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# Nucleon and Hyperon Form Factors from Lattice QCD

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# Outline

## ➤ Background on Lattice QCD

## ➤ Nucleon Properties

- Axial coupling charge
- Structure functions
- Charge radius and Goldberger-Treiman relation ( $N_f = 2$ )
- Summary/Outlook: I



## ➤ From Hyperon Analysis

- Proton strangeness magnetic moment
- $\Sigma$  and  $\Xi$  axial coupling prediction
- Summary/Outlook: II



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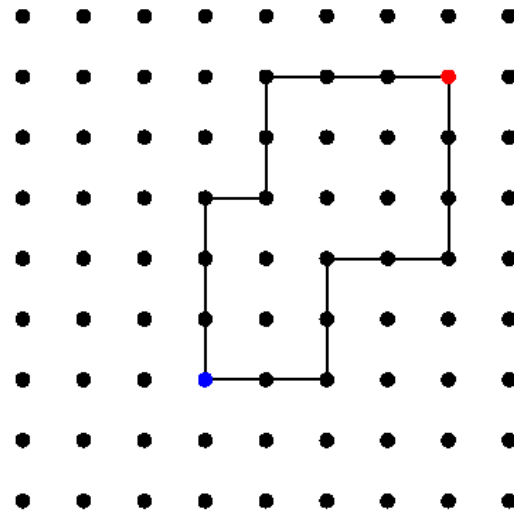
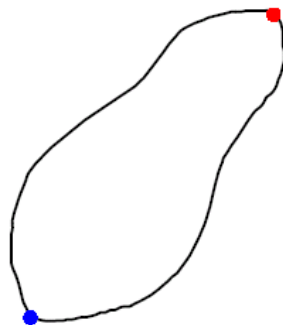
# Background: Lattice 101

# Lattice Gauge Theory

- Physical observables are calculated from the path integral

$$\langle \Omega | O | \Omega \rangle = \frac{1}{Z} \int [dU][d\bar{\psi}][d\psi] O(U, \bar{\psi}, \psi) e^{i \int d^4x [S_F(U, \bar{\psi}, \psi) + S_G(U)]}$$

- Lattice QCD is a discrete version of continuum QCD theory



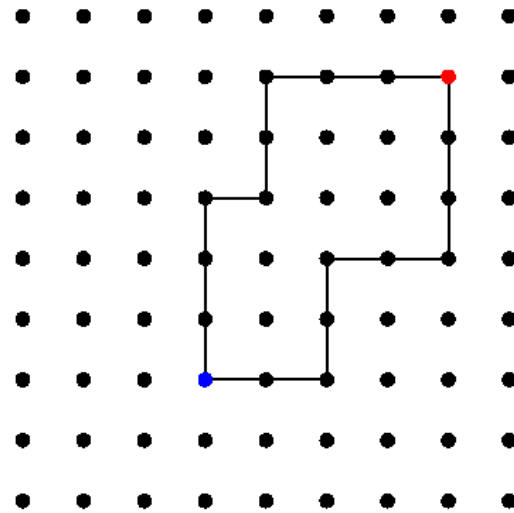
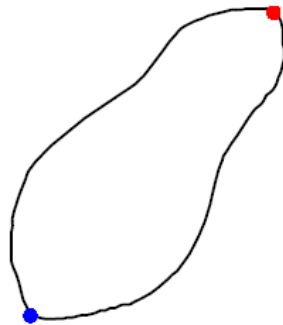
- Use Monte Carlo integration combined with the “importance sampling” technique to calculate the path integral.
- Take  $a \rightarrow 0$  and  $V \rightarrow \infty$  for the “continuum limit”

# Lattice Gauge Theory

- Physical observables are calculated from the path integral

$$\langle \Omega | O | \Omega \rangle = \frac{1}{Z} \int [dU][d\bar{\psi}][d\psi] O(U, \bar{\psi}, \psi) e^{i \int d^4x [S_F(U, \bar{\psi}, \psi) + S_G(U)]}$$

- Lattice QCD is a discrete version of continuum QCD theory



- Use Monte Carlo integration combined with the “importance sampling” technique to calculate the path integral.
- Chiral extrapolation  $m_\pi \rightarrow (m_\pi)_{\text{phys}}$

# Lattice Fermion Actions

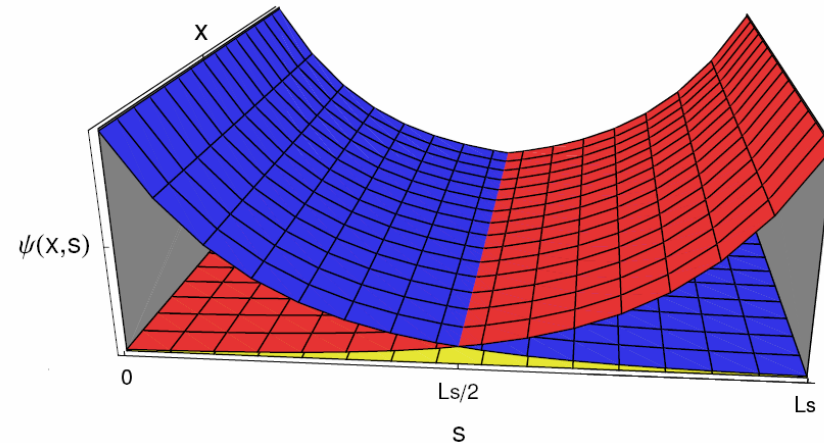


## Chiral fermions (e.g., Domain-Wall/Overlap):

- Automatically  $O(a)$  improved, suitable for spin physics and weak matrix elements
- Expensive

$$D_{x,s;x',s'} = \delta_{x,x'} D_{s,s'}^\perp + \delta_{s,s'} D_{x,x'}^\parallel$$

$$D_{s,s'}^\perp = \frac{1}{2} [(1 - \gamma_5) \delta_{s+1,s'} + (1 + \gamma_5) \delta_{s-1,s'} - 2\delta_{s,s'}] \\ - \frac{m_f}{2} [(1 - \gamma_5) \delta_{s,L_s-1} \delta_{0,s'} + (1 + \gamma_5) \delta_{s,0} \delta_{L_s-1,s'}]$$



## (Improved) Staggered fermions (asqtad):

- Relatively cheap for dynamical fermions (good)
- Mixing among parities and flavours or “tastes”
- Baryonic operators a nightmare — not suitable

## Mixed actions:

- Staggered sea (cheap) with Domain-Wall valence (chiral)
- Match the sea Goldstone pion mass to the DWF pion



# Green Functions

- Interpolating field for a baryon

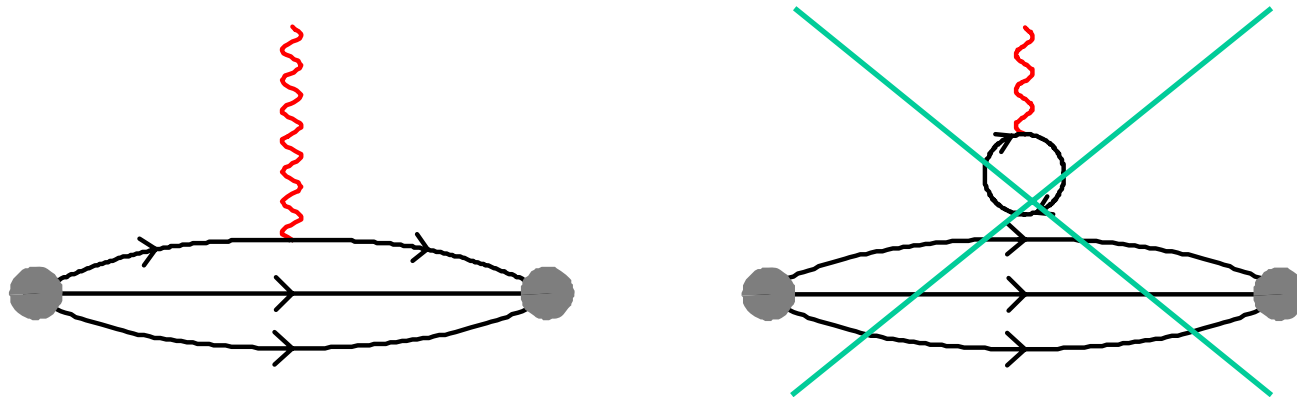
$$J_\alpha(\vec{p}, t) = \sum_{\vec{x}, a, b, c} e^{i\vec{p}\cdot\vec{x}} \epsilon^{abc} [u_a^T(y_1, t) C \gamma_5 d_b(y_2, t)] u_{c,\alpha}(y_3, t) \phi(y_1 - x) \phi(y_2 - x) \phi(y_3 - x)$$

- Two-point function with projection

$$C_{2\text{pt}}(\vec{p}, t) = \sum_{\alpha, \beta} \left( \frac{1 + \gamma_4}{2} \right)_{\alpha\beta} \langle J_\beta(\vec{p}, t) \bar{J}_\alpha(\vec{p}, 0) \rangle$$

- Three-point function with connected piece only

$$C_{3\text{pt}}^{\Gamma, \mathcal{O}}(\vec{p}, t, \tau) = \sum_{\alpha, \beta} \Gamma^{\alpha, \beta} \langle J_\beta(\vec{p}, t) \mathcal{O}(\tau) \bar{J}_\alpha(\vec{p}, 0) \rangle$$



# Ratio Construction

## Two-point function

$$\begin{aligned} \Gamma_{AB}^{NN}(t_i, t_f, \vec{p}; T) &\rightarrow \\ &a^6 Z_B^N(p) Z_A^N(p) \sum_s T_{\alpha\beta} \bar{u}_\alpha(\vec{p}, s) u_\beta(\vec{p}, s) \\ &\times \frac{m_N}{E_N(\vec{p})} e^{-(t_f-t_i)E_N(\vec{p})} \\ &= \left( \frac{E_N(\vec{p}) + m_N}{2E_N(\vec{p})} \right) e^{-(t_f-t_i)E_N(\vec{p})} \end{aligned}$$

## Three-point function

$$\begin{aligned} \Gamma_{\mu,AB}^{NN}(t_i, t, t_f, \vec{p}_i, \vec{p}_f; T) &\rightarrow \\ &= \frac{m_N^2}{E_N(\vec{p}_f) E_N(\vec{p}_i)} e^{-(t_f-t)E_N(\vec{p}_f)} e^{-(t-t_i)E_N(\vec{p}_i)} \\ &\sum_{s,s'} T_{\alpha\beta} Z_B(p_f) u_\beta(\vec{p}_f, s') \\ &\langle N(\vec{p}_f, s') | j_\mu(0) | N(\vec{p}_i, s) \rangle \bar{u}_\alpha(\vec{p}_i, s) Z_A(p_i) \end{aligned}$$

Ratio cancels out  $t$  and  $Z$  dependence

$$\begin{aligned} R_{j\mu} &= \frac{Z_V \Gamma_{\mu,GG}^{(3),P}(t_{\text{src}}, t, t_{\text{snk}}, \vec{p}_{\text{src}}, \vec{p}_{\text{snk}})}{\Gamma_{GG}^{(2),P+}(t_{\text{src}}, t_{\text{snk}}, \vec{p}_{\text{snk}})} \sqrt{\frac{\Gamma_{LG}^{(2),P+}(t_{\text{src}}, t_{\text{snk}}, \vec{p}_{\text{src}})}{\Gamma_{LG}^{(2),P+}(t_{\text{src}}, t_{\text{snk}}, \vec{p}_{\text{snk}})}} \\ &\times \sqrt{\frac{\Gamma_{LG}^{(2),P+}(t_{\text{src}}, t_{\text{snk}}, \vec{p}_{\text{snk}})}{\Gamma_{LG}^{(2),P+}(t_{\text{src}}, t_{\text{snk}}, \vec{p}_{\text{src}})}} \sqrt{\frac{\Gamma_{GG}^{(2),P+}(t_{\text{src}}, t, \vec{p}_{\text{snk}})}{\Gamma_{GG}^{(2),P+}(t_{\text{src}}, t, \vec{p}_{\text{src}})}} \end{aligned}$$



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# Nucleon Structure

(work in progress)

in collaboration with

*Tom Blum, Shigemi Ohta, Kostas Orginos,  
Shoichi Sasaki and Takeshi Yamazaki*



# 2+1-flavor RBC/UKQCD Ensembles

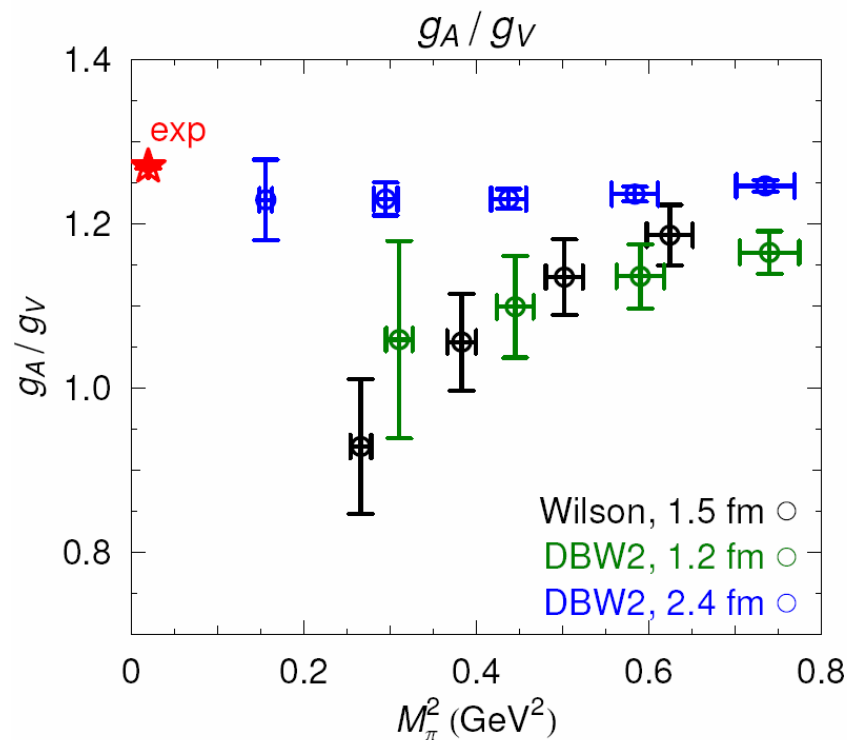
- Generated jointly by RBC and UKQCD
  - Iwasaki gauge action ( $c_1 = -0.331$ )
  - Pion mass range: 300–625 MeV
  - $V_{\text{lat}} = 24^3 \times 64$ ,  $a \approx 0.125$  fm,  $L_s = 16$ ,  $M_5 = 1.8$

FSE (RBCK quenched study)

TABLE I: Gaussian-smeared source parameters

$m_{\text{sea}}$	0.01	0.02	0.03
$t_{\text{snk}} - t_{\text{src}}$	12	12	12
$t_{\text{src}}$	0, 16, 32, 48	0, 16, 32, 48	0, 16, 32, 48
# of conf.	119	49	54
$m_\pi$ (GeV)	0.399(3)	0.535(3)	0.625(3)
$m_N$ (GeV)	1.169(19)	1.204(13)	1.474(18)

*C. Allton, et al., RBC/UKQCD  
collaborations, coming soon*



# Axial Charge Coupling

## ➤ Isovector vector and axial-vector current

$$\langle p|V_\mu^\dagger(0)|n\rangle = \bar{u}_p[\gamma_\mu g_V(q^2) - q_\mu \sigma_{\mu\nu} g_T(q^2)]u_n$$

$$\langle p|A_\mu^\dagger(0)|n\rangle = \bar{u}_p[\gamma_\mu \gamma_5 g_A(q^2) - iq_\mu \gamma_5 g_P(q^2)]u_n$$

## ➤ Chiral symmetry gives $Z_A = Z_V = 1/g_V$

## ➤ Continuum $\chi$ PT extrapolation (SSE scheme)

*T. R. Hemmert et al., J. Phys. G24, 1831 (1998)*

*T. R. Hemmert et al., Phys. Rev. D68, 075009 (2003)*

$$\begin{aligned} g_A^{\text{SSE}}(m_\pi^2) = & g_A^0 + \left[ 4C_{\text{SSE}}(\lambda) - \frac{(g_A^0)^3}{16\pi^2 f_\pi^2} - \frac{25c_A^2 g_1}{324\pi^2 f_\pi^2} + \frac{19c_A^2 g_A^0}{108\pi^2 f_\pi^2} \right] m_\pi^2 \\ & - \frac{m_\pi^2}{4\pi^2 f_\pi^2} \left[ (g_A^0)^3 + \frac{1}{2} g_A^0 \right] \ln \frac{m_\pi}{\lambda} + \frac{4c_A^2 g_A^0}{27\pi \Delta_0 f_\pi^2} m_\pi^3 \\ & + \left[ 25c_A^2 g_1 \Delta_0^2 - 57c_A^2 g_A^0 \Delta_0^2 - 24c_A^2 g_A^0 m_\pi^2 \right] \frac{\sqrt{m_\pi^2 - \Delta_0^2}}{81\pi^2 f_\pi^2 \Delta_0} \arccos \frac{\Delta_0}{m_\pi} \\ & + \frac{25c_A^2 g_1 (2\Delta_0^2 - m_\pi^2)}{162\pi^2 f_\pi^2} \ln \frac{2\Delta_0}{m_\pi} + \frac{c_A^2 g_A^0 (3m_\pi^2 - 38\Delta_0^2)}{54\pi^2 f_\pi^2} \ln \frac{2\Delta_0}{m_\pi} + \mathcal{O}(\epsilon^4) \end{aligned}$$

# Axial Charge Coupling

## ➤ Isovector vector and axial-vector current

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## ➤ Chiral symmetry gives $Z_A = Z_V = 1/g_V$

## ➤ Continuum $\chi$ PT extrapolation (SSE scheme)

$$\begin{aligned} \delta_L(\Gamma_{NN}) \equiv \delta g_A &= \frac{m_\pi^2}{3\pi^2 f^2} \left[ g_A^3 \mathbf{F}_1 \right. \\ &+ \left( g_{\Delta N}^2 g_A + \frac{25}{81} g_{\Delta N}^2 g_{\Delta\Delta} \right) \mathbf{F}_2 \\ &+ g_A \mathbf{F}_3 + g_{\Delta N}^2 g_A \mathbf{F}_4 \left. \right] \\ \mathbf{F}_2(m, \Delta, L) &= - \sum_{\mathbf{n} \neq 0} \left[ \frac{K_1(mL|\mathbf{n}|)}{mL|\mathbf{n}|} \right. \\ &+ \frac{\Delta^2 - m^2}{m^2} K_0(mL|\mathbf{n}|) - \frac{\Delta}{m^2} \int_m^\infty d\beta \\ &\times \left. \frac{2\beta K_0(\beta L|\mathbf{n}|) + (\Delta^2 - m^2)L|\mathbf{n}| K_1(\beta L|\mathbf{n}|)}{\sqrt{\beta^2 + \Delta^2 - m^2}} \right] \end{aligned}$$

*S. R. Beane et al., Phys. Rev. D70, 074029 (2004)*

$$\mathbf{F}_1(m, L) = \sum_{\mathbf{n} \neq 0} \left[ K_0(mL|\mathbf{n}|) - \frac{K_1(mL|\mathbf{n}|)}{mL|\mathbf{n}|} \right]$$

$$\mathbf{F}_3(m, L) = -\frac{3}{2} \sum_{\mathbf{n} \neq 0} \frac{K_1(mL|\mathbf{n}|)}{mL|\mathbf{n}|};$$

$$\begin{aligned} \mathbf{F}_4(m, \Delta, L) &= \frac{8}{9} \sum_{\mathbf{n} \neq 0} \left[ \frac{K_1(mL|\mathbf{n}|)}{mL|\mathbf{n}|} - \frac{\pi e^{-mL|\mathbf{n}|}}{2\Delta L|\mathbf{n}|} \right. \\ &\left. - \frac{\Delta^2 - m^2}{m^2 \Delta} \int_m^\infty d\beta \frac{\beta K_0(\beta L|\mathbf{n}|)}{\sqrt{\beta^2 + \Delta^2 - m^2}} \right], \end{aligned}$$

# Axial Charge Coupling

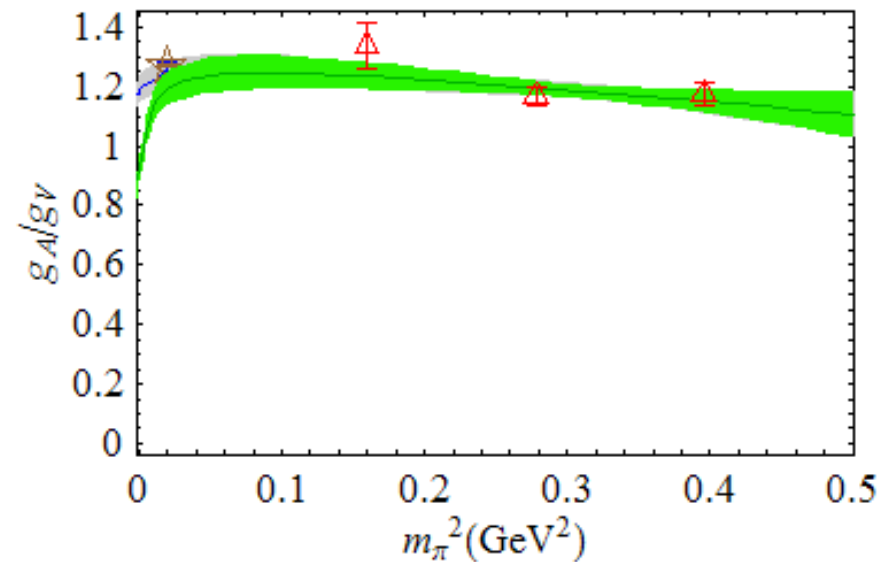
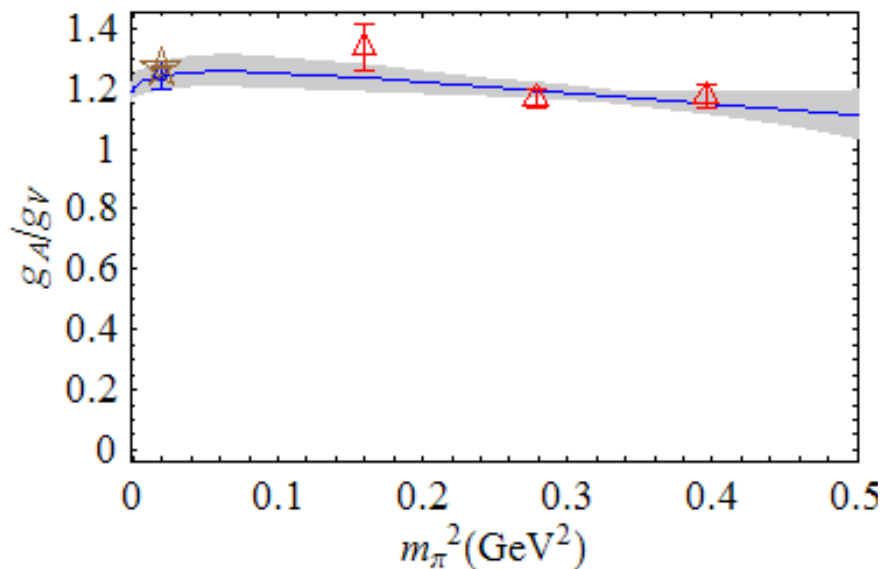
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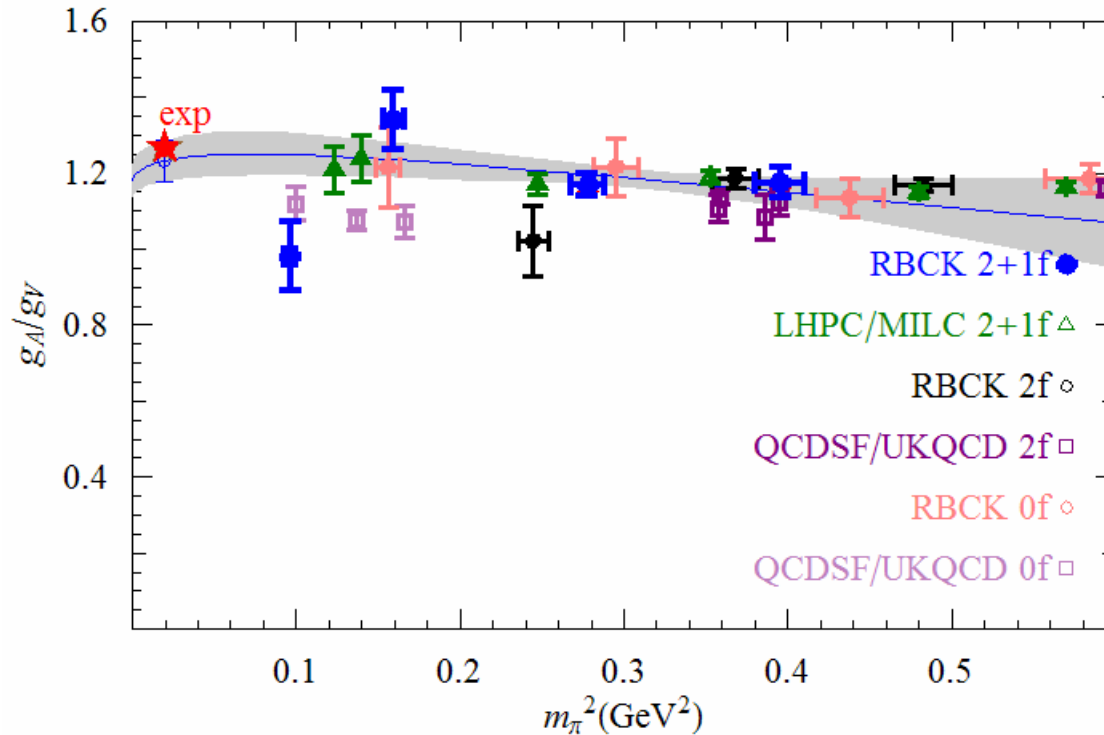
## ◆ Chiral symmetry gives $Z_A = Z_V = 1/g_V$

## ◆ Continuum $\chi$ PT extrapolation (SSE scheme)



# Axial Charge Coupling: Global View

- Comparison among various lattice results (Lattice 2006)
- Add in the lightest pion mass point (310 MeV)



*K. Orginos et al., Phys.Rev.D73:094507, 2005; H.-W. Lin et al., coming soon*

*M. Guertler et al., PoS(LAT2006)107; D. Pleiter et al., PoS(LAT2006)120*

*D. Renner et al., PoS(LAT2006)121*

# Nucleon Structure Functions

## ➤ List of Operators: lowest moments only

$\langle x \rangle_q$	$\langle x \rangle_{\Delta q}$
momentum fraction	helicity distribution
$\mathcal{O}_{44}^q = \bar{q} \left[ \gamma_4 \vec{D}_4 - \frac{1}{3} \sum_k \gamma_k \vec{D}_k \right] q$	$\mathcal{O}_{\{34\}}^{5q} = i\bar{q}\gamma_5 \left[ \gamma_3 \vec{D}_4 + \gamma_4 \vec{D}_3 \right] q$
$\mathbf{3}_1^+$	$\mathbf{6}_3^-$
$R_{\langle x \rangle_q} = \frac{C_{3pt}^{\Gamma, \mathcal{O}_{44}^q}}{C_{2pt}} = m_N \langle x \rangle_q$	$R_{\langle x \rangle_{\Delta q}} = \frac{C_{3pt}^{\Gamma, \mathcal{O}_{\{34\}}^{5q}}}{C_{2pt}} = m_N \langle x \rangle_{\Delta q}$
$\mathcal{P}_{44}^{q-1} = \gamma_4 p_4 - \frac{1}{3} \sum_{i=1,3} \gamma_i p_i$	$\mathcal{P}_{34}^{5q-1} = i\gamma_5 (\gamma_3 p_4 + \gamma_4 p_3)$
$\langle 1 \rangle_{\delta q}$	$d_1$
transversity	twist-3 matrix element
$\mathcal{O}_{34}^{\sigma q} = \bar{q}\gamma_5\sigma_{34}q$	$\mathcal{O}^5_{q[34]} = i\bar{q}\gamma_5 \left[ \gamma_3 \vec{D}_4 - \gamma_4 \vec{D}_3 \right] q$
$\mathbf{6}_1^+$	$\mathbf{6}_1^+$
$R_{\langle 1 \rangle_{\delta q}} = \frac{C_{3pt}^{\Gamma, \mathcal{O}_{34}^{\sigma q}}}{C_{2pt}} = \langle 1 \rangle_{\delta q}$	$R_{d_1} = \frac{C_{3pt}^{\Gamma, \mathcal{O}^5_{q[34]}}}{C_{2pt}} = d_1$
$\mathcal{P}_{34}^{\sigma q-1} = \gamma_5 \sigma_{34}$	$\mathcal{P}_{[34]}^{5q-1} = i\gamma_5 (\gamma_3 p_4 - \gamma_4 p_3)$

# Nucleon Structure Functions

- Chiral extrapolation formulae for each quantity

*Chen et al., Nucl.Phys. A707, 452 (2002); Phys. Lett. B523, 107 (2001)*

$$\langle x \rangle_{u-d} = C \left[ 1 - \frac{3g_A^2 + 1}{(4\pi f_\pi)^2} m_\pi^2 \ln \left( \frac{m_\pi^2}{\mu^2} \right) \right] + e(\mu^2) \frac{m_\pi^2}{(4\pi f_\pi)^2}$$
$$\langle x \rangle_{\Delta u - \Delta d} = \tilde{C} \left[ 1 - \frac{2g_A^2 + 1}{(4\pi f_\pi)^2} m_\pi^2 \ln \left( \frac{m_\pi^2}{\mu^2} \right) \right] + \tilde{e}(\mu^2) \frac{m_\pi^2}{(4\pi f_\pi)^2}$$

Linear ansatz

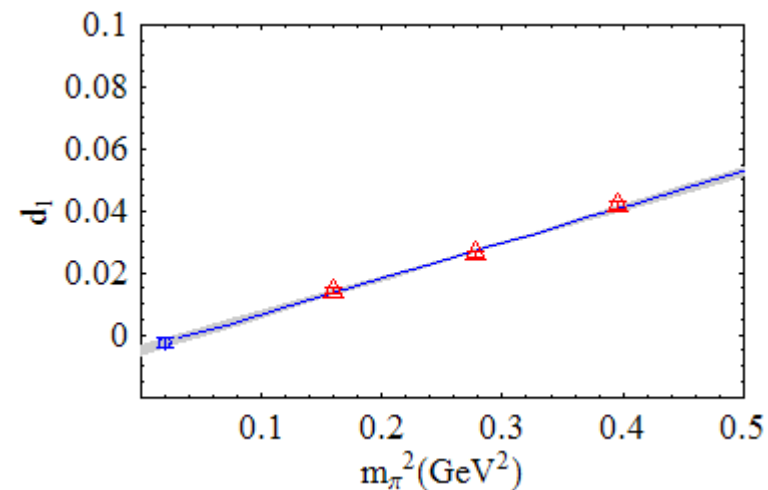
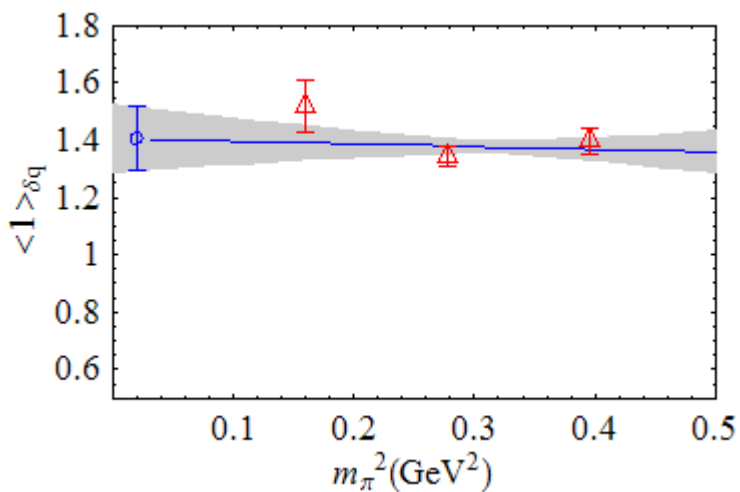
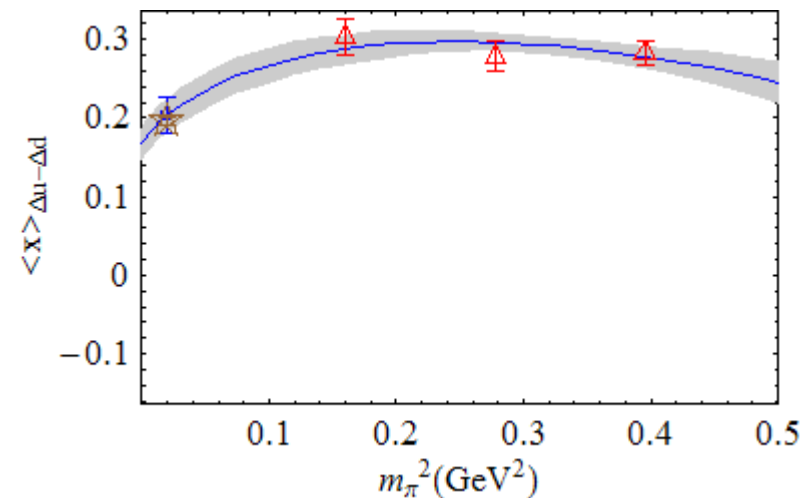
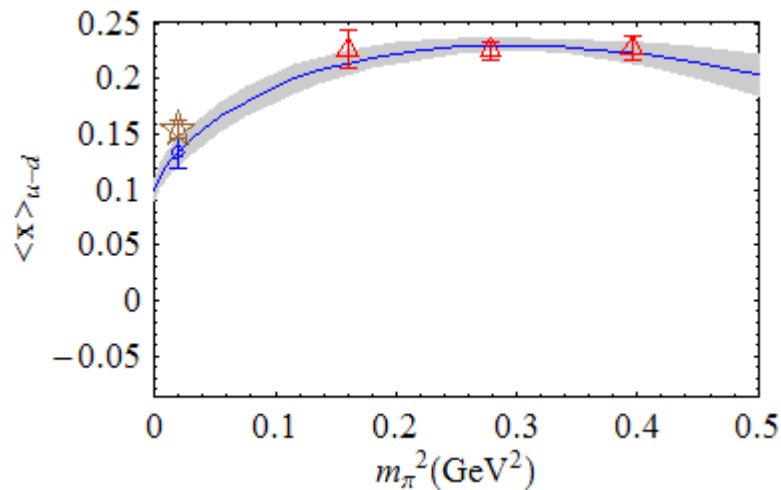
Linear ansatz

- RI/MOM-scheme nonperturbative renormalization (except for  $d_1$ )



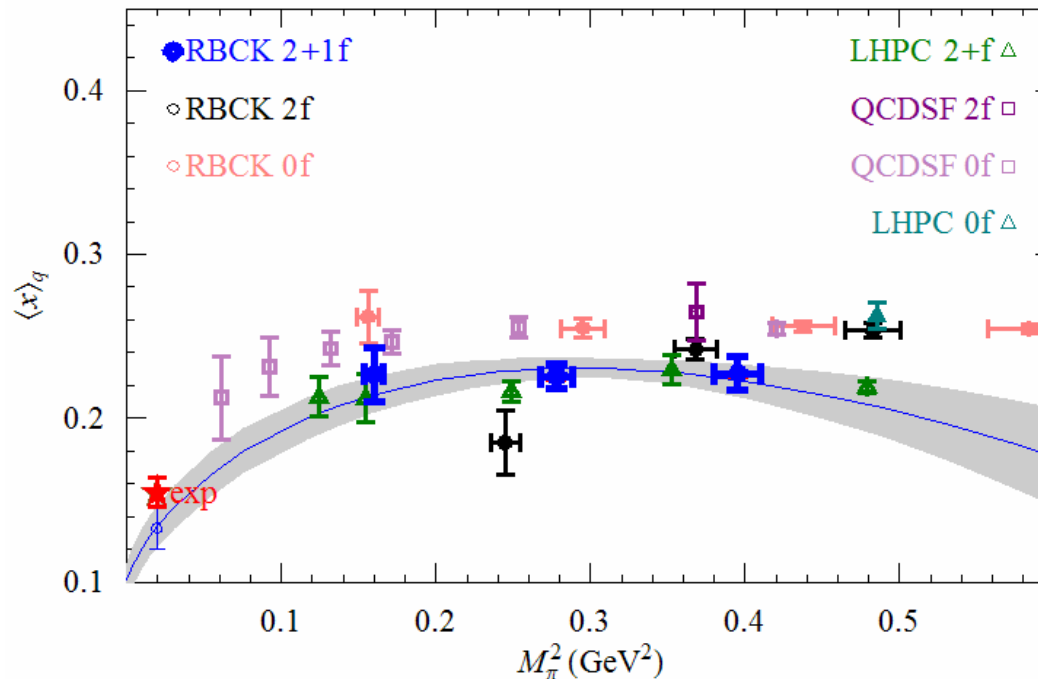
# Nucleon Structure Functions

➡ Chiral extrapolations: lowest moments only



# Nucleon Structure Functions

- Comparison among calculations of the first moment of the momentum fraction



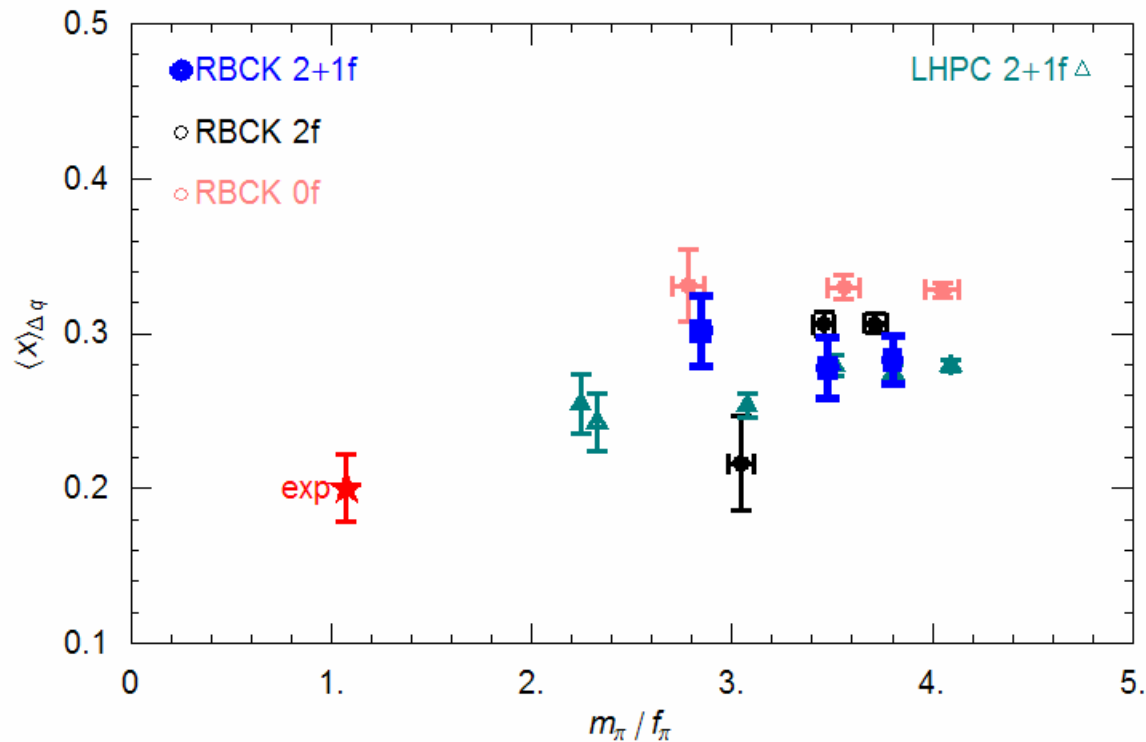
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*M. Guertler et al., PoS(LAT2006)107; D. Pleiter et al., PoS(LAT2006)120*

*D. Renner et al., PoS(LAT2006)121*

# Nucleon Structure Functions

- ➔ Comparison among calculations of the first moment of the helicity distribution

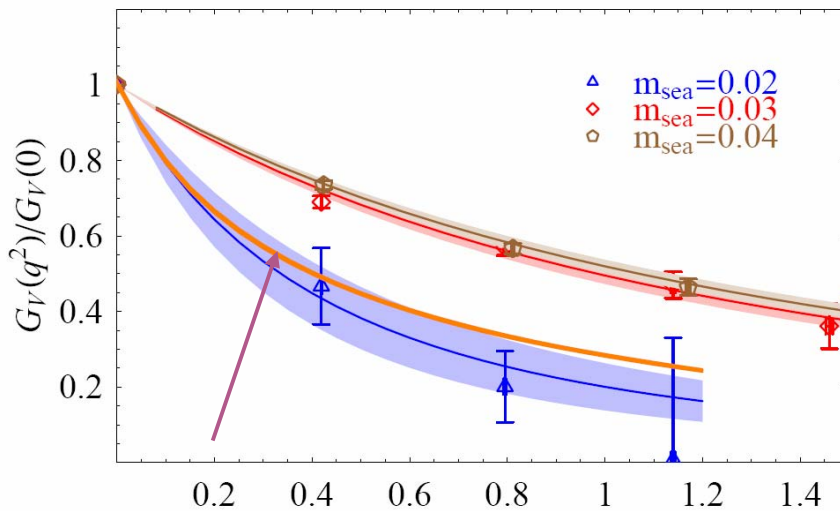


*K. Orginos et al., Phys.Rev.D73:094507, 2005; H.-W. Lin et al., coming soon*

*D. Renner et al., PoS(LAT2006)121*

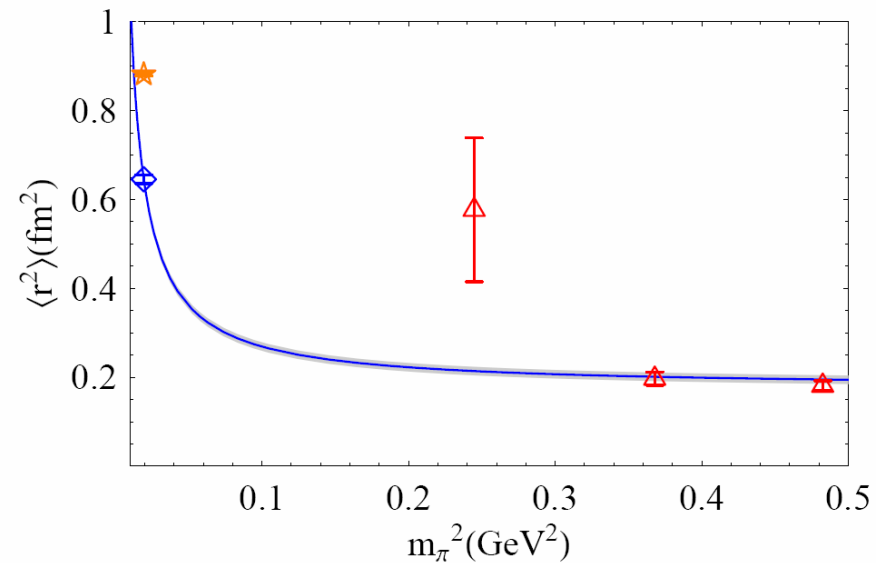
# Nucleon Isovector Form Factor ( $F_1^{u-d}$ )

- ➡ 2+1-flavor analysis not yet confirmed
- ➡ **2-flavor** DWF data with pion mass: 500–700 MeV



*J. J. Kelley, Phys. Rev. C70, 068202 (2004)*

Dipole-form extrapolation



$$\langle r^2 \rangle = c_1 - \frac{1 + 5g_A^2}{(4\pi f_\pi)^2} \log \frac{m_\pi^2}{m_\pi^2 + \Lambda^2}$$

*G. V. Dunne et al., Phys. Lett. B531, 77 (2002)*

# Goldberger-Treiman Relation

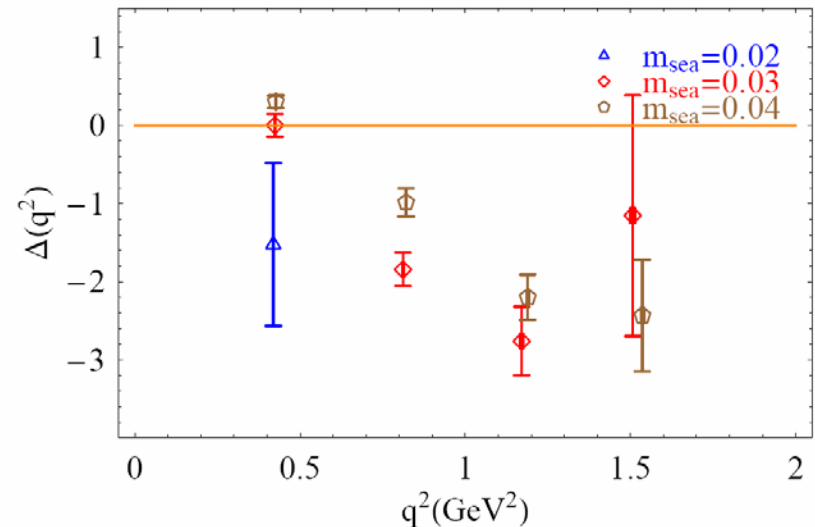
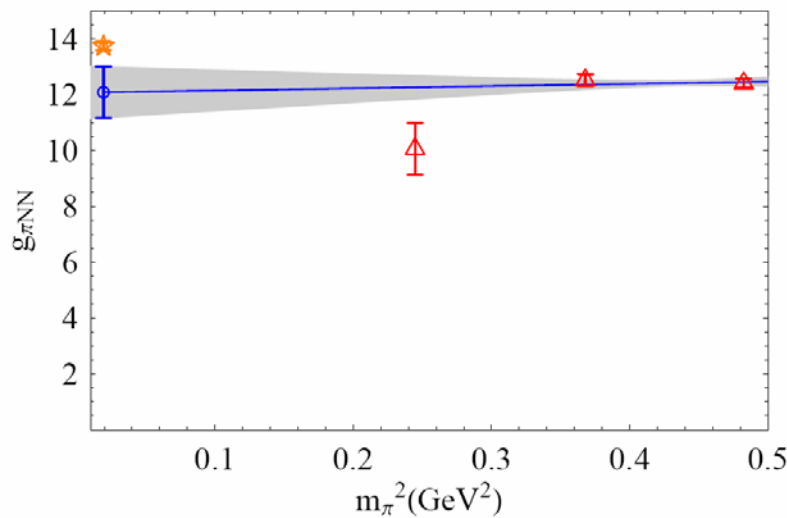
- The Goldberger-Treiman relation states:

$$q^2 \frac{G_P(q^2)}{2m_N} - 2m_N G_A(q^2) = -\frac{g_{\pi NN} F_\pi m_\pi^2}{q^2 + m_\pi^2}$$

- A measure of the discrepancy

$$\Delta_{GT}(q^2) = 1 - \frac{q^2 + m_{\pi,\text{lat}}^2}{2m_{N,\text{lat}}} \frac{q^2 G_{P,\text{lat}}(q^2) - 4m_{N,\text{lat}}^2 G_{A,\text{lat}}(q^2)}{g_{\pi NN,\text{lat}} F_{\pi,\text{lat}} m_{\pi,\text{lat}}^2}$$

$$\text{with } g_{\pi NN}^{\text{lat}} = \frac{2m_N^{\text{lat}} g_A^{\text{lat}}}{F_\pi^{\text{lat}}}$$



# Summary/Outlook: I

## ➤ Nucleon structure functions and form factors

- Work in progress: Full-QCD calculation with pion masses 400–600 MeV
- **Preliminary** study shows good agreement with experiment, even for notoriously difficult quantities, such as momentum fraction and helicity distribution

## ➤ In the near future

- 300 MeV pion analysis is on the way (**Lattice 2007**)
- Taking more statistics at each pion mass
- Finite-volume studied (combined UKQCD  $16^3 \times 32$  data)
- Lattice discretization effects will be examined ( $32^3 \times 64$ ,  $a \sim 0.09$  fm lattices are on the way)

## ➤ Within a few years or so

- < 200 MeV full-QCD gauge generation proposal

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# Hyperon Channel (preliminary)

in collaboration with

*Kostas Orginos*



# Parameters

## ➔ This calculation:

- ➔ Mixed action (staggered sea with DWF valence), 2+1-flavor
- ➔ Pion mass range: 360–700 MeV
- ➔ Strange-strange Goldstone fixed at 763(2) MeV
- ➔ Volume fixed at 2.6 fm
- ➔  $a \approx 0.125$  fm,  $L_s = 16$ ,  $M_5 = 1.7$
- ➔ HYP-smearred gauge, box size of  $20^3 \times 32$

Label	$m_\pi$ (MeV)	$m_K$ (MeV)
m010	358(2)	605(2)
m020	503(2)	653(2)
m030	599(1)	688(2)
m040	689(2)	730(2)



# Strangeness Magnetic Moment of Nucleon

- Disconnected diagrams are challenging
- Much effort has been put into resolving this difficulty
- Alternative approach:

*D. B. Leinweber, Phys. Rev. D 53, 5115 (1996).*

- Assume charge symmetry:

$$\begin{aligned}
 p &= e_u u^p + e_d d^p + O_N; & n &= e_d u^p + e_u d^p + O_N, \\
 \Sigma^+ &= e_u u^\Sigma + e_s s^\Sigma + O_\Sigma; & \Sigma^- &= e_d u^\Sigma + e_s s^\Sigma + O_\Sigma, \\
 \Xi^0 &= e_s s^\Xi + e_u u^\Xi + O_\Xi; & \Xi^- &= e_s s^\Xi + e_d u^\Xi + O_\Xi.
 \end{aligned}$$

- The disconnected piece for the proton is  $O_N = \frac{2}{3} {}^l G_M^u - \frac{1}{3} {}^l G_M^d - \frac{1}{3} {}^l G_M^s$ .

- The strangeness contribution is

$$G_M^s = \left( \frac{{}^l R_d^s}{1 - {}^l R_d^s} \right) \left[ 2p + n - \frac{u^p}{u^\Sigma} (\Sigma^+ - \Sigma^-) \right]$$

$$G_M^s = \left( \frac{{}^l R_d^s}{1 - {}^l R_d^s} \right) \left[ p + 2n - \frac{u^n}{u^\Xi} (\Xi^0 - \Xi^-) \right] \quad \text{with} \quad {}^l R_d^s \equiv {}^l G_M^s / {}^l G_M^d$$

# Strangeness Magnetic Moment of Nucleon

- Disconnected diagrams are challenging
- Much effort has been put into resolving this difficulty
- Alternative approach:

*D. B. Leinweber, Phys. Rev. D 53, 5115 (1996).*

- Assume charge symmetry:

$$\begin{aligned}
 p &= e_u u^p + e_d d^p + O_N; & n &= e_d u^p + e_u d^p + O_N, \\
 \Sigma^+ &= e_u u^\Sigma + e_s s^\Sigma + O_\Sigma; & \Sigma^- &= e_d u^\Sigma + e_s s^\Sigma + O_\Sigma, \\
 \Xi^0 &= e_s s^\Xi + e_u u^\Xi + O_\Xi; & \Xi^- &= e_s s^\Xi + e_d u^\Xi + O_\Xi.
 \end{aligned}$$

- The disconnected piece for the proton is  $O_N = \frac{2}{3} {}^l G_M^u - \frac{1}{3} {}^l G_M^d - \frac{1}{3} {}^l G_M^s$ .

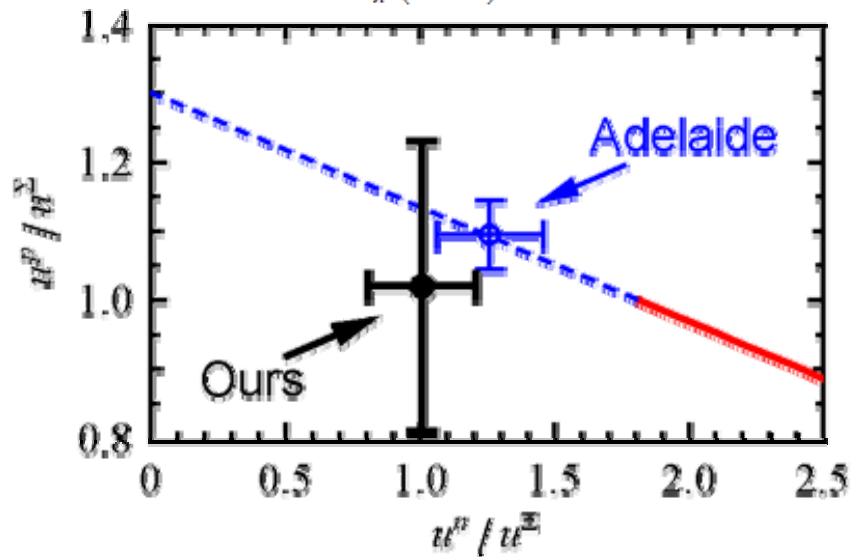
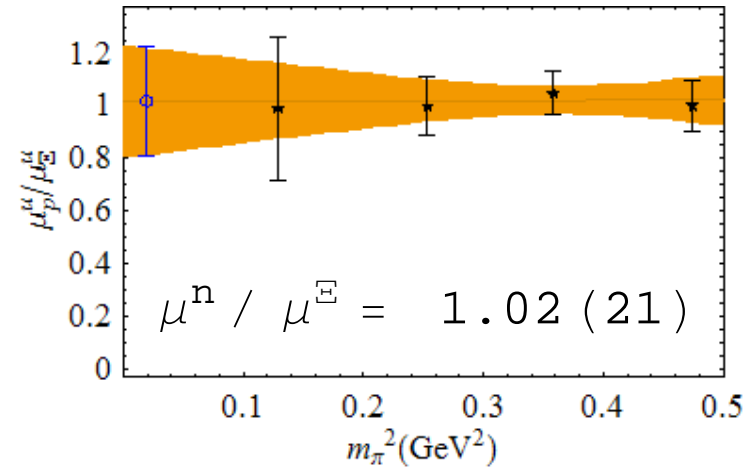
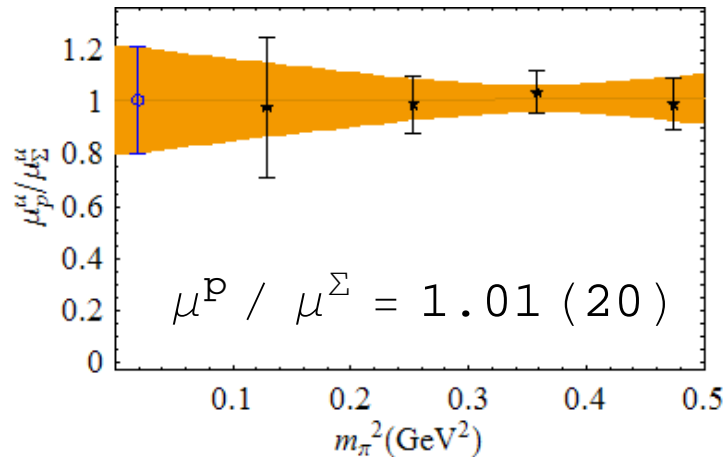
- The strangeness contribution is

$$G_M^s = \left( \frac{{}^l R_d^s}{1 - {}^l R_d^s} \right) \left[ 3.673 - \frac{u^p}{u^\Sigma} (3.618) \right] \mu_N \quad \textit{Need better statistics}$$

$$\boxed{G_M^s = \left( \frac{{}^l R_d^s}{1 - {}^l R_d^s} \right) \left[ -1.033 - \frac{u^n}{u^\Xi} (-0.599) \right] \mu_N} \quad \text{with } {}^l R_d^s \equiv {}^l G_M^s / {}^l G_M^d$$

# Strangeness Magnetic Moment of Nucleon

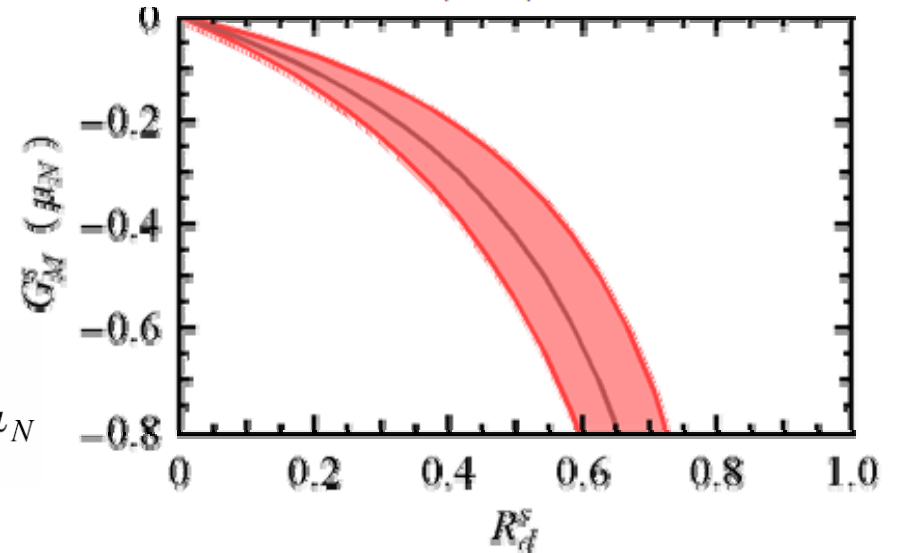
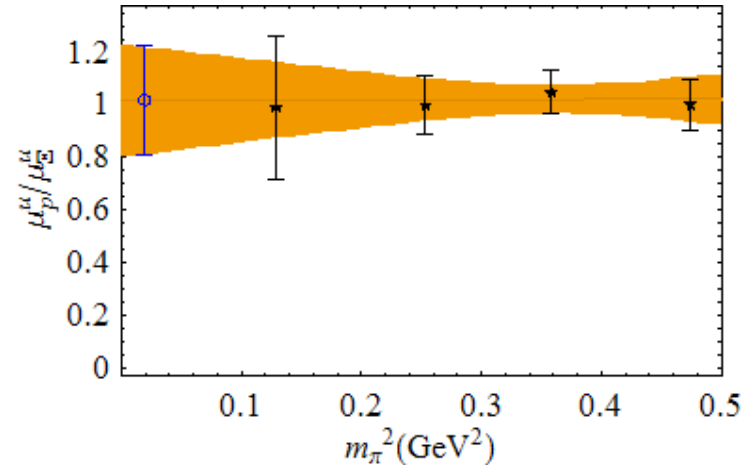
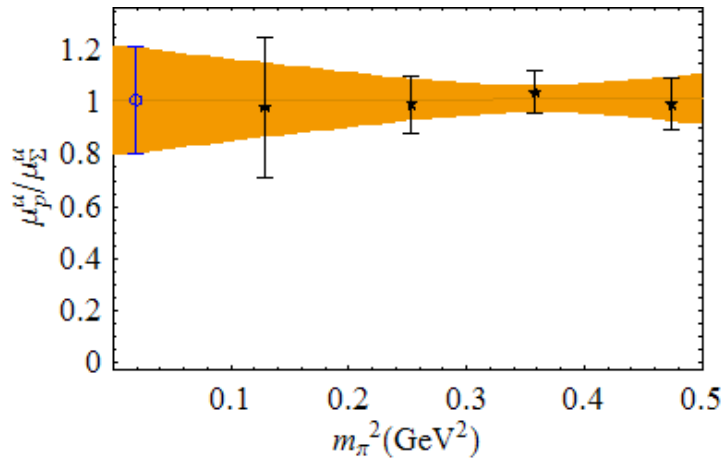
- Dipole-form extrapolation to  $q^2 = 0$
- Magnetic-moment ratios (linear extrapolation, for now)



*D. B. Leinweber et al., Phys. Rev. Lett. 94, 212001 (2004).*

# Strangeness Magnetic Moment of Nucleon

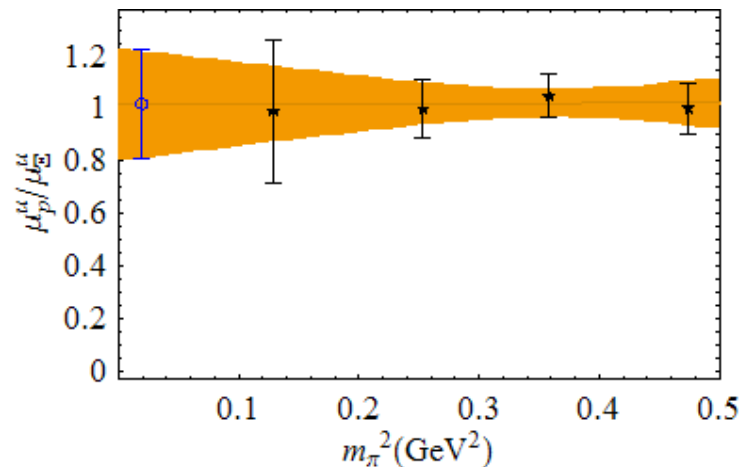
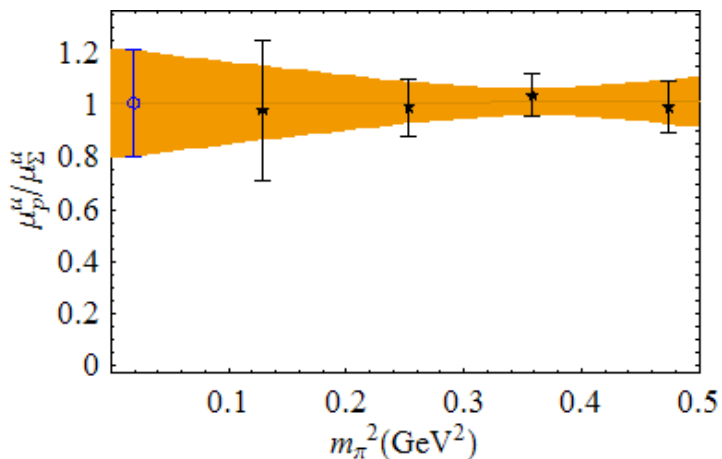
- ➔ Dipole-form extrapolation to  $q^2 = 0$
- ➔ Magnetic-moment ratios (linear extrapolation, for now)



$$G_M^s = \left( \frac{{}^l R_d^s}{1 - {}^l R_d^s} \right) \left[ -1.033 - \frac{u^n}{u^\Xi} (-0.599) \right] \mu_N$$

# Strangeness Magnetic Moment of Nucleon

- Dipole-form extrapolation to  $q^2 = 0$
- Magnetic-moment ratios (linear extrapolation, for now)

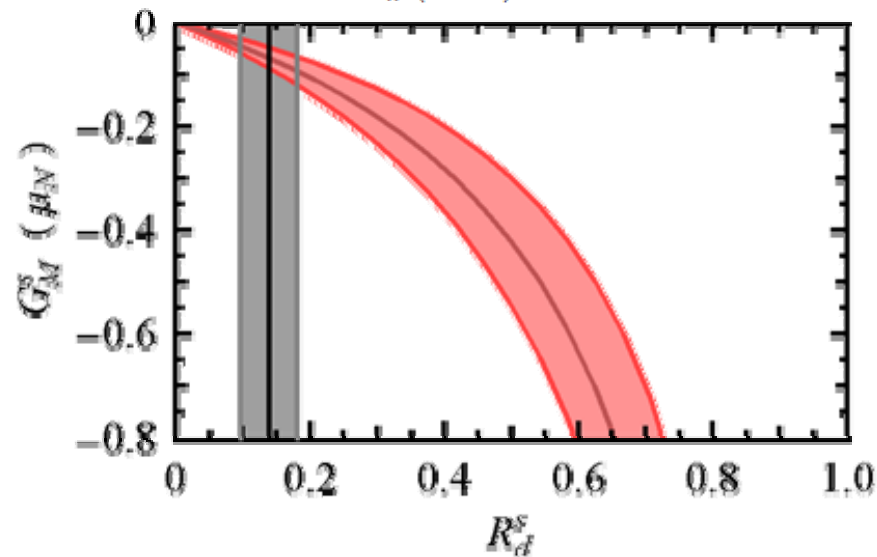


*D. B. Leinweber et al., Phys. Rev. Lett. 94, 212001 (2004).*

$$R_d^S = 0.139(42)$$

We find

$$G_M^S = -0.07(3)$$



# Axial Coupling Constants: $g_{\Xi\Xi}$ and $g_{\Sigma\Sigma}$

- Has applications such as hyperon scattering, non-leptonic decays, etc.
- Cannot be determined by experiment
- Existing predictions from  $\chi$ PT and large- $N_c$  calculations

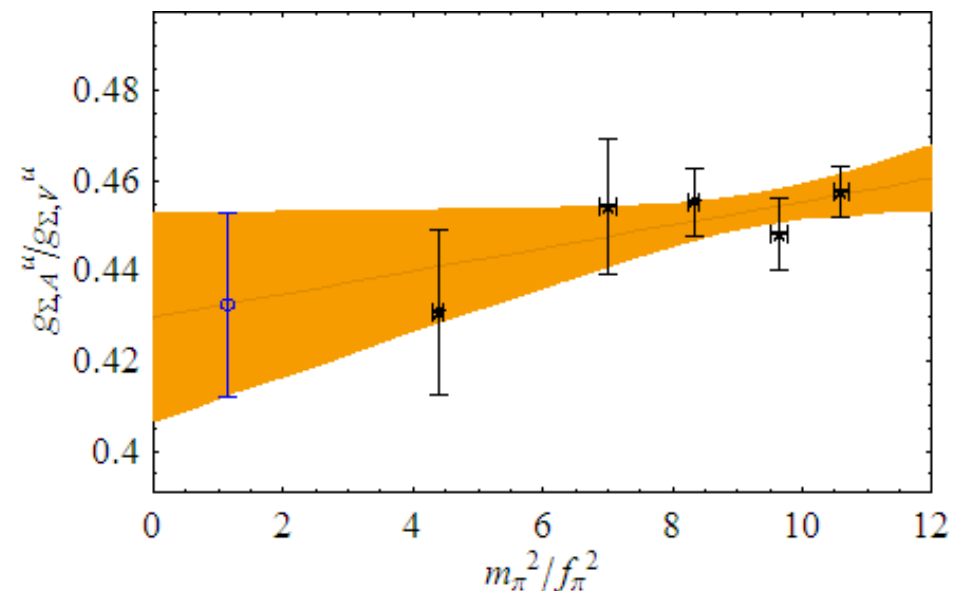
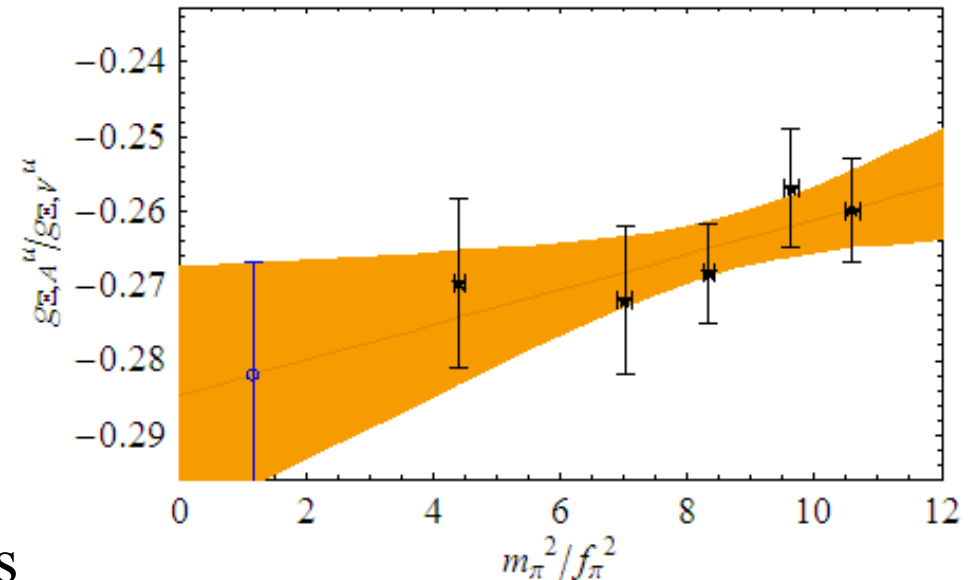
*M. J. Savage et al., Phys. Rev. D55, 5376 (1997);*

*R. Flores-Mendieta et al., Phys. Rev. D58, 094028(1998);*

$$0.18 < -g_{\Xi\Xi} < 0.36$$

$$0.30 < g_{\Sigma\Sigma} < 0.55$$

- We find consistent numbers with much smaller errors



# Summary/Outlook: II

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## ➤ From hyperon analysis

- **Preliminary** estimate of the proton strange magnetic moment directly from full QCD:  $-0.07(3)$
- Predictions for  $g_{\Sigma\Sigma} = 0.441(14)$  and  $g_{\Xi\Xi} = -0.277(11)$
- DWF are too expensive to get small errors with light pion masses, especially as the  $< 300$  MeV era approaches

## ➤ In the future

- Anisotropic 2+1-flavor clover lattices will be started soon
- Proposed light pion mass  $< 200$  MeV in a few years
- Obvious cost benefit right away; share propagators
- Anisotropic clover is good for separating excited contributions; thus cleaner ground-state signal for precision calculation