



Simulations with N_f =2+1 Flavors of Anisotropic Clover Fermions

Huey-Wen Lin



Perspectives and Challenges for Full-QCD Lattice Calculations ECT, Trento, Italy May 07, 2008

Physics Research Directions

Wanted:

Spectrum:

Excited-state baryon resonances (Hall B)

Conventional and exotic (hybrid) mesons (Hall D)



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Form factors: ground-state and excited-state form factors and transition form factors



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Excited-state baryon resonances (Hall B)

Conventional and exotic (hybrid) mesons (Hall D)

Form factors: ground-state and excited-state form factors and transition form factors

Solution: increase resolution

Anisotropic lattices ($a_t < a_{x,y,z}$)



Only Interested in Ground State?

- Going to larger t does not always work well with three-point correlators Example: $\langle x \rangle_q^{(b)}$ Quark helicity distribution LHPC & SESAM 2 10 12 8 Phys. Rev. D 66, 034506 (2002) 80 50% increase in error $\langle x \rangle_q^{(b)}$ budget at $t_{sep} = 14$ 0.0 2 10
 - Confronting the excited states might be a better solution than avoiding them.

Actions

Anisotropic Symanzik gauge action with bare anisotropy γ_g $S_G = \frac{\beta}{N_c \gamma_g} \left\{ \sum_{x,s>s'} \left[\frac{5}{3} \mathcal{P}_{ss'} - \frac{1}{12} \mathcal{R}_{ss'} \right] + \sum_{x,s} \left[\frac{4}{3} \mathcal{P}_{st} - \frac{1}{12} \mathcal{R}_{st} \right] \right\}$

(Morningstar, Peardon '99)

Anisotropic clover fermion action with 3d stout-link smeared U's (spatially smeared only) $S_F^{SW} = \overline{\psi} \left[m_0 + D_t(U') + \frac{1}{\gamma_f} D_s(U') \right] \psi - \overline{\psi} \left[c_t \sum_s \sigma_{ts} F_{ts}(U') + \frac{c_s}{\gamma_g} \sum_{rs} \sigma_{rs} F_{rs}(U') \right] \psi$ Tree-level values for c_t and c_s (*P. Chen 2001*) $c_s = \frac{\gamma_g}{\gamma_f}$, $c_t = \frac{1}{2} \left(\frac{\gamma_g}{\gamma_f} + 1/\xi \right)$

Tadpole improvement factors u_s (gauge) and u'_s (fermion)
Coefficients to tune: γ_g, γ_f, m₀, β

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Anisotropic Symanzik gauge action with bare anisotropy γ_g $S_G = \frac{\beta}{N_c \gamma_g} \left\{ \sum_{x,s>s'} \left[\frac{5}{3} \mathcal{P}_{ss'} - \frac{1}{12} \mathcal{R}_{ss'} \right] + \sum_{x,s} \left[\frac{4}{3} \mathcal{P}_{st} - \frac{1}{12} \mathcal{R}_{st} \right] \right\}$

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- Tadpole improvement factors u_s (gauge) and u'_s (fermion)
- Coefficients to tune: γ_g , γ_f , m_0 , β
- "SLAC" = Stout Link Anisotropic Clover

3D Stout-Link Smearing

Morningstar, Peardon '04

- Smoothes out dislocations; impressive glueball results
- ♦ Updating spatial links ⇒ = → + ¹/₂ ∑_{v≠µ} ρ_{µv} { ¹/₊ + ¹/₊ ¹/₊
 ♦ Differentiable!
 Direct implementation for

dynamical simulation

Why 3d Stout-link smearing?

- i. Still have positive-definite transfer matrix in time (good for spectroscopy with multiple excited states)
- ii. Light quark action (more) stable
- iii. Tadpole $c_{s,t}$ is closer to nonperturbative one

3D Stout-Link Smearing



Computational Facilities

Two major resources:

USQCD





7n cluster (13 TF) @ JLab

Jaguar cluster (119 TF) @ ORNL

Huey-Wen Lin — INT & UW

Algorithm

- Rational Hybrid Monte Carlo (RHMC)
- Multi-scale anisotropic molecular dynamics update
- Even-odd preconditioning for the clover term
- Stout-link smearing in fermion actions

$$\frac{d\tilde{Q}}{dU_{\rm thin}} = \frac{d\tilde{Q}}{dU_{\rm stout}} \frac{dU_{\rm stout}}{dU_{\rm thin}}$$

- Split gauge term
- Three time scales
 - δt_1 : Omelyan integrator for tr log A_{ee} and $\phi_i^{\dagger} r^{-\frac{1}{2}}(\tilde{Q})\phi$
 - ♦ δt_2 : Leapfrog integrator $S_{G,(S)}$
 - δt_3 : Leapfrog integrator $S_{G,(T)}$
 - Choice: $(\delta t_1, \delta t_2, \delta t_3) = (1/4, 1/4, 1/3)$ for $12^3 \times 96$ (1/5, 1/3, 1/2) for $12^3 \times 32$

• Acceptance rate: 60–70%

Dynamical Generation Costs



- Extra cost a dimension taken to (near) continuum limit!
- Improvement: Temporal preconditions of clover Dirac operator (Edwards, Joo, Peardon, work in progress)
 - Gain factor of 2.5 in quenched study
 - Ready to implement on next anisotropic runs

Tadpole Factors and Stout Smearing



- Nonperturbatively determine γ_g , γ_f , m_0 on anisotropic lattice $S_F^{SW} = \overline{\psi} \left[m_0 + D_t(U') + \frac{1}{\gamma_f} D_s(U') \right] \psi - \overline{\psi} \left[c_t \sum_s \sigma_{ts} F_{ts}(U') + \frac{c_s}{\gamma_g} \sum_{rs} \sigma_{rs} F_{rs}(U') \right] \psi$
- Three calculations:

b Background field in time: PCAC gives M_t

Nonperturbatively determine γ_g , γ_f , m_0 on anisotropic lattice $S_F^{SW} = \overline{\psi} \left[m_0 + D_t(U') + \frac{1}{\gamma_\ell} D_s(U') \right] \psi - \overline{\psi} \left[c_t \sum_s \sigma_{ts} F_{ts}(U') + \frac{c_s}{\gamma_s} \sum_s \sigma_{rs} F_{rs}(U') \right] \psi$ Three calculations: \Rightarrow Background field in time: PCAC gives M_t Background field in space: sideways potential gives $\gamma_{g,R}$ Klassen Method: ratio of Wilson loops Wanted: $V_{\rm s}(ya_{\rm s}) = V_{\rm s}(ta_{\rm s}/\xi_{\rm R})$ \Rightarrow Condition: $R_{ss}(x,y) = R_{st}(x,t)$ 4.0 ξ_a^{PBC} $L(\xi_g) = \sum \frac{(R_{ss}(x, y) - R_{st}(x, \xi_g y))^2}{(\Delta R_s)^2 + (\Delta R_t)^2}$ 3.8 3.6 3.4 $\xi_a^{\rm SF}$ Comparison with PBC result 3.2 Example: 3.0 $(\gamma_{g} = 4.4, \gamma_{f} = 3.4, m_{0} = -0.0570, \beta = 1.5)$ 2 3 5 6 4 min(xy)

- $\text{Nonperturbatively determine } \gamma_g, \gamma_f, m_0 \text{ on anisotropic lattice}$ $S_F^{SW} = \overline{\psi} \left[m_0 + D_t(U') + \frac{1}{\gamma_f} D_s(U') \right] \psi \overline{\psi} \left[c_t \sum_s \sigma_{ts} F_{ts}(U') + \frac{c_s}{\gamma_g} \sum_{rs} \sigma_{rs} F_{rs}(U') \right] \psi$
- Three calculations:
 - **b** Background field in time: PCAC gives M_t
 - Background field in space: sideways potential gives $\gamma_{g,R}$
 - Antiperiodic in time: dispersion relation gives $\gamma_{f,R}$, $(m_0, r_0, \text{ etc.})$



- ♦ Nonperturbatively determine γ_g , γ_f , m_0 on anisotropic lattice $S_F^{SW} = \overline{\psi} \left[m_0 + D_t(U') + \frac{1}{\gamma_f} D_s(U') \right] \psi - \overline{\psi} \left[c_t \sum_s \sigma_{ts} F_{ts}(U') + \frac{c_s}{\gamma_g} \sum_{rs} \sigma_{rs} F_{rs}(U') \right] \psi$
- Three calculations:
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m_0	γ_g	γf	M_t	ΔM_t	$\Delta M_t^{(0)}$	m_0	$\gamma_g \gamma_f$	ξ_g	_	<i>m</i> ₀	$\gamma_g \gamma_f$	ξf	m_{π}	m_{ρ}	m_{π}/m_{ρ}
-0.0950	4.3 3	3.5	-0.0122(9)	0.0003(10)	-0.001547	-0.0950	4.3 3.5	3.48(5)		-0.0743	4.3 3.4	3.43(4)	0.1501(9)	0.222(3)	0.677(9)
-0.0950	4.3 3	3.4	-0.0121(9)	0.0003(9)	-0.00167	-0.0950	$4.3 \ 3.4$	3.42(3)		-0.0618	4.2 3.5	3.62(5)	0.2830(9)	0.348(2)	0.814(5)
-0.0950	4.3 3	3.3	-0.0141(8)	0.0002(9)	-0.001798	-0.0950	$4.3 \ 3.3$	3.40(3)		-0.0618	4.2 3.4	3.38(4)	0.2753(10)	0.337(2)	0.816(5)
-0.0734	4.3 3	3.5	0.0160(9)	-0.0007(9)	-0.001547	-0.0743	$4.3 \ 3.4$	3.47(10)		-0.0618	4.2 3.3	3.18(3)	0.2604(10)	0.319(5)	0.817(11)
-0.0734	4.3 3	3.4	0.0149(7)	-0.0007(6)	-0.00167	-0.0734	$4.3 \ 3.4$	3.46(4)		-0.0618	4.3 3.4	3.47(6)	0.2232(15)	0.290(4)	0.769(9)
-0.0734	4.3 3	3.3	0.0139(7)	0.0006(11)	-0.001798	-0.0618	$4.2 \ 3.5$	3.48(4)		-0.0618	4.4 3.3	3.25(6)	0.1639(17)	0.217(5)	0.754(16)
-0.0618	4.2 3	3.5	0.0431(11)	0.0002(4)	-0.001427	-0.0618	$4.2 \ 3.4$	3.41(3)		-0.0570	4333	3.23(4)	0.2401(13)	0.299(4)	0.804(8)
-0.0618	4.2 3	3.4	0.036(2)	-0.0004(8)	-0.001545	-0.0618	$4.2 \ 3.3$	3.42(2)		-0.0570	4.3 3.2	3.19(5)	0.2290(16)	0.292(4)	0.784(10)
-0.0618	4.2 3	3.3	0.0339(11)	0.0004(8)	-0.001672	-0.0618	$4.3 \ 3.5$	3.50(4)		-0.0570	4331	2.99(4)	0.2164(18)	0.261(7)	0.828(20)
-0.0618	4.3 3	3.5	0.0321(9)	0.0003(5)	-0.001547	-0.0618	$4.3 \ 3.4$	3.47(4)		-0.0570	4.0 0.1	3.43(6)	0.103(3)	0.255(6)	0.020(20) 0.758(17)
-0.0618	4.3 3	3.4	0.0303(6)	-0.0001(5)	-0.00167	-0.0618	4.3 3.3	3.43(4)		-0.0570	4.4 3.9	3.22(8)	0.155(3)	0.233(5)	0.730(17) 0.780(17)
-0.0618	4.3 3	3.3	0.0297(6)	-0.0003(4)	-0.001798	-0.0618	4.3 3.2	3.38(7)		0.0570	4431	2.01(11)	0.151(9)	0.104(6)	0.78(2)
-0.0618	4.4 3	3.4	0.0218(5)	-0.0004(5)	-0.001791	-0.0618	4.4 3.3	3.47(5)		-0.0370	4.4 3.1	2.91(11)	0.151(5)	0.194(0)	0.76(2)
-0.0618	4.4 3	3.3	0.0213(7)	0.0001(6)	-0.001923	-0.0570	4.3 3.4	3.48(7)							
-0.0570	4.3 3	3.4	0.0349(7)	0.0002(5)	-0.00167	-0.0570	4.3 3.3	3.43(9)							
-0.0570	4.3 3	3.3	0.0342(10)	0.0004(7)	-0.001798	-0.0570	4.3 3.2	3.39(3)							
-0.0570	4.3 3	3.2	0.0311(12)	0.0005(8)	-0.001935	-0.0570	4.3 3.1	3.36(4)							
-0.0570	4.3 3	3.1	0.0300(9)	0.0025(8)	-0.002078	-0.0570	4.4 3.3	3.50(4)							
-0.0570	4.4 3	3.3	0.0248(12)	0.0002(8)	-0.001923	-0.0570	4.4 3.2	3.54(5)							
-0.0570	4.4 3	3.2	0.0239(6)	0.0005(7)	-0.002065	-0.0570	4.4 3.1	3.399(16)							
-0.0570	4.4 3	3.1	0.0220(7)	0.0012(11)	-0.002212				=						
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Nonperturbatively determine γ_{g} , γ_{f} , m_{0} on anisotropic lattice $S_F^{SW} = \overline{\psi} \left[m_0 + D_t(U') + \frac{1}{\gamma_f} D_s(U') \right] \psi - \overline{\psi} \left[c_t \sum_{s} \sigma_{ts} F_{ts}(U') + \frac{c_s}{\gamma_s} \sum_{s} \sigma_{rs} F_{rs}(U') \right] \psi$ $\blacktriangleright N_f = 3, \, \xi = 3.5, \, \beta = 1.5$ arxiv:0803.3960 Plot of γ_g and γ_f versus input current quark mass • Mild dependence on quark mass; fixed γ_g and γ_f PCAC mass measured in SF scheme: $m_{\rm cr} = -0.0854(5)$ 0.05 0.04 0.03 M_t 0.02 0.01 0.00 -0.01-0.10-0.09-0.08-0.06-0.05-0.04-0.07 m_0

 Nonperturbatively determine γ_g, γ_f, m₀ on anisotropic lattice S^{SW}_F = ψ [m₀ + D_t(U') + ¹/_{γf}D_s(U')] ψ-ψ [c_t ∑_s σ_{ts}F_{ts}(U') + ^{c_s}/_{γg} ∑_s σ_{rs}F_{rs}(U')] ψ

 N_f = 3, ξ = 3.5, β = 1.5 arxiv:0803.3960

 Plot of γ_g and γ_f versus input current quark mass
 Mild dependence on quark mass; fixed γ_g and γ_f

 PCAC mass measured in SF scheme: m_{cr} = -0.0854(5)

• Check NP $c_{s,t}$ condition in SF scheme



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Mass-independent scheme (fixed β=1.5 approach)
Scale and masses are defined in chiral limit

(Edwards, Joo, Lin, Peardon, work in progress)

L_x	L_t	m_l	m_s	L (fm)	m_{π} L	$m_{\pi}(MeV)$
12	96	-0.0540	-0.0540	1.44	11.6	~ 1600
12	96	-0.0699	-0.0540	1.44	8.3	
12	96	-0.0794	-0.0540	1.44	5.9	
12	96	-0.0826	-0.0540	1.44	9.6	
16	96	-0.0826	-0.0540	1.92	6.3	~ 660
12	96	-0.0618	-0.0618	1.44	9.7	~ 1340
16	128	-0.0743	-0.0743	1.92	8.1	~ 850
16	128	-0.0808	-0.0743	1.92	5.6	
16	128	-0.0830	-0.0743	1.92	4.5	
16	128	-0.0840	-0.0743	1.92	3.8	
24	128	-0.0840	-0.0743	2.88	5.7	~ 390

Algorithm

- Rational Hybrid Monte Carlo (RHMC)
 - Multi-scale anisotropic molecular dynamics update
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- Split gauge term
- Three time scales
 - δt_1 : Omelyan integrator for tr log A_{ee} and $\phi_i^{\dagger} r^{-\frac{1}{2}}(\tilde{Q})\phi$
 - δt_2 : Omelyan integrator $S_{G,(S)}$
 - δt_3 : Omelyan integrator $S_{G,(T)}$
 - Choice: $\delta t_1 = \delta t_2$
- Acceptance rate: 75%

Autocorrelation



Lowest Eigenvalue

- Example: $24^3 \times 128$ volume, with pion mass 315 MeV
- Histogram distributions



- Preliminary spectroscopic measurements
 - 3 Gaussian smearing parameters + Point/Smeared sink
 - Average over 4 time sources
 - (w/ eigcg solver 0707.0131 [hep-lat])
 - $\{0,0,0,0\}, \{8,8,8,32\}, \{0,0,0,64\}, \{8,8,8,128\}$
 - Ground states obtained from 2-state fits
 - For example: 12 pion correlators



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 - 3 Gaussian smearing parameters + Point/Smeared sink
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 - $\{0,0,0,0\}, \{8,8,8,32\}, \{0,0,0,64\}, \{8,8,8,128\}$
 - Ground states obtained from 2-state fits
 - For example: 6 Lambda correlators



More spectroscopy results by Saul Cohen









Strange-Quark Mass

- Difficult: Chiral extrapolation to obtain m_s and r_0/a_s
- Strange-quark tuning
 - **\diamond** Candidates: kaon, φ , Ω mass, etc.
 - Example: at a fixed m_s , see 30% variation in phi mass



Need multiple 2+1 runs to obtain reasonable m_s

- Better description for s-quark tuning
 - Use ratio of hadron masses to eliminate lattice spacing
 - Leading-order XPT $l_{\Omega} = (9/4) m_{\pi}^2/m_{\Omega}^2$ and

 $s_{\Omega} = (9/4) m_{\pi}/m_{\Omega} \text{ and} s_{\Omega} = (9/4) (2m_K^2 - m_{\pi}^2)/m_{\Omega}^2$

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 - Tune $N_f = 3$ quark mass until physical s_{Ω} achieved



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 - 2+1f comparison plot
 Aniso-clover, a_s = 0.13 fm
 DWF on asqtad, a = 0.125 fm (LHPC)
 DWF on DWF, a = 0.116 fm
 (RBC+UKQCD)



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 Aniso-clover, a_s ~ 0.13 fm
 DWF on asqtad, a = 0.125 fm (LHPC)
 DWF on DWF, a = 0.116 fm
 (RBC+UKQCD)
 - LO Extrapolation Strange is off by 4–6%
 - NLO correction?



Future Dynamical Generation

Where we stand now:

- Scaling based on actual (24³) runs down to ~170 MeV $Cost_{traj} = \xi^{1.25} \left(\frac{fm}{a_s}\right)^6 \cdot \left[\left(\frac{L_s}{fm}\right)^3 \left(\frac{L_t}{fm}\right) \right]^{5/4} \cdot [C_1 + C_2/m_l].$
- Currently, ~5k traj @ 875, 580, ~3k @ 456 MeV (16³):
 ~6k by July 1
- ◆ 315 MeV (24³), currently 3k traj, get ~1k traj/week
- ◆ 24³ 315 MeV ORNL runs underway now
- ◆ 24³ and 32³ 250 MeV for the near future

Future plans:

- 200 MeV generation possible within next year or two
- Excited single-particle state vs. multi-particle ones
- Multiple lattice spacings

