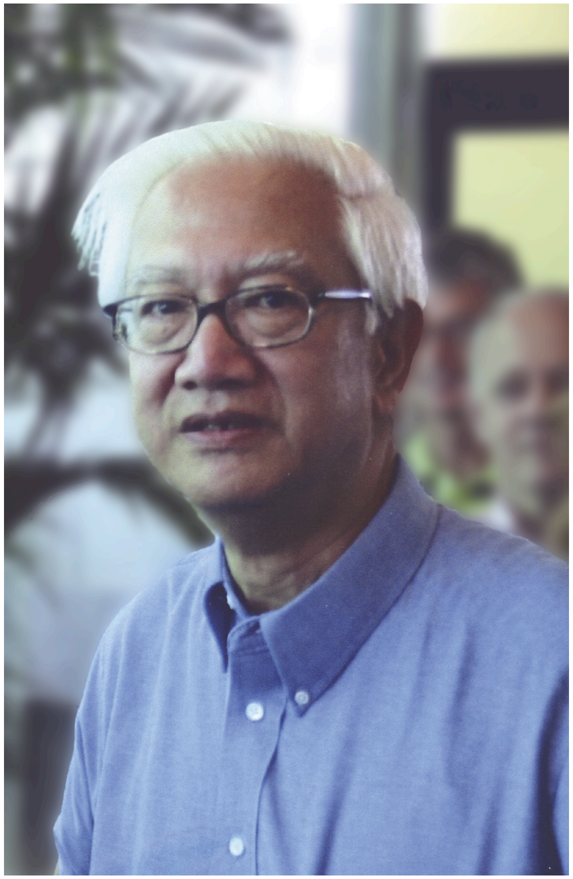
Non-perturbative methods in relativistic field theory

APFB2011, Seoul, Korea, August 23, 2011

Franz Gross

JLab and W&M

- ★ Remembrance of John
- ★ Bound state methods:
  - Bethe-Salpeter
  - Covariant Spectator Theory
- ★ Lessons from Feynman-Schwinger
- ★ Conclusions



Thanks to: John Tjon and Cetin Savkli

In Memoriam: John Tjon (7 Dec 1937 -- 20 Sept 2010)

- 7 Dec 1937 - Paramaribo, the capital of Surinam, in Northern South America
- 1964 - PhD from Utrecht (statistical physics)
- 1969 (with Rudi Malfliet) - 1st solution of the Faddeev equations using a local potential
- 1971- full professor at Utrecht
- 1980 (with Zuilhof) - Deuteron and BS equation with OBE
- 1993 (with Simonov) - Feynman-Schwinger representation for 2-body amplitude
- 1996 (with Nieuwenhuie) - generalized ladder graphs in $\varphi^2\chi$ theory
- 1997 - elected membr of the Royal Netherlands Academy
- 2003 (with Blunden and Melnitchouk) - 2 photon exchange
- 20 Sept 2010 - Bilthoven, NL

1993 -- during the Few Body conference in Amsterdam (age 55)

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1978 - Meet John at the Graz conference
Learn about the singularities in my eq.

1989 - first paper with John (and S. Wallace)
pN scattering

1990 (summer) - Five months at Utrecht

1999-2005 - FS calculations with John and
Cetin Savkli

- 2003 (with Blunden and Melnitchouk) - 2 photon exchange
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John Tjon's physics (as I understood it)

- ★ John believed that relativistic field theory was the best tool for the development of Few-Body physics
- ★ He always sought out new directions, and liked doing something no one had tried before; he was an excellent calculator and applied his skills to physics problems of interest
- ★ John favored using the Bethe-Salpeter equation in the ladder approximation, but was quite willing to make approximations -- his favorite was what he called the Equal-Time (ET) approximation
- ★ He was selected several times to give the standard "relativistic effects" talk that used to have a place at all Few-Body conferences
- ★ He was very honest in his assessment, and was careful not to oversell his work.

Non-perturbative physics in RFT (I)

and our dispute between

Bethe Salpeter equation, and the

Covariant Spectator Theory© (CST)

(1970 - 1990)

Why go relativistic? (My words, but I think John would agree)

- ★ Intellectual: to preserve an exact symmetry (Poncare' invariance)
- ★ Practical: to calculate boosts and Lorentz kinematics consistently to all orders (essential when energies are of the order of 1 GeV)
- ★ Consistent: to use field theory for guidance in the construction of
  - forces (2⇔3 body consistency)
  - currents consistent with forces
- ★ Conceptual: for "phenomenological economy", and to understand the non relativistic limit:
  - spin 1/2 particles (Dirac equation)
  - interpretation of L•S forces (covariant scalar-vector theory of N matter)
  - efficient one boson exchange models of NN forces (?)

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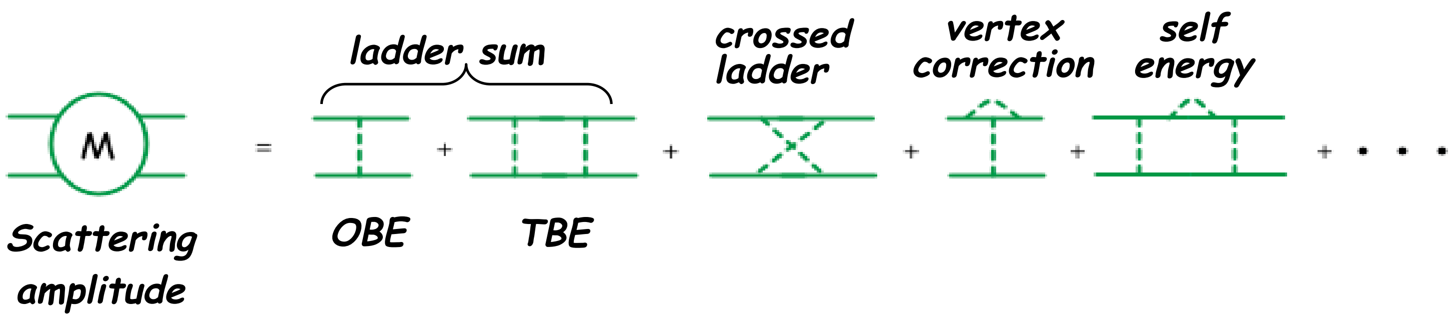
Diagrammatic derivation of field theory scattering equations

★ Step 1: The exact scattering amplitude is the sum of all Feynman diagrams



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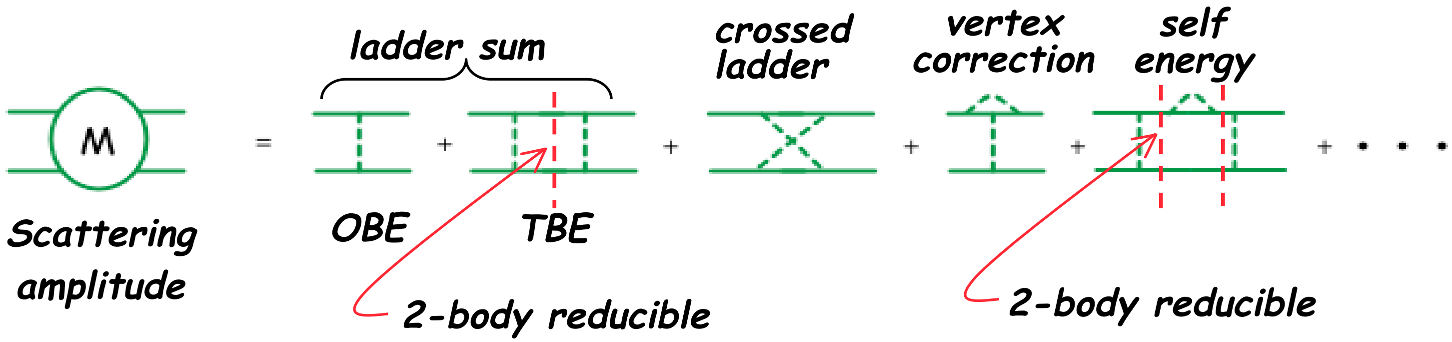
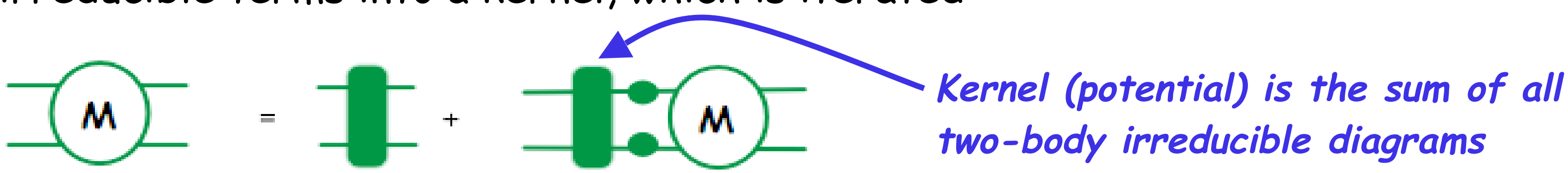


★ Step 2: Divide the sum into irreducible and 2-body reducible terms, and collect the irreducible terms into a kernel, which is iterated



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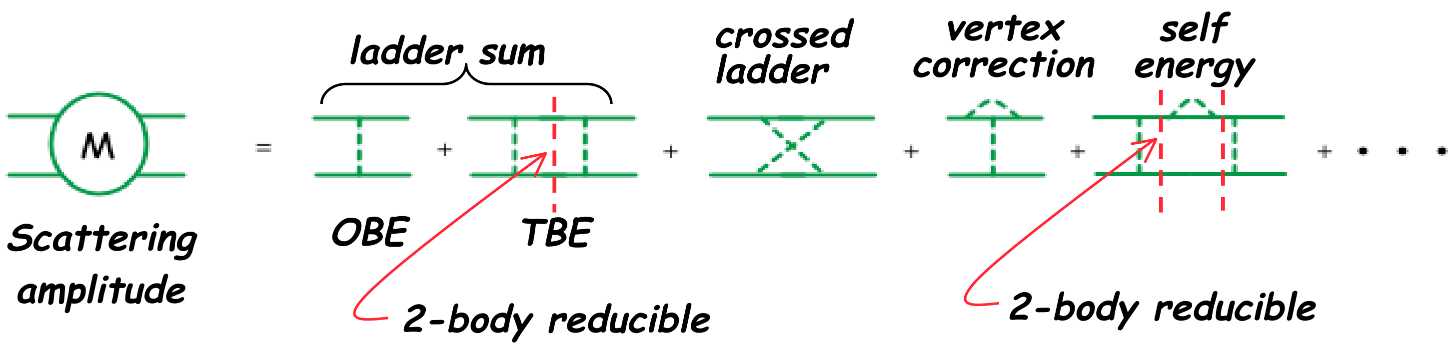

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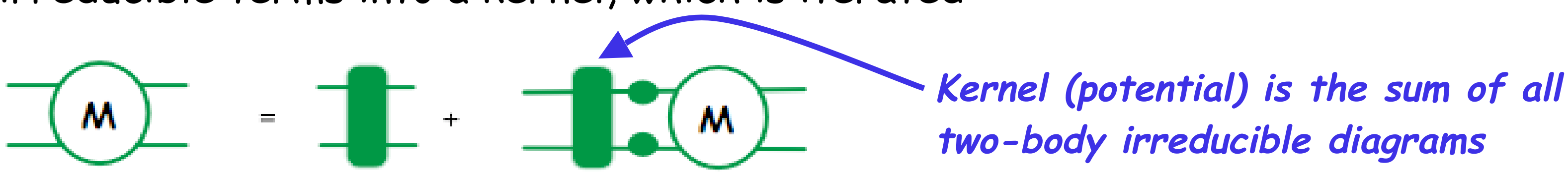

★ Step 3: *Field theory* becomes *field dynamics* when the kernel is phenomenological

$$M(p',p;P) = V(p',p;P) + \int V(p',k;P)G(k;P)M(k,p;P)$$

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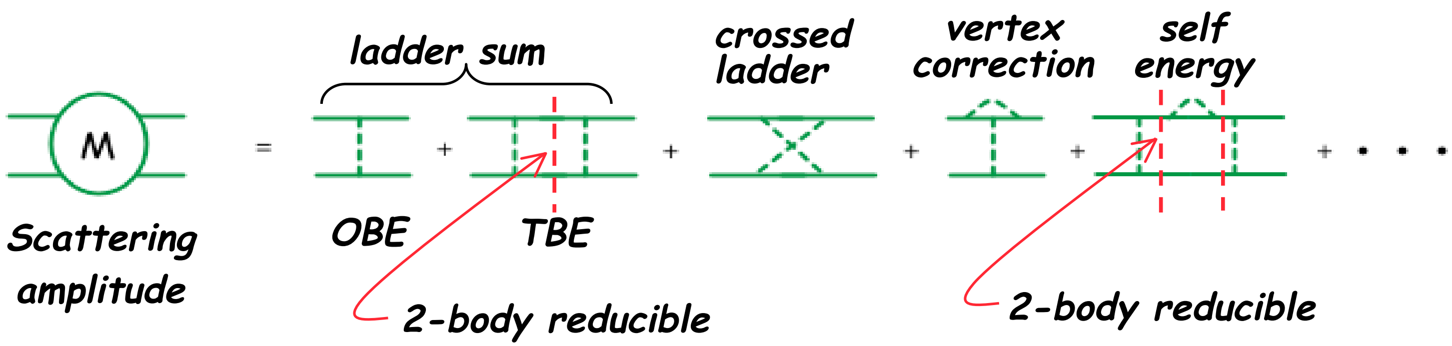

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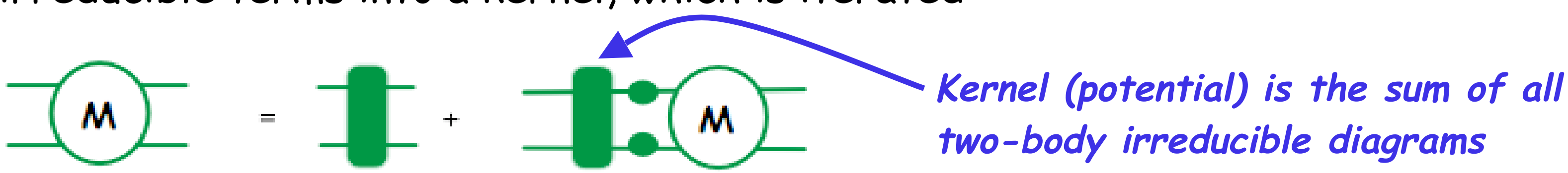

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- Spin 1/2 particles have a Dirac structure

When is it necessary to sum diagrams to all orders?

★ In schematic form, the integral equation looks like

$$M = V + VgM = V + VgV + VgVgV + \bullet\bullet\bullet = \frac{V}{1-gV}$$

★ must sum when $gV \approx 1$ (even if V is small, g might be large)

- example: atomic states

★ to describe bound states

- *A* bound state is a new particle (not in the Lagrangian).
- shows up as a pole in the scattering matrix (i.e. $gV = 1$)
- generated *non-perturbatively* from the sum of an infinite number of diagrams

★ to describe unitarity

$$M = \frac{V}{1-igV} \quad \Rightarrow \quad \mathrm{Im}\, M = \frac{gV^2}{1+(gV)^2} = g|M|^2$$

Relation between the BS and CST© two-body equations

★ The Bethe-Salpeter (BS) propagator depends on all four components of the relative momentum, $\{k_0,\mathbf{k}\}$. For two spinor particles it is

BS

$$G_{BS}(k;P)=\frac{1}{\left(m_1-\not p_1-\Sigma(\not p)-i\varepsilon\right)\left(m_2-\not p_2-\Sigma(\not p_2)-i\varepsilon\right)} \quad \textbf{with} \quad \begin{cases} p_1=\tfrac{1}{2}P+k \\ p_2=\tfrac{1}{2}P-k \end{cases}$$

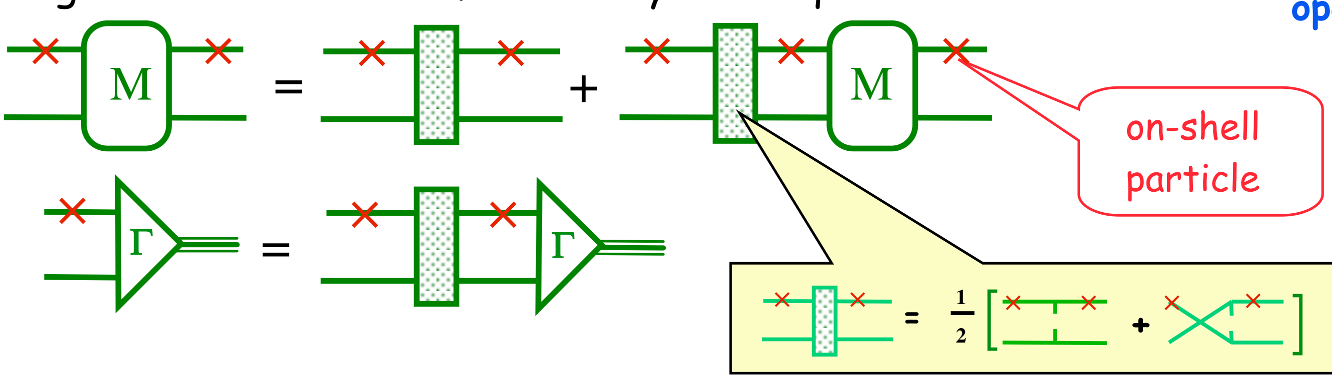
★ The Covariant Spectator Theory© propagator depends on only three components of the relative momentum, $\mathbf{k}$. One particle is on-shell

CS

$$G_{CS}(k;P)=\frac{2\pi i\,\delta_+\left(m_1^2-p_1^2\right)\left[m_1+\hat{\not p}_1\right]}{\left(m_2-\not p_2-\Sigma(\not p_2)-i\varepsilon\right)}=\frac{2\pi i\,\delta\left(p_0-E_1\right)}{\left(m_2-\not p_2-\Sigma(\not p_2)-i\varepsilon\right)}\frac{m_1}{E_1}\sum_s u(\mathbf{p}_1,s)\bar{u}(\mathbf{p}_1,s)$$

on-shell projection operator

★ Diagrammatic notation for 2-body CST equations:

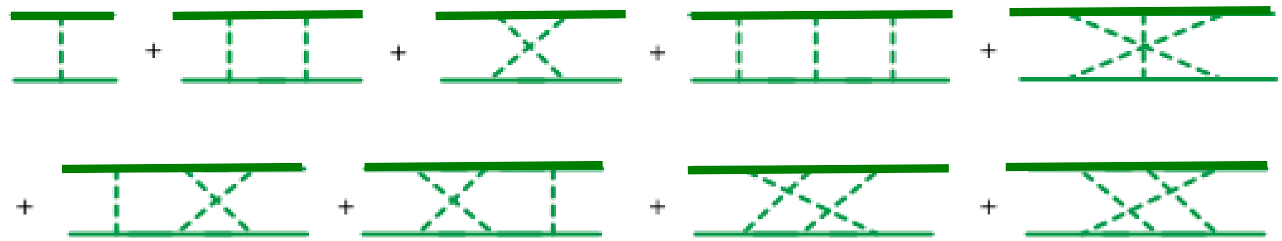


CST: must explicitly (anti)symmetrize the kernel

BS and CST are equivalent when both are solved exactly

- To 6th order, the generalized ladder sum is M > m

M
m



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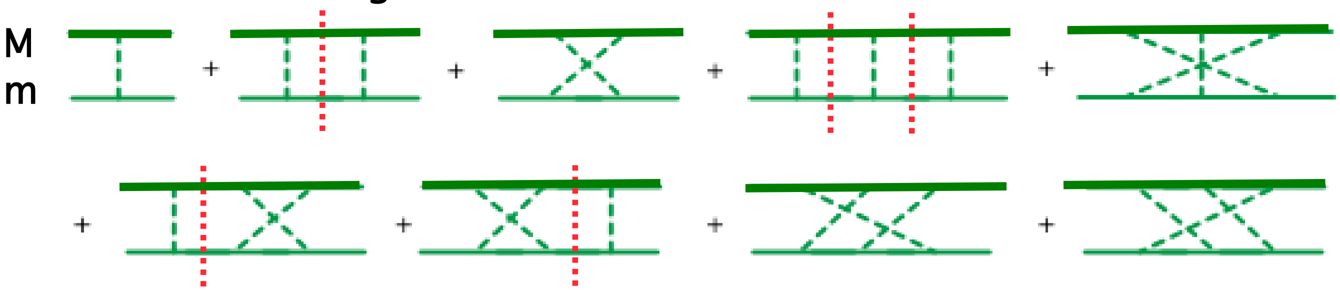
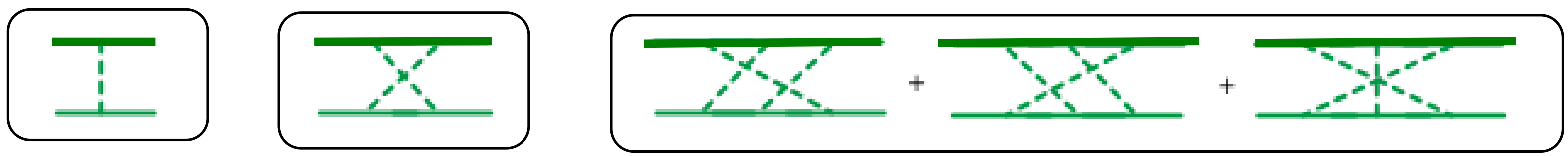


- In the BS theory, these terms require the following *irreducible* kernel:



2nd order ***4th order*** ***6th order***

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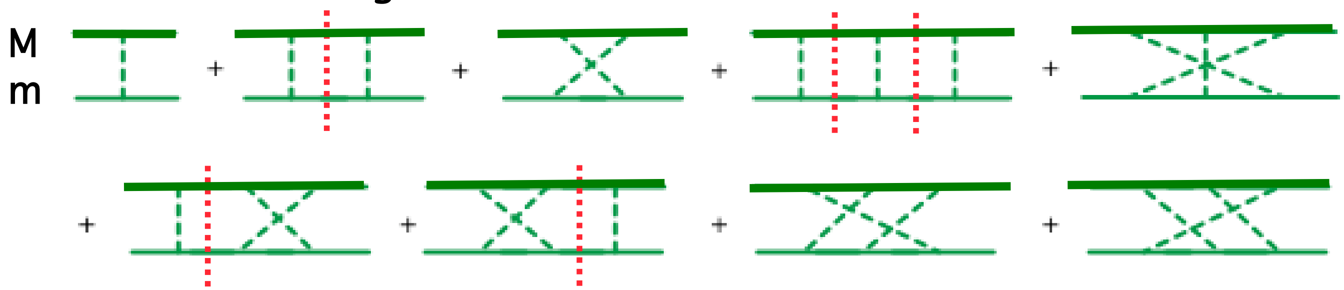

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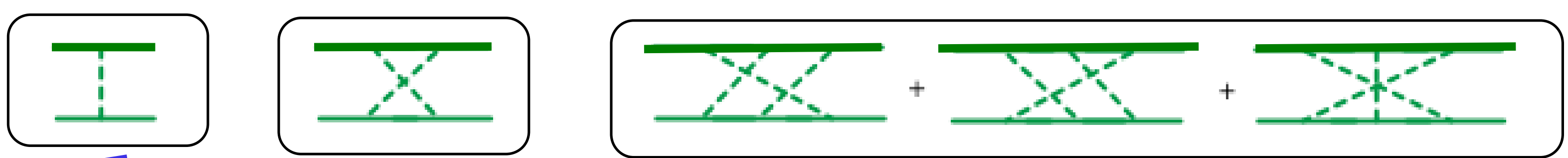
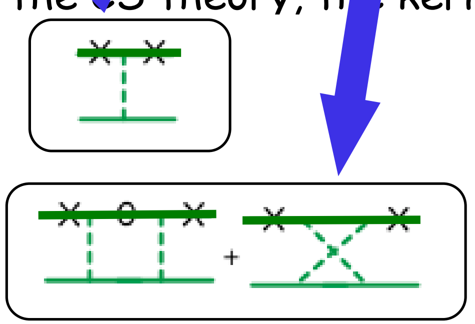
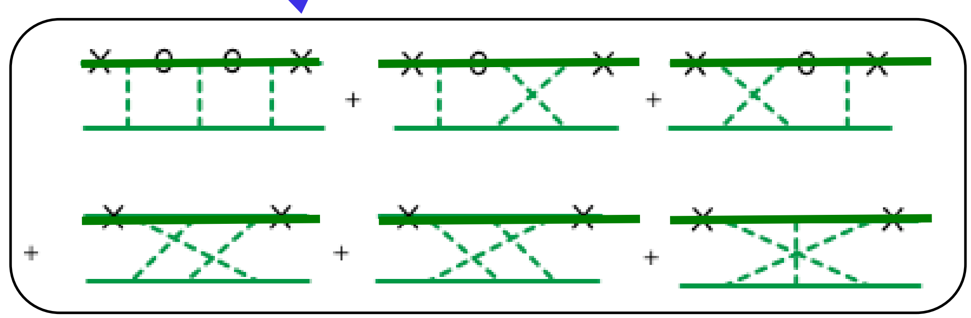


2nd order *4th order* *6th order*

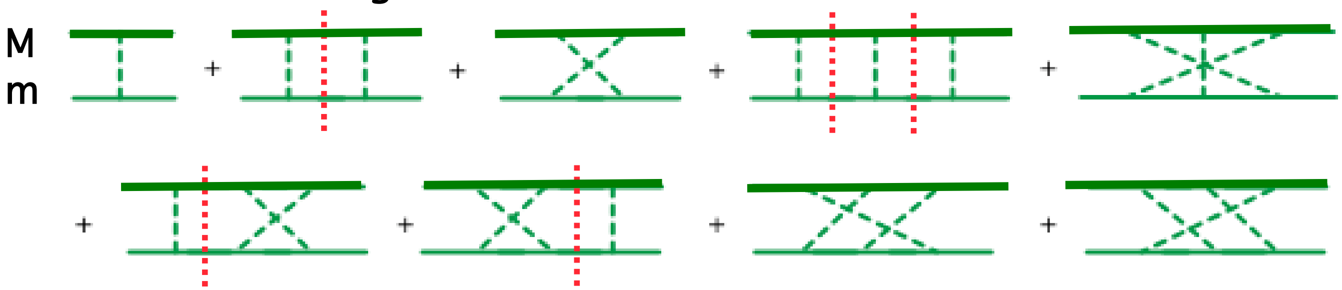
- In the CS theory, the kernel is





BS and CST are equivalent when both are solved exactly

★ To 6th order, the generalized ladder sum is M > m



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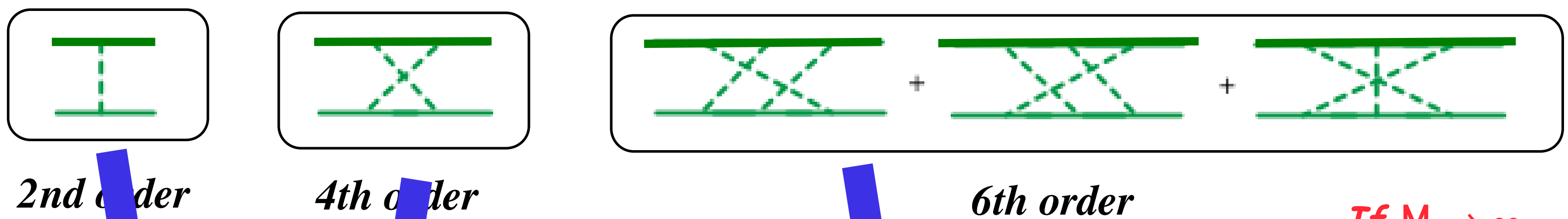
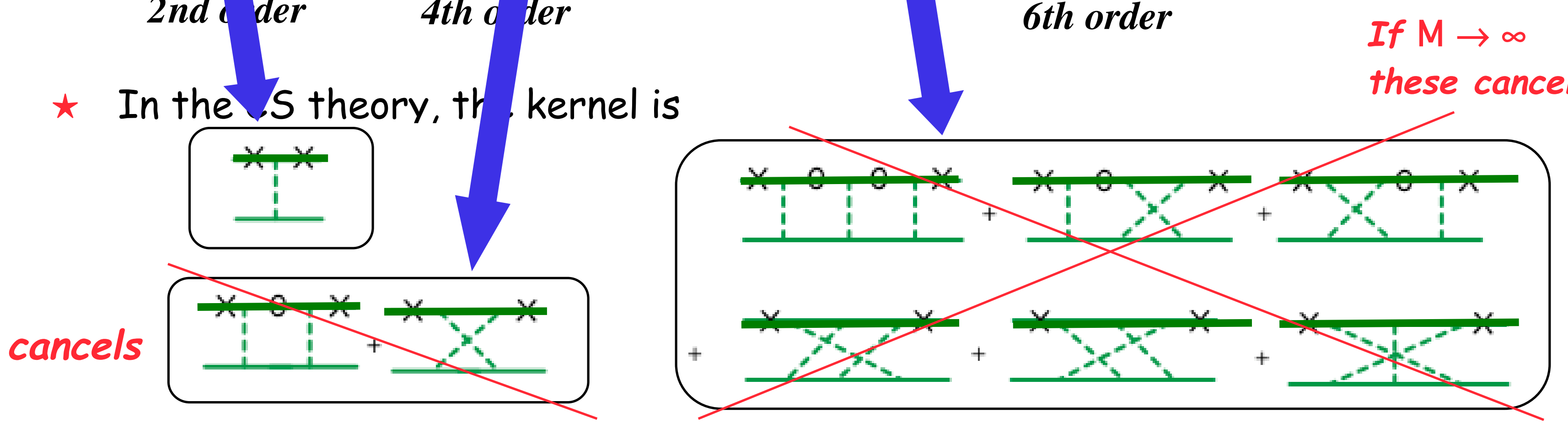


★ In the CST theory, the kernel is

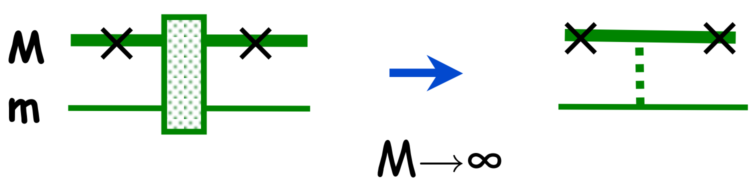


The One-Body limit

★ The One-Body limit is the following principal:

> *A relativistic equation for the interaction of heavy scalar particle (mass M) with a light particle (mass m) must reduce, in the limit of $M \to \infty$ to*
>
> - a Dirac equation for the light particle, if it has spin 1/2;
> - a Klein Gordon equation for the light particle, if it has spin 0

★ The CS equation has this limit, and in fact



★ The BS equation (in ladder approximation) does not.

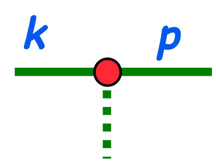
The debate: BS vs CS (an example only)

- ★ Franz: I have the one-body limit!
- ★ John: That is true only if one particle is scalar, and M » m. The physical world, and particular the NN system does not have this property.
- ★ Franz: Yes, but surely it motivates the use of my equation!
- ★ John: Your equation has spurious singularities!
- ★ Franz: Yikes! I did not know that. You must publish that (he did, and I have spent years trying to find the best way to fix this).
- ★ John: Your equation has spurious bound states when the total rest energy $W \to 0$.
- ★ Franz: Yes, but it is not meant to be used in that region, and if it is, this can be fixed by adding more channels (as was done in applications to the study of the pion as a qq system in the $m_\pi \to 0$ limit).

Applications to the NN and 3N systems (selected results)

- CST OBE model gives an excellent quantitative fit to the NN data below 350 MeV
  - χ^2/datum ~ 1 (1.06) for the 2007 np data set (3788 data points)
  - simplest model (WJC-2) with only 15 parameters fits with χ^2/datum =1.12
  - interesting differences from the Nijmegen 93 phase shift analysis -- we have a new phase shift analysis!
  - we use off-shell couplings, for example the σNN coupling is:

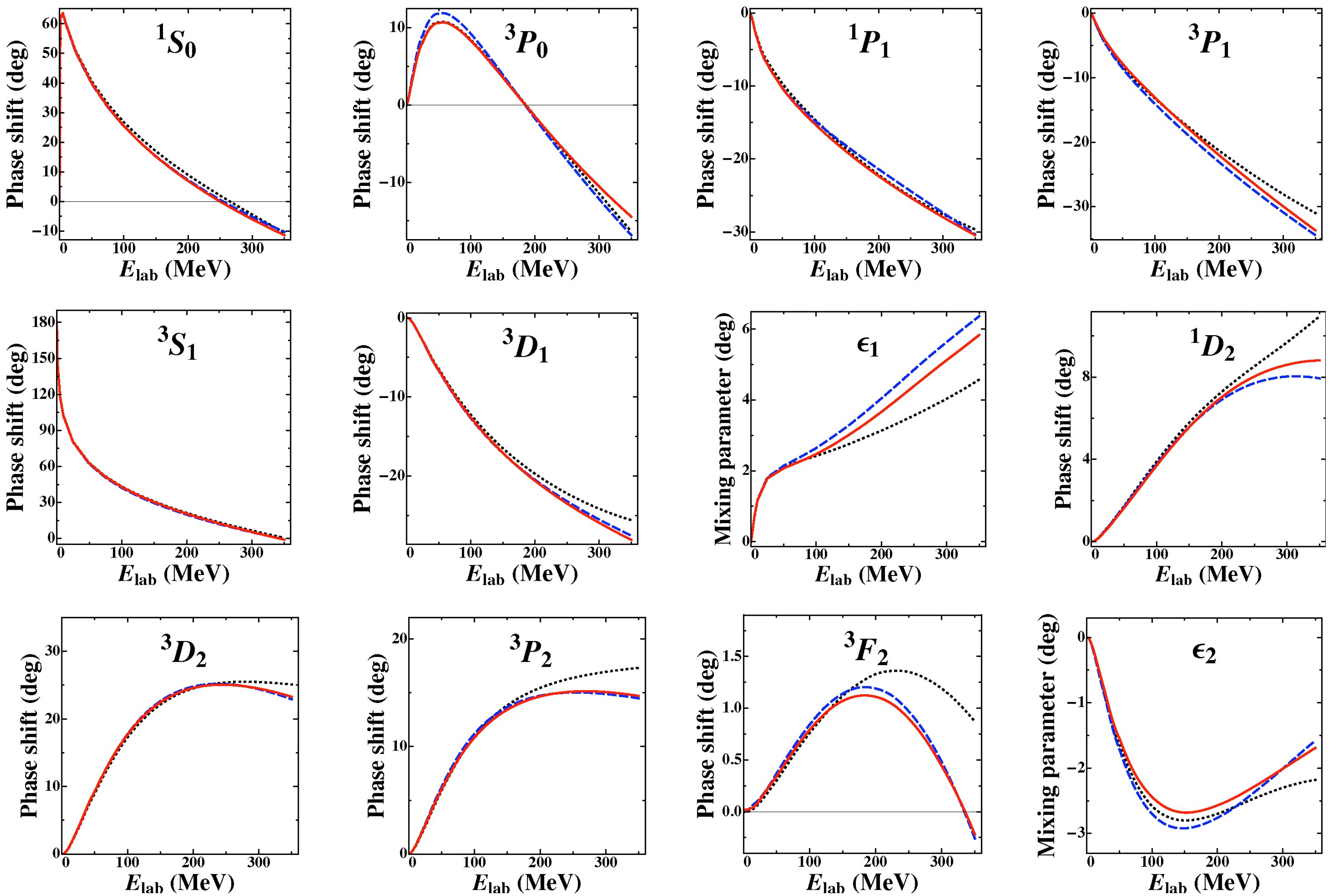
$$\Lambda(k,p) = g_s + \nu_s\left[\theta(k) + \theta(p)\right] \quad \textit{where} \quad \theta(p) = \frac{m - \not{p}}{2m} \quad \textit{vanishes on-shell}$$



- Three body binding energy
  - correctly predicted without any irreducible 3-body forces
- Deuteron form factors
  - were calculated using older models
  - give a very good explanation of the data to the highest Q^2
  - results for newer models now be calculated
- 3N form factors
  - good qualitative results but no exchange currents yet

J=0, 1, 2 Phases

WJC-1 — WJC-2 — Nijmegen PSA93

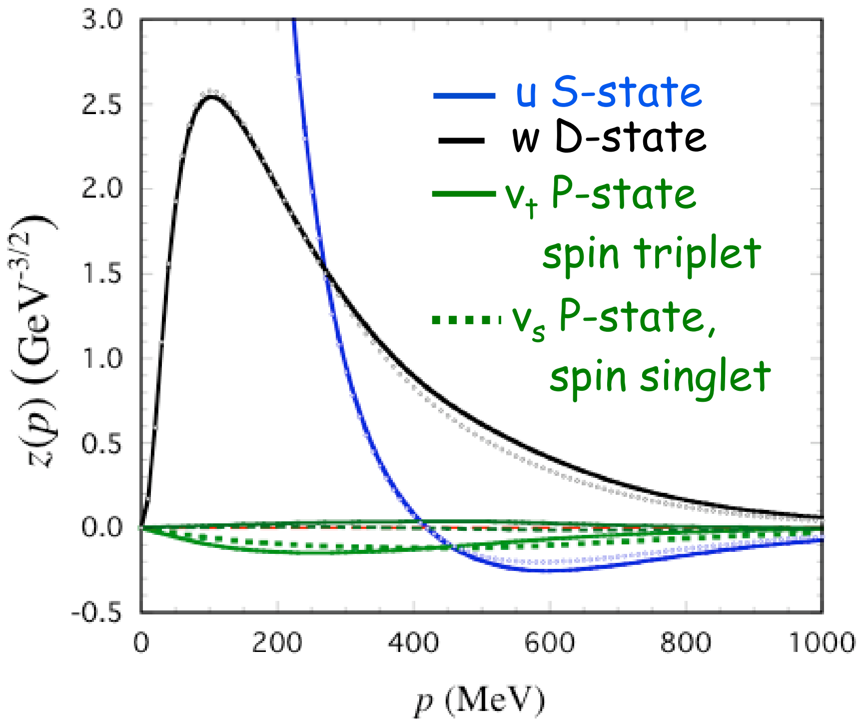


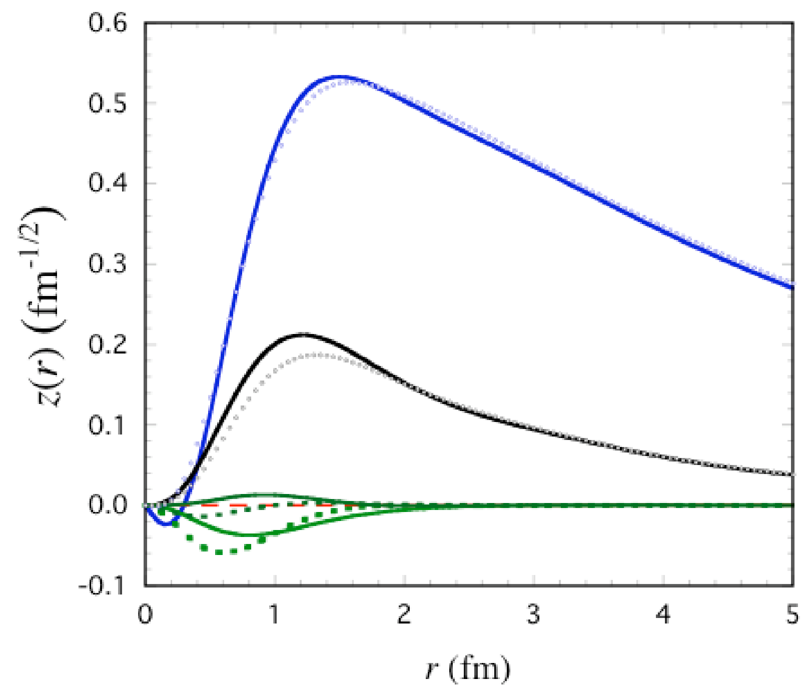
Deuteron wave functions

Probability	WJC-1 exact	WJC-1 scaled	WJC-2 exact	WJC-2 scaled
P_S	97.3876	92.3330	95.7607	93.5985
P_D	7.7452	7.3432	6.5301	6.3827
P_{Vt}	0.1180	0.1119	0.0103	0.0101
P_{Vs}	0.2234	0.2118	0.0090	0.0088
ΣP	105.4743	100.0000	102.3101	100.0000
$\langle V' \rangle$	-5.4743	-----	-2.3101	------
total	100.0000		100.0000	

Normalization condition

$$\int_0^\infty dr \left\{ u^2 + w^2 + v_t^2 + v_s^2 \right\} + \langle V' \rangle = 1$$





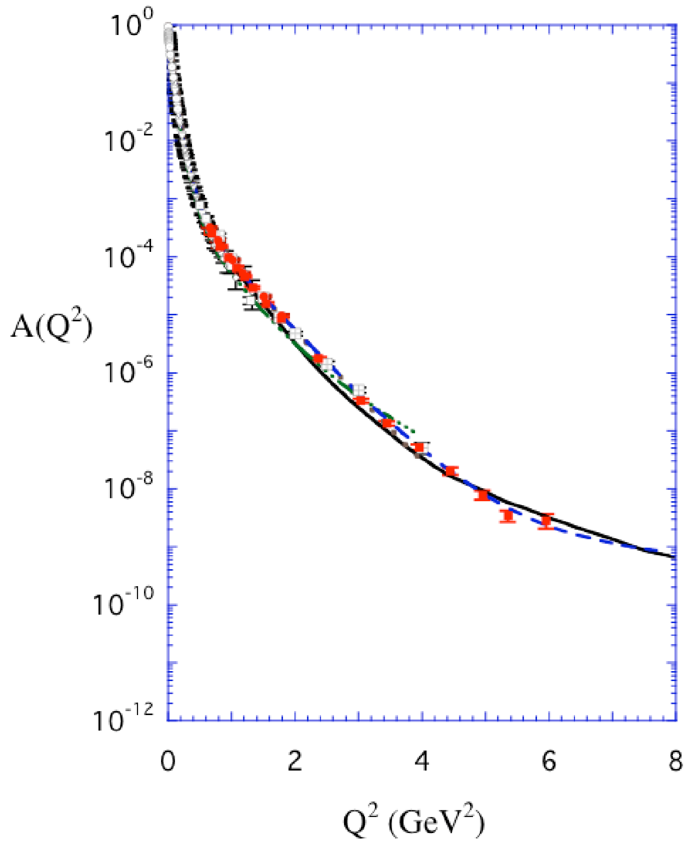
Deuteron Form Factors (comparisons)

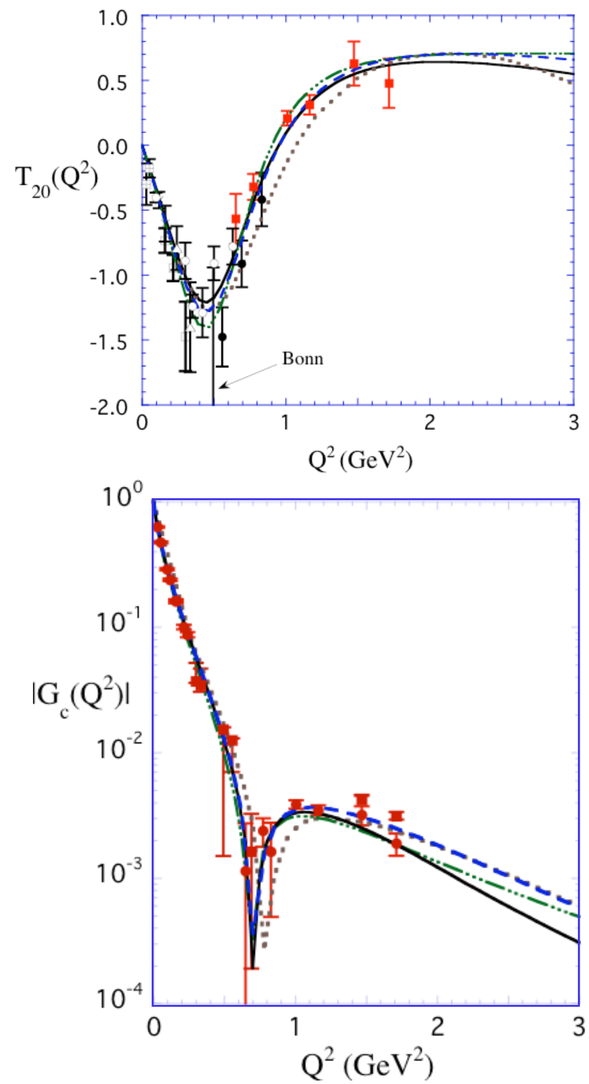
CST-IIB (van Orden, Devine, FG)

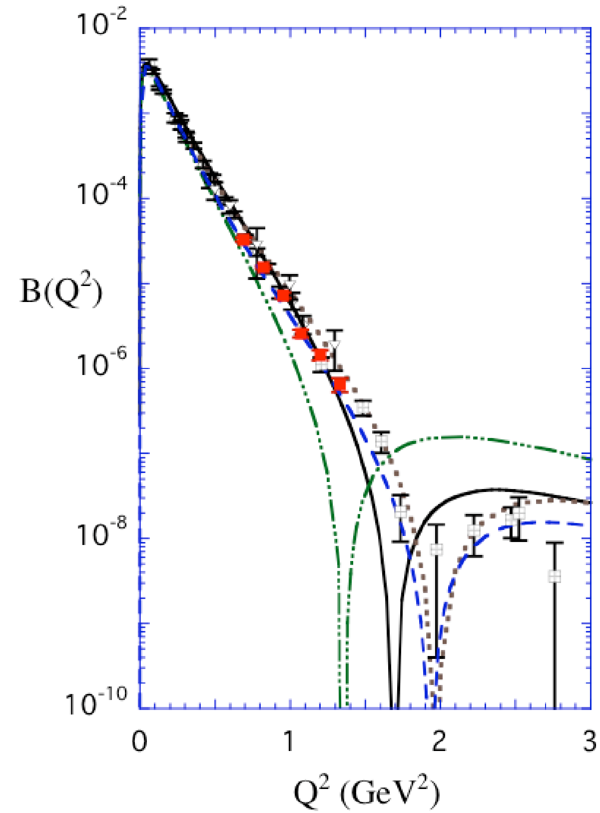
LF-HD (Huang and Polyzou)

MW (Phillips, Mandelsweig and Wallace)

QCB (Dijk and Bakker)

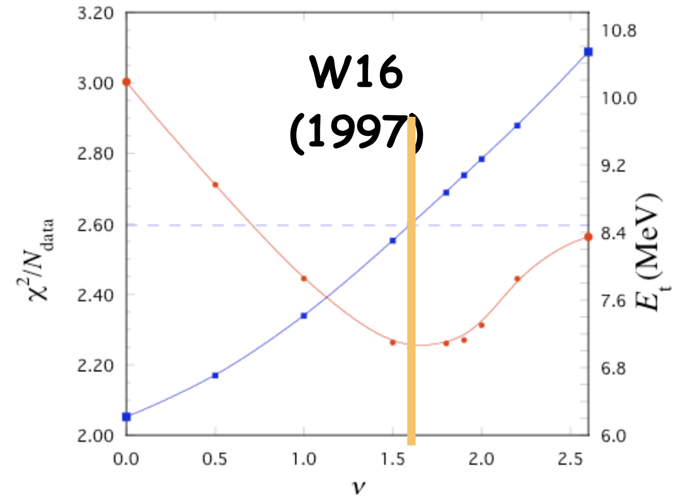






B structure function most sensitive

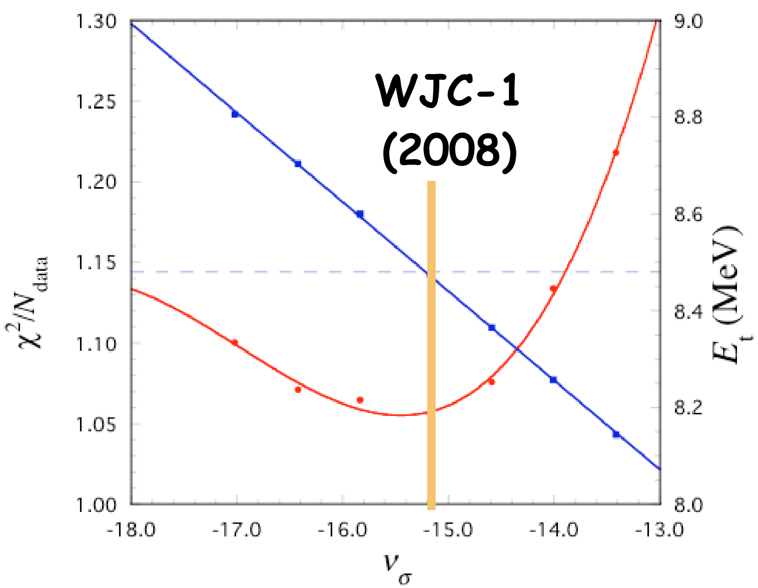
How well can the CST OBE model predict the ^{3}H binding energy?

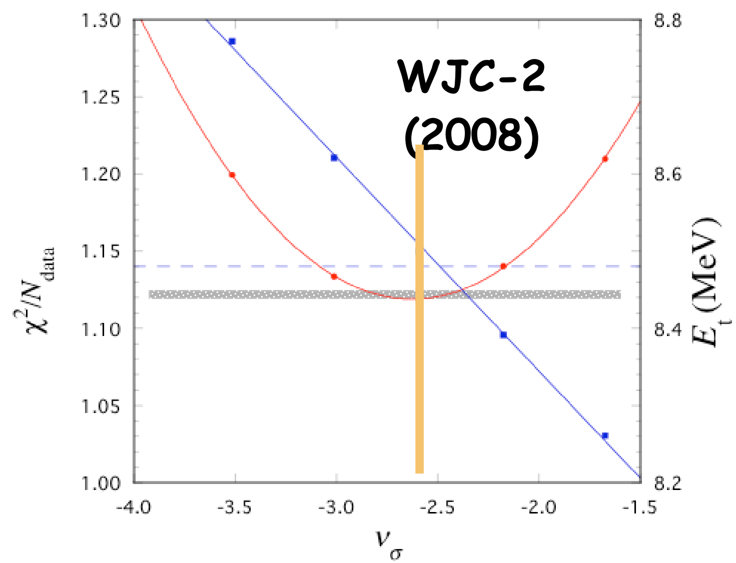


- ★ Recall the off-shell scalar coupling

$$\Lambda(k,p) = g_s + \nu_s\left[\theta(k) + \theta(p)\right]$$

- ★ In ALL three cases we found that the best value of ν_σ also gave the correct binding energy for ^{3}H !

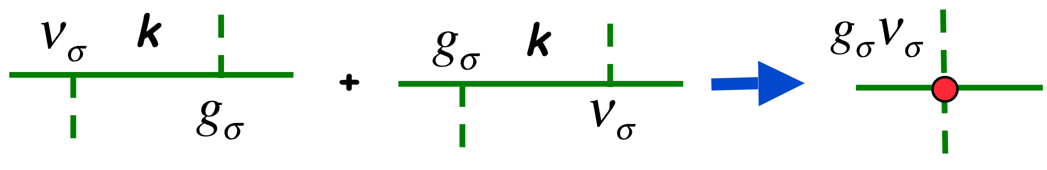




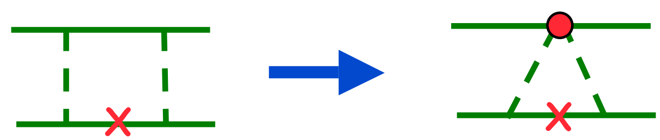
The right binding with NO irreducible 3-body force? HOW?

- ★ Off-shell couplings remove the off-shell propagator, contracting the interaction to a point

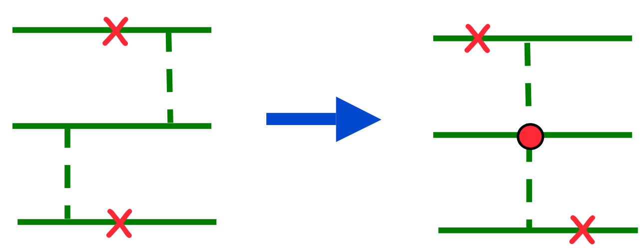
$$v_\sigma \frac{m-\not{k}}{2m}\left(\frac{1}{m-\not{k}}\right) g_\sigma + g_\sigma\left(\frac{1}{m-\not{k}}\right)\frac{m-\not{k}}{2m} v_\sigma = \frac{g_\sigma v_\sigma}{m}$$



- ★ In the 2-body space, off-shell couplings are equivalent to effective non-OBE type interactions with loops



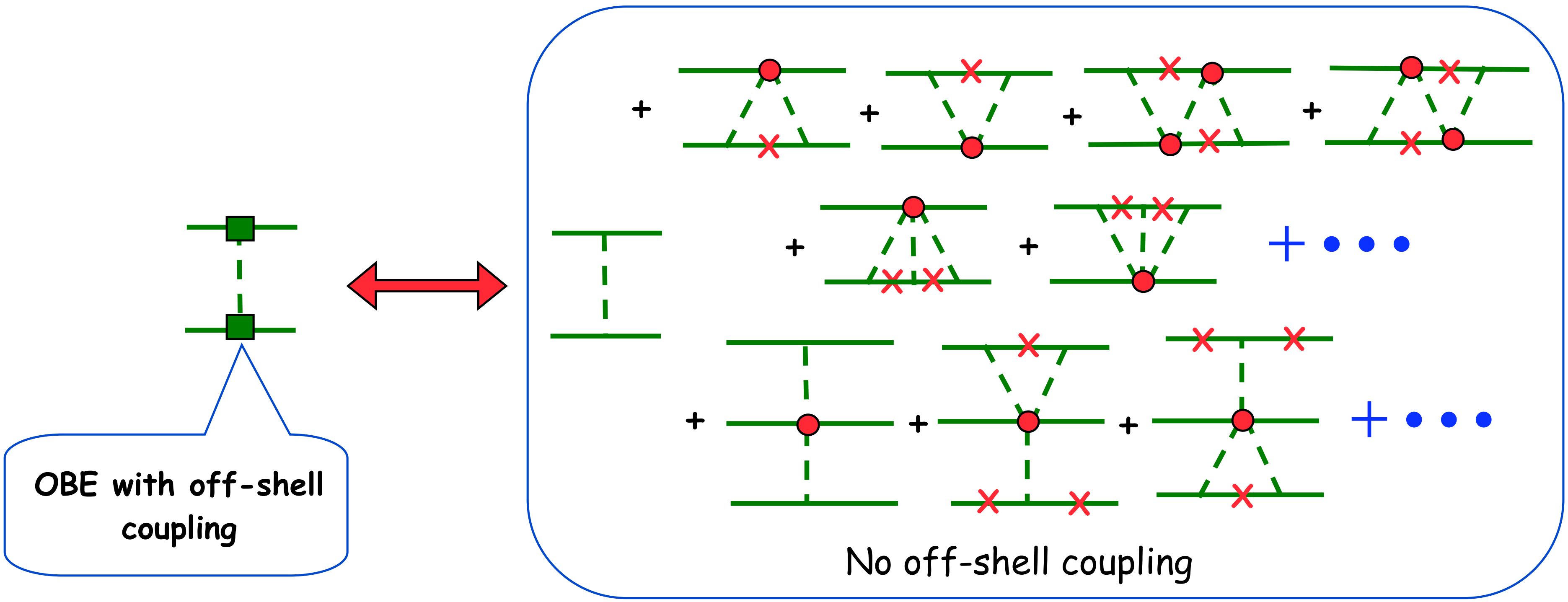
- ★ In the 3-body space, off-shell couplings are equivalent to 3-body forces



Equivalence theorem

OBE with off-shell couplings ⟺ **OBE without off-shell couplings PLUS a specific set of N-meson exchange and N-body forces**

★ The equivalence is very complicated !!



Discussion with John

- ★ John: the off-shell couplings might produce very large effects in nuclear matter

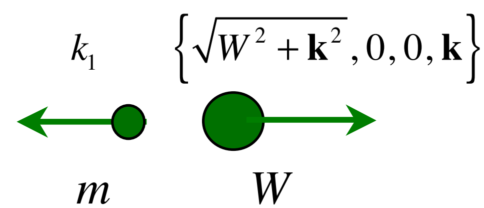
- ★ Franz: maybe a new doubly off-shell term might provide stability. The most general off-shell sigma coupling is

  $$\Lambda(k,p) = g_s + \nu_s\left[\theta(k)+\theta(p)\right] + \kappa_s\theta(k)\theta(p)$$

  The κ term has a very small effect in the NN sector, but might be important in the AN sector(?). We need a nuclear matter calculation (which is easier than the 3N calculation already done).

- ★ John: in the three body sector, when the energy of the off-shell particle gets very large, you are forced to a region where the mass W of the interacting NN pair is zero, and then imaginary.

  $$W^2 = \left(P_T - k_1\right)^2 = M_T^2 + m^2 - 2M_T\sqrt{m^2+\mathbf{k}^2}$$

  How do you handle this?

- ★ Franz: I cut off the energy integral at the critical point where the mass W goes to zero (the integrand is zero there anyway)

Non-perturbative physics in RFT (II)

lessons from the

Feynman-Schwinger calculations

(1993 - 2005)

Scalar $\chi^2 \phi$ theory -- an example

- Fields: $\chi(x)$ charged scalar particles of mass m: the "matter" field
  $\phi(x)$ massive neutral "photon" field with mass μ
- Lagrangian and Action (in Euclidean space)

$$\mathcal{L}_E(z) = \chi^* \left[m^2 - \partial^2 + g\phi \right] \chi + \frac{1}{2}\phi\left(\mu^2 - \partial^2\right)\phi$$

$$S_E[\phi,\chi] = \int d^4 z \, \mathcal{L}_E(z)$$

- Two-body Green's function for the transition from an initial state $\Phi_i = \chi^*(x)\chi(x')$ to final state $\Phi_f = \chi^*(y)\chi(y')$ is

$$G\left(y, y' \mid x, x'\right) = N \int \mathrm{D}\chi^* \int \mathrm{D}\chi \int \mathrm{D}\phi \, \Phi_f^* \Phi_i \, e^{-S_E}$$

- Integrate over the "matter" fields χ and χ^*, exponentiate the propagator, and integrate over the "photon" field (after working out some subtleties) to get the final answer.

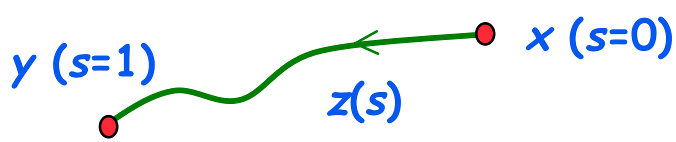
Feynman-Schwinger -- Final result

★ Assembling the final result gives

integrate over all possible trajectories of the two particles

$$G(yy'|xx') = \int_0^\infty ds \int_0^\infty ds' \iint \left[Dz\right]_{yx} \left[Dz'\right]_{y'x'} \exp\left\{-K(z,s) - K(z',s') + \langle W(C)\rangle\right\}$$

$$\begin{array}{lll} z(s): & z(0) = x; & z(1) = y \\ z'(s'): & z'(0) = x'; & z'(1) = y' \end{array}$$



with

$$K(z,s) = m^2 s + \frac{1}{4s}\int_0^1 d\tau\, \dot{z}^2(\tau)$$ (kinetic energy term)

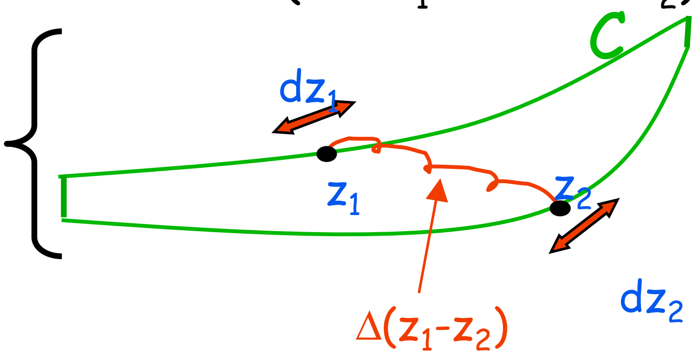
$$\langle W(C)\rangle = \tfrac{1}{2} g^2 ss' \int_C d\tau \int_C d\tau' \Delta(z(\tau) - z'(\tau'), \mu)$$ (interaction term)

$$\Delta(x,\mu) = \int \frac{d^4 p}{(2\pi)^4} \frac{e^{ip\cdot x}}{p^2 + \mu^2} = \frac{\mu}{4\pi^2 |x|} K_1\left(\mu |x|\right)$$ ("photon" propagator)

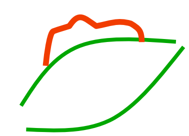
Feynman-Schwinger -- interpretation of final result

Interpretation of the result ($z \rightarrow z_1$ and $z' \rightarrow z_2$)

$$\int [Dz_i]\; e^{-\{\;\cdots\;\}} = e^{-C1} + e^{-C2} + \ldots$$



(i) The action is integrated over ALL closed trajectories *C*



(ii) Because of the exponent, all orders are computed.

(iii) Self-energies come if z_1 and z_2 are limited to the same "side" of *C*.

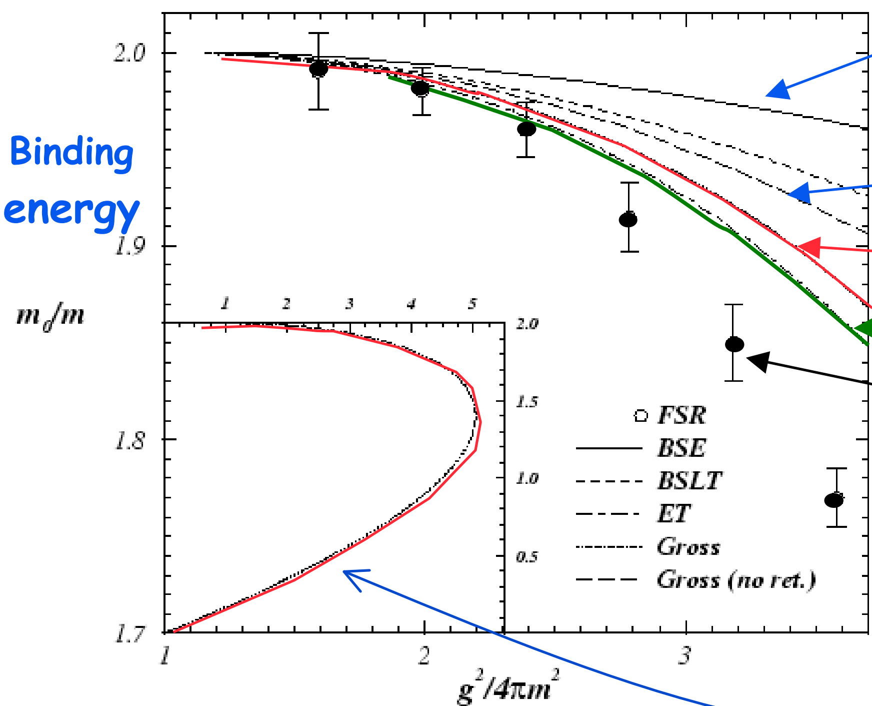
(iv) Exchange interactions come if z_1 and z_2 are limited to different "sides" of *C*. This gives the sum of all ladders and crossed ladders.

(v) If z_1 and z_2 are unrestricted the result gives self-energy, exchange, and vertex corrections (everything).

Results 1: ladders and crossed ladders in scalar $\chi^2\phi$ theory

★ The generalized ladder sum in scalar $\chi^2\phi$ theory can be evaluated exactly

$m_1 = m_2$



Bethe-Salpeter equation in ladder approximation

CS (without retardation)

CS (with retardation)

Tjon's equal time approximation

Exact

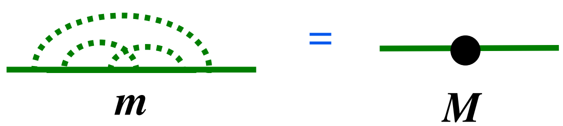
The strange behavior of the CS equation when $m_0 \to 0$ is due to a insularity in the propagator (removed when a second channel is added).

★ Crossed ladders are important

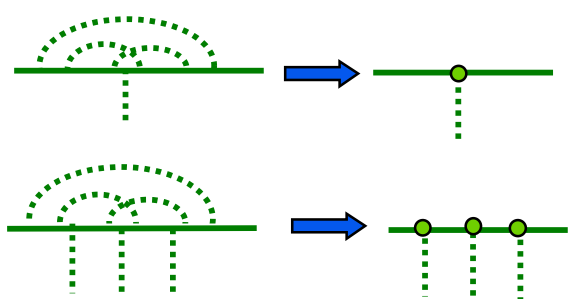
*Taco Nieuwenhuis, Ph. D. thesis; PRL 77 (1996) 814
Cetin Savkli, FG, and J. Tjon, Phys.Atom. Nucl. 68 (2005) 842

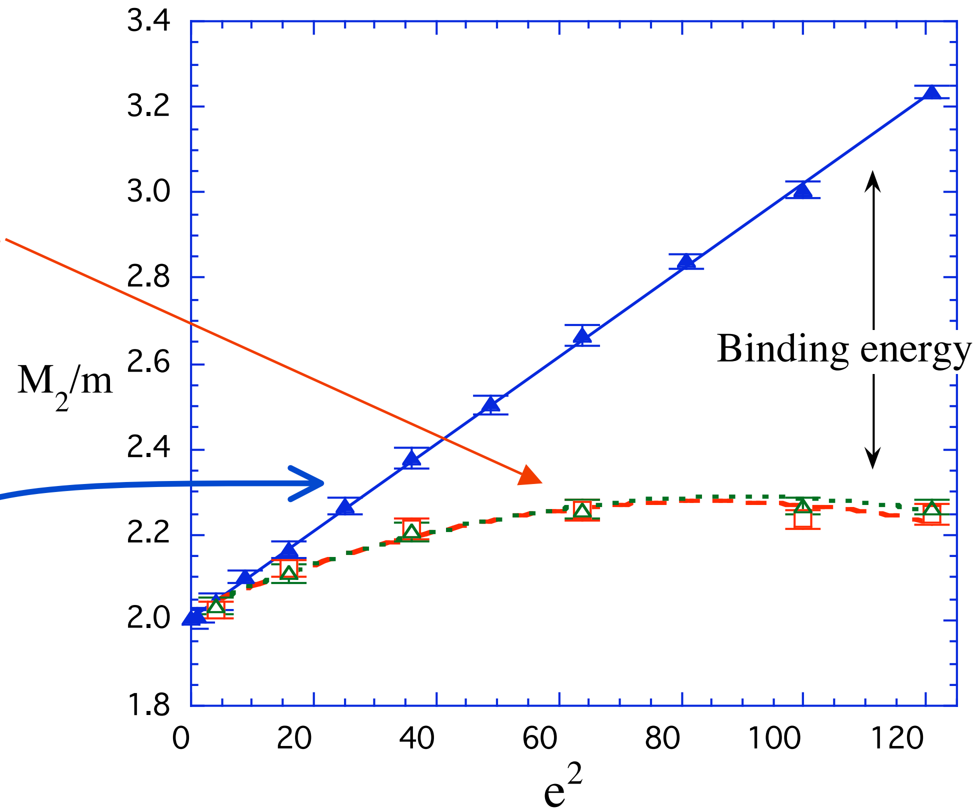
Results 2: Cancellations of vertex corrections in SQED*

- ★ Two results are equal:
  - *exact* answer, and
  - sum of generalized ladders with *constant* dressed mass and *no* vertex corrections
- ★ Dressed mass is linear in e^2



- ★ vertex corrections and self energies cancel





results for 1+3 dimensions and μ/m =0.15

*Cetin Savkli, FG, and John Tjon, PLB **531**, 161 (2002)

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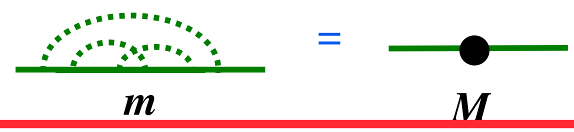

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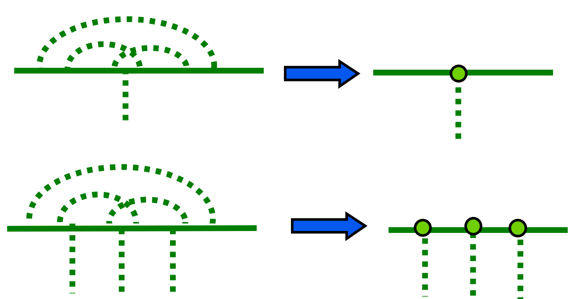

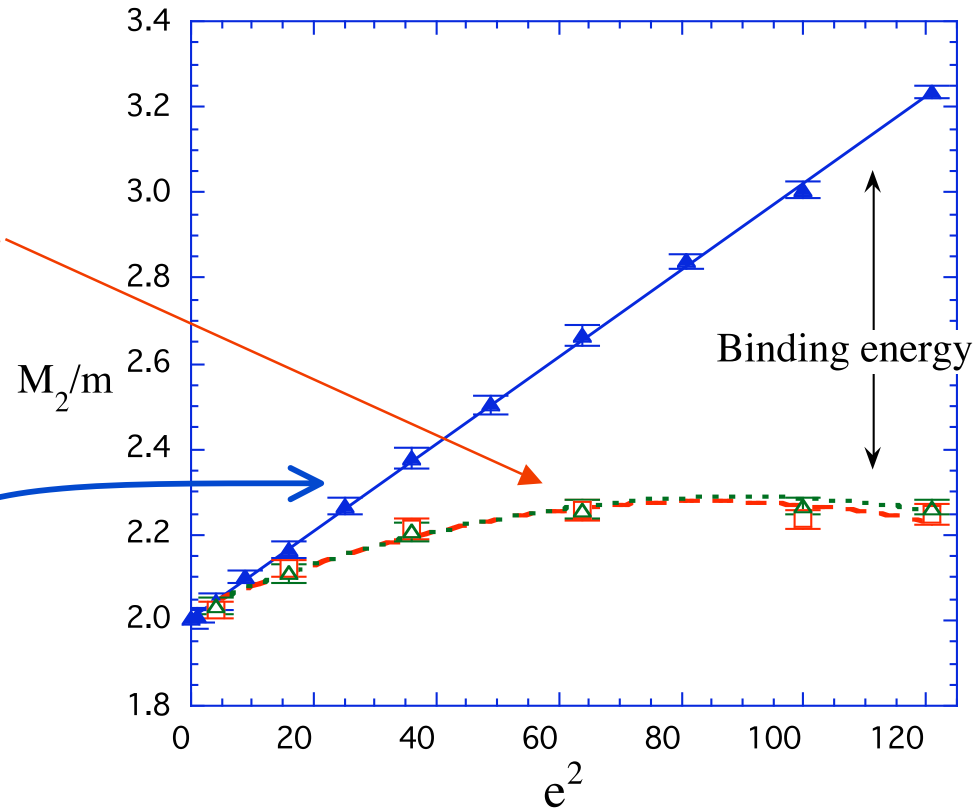

results for 1+3 dimensions and μ/m =0.15

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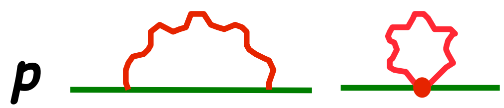
*Cetin Savkli, FG, and John Tjon, PLB **531**, 161 (2002)

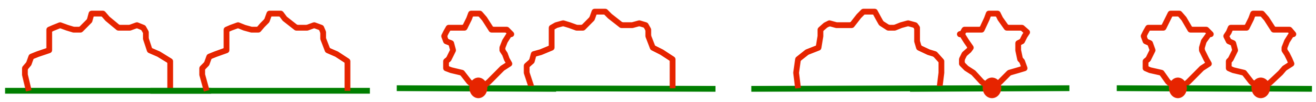
Result (that dressed mass is linear in e^2) is remarkable!

★ One body propagator - dressed masses up to 4th order $\Sigma(p^2)$

2nd order: all irreducible

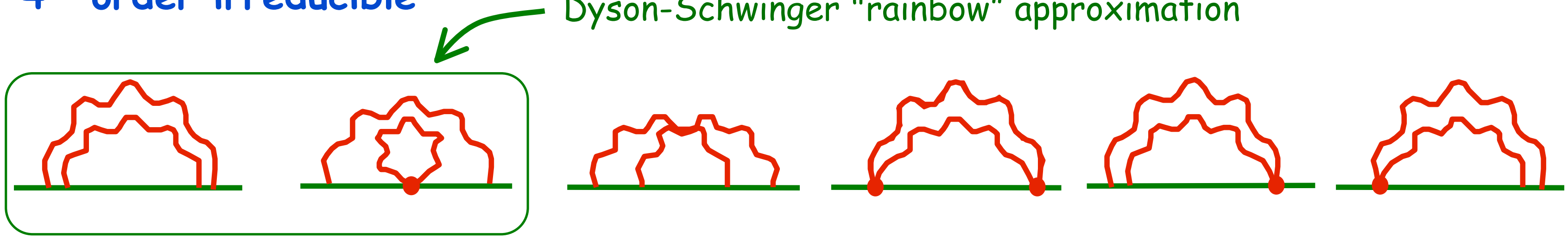


4th order: reducible (iteration of 2nd order)



4th order irreducible

Dyson-Schwinger "rainbow" approximation



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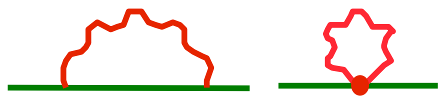
p



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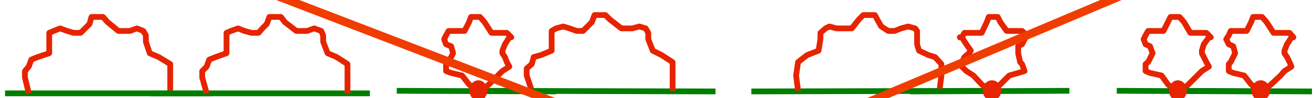

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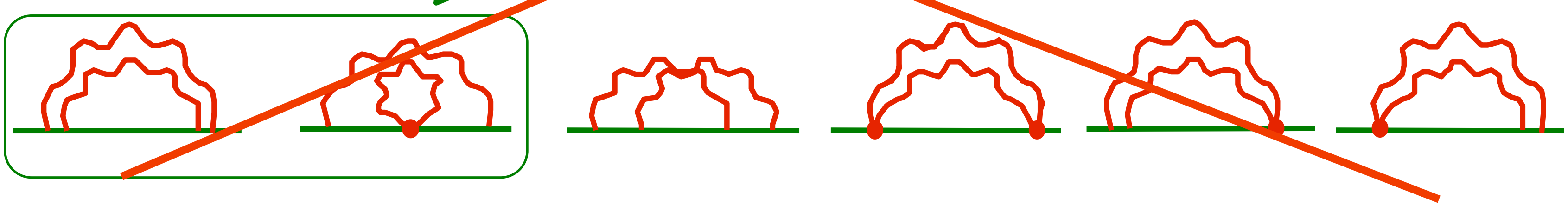
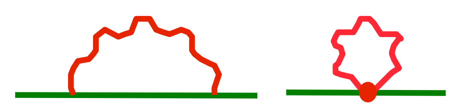


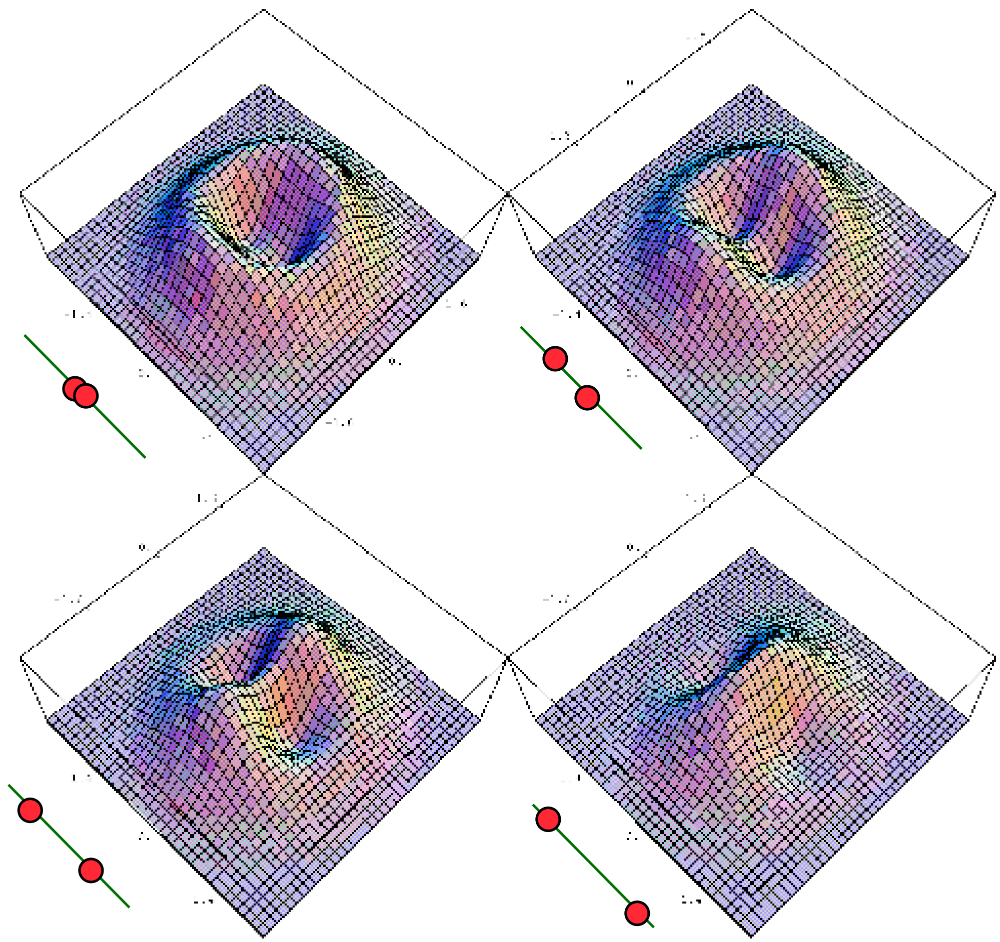
★ Forget the partial sums! The best result is simply $\Sigma(m^2)$

$$\Sigma(m^2) = \lim_{p^2 \to m^2} \{ \quad \}$$



Result 3: Three-body bound state wave functions*

- Bound state wave function can be determined by looking at the distribution of probabilities at $t = t_{final}$
- These figures show the distribution of the third particle when the other two are fixed at various separations along the z axis
- This work was just started -- there is a LOT more to do!



*C. Savkli, Czech.J.Phys. **51**, B71 (2001)

Insights from FS calculations

- Other results
  - famous instability of $\chi^2\phi$ type theories is due to the production of an infinite number of $\chi\bar{\chi}$ loops and hence generalized ladder sums, including Z diagrams, are not unstable.
- Great insight from the exact FS calculations
  - Many cancellations occur between different perturbative diagrams; it is dangerous to ignore these cancellations:
    - crossed ladders and crossed bubbles are important
    - vertex corrections are less important
  - Quasi-potential equations can represent RFT, at least in certain cases
- MUCH to be done -- an excellent area for new study
  - spin 1/2 particles
  - many body systems (three body forces?)
- In my view, one of John Tjon's signature programs; untimely interruption !
- There is still room for creative new ideas on how best to treat non-perturbative physics in RFT

Conclusions

- ★ The practical solution of nonperturbative problems in RFT is very challenging
- ★ John Tjon was a major force in understanding the issues, and made substantial contributions
  - His practical program of Feynman-Schwinger calculations is certainly among his most original contributions to this area
- ★ The use of quasi-potential equations, either his ET or my CS, are both very effective, practical ways to address these problems.
- ★ However, we probably can do better! (Some would say light cone methods are the answer, but …)
- ★ He is greatly missed
  - He showed me many ways to calculate more efficiently
  - He always provided an honest assessment, and his insight was helpful
- ★ I thank the organizers for recognizing him at this conference.