Nucleon Physics from Lattice QCD (2)

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Outline

- Lecture #1
  - Lattice QCD overview
    Background, actions, observables
  - Baryon spectroscopy
    Group theory, operator design, spectroscopy results
  - Nucleon Structure Functions
- Lecture #2
  - Axial charge couplings and form factors
  - Generalized Parton Distributions (GPDs)
  - Strangeness in the nucleon

Chiral Perturbation Theory

- Effective field theory for the low-energy regime of QCD
  - Parameterized in terms of low-energy constants (LECs)
  - Lattice QCD can make calculations at different pion masses to get these constants accurately

- Meson SU(3) case: straightforward
  - Dynamical variables

$$
\begin{aligned}
U(x) &= \exp\left(i\frac{\phi(x)}{F_0}\right), \\
\phi(x) &= \sum_{a=1}^{8}\lambda_a\phi_a(x) \equiv \begin{pmatrix} \pi^0+\frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0+\frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}
\end{aligned}
$$

  - Lagrangian

$$
\mathcal{L}_{\text{eff}} = \frac{F_0^2}{4}\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \cdots
$$

  - Scale from dimension counting $\Lambda_{XSB} \sim 4\pi F_0 \sim 1$ GeV

Chiral Perturbation Theory

- Baryon case
  - Can write down the variables; octet field

$$B = \sum_{a=1}^{8} \lambda_a B_a = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$$

  - Tree-level, gives nice mass relation $\Delta_{GMO} = \frac{3}{4}M_\Lambda + \frac{1}{4}M_\Sigma - \frac{1}{2}M_N - \frac{1}{2}M_\Xi$
  - At loop level, $M_B/4\pi F_0 \sim 1 \rightarrow$ non-convergence problem

- Solutions (SU(2)): expand in different parameters, different power count
  - $\Delta(1232)$ freedom: $\Delta = M_\Delta - M_N \sim 300$ MeV $\sim O(M_\pi)$
  - Scales $\varepsilon \equiv m_\pi/\Lambda_{\chi SB}$ $\delta \equiv \Delta/\Lambda_{\chi SB}$
  - Small-scale expansion (SSE)
  - Heavy-baryon ChPT (HBChPT)
  - Finite-range regularization (FRR): Λ_{FRR} in self-energy

Form Factors

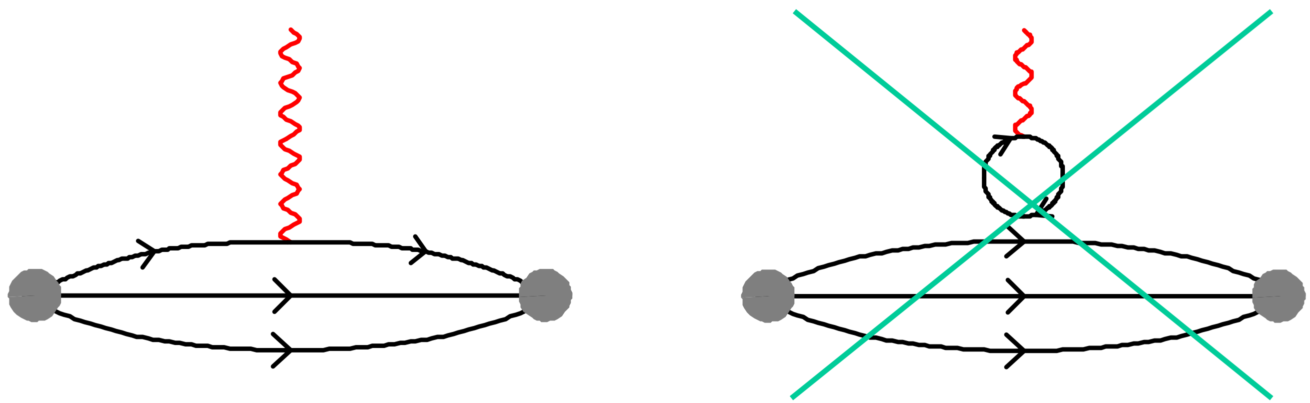
Green Functions

- Three-point function with connected piece only

$$C^{\Gamma,\mathcal{O}}_{\text{3pt}}\left(\vec{p},t,\tau\right) = \sum_{\alpha,\beta}\Gamma^{\alpha,\beta}\langle J_\beta\left(\vec{p},t\right)\mathcal{O}(\tau)\overline{J}_\alpha\left(\vec{p},0\right)\rangle$$

O : $V_\mu = \bar{q}\gamma_\mu q$, $A_\mu = \bar{q}\gamma_\mu\gamma_5 q$

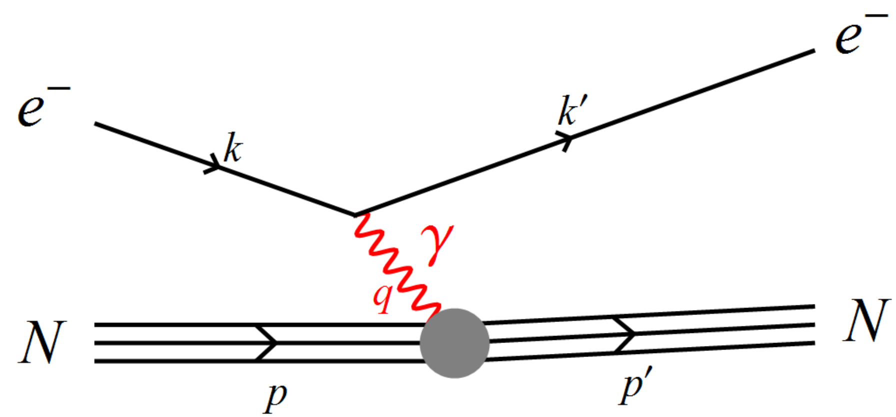
- Two topologies:



- Isovector quantities $O^{u\text{-}d}$: disconnected diagram cancelled

Axial Couplings and Form Factors

- Elastic scattering process
  - Axial couplings
  - Vector and Axial form factors
  - Magnetic moments
  - Charge radii



- For octet baryons

$$\langle B\,|V_\mu|\,B\rangle(q) \;=\; \overline{u}_B(p')\left[\gamma_\mu F_1(q^2)+\sigma_{\mu\nu}q_\nu\frac{F_2(q^2)}{2M_B}\right]u_B(p)$$

$$\langle B\,|A_\mu(q)|\,B\rangle \;=\; \overline{u}_B(p')\left[\gamma_\mu\gamma_5 G_A(q^2)+\gamma_5 q_\nu\frac{G_P(q^2)}{2M_B}\right]u_B(p)$$

Axial Charge Coupling

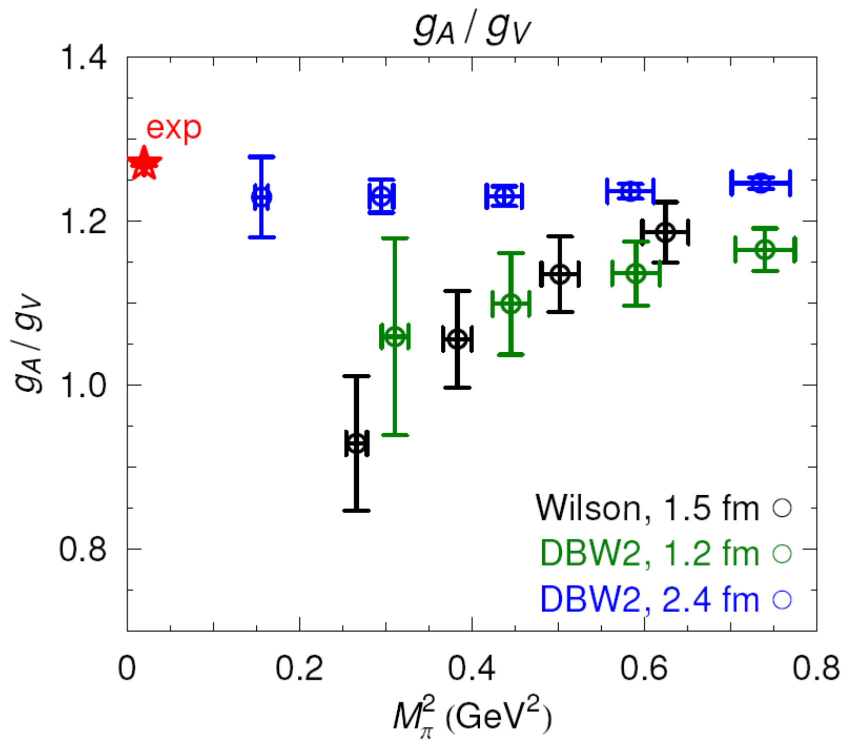
- Axial-vector current matrix element

$$\langle B\,|A_\mu(q)|\,B\rangle \;=\; \overline{u}_B(p')\left[\gamma_\mu\gamma_5 G_A(q^2)+\gamma_5 q_\nu \frac{G_P(q^2)}{2M_B}\right]u_B(p)$$

  and axial charge coupling $g_A = G_A^{u-d}(Q^2{=}0)$
- Well-measured experimentally from neutron beta decay
- No disconnected-diagram contribution
- Should be able to reproduce the experimental number and understand the systematic effects

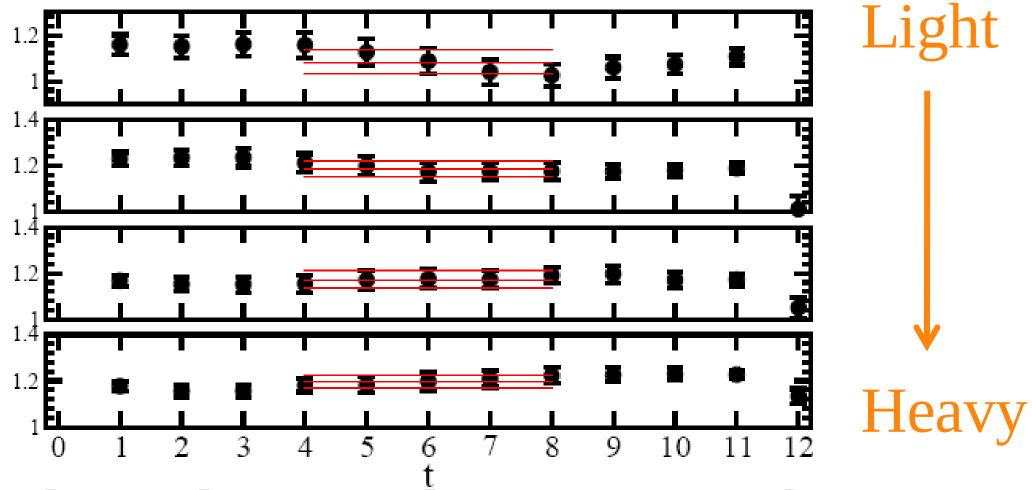
Finite-Volume Effect

- Example from axial coupling constant Phys.Rev.D68:054509,2003
  - Quenched approximation
  - Pion mass range: 300–650 MeV
  - DWF, $L_s = 16$, $M_5 = 1.8$, $a = 0.15$ fm



Axial Charge Coupling

- Example: 2+1 DWF, $M_\pi \sim 320$–620 MeV, $a \sim 0.12$ fm, $L \sim 3$ fm
  - Plateaux



  - Pion-mass dependence and XPT extrapolation

Axial Charge Coupling

- Continuum XPT extrapolation (SSE scheme)

T. R. Hemmert et al., J. Phys. G24, 1831 (1998);Phys. Rev. D68, 075009 (2003)

$$\begin{aligned} g_A^{\mathrm{SSE}}(m_\pi^2) \;=\; & g_A^0 + \left[4C_{\mathrm{SSE}}(\lambda) - \frac{(g_A^0)^3}{16\pi^2 f_\pi^2} - \frac{25c_A^2 g_1}{324\pi^2 f_\pi^2} + \frac{19c_A^2 g_A^0}{108\pi^2 f_\pi^2}\right] m_\pi^2 \\ & - \frac{m_\pi^2}{4\pi^2 f_\pi^2}\left[(g_A^0)^3 + \frac{1}{2}\, g_A^0\right] \ln\frac{m_\pi}{\lambda} + \frac{4c_A^2 g_A^0}{27\pi\Delta_0 f_\pi^2}\, m_\pi^3 \\ & + \left[25c_A^2 g_1\Delta_0^2 - 57c_A^2 g_A^0\Delta_0^2 - 24c_A^2 g_A^0 m_\pi^2\right]\frac{\sqrt{m_\pi^2-\Delta_0^2}}{81\pi^2 f_\pi^2\Delta_0}\arccos\frac{\Delta_0}{m_\pi} \\ & + \frac{25c_A^2 g_1\left(2\Delta_0^2 - m_\pi^2\right)}{162\pi^2 f_\pi^2}\ln\frac{2\Delta_0}{m_\pi} + \frac{c_A^2 g_A^0\left(3m_\pi^2 - 38\Delta_0^2\right)}{54\pi^2 f_\pi^2}\ln\frac{2\Delta_0}{m_\pi} + \mathcal{O}(\epsilon^4) \end{aligned}$$

Axial Charge Coupling

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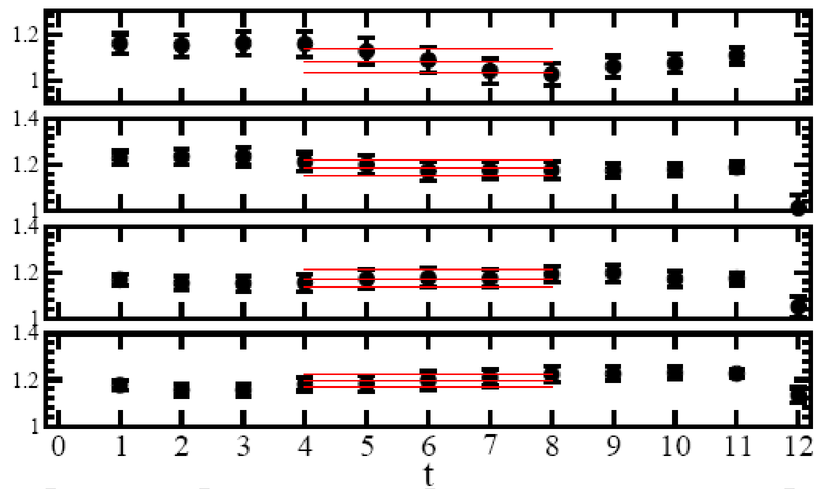
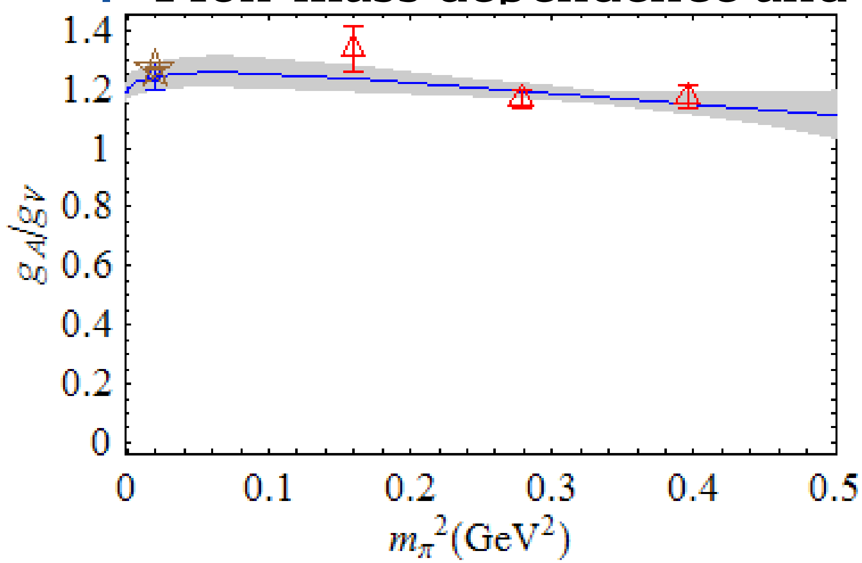
  - Plateaux



Light

Heavy

  - Pion-mass dependence and XPT extrapolation



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& - \frac{m_\pi^2}{4\pi^2 f_\pi^2}\left[(g_A^0)^3 + \frac{1}{2}g_A^0\right]\ln\frac{m_\pi}{\lambda} + \frac{4c_A^2 g_A^0}{27\pi\Delta_0 f_\pi^2} m_\pi^3 \\
& + \left[25c_A^2 g_1\Delta_0^2 - 57c_A^2 g_A^0\Delta_0^2 - 24c_A^2 g_A^0 m_\pi^2\right]\frac{\sqrt{m_\pi^2-\Delta_0^2}}{81\pi^2 f_\pi^2\Delta_0}\arccos\frac{\Delta_0}{m_\pi} \\
& + \frac{25c_A^2 g_1\left(2\Delta_0^2 - m_\pi^2\right)}{162\pi^2 f_\pi^2}\ln\frac{2\Delta_0}{m_\pi} + \frac{c_A^2 g_A^0\left(3m_\pi^2 - 38\Delta_0^2\right)}{54\pi^2 f_\pi^2}\ln\frac{2\Delta_0}{m_\pi} + \mathcal{O}(\epsilon^4)
\end{aligned}
$$

with finite-volume correction

S. R. Beane et al., Phys. Rev. D70, 074029 (2004)

$$
\begin{aligned}
\delta_L\left(\Gamma_{NN}\right) \equiv \delta g_A \;=\; & \frac{m_\pi^2}{3\pi^2 f^2}\Big[\; g_A^3\mathbf{F_1} \\
& + \left(g_{\Delta N}^2 g_A + \frac{25}{81}g_{\Delta N}^2 g_{\Delta\Delta}\right)\mathbf{F_2} \\
& + \; g_A\mathbf{F_3} + g_{\Delta N}^2 g_A\mathbf{F_4}\;\Big]
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{F_2}(m,\Delta,L) = & -\sum_{\mathbf{n}\neq\mathbf{0}}\left[\frac{K_1(mL|\mathbf{n}|)}{mL|\mathbf{n}|}\right. \\
& + \frac{\Delta^2-m^2}{m^2}K_0(mL|\mathbf{n}|) - \frac{\Delta}{m^2}\int_m^\infty d\beta \\
& \left.\times\frac{2\beta\, K_0(\beta L|\mathbf{n}|) + (\Delta^2-m^2)L|\mathbf{n}|\; K_1(\beta L|\mathbf{n}|)}{\sqrt{\beta^2+\Delta^2-m^2}}\right]
\end{aligned}
$$

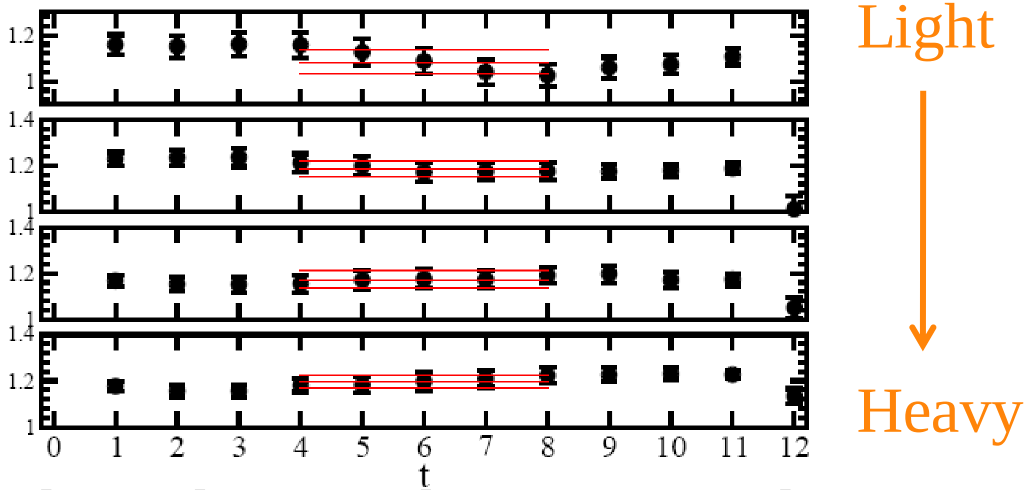
$$
\mathbf{F_1}(m,L) = \sum_{\mathbf{n}\neq\mathbf{0}}\left[K_0(mL|\mathbf{n}|) - \frac{K_1(mL|\mathbf{n}|)}{mL|\mathbf{n}|}\right]
$$

$$
\mathbf{F_3}(m,L) = -\frac{3}{2}\sum_{\mathbf{n}\neq\mathbf{0}}\frac{K_1(mL|\mathbf{n}|)}{mL|\mathbf{n}|};
$$

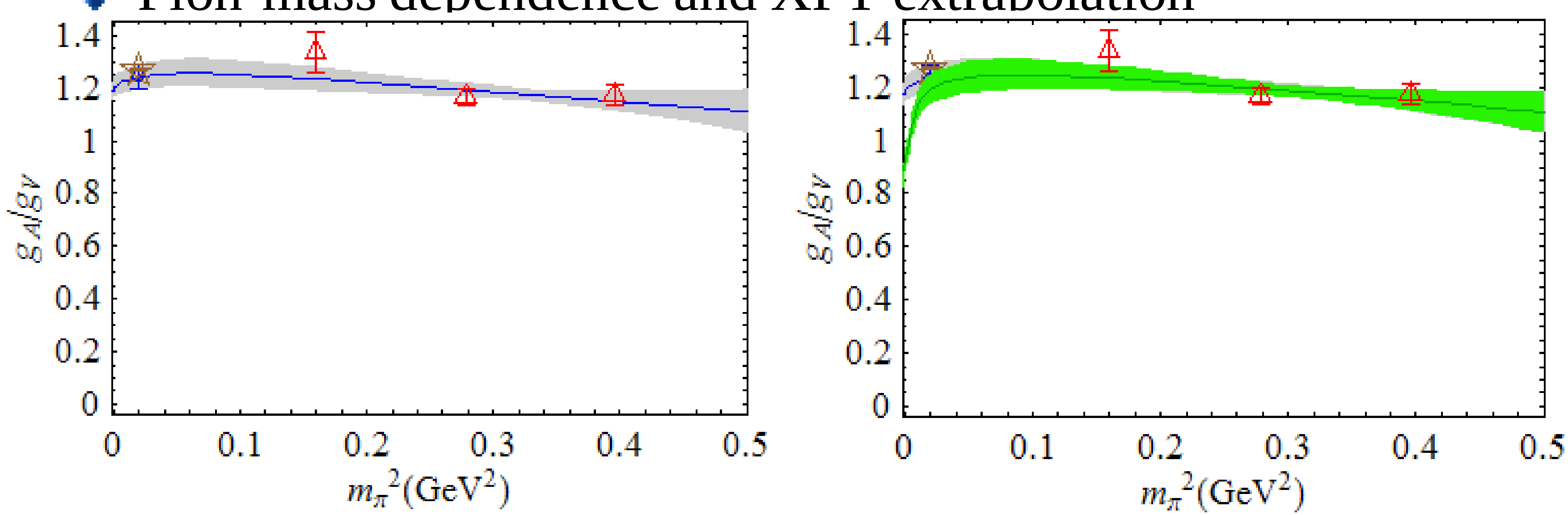
$$
\begin{aligned}
\mathbf{F_4}(m,\Delta,L) = & \frac{8}{9}\sum_{\mathbf{n}\neq\mathbf{0}}\left[\frac{K_1(mL|\mathbf{n}|)}{mL|\mathbf{n}|} - \frac{\pi e^{-mL|\mathbf{n}|}}{2\Delta L|\mathbf{n}|}\right. \\
& \left. - \frac{\Delta^2-m^2}{m^2\Delta}\int_m^\infty d\beta\,\frac{\beta\, K_0(\beta L|\mathbf{n}|)}{\sqrt{\beta^2+\Delta^2-m^2}}\right],
\end{aligned}
$$

Axial Charge Coupling

- Example: 2+1 DWF, M_π ~ 320–620 MeV, a ~ 0.12 fm, L ~ 3 fm
  - Plateaux

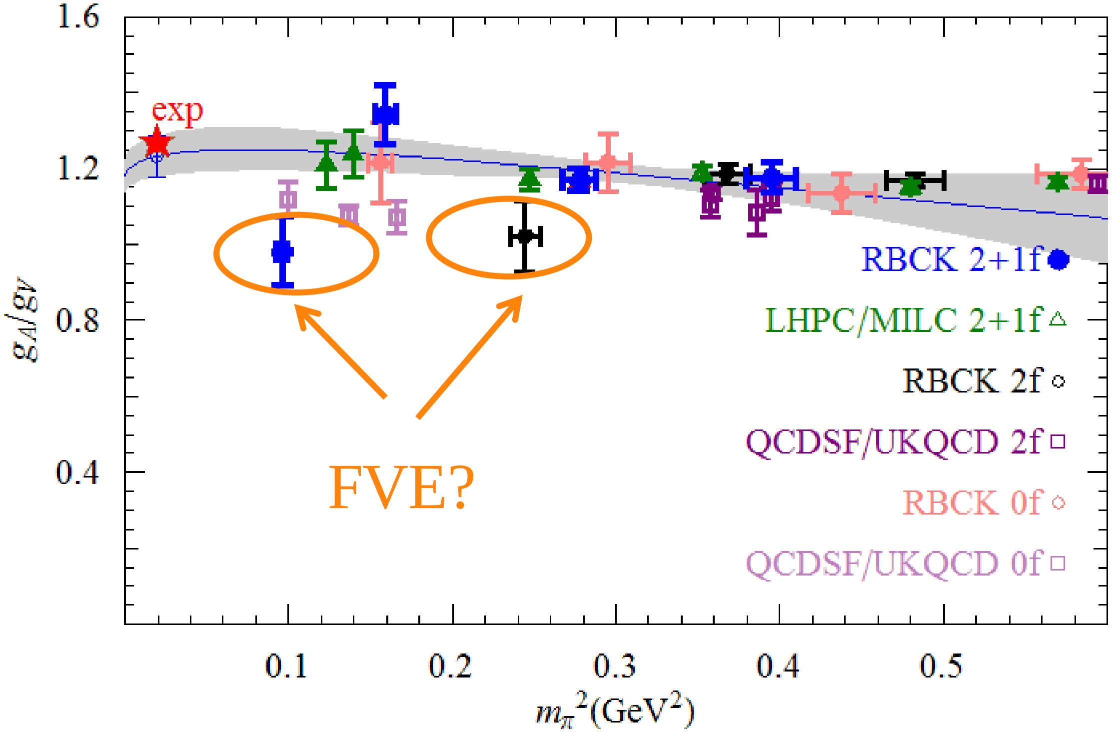


  - Pion-mass dependence and XPT extrapolation



Axial Charge Coupling: Global View

- World data: statistical error-bars only



HWL et al., 0802.0863[hep-lat]; M. Guertler et al., PoS(LAT2006)107;
D. Pleiter et al., PoS(LAT2006)120 ; K. Orginos et al., Phys.Rev.D73:094507, 2005;
D. Renner et al., PoS(LAT2006)121; D. Dolgov et al., Phys. Rev. D66, 034506 (2002)

Axial Coupling Constants: $g_{\Xi\Xi}$ and $g_{\Sigma\Sigma}$

HWL and K. Orginos, arXiv:0712.1214

- Pion mass: 350–750 MeV
- First lattice calculation of these quantities; mixed-action full-QCD

	m010	m020	m030	m040	m050
m_π (MeV)	354.2(8)	493.6(6)	594.2(8)	685.4(19)	754.3(16)
m_π/f_π	2.316(7)	3.035(7)	3.478(8)	3.822(23)	4.136(20)
m_K/f_π	3.951(14)	3.969(10)	4.018(11)	4.060(26)	4.107(21)
confs	612	345	561	320	342
$g_{A,N}$	1.22(8)	1.21(5)	1.195(17)	1.150(17)	1.167(11)
$g_{\Sigma\Sigma}$	0.418(23)	0.450(15)	0.451(7)	0.444(8)	0.453(5)
$g_{\Xi\Xi}$	−0.262(13)	−0.270(10)	−0.269(7)	−0.257(9)	−0.261(7)

- Combine with g_A for study of
  - SU(3) symmetry breaking
  - SU(3) simultaneous fits among three coupling constants
    $\Longrightarrow$ *D*, *F*, and other low-energy constants

g_A's SU(3) Partners: $g_{\Xi\Xi}$ and $g_{\Sigma\Sigma}$

- Has applications such as hyperon scattering, non-leptonic decays, …
- Cannot be determined by experiment
- Existing theoretical predictions:
  - Chiral perturbation theory

$$0.35 \le g_{\Sigma\Sigma} \le 0.55 \qquad 0.18 \le -g_{\Xi\Xi} \le 0.36$$

M. J. Savage et al., Phys. Rev. D55, 5376 (1997);

  - Large-N_c

$$0.30 \le g_{\Sigma\Sigma} \le 0.36 \qquad 0.26 \le -g_{\Xi\Xi} \le 0.30$$

R. Flores-Mendieta et al., Phys. Rev. D58, 094028 (1998);

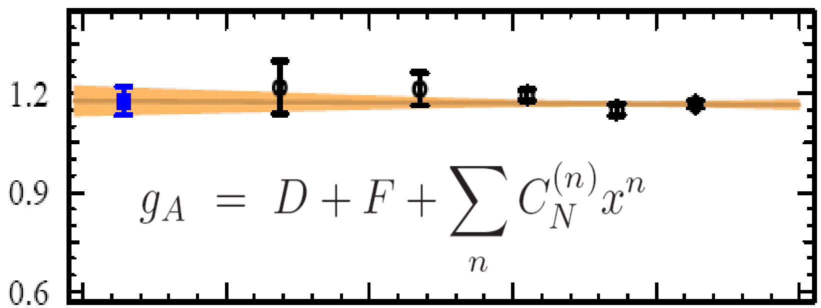
- Loose bounds on the values
- Lattice QCD can provide substantial improvement

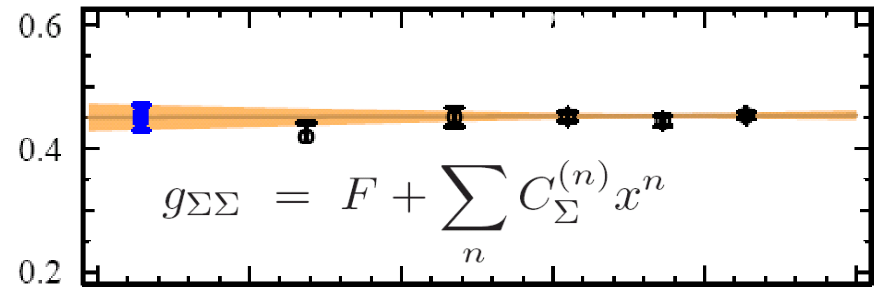
g_A's SU(3) Partners: $g_{\Xi\Xi}$ and $g_{\Sigma\Sigma}$

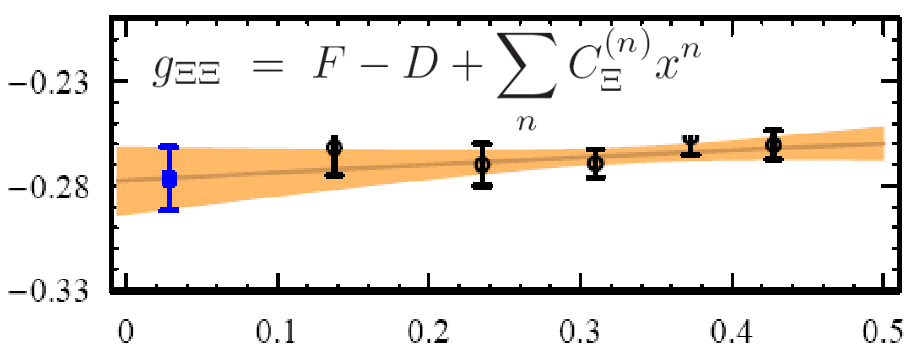
- Simultaneous SU(3) fit
  - SU(3) HBXPT (with 8 parameters)

    W. Detmold and C. J. D. Lin, Phys. Rev. D71, 054510 (2005)

    which fails to describe the data
  - Simple chiral form







- Systematic errors: finite volume + finite *a*

$$g_A = 1.18(4)_{\rm stat}(6)_{\rm syst}$$
$$g_{\Sigma\Sigma} = 0.450(21)_{\rm stat}(27)_{\rm syst}$$
$$g_{\Xi\Xi} = -0.277(15)_{\rm stat}(19)_{\rm syst}$$

$$D = 0.715(6)_{\rm stat}(29)_{\rm syst} \quad F = 0.453(5)_{\rm stat}(19)_{\rm syst}$$

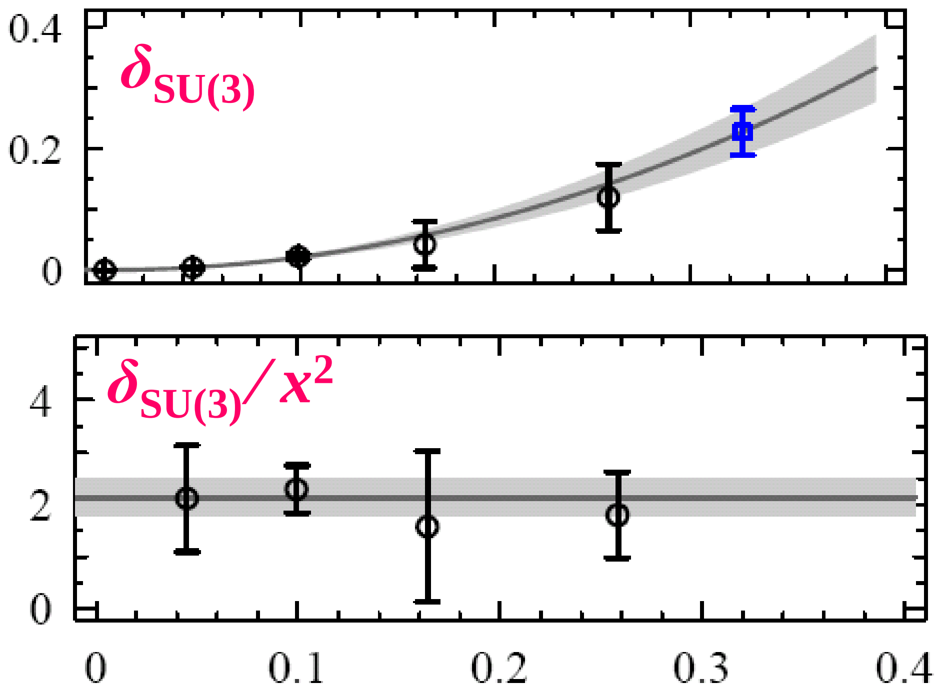
Jefferson Lab
Exploring the Nature of Matter

g_A's SU(3) Partners: $g_{\Xi\Xi}$ and $g_{\Sigma\Sigma}$

- SU(3) symmetry breaking

$$\delta_{\mathrm{SU(3)}} = g_{\mathrm{A}} - 2.0 \times g_{\Sigma\Sigma} + g_{\Xi\Xi}$$
$$= \sum_n c_n x^n \quad \text{with} \quad x = (m_K^2 - m_\pi^2)/(4\pi f_\pi^2)$$

- Quadratic behaviour is observed



- Not predicted by any theorem nor chiral perturbation theory
  $\Longrightarrow$ coincidence?

- 20% breaking at physical point

Electromagnetic Form Factors

- Two definitions
  - Dirac and Pauli form factors F_1, F_2

$$\langle N | V_\mu | N \rangle(q) = \overline{u}_N(p') \left[\gamma_\mu F_1(q^2) + \sigma_{\mu\nu} q_\nu \frac{F_2(q^2)}{2m} \right] u_N(p)$$

at $Q^2=0$

$$F_{1p}(0) = 1,\, F_{2p}(0) = \kappa_p,\, F_{1n}(0) = 0,\, F_{2n}(0) = \kappa_n$$

  - Sachs form factors G_E, G_M

$$\begin{aligned} G_E(q^2) &= F_1(q^2) + \frac{q^2}{(2M_N)^2} F_2(q^2) \\ G_M(q^2) &= F_1(q^2) + F_2(q^2) \quad . \end{aligned}$$

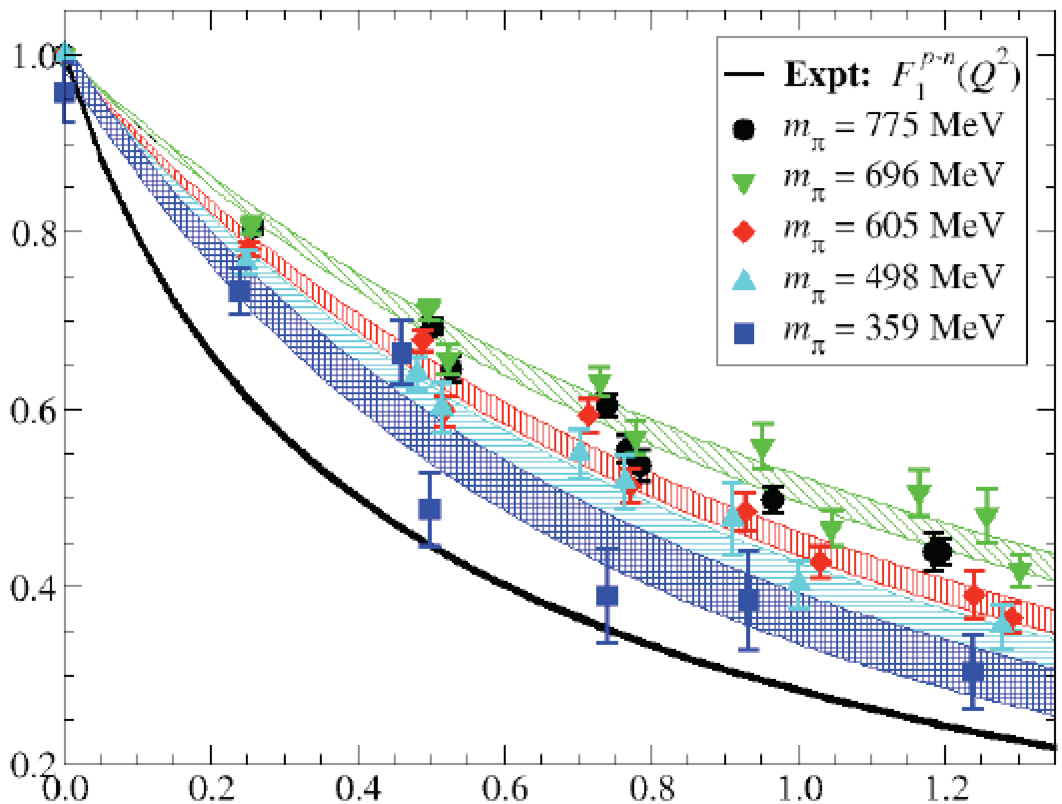
at $Q^2=0$

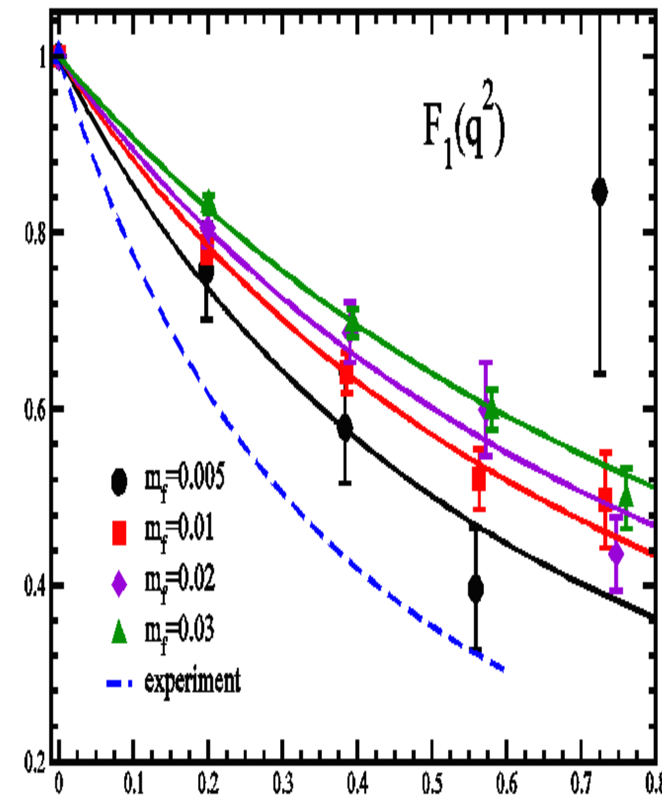
$$G_{Ep}(0) = 1,\, G_{Mp}(0) = \mu_p,\, G_{En}(0) = 0,\, G_{Mn}(0) = \mu_n$$

- Isovector quantities only

EM Form Factors: F_1 vs Q^2

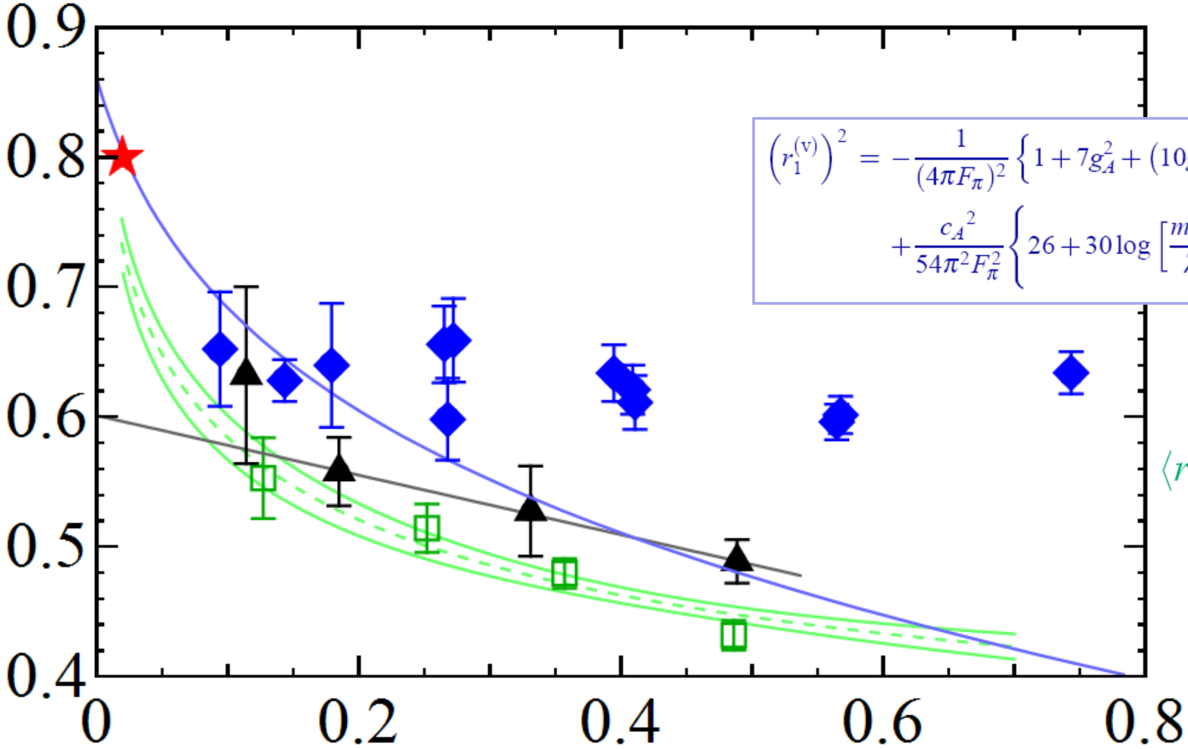
- 2+1 examples:
  - LHPC mixed action, $M_\pi \sim 350\text{–}760$ MeV
  - RBC DWF, $M_\pi \sim 320\text{–}620$ MeV





Charge Radii

- Definition: $F_i(q^2) = F_i(0)\left[1 + \frac{1}{6} r_i^2 q^2 + \mathcal{O}(q^4)\right]$
- Examples:
  - QCDSF: 2f clover $M_\pi \sim$ 350–950 MeV, SSE extrapolation
  - RBC DWF, $M_\pi \sim$ 320–620 MeV, linear extrapolation
  - LHPC mixed action, $M_\pi \sim$ 350–760 MeV, FRR extrapolation



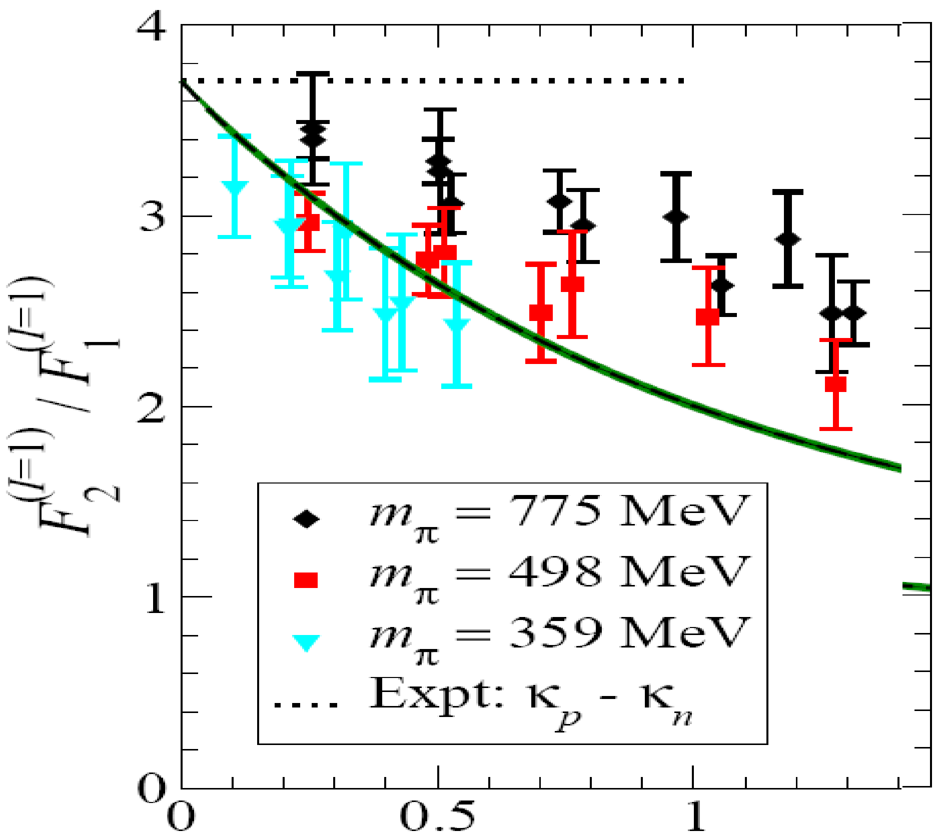
$$\left(r_1^{(v)}\right)^2 = -\frac{1}{(4\pi F_\pi)^2}\left\{1 + 7g_A^2 + \left(10g_A^2 + 2\right)\log\left[\frac{m_{PS}}{\lambda}\right]\right\} + \frac{c_A{}^2}{54\pi^2 F_\pi^2}\left\{26 + 30\log\left[\frac{m_{PS}}{\lambda}\right] + 30\frac{\Delta}{\sqrt{\Delta^2 - m_{PS}^2}}\log\left[\frac{\Delta}{m_{PS}} + \sqrt{\frac{\Delta^2}{m_{PS}^2} - 1}\right]\right\}$$

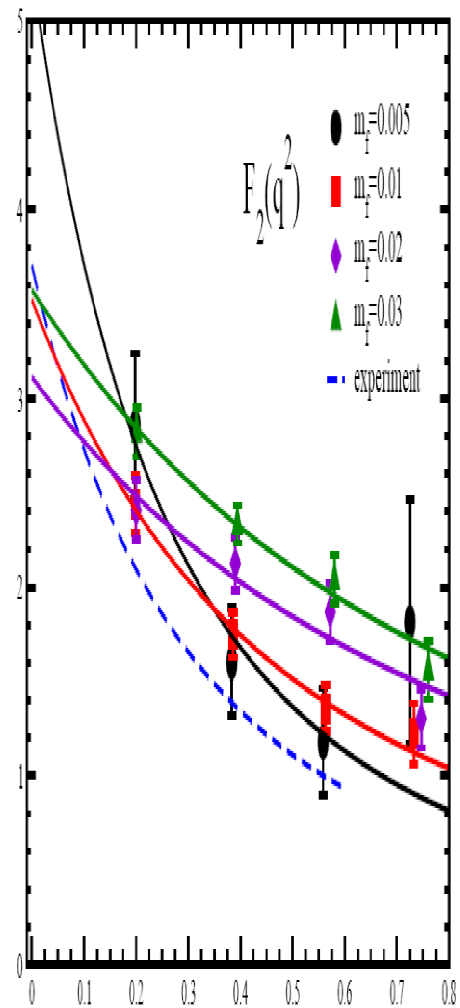
$$\langle r^2\rangle^{u-d} = a_0 - \frac{1+5g_A^2}{(4\pi f_\pi)^2}\log\left(\frac{m_\pi^2}{m_\pi^2 + \Lambda^2}\right)$$

M_π^2(GeV^2)

EM Form Factors: F_2 vs Q^2

- 2+1 examples:
  - LHPC mixed action, $M_\pi \sim 350–760$ MeV
  - RBC DWF, $M_\pi \sim 320–620$ MeV



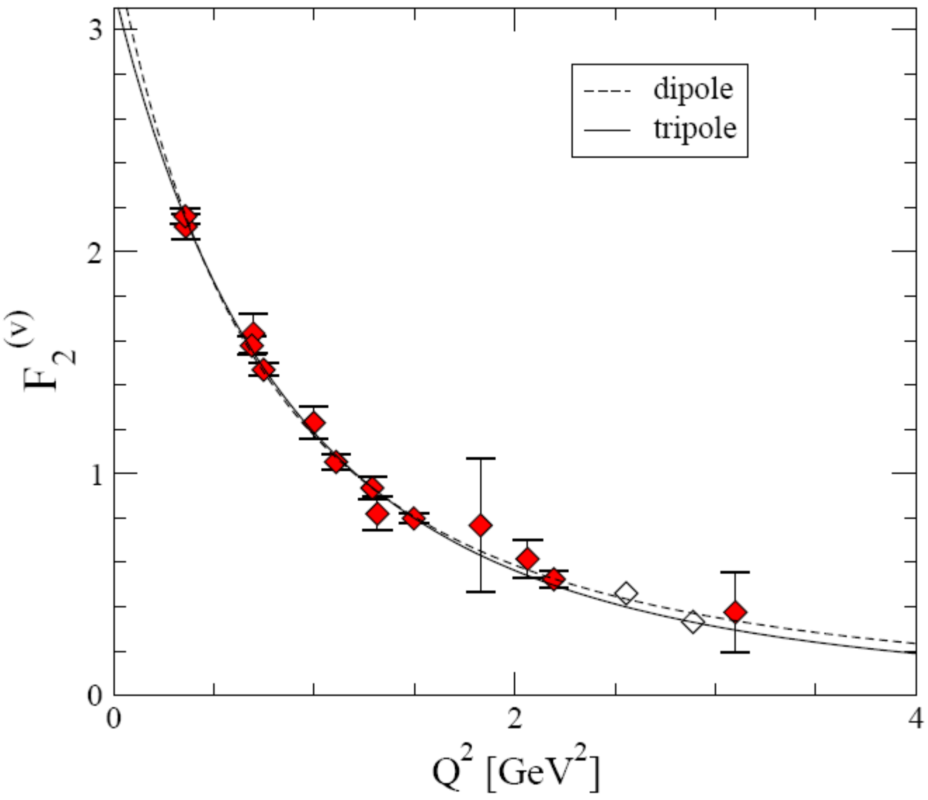


Magnetic Moment

- Definition: $\kappa_p = \mu_p - 1 = F_{p,2}(0)$, $\kappa_n = \mu_n = F_{n,2}(0)$

$$\kappa_{\text{iso}} = \kappa_{p-n} = \mu_p - \mu_n - 1$$

- Examples:
  - QCDSF: 2f clover $M_\pi \sim 350$–950 MeV, tripole-fits



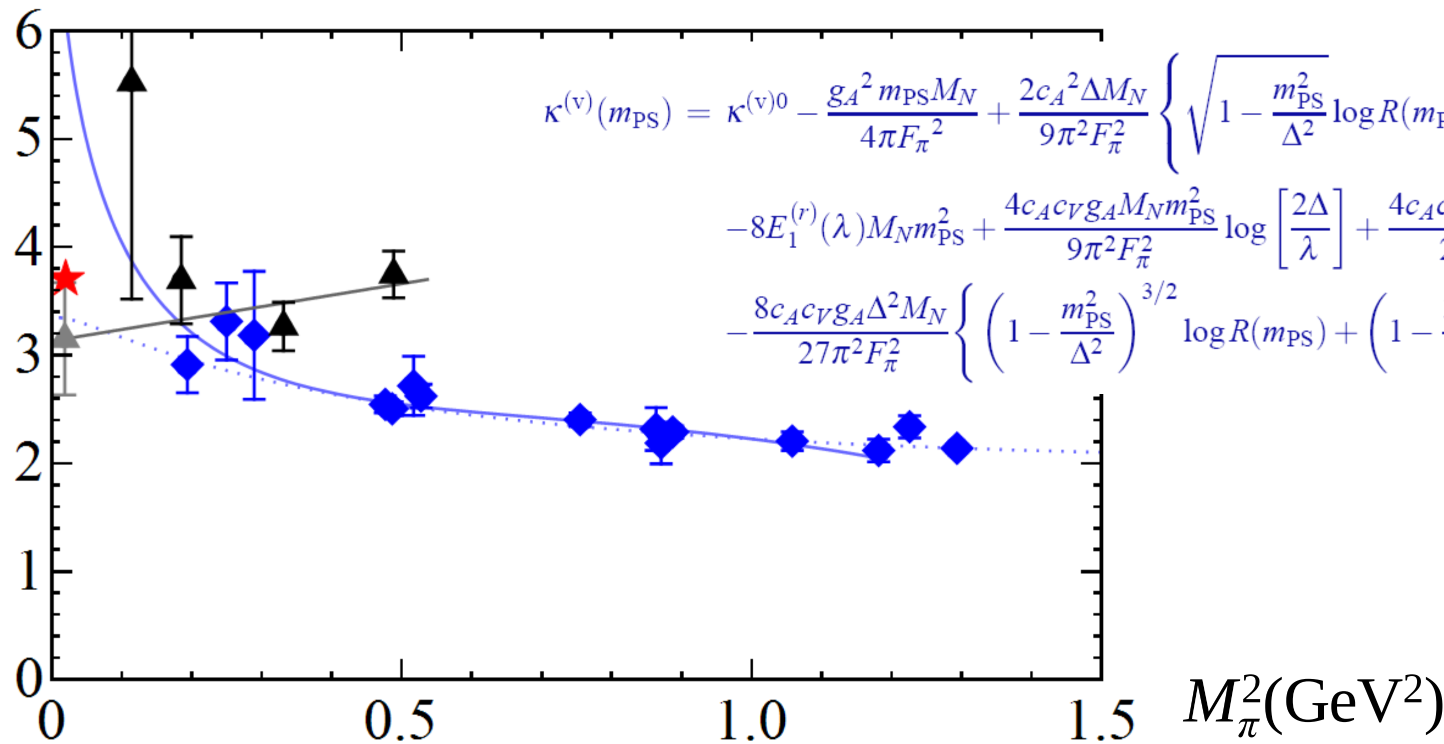
$M_\pi \sim 400$ MeV

Magnetic Moment

- Definition: $\kappa_p = \mu_p - 1 = F_{p,2}(0)$, $\kappa_n = \mu_n = F_{n,2}(0)$

$$\kappa_{\rm iso} = \kappa_{p-n} = \mu_p - \mu_n - 1$$

- Examples:
  - QCDSF: 2f clover M_π ~ 350–950 MeV, tripole fit, SSE extrapolation
  - RBC DWF, M_π ~ 320–620 MeV, dipole



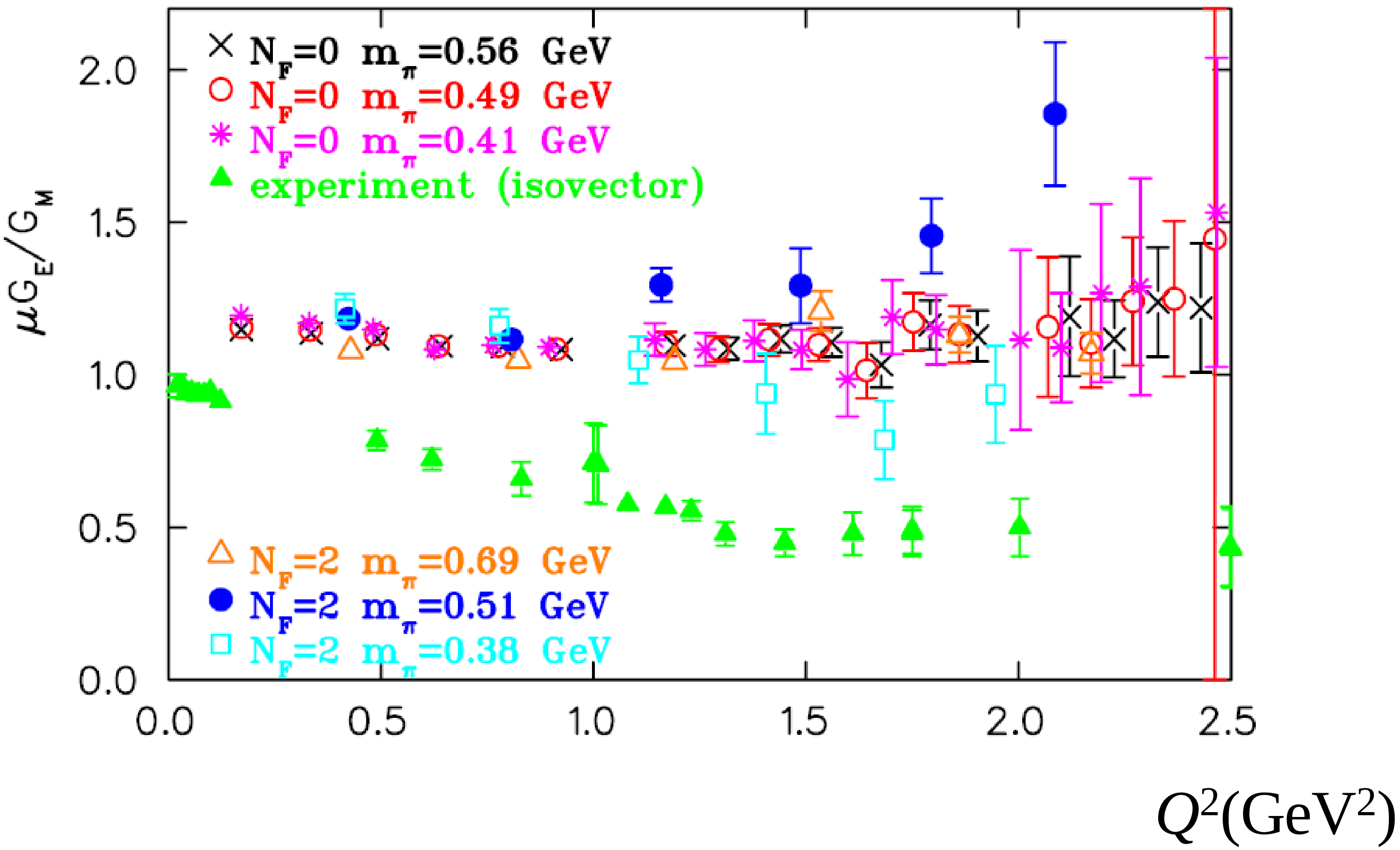
$$\kappa^{(v)}(m_{\rm PS}) = \kappa^{(v)0} - \frac{g_A{}^2 m_{\rm PS} M_N}{4\pi F_\pi{}^2} + \frac{2c_A{}^2 \Delta M_N}{9\pi^2 F_\pi^2}\left\{\sqrt{1-\frac{m_{\rm PS}^2}{\Delta^2}}\log R(m_{\rm PS}) + \log\left[\frac{m_{\rm PS}}{2\Delta}\right]\right\}$$

$$-8E_1^{(r)}(\lambda) M_N m_{\rm PS}^2 + \frac{4c_A c_V g_A M_N m_{\rm PS}^2}{9\pi^2 F_\pi^2}\log\left[\frac{2\Delta}{\lambda}\right] + \frac{4c_A c_V g_A M_N m_{\rm PS}^3}{27\pi F_\pi^2 \Delta}$$

$$-\frac{8c_A c_V g_A \Delta^2 M_N}{27\pi^2 F_\pi^2}\left\{\left(1-\frac{m_{\rm PS}^2}{\Delta^2}\right)^{3/2}\log R(m_{\rm PS}) + \left(1-\frac{3m_{\rm PS}^2}{2\Delta^2}\right)\log\left[\frac{m_{\rm PS}}{2\Delta}\right]\right\}$$

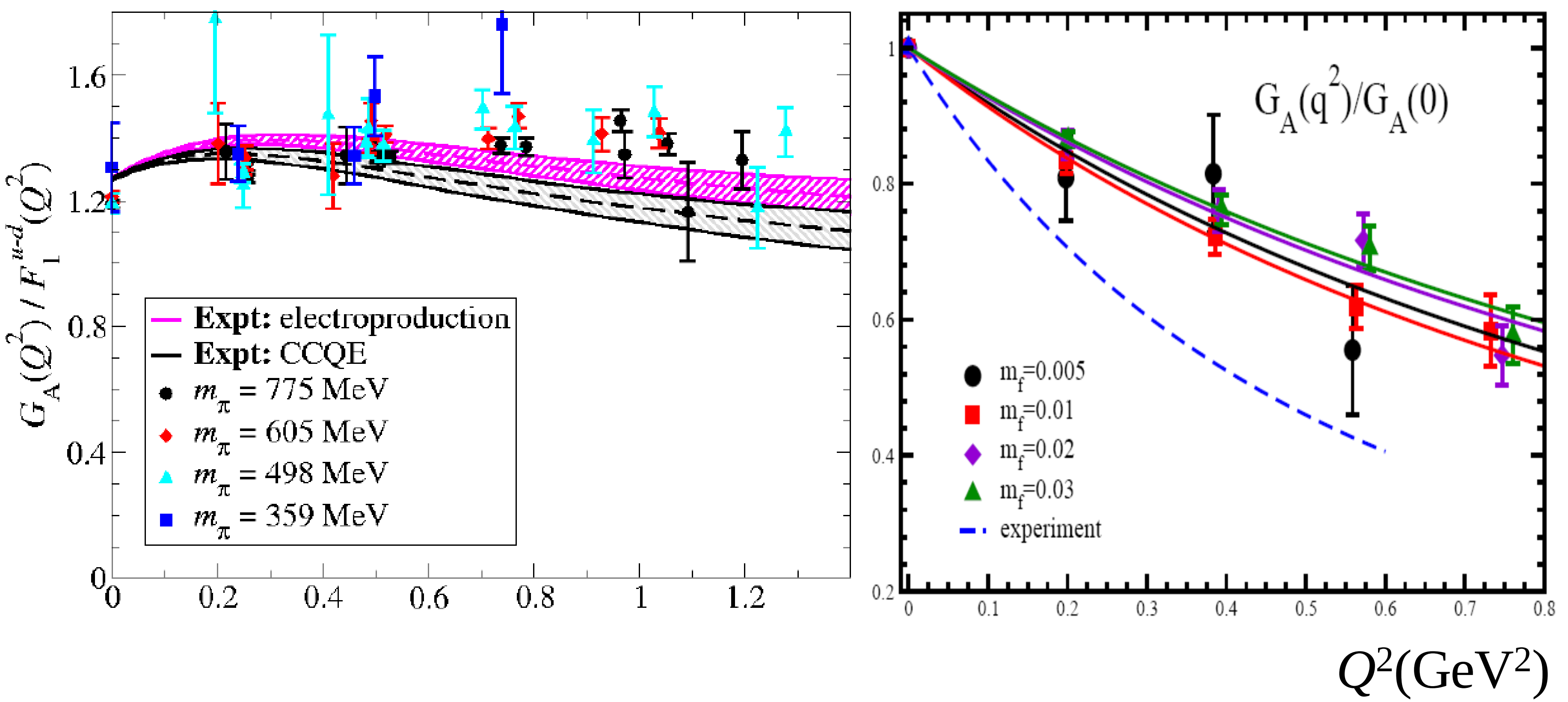
EM Form Factors: G_E/G_M vs Q^2

- 0f & 2f example:
  - MIT/Cyprus, Wilson action, $M_\pi \sim$ 380–690 MeV
  - Take experimental $G_E^{p,n}$ and $G_M^{p,n}$ to construct isovector form factor



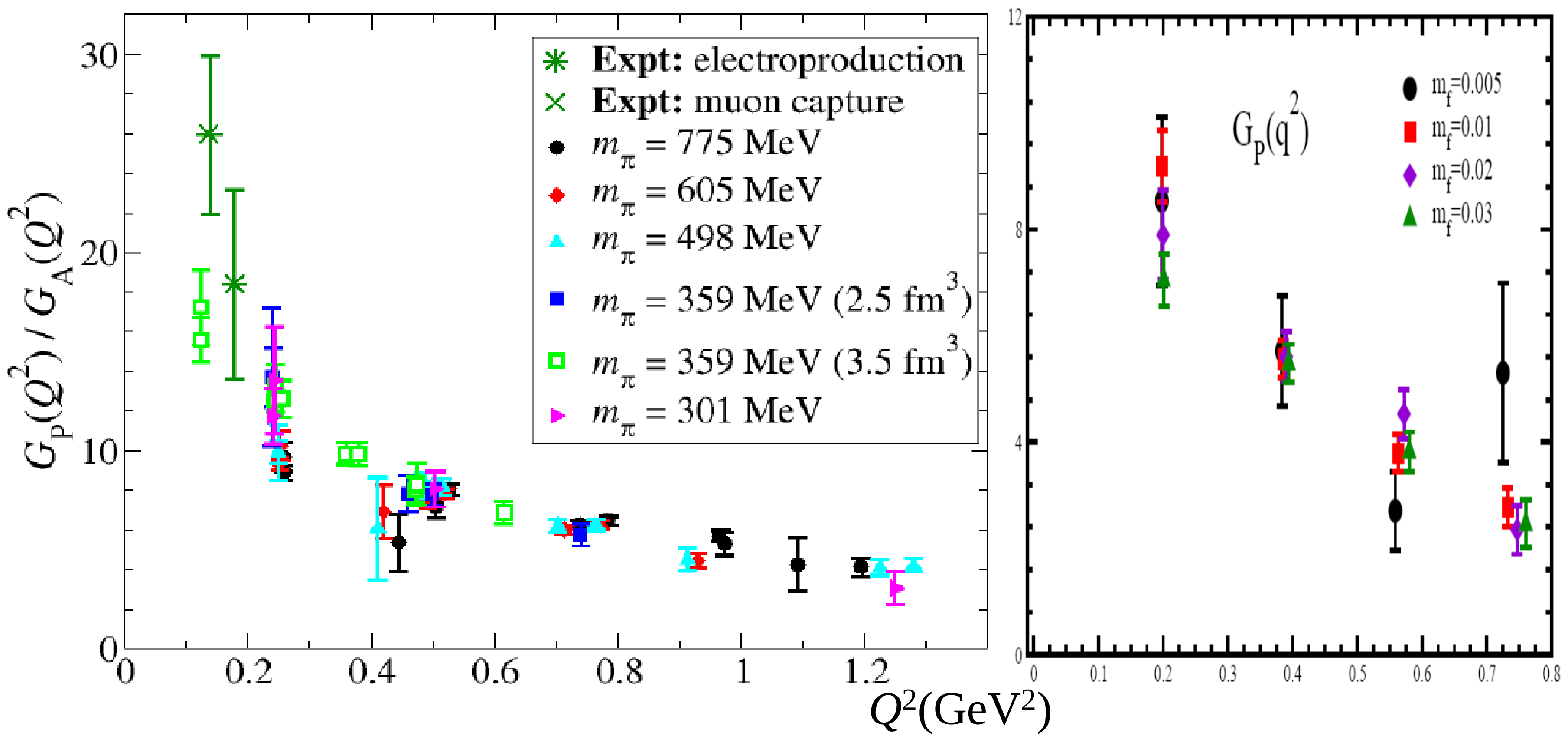
Axial Form Factors: G_A

- 2+1f examples:
  - LHPC mixed action, $M_\pi \sim 350$–760 MeV
  - RBC DWF, $M_\pi \sim 320$–620 MeV



Axial Form Factors: G_P

- 2+1f examples:
  - LHPC mixed action, $M_\pi \sim 300–760$ MeV
  - RBC DWF, $M_\pi \sim 320–620$ MeV
  - Convention: $G_p^{\mathrm{LHPC}} = 2m_N G_p^{\mathrm{RBC}}$



Goldberger-Treiman Relation

- The Goldberger-Treiman (GT) relation states

$$G_A(q^2) + \frac{q^2}{4m_N^2} G_p(q^2) = \frac{1}{2m_N} \frac{2G_{\pi NN}(q^2) f_\pi m_\pi^2}{m_\pi^2 - q^2}$$

- If pion-pole dominance

$$\frac{1}{2m_N} G_p(q^2) \sim \frac{2G_{\pi NN}(q^2) f_\pi}{m_\pi^2 - q^2}$$

- Replacing $G_{\pi NN}$

$$G_A \sim (m_\pi^2 - q^2) G_P / 4m_N^2$$

- Replacing G_P

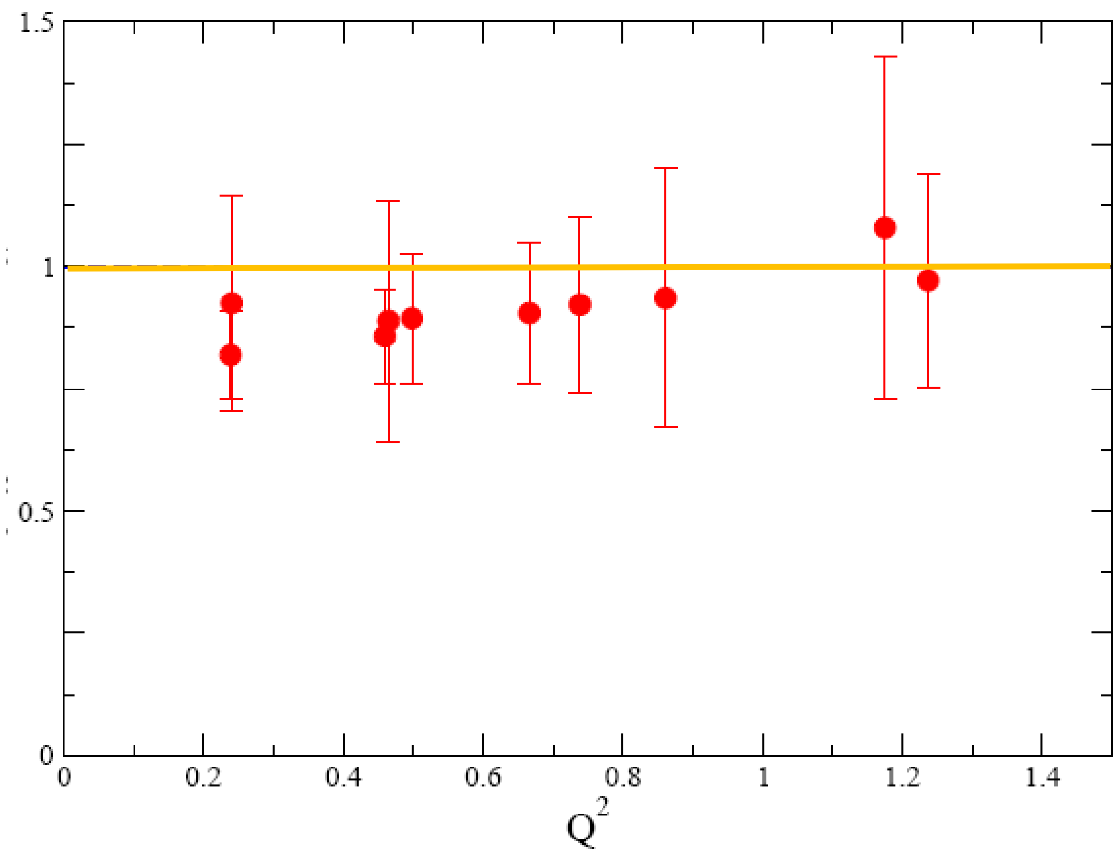
$$G_{\pi NN} f_\pi \sim m_N G_A$$

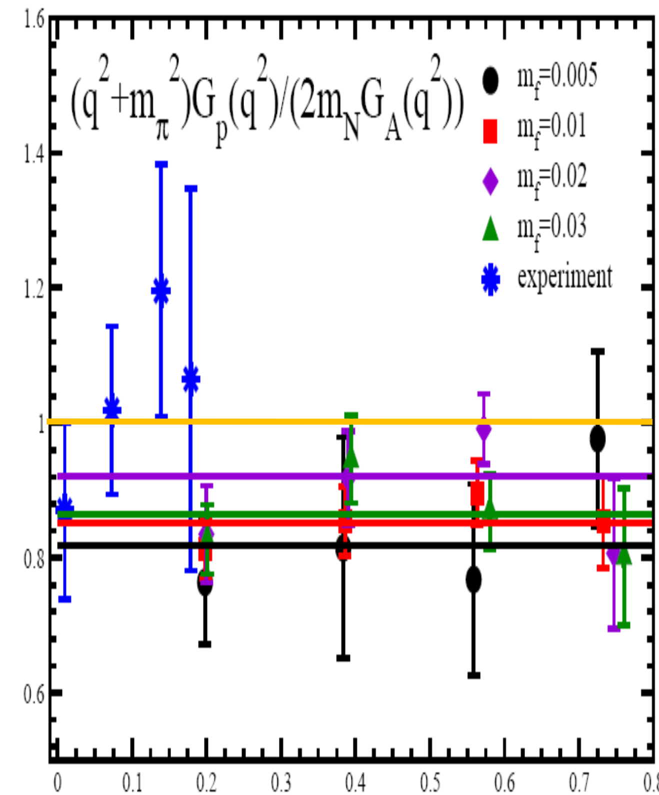
Goldberger-Treiman Relation

- Check the following

$$(Q^2+m_\pi^2)\ G_P\ /4m_N^2 G_A \sim 1$$

  - Examples: LHPC mixed action, $M_\pi \sim 350$ MeV
    RBC DWF, $M_\pi \sim 320$–620 MeV



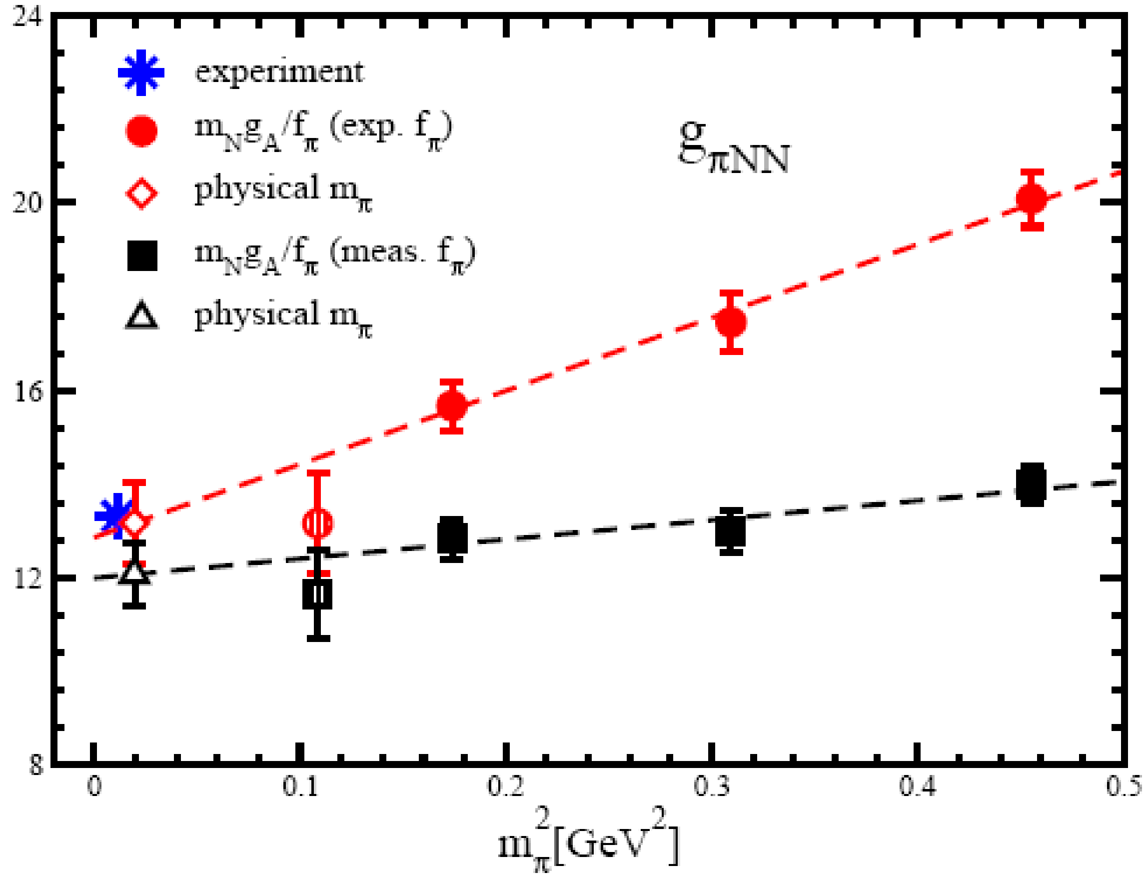


πNN Coupling

- Definition:

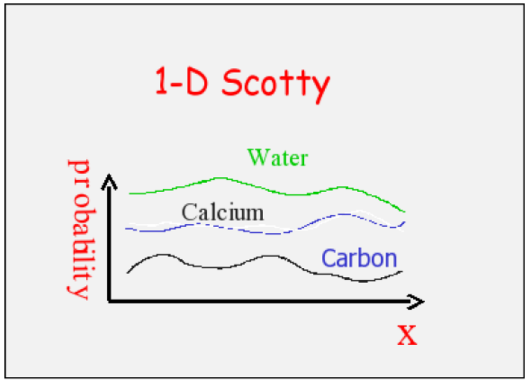
$$G_{\pi NN}(Q^2=0) = g_{\pi NN} \sim m_N G_A(Q^2=0) \,/\, f_\pi$$

  - Example: RBC DWF, $M_\pi \sim 320–620$ MeV



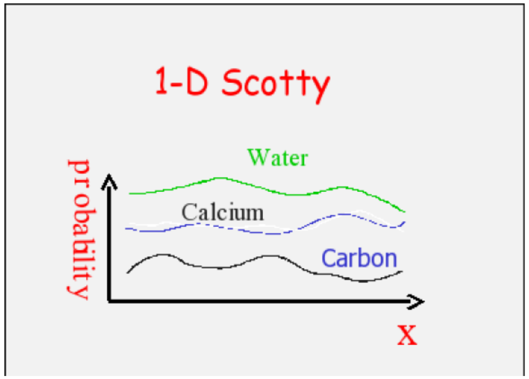
Generalized Parton Distribution

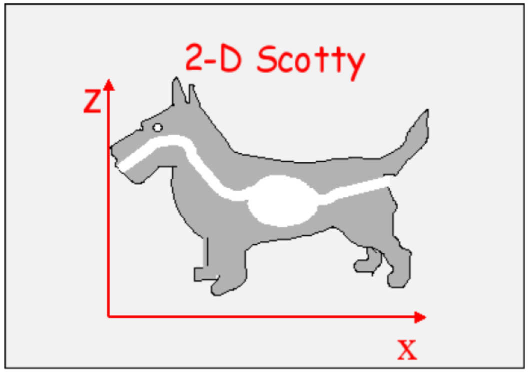
- Structure function/distribution functions
  - deep inelastic scattering
  - $\langle x^n \rangle_q, \langle x^n \rangle_{\Delta q}, \langle x^n \rangle_{\delta q}$



Generalized Parton Distribution

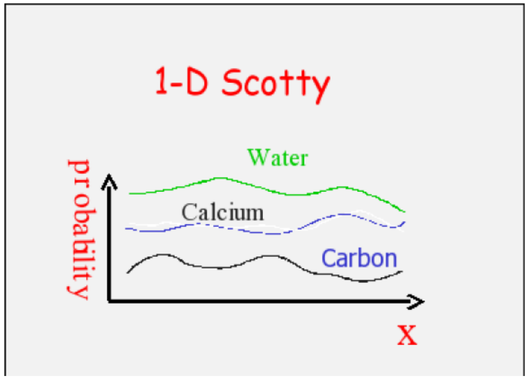
- Structure function/distribution functions
  - deep inelastic scattering
  - $\langle x^n \rangle_q, \langle x^n \rangle_{\Delta q}, \langle x^n \rangle_{\delta q}$
- Form factors
  - elastic scattering
  - $F_1(Q^2), F_2(Q^2), G_A(Q^2), G_P(Q^2)$

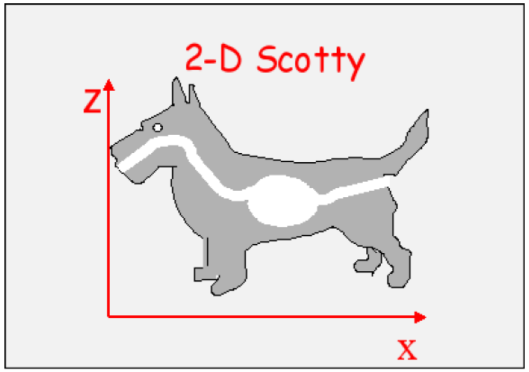


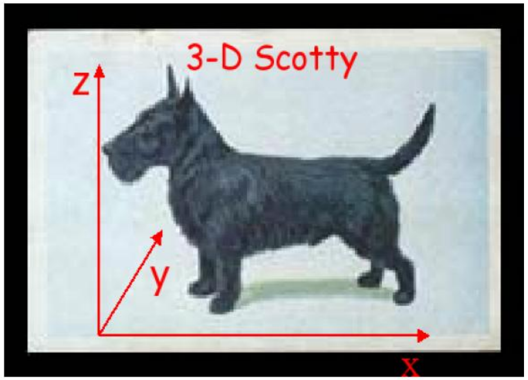


Generalized Parton Distribution

- Structure function/distribution functions
  - deep inelastic scattering
  - $\langle x^n \rangle_q, \langle x^n \rangle_{\Delta q}, \langle x^n \rangle_{\delta q}$
- Form factors
  - elastic scattering
  - $F_1(Q^2), F_2(Q^2), G_A(Q^2), G_P(Q^2)$
- Generalized Parton Distribution
  - DVCS
  - $\langle x^{n-1} \rangle_q = A_{n0}(0), \langle x^{n-1} \rangle_{\Delta q} = \tilde{A}_{n0}(0),$
    $\langle x^n \rangle_{\delta q} = A_{Tn0}(0),$
  - $F_1(Q^2) = A_{10}(Q^2), F_2(Q^2) = B_{20}(Q^2),$
    $G_A(Q^2) = \tilde{A}_{10}(Q^2), G_P(Q^2) = \tilde{B}_{10}(Q^2)$
  - Nucleon spin and transverse structure







Graphics from G. Fleming

Generalized Parton Distribution

- GPD gives off-forward light-cone operator of the form

$$\mathcal{O}_\Gamma(x) = \frac{1}{2}\int \frac{d\lambda}{2\pi}\, e^{ix\lambda}\, \overline{q}(-\lambda n/2)\,\Gamma\,\mathcal{P}e^{-ig\int_{\lambda n/2}^{-\lambda n/2} dz\, n\cdot A(z)}\, q(\lambda n/2)$$

Momentum fraction $n_\mu\gamma^\mu, n_\mu\gamma^\mu\gamma^5, n_\mu\sigma^{\mu\nu}$ Light-like vector

- Cannot directly calculate the matrix element $\langle P', S' | O_\Gamma(x) | P, S\rangle$
- LQCD: Use operator production expansion (OPE)

$$O_q^{\mu_1\cdots\mu_n} = \overline{q}\, iD^{(\mu_1}\cdots iD^{\mu_{n-1}}\,\gamma^{\mu_n)}\, q$$

$$\tilde{O}_q^{\mu_1\cdots\mu_n} = \overline{q}\, iD^{(\mu_1}\cdots iD^{\mu_{n-1}}\gamma^{\mu_n)}\,\gamma^5\, q$$

$$O_{Tq}^{\mu_1\cdots\mu_n\alpha} = \overline{q}\, iD^{(\mu_1}\cdots iD^{\mu_{n-1}}\,\sigma^{\mu_n)\alpha}\, q$$

and obtain GFF $A_{ni}(0)$, $B_{ni}(0)$, $C_n(0)$, $\tilde{A}_{ni}(0)$, $\tilde{B}_{ni}(0)$
$A_{Tni}(0)$, $B_{Tni}(0)$, $\tilde{A}_{Tni}(0)$, $\tilde{B}_{Tni}(0)$

Generalized Parton Distribution

- Cannot directly calculate the matrix element $\langle P', S' | O_\Gamma(x) | P, S \rangle$
- Use operator production expansion (OPE)
- Definition of GFF:

$$\langle P'|\mathcal{O}^{\mu_1}|P\rangle = \langle\langle \gamma^{\mu_1}\rangle\rangle A_{10}(t) + \frac{i}{2m}\langle\langle \sigma^{\mu_1\alpha}\rangle\rangle \Delta_\alpha B_{10}(t)\,,$$

Only even n

$$\langle P'|\mathcal{O}^{\{\mu_1\mu_2\}}|P\rangle = \bar{P}^{\{\mu_1}\langle\langle \gamma^{\mu_2\}}\rangle\rangle A_{20}(t) + \frac{i}{2m}\bar{P}^{\{\mu_1}\langle\langle \sigma^{\mu_2\}\alpha}\rangle\rangle \Delta_\alpha B_{20}(t) + \frac{1}{m}\Delta^{\{\mu_1}\Delta^{\mu_2\}} \boxed{C_{20}(t)}\,,$$

$$\begin{aligned}\langle P'|\mathcal{O}^{\{\mu_1\mu_2\mu_3\}}|P\rangle &= \bar{P}^{\{\mu_1}\bar{P}^{\mu_2}\langle\langle \gamma^{\mu_3\}}\rangle\rangle A_{30}(t) + \frac{i}{2m}\bar{P}^{\{\mu_1}\bar{P}^{\mu_2}\langle\langle \sigma^{\mu_3\}\alpha}\rangle\rangle \Delta_\alpha B_{30}(t) \\ &+ \Delta^{\{\mu_1}\Delta^{\mu_2}\langle\langle \gamma^{\mu_3\}}\rangle\rangle A_{32}(t) + \frac{i}{2m}\Delta^{\{\mu_1}\Delta^{\mu_2}\langle\langle \sigma^{\mu_3\}\alpha}\rangle\rangle \Delta_\alpha B_{32}(t),\end{aligned}$$

with $\overline{P} = (P' + P)/2 \qquad \Delta = P' - P \qquad t = \Delta^2$

- List of GFF:
  - Polarized: $\tilde{A}_{ni}(0)$, $\tilde{B}_{ni}(0)$
  - Unpolarized: $A_{ni}(0)$, $B_{ni}(0)$, $C_n(0)$
  - Transverse (+ pol.): $A_{Tni}(0)$, $B_{Tni}(0)$ ($\tilde{A}_{Tni}(0)$, $\tilde{B}_{Tni}(0)$)

Generalized Parton Distribution

- List of GFF:
  - Polarized: $\tilde{A}_{ni}(0)$, $\tilde{B}_{ni}(0)$
  - Unpolarized: $A_{ni}(0)$, $B_{ni}(0)$, $C_n(0)$
  - Transverse (+ pol.): $A_{Tni}(0)$, $B_{Tni}(0)$ ($\tilde{A}_{Tni}(0)$, $\tilde{B}_{Tni}(0)$)

- Link to Mellin moments
  - Example: unpolarized case

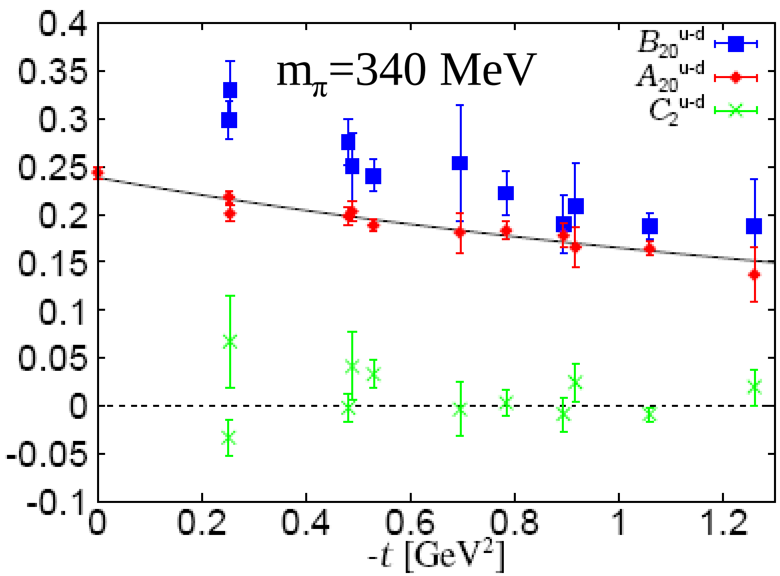
$$\int_{-1}^{1} dx x^{n-1} \begin{bmatrix} H(x,\xi,t) \\ E(x,\xi,t) \end{bmatrix} = \sum_{k=0}^{[(n-1)/2]} (2\xi)^{2k} \begin{bmatrix} A_{n,2k}(t) \\ B_{n,2k}(t) \end{bmatrix} \pm \delta_{n,\mathrm{even}} (2\xi)^n C_n(t)$$

$$\xi = -n \cdot \Delta/2$$

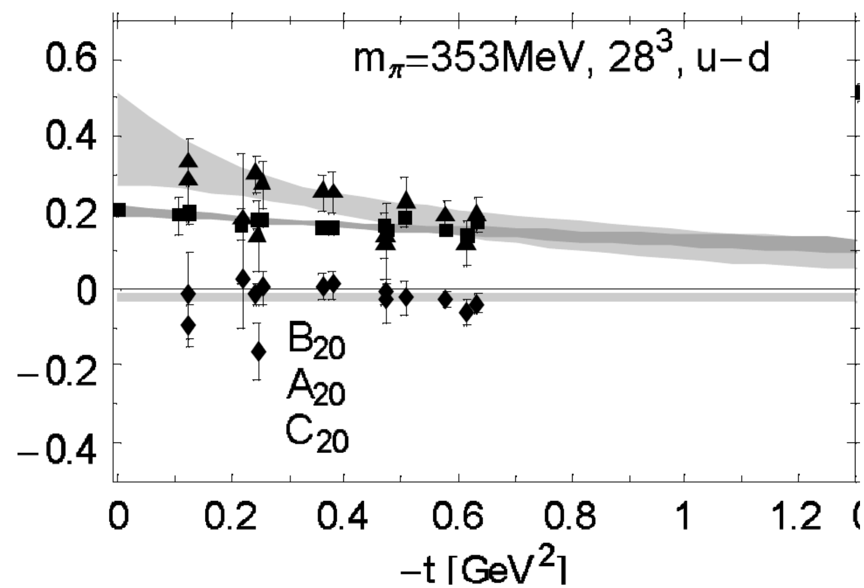
Generalized Parton Distribution

- Two major groups:
  - LHPC: N_f = 2+1 mixed action, M_π ~ 350–760 MeV
  - QCDSF: N_f = 2 clover action, M_π ~ 340–950 MeV
- Decomposition: **Isovector** ($u-d$) and **isoscalar** ($u+d$, which contains disconnected uncertainty)
- Observe: $A_{10}(Q^2)$, $A_{20}(Q^2)$, $B_{10}(Q^2)$: dipole behavior
  $C_2(Q^2) \sim 0$, $B_{20}(Q^2)$: dipole-tripole

QCDSF



LHPC



Transverse Quark Distribution

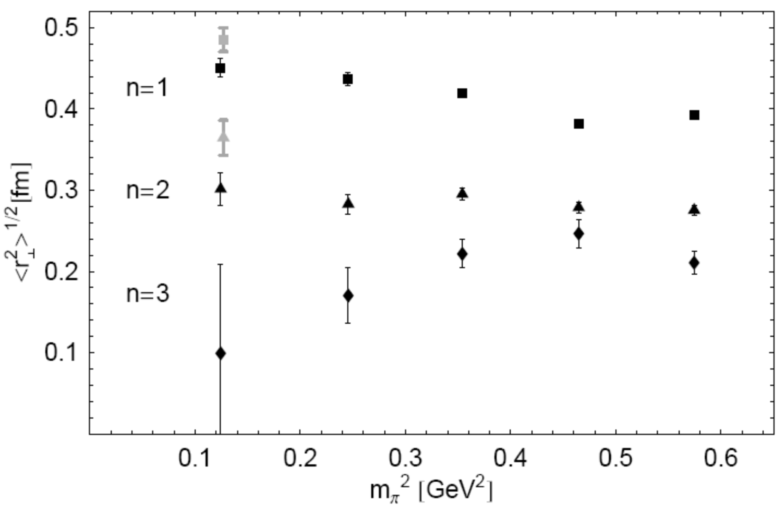
- Slopes of *A*'s are related to the transverse size of nucleon

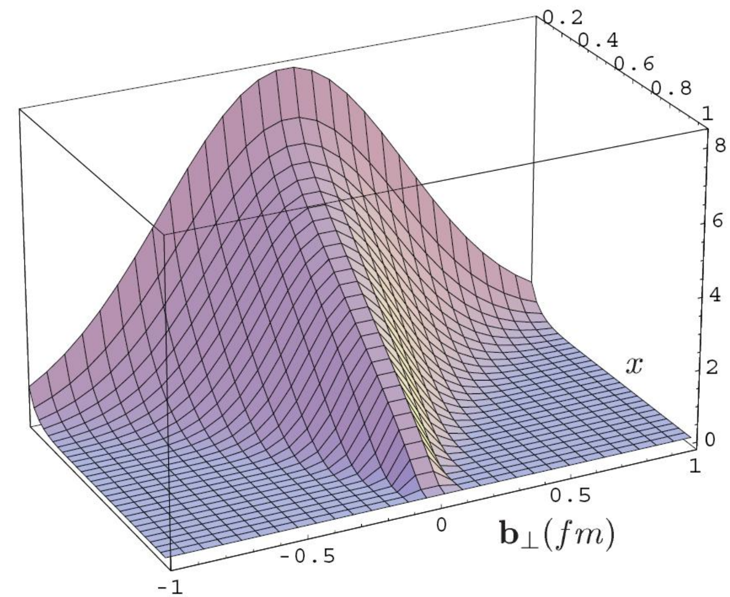
$$q(x,\vec{b}_\perp) \;=\; \int \frac{d^2\Delta_\perp}{(2\pi)^2}\, e^{-i\vec{b}_\perp\cdot\Delta_\perp} H^q(x,0,-\vec{\Delta}_\perp^2)$$

- Simple exercise: M. Burkardt, Int.J.Mod.Phys.A18:173 (2003)
  - Ansatz $H_q(x,0,-\mathbf{\Delta}_\perp^2) = q(x)e^{-a\mathbf{\Delta}_\perp^2\frac{1-x}{x}}$ $\quad q(x) \sim (1-x)^{2n_s-1}$
  - Distribution

$$q(x,\mathbf{b}_\perp) = q(x)\frac{1}{4\pi a(1-x)\ln\frac{1}{x}}e^{-\frac{\mathbf{b}_\perp^2}{4a(1-x)\ln\frac{1}{x}}}$$

- Lattice result LHPC





The Proton Spin Crisis

- Long, long ago in a kingdom far, far away…
  - Naïve parton model: proton = 2 up + down
  - Successful in explaining and predicting hadron spectroscopy
  - Proton spin comes from quark spin: $2 \times +\frac{1}{2} + (-\frac{1}{2}) = \frac{1}{2}$.
- Then in the late 80's...
  - European Muon Collaboration (EMC) performed polarized muon-nucleon scattering experiments:
    Only a small fraction (12±16%) of proton spin is quark spin!
  - **The proton spin crisis begins!!**
- What we know now:
  - Quark orbital angular momentum
  - Gluonic contributions
  - Other interactions?

Nucleon Spin

- Decomposition according to sum rule

Xiang-Dong Ji, Phys. Rev. Lett., 78:610-613, 1997.

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma^{u+d} + L^{u+d} + J^g$$

quark spin fraction

Quark orbital angular momentum

Total gluonic contribution

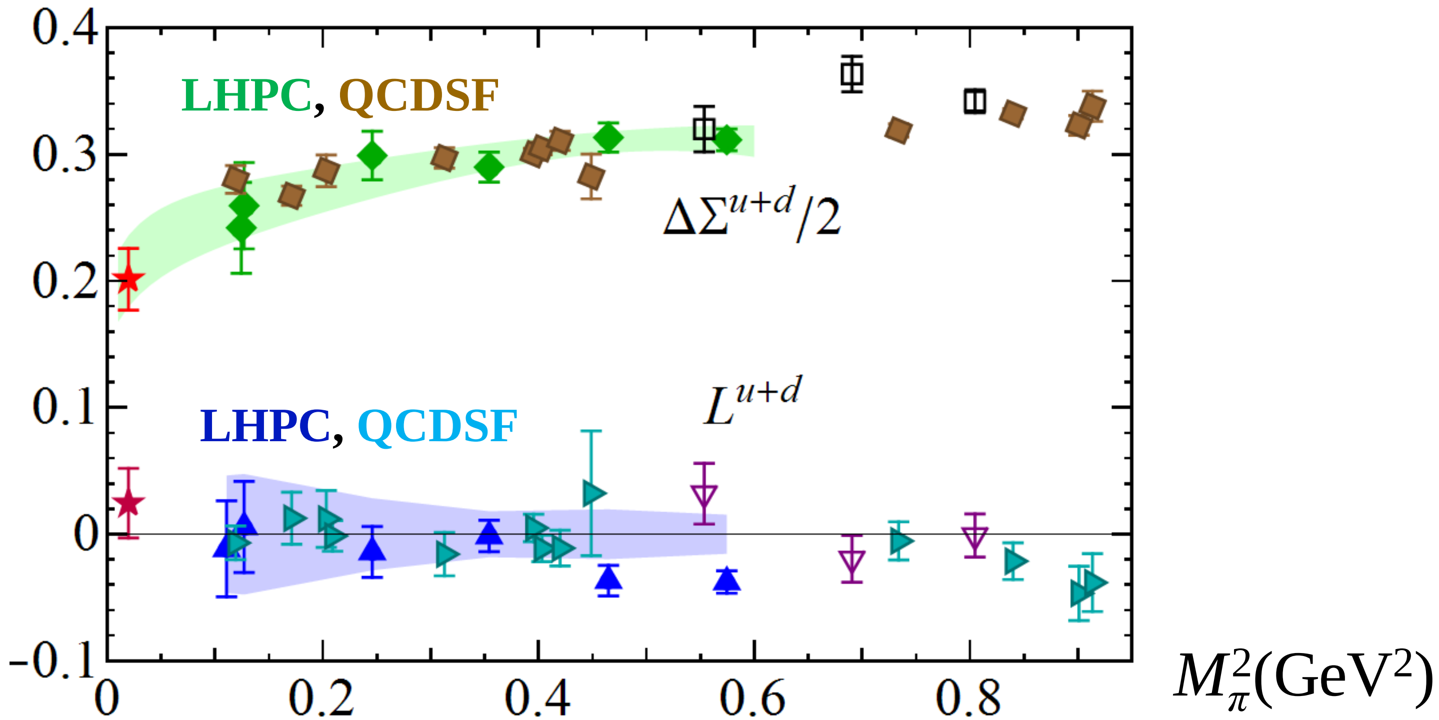
$$\frac{1}{2}\Delta\Sigma^q = \frac{1}{2}\tilde{A}^q_{10}(0)$$

$$J^g = \frac{1}{2} - J^{u+d}$$

$$L^q = \frac{1}{2}\left[A^q_{20}(0) + B^q_{20}(0)\right] - \frac{1}{2}\Delta\Sigma^q$$

Nucleon Spin

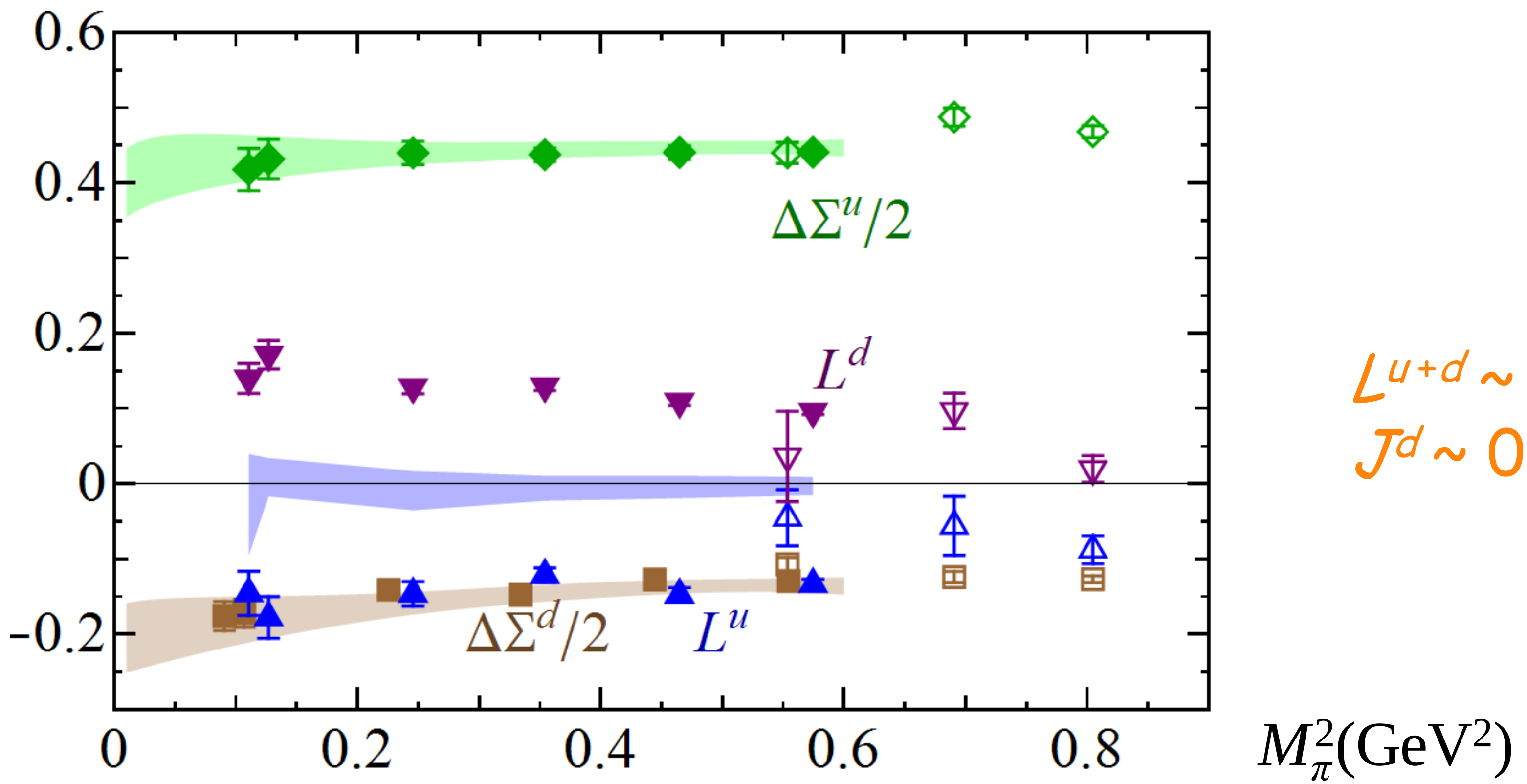
- Decomposition according to spin and orbital angular momentum:
  - LHPC: $N_f = 2+1$ mixed action, $M_\pi \sim 350–760$ MeV



Ph. Hagler, 0705.4295 et. al,; M. Ohtani et al, PoS Lat2007(2007)

Nucleon Spin

- Decomposition according to quark flavor:
  - LHPC: $N_f = 2+1$ mixed action, $M_\pi \sim 350–760$ MeV
  - QCDSF: $N_f = 2$ clover action, $M_\pi \sim 340–950$ MeV

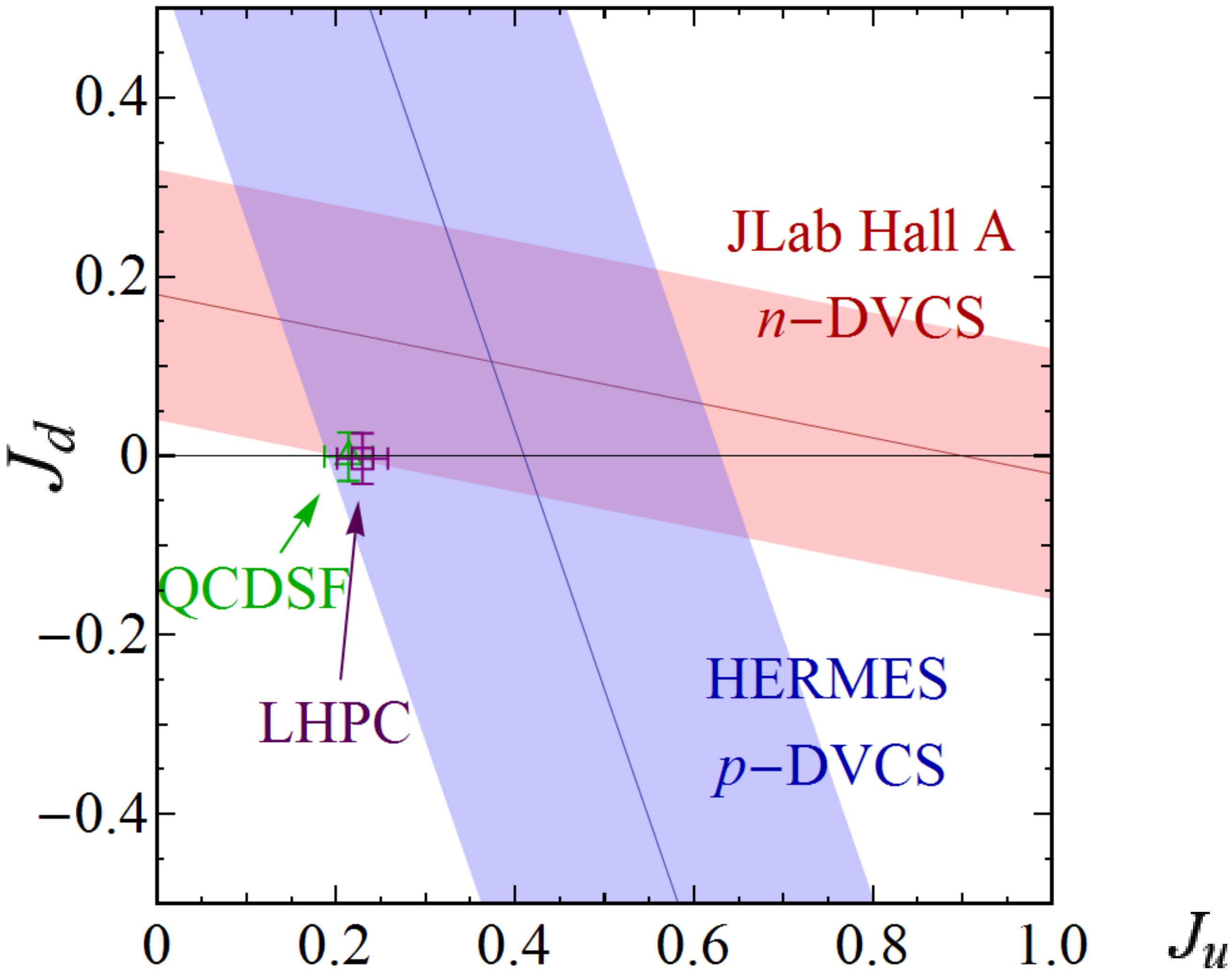


$L^{u+d} \sim 0$
$J^d \sim 0$

Ph. Hagler et. al, 0705.4295

Nucleon Spin

- J_u-J_d plot with experiments



Strangeness in Nucleon

- Overview

Momentum	$\int x(\bar{s}+s)\,dx$	DIS ν,μ,e	2–4%
Mass	$m_s \langle N\vert \bar{s}s \vert N\rangle$	πN-scatt. $\Sigma_{\pi N}$-Term	220 MeV
Spin	$\langle N\vert \bar{s}\gamma_\mu\gamma_5 s \vert N\rangle$	pol. DIS	10%
EM FF G_E^s, G_M^s	$\langle N\vert \bar{s}\gamma_\mu s \vert N\rangle$	PV electron scattering	0??

Strange Magnetic Moment of Nucleon

- Purely sea-quark effect
- First strange magnetic moment was measured by SAMPLE

$$G_M^s(Q^2 = 0.1 \; GeV^2) = 0.23(37)(25)(29)$$

B. Mueller et al. (SAMPLE) Phys. Rev. Lett. 78, 3824 (1997)

- New data, still being collected, suggests the value is non-zero.

HAPPEX and G0 collaborations at Jefferson Lab, SAMPLE at MIT-BATES, and A4 at Mainz

- Lattice calculations

$$\langle B \,|V_\mu|\, B\rangle(q) = \overline{u}_B(p') \left[\gamma_\mu F_1(q^2) + \sigma_{\mu\nu} q_\nu \frac{F_2(q^2)}{2M_B}\right] u_B(p)$$

the disconnected diagram is a must

- Done in quenched approximation
  - Direct: Noisy (Z_2) estimator — Kentucky Field Theory group (1997–2001)

    −0.28(10) to +0.05(6)
  - Indirect: Charge symmetry — Adelaide-JLab group (2006)

    −0.046(19)

Strange Magnetic Moment of Nucleon

- Disconnected diagrams are challenging
- Much effort has been put into resolving this difficulty
- Alternative approach:

D. B. Leinweber, Phys. Rev. D 53, 5115 (1996).

- Assume charge symmetry (for example, $d^n = u^p$):

$$p = e_u u^p + e_d d^p + O_N; \qquad n = e_d u^p + e_u d^p + O_N,$$
$$\Sigma^+ = e_u u^\Sigma + e_s s^\Sigma + O_\Sigma; \qquad \Sigma^- = e_d u^\Sigma + e_s s^\Sigma + O_\Sigma,$$
$$\Xi^0 = e_s s^\Xi + e_u u^\Xi + O_\Xi; \qquad \Xi^- = e_s s^\Xi + e_d u^\Xi + O_\Xi.$$

- The disconnected piece for the proton is $O_N = \frac{2}{3}\,{}^lG_M^u - \frac{1}{3}\,{}^lG_M^d - \frac{1}{3}\,{}^lG_M^s$
- The strangeness contribution is

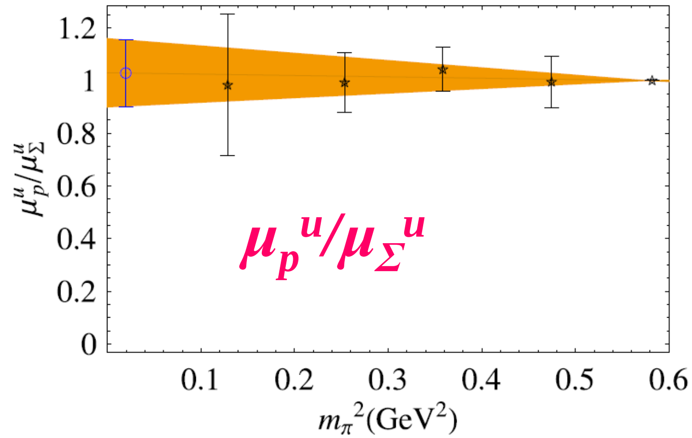
$$G_M^s = \left(\frac{{}^lR_d^s}{1 - {}^lR_d^s}\right)\left[\boxed{2p + n} - \frac{u^p}{u^\Sigma}\boxed{(\Sigma^+ - \Sigma^-)}\right]$$

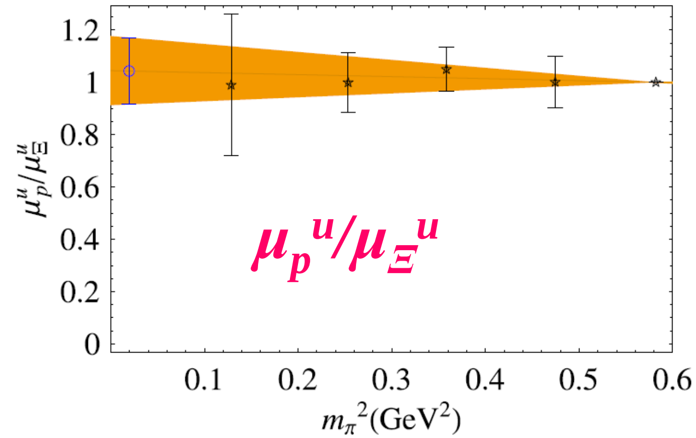
$$G_M^s = \left(\frac{{}^lR_d^s}{1 - {}^lR_d^s}\right)\left[\boxed{p + 2n} - \frac{u^n}{u^\Xi}\boxed{(\Xi^0 - \Xi^-)}\right]$$

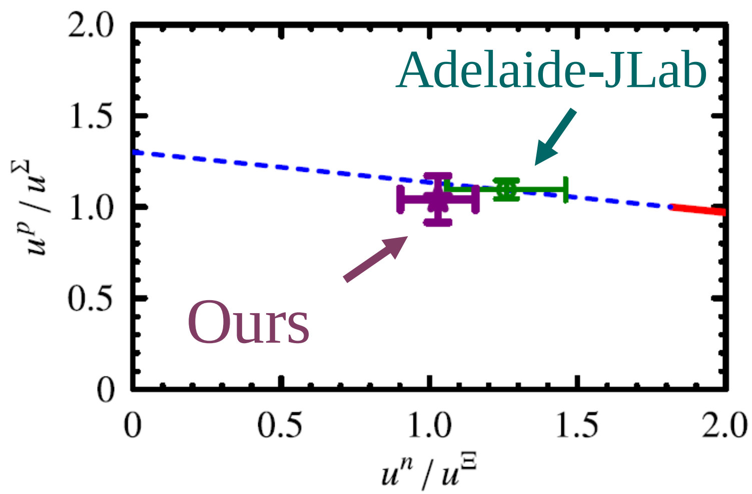
with $\quad {}^lR_d^s \equiv {}^lG_M^s / {}^lG_M^d$

Strange Magnetic Moment of Nucleon

- Dipole-form extrapolation to $q^2 = 0$
- Magnetic-moment ratios (linear extrapolation, for now)







D. B. Leinweber et al., Phys. Rev. Lett. 94, 212001 (2004).

Strange Magnetic Moment of Nucleon

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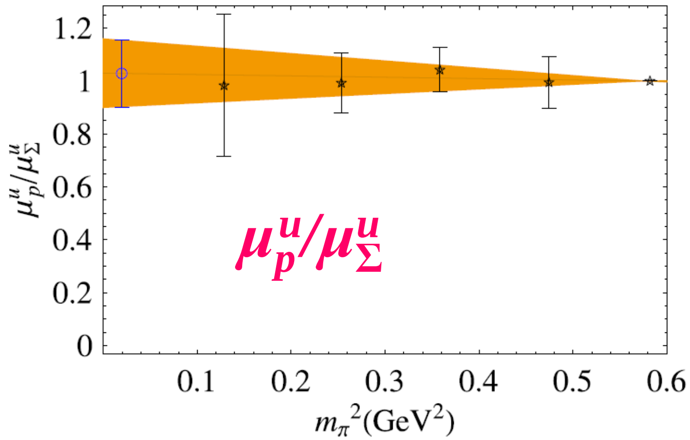
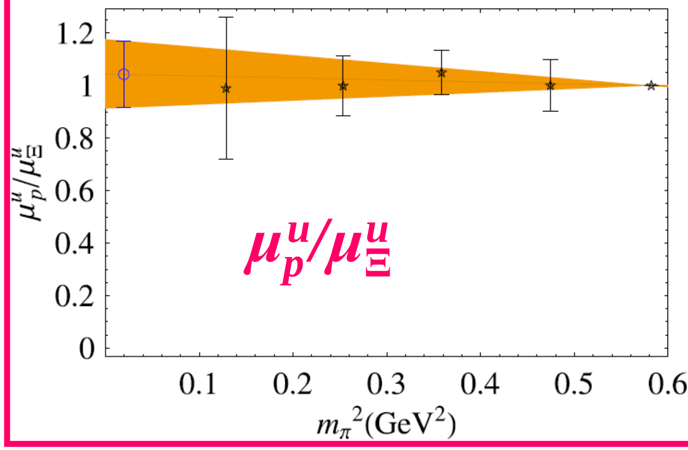


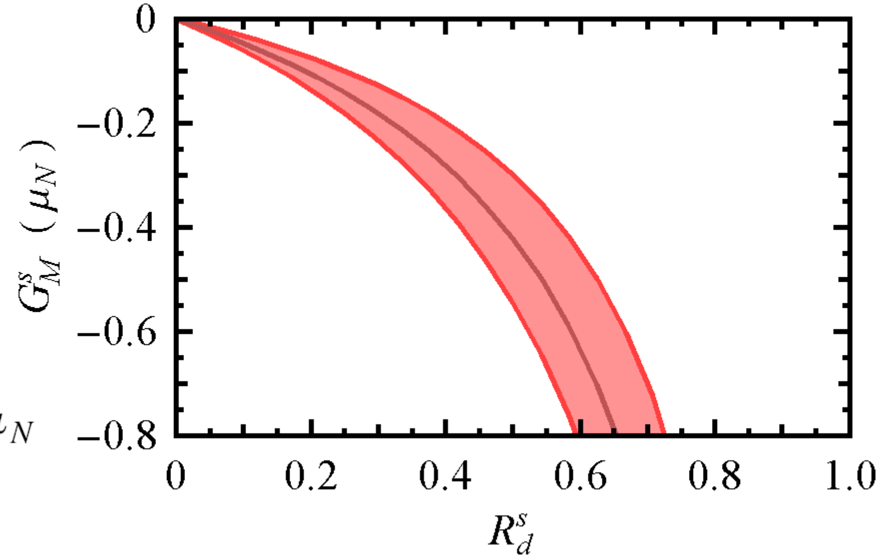


Needs better statistics

$$G_M^s = \left(\frac{{}^\ell R_d^s}{1-{}^\ell R_d^s}\right)\left[3.673 - \frac{u^p}{u^\Sigma}(3.618)\right]$$

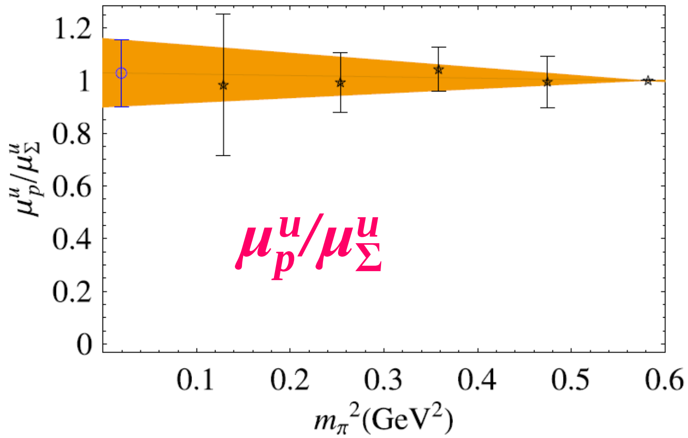
Using the following

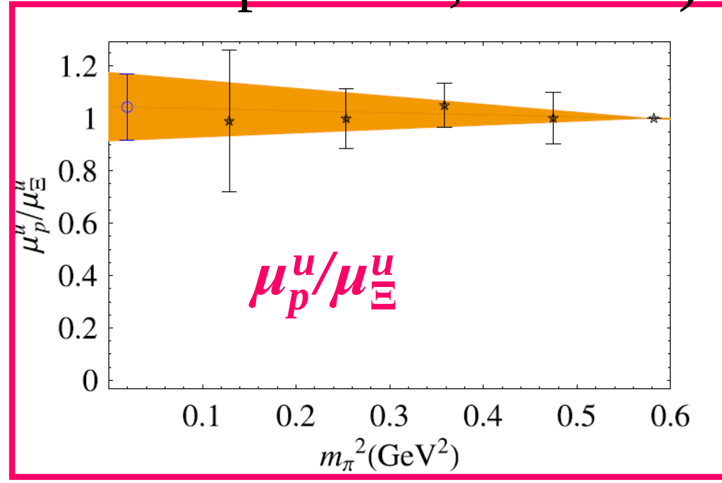
$$G_M^s = \left(\frac{{}^l R_d^s}{1-{}^l R_d^s}\right)\left[-1.033 - \frac{u^n}{u^\Xi}(-0.599)\right]\mu_N$$



Strange Magnetic Moment of Nucleon

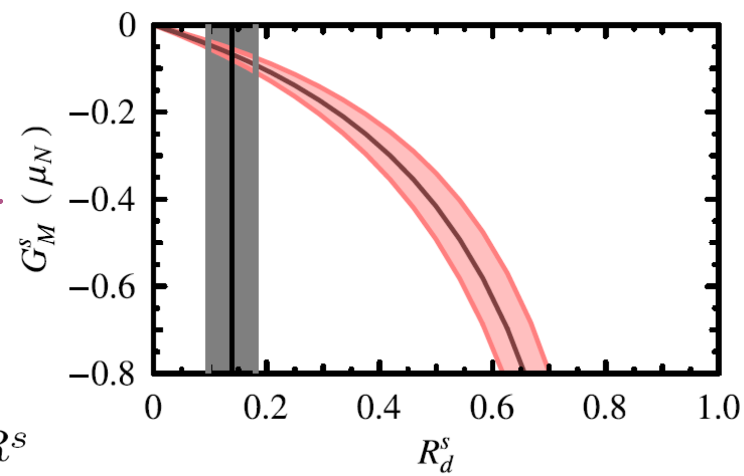
- Dipole-form extrapolation to $q^2 = 0$
- Magnetic-moment ratios (linear extrapolation, for now)





$$R_d^s = 0.139(42)$$

D. B. Leinweber et al.,
Phys. Rev. Lett. 94, 212001 (2004).



We find

$$G_M^s = -0.066(12)_{\mathrm{stat}}(23)_{R^s}$$

HWL, arXiv:0707.3844 [hep-lat]

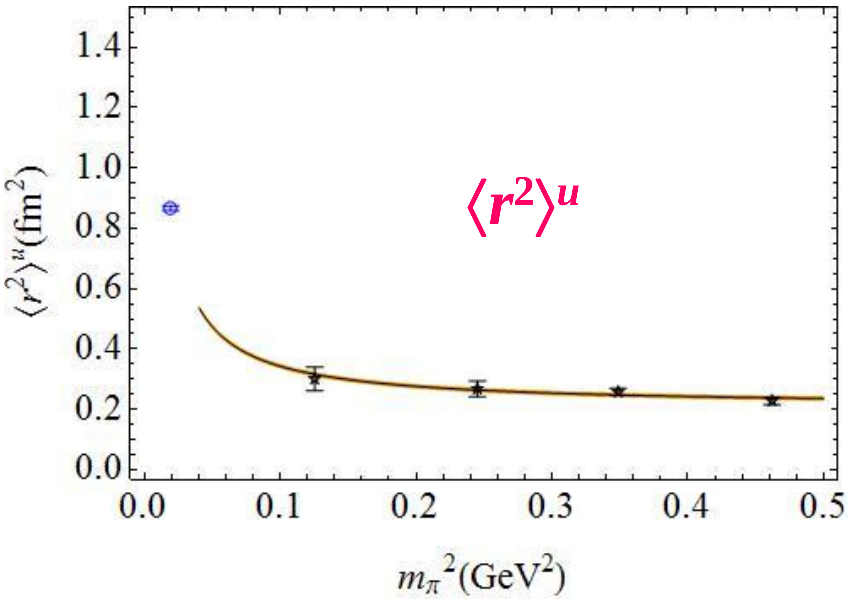
Strange Electric Moment of Nucleon

- G_E^s is proportional to $Q^2\langle r^2\rangle^s$
- Charge symmetry: D. B. Leinweber et al., Phys. Rev. Lett. 94, 212001 (2004).

$$\langle r^2\rangle^s = \frac{r_d^s}{1-r_d^s}\left[2\langle r^2\rangle^p + \langle r^2\rangle^n - \langle r^2\rangle^u\right] \qquad r_d^s = 0.16(4)$$

- u-quark form contribution of vector form factors

HWL, arXiv:0707.3844[hep-lat]



- Need more literature research on chiral extrapolation
- Using an extrapolation of the form

$$\langle r^2\rangle^u = a_0 - \frac{1+5g_A^2}{(4\pi f_\pi)^2}\log\left(\frac{m_\pi^2}{m_\pi^2+\Lambda^2}\right)$$

We find

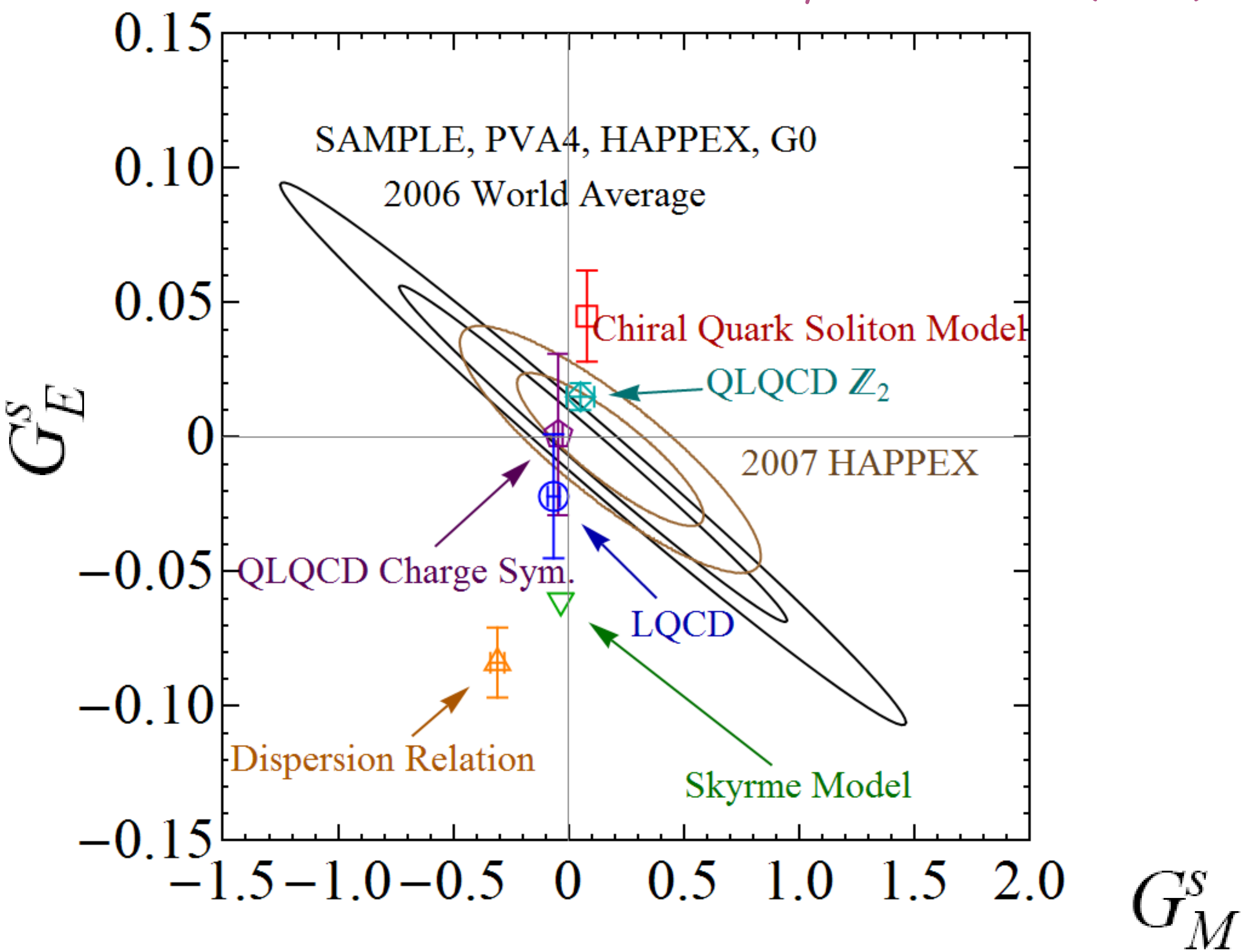
$$G_E^s(Q^2{=}0.1\ \mathrm{GeV}^2) = -0.022(23)$$

Preliminary

World Plot at $Q^2 \sim 0.1$ GeV^2

- G_E^s-G_M^s plots

HAPPEX: Phys.Rev.Lett.98:032301, 2007
SAMPLE, PVA4,HAPPEX,G0: Phys.Rev.Lett. 97 (2006) 102002



Not enough time to mention…

- Meson (π) form factors and structure functions
- Polarizabilities:
  - Electric polarizabilities of neutral hadrons
  - Magnetic polarizabilities of hadrons
  - Spin polarizabilities and electric polarizabilities of charged particles
- Vector meson (ρ, K^*) quadrupole moments
- Transition form factors
  - $\gamma^* N \to \Delta;\ \gamma^* N \to R$
- π, K, K^* distribution amplitudes
- Transverse spin densities
- Neutron electric dipole moment
- Hadron interactions (π-π, π-K, N-N, N-Y scattering lengths)
- On-going efforts: Direct calculation of disconnected contributions and gluonic matrix elements