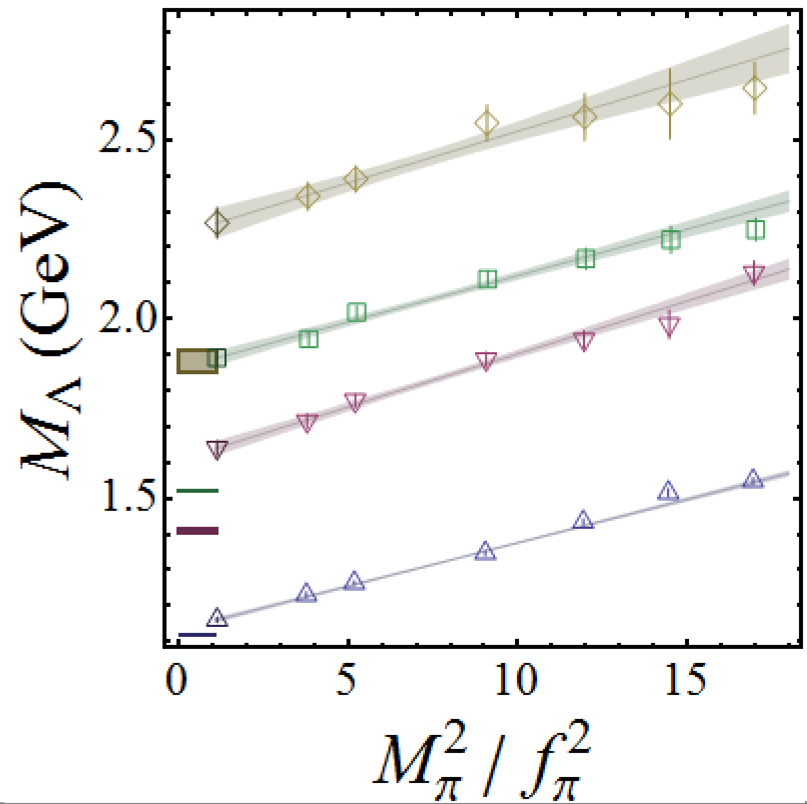


General Spectroscopy

- The singly strange baryons: (Σ and Λ)
- Symbols: $J^P = 1/2^+$ △, $1/2^-$ ▽, $3/2^+$ ◇, $3/2^-$ □

$1/2^+$	$1/2^-$	$3/2^+$	$3/2^-$
Σ	Σ(1620)	Σ^*	Σ(1580)
Λ	Λ(1405)	Λ(1890)	Λ(1520)

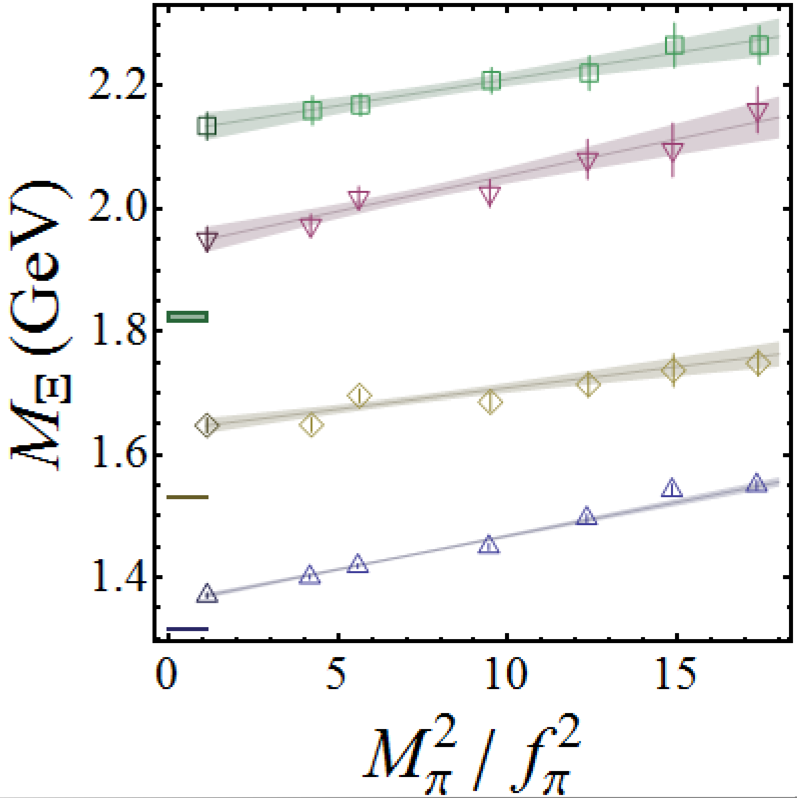




General Spectroscopy

- The less known baryons (Ξ)
- Symbols: J^P = 1/2⁺ △ , 1/2⁻ ▽ , 3/2⁺ ◇ , 3/2⁻ □

Ξ Ξ(1690)? Ξ(1530) Ξ(1820)



General Spectroscopy

- The less known baryons (Ξ)
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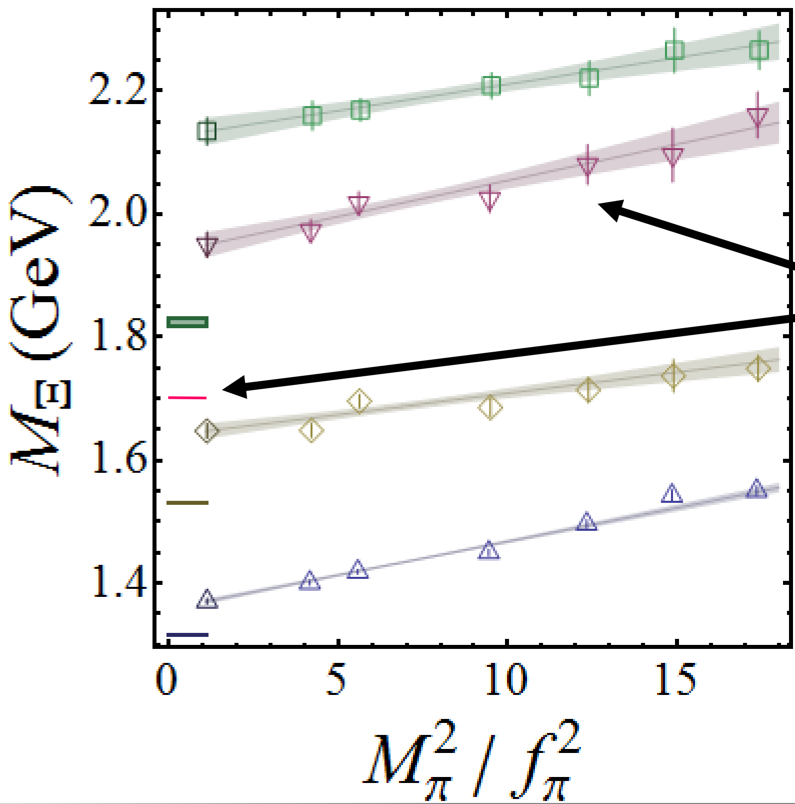


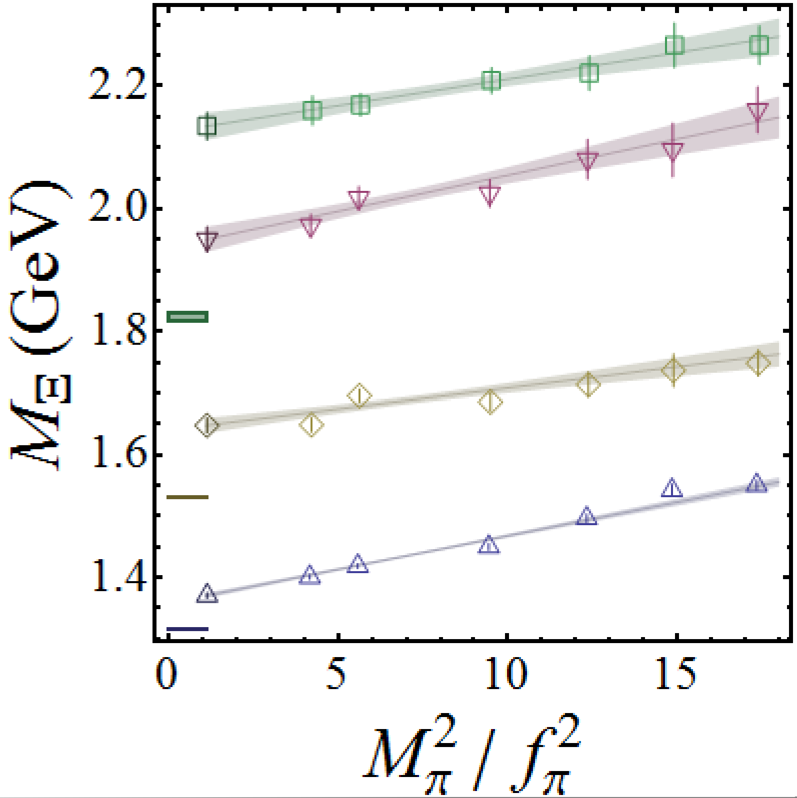
- Babar at MENU 2007: Ξ(1690)⁰ negative parity −1/2

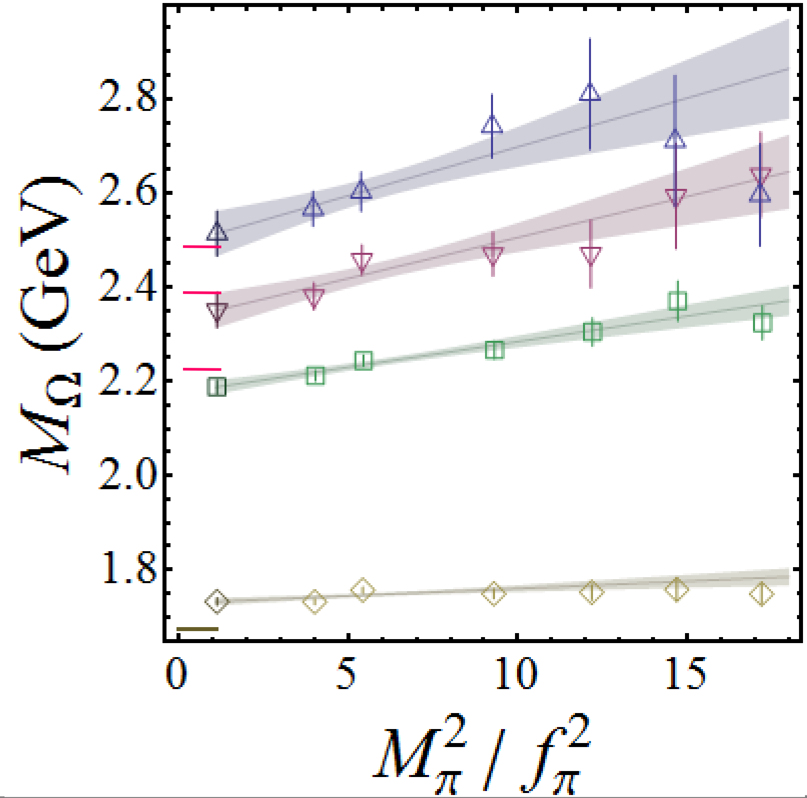
General Spectroscopy

- The less known baryons (Ξ and Ω)
- Symbols: J^P = 1/2$^+$ △, 1/2$^-$ ▽ , 3/2$^+$ ◇, 3/2$^-$ □

  Ξ Ξ(1690)? Ξ(1530) Ξ(1820)

  Could they be Ω(2250), Ω(2380), Ω(2470)?





Multiplet Mass Relations

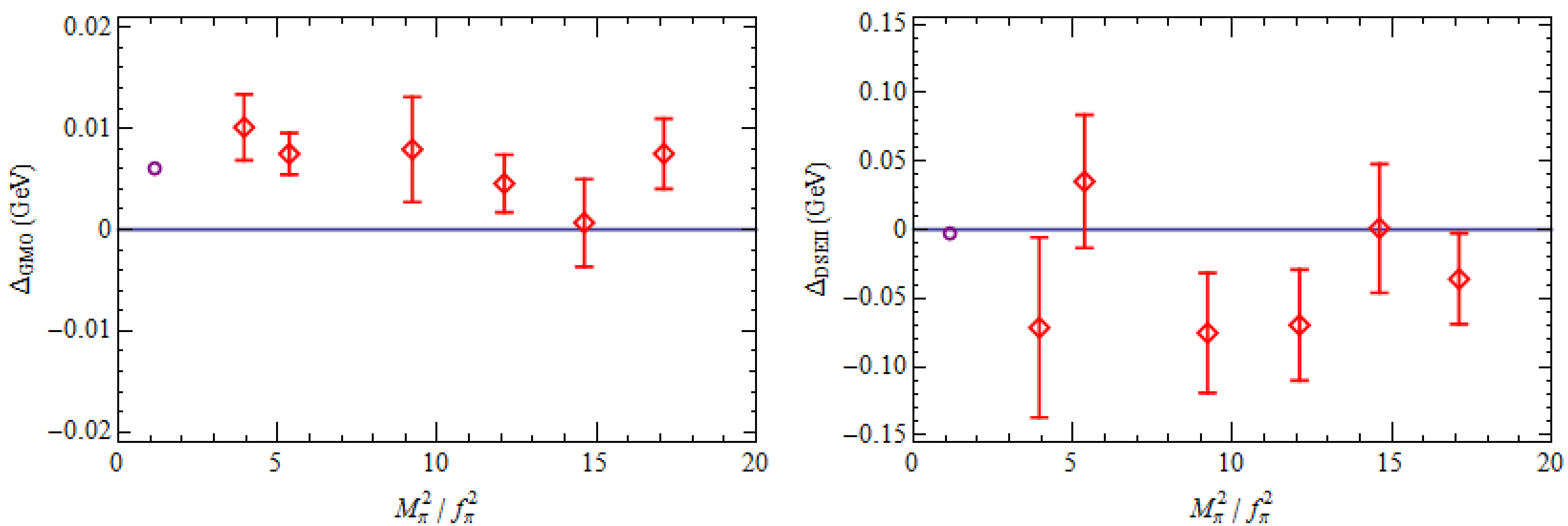
- SU(3) flavor symmetry breaking
  - Gell-Mann-Okubo relation

$$\Delta_{GMO} = \frac{3}{4}M_\Lambda + \frac{1}{4}M_\Sigma - \frac{1}{2}M_N - \frac{1}{2}M_\Xi$$

  - Decuplet Equal Spacing relation

$$\Delta_{DESII} = \frac{1}{2}(M_{\Sigma^*} - M_\Delta) + \frac{1}{2}(M_\Omega - M_{\Xi^*}) - M_{\Xi^*} + M_{\Sigma^*}$$

- Mass differences are close to experimental numbers



Summary/Outlook — I

- What we have done:
  - 2+1-flavor calculations with volume around 2.6 fm
  - Ground states of $G_{1g/u}$ and $H_{g/u}$ for each flavor
  - Preliminary study with lightest pion mass 300 MeV
  - Correct mass-ordering pattern is seen
- Currently in progress:
  - Mixed action chiral extrapolation for octet and decuplet
  - Open-minded for extrapolation to physical pion mass for other states
- In the future:
  - Lower pion masses to confirm chiral logarithm drops
  - Finer lattice spacing for excited states

Three-Point Green Functions

in collaboration with

Kostas Orginos

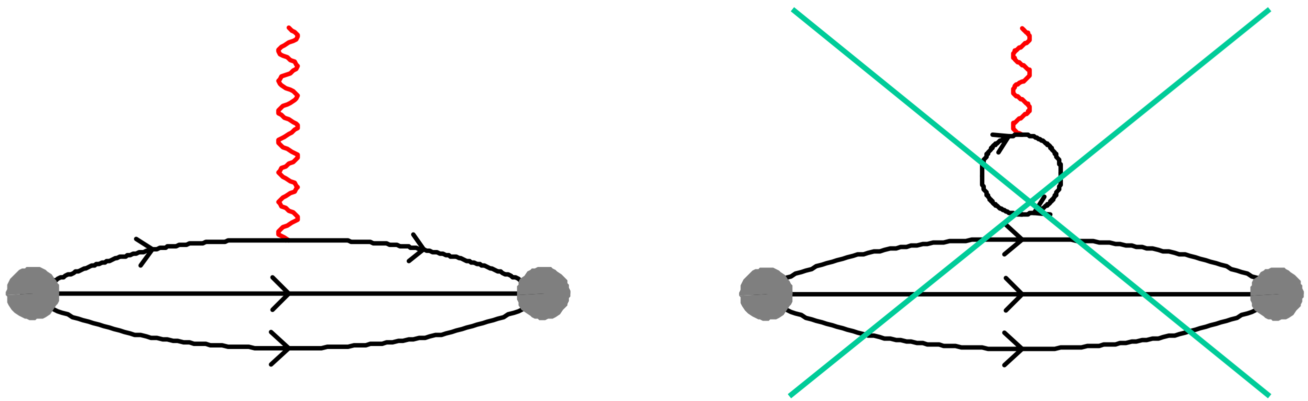
- Hyperon axial coupling constants
- Strangeness in nucleon magnetic and electric moments
- Semi-leptonic decays

Green Functions

- Three-point function with connected piece only

$$C^{\Gamma,\mathcal{O}}_{3\text{pt}}\left(\vec{p},t,\tau\right) = \sum_{\alpha,\beta} \Gamma^{\alpha,\beta}\langle J_\beta\left(\vec{p},t\right)\mathcal{O}(\tau)\overline{J}_\alpha\left(\vec{p},0\right)\rangle$$

- Two constructions:



- Isovector quantities
- Use ratios to cancel out the unwanted factors

$$\frac{\Gamma^{BB}_{\mu,GG}(t_i,t,t_f,\vec{p}_i,\vec{p}_f;T)}{\Gamma^{BB}_{GG}(t_i,t_f,\vec{p}_f;T)}\sqrt{\frac{\Gamma^{BB}_{PG}(t,t_f,\vec{p}_i;T)}{\Gamma^{BB}_{PG}(t,t_f,\vec{p}_f;T)}}\sqrt{\frac{\Gamma^{BB}_{GG}(t_i,t,\vec{p}_f;T)}{\Gamma^{BB}_{GG}(t_i,t,\vec{p}_i;T)}}\sqrt{\frac{\Gamma^{BB}_{PG}(t_i,t_f,\vec{p}_f;T)}{\Gamma^{BB}_{PG}(t_i,t_f,\vec{p}_i;T)}}$$

Axial Coupling Constants: $g_{\Xi\Xi}$ and $g_{\Sigma\Sigma}$

- Defined $\langle B|A_\mu(q)|B\rangle = \overline{u}_B(p')\left[\gamma_\mu\gamma_5 G_A(q^2) + \gamma_5 q_\nu \frac{G_P(q^2)}{2M_B}\right]u_B(p)e^{-iq\cdot x}$
- Has applications such as hyperon scattering, non-leptonic decays, …
- Cannot be determined by experiment
- Existing theoretical predictions:
  - Chiral perturbation theory

$$0.35 \le g_{\Sigma\Sigma} \le 0.55 \qquad 0.18 \le -g_{\Xi\Xi} \le 0.36$$

M. J. Savage et al., Phys. Rev. D55, 5376 (1997);

  - Large-N_c

$$0.30 \le g_{\Sigma\Sigma} \le 0.36 \qquad 0.26 \le -g_{\Xi\Xi} \le 0.30$$

R. Flores-Mendieta et al., Phys. Rev. D58, 094028 (1998);

- Loose bounds on the values
- Lattice QCD can provide substantial improvement

Axial Coupling Constants: $g_{\Xi\Xi}$ and $g_{\Sigma\Sigma}$

- Pion mass: 350–750 MeV

HWL and K. Orginos, arXiv:0712.1214

- First lattice calculation of these quantities; mixed-action full-QCD

	m010	m020	m030	m040	m050
m_π (MeV)	354.2(8)	493.6(6)	594.2(8)	685.4(19)	754.3(16)
m_π/f_π	2.316(7)	3.035(7)	3.478(8)	3.822(23)	4.136(20)
m_K/f_π	3.951(14)	3.969(10)	4.018(11)	4.060(26)	4.107(21)
confs	612	345	561	320	342
$g_{A,N}$	1.22(8)	1.21(5)	1.195(17)	1.150(17)	1.167(11)
$g_{\Sigma\Sigma}$	0.418(23)	0.450(15)	0.451(7)	0.444(8)	0.453(5)
$g_{\Xi\Xi}$	−0.262(13)	−0.270(10)	−0.269(7)	−0.257(9)	−0.261(7)

- Combine with g_A for study of
  - SU(3) symmetry breaking
  - SU(3) simultaneous fits among three coupling constants

    $\Longrightarrow$ *D*, *F*, and other low-energy constants

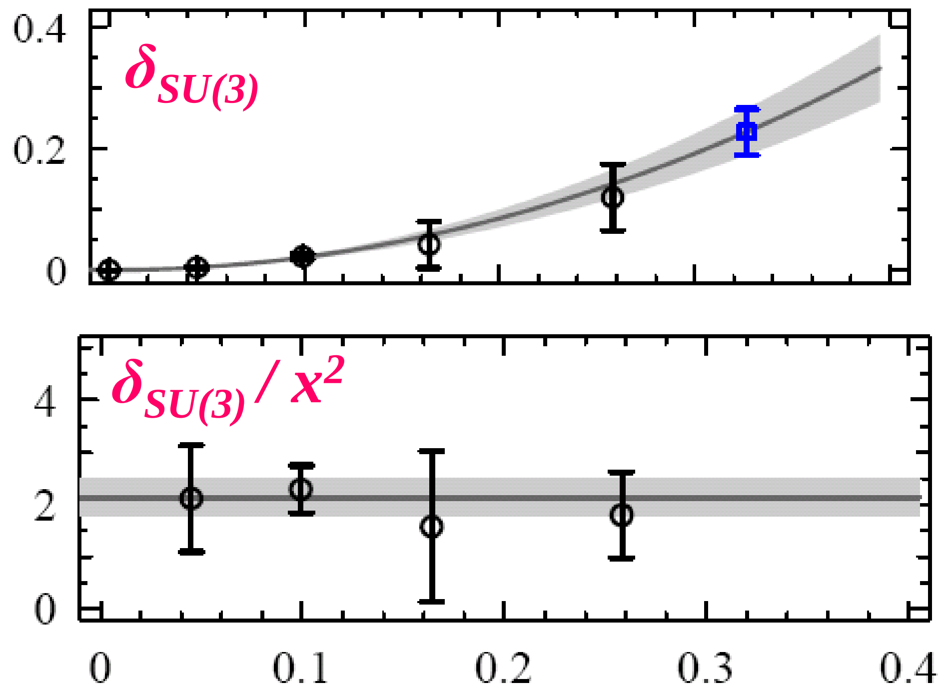
Axial Coupling Constants: $g_{\Xi\Xi}$ and $g_{\Sigma\Sigma}$

- SU(3) symmetry breaking

$$\delta_{\mathrm{SU}(3)} = g_{\mathrm{A}} - 2.0 \times g_{\Sigma\Sigma} + g_{\Xi\Xi}$$

$$= \sum_n c_n x^n \quad \text{with} \quad x = (m_K^2 - m_\pi^2)/(4\pi f_\pi^2)$$

- Quadratic behaviour is observed



- Not predicted by any theorem nor chiral perturbation theory $\Longrightarrow$ coincidence?

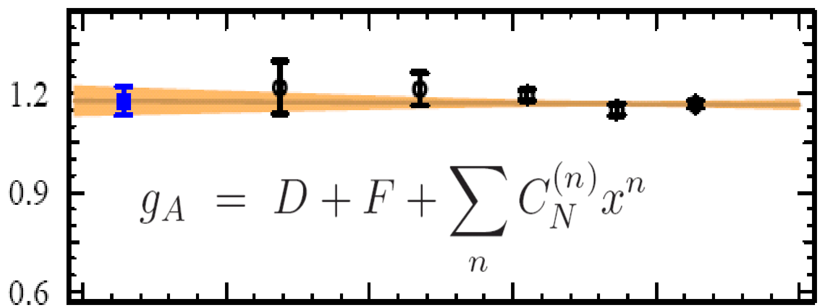
- 20% breaking at physical point

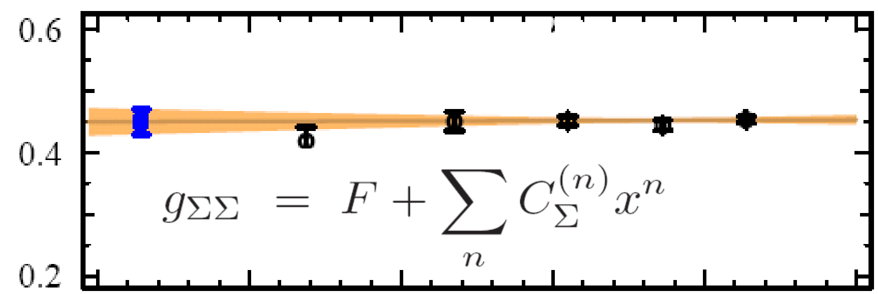
Axial Coupling Constants: $g_{\Xi\Xi}$ and $g_{\Sigma\Sigma}$

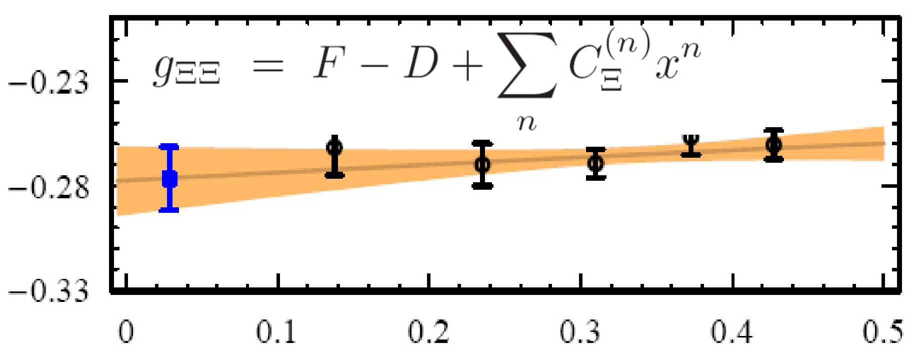
- Simultaneous SU(3) fit
  - SU(3) chiral perturbation theory (with 8 parameters)

    W. Detmold and C. J. D. Lin, Phys. Rev. D71, 054510 (2005)

    which fails to describe the data
  - Simple chiral form







- Systematic errors: finite volume + finite *a*

$$g_A = 1.18(4)_{\rm stat}(6)_{\rm syst}$$
$$g_{\Sigma\Sigma} = 0.450(21)_{\rm stat}(27)_{\rm syst}$$
$$g_{\Xi\Xi} = -0.277(15)_{\rm stat}(19)_{\rm syst}$$

$$D = 0.715(6)_{\rm stat}(29)_{\rm syst} \quad F = 0.453(5)_{\rm stat}(19)_{\rm syst}$$

Strange Magnetic Moment of Nucleon

- Purely sea-quark effect
- First strange magnetic moment was measured by SAMPLE

$$G_M^s(Q^2 = 0.1\ GeV^2) = 0.23(37)(25)(29)$$

B. Mueller et al. (SAMPLE) Phys. Rev. Lett. 78, 3824 (1997)

- More data is being collected today

HAPPEX and G0 collaborations at Jefferson Lab, SAMPLE at MIT-BATES, and A4 at Mainz

- Lattice calculations

$$\langle B\,|V_\mu|\,B\rangle(q) = \overline{u}_B(p')\left[\gamma_\mu F_1(q^2) + \sigma_{\mu\nu} q_\nu \frac{F_2(q^2)}{2M_B}\right] u_B(p) e^{-iq\cdot x}$$

the disconnected diagram is a must

- Noisy estimator — Kentucky Field Theory group (1997–2001)

−0.28(10) to +0.05(6)

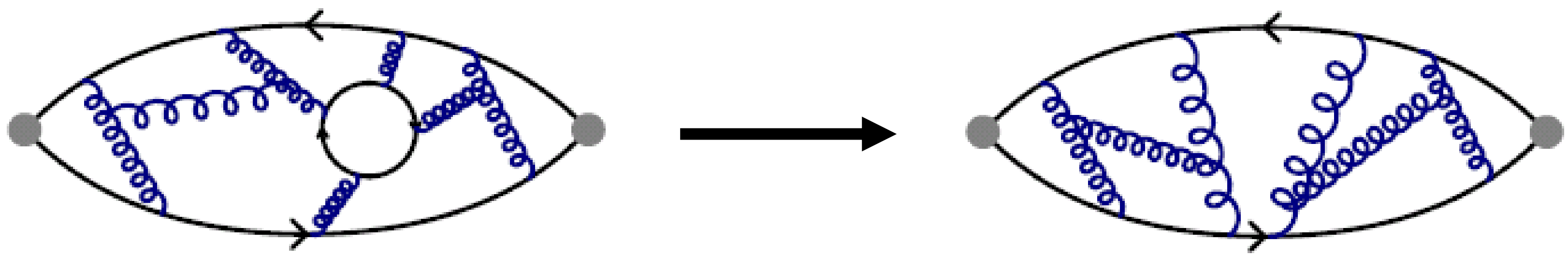
- Help with chiral perturbation theory — Adelaide-JLab group (2006)

−0.046(19)

in quenched approximation

Quenched Approximation

- Full QCD: $$\langle O \rangle = \frac{1}{Z}\int [dU][d\psi][d\overline{\psi}]e^{-S_F(U,\psi,\overline{\psi})-S_G(U)}O(U,\psi,\overline{\psi}) = \frac{1}{Z}\int [dU]\det M e^{-S_G(U)}O(U)$$
- Quenched: Take det M = constant.



- Historically used due to the lack of computation power

- Bad: Uncontrollable systematic error
- Good? Cheap exploratory studies to develop new methods

Strange Magnetic Moment of Nucleon

- Disconnected diagrams are challenging
- Much effort has been put into resolving this difficulty
- Alternative approach:

D. B. Leinweber, Phys. Rev. D 53, 5115 (1996).

  - Assume charge symmetry:

$$p = e_u u^p + e_d d^p + O_N; \qquad n = e_d u^p + e_u d^p + O_N,$$
$$\Sigma^+ = e_u u^\Sigma + e_s s^\Sigma + O_\Sigma; \qquad \Sigma^- = e_d u^\Sigma + e_s s^\Sigma + O_\Sigma,$$
$$\Xi^0 = e_s s^\Xi + e_u u^\Xi + O_\Xi; \qquad \Xi^- = e_s s^\Xi + e_d u^\Xi + O_\Xi.$$

  - The disconnected piece for the proton is $O_N = \frac{2}{3}\,{}^lG_M^u - \frac{1}{3}\,{}^lG_M^d - \frac{1}{3}\,{}^lG_M^s$
  - The strangeness contribution is

$$G_M^s = \left(\frac{{}^lR_d^s}{1 - {}^lR_d^s}\right)\left[\boxed{2p + n} - \frac{u^p}{u^\Sigma}\boxed{(\Sigma^+ - \Sigma^-)}\right]$$

$$G_M^s = \left(\frac{{}^lR_d^s}{1 - {}^lR_d^s}\right)\left[\boxed{p + 2n} - \frac{u^n}{u^\Xi}\boxed{(\Xi^0 - \Xi^-)}\right] \quad \text{with} \quad {}^lR_d^s \equiv {}^lG_M^s / {}^lG_M^d$$

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  - The strangeness contribution is

$$G_M^s = \left(\frac{{}^lR_d^s}{1 - {}^lR_d^s}\right)\left[3.673 - \frac{u^p}{u^\Sigma}(3.618)\right]\mu_N$$

Needs better statistics

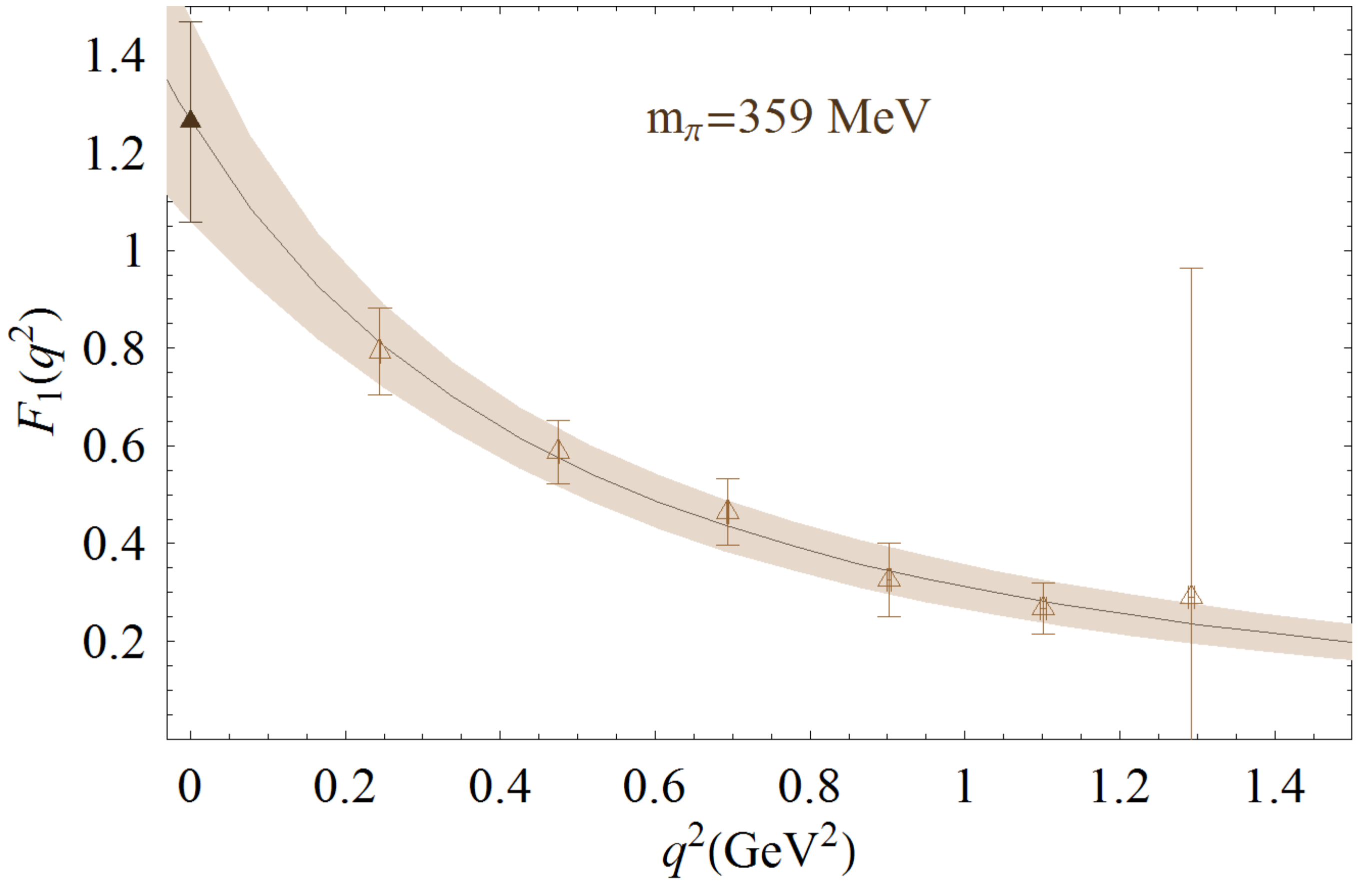
$$G_M^s = \left(\frac{{}^lR_d^s}{1 - {}^lR_d^s}\right)\left[-1.033 - \frac{u^n}{u^\Xi}(-0.599)\right]\mu_N$$

with $\quad {}^lR_d^s \equiv {}^lG_M^s / {}^lG_M^d$

Strange Magnetic Moment of Nucleon

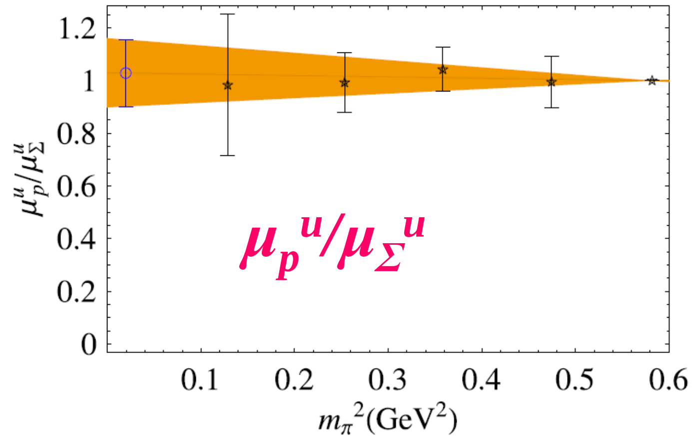
- Magnetic moment $\mu_B = F_2(q^2=0)$
- Dipole-form extrapolation to $q^2 = 0$

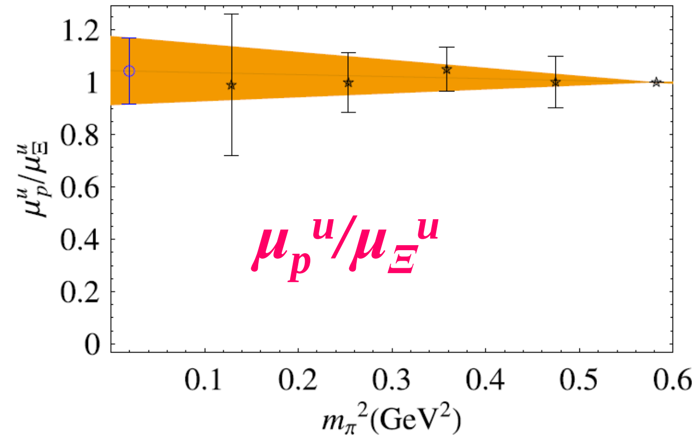
  Example: u-quark contribution in Σ form factor $F_2(q^2)$

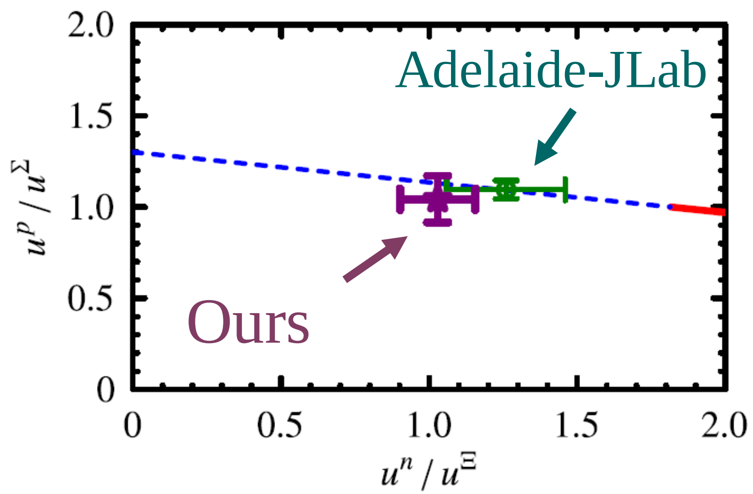


Strange Magnetic Moment of Nucleon

- Dipole-form extrapolation to $q^2 = 0$
- Magnetic-moment ratios (linear extrapolation, for now)







D. B. Leinweber et al., Phys. Rev. Lett. 94, 212001 (2004).

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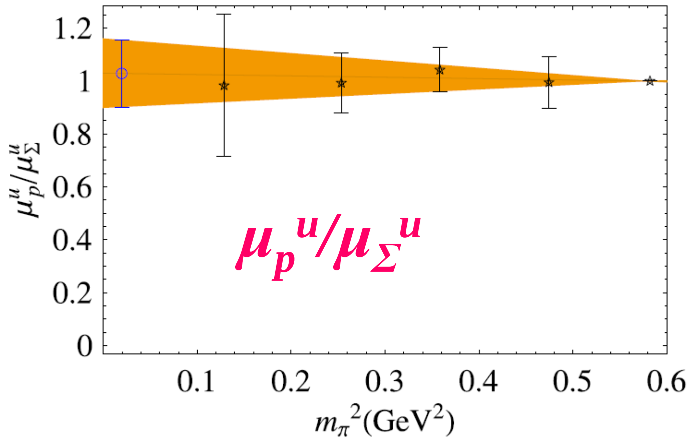
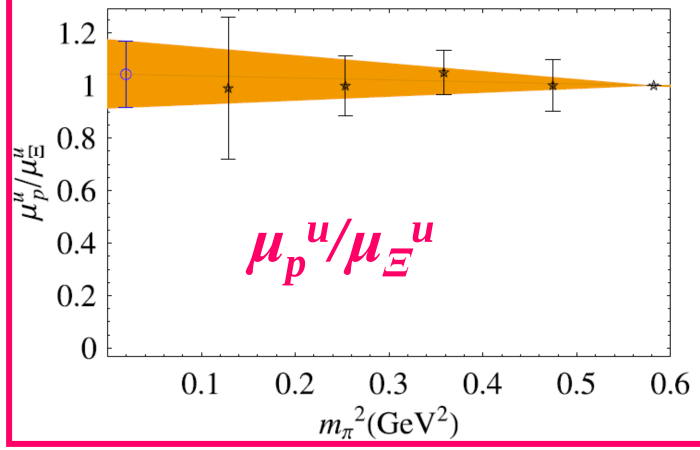
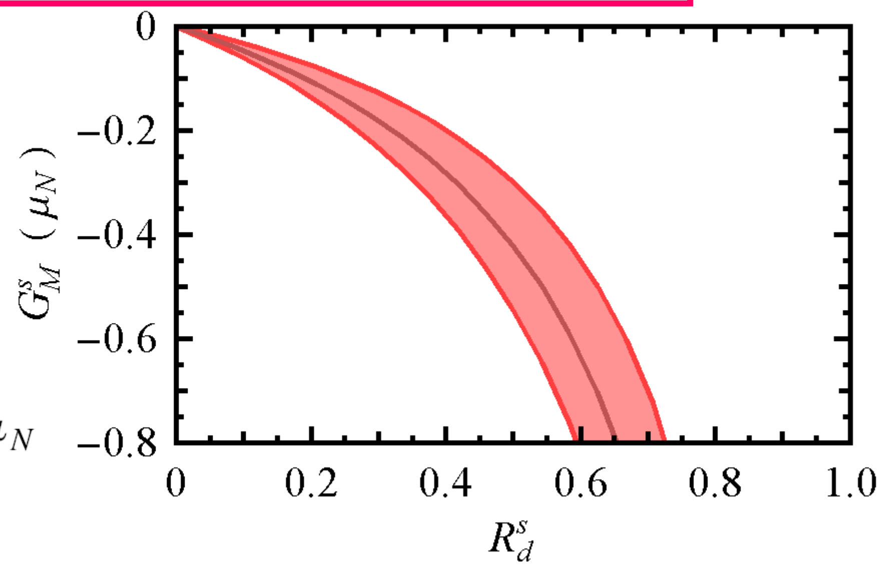




$$G_M^s = \left(\frac{{}^l R_d^s}{1 - {}^l R_d^s}\right)\left[-1.033 - \frac{u^n}{u^\Xi}(-0.599)\right]\mu_N$$



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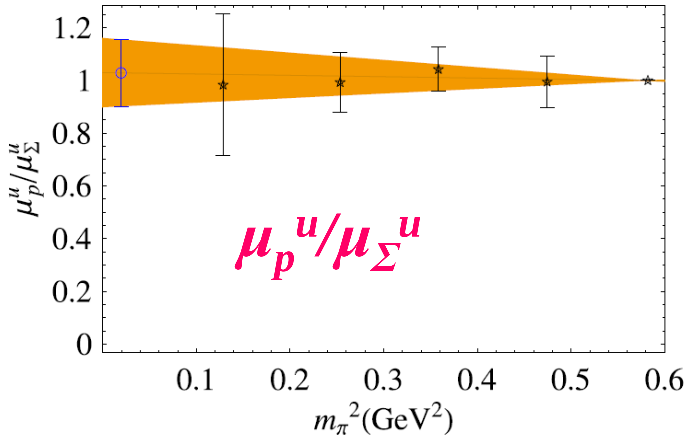
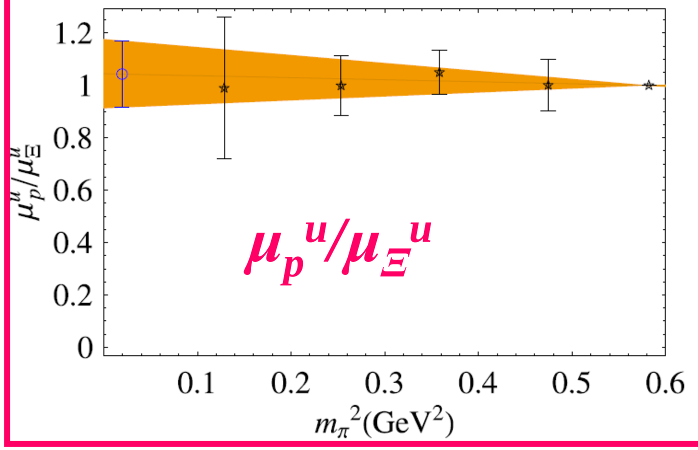


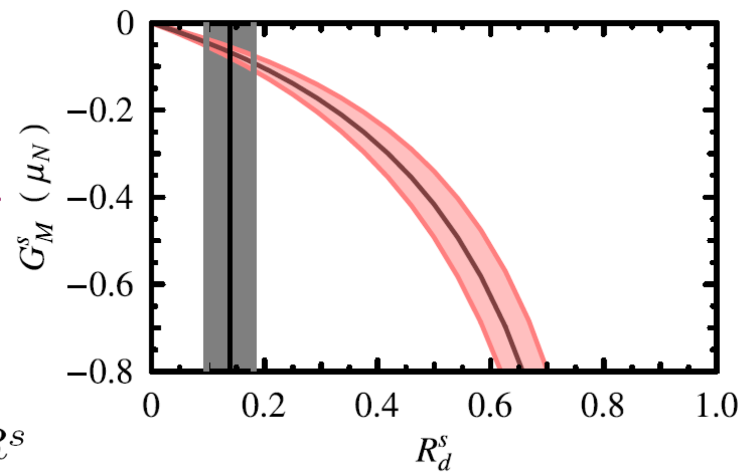


$$R_d^s = 0.139(42)$$

D. B. Leinweber et al.,
Phys. Rev. Lett. 94, 212001 (2004).

We find

$$G_M^s = -0.066(12)_{\mathrm{stat}}(23)_{R^s}$$



HWL, arXiv:0707.3844 [hep-lat]

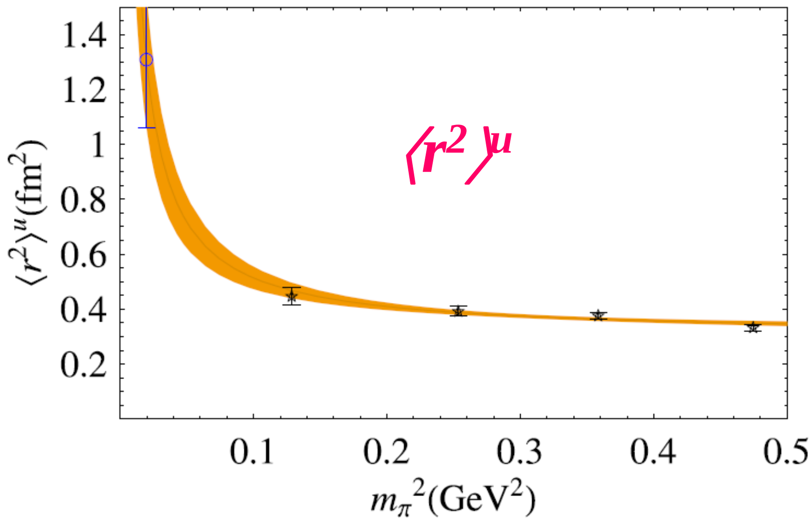
Strange Electric Moment of Nucleon

- G_E^s is proportional to $Q^2\langle r^2\rangle^s$
- Charge symmetry: D. B. Leinweber et al., Phys. Rev. Lett. 94, 212001 (2004).

$$\langle r^2\rangle^s = \frac{r_d^s}{1-r_d^s}\left[2\langle r^2\rangle^p + \langle r^2\rangle^n - \langle r^2\rangle^u\right] \qquad r_d^s = 0.16(4)$$

- u-quark form contribution of vector form factors

HWL, arXiv:0707.3844[hep-lat]



- Need more literature research on chiral extrapolation
- Using an extrapolation of the form

$$\langle r^2\rangle^u = a_0 - \frac{1+5g_A^2}{(4\pi f_\pi)^2}\log\left(\frac{m_\pi^2}{m_\pi^2+\Lambda^2}\right)$$

We find

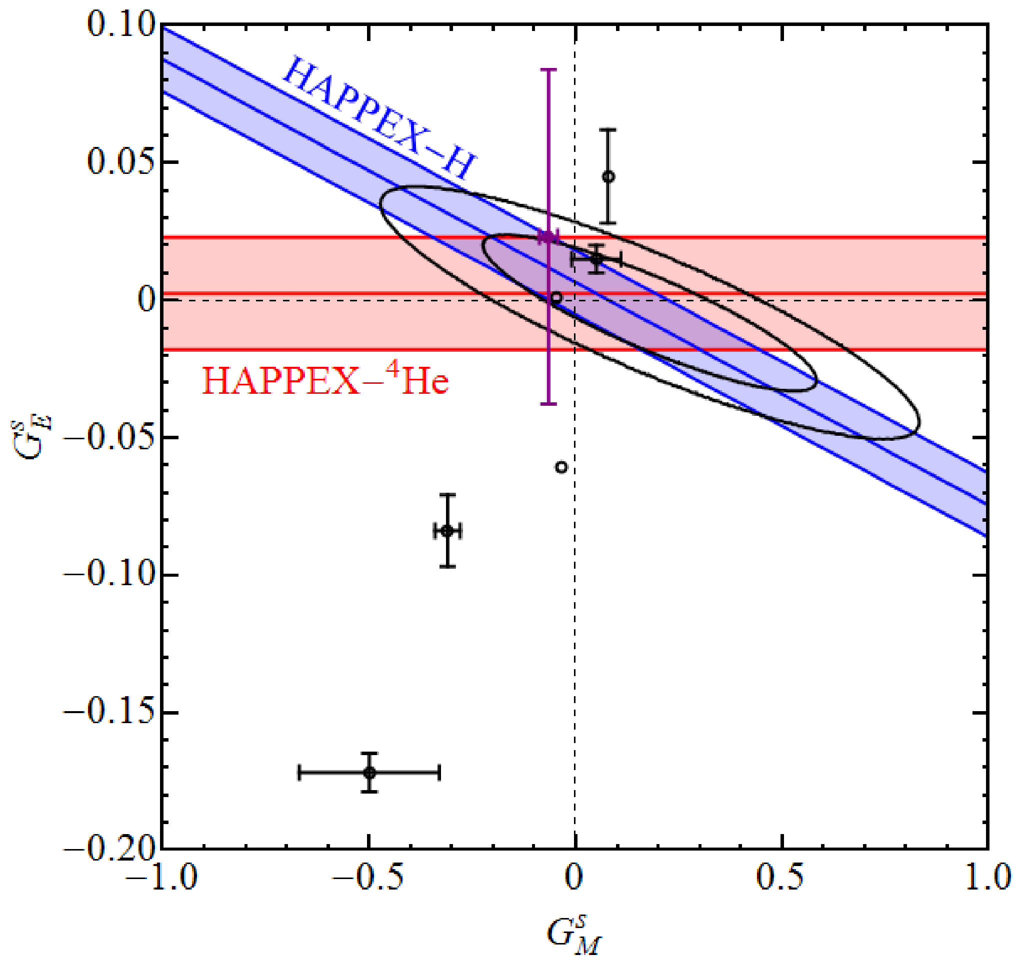
$$G_E^s(Q^2 = 0.1\ \mathrm{GeV}) = 0.022(61)$$

Preliminary

Strangeness

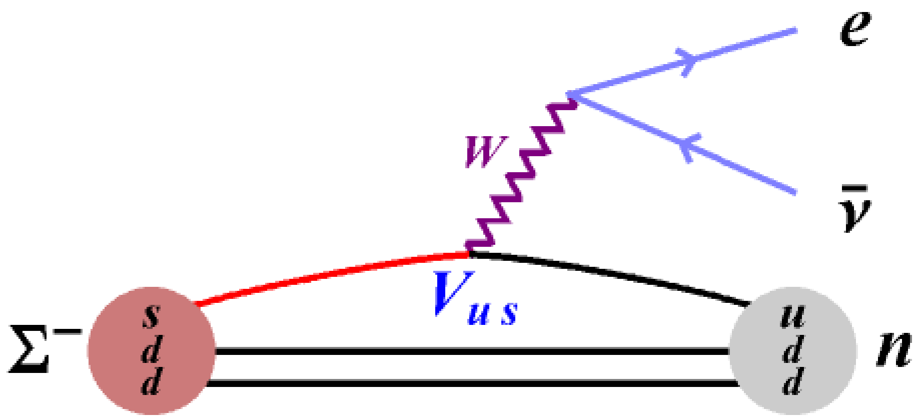
- G_E^s-G_M^s plots

A. Acha et al., Happex Collaboration, Phys.Rev.Lett.98:032301, 2007



Hyperon Decays

- Matrix element of the hyperon β-decay process $B_1 \to B_2 e^- \overline{\nu}$



$$\mathcal{M} = \frac{G_s}{\sqrt{2}} \overline{u}_{B_2} (O_\alpha^{\rm V} + O_\alpha^{\rm A}) u_{B_1} \overline{u}_e \gamma^\alpha (1+\gamma_5) v_\nu$$

with

$$O_\alpha^{\rm V} = f_1(q^2)\gamma^\alpha + \frac{f_2(q^2)}{M_{B_1}} \sigma_{\alpha\beta} q^\beta + \frac{f_3(q^2)}{M_{B_2}} q_\alpha$$

$$O_\alpha^{\rm A} = \left(g_1(q^2)\gamma^\alpha + \frac{g_2(q^2)}{M_{B_1}} \sigma_{\alpha\beta} q^\beta + \frac{g_3(q^2)}{M_{B_2}} q_\alpha \right) \gamma_5$$

Hyperon Decay Experiments

- Experiments: CERN WA2, Fermilab E715, BNL AGS, Fermilab KTeV, CERN NA48

- Summary N. Cabibbo et al. 2003
  with f_2/f_1 and f_1 at the SU(3) limit

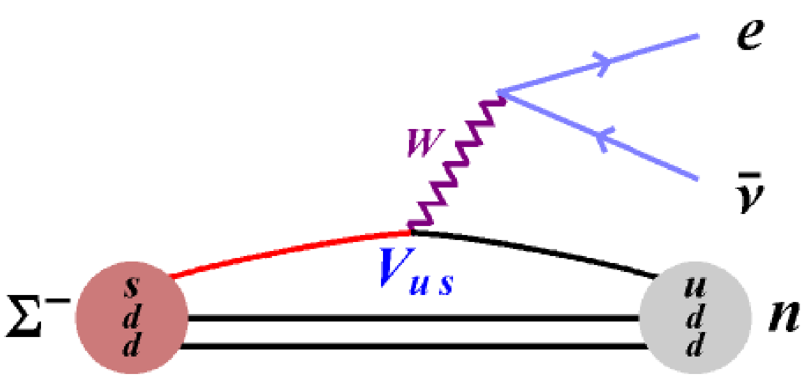
Decay	**Rate (μs^{-1})**	g_1/f_1	V_{us}
$\Lambda \to p e^- \bar{\nu}$	3.161(58)	0.718(15)	0.2224 ± 0.0034
$\Sigma^- \to n e^- \bar{\nu}$	6.88(24)	−0.340(17)	0.2282 ± 0.0049
$\Xi^- \to \Lambda e^- \bar{\nu}$	3.44(19)	0.25(5)	0.2367 ± 0.0099
$\Xi^0 \to \Sigma^+ e^- \bar{\nu}$	0.876(71)	1.32(+.22/−.18)	0.209 ± 0.027
Combined	—	—	0.2250 ± 0.0027

PDG 2006 number

- Better g_1/f_1 from lattice calculations?

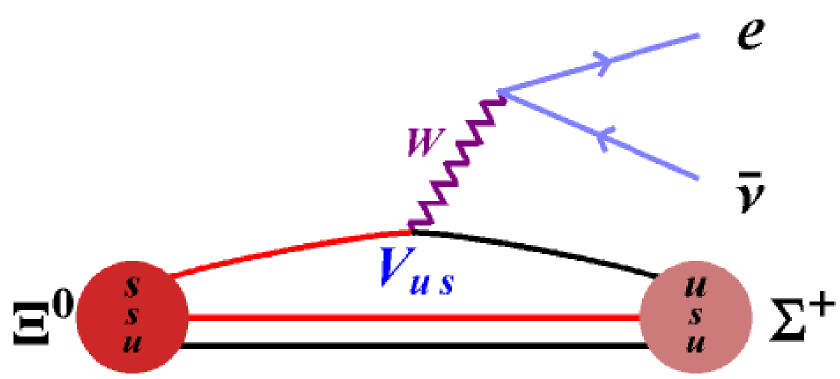
$|V_{us}|$ from Hyperons Decays

- Two quenched calculations, different channels



- Pion mass > 700 MeV
- $f_1(0) = -0.988(29)_{\text{stat.}}$
- $|V_{us}| = 0.230(5)_{\text{exp}}(7)_{\text{lat}}$

Guadagnoli et al.



- Pion mass ≈ 530–650 MeV
- $f_1(0) = 0.953(24)_{\text{stat}}$
- $|V_{us}| = 0.219(27)_{\text{exp}}(5)_{\text{lat}}$

Sasaki et al.

No systematic error estimate from **quenching** effects!

Ademollo-Gatto Theorem

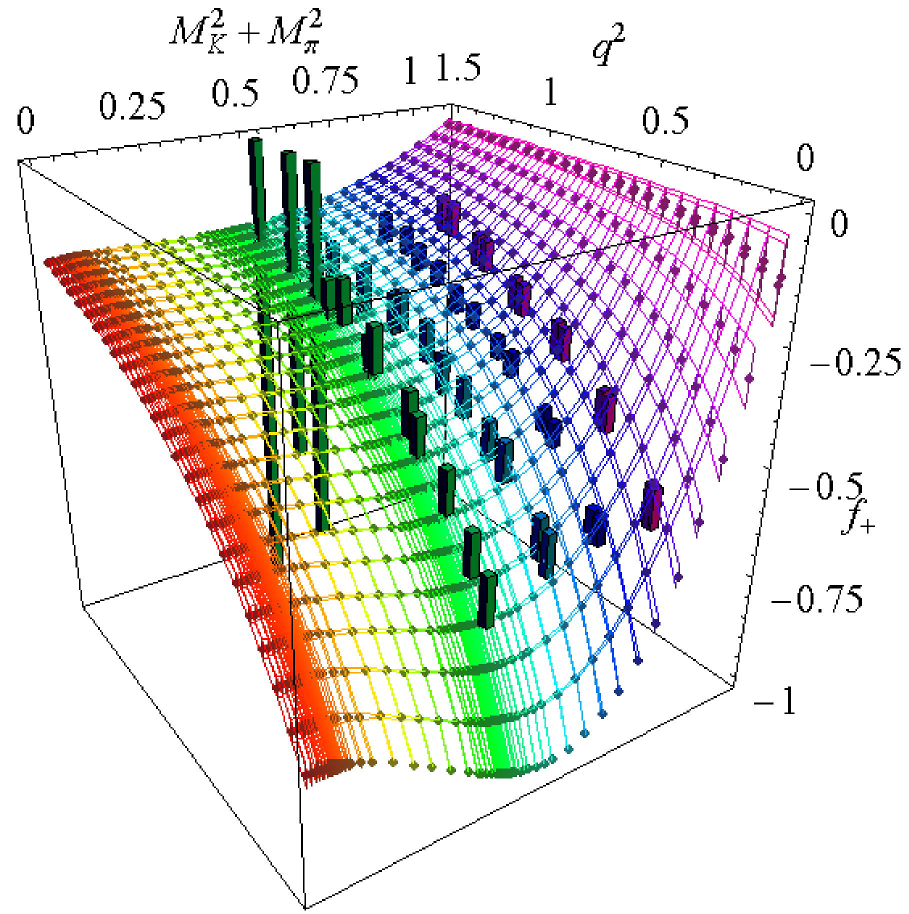
- Chiral extrapolation:
  - SU(3) symmetry-breaking Hamiltonian
    $$H' = \frac{1}{\sqrt{3}}\left(m_s - \frac{m_d + m_u}{2}\right)\bar{q}\lambda^8 q$$
  - There is no first-order correction $O(H')$ to $f_1(0)$; thus
    $$f_1(0) = f_1^{SU(3)}(0) + O(H'^2)$$
  - Common choice of observable for H' : $M_K{}^2 - M_\pi{}^2$
  - Step I: $$R(M_K, M_\pi) = \frac{1 - |f'(0)|}{a^4(M_K^2 - M_\pi^2)^2}$$
  - Step II: $$R(M_K, M_\pi) = b_0 + b_1 a^2(M_K^2 + M_\pi^2)$$
- Obtain $|V_{us}|$ from

$$\Gamma = G_F^2 |V_{us}|^2 \frac{\Delta m^5}{60\pi^3}(1 + \delta_{\rm rad})$$
$$\times\left[\left(1 - \frac{3}{2}\beta\right)\left(|f_1|^2 + |g_1|^2\right) + \frac{6}{7}\beta^2\left(|f_1|^2 + 2|g_1|^2 + {\rm Re}(f_1 f_2^*) + \frac{2}{3}|f_2^2|\right) + \delta_{q^2}\right]$$

with g_1/f_1 (exp) and f_2/f_1 (SU(3) value)

Simultaneous Fit

- Combined momentum and mass dependence



$f_1(0) = -0.88(15)$ (Preliminary)

Summary/Outlook — II

- From hyperon analysis
    - Predictions for $g_{\Sigma\Sigma}$ = 0.450(21)(27) and $g_{\Xi\Xi}$ = −0.277(15)(19)
    - Preliminary **proton strange magnetic and electric moments** directly from full QCD: −0.066(12)(23) and −0.022(61)
    - Looking for improvements in G_E^s
- More work to be done in hyperon semi-leptonic decay
    - First dynamical calculation
    - Preliminary result from Lin-Orginos is consistent with the previous calculation
    - We need much higher statistics for a lighter-pion mass calculation (compared with the quenched one)
    - Higher precision g_1/f_1:
      Will make the $|V_{us}|$ equivalent to or better than the one from K_{l3} channel