Quark-Gluon Vertex Dressing And Meson Masses Beyond Ladder-Rainbow Truncation.

nucl-th/0605057

H.H. Matevosyan[1,2], A.W. Thomas[2], P.C. Tandy[3]

[1]Department of Physics & Astronomy,
Louisiana State University

[2]Thomas Jefferson National Accelerator Facility

[3]Center for Nuclear Research,
Department of Physics,
Kent State University

Seminar at Nuclear Theory Center, Indiana University, 03/22/2007

Outline of Part I

Outline of Part II

Part I

Improving The Ladder-Summer Quark-Gluon Vertex

Motivation

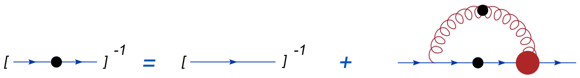
- Dyson-Schwinger Equations - tool for exploring non-perturbative hadron structure.
- Fully covariant Bethe-Salpeter description of bound states.
- Rarely one goes beyond Ladder-Rainbow (LR) truncation.
- How good or bad is LR truncation?
- Need to study the ladder truncation of more complete solution.
- Employ a simple model that can solve to high order and give some insight.

GAP Equation

R. Alkofer and L. von Smekal, Phys. Rept. 353:28 (2001).

The DSE for the quark propagator (GAP equation):

- $S^{-1}(p) = Z_2\, S_0^{-1}(p) + C_{\rm F}\, Z_1 \int_q^\Lambda g^2 D_{\mu\nu}(p-q)\, \gamma_\mu S(q) \Gamma_\nu(q,p)$
- $S_0^{-1}(p) = i\not p + m_{bm}$
- $S(p)^{-1} = i\not p\, A(p^2,\mu^2) + B(p^2,\mu^2) = \frac{1}{Z(p^2,\mu^2)}[i\not p + M(p^2)]$



Renormalization Condition At The Scale $p^2 = \mu^2$:

- $S(p)^{-1} \to i\not p + m(\mu)$

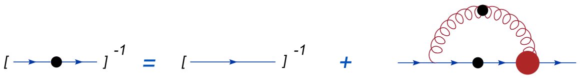
Need the dressed quark-gluon vertex and gluon 2-point functions to solve the GAP eq.

GAP Equation

R. Alkofer and L. von Smekal, Phys. Rept. 353:28 (2001).

The DSE for the quark propagator (GAP equation):

- $S^{-1}(p) = Z_2\, S_0^{-1}(p) + C_{\mathrm{F}}\, Z_1 \int_q^\Lambda g^2 D_{\mu\nu}(p-q)\, \gamma_\mu S(q) \Gamma_\nu(q,p)$
- $S_0^{-1}(p) = i\not{p} + m_{bm}$
- $S(p)^{-1} = i\not{p}\, A(p^2,\mu^2) + B(p^2,\mu^2) = \frac{1}{Z(p^2,\mu^2)}[i\not{p} + M(p^2)]$



Renormalization Condition At The Scale $p^2 = \mu^2$:

- $S(p)^{-1} \to i\not{p} + m(\mu)$

Need the dressed quark-gluon vertex and gluon 2-point functions to solve the GAP eq.

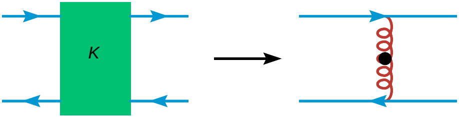
Ladder-Rainbow Truncation

P. Maris and C. D. Roberts, Int. J. Mod. Phys. E12, 297 (2003).

- GAP Eq. interaction kernel in Ultra-Violet:

$$Z_1 \gamma_\mu g^2 D_{\mu\nu}(k) \Gamma_\nu(q,p) \rightarrow 4\pi\alpha(k^2)\gamma_\mu D^{\text{free}}_{\mu\nu}(k)\gamma_\nu$$

- Rainbow truncation of GAP eq. : $\alpha(k^2) \rightarrow \alpha_{eff}(k^2)$ for all k^2 and fit it to one or more chiral observables.
- Ladder approximation of Bethe-Salpeter (BS) scattering kernel - one dressed gluon exchange:

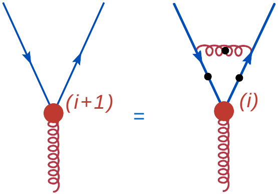


- Inadequate description of scalar and flavor-singlet pseudoscalar mesons, admits colored diquark bound states.

Ladder-Summed Quark-Gluon Vertex Function

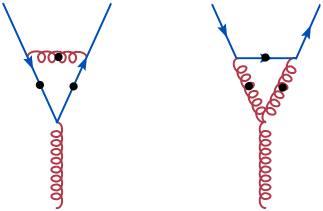
A. Bender et al., Phys. Rev. C65, 065203 (2002).

- Only 2-point gluon function were considered in dressing the quark-gluon vertex.
- $\Gamma_\mu = \sum_{i=0} \Gamma^i_\mu$, the ladder dressing scheme was used:



- Ansatz gluon 2-point function was used - algebraic model.
- Chiral-Symmetry preserving Bethe-Salpeter (BS) scattering kernel has been constructed and the model was solved to obtain physical observables (meson masses, etc).
- Results don't agree with lattice-QCD data on vertex function and 1-loop pQCD analysis .

1-loop pQCD Analysis



(a) $-\frac{1}{2N_c}$ (b) $\frac{N_c}{2}$

- Satisfies Slavnov-Taylor Id to $\mathcal{O}(g^3)$

$$k_\mu i\Gamma_\mu(p+k,p) = G(k^2)\left\{(1-B)S(p+k)^{-1} - S(p)^{-1}(1-B)\right\}$$

- Both in effective model: Ladder-Summed with $\mathcal{C}C_F$ color factor for each rung, $-\frac{1}{8} < \mathcal{C} < 1$.

Effective 3-gluon Coupling in Ladder-Summed Vertex

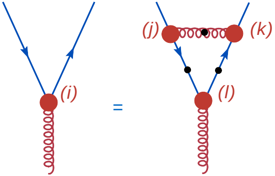
M. S. Bhagwat et al., Phys. Rev. C70, 035205 (2004).

- Implemented in DSE_q and meson BSE via (algebraic) MN model.
- $\mathcal{C}$ fitted to best reproduce lattice data - reasonably good agreement with the data achieved.
- Compared to LR: 30% reduction in M_V, minor change in M_{PS}.

Self-Consistent Dressing

H.M. et al., nucl-th/0605057.

- Include all lower-order vertices in the ladder dressing scheme.



- Use effective color factor $\mathcal{C}$ to account for 3-gluon coupling.

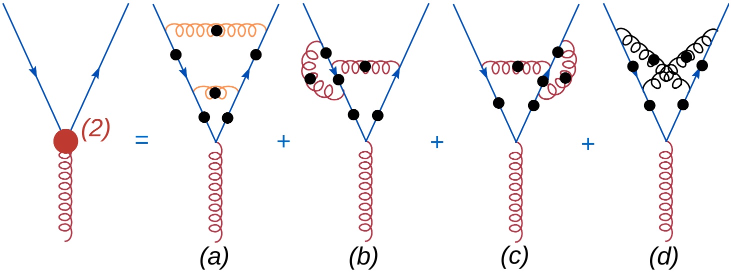
$$\Gamma^i_\mu(p_+, p_-) = -\mathcal{C}\, C_{\rm F} \sum_{\substack{j,k,l \\ i=j+k+l+1}} \int_l^\Lambda g^2 D_{\sigma\nu}(p-l) \times \Gamma^j_\sigma(p_+, l_+) S(l_+) \Gamma^l_\mu(l_+, l_-) S(l_-) \Gamma^k_\nu(l_-, p_-).$$

The Wider Class vs Ladder-Summed Vertex

The number of diagrams in a vertex with up to n gluon lines

- Improved scheme: $1 + n(n+1)(n+2)/6$.
- Ladder-Summed vertex: a subset of $(n+1)$.

For Example:



Meson Bethe-Salpeter Equation (BSE)

The renormalized homogeneous Bethe-Salpeter equation (BSE) for the quark-antiquark channel, denoted by M:

$$[\Gamma_M(k;P)]_{EF} = \int_q^{\Lambda} [K(k,q;P)]_{EF}^{GH} \, [\chi_M(q;P)]_{GH} \, ,$$

where

- $\Gamma_M(k;P)$ - meson Bethe-Salpeter amplitude (BSA).
- $\chi_M(k;P) = S(k_+)\Gamma_M(k;P)S(k_-)$ - BS wavefunction.
- K - amputated quark-antiquark scattering kernel.

Dressed-gluon ladder-truncation:

$$[K(k,q;P)]_{EF}^{GH} = D_{\mu\nu}(k-q)\,[l^a\gamma_\mu]_{EG}\,[l^a\gamma_\nu]_{HF}$$

Symmetry-Preserving Bethe-Salpeter Kernel

A systematic procedure has been developed for obtaining chiral-symmetry preserving K_{BSE} from Σ_{GAP} (*H. J. Munczek, Phys. Rev. D52, 4736 (1995).*):

- $K_{EF}^{GH} = -\frac{\delta \Sigma_{EF}}{\delta S_{GH}}$.
- This kernel preserves the Axial-Vector Ward-Takahashi Identity - ensures that chiral pseudoscalars remain massless independent of model details.

Using our model vertex to decompose the self-energy:

- $\Sigma(k) = \sum_{n=0}^{\infty} \Sigma^n(k)$.

The Model Interaction Kernel

The meson BSE corresponding to the extended class of vertex dressing:

$$\Gamma_M(k;P) = -C_{\mathrm{F}} \int_q^{\Lambda} g^2 D_{\mu\nu}(k-q)\gamma_\mu \times \left[\chi_M(q;P)\Gamma_\nu(q_-,k_-) + S(q_+)\Lambda_{M\nu}(q,k;P)\right],$$

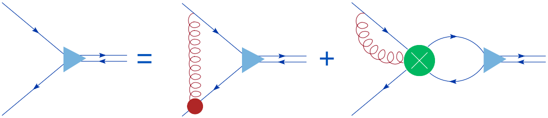
where

$$\Lambda_{M\nu}(q,k;P) = \sum_{n=0}^{\infty} \Lambda^n_{M\nu}(q,k;P),$$

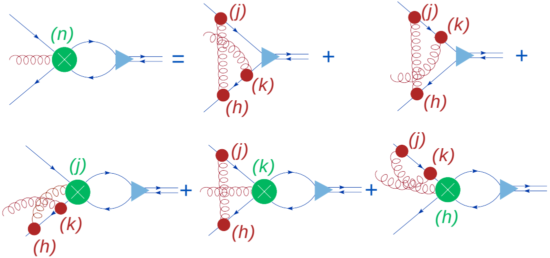
with

$$[\Lambda^n_{M\nu}(q,k;P)]_{LF} = \int_l^{\Lambda} \frac{\delta}{\delta S_{GH}(l_\pm)} [\Gamma^n_\nu(q_-,k_-)]_{LF} \times [\chi_M(l;P)]_{GH} .$$

The Model Interaction Kernel



(c) Kernel decomposition.



(d) Λ function decomposition.

Figure: BSE corresponding to the extended class of vertex dressing.

The Munczek-Nemirovsky Interaction Kernel

- Munczek-Nemirovsky Ansatz for the interaction kernel in Landau gauge:

$$g^2 D_{\mu\nu}(k) \to \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)(2\pi)^4 \mathcal{G}^2 \delta^4(k)$$

$\mathcal{G}^2$ - integrated kernel strength.

H. J. Munczek and A. M. Nemirovsky, Phys. Rev. D28, 181 (1983).

- Yields Ultra-Violet Finite DSEs: $Z_1 = Z_2 = 1$.
- Integral Equations $\Rightarrow$ Algebraic Equations

Computer-Algebraic Evaluation Of The Dirac Algebra

- In the case of a limitation to a strict ladder summation with bare internal vertices, closed form expression for the vertex function in terms of A and B is obtainable.
- With the enlarged class of dressing considered here, corresponding closed form expressions have not been obtained.
- Numerical solution of the simultaneous algebraic equations for the vertex and propagator is carried out here using the algebraic and numerical tools of *Mathematica (5.2)* with the assistance of the *FeynCalc* (*R. Mertig et al., Comput. Phys. Commun. 64 (1991),345-359. "http://www.feyncalc.org"*) package used for computer-algebraic evaluation of the Dirac algebra.

$\mathcal{C}$ From Fits To Lattice QCD Data

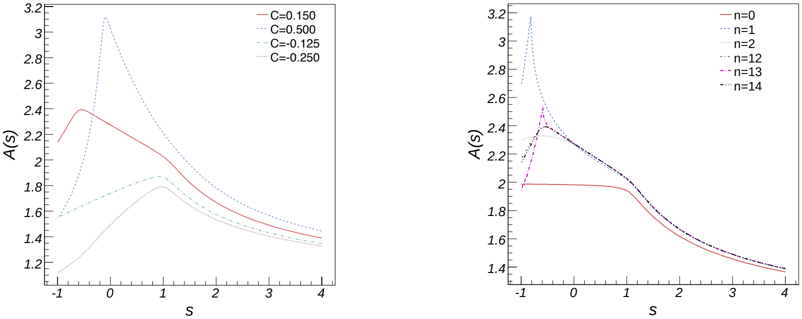
We fit $\mathcal{C}$ to best reproduce $p = 0$ extrapolations of lattice-QCD calculated:

- Quark propagator functions $A(0)$ and $B(0)$ (*P. O. Bowman et al., Nucl. Phys. Proc. Suppl. 119, 323 (2003).*)
- Invariant amplitudes $\alpha_i(0)$ of quark-gluon vertex function (*J. I. Skullerud et al., JHEP 04, 047 (2003).*)

The best fit to these quantities gives:

- $\mathcal{C} = 0.34$, $\bar{r} = 24$ % and $\sigma_r = 70$ %.
- The quality of fit is about the same as in *Bhagwat et al.*, and changes $\Delta\mathcal{C} \approx \pm 0.2$ are not significant in this regard.
- $\mathcal{C} = 0.15$ leads to $\bar{r} = 39$ % and $\sigma_r = 72$ %. will be used, because the resulting vertex at timelike p^2 is more convergent with respect to increasing order of dressing. (Note: $\mathcal{C} = 0.51$ in the prev. work)
- $\mathcal{C} \gg \mathcal{C}_{SLR} = -1/8$ - The attraction provided by the 3-gluon coupling is important for the vertex.

Solutions For $A(p^2)$

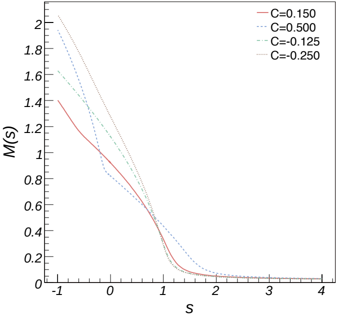


(a) $\mathcal{C}$ dependence calculated with converged summation of vertex dressing

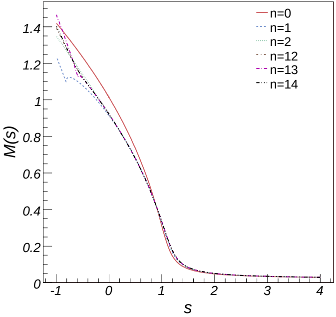
(b) Influence of vertex dressing to order n for $\mathcal{C} = 0.15$.

Figure: Quark propagator amplitude $A(s)$ versus Euclidean $s = p^2$ for $\mathcal{G} = 1$ GeV and $m = 0.0183\,\mathcal{G} = 18.3$ MeV.

Solutions For $M(p^2)$



(a) $\mathcal{C}$ dependence calculated with converged summation of vertex dressing



(b) Influence of vertex dressing to order n for $\mathcal{C} = 0.15$.

Figure: Quark propagator amplitude $M(s)$ versus Euclidean $s = p^2$ for $\mathcal{G} = 1$ GeV and $m = 0.0183\ \mathcal{G} = 18.3$ MeV.

Vertex Dressing Effect on m_π and m_ρ.

Meson masses are in GeV:

Vertex Dressing	m_π	m_ρ	Δm_ρ	$\frac{\Delta m_\rho}{m_\rho}$	$\frac{\Delta m_\rho}{m_\rho}$ (prev.)
$n = 0$ (LR)	0.140	0.850	+0.074	+0.095	+0.295
$n = 1$ (1-loop)	0.135	0.759	-0.017	-0.022	——
$n = 2$	0.135	0.781	+0.005	+0.006	+0.096
$n = 3$	0.135	0.772	-0.004	-0.005	N/A
$n = 4$	0.135	0.778	+0.002	+0.003	N/A
$n = \infty$ (full model)	0.135	0.776	0.0	0.0	0.0

$\mathcal{G} = 0.59$ GeV, $m = 0.0183\,\mathcal{G} = 11$ MeV and $\mathcal{C} = 0.15$.

LR Truncation Accuracy vs. Current Quark Mass

	ladder-rainbow $n = 0$	full model $n = \infty$	LR % error this model	(prev.)
$m_{u,d} = 0.011$				
m_ρ	0.850	0.776	9.5%	30%
$\mathcal{BE}_\rho$	0.346	0.311	11%	
$m_s = 0.165$				
m_ϕ	1.08	1.02	6.0%	21%
$\mathcal{BE}_\phi$	0.350	0.320	9.0%	
$m_c = 1.35$				
$m_{J/\psi}$	3.11	3.09	0.3%	3.5%
$\mathcal{BE}_{J/\psi}$	0.260	0.260	0%	
$m_b = 4.64$				
m_Υ	9.46	9.46	0%	0%
$\mathcal{BE}_\Upsilon$	0.100	0.100	0%	

Heavy Pseudoscalar and Vector Mesons

Meson masses (in GeV) calculated for u/d, s , c and b quarks:

$m_{u,d} = 0.011$	$m_s = 0.165$	$m_c = 1.35$	$m_b = 4.64$
$m_\rho = 0.776$	$m_\phi = 1.02$	$m_{J/\psi} = 3.09$	$m_{\Upsilon(1S)} = 9.46$
$\mathcal{BE}_\rho = 0.311$	$\mathcal{BE}_\phi = 0.320$	$\mathcal{BE}_{J/\psi} = 0.260$	$\mathcal{BE}_\Upsilon = 0.100$
$m_\pi = 0.135$	$m_{0^-_{s\bar{s}}} = 0.61$	$m_{\eta_c} = 2.97$	$m_{\eta_b} = 9.43$
$\mathcal{BE}_\pi = 0.953$	$\mathcal{BE}_{0^-} = 0.727$	$\mathcal{BE}_{\eta_c} = 0.380$	$\mathcal{BE}_{\eta_b} = 0.130$

Note:

Experimentally $m_{\eta_c} = 2.9797 \pm 0.00015$ and $m_{\eta_b} = 9.30 \pm 0.03$*. The fictitious pseudoscalar $0^-_{s\bar{s}}$ is included for comparison with previous studies (0.63 in *Bhagwat et al.*).

* To be confirmed

Heavy Pseudoscalar and Vector Mesons

Meson masses (in GeV) calculated for u/d, s , c and b quarks:

$m_{u,d} = 0.011$	$m_s = 0.165$	$m_c = 1.35$	$m_b = 4.64$
$m_\rho = 0.776$	$m_\phi = 1.02$	$m_{J/\psi} = 3.09$	$m_{\Upsilon(1S)} = 9.46$
$\mathcal{BE}_\rho = 0.311$	$\mathcal{BE}_\phi = 0.320$	$\mathcal{BE}_{J/\psi} = 0.260$	$\mathcal{BE}_\Upsilon = 0.100$
$m_\pi = 0.135$	$m_{0^-_{s\bar{s}}} = 0.61$	$m_{\eta_c} = 2.97$	$m_{\eta_b} = 9.43$
$\mathcal{BE}_\pi = 0.953$	$\mathcal{BE}_{0^-} = 0.727$	$\mathcal{BE}_{\eta_c} = 0.380$	$\mathcal{BE}_{\eta_b} = 0.130$

Note:

Experimentally $m_{\eta_c} = 2.9797 \pm 0.00015$ and $m_{\eta_b} = 9.30 \pm 0.03$*. The fictitious pseudoscalar $0^-_{s\bar{s}}$ is included for comparison with previous studies (0.63 in *Bhagwat et al.*).

* To be confirmed

Summary - Part I

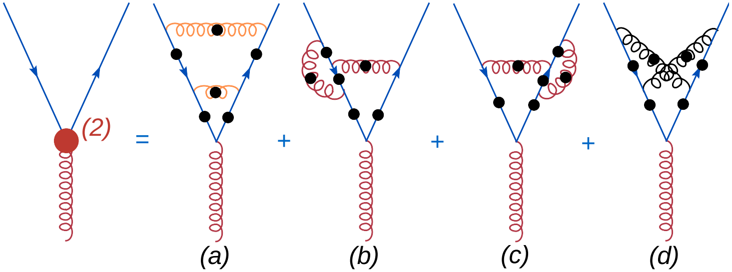
- We included self-consistent dressing on all available vertices in ladder dressing scheme of the quark-gluon vertex function and constructed chiral symmetry-preserving BSE kernel.
- We used a model gluon propagator to solve the GAP and the BS equations.
- Resulting vector meson masses were compared to ladder-rainbow truncation: 10% difference for m_ρ decreasing to $< 1\%$ for J/ψ and Υ.

Part II

Consequences Of Fully Dressing Quark-Gluon Vertex Function With Two-Point Gluon Lines.

The Non-Planar Diagrams

- We previously neglected all the non-planar diagrams in the vertex dressing supposing their smallness.
- Figure (d) is the lowest-order non-planar diagram:



- They prove to be significant in the dressing as n increases*!

* One should be careful when implementing the large N_c counting in here.

The Non-Planar Diagrams - How to Generate Them?

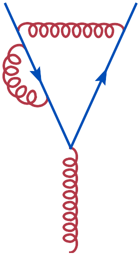
Recurrent Algorithm For Constructing The Full Vertex

First we construct all possible diagrams for the vertex function with exactly n gluon lines:

- We construct them by considering every diagram with n-1 gluon lines from the full set of all such vertices.
- For each diagram we make all possible insertions of a single gluon propagator on the quark line, so that there is at least one other quark-gluon vertex in-between the gluon line's endpoints.
- We check if the resulting vertex is not redundant with already produced ones.

Recurrent Algorithm For Constructing The Full Vertex - Cont.

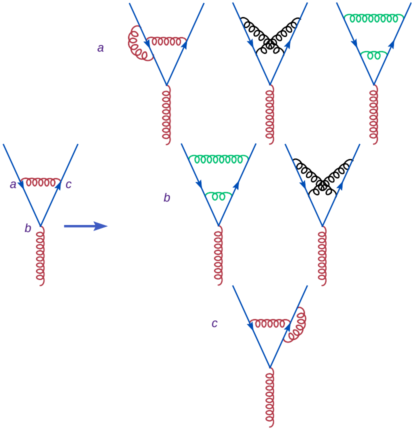
- This procedure guaranties that no quark self-energy type diagrams are produced!



- We generate ALL THE DIAGRAMS - it is easy to prove by mathematical induction!

Illustration

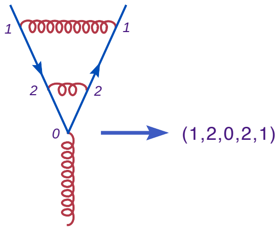
Generating all the 2-nd order diagrams form the only 1-st order diagram:



The Numerical Implementation

We implement the algorithm by first constructing a unique set of numbers for each diagram:

- We build the set by enumerating the bare quark-gluon vertices in a diagram with n-1 gluon lines from 1 to n-1 and assigning the same numbers to the vertices attached to the same gluon propagators. We assign 0 to the external gluon vertex.



The Numerical Implementation - Cont.

To construct the vertices with n gluon lines:

- We insert a pair of n into the set, so that we will not have them next to each other:

n n

$$\left(\quad 1, \; 2, \; 1, \; \ldots, \; 0, \; \ldots, \; n-5, \; n-1 \; \right)$$

- We relabel the resulting set in the ascending order.
- We check if the final set was already generated.

The Challenges

We encounter a skyrocketing number of diagrams!

n	*LR Summed*	*Improved*	*Full*
2	1	3	4
3	1	6	27
6	1	21	38232
7	1	28	$\sim 5 * 10^5$

- We need to calculate the color factors for all the diagrams.
- The evaluation of the Dirac algebra will be unaffordable on a single PC.

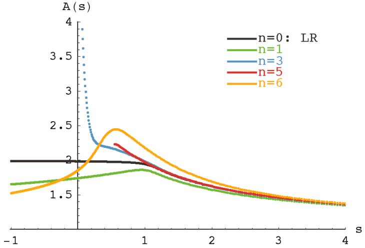
The Color Factors

- We lacked analytic tools for calculating the color factors for diagrams with over n=2 gluon lines.
- We used simple numeric contractions of SU(3) color matrices to evaluate the color factors. The calculation speed on an ordinary PC was sufficient for the vertex truncation order we used ($\sim$ one week for n=6).
- Random checks with the results from the program Colour (*J. Hakkinen et al, arXiv:hep-ph/9603229*, discovered only after the completion of the project :)) and easily reducible cases were positive.

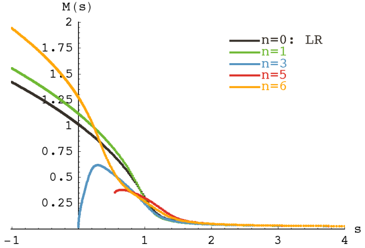
Evaluation Of The Dirac Algebra

- We used *FeynCalc* package for computer-algebraic evaluation of the Dirac algebra.
- We used JLab's Scientific Computing Farm for parallel analytic computation of the vertex functions.
- For vertices with n=6 lines the code ran $\sim$ one week on 10 machines simultaneously.
- We used the produced vertices to construct and solve the quark GAP equation for the propagator functions A and B.

The Solutions For The Quark Propagator



(a) Quark propagator amplitude A(s) versus Euclidean $s = p^2$.



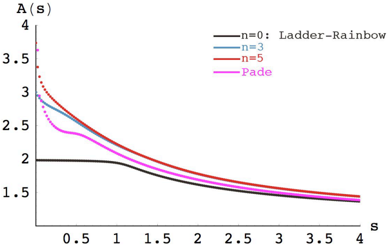
(b) Quark mass function M(s) versus Euclidean $s = p^2$.

Figure: The influence of the vertex dressing to a finite order n: 0(LR)-Black, 1-Green, 3-Blue, 5-Red, 6-Orange. We used $\mathcal{G} = 1$ GeV and $m = 0.0175\,\mathcal{G} = 17.5$ MeV.

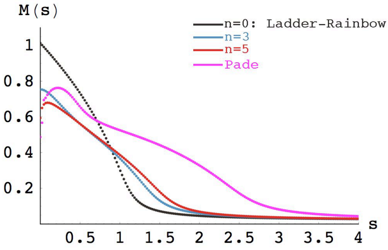
Effective 3-point gluon function dressing and Pade Approximant

- We introduced the phenomenological parameter $-\frac{1}{8} \leqslant \mathcal{C} \leqslant 1$ in counting the color factors to account for 3-gluon dressing and chose to implement it only for the sub-class of diagrams that were included in the improved LR vertex to be unambiguous.
- The results show that for the range of parameter $\mathcal{C} \in (0.375, 0.8)$ the solutions of the GAP equation are in Nambu-Goldstone mode at every calculated order up to n = 6.
- In order to draw any reliable conclusions we employed a Pade approximant to re-sum the perturbative solutions of the GAP equation and yield a solution at $n = \infty$.

The Converged Solutions



(a) Propagator amplitude A(s) versus Euclidean $s = p^2$.



(b) Mass function M(s) versus Euclidean $s = p^2$.

Figure: The influence of the vertex dressing to a finite order n: 0(LR)-Black, 3-Blue, 5-Red. Pade Approximant - Magenta. We used $\mathcal{C} = 0.375$ and $m = 0.0175\ \mathcal{G} = 17.5$ MeV.

Summary - Part II

- We employed the most general quark-gluon vertex dressing scheme with only 2-point gluon functions to solve the GAP eqn.
- The solutions of the GAP eqn. didn't converge with the maximum number of gluon lines included and showed significant deviation from those calculated previously.
- The solutions for the vertices with the maximum odd number of gluon lines yield solutions in Wigner-Weyl mode.
- A phenomenological inclusion of 3-point gluon function dressing and employment of a Pade approximate yield a converged solutions in Nambu-Goldstone mode that show significant deviations from the solutions with rainbow truncated vertex.

- Outlook
  - Use more realistic 2-point gluon functions - Thinking about.
  - Explicitly include 3- and 4-point gluon functions - Dreaming about :).