
Semileptonic Hyperon Decays in Full QCD

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in collaboration with Kostas Orginos

Outline

- Motivation/Background
- Quick Review of Lattice Calculations
- Lattice Techniques/Parameters
- Numerical Results
- Summary and Outlook

Semileptonic Decays and CKM Matrix

- ◆ 1963: Cabibbo proposes current theory $J_\alpha^i = V_\alpha^i + A_\alpha^i$

$$J_\alpha = \cos \theta_C (J_\alpha^1 + iJ_\alpha^2) + \sin \theta_C (J_\alpha^4 + iJ_\alpha^5)$$

to explain semileptonic decays.

- ◆ 1973: Kobayashi and Maskawa add mixing of three generations of quarks:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

before the discovery of the b quark.

- ◆ Cabibbo angle in CKM matrix: $\tan \theta_C = \frac{V_{us}}{V_{ud}}$

$|V_{us}|$ in CKM Matrix

➤ Unitarity constraint: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1.$

➤ PDG 2006 gives

➤ Well determined $|V_{ud}| = 0.97377(27)$

➤ Very small $|V_{ub}| = 4.31(30) \times 10^{-3}$

➤ Less known $|V_{us}| = 0.2257(21)(2006)$

$|V_{us}| = 0.2196(23)(2003)$

➤ Experimentally,

➤ $\bar{K} \rightarrow \pi l \nu$

➤ Hyperon decay $0.2250(27)$

$\Lambda \rightarrow p e^- \bar{\nu}, \Sigma^- \rightarrow n e^- \bar{\nu},$

$\Xi^- \rightarrow \Lambda e^- \bar{\nu}, \Xi^0 \rightarrow \Lambda^+ e^- \bar{\nu}$

➤ Leptonic decay ratios: $0.2234^{(+12)}_{(-31)}$

➤ Hadronic τ decays: $0.2208(34)$

Lattice Calculation of $|V_{us}|$ – Meson

- Calculate $K \rightarrow \pi$ matrix element
- Lorentz invariance

$$\langle \pi(p') | V_\mu | K(p) \rangle = (p_\mu + p'_\mu) f_+(q^2) + (p_\mu - p'_\mu) f_-(q^2)$$

with $V_\mu = \bar{s} \gamma_\mu u$

- Calculate the scalar form factor

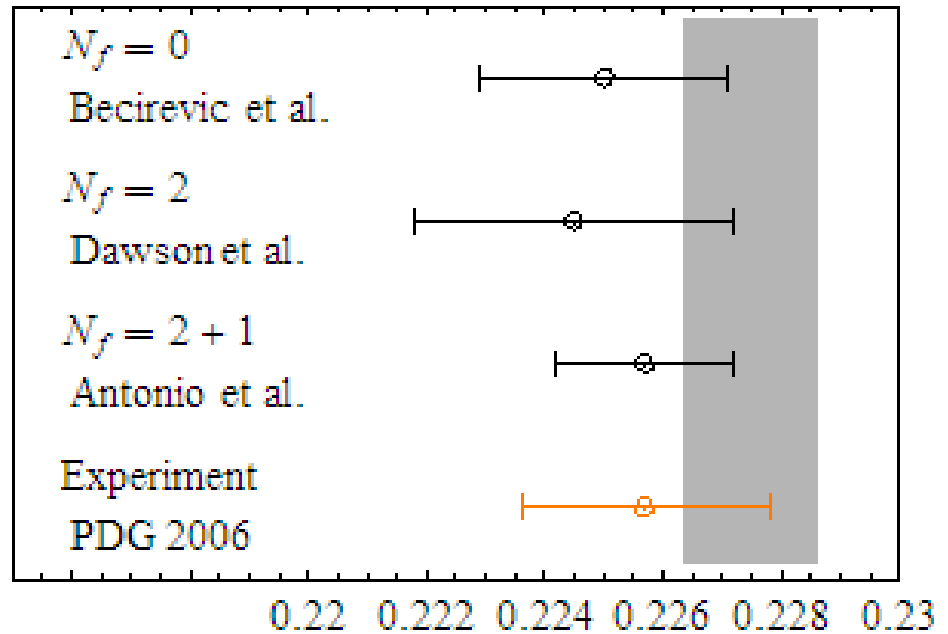
$$f_0(q^2) = f_+(q^2) + \frac{q^2}{m_K^2 - m_\pi^2} f_-(q^2)$$

- Extrapolate to $q = 0$ point in dipole
- Obtain $|V_{us}|$ from

$$\Gamma(K_{\ell 3}) = \frac{G_F^2 M_K^5}{128\pi^3} |V_{us}|^2 S_{\text{ew}} |f_+^{K^0\pi^-}(0)|^2 C_K^2 I_K^\ell(\lambda_i) [1 + \delta_{SU(2)}^K + \delta_{\text{em}}^{K\ell}]^2$$

Lattice Calculation of $|V_{us}|$ – Meson

Multiple K_{l3} decay calculations:



$|V_{us}|$
Guadagnoli et al. (2004)

- Twisted-BC approach

$$\tilde{\psi}(x + \hat{e}_j L) = e^{2\pi i \theta_j} \tilde{\psi}(x)$$

momentum changes by $\tilde{p}_j = \theta_j \frac{2\pi}{L} + n_j \frac{2\pi}{L}$

Tune θ_j to cancel out the mass difference.

No extrapolation in momentum is needed!

Baryon Matrix Elements

- Matrix element of hyperon β decay $B_1 \rightarrow B_2 e^- \bar{\nu}$

$$\mathcal{M} = \frac{G_s}{\sqrt{2}} \bar{u}_{B_2} (O_\alpha^V + O_\alpha^A) u_{B_1} \bar{u}_e \gamma^\alpha (1 + \gamma_5) v_\nu$$

with

$$O_\alpha^V = f_1(q^2) \gamma^\alpha + \frac{f_2(q^2)}{M_{B_1}} \sigma_{\alpha\beta} q^\beta + \frac{f_3(q^2)}{M_{B_2}} q_\alpha$$

$$O_\alpha^A = \left(g_1(q^2) \gamma^\alpha + \frac{g_2(q^2)}{M_{B_1}} \sigma_{\alpha\beta} q^\beta + \frac{g_3(q^2)}{M_{B_2}} q_\alpha \right) \gamma_5$$

- The decay rate is

$$\Gamma = G_F^2 |V_{us}|^2 \frac{\Delta m^5}{60\pi^3} (1 + \delta_{\text{rad}}) \times \left[\left(1 - \frac{3}{2} \beta\right) (|f_1|^2 + |g_1|^2) + \frac{6}{7} \beta^2 \left(|f_1|^2 + 2|g_1|^2 + \text{Re}(f_1 f_2^*) + \frac{2}{3} |f_2|^2 \right) + \delta_{q^2} \right]$$

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- The vector form factor $f_1(0)$ links to $|V_{us}|$

More than just an alternative way to get $|V_{us}|$!

- $g_1(0)/f_1(0)$ gives information about **strangeness** content.
- $g_2(0)$ and $f_3(0)$ vanish in the SU(3) limit \rightarrow Symmetry-breaking measure

Hyperon Experiments

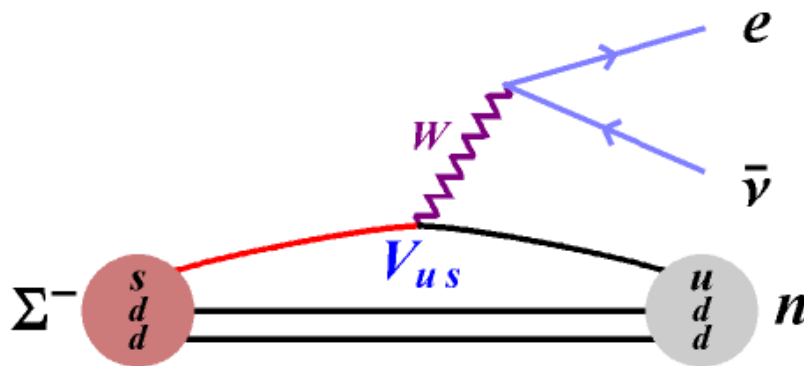
- Experiments: CERN WA2, Fermilab E715, BNL AGS, Fermilab KTeV, CERN NA48
- Summary

Channel	$f_1^{SU(3)}$	$ f_1 V_{us} $	$(g_1/f_1)^{SU(3)}$	$(g_1/f_1)^{\text{exp}}$
$n \rightarrow p$	1	n/a	$F + D$	1.2670(30)
$\Lambda \rightarrow p$	$-\sqrt{3}/2$	0.2221(33)	$F + D/3$	0.718(15)
$\Sigma^- \rightarrow n$	-1	0.2274(49)	$F - D$	-0.340(17)
$\Xi^- \rightarrow \Lambda$	$\sqrt{3}/2$	0.2367(97)	$F - D/3$	0.25(5)
$\Xi^- \rightarrow \Sigma^0$	$\sqrt{1/2}$	n/a	$F + D$	n/a
$\Xi^0 \rightarrow \Sigma^+$	1	0.216(33)	$F + D$	1.32(22)

- \equiv measurements are still active!

Lattice Calculation of $|V_{us}|$ — Baryon

Two quenched calculations, different channels

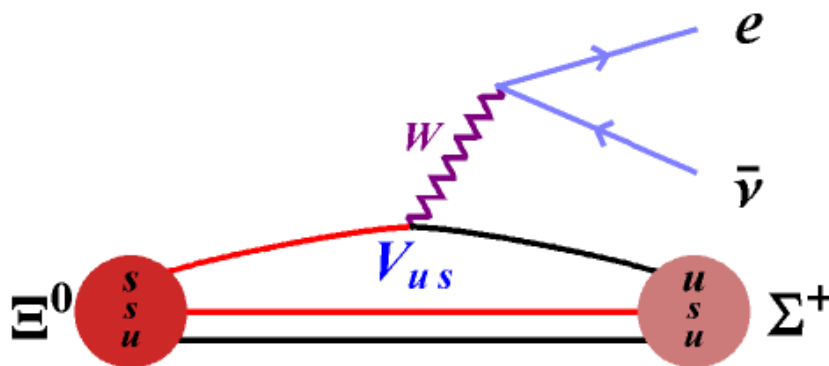


* Pion mass > 700 MeV

* $f_1(0) = -0.988(29)_{\text{stat.}}$

* $|V_{us}| = 0.230(8)$

Guadagnoli et al.



* Pion mass ≈ 530 – 650 MeV

* $f_1(0) = -0.953(24)_{\text{stat}}$

* $|V_{us}| = 0.219(27)$

Sasaki et al.

Lattice Actions

- (Improved) Staggered fermions (asqtad):
 - Relatively cheap for dynamical fermions (good)
 - Mixing among parities and flavors or *tastes*
 - Baryonic operators a nightmare — not suitable
- Chiral fermions (e.g., Domain-Wall/Overlap):
 - Automatically $O(a)$ improved, suitable for spin physics and weak matrix elements
 - Expensive

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 - Automatically $O(a)$ improved, suitable for spin physics and weak matrix elements
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- Mixed actions:
 - Match the sea Goldstone pion mass to the DWF pion
 - Pion masses as low as 260 MeV
 - Volume: 2.6-3.5 fm
 - Free light quark propagators

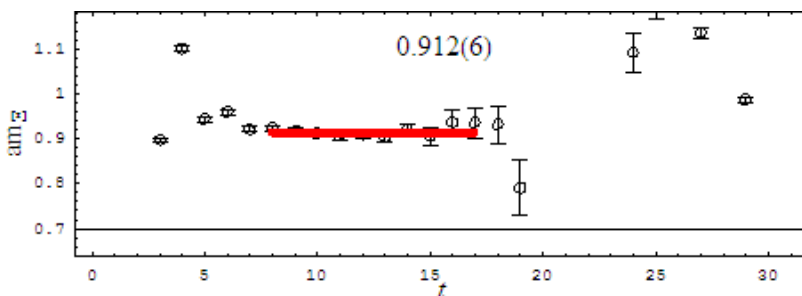
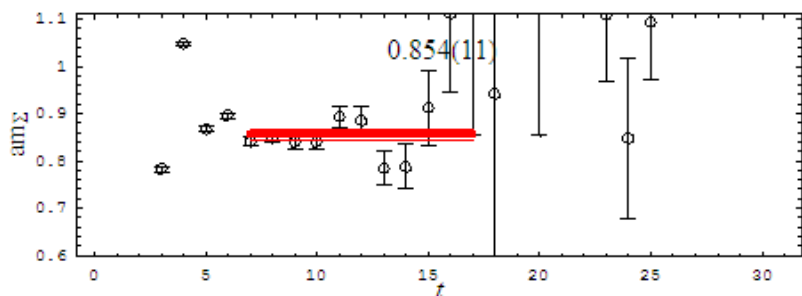
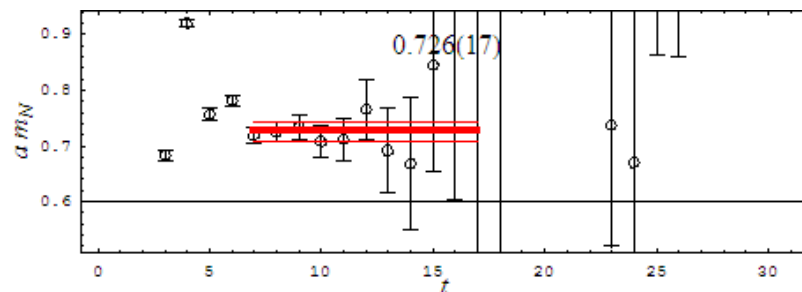
Parameters

- ◆ This calculation:
 - ◆ Pion mass range: 360-700 MeV
 - ◆ Strange-strange Goldstone fixed at 763(2) MeV
 - ◆ Volume fixed at 2.6 fm
 - ◆ $a \approx 0.125$ fm, $L_s = 16$, $M_5 = 1.7$
 - ◆ HYP-smearred gauge, box size of $20^3 \times 32$

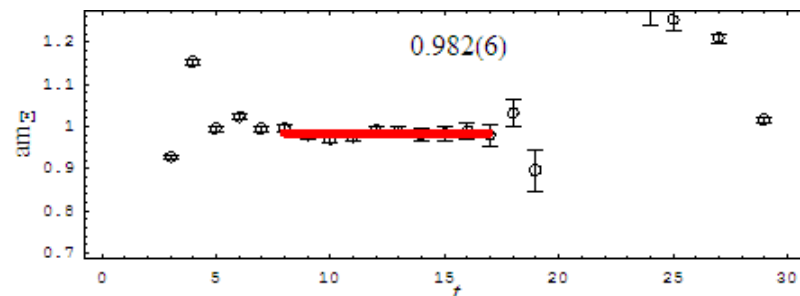
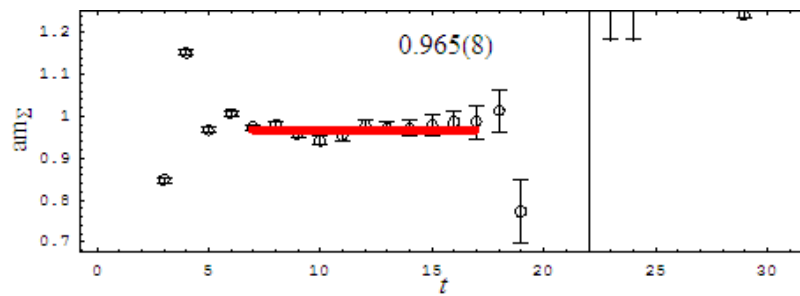
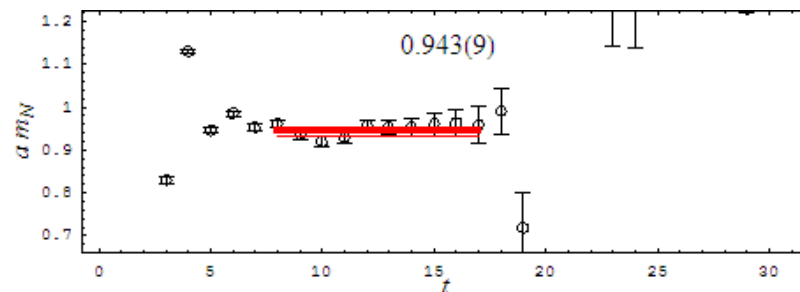
Label	m_π (MeV)	m_K (MeV)	$\Sigma^- \rightarrow n$ conf.
m010	358(2)	605(2)	600
m020	503(2)	653(2)	420
m030	599(1)	688(2)	561
m040	689(2)	730(2)	306

Effective Mass Plots

➡ The worst set

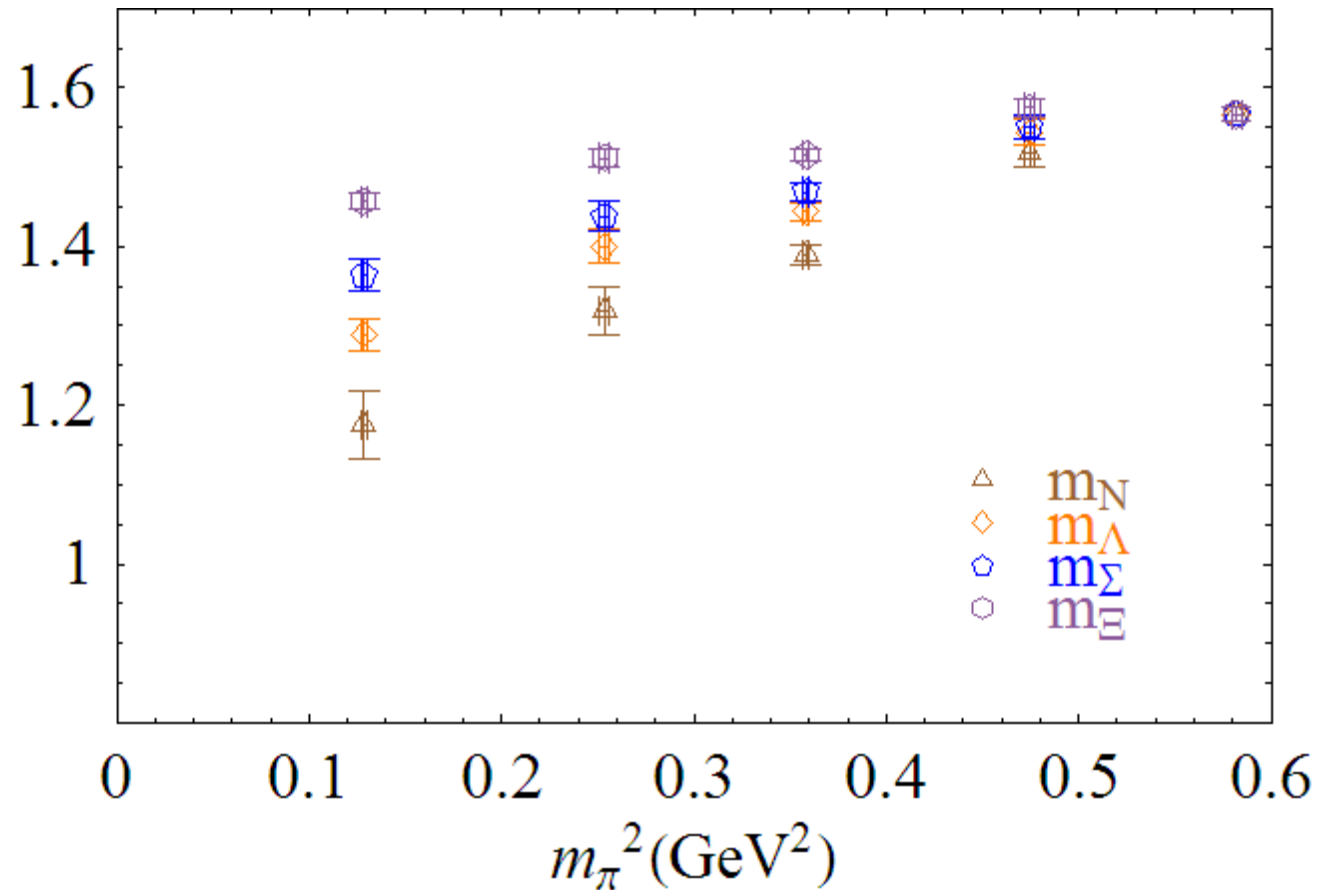


➡ The best set



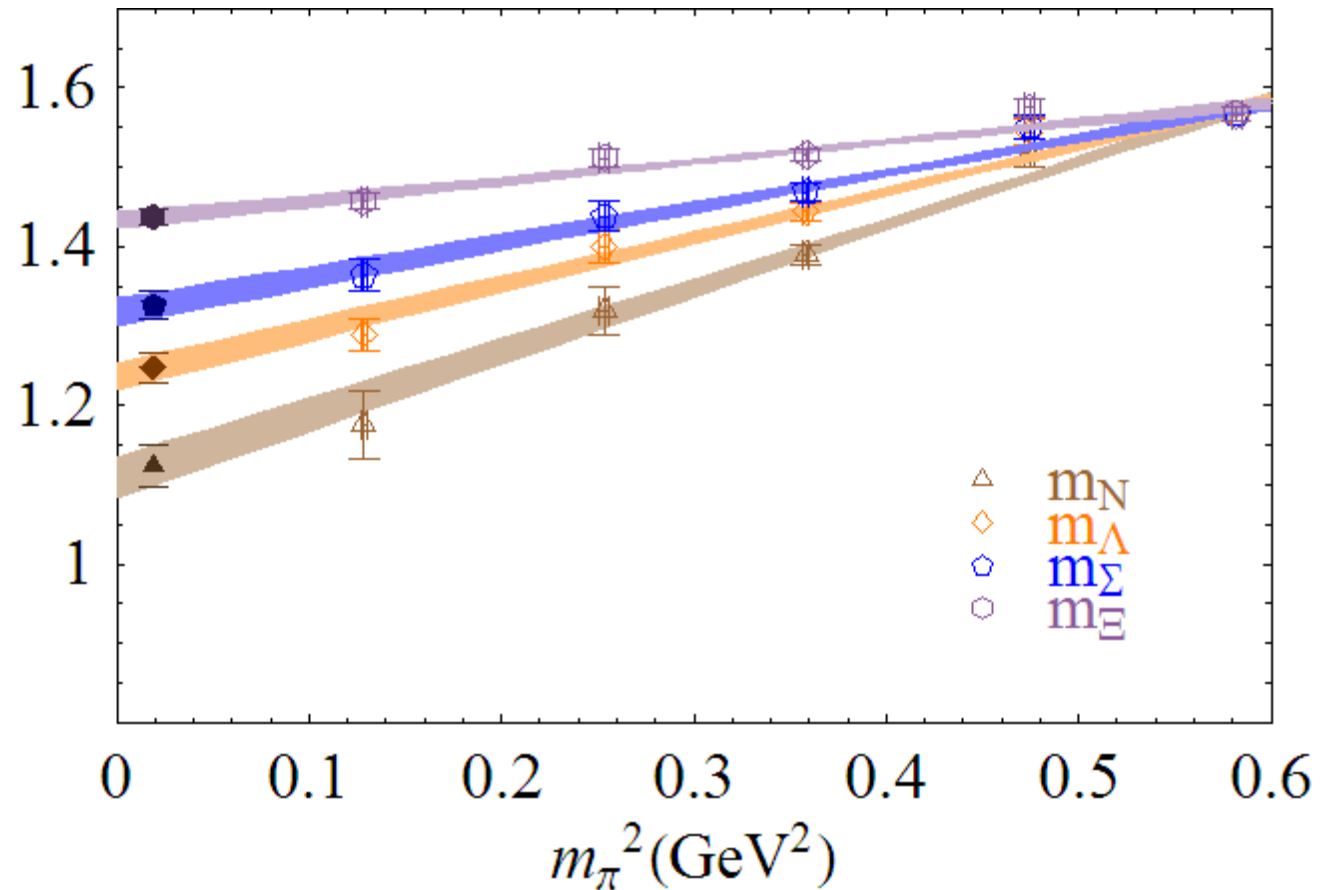
Octet Spectrum

Summary



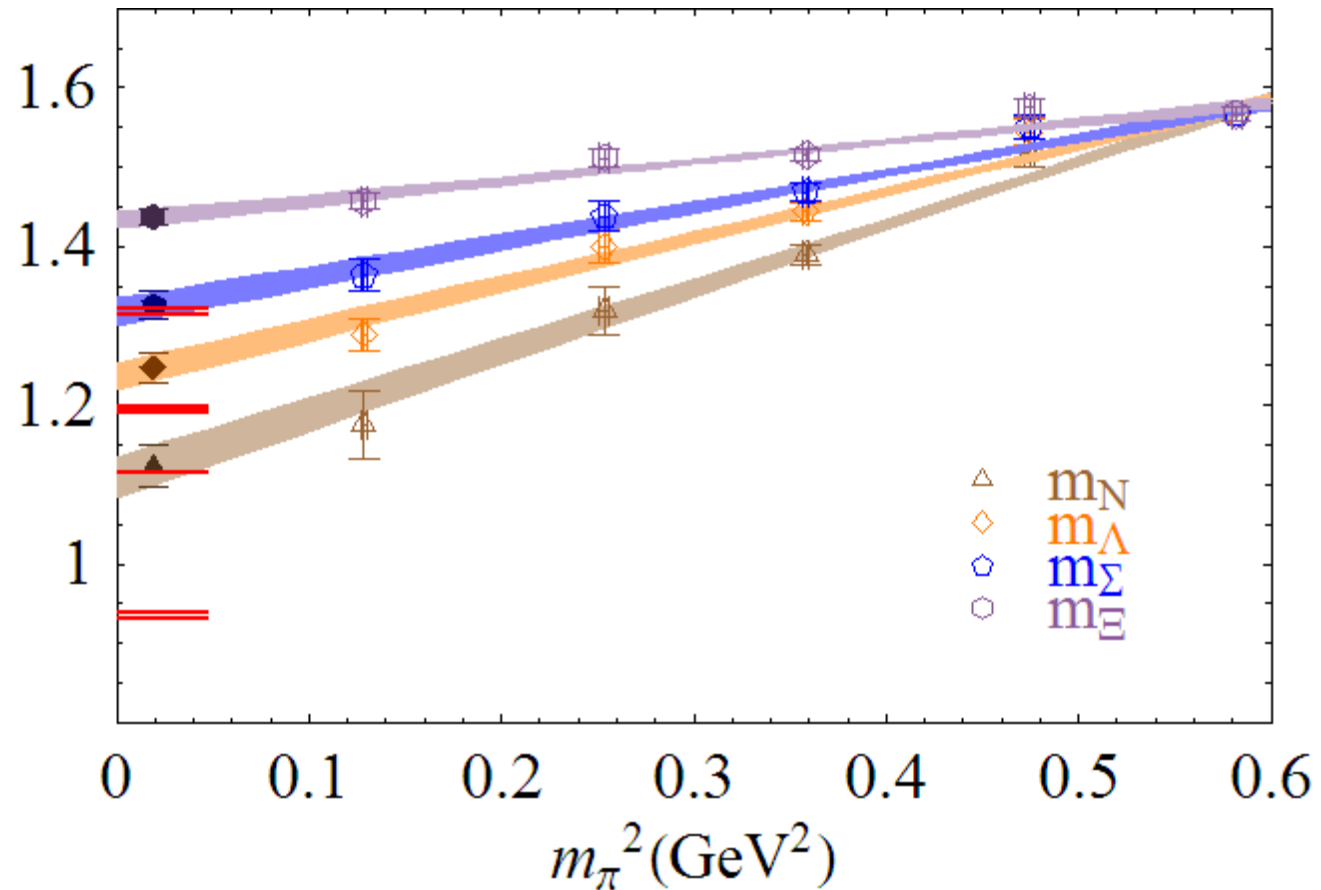
Octet Spectrum

Summary and extrapolation



Octet Spectrum

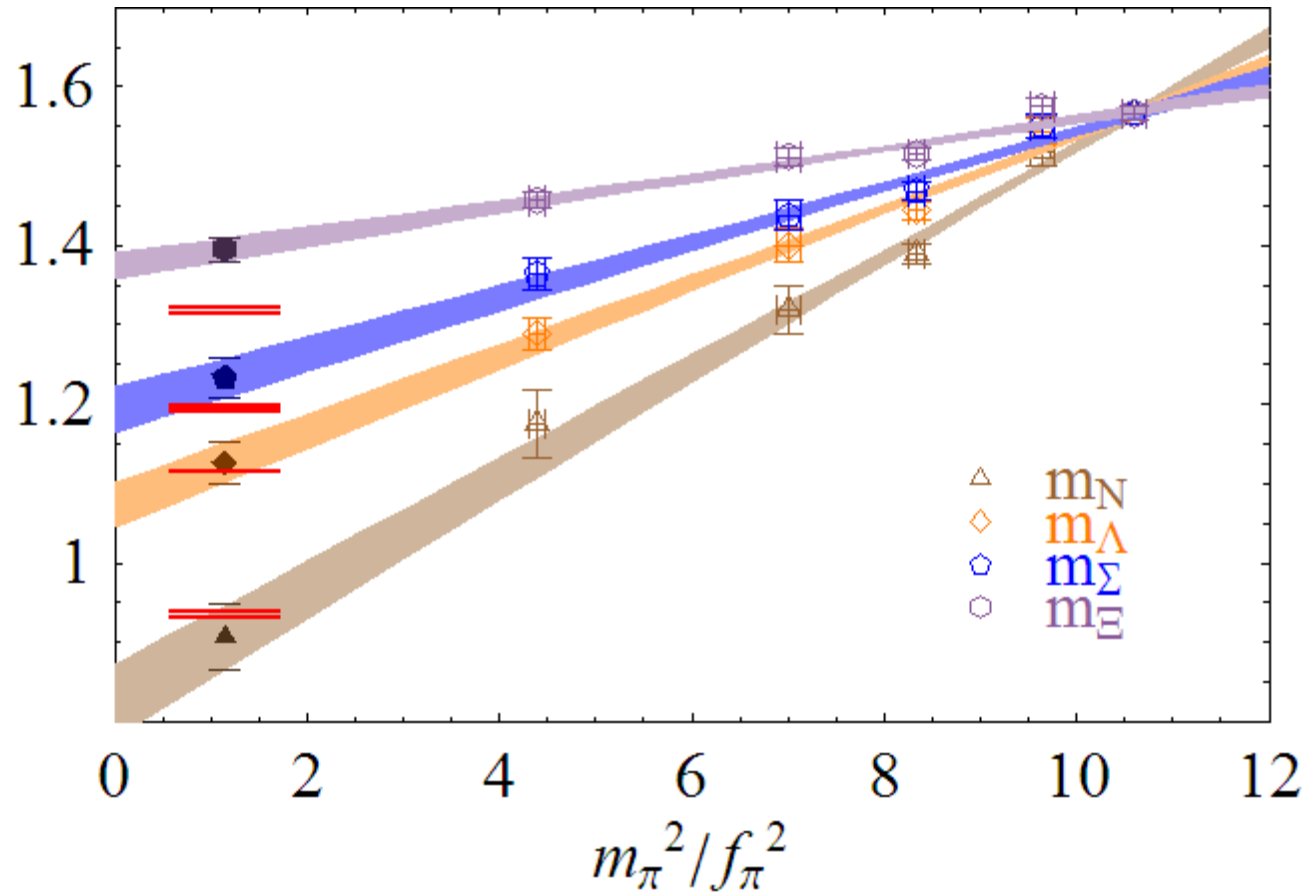
Summary and extrapolation



Disappointed ☹

Octet Spectrum

Alternative extrapolations



Better agreement with experiments

Constructions

Two-point function

$$\begin{aligned}\Gamma_{AB}^{NN}(t_i, t_f, \vec{p}; T) &\rightarrow \\ &a^6 Z_B^N(p) Z_A^N(p) \sum_s T_{\alpha\beta} \bar{u}_\alpha(\vec{p}, s) u_\beta(\vec{p}, s) \\ &\times \frac{m_N}{E_N(\vec{p})} e^{-(t_f - t_i) E_N(\vec{p})} \\ &= \left(\frac{E_N(\vec{p}) + m_N}{2E_N(\vec{p})} \right) e^{-(t_f - t_i) E_N(\vec{p})}\end{aligned}$$

Three-point function

$$\begin{aligned}\Gamma_{\mu, AB}^{NN}(t_i, t, t_f, \vec{p}_i, \vec{p}_f; T) &\rightarrow \\ &= \frac{m_N^2}{E_N(\vec{p}_f) E_N(\vec{p}_i)} e^{-(t_f - t) E_N(\vec{p}_f)} e^{-(t - t_i) E_N(\vec{p}_i)} \\ &\sum_{s, s'} T_{\alpha\beta} Z_B(p_f) u_\beta(\vec{p}_f, s') \\ &\langle N(\vec{p}_f, s') | j_\mu(0) | N(\vec{p}_i, s) \rangle \bar{u}_\alpha(\vec{p}_i, s) Z_A(p_i)\end{aligned}$$

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Ratio cancels out t and Z dependence

$$\begin{aligned} R_{j_\mu} &= \frac{Z_V \Gamma_{\mu, AB}^{\Sigma N}(t_i, t, t_f, \vec{p}_i, \vec{p}_f; T)}{\Gamma_{BC}^{NN}(t_i, t_f, \vec{p}_f; T)} \sqrt{\frac{\Gamma_{DE}^{\Sigma\Sigma}(t_i, t_f, \vec{p}_i; T)}{\Gamma_{FH}^{NN}(t_i, t_f, \vec{p}_f; T)}} \\ &\times \sqrt{\frac{\Gamma_{CC}^{NN}(t_i, t, \vec{p}_f; T)}{\Gamma_{AA}^{\Sigma\Sigma}(t_i, t, \vec{p}_i; T)}} \sqrt{\frac{\Gamma_{FH}^{NN}(t_i, t_f, \vec{p}_f; T)}{\Gamma_{DE}^{\Sigma\Sigma}(t_i, t_f, \vec{p}_i; T)}}, \end{aligned}$$

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Solve for Form factors

- ➔ Redefine matrix element as

$$\langle B_2 | V_\mu | B_1 \rangle(q) = \bar{u}_{B_2}(p') \left[\gamma_\mu F_1(q^2) + \sigma_{\mu\nu} q_\nu \frac{F_2(q^2)}{M_{B_1} + M_{B_2}} - i q_\mu \frac{F_3(q^2)}{M_{B_1} + M_{B_2}} \right] u_{B_1}(p) e^{-iq \cdot x}$$

$$\langle B_2 | A_\mu | B_1 \rangle_\mu(q) = \bar{u}_{B_2}(p') \left[\gamma_\mu G_1(q^2) + \sigma_{\mu\nu} q_\nu \frac{G_2(q^2)}{M_{B_1} + M_{B_2}} - i q_\mu \frac{G_3(q^2)}{M_{B_1} + M_{B_2}} \right] \gamma_5 u_{B_1}(p) e^{-iq \cdot x}$$

- ➔ Use “mixed” projection operator

$$T = \frac{1}{8} (1 + \gamma_4) (1 + i\gamma_5 \gamma_3) (1 + \gamma_4)$$

- ➔ Solve the following:

$$\frac{1}{\sqrt{2E_{m_\Sigma} (m_\Sigma + E_{m_\Sigma}) (m_N + m_\Sigma)}} \times$$

$$\left\{ \begin{aligned} & F_1 (m_N + m_\Sigma) (p_y - ip_x) + F_2 (-im_N p_x + iE_{m_\Sigma} p_x + m_N p_y + m_\Sigma p_y) + F_3 (-2m_\Sigma p_x - 2E_{m_\Sigma} p_x), \\ & F_1 (-m_N - m_\Sigma) (p_x + ip_y) + F_2 (-m_N p_x - m_\Sigma p_x - im_N p_y + ip_y E_{m_\Sigma}) + F_3 (-2m_\Sigma p_y - 2E_{m_\Sigma} p_y), \\ & -iF_1 (m_N + m_\Sigma) p_z - iF_2 (m_N - E_{m_\Sigma}) p_z - iF_3 (-2im_\Sigma - 2iE_{m_\Sigma}) p_z, \\ & F_1 (m_\Sigma (m_N + m_\Sigma) + E_{m_\Sigma} (m_N + m_\Sigma)) - F_2 \vec{p}^2 + F_3 (2i\vec{p}^2 + 2im_\Sigma^2 - 2im_N m_\Sigma - 2i(m_N - m_\Sigma) E_{m_\Sigma}) \end{aligned} \right\}$$

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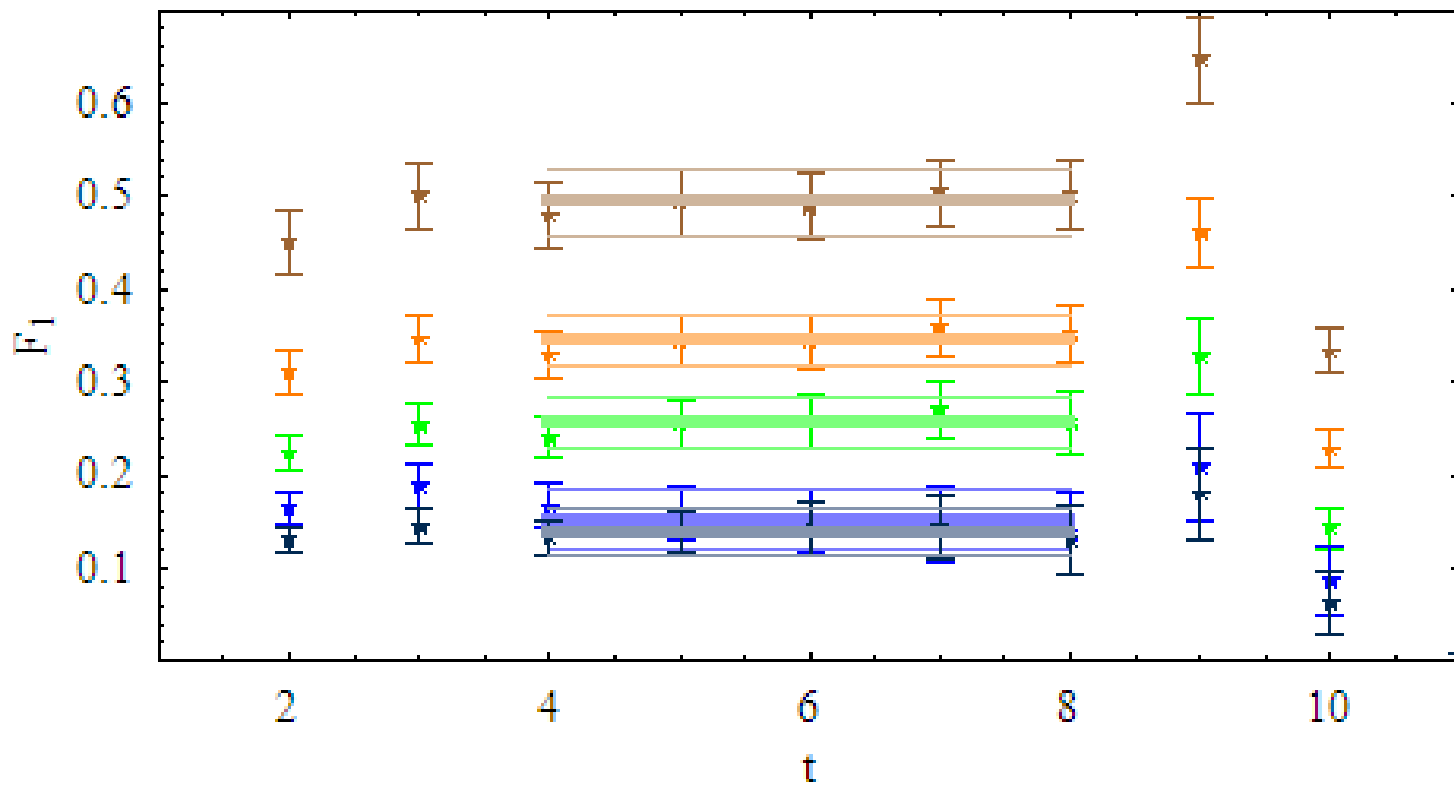
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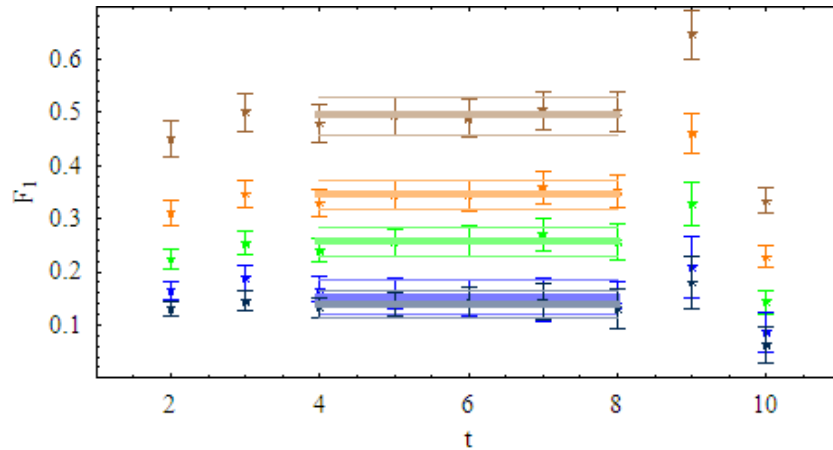
Three-Point Plateau

◆ $m_\pi = 358(2) \text{ MeV}$

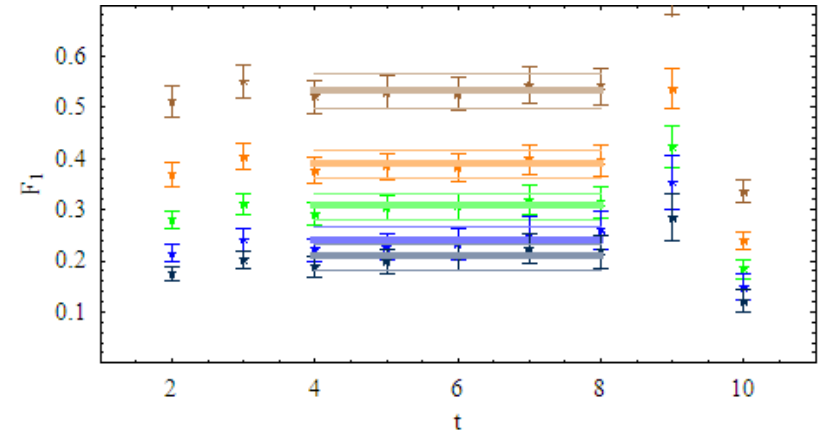


Three-Point Plateau

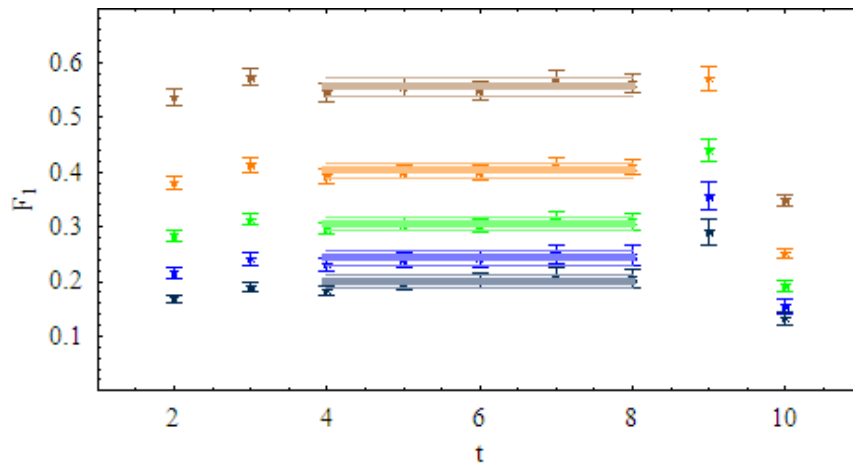
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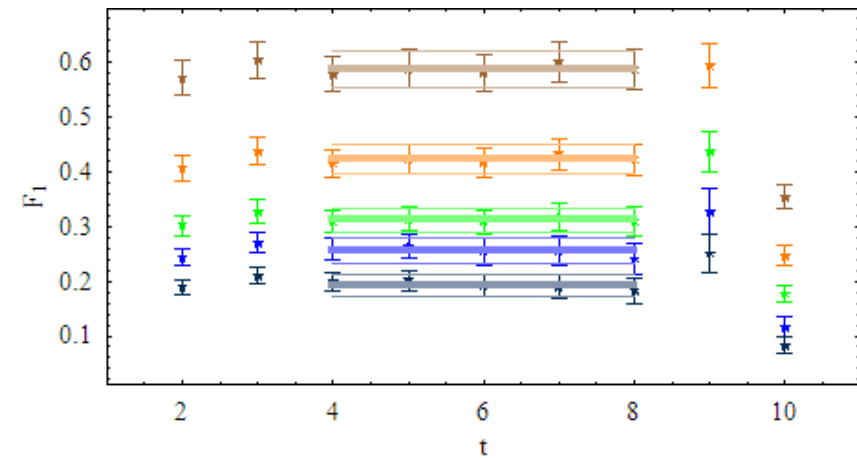
◆ $m_\pi = 503(2) \text{ MeV}$



◆ $m_\pi = 599(2) \text{ MeV}$

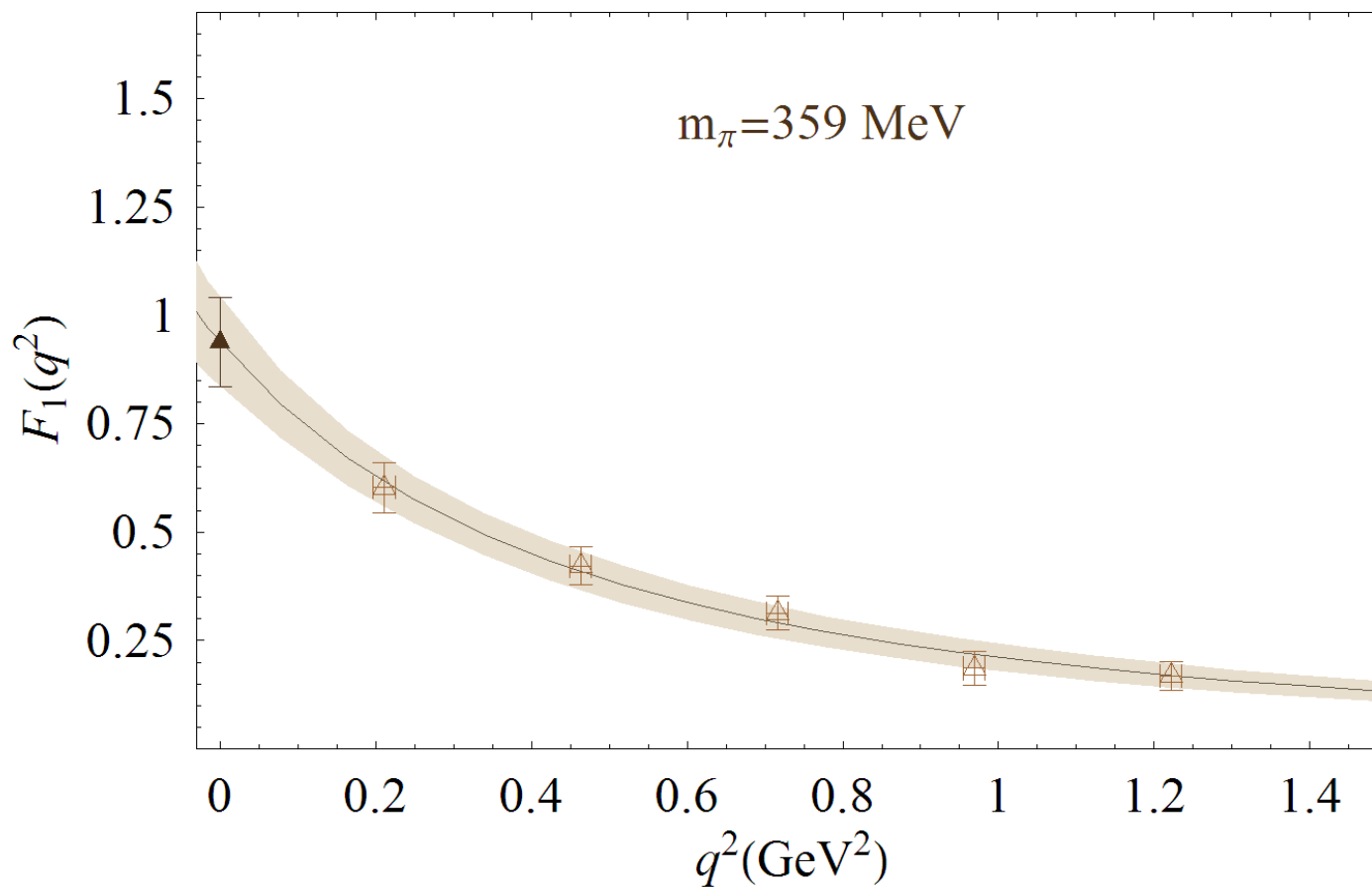


◆ $m_\pi = 689(2) \text{ MeV}$



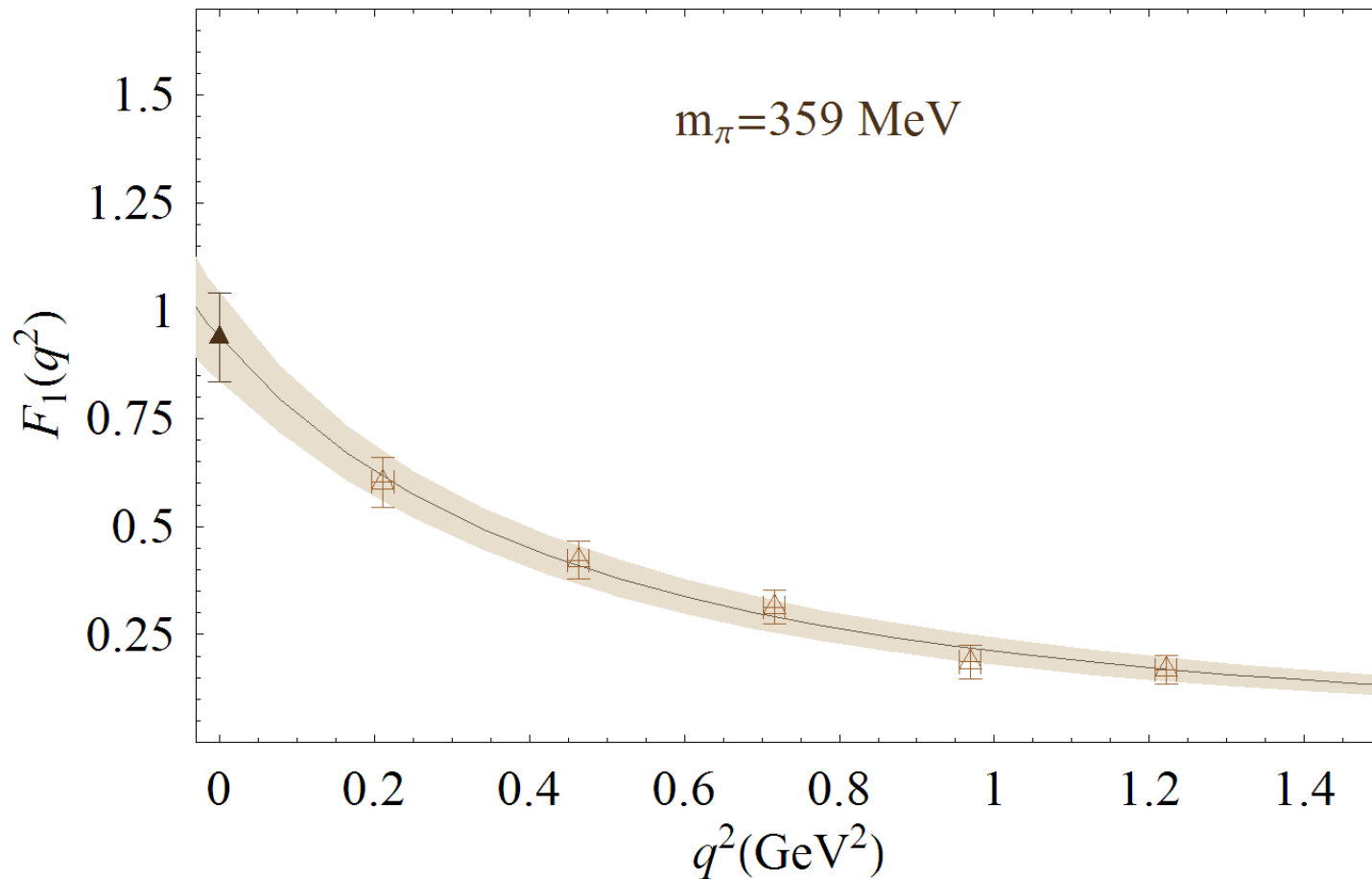
Momentum Extrapolation

- Fit to the dipole form



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Ademollo-Gatto Theorem

- ◆ Symmetry-breaking Hamiltonian

$$H' = \frac{1}{\sqrt{3}} \left(m_s - \frac{m_d + m_u}{2} \right) \bar{q} \lambda^8 q$$

- ◆ Long story short,

There is no first order correction $O(H')$ to $f_1(0)$; thus

$$f_1(0) = f_1^{SU(3)}(0) + O(H'^2)$$

Ademollo-Gatto Theorem

- Symmetry-breaking Hamiltonian

$$H' = \frac{1}{\sqrt{3}} \left(m_s - \frac{m_d + m_u}{2} \right) \bar{q} \lambda^8 q$$

- The theorem tells us that

There is no first order correction $O(H')$ to $f_1(0)$; thus

$$f_1(0) = f_1^{SU(3)}(0) + O(H'^2)$$

- Choices of observable for H' :

- $m_K^2 - m_\pi^2$

- $(m_\Sigma - m_N) / m_\Sigma$

- Others...

Mass Dependence – I

What has been done in the past...

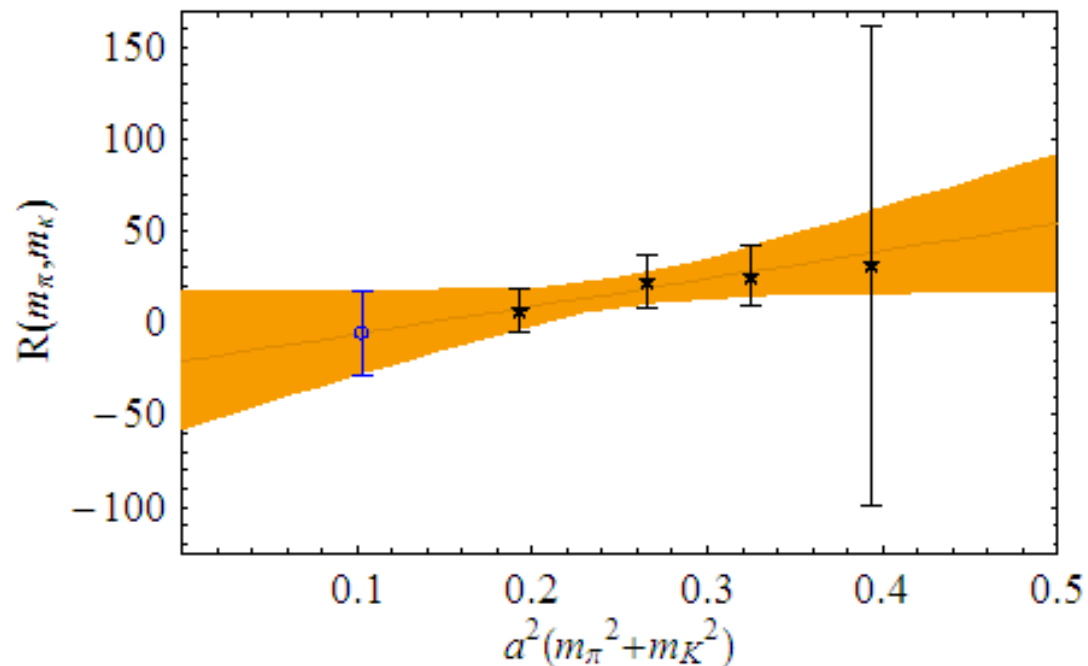
➤ *Guadagnoli et al.* $\Sigma^- \rightarrow n$

construct an AG ratio

$$R(M_K, M_\pi) = \frac{1 - f_1(0)}{a^4(M_K^2 - M_\pi^2)^2}$$

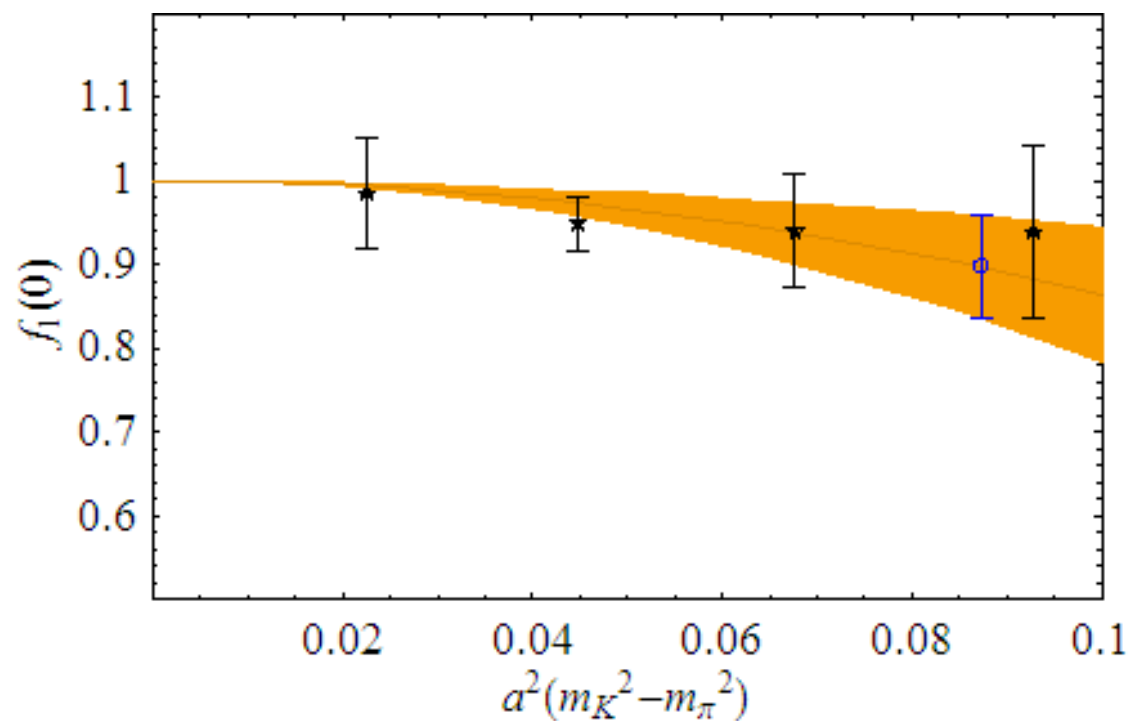
and extrapolate mass dependence as

$$R(M_K, M_\pi) = b + ca^2(M_K^2 + M_\pi^2)$$



Mass Dependence – II

- Use $\delta = a^2(M_K^2 - M_\pi^2)$ to describe the SU(3) symmetry breaking



$$f_1(0) = 0.90(7) \text{ (Preliminary)}$$

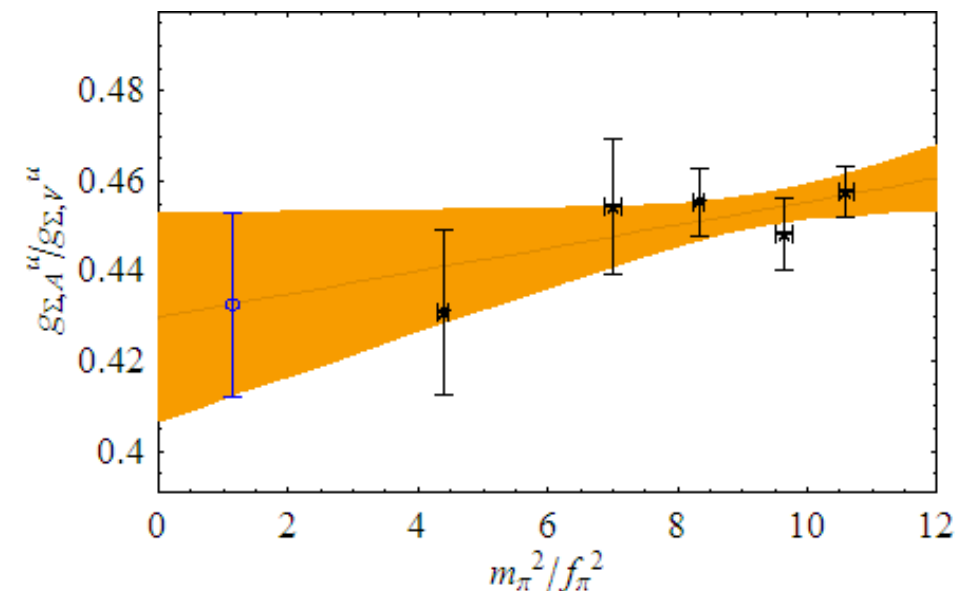
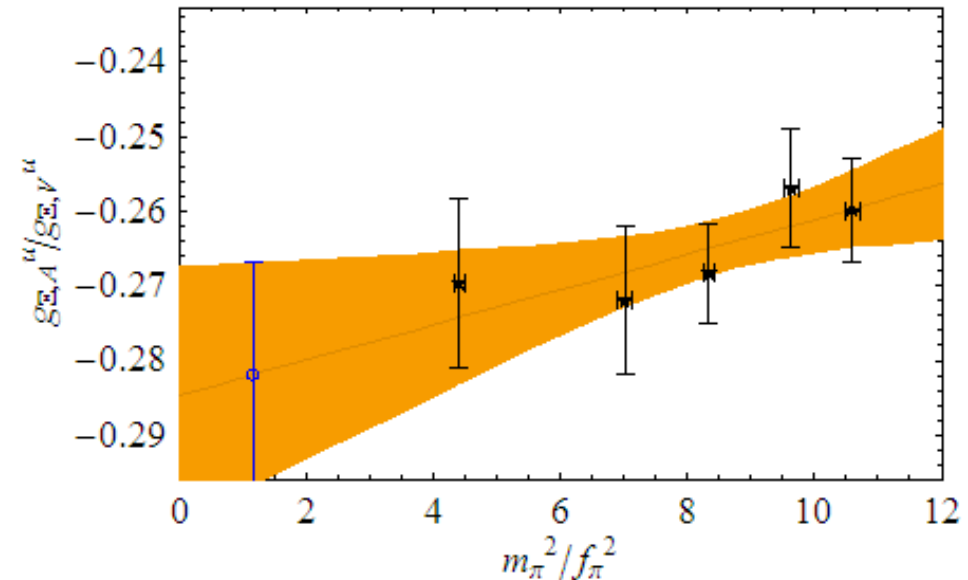
Axial Coupling Constants: $g_{\Xi\Xi}$ and $g_{\Sigma\Sigma}$

- Cannot be determined by exp.
- Existing predictions from χ PT and large N_c calculations

$$0.18 < -g_{\Xi\Xi} < 0.36$$

$$0.30 < g_{\Sigma\Sigma} < 0.55$$

- Applications such as hyperon scattering, non-leptonic decays, etc.



Summary/Outlook

- First non-quenched calculation of hyperon semileptonic decays
- Lighter pion masses as low as 350 MeV
- **Preliminary** results show $|V_{us}|$ (from $\Sigma \rightarrow n$)
 - * Consistent with the previous lattice measurement
 - * Larger error due to lighter pion mass
 - * More statistics needed for lightest point!

In the near future:

- Finish the semileptonic form factor analysis, including $\Xi \rightarrow \Sigma$ channel
- Σ and Ξ structure-function form factors
- Possibly take $\Lambda \rightarrow p$ data, if time allows