Paving the Relativistic Way: Past, Present, Future

ETC* workshop, October 19, 2009

Franz Gross

JLab and W&M

- ★ Part I -- Past: selected accomplishments of the last 60 years
- ★ Part II -- Present: what do we know; what is the state-of-the-art?
- ★ Part III -- Future: where should we go from here?

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If this field is to realize its full potential, we MUST learn to understand and compare different approaches. This workshop provides an opportunity!

Part I -- Past: selected accomplishments of the last 60 years

- ★ Non-relativistic nuclear physics matures
  - Precision description of the nuclear force with OPE
  - Explanation of the mass spectrum of nuclei with $A \leq 12$
  - Precision description of three-body scattering; the A_y puzzle
- ★ Chiral effective field theory

Discuss now

- ★ Development of Hamiltonian dynamical schemes
  - Three methods based on Dirac's famous (1949) paper -- 60 years
  - Field theory NOT essential for relativistic quantum mechanics
  - Emergence of light-front quantum mechanics as a powerful and practical technique for describing physics at all energy scales
- ★ Progress with hadronic field theory
  - Bound states: require a nonperturbative approach
  - Introduction of the Bethe-Salpeter equation and new methods for its solution
  - Introduction of Quasipotential methods; quantitative application of the Covariant Spectator Theory (CST) to two and three body problems
  - Treatment of current conservation in the presence of composite systems

Discuss in Part II

Non-relativistic nuclear physics matures

-- Precision description of the nuclear force below 350 MeV (1)

- Potential models have been found that give essentially perfect fits to NN data

Models			χ^2/Ndata(Ndata)		
Ref.	#	year	1993	2000	2007
PWA93	39	1993	0.99(2514)		
			1.09(3011)	1.12(3336)	1.13(3788)
Nijm I	41	1993	1.03(2514)		
AV18	40	1995	1.06(2526)		
CD-Bonn	43	2000	--	1.02(3058)	

3 data sets
-- 1993 (PWA)
-- 2000 (Bonn)
-- 2007 (WJC)

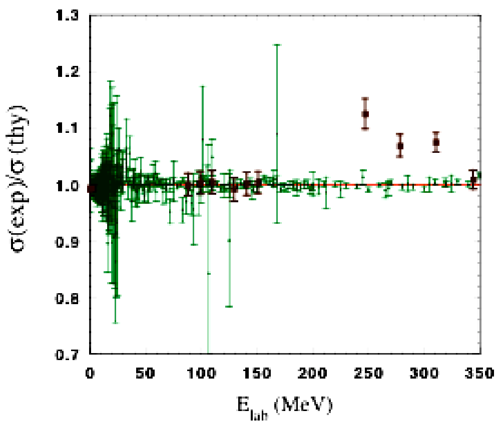
Red χ^2 for 2007set

- Relativistic models (WJC is just as good)

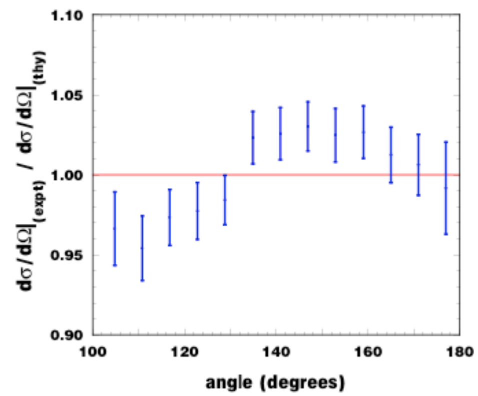
WJC-1	27	2007	1.03(3011)	1.05(3336)	1.06(3788)
WJC-2	15	2007	1.09(3011)	1.11(3336)	1.12(3788)

-- Precision description of the nuclear force below 350 MeV (2)

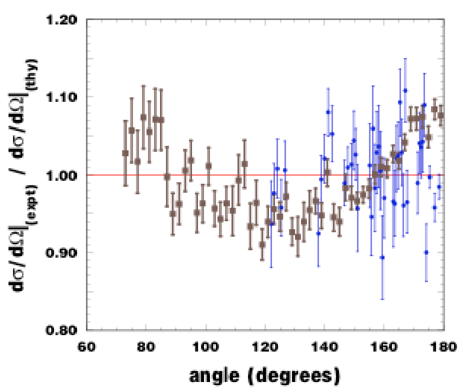
- Fits to the data are excellent; all data shown are scaled by the fit; some data with large systematic errors is excluded



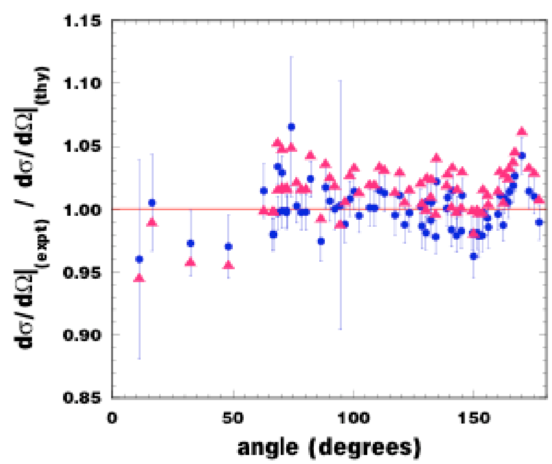
total cross section; entire energy range



194 MeV differential cross section

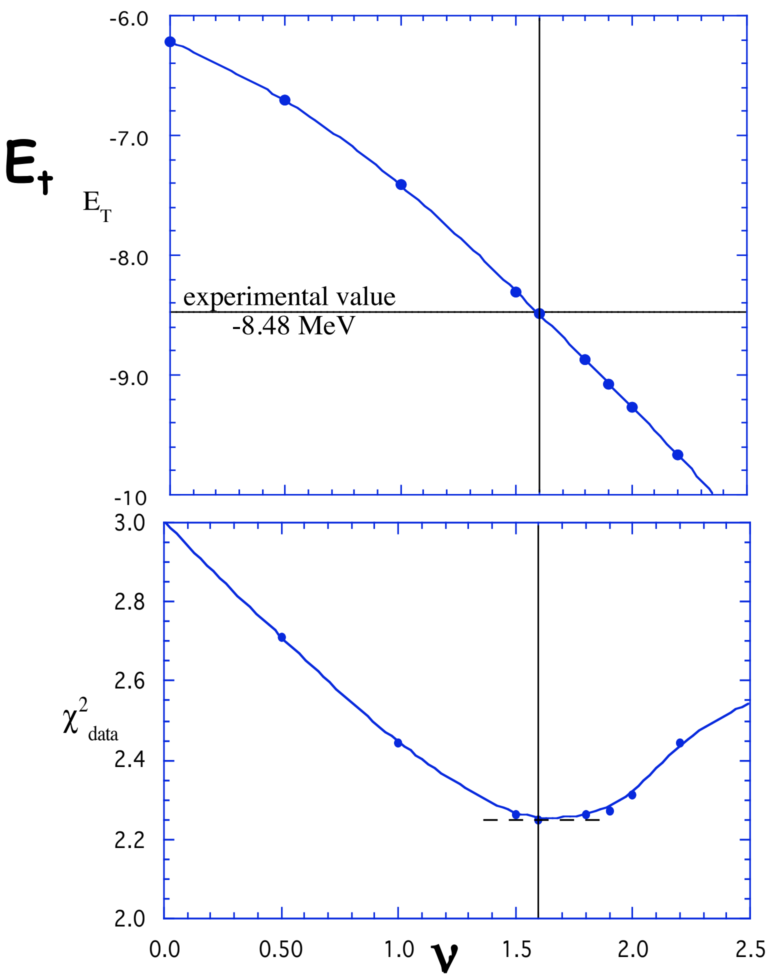


162 MeV differential cross section; brown data excluded



319 MeV differential cross section; shows scaling permitted by systematic errors

Relativistic effects in ^3H binding*



It turns out that the relativistic calculation of the three body binding energy is sensitive to a new, relativistic off-shell coupling (described by the parameter ν). Non-zero ν is equivalent to effective three-body (and n-body forces).

*three body calculations FG and Alfred Stadler, Phys. Rev. Letters **78**, 26 (1997)

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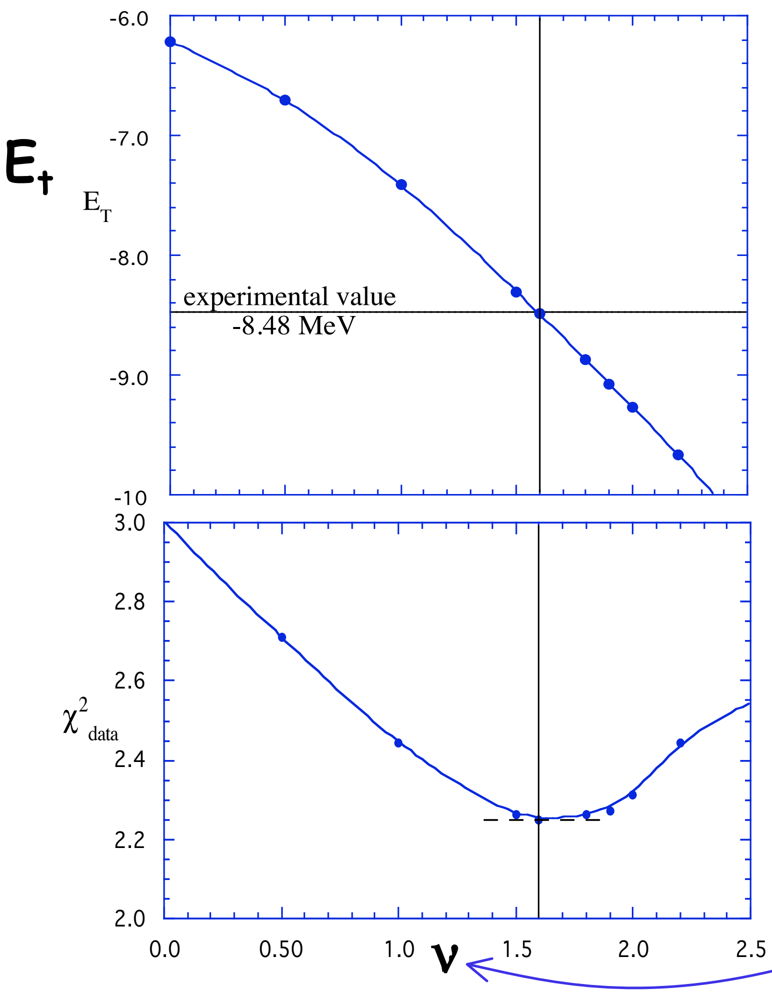

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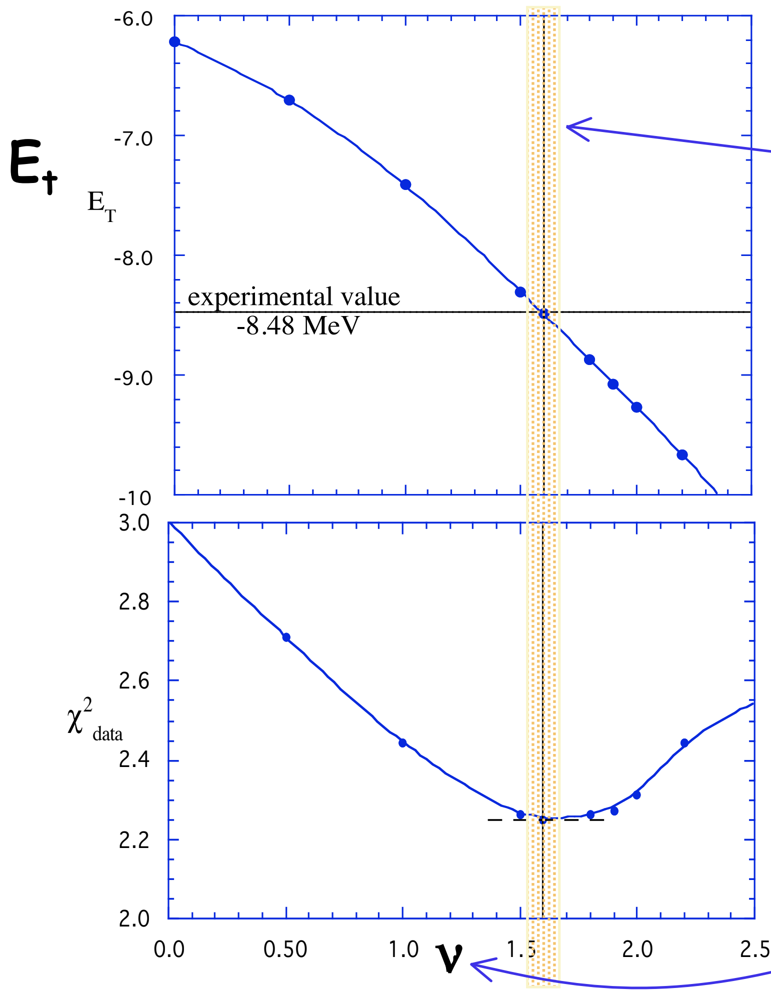

The value of ν that gives the correct binding energy is close to the value that gives the best fit to the two-body data!

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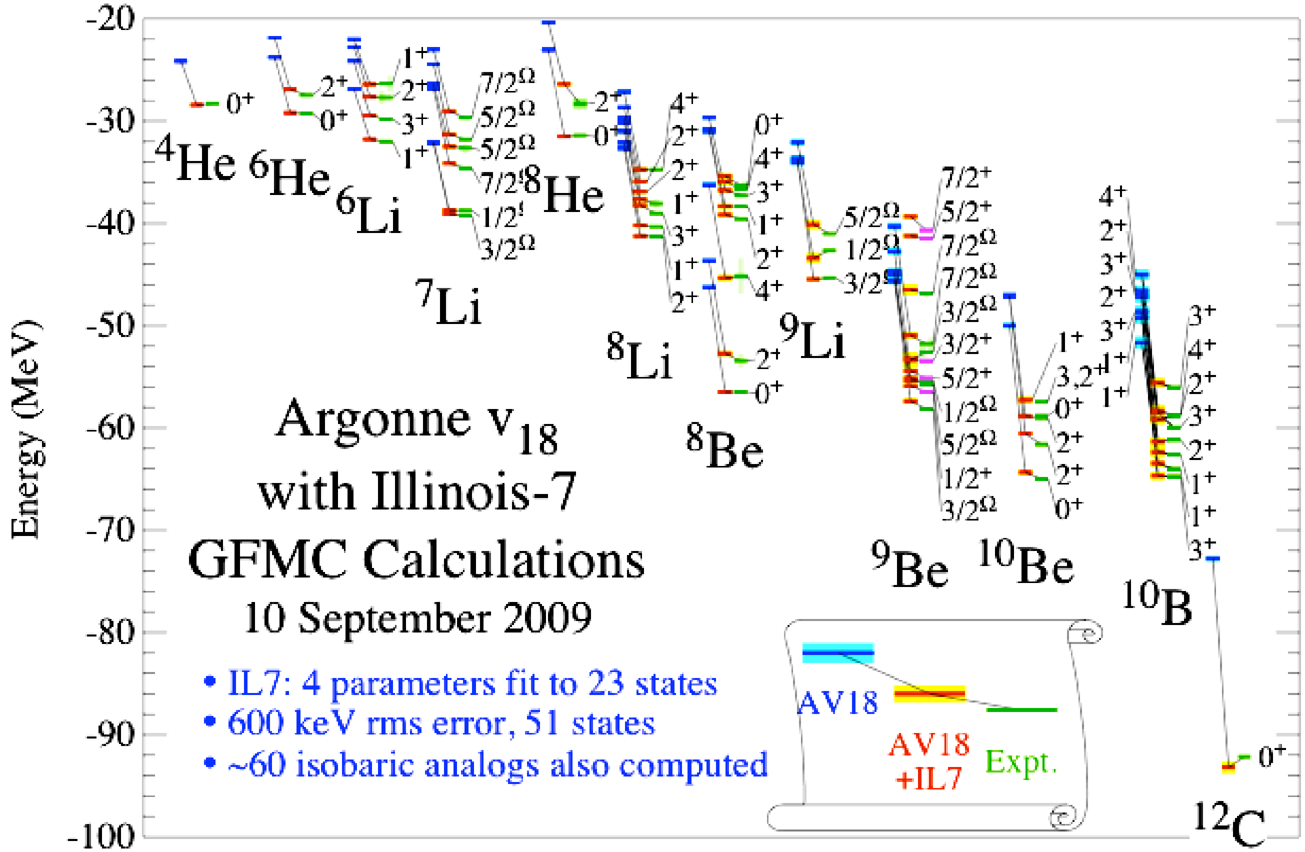
Non-relativistic nuclear physics matures

--Explanation of the mass spectrum of nuclei with $A \leq 12$*

★ Greens function Monte Carlo (GFMC) shows good agreement with 51 states for $A \leq 12$



*from Bob Wiringa (winner of the Bonner prize, together with Steve Pieper), Oct. 11, 2009

Non-relativistic nuclear physics matures

-- Precision description of 3-body scattering (slides from 1995)

- ★ Famous calculations first presented by Glöckle and collaborators*
  - many three body observables agree with data to excellent precision
  - all precision potential models agree
  - full Faddeev calculations needed
- ★ Example: excellent agreement for *pD* elastic scattering at 6, 16, and 22.7 MeV
- ★ Disagreement in *nD* analyzing power (A_y) a "puzzle" only because of the precision of other calculations
- ★ Still problems with some breakup observables. Worst case: breakup p+d-> p_1+p_2+n along the "S" curve (θ_1+ θ_2 fixed)>

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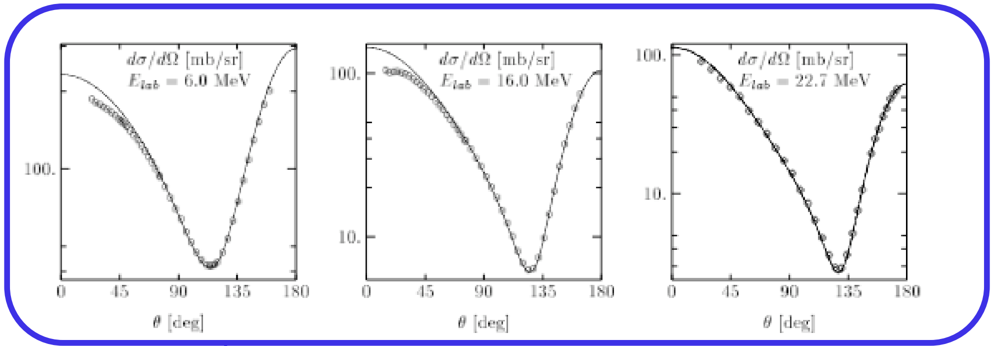

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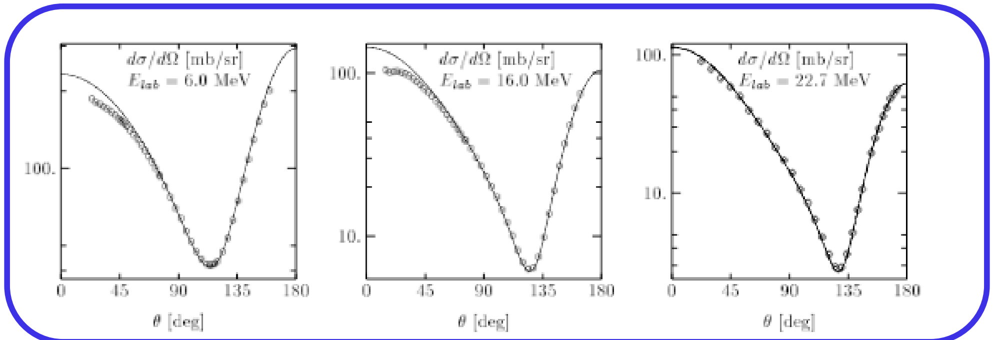

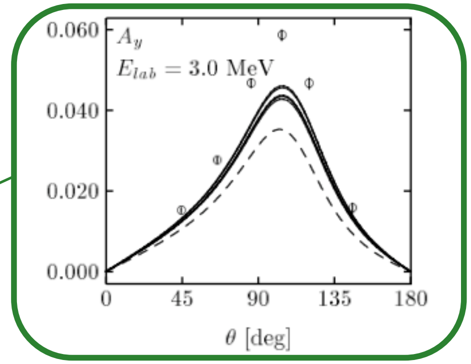

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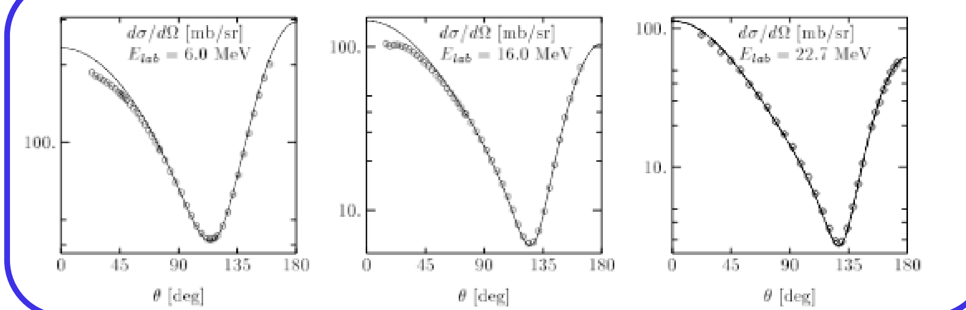

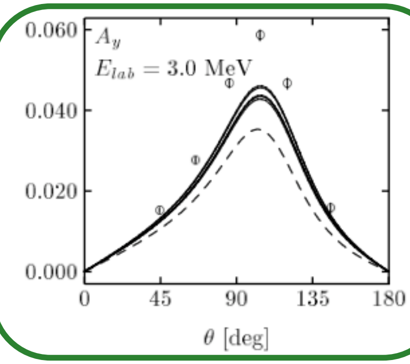

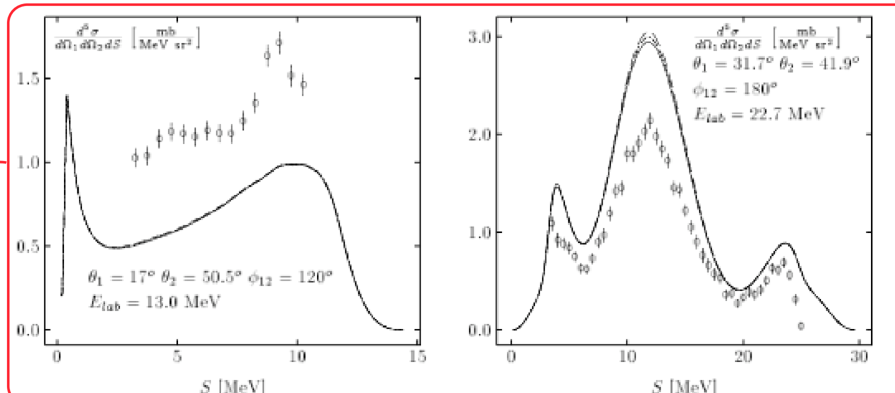

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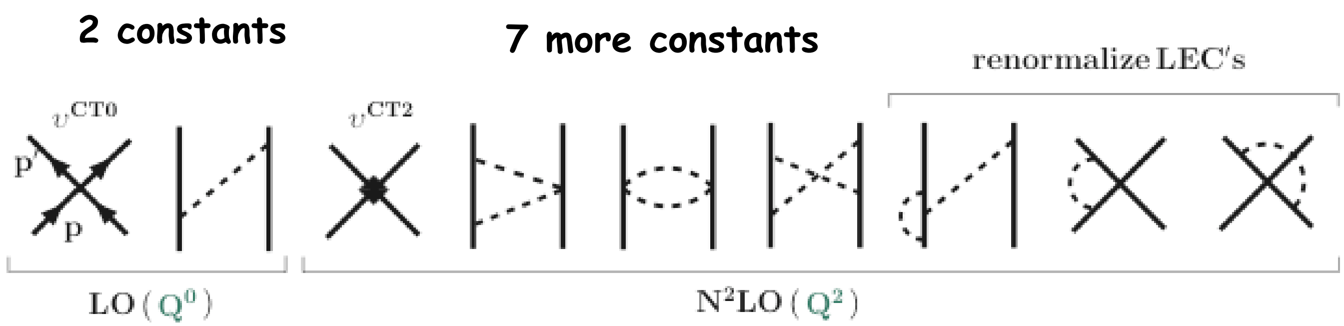
Chiral effective field theory

- Features
  - A perturbation expansion for the potential based on chiral symmetry
  - Short range physics parameterized by unknown constants
  - Power counting scheme depends on momentum scale Q
- Logical development (not historical)
  - KSW (Kaplan, Savage, and Weiss) works for the "pionless" theory.
    - leading term is $C_S + C_T\ \sigma_1 \cdot \sigma_2$ where C's are $O(Q^{-1})$
    - pion exchange is $O(Q^0)$ $V_{OPE} = -\frac{g^2}{(2m)^2}\tau_1 \cdot \tau_2 \frac{\sigma_1 \cdot \mathbf{q}\, \sigma_2 \cdot \mathbf{q}}{m_\pi^2 + \mathbf{q}^2}$
    - Bubbles involving C_S and C_T can be regularized and summed to all orders giving a generalized effective range theory. ALL cutoffs absorbed into renormalized C's. Pions were added perturbatively.
  - KSW breaks down at momenta well below the pion mass (140 MeV). For modern applications need the Weinberg counting scheme.
    - C_S and C_T demoted to order Q^0 and together with OPE, are LO.
    - NLO includes pion loops
    - Resulting potential (to any order) is inserted into the Schrödinger equation.
    - Cutoffs are needed!

Weinberg power counting

★ The power counting as used modern NN calculations:



(from R. Schiavilla)

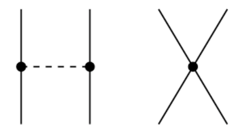
- Many more diagrams and 15 more constants at N^4LO (terms up to Q^4 - see next slide)
- Calculations sensitive to the cutoffs
- Consistent currents are complicated (Schiavilla)

★ At N^4LO, a total of 24 unknown constants, cutoffs, and still the calculations do not reproduce phase shifts well above ≈ 200 MeV!

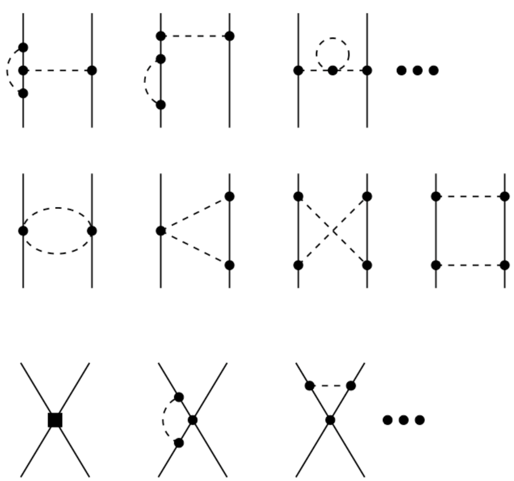
★ Still, a great intellectual advance because of close connection to QCD and possibility to estimate errors.

Diagrams to N^{4}LO*

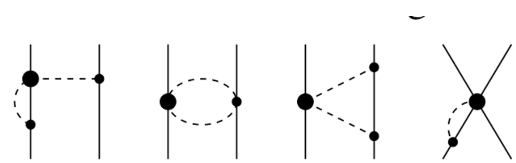
LO



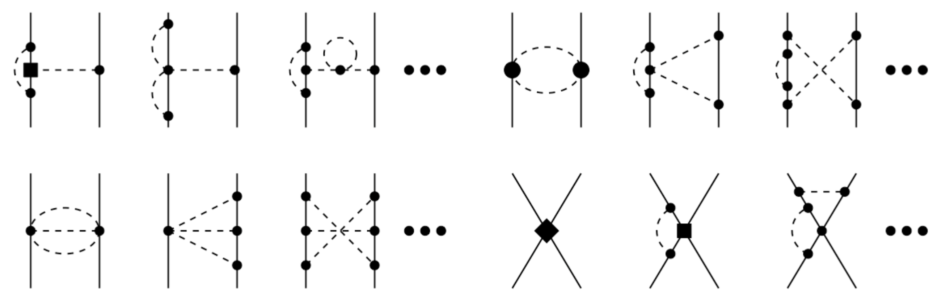
N^{2}LO



N^{3}LO



N^{4}LO



***E. Epelbaum, H.-W. Hammer, Ulf-G. Meißner, Reviews of Modern Physics, arXiv:0811.1338 [nucl-th]**

Conclusions -- Part I

- Non-relativistic nuclear physics now on a solid footing;
  - Precise 2 (and 3 and ?) body potentials can describe nuclei (for all A?).
  - Low energy few body scattering largely explained by the same potentials
- Chiral perturbation theory establishes the close connection between QCD (through chiral symmetry) and nuclear physics based on hadronic degrees of freedom.

Part II -- Present:

what do we know;

what is the state-of-the-art?

(This part is larger a primer of

elementary concepts.

Many topics are omitted.)

Part II-- Present:
what do we know; what is the state-of-the-art?

- ★ Lessons (largely ignored) from the Dirac equation
- ★ Pictorial discussion of Hamiltonian vs. field dynamics for fixed numbers of particles
  - Use simple ϕ^4-type field theory (in second order) to illustrate general principles
  - Time and tau ordered diagrams
  - Role of u-channel diagrams
  - One body limit
  - Cancellation theorem
- ★ Construction of the non-perturbative equations of field dynamics (many parallels with Hamiltonian dynamics)
  - Bethe-Salpeter and spectator equations
  - Definitions of bound states and normalization
- ★ Interaction currents
  - Consistency and uniqueness

Lessons (largely ignored) from the Dirac equation

Lessons from the Dirac equation (1)

- The Dirac equation for the coulomb interaction (with $A^{\mu} = \{\phi, \mathbf{A}\}$) is

$$i\frac{\partial}{\partial t}\Psi = \left(\alpha\cdot(\mathbf{p}-e\mathbf{A})+\beta m+e\phi\right)\Psi$$

- Taking the non-relativistic limit gives [to order $(v/c)^2$]

$$i\frac{\partial}{\partial t}\psi = \left\{\frac{(\mathbf{p}-e\mathbf{A})^2}{2m}-\frac{\mathbf{p}^4}{8m^3}-\frac{e}{2m}\sigma\cdot\mathbf{B}+\frac{e\left[\nabla^2\phi\right]}{8m^2}+\frac{e}{4m^2 r}\frac{d\phi}{dr}\sigma\cdot\mathbf{L}\right\}\psi$$

- Each of these terms has a special history:

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$$-\frac{e}{2m}(\mathbf{p}\cdot\mathbf{A}+\mathbf{A}\cdot\mathbf{p}) - \frac{e}{2m}\sigma\cdot\mathbf{B} = -\frac{e}{2m}\mathbf{B}\cdot(\mathbf{L}+\sigma) = -\frac{e}{2m}\mathbf{B}\cdot(\mathbf{L}+2\mathbf{S})$$ **Zeeman effect (with the gyromagnetic ratio of 2)**

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Why don't we use the Dirac equation?

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Why don't we use the Dirac equation?

Lessons from the Dirac equation (2)

- The Dirac equation has negative energy solutions -- that can be reinterpreted as anti-particle states when we use field theory
- Two choices (or points of view):
  - Avoid the Dirac equation, because
    - We abhor negative energy states; they are unphysical
    - Hard (maybe impossible??) to extend to 2+ body sector; what is the Hilbert space?
    - Return to Schrodinger equation quantum mechanics with the familiar Hilbert space with a fixed number of particles, and
    - Treat relativity using Hamiltonian dynamics.
  - Keep the Dirac equation, because
    - We are impressed with the physics it contains
    - We are willing to truncate the field theory (i.e. invent a new "field dynamics" which may require uncontrolled approximations),
    - We are willing to have a formalism with "off-shell" particles or and negative energy states
    - We are willing to give up the variational principle (?).

Overview of relativistic methods: Two "schools"

Relativity
with a fixed number of particles

Hamiltonian dynamics
Hilbert space of on-shell particles
\+ no negative energy states
– lose the Dirac equation

Polyzou

- Point form: Schweiger, Klink
- Front form: Bakker, Mathiot, Salme, Smirnov, Tsirova
- Instant form: Arriaga, Carlson, Elster, Kamada, Schiavilla, Witala

Field dynamics
Off-shell particles - no Hilbert space(?)
\+ manifest covariance and locality
– keep the Dirac equation

Equal Time (ET)

- BSLT* (1966): *Blankenbecler, Sugar, Logunov, Tavkhelidze
- PWM†: †Phillips, Wallace, Mandelzweig

manifest covariance

- Front form: Frederico, Maris, Miller
- Spectator (1969): Gross, Jeschonnek, Pena, Stadler
- Bethe-Salpeter (1951): Carbonell, Kaptari, Karmanov, Rogochaya, Sauli
- Dyson-Schwinger: Nicmorus

Hamiltonian vs. field dynamics

Simple discussion using ϕ^4-type theory as an example

Illustrate general principles with ϕ^4 as an example (1):
-- Feynman vs. time ordered diagrams

- Example: ϕ^4-type interactions

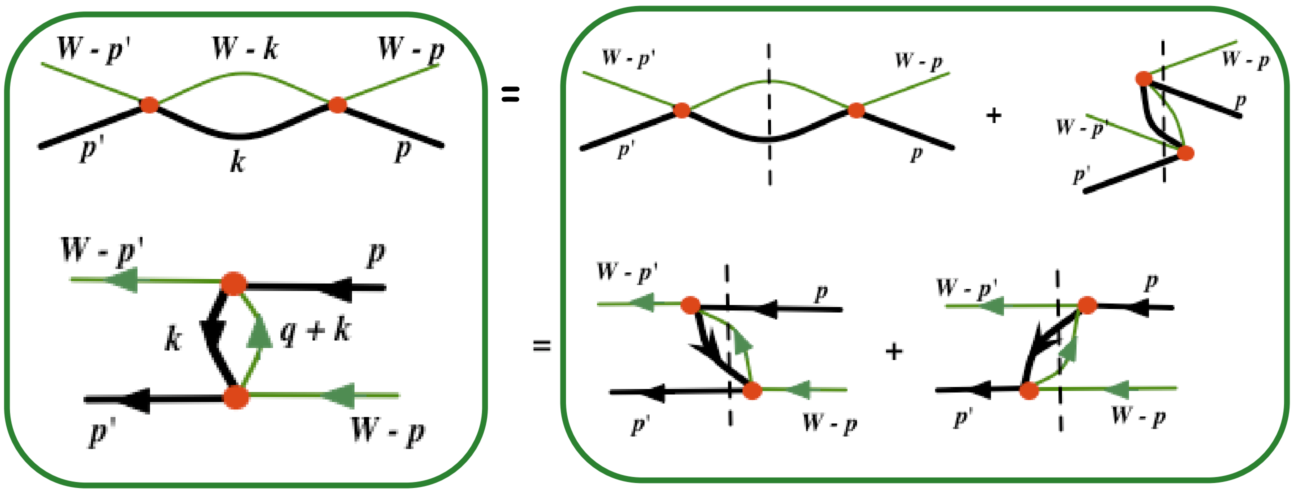
  Consider the interaction $-\lambda\phi^{\dagger}\phi\psi^{2}$ where

  ϕ is a "light" charged scalar field of mass *m;*

  ψ is a "heavy" neutral scalar field of mass *M*

- Feynman diagrams = sums of all time (or tau) ordered diagrams

- Field theory to second order (λ^2) has 2 Feynman = 4 time-ordered



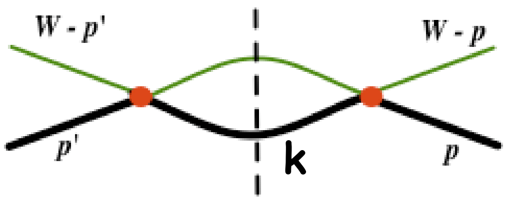
Feynman diagrams

Time-ordered diagrams

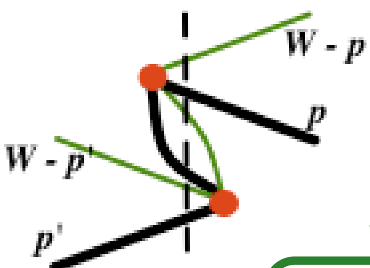
Lessons illustrated by ϕ^4 (2):

-- s-channel diagram contains quantum mechanics

- ★ Regularization an issue -- here take a cutoff in **k**
- ★ s-channel diagrams (**p**=0): $W = M + \alpha m; \quad \alpha \cong 1; \quad E_k = \sqrt{M^2 + \mathbf{k}^2}, \quad e_k = \sqrt{m^2 + \mathbf{k}^2}$



$$\int_{\mathbf{k}} \frac{\lambda^2}{4E_k e_k \left(e_k + E_k - W\right)} \underset{M\to\infty}{\Rightarrow} \int_{\mathbf{k}} \frac{\lambda^2}{4M\, e_k \left(e_k - \alpha\, m\right)}$$



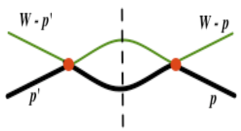
$$\int_{\mathbf{k}} \frac{\lambda^2}{4E_k e_k \left(e_k + E_k + W\right)} \underset{M\to\infty}{\Rightarrow} \int_{\mathbf{k}} \frac{\lambda^2}{8M^2\, e_k} \Rightarrow 0$$

This is "quantum mechanics" with a fixed number of particles
→ 2 particles in positive energy states

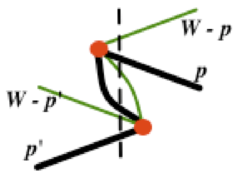
Lessons illustrated by ϕ^4 (3):

-- (aside) Z-diagrams give manifest covariance

- Add and subtract a term to aid in the comparison:

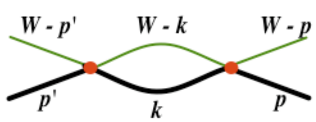


$$\int_{\mathbf{k}} \frac{\lambda^2}{4E_k e_k\left(e_k + E_k - W\right)} + \int_{\mathbf{k}} \cancel{\frac{\lambda^2}{4E_k e_k\left(e_k + E_k + W\right)}} = \int_{\mathbf{k}} \frac{\lambda^2}{2E_k\left(e_k^2 - \left(W - E_k\right)^2\right)}$$



$$\int_{\mathbf{k}} \frac{\lambda^2}{4E_k e_k\left(e_k + E_k + W\right)} - \int_{\mathbf{k}} \cancel{\frac{\lambda^2}{4E_k e_k\left(e_k + E_k + W\right)}} = \int_{\mathbf{k}} \frac{\lambda^2}{2e_k\left(E_k^2 - \left(e_k + W\right)^2\right)}$$

- And, note that these two terms come from integrating the Feynman diagram over k_0



$$B(W^2) = -i\int_{\mathbf{k}}\int \frac{dk_0}{2\pi} \frac{\lambda^2}{\left(m^2 - (W-k)^2 - i\varepsilon\right)\left(M^2 - k^2 - i\varepsilon\right)}$$

$$= \int_{\mathbf{k}} \frac{\lambda^2}{2E_k\left(e_k^2 - \left(W - E_k\right)^2\right)} + \int_{\mathbf{k}} \frac{\lambda^2}{2e_k\left(E_k^2 - \left(e_k + W\right)^2\right)}$$

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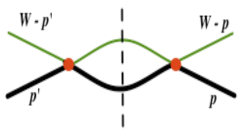
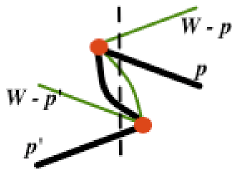


$$\int_{\mathbf{k}} \frac{\lambda^2}{4E_k e_k\left(e_k + E_k - W\right)} + \int_{\mathbf{k}} \frac{\lambda^2}{4E_k e_k\left(e_k - E_k + W\right)} = \int_{\mathbf{k}} \frac{\lambda^2}{2E_k\left(e_k^2 - \left(W - E_k\right)^2\right)}$$



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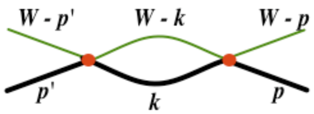

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Lessons illustrated by ϕ^4 (4):

-- BUT: Z-diagrams NOT needed for covariance (Hamiltonian dynamics)!

- ★ Non-relativistic quantum mechanics can be made Poincaré invariant.
- ★ Here the integrals are functions of $(\mathbf{k}_R)^2$ ($\mathbf{k}_R$ is the three vector in the rest system).
  - Define $\mathbf{k}_R^2 = -k^2 + \frac{(k\cdot P)^2}{P^2}$, with $k_R^2 = k^2 = M^2$ and $W = \sqrt{P^2}$. This is covariant.
  - $(\mathbf{k}_R)^2 = \mathbf{k}^2$ when the system is at rest (when P={W,**0**})
  - If $P=\{P_0,0,0,P_z\}$, then,

$$\mathbf{k}_R^2 = -M^2 + \frac{\left(P_0 E_k - k_z P_z\right)^2}{W^2}; \quad P_0 = \sqrt{W^2 + P_z^2}, \quad E_k = \sqrt{M^2 + \mathbf{k}^2}$$

where **k** is the three momentum in the moving system.

- ★ BUT, this same result can be gotten from the boost operator

$$\boxed{(P_0 k_z - P_z E_k)^2 = \left(P_0 E_k - k_z P_z\right)^2 - \left(P_0^2 - P_z^2\right)(M^2 + \mathbf{k}_\perp^2)}$$

$$k_R = B(-P)k = \frac{1}{W}\begin{pmatrix} P_0 & 0 & 0 & -P_z \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -P_z & 0 & 0 & P_0 \end{pmatrix}\begin{pmatrix} E_k \\ k_x \\ k_y \\ k_z \end{pmatrix} \Rightarrow \mathbf{k}_R^2 = \mathbf{k}_\perp^2 + \frac{(P_0 k_z - P_z E_k)^2}{W^2} = -M^2 + \frac{\left(P_0 E_k - k_z P_z\right)^2}{W^2}$$

AGREES!

Lessons illustrated by ϕ^4 (5):

-- Hamiltonian dynamics from field theory; light-front

- ★ Leading contribution from s-channel diagram ⇔ quantum mechanics.
- ★ Z-diagrams give covariance, but diagram can also be made covariant using Hamiltonian dynamics. How do these compare?
- ★ Instant-form and front-form give different results. The s-channel bubble in front-form is obtained by integrating over $k_+ = k_0 + k_z$ instead of k_0, and gives the *exact* result for the bubble :

$$B(s) = \frac{-i\lambda^2}{2(2\pi)^4} \int \frac{dk_+ dk_- d^2 k_\perp}{\left(m^2 + \mathbf{k}_\perp^2 - (W-k)_+ (W-k)_- - i\varepsilon\right)\left(M^2 + \mathbf{k}_\perp^2 - k_+ k_- - i\varepsilon\right)}$$

$$= \frac{\lambda^2}{2(2\pi)^3} \int d^2 k_\perp \int_0^W \frac{dk_-}{k_-} \frac{1}{\left(m^2 + \mathbf{k}_\perp^2 - \left(W - \frac{M^2 + \mathbf{k}_\perp^2}{k_-}\right)(W - k_-) - i\varepsilon\right)}$$

$$= \lambda^2 \int_{k_\perp} \int_0^1 \frac{dx}{x(1-x)} \frac{1}{\left(\frac{m^2 + \mathbf{k}_\perp^2}{(1-x)} + \frac{M^2 + \mathbf{k}_\perp^2}{x} - W^2\right)} \quad \text{where } k_- = xW$$

- ★ Does this the exact result make the front-form better?

Lessons illustrated by ϕ^4 (5):

-- Hamiltonian dynamics from field theory; light-front

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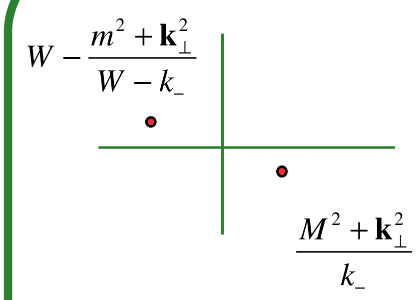


Only 2 poles; lying in the opposite planes iff $W > k_- > 0$

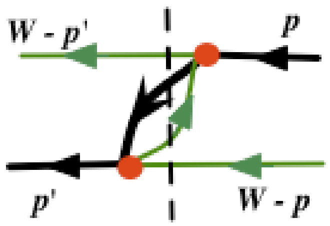
- Does this the exact result make the front-form better?

SO -- whats the difference between Hamiltonian dynamics and Field dynamics?

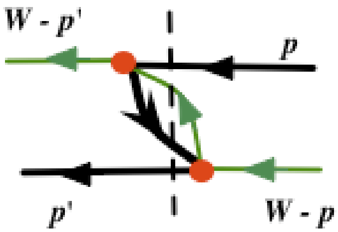
Lessons illustrated by ϕ^4 (6):

-- BUT: Field dynamics requires u-channel diagrams:

- For **p**=0 and $W = M + \alpha m$ as before



$$\int_{\mathbf{k}} \frac{\lambda^2}{4E_k e_k\left(E_k + e_k + 2\alpha m - W\right)} \underset{M\to\infty}{\Rightarrow} \int_{\mathbf{k}} \frac{\lambda^2}{4M\, e_k\left(e_k + \alpha\, m\right)}$$



$$\int_{\mathbf{k}} \frac{\lambda^2}{4E_k e_k\left(E_k + e_k + 2M - W\right)} \underset{M\to\infty}{\Rightarrow} \int_{\mathbf{k}} \frac{\lambda^2}{8M^2\, e_k} \Rightarrow 0$$

- u-channel contributions are NEW terms not included in "quantum mechanics"

Lessons illustrated by ϕ^4 (6):

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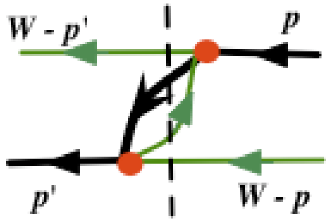

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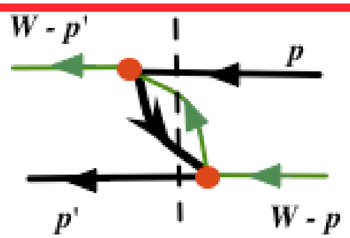


$$\int_{\mathbf{k}} \frac{\lambda^2}{4E_k e_k\left(E_k + e_k + 2M - W\right)} \underset{M\to\infty}{\Rightarrow} \int_{\mathbf{k}} \frac{\lambda^2}{8M^2\, e_k} \Rightarrow 0$$

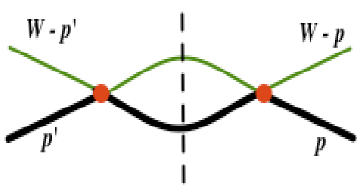
This term does NOT vanish in the *M* → ∞ limit!

★ u-channel contributions are NEW terms not included in "quantum mechanics"

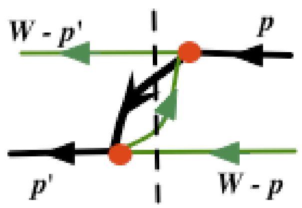
Lessons illustrated by ϕ^4 (7):

-- Furthermore, field dynamics satisfies the one-body limit

★ The leading contributions as $M \to \infty$ are



(A)



(B)

$$= \int_{\mathbf{k}} \frac{\lambda^2}{4M\, e_k} \left\{ \underbrace{\frac{1}{e_k - \alpha m}}_{(A)} + \underbrace{\frac{1}{e_k + \alpha m}}_{(B)} \right\} = \frac{\lambda^2}{2M} \int_{\mathbf{k}} \frac{1}{e_k^2 - \alpha^2 m^2}$$

Klein-Gordon propagator

$$e_k^2 - \alpha^2 m^2 = m^2 + \mathbf{k}^2 - (W - M)^2$$

★ This is the second order result for scattering from the Klein-Gordon equation:

$$\left[m^2 + \mathbf{k}^2 - (W - M)^2 \right] \Psi(\mathbf{k}) = \lambda\, \Psi(\mathbf{k})$$

One-body limit: as $M \to \infty$, the light particle satisfies a one-body equation (Klein-Gordon) with an effective potential (the constant λ in this case).

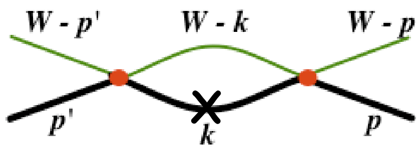
★ The s-channel bubbles do NOT have a one-body limit.

★ Crossed diagrams (u-channel exchanges) are needed. (For meson exchange: BOTH ladder and crossed ladder diagrams are needed).

Lessons illustrated by ϕ^4 (8):

-- Cancellation theorem in field dynamics

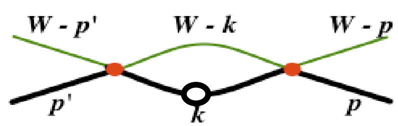
★ The Covariant Spectator Theory (CST), which picks up the positive energy pole of the heavy particle, has a one-body limit



$$\int_{\mathbf{k}} \frac{\lambda^2}{2E_k\left(e_k^2-\left(W-E_k\right)^2\right)} \underset{M\to\infty}{\Rightarrow} \frac{\lambda^2}{2M}\int_{\mathbf{k}} \frac{1}{e_k^2-\alpha^2 m^2}$$

★ This implies that the rest of the second order terms cancel in the limit $M \to \infty$

$\lim\limits_{M\to\infty}$ {



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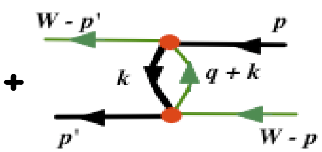


} $\Rightarrow \quad 0$

This is the Cancellation Theorem

★ Conclusions:

- Both the Bethe-Salpeter equation (*in ladder approximation*), and the Schrodinger equation do NOT have the one-body limit !
- The CST DOES have the one-body limit
- Caveat: How important is the one-body limit? (Return to this in Part III)

Lessons illustrated by ϕ^4 (9):

-- Conclusions - Hamiltonian vs. field dynamics

- ★ Field dynamics includes contributions (i.e. u-channel, or crossed ladder diagrams) not included in "quantum mechanics"
  - some of these contributions involve virtual antiparticles
- ★ These other diagrams are NOT needed for Poincaré invariance
  - Hamilton dynamics incorporates exact Poincaré invariance into quantum mechanics of a fixed number of particles
- ★ Is the physics described by the "other diagrams" of field dynamics important?
  - They give a one-body limit, showing the connection between one body relativistic equations and two (and many) body theory
  - Their contributions are not small.

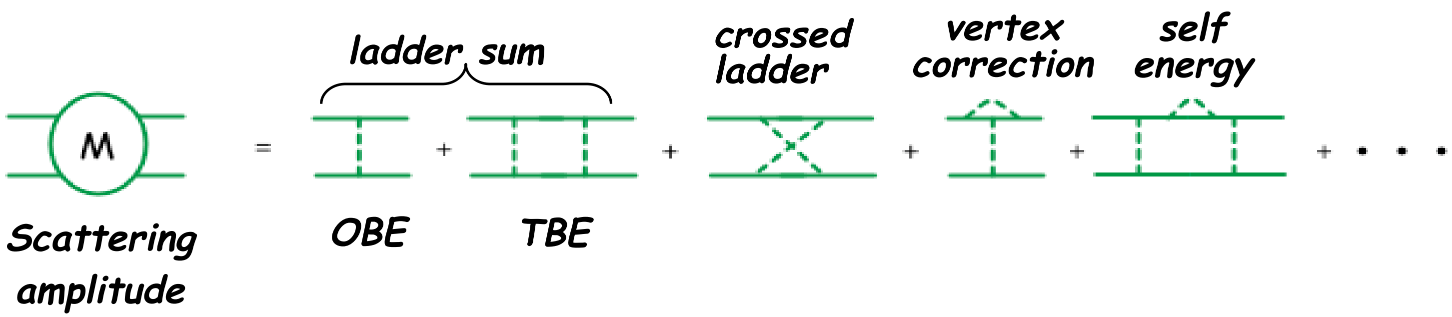
Construction of field dynamical equations

(Many of these ideas also apply to Hamiltonian dynamics, and even Schrödinger theory)

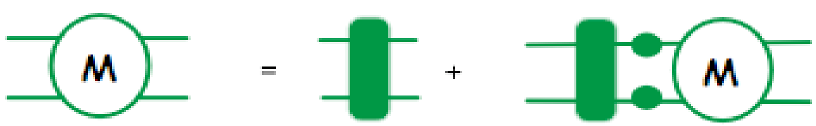
Construction of field dynamical equations (similarities with Hamiltonian dynamics) (1)

-- Diagrammatic derivation of the 2-body scattering equations

★ Step 1: The exact scattering amplitude is the sum of all Feynman diagrams



★ Step 2: Divide the sum into irreducible and 2-body reducible terms, and collect the irreducible terms into a kernel, which is iterated



★ Step 3: Field *theory* becomes field *dynamics* when the kernel is phenomenological

$$M(p',p;P)=V(p',p;P)+\int V(p',k;P)G(k;P)M(k,p;P)$$

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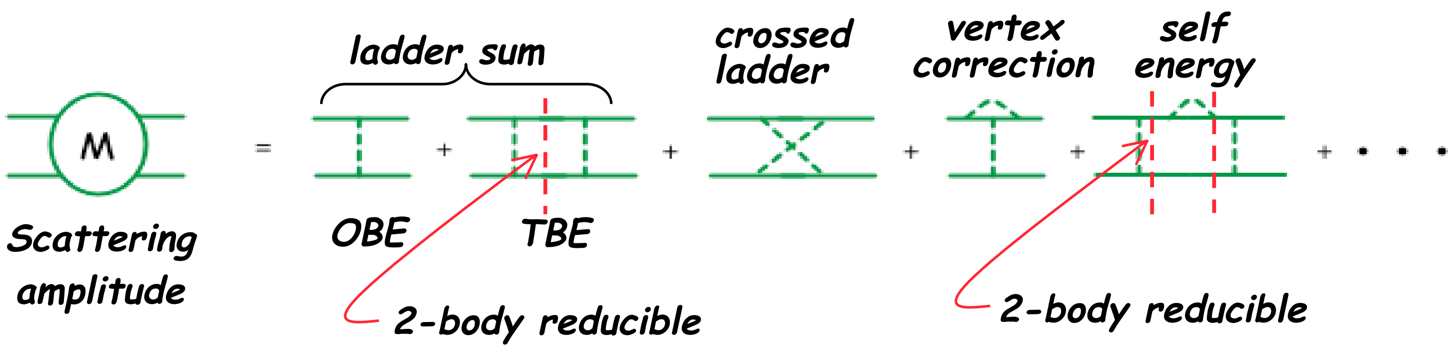

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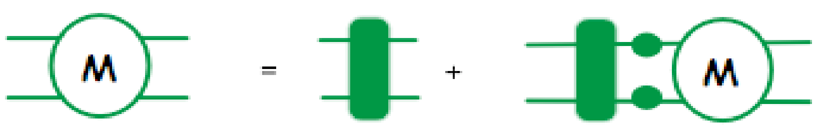

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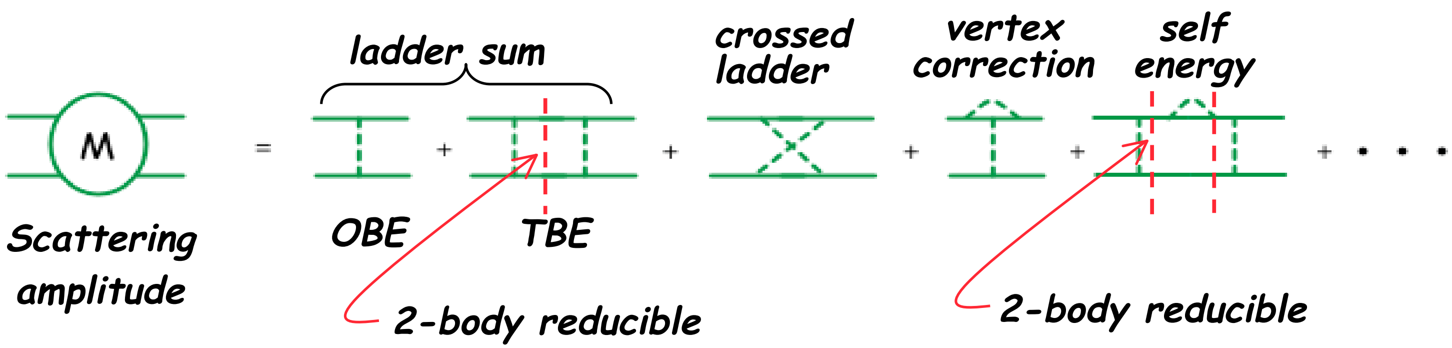

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$$M(p',p;P) = V(p',p;P) + \int V(p',k;P)\,G(k;P)\,M(k,p;P)$$

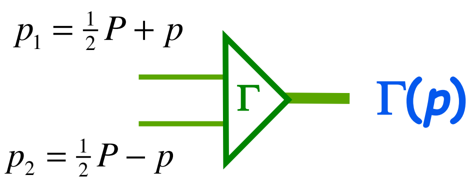
Field dynamics (2)

-- Bound state equations emerge automatically: NO extra assumptions

- A bound state is a new particle (not in the Lagrangian). It is generated *non-perturbatively* from the sum of an infinite number of diagrams much as the geometric series generates a pole at *z*=1:

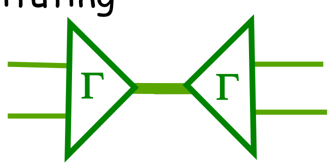
$$M = V + \int VGM \;\Rightarrow\; z + zM = z + z^2 + z^3 + \bullet\bullet\bullet = \frac{z}{1-z}$$

- The vertex function Γ describes how the bound state couples to particles in the Lagrangian:



Notation:
P=total momentum (always conserved)
p relative momentum

- The bound state equation follows from the assumption the M matrix has a pole, and substituting



$$M(p',p;P) = \frac{\Gamma(p')\overline{\Gamma}(p)}{M_B{}^2 - P^2} + R(p',p;P)$$

$$M(p',p;P) = \frac{\Gamma(p')\overline{\Gamma}(p)}{M_B^2 - P^2} + R(p',p;P) = V(p',p;P) + \int V(p',k;P)G(k;P)\left\{\frac{\Gamma(k)\overline{\Gamma}(p)}{M_B^2 - P^2} + R(k,p;P)\right\}$$

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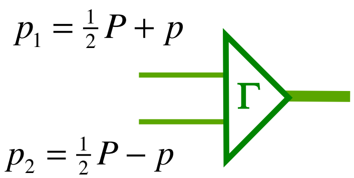

$\Gamma(\boldsymbol{p})$

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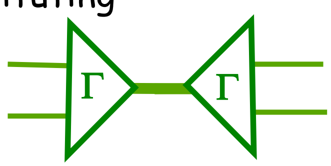


$$M(p',p;P) = \frac{\Gamma(p')\overline{\Gamma}(p)}{M_B{}^2 - P^2} + R(p',p;P)$$

$$M(p',p;P) = \frac{\Gamma(p')\overline{\Gamma}(p)}{M_B^2 - P^2} + R(p',p;P) = V(p',p;P) + \int V(p',k;P)G(k;P)\left\{\frac{\Gamma(k)\overline{\Gamma}(p)}{M_B^2 - P^2} + R(k,p;P)\right\}$$

at the pole: $\Gamma(p') = \int V(p',k;P)G(k;P)\Gamma(k)$

Field dynamics (3)
-- Two (of many) types of field dynamical equations

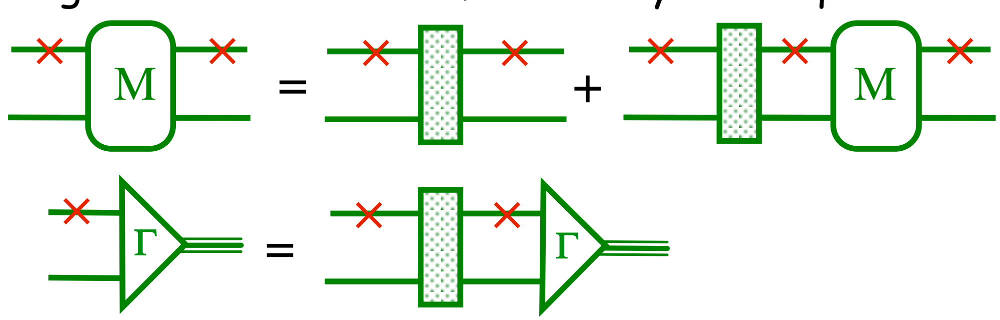
★ The Bethe-Salpeter (BS) propagator depends on all four components of the relative momentum, $\{k_0, \mathbf{k}\}$. For two spinor particles it is

$$G_{BS}(k;P) = \frac{1}{\left(m_1 - \not{p}_1 - \Sigma(\not{p}) - i\varepsilon\right)\left(m_2 - \not{p}_2 - \Sigma(\not{p}_2) - i\varepsilon\right)} \quad \textbf{with} \quad \begin{cases} p_1 = \frac{1}{2}P + k \\ p_2 = \frac{1}{2}P - k \end{cases}$$

★ The Covariant Spectator Theory© propagator depends on only three components of the relative momentum, **k**. One particle is on-shell

$$G_{CS}(k;P) = \frac{2\pi i\,\delta_+\left(m_1^2 - \left(\frac{1}{2}P + k\right)^2\right)\left[m_1 + \hat{\not{p}}_1\right]}{\left(m_2 - \not{p}_2 - \Sigma(\not{p}_2) - i\varepsilon\right)} = \frac{2\pi i\,\delta\left(k_0 - E_1 + \frac{1}{2}P_0\right)}{\left(E_2^2 - \left(P_0 - E_1\right)^2 - \Sigma(\not{p}_2) - i\varepsilon\right)} \frac{m_1}{E_1} \sum_s u(\mathbf{k},s)\bar{u}(\mathbf{k},s)$$

★ Diagrammatic notation for 2-body CST equations:



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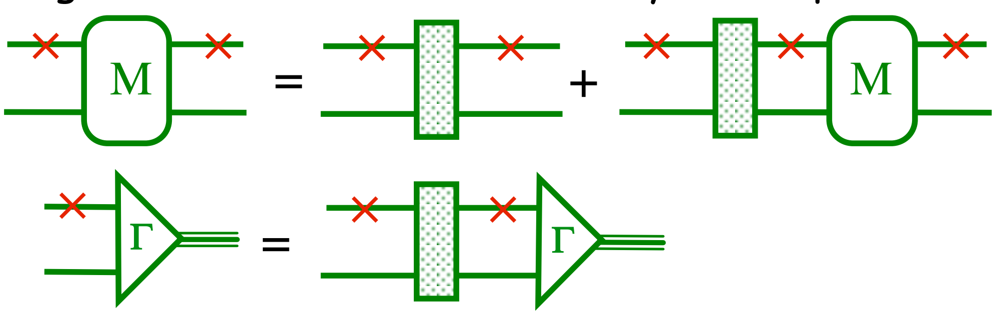

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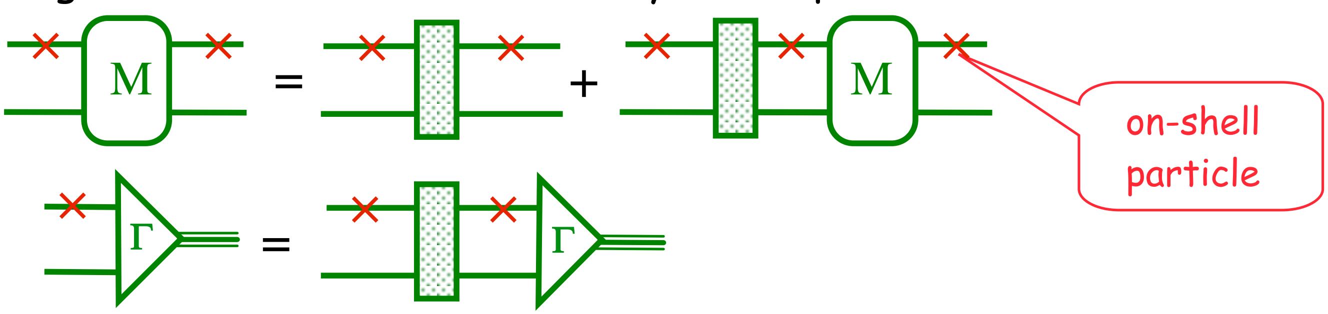

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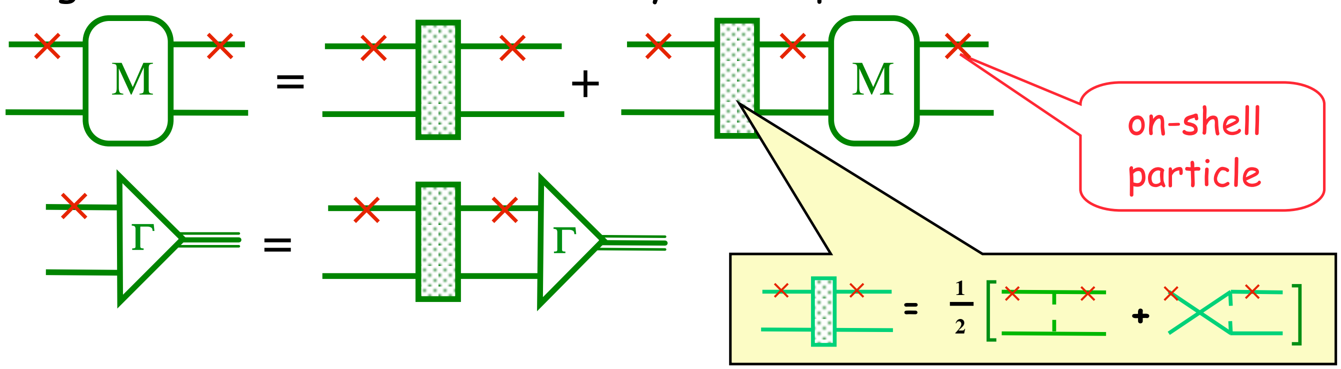
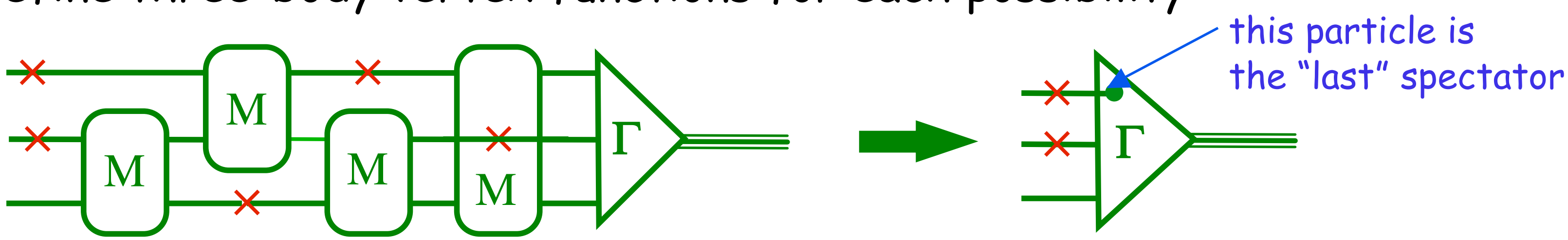


CST: must explicitly (anti)symmetrize the kernel

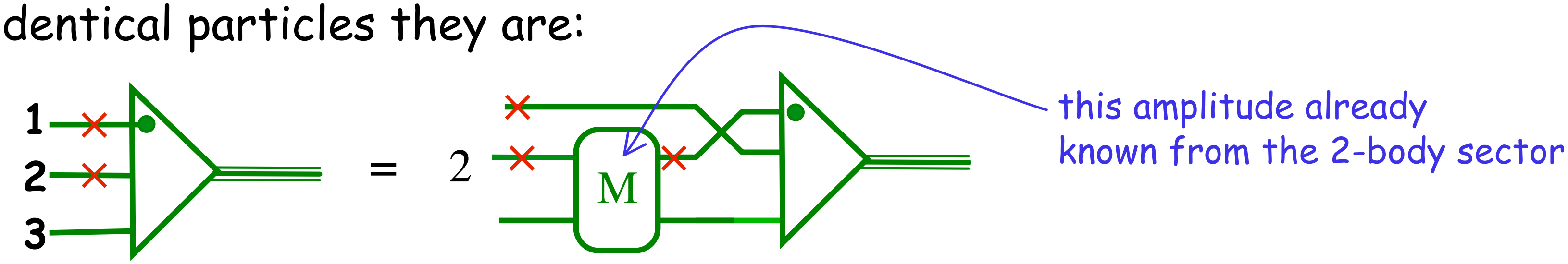
Field dynamics (4)

-- CST equations for three-body bound state*

- Define three-body vertex functions for each possibility



- then three body Faddeev-like equations emerge automatically. For identical particles they are:



$$\left|\Gamma^1_2\right\rangle = 2M^1_{22}G^1_2P_{12}\left|\Gamma^1_2\right\rangle$$

---

*Alfred Stadler, FG, and Michael Frank, Phys. Rev. C **56**, 2396 (1997)

Field dynamics (5)

-- Normalization conditions obtained directly from the CST equations

- ★ Covariant bound state normalization conditions follow from examination of the residue of the bound state pole
  - 2-body case

$$1=\langle\Gamma|\frac{dG}{dM_d^2}|\Gamma\rangle-\langle\Gamma|G\frac{dV}{dM_d^2}G|\Gamma\rangle$$

  - 3-body case

$$1=3\left\langle\Gamma_2^1\right|(1+2P_{12})\frac{dG_2^1}{dM_d^2}\left|\Gamma_2^1\right\rangle-3\left\langle\Gamma_2^1\right|(1+2P_{12})G_2^1\frac{dV_{22}^1}{dM_d^2}G_2^1(1+2P_{12})\left|\Gamma_2^1\right\rangle$$

- ★ Define the 2-body relativistic wave function: $|\Psi\rangle=G|\Gamma\rangle$. Then, if $\frac{dV}{dM_d^2}=0$,

$$\boxed{2M_d=\langle\Psi|\gamma^0|\Psi\rangle}\quad\left(\text{because }\frac{dG}{dM_d^2}=\frac{1}{2M_d}\frac{dG}{dM_d}=\frac{1}{2M_d}G\gamma^0G\right).$$

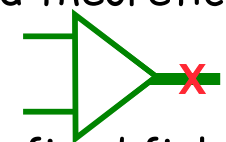
  - Identical to the normalization condition for the Dirac equation
- ★ Similar interpretation for the 3-body normalization condition
- ★ Similar derivation for the Bethe-Salpeter (and Schrödinger) equation

Field dynamics (6)

-- Close connection between field *dynamics* and field *theory*

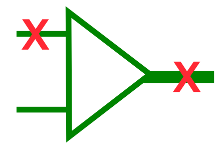
★ The Bethe-Salpeter amplitude is a well defined field theoretic matrix element:

$$\Psi(x_1,x_2)=\langle 0|T\left(\psi(x_1)\psi(x_1)\right)|d\rangle$$



★ The Covariant Spectator© amplitude is *also* a well defined field theoretic amplitude:

$$\Psi(x_2)=\langle N|\psi(x_2)|d\rangle$$



★ Equations for the Bethe-Salpeter and the Spectator* amplitudes can be derived from field theory

★ It is only the fact that the kernels are approximate that makes them a *dynamics* and not a *theory* ! Field dynamics is merely a relativistic generalization of QM.

★ The close connection to field theory provides guidance for

- construction of new channels
- construction of the current operator

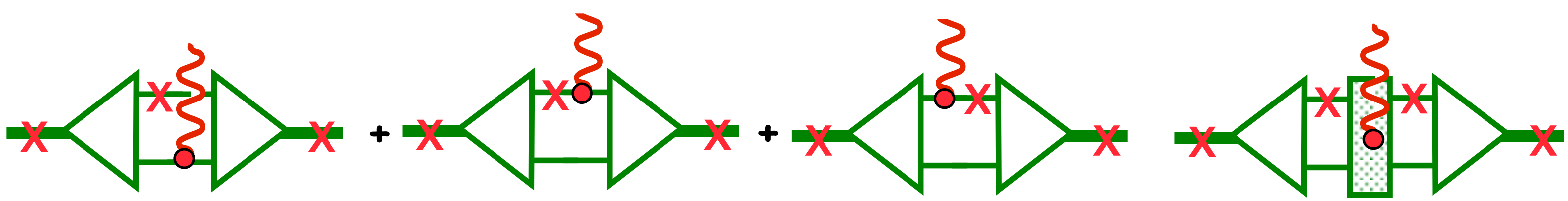
***O. W. Greenberg's "n-quantum approximation"**

Gauge invariant currents for CST

Also true (with modifications) for BS and quantum mechanics

Construction of the current operator in CST©
-- Gauge invariant* two body current operator

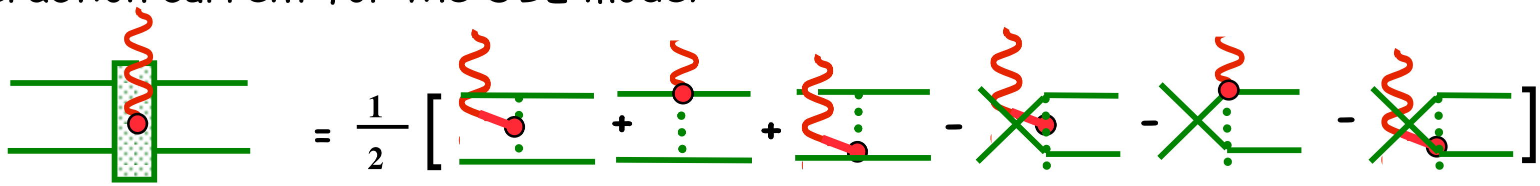
- Exact gauge invariance currents can be constructed following the method of FG and Riska,* and these have been used for both relativistic and non-relativistic calculations
- Proceed in two steps:
  - Step 1: construct one body currents that satisfy the Ward-Takahashi identity
  - Step 2: couple these currents to all charges (or momentum dependent couplings) in ALL of the infinite number of diagrams under consideration.
- Step 2: coupling to ALL charges not so difficult -- if the equations are used. The diagrams for the elastic two-body current, in CST, are



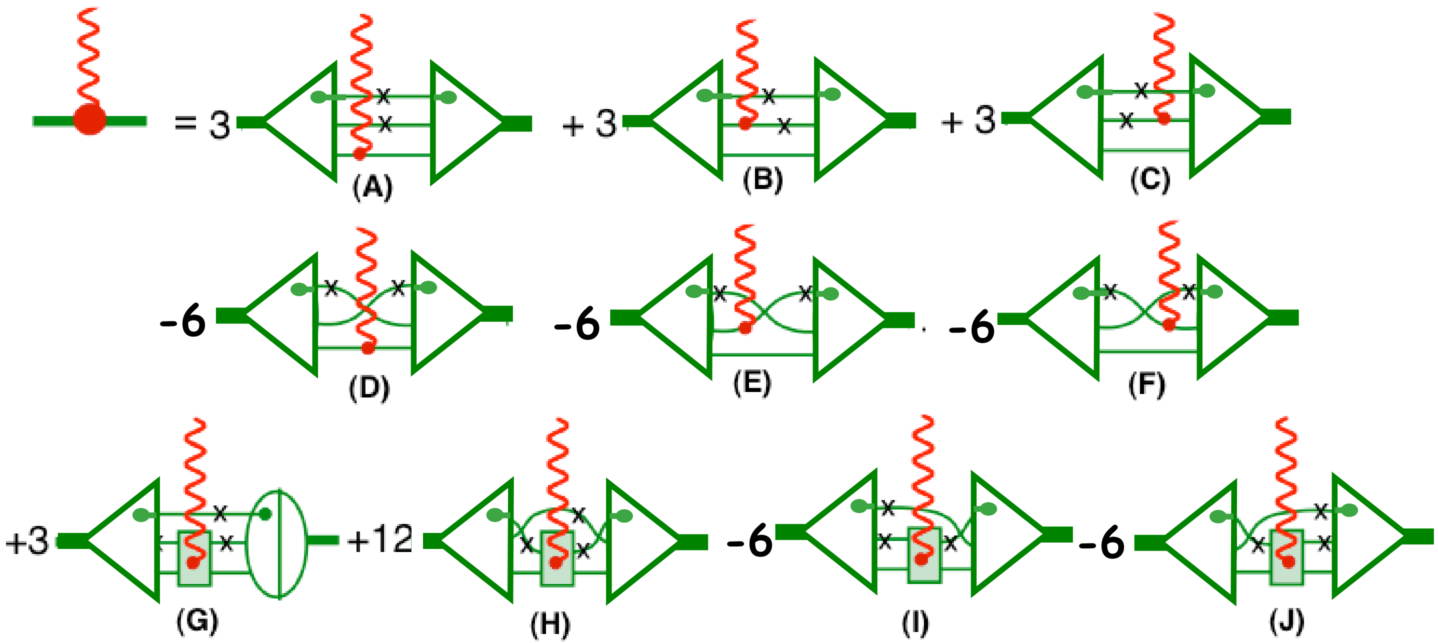
*FG, and D. O. Riska, PRC 36, 1928 (1987)

Construction of the current operator in CST©
-- Gauge invariant* three body current operator

- ★ Interaction current for the OBE model:



- ★ Current for three body elastic scattering



Construction of the current operator in CST©
-- Gauge invariant* three body current operator

- Step 1: to conserve current, the one body current operators must satisfy the WT identity. Example: the nucleon:

$$\boxed{q_\mu j_N^\mu(p',p) = S^{-1}(p) - S^{-1}(p')} \longrightarrow S(p) = \frac{h^2(p)}{m-p} = \frac{h^2(p)}{\Delta_-(p)}$$

- The spectator models use a *nucleon form factor, h(p).* This means that the nucleon propagator can be considered to be dressed. One solution (the simplest) is

$$j^\mu(p',p) = F_0\left\{\gamma^\mu + (F_1 - 1)\left(\gamma^\mu - \frac{\not{q}q^\mu}{q^2}\right) + F_2\frac{i\sigma^{\mu\nu}q_\nu}{2m}\right\} + G_0\,\Lambda_-(p')\left\{\gamma^\mu + (F_3 - 1)\left(\gamma^\mu - \frac{\not{q}q^\mu}{q^2}\right)\right\}\Lambda_-(p)$$

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$$F_0 = \frac{h(p)}{h(p')}\left(\frac{m^2 - p'^2}{p^2 - p'^2}\right) - \frac{h(p')}{h(p)}\left(\frac{m^2 - p^2}{p^2 - p'^2}\right) \qquad G_0 = \left(\frac{h(p')}{h(p)} - \frac{h(p)}{h(p')}\right)\frac{4m^2}{p^2 - p'^2}$$

- $F_3(Q^2)$ is unknown, except $F_3(0)=1$. This freedom can be exploited.

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purely transverse

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Part III: Future

Where do we go from here?

This part is designed to stimulate discussion!

Part III: Future

- Limitations of hadronic field theory
  - Structure vs renormalization in hadronic field theory; scales
  - Including excited baryons; coupled channels
  - Degrees of freedom: quarks vs hadrons
- Issues:
  - Extensions of the cancellation theorem
  - Interpretation of angular momentum on the light cone
  - Does the boost operator create particles?
- Low Q^2 form factors and study of relativistic effects (discuss in the working group)
  - an opportunity to benchmark relativistic calculations
- The landscape of electron scattering: Q^2 versus W

Future

-- Limitations of hadronic field theory

- ★ Hadrons are composite. This alone is not the problem, because
  - QED works in atomic physics even though the nucleus is composite
  - Atomic bound states are non-perturbative, but we can treat higher order corrections perturbatively.
  - QED works the way we hoped χPT would!
- ★ Fundamental differences between QCD and QED:
  - We know empirically that momentum scales of 700-800 MeV are important in nuclear physics
  - BUT, strong forces are perturbative only at very low momentum scales
    - much less than the pion mass (because χPT does not work for the pion tensor force);
- ★ Way out?
  - Assume X GeV (where X is some momentum >> 1 GeV) is the scale for the emergence of quark core effects
  - Use regularized hadronic field theory (i.e. field dynamics) for all scales up to X GeV.
- ★ Is this a good way out? If not, what should we do?

Future

-- Implications of field dynamics with X ≥ 1 Gev

- If X > 1, its allowed to choose a phenomenological OBE kernel with contact terms and boson masses < 1 GeV
- The hadrons are considered point-like; their interactions are regularized using Pauli-Villiars type subtractions with masses ≥ 1 GeV.
- For example, recent CST fit uses a regularized pion propagator

$$\Delta_\pi(q) = \frac{f(q)}{m_\pi^2 + |q^2|} = \left[\frac{1}{m_\pi^2 + |q^2|}\right]\left(\frac{\Lambda_\pi^2}{\Lambda_\pi^2 + |q^2|}\right)^4 \quad \text{where } \Lambda_\pi \approx 4 \text{ GeV}$$

replacement of $-q^2 \to |q^2|$ discussed by Stadler

This hard cutoff is a regularization parameter, NOT the "size" of the pion!

- In this language, nucleon charge form factors must be described by vector dominance; otherwise their "small" size violates the field dynamical model
- Current conservation can be exactly (but not uniquely) described using the ideas of Riska and FG.

Future

-- Including excited baryons

- ★ Excited baryons (masses ≤ 1 + X GeV) are treated as point like particles. Inclusion of the Δ will extend the NN scattering to 500 MeV lab energy
- ★ Treat the spin 3/2 particles using the formalism of Pascalutsa.*
  - The spin 3/2 propagator includes (well known) spurious spin 1/2 parts

$$S_{\mu\nu}(P) = \frac{1}{M - \not{P} - i\varepsilon} P^{(3/2)}_{\mu\nu} + S^{(1/2)}_{\mu\nu}(P) \text{ (spurious spin } \tfrac{1}{2} \text{ part)}$$

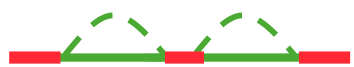
  - These are eliminated by the strong vertex $\Theta^{\mu\nu}$ (invariant under a strong gauge transformation). The vertex has the property

$$\Theta^{\mu\nu} S^{(1/2)}_{\nu\lambda}(P) = 0$$

  - The pion bubble contribution then becomes a pure spin 3/2 structure

$$\Rightarrow \int \Theta_{\mu\nu} \Theta^{\nu}{}_{\lambda}\, I = B P^{(3/2)}_{\mu\lambda}$$

  - and the dressing of the Δ pole contributions is easily summed to all orders

 +  + 

$$P^{(3/2)}_{\mu\nu} + B P^{(3/2)}_{\mu\nu} + B P^{(3/2)}_{\mu\nu'} P^{\nu'\mu'(3/2)} B P^{(3/2)}_{\mu'\nu} + \cdots = \frac{P^{(3/2)}_{\mu\nu}}{1 - B}$$

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*Pascalutsa, Phys. Rev. D **58**, 096002 (1998); Pascalutsa and Timmermans, Phys. Rev. C **60**, 042201 (1999); Pascalutsa and Phillips, Phys. Rev. C **68**, 055205 (2003); Pascalutsa and Vanderhaeghen, Phys.Lett. **B63**, 31 (2006)

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This solution of the spin 3/2 problem is a breakthrough

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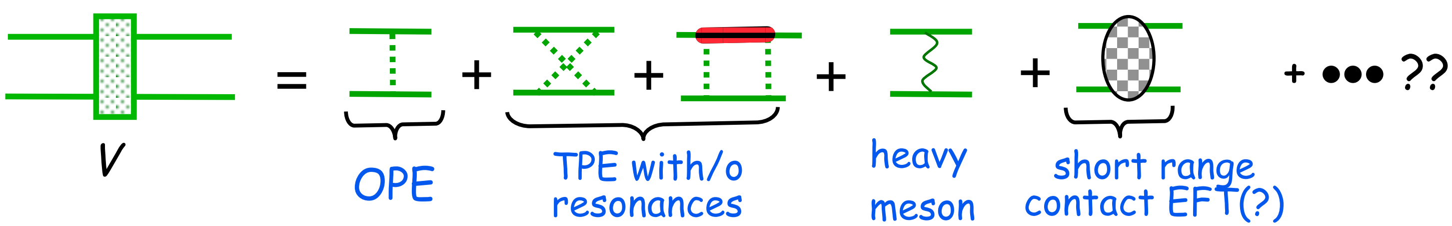
-- Where are the quarks?

- In this picture the quarks are "frozen" out, and do not need to be included explicitly until momenta >> X GeV
- Still, quarks might "explain," through duality, the relative strength of meson exchange models:

The "exact kernel is a complicated sum of many contributions



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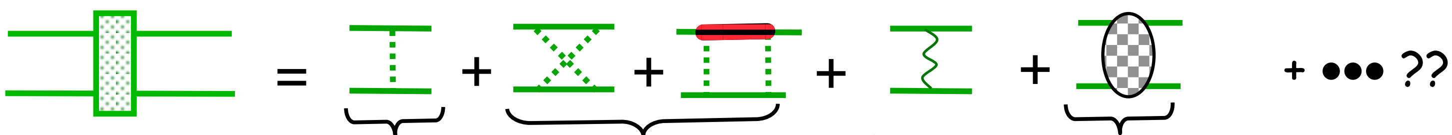

these can be replaced by $I = 0$ and I = 1 scalar exchanges (σ and δ)*

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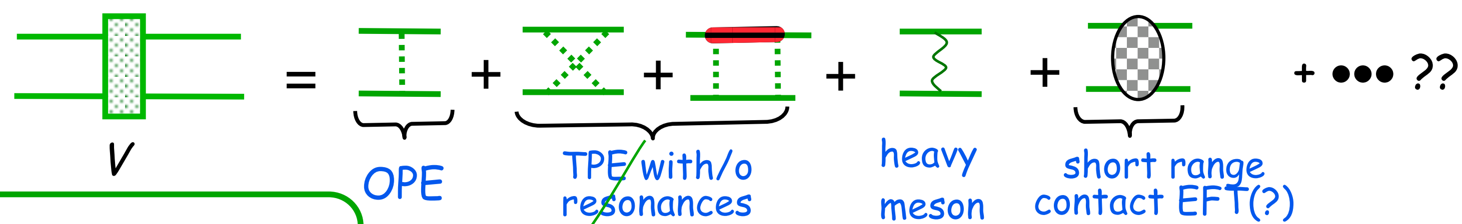

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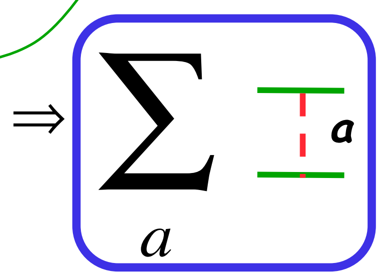


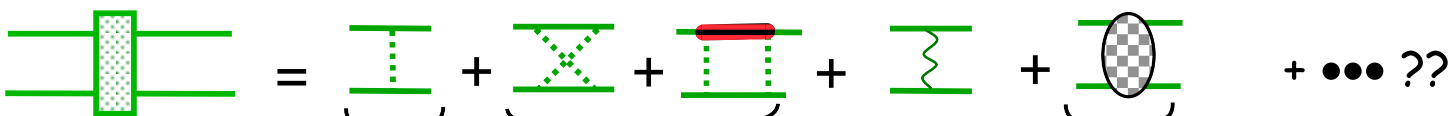
this whole sum is approximated By the OBE model

Future

-- Where are the quarks?

- ★ In this picture the quarks are "frozen" out, and do not need to be included explicitly until momenta >> X GeV
- ★ Still, quarks might "explain," through duality, the relative strength of meson exchange models:

The "exact kernel is a complicated sum of many contributions



V OPE TPE with/o resonances heavy meson short range contact EFT(?)

these can be replaced by I = 0 and I = 1 scalar exchanges (σ and δ)*

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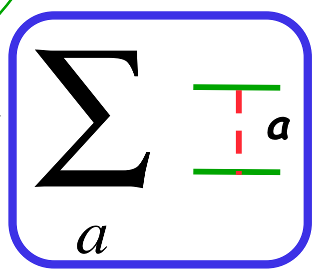
⇒



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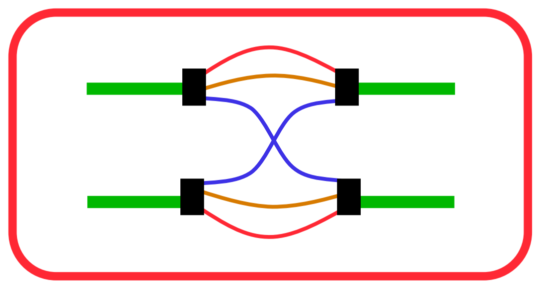

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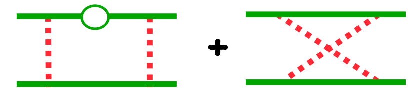
quark exchange might approximate the OBE model

Future

-- Extensions of the cancellation theorem

★ Has been proved only for scalar theories and QED



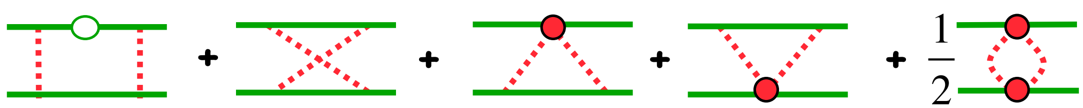
$$-\frac{N}{m_X^2 - t} + \frac{N}{m_X^2 - t} \cong 0$$

★ For pion exchanges with chiral symmetry treated as in the sigma model (i.e. γ^5 coupling with sigma type contact term δ^{ij}/m required by chiral symmetry, and use

$$\bar{u}\left[\gamma^5 \frac{1}{(m-\not{p})}\gamma^5\right]u = \bar{u}\gamma^5\left(\frac{m}{E_p}\right)\left\{\frac{u\bar{u}}{E_p - p_0} - \frac{v\bar{v}}{E_p + p_0}\right\}\gamma^5 u \Rightarrow -\frac{1}{2m} = -\eta$$

The 4th order kernel becomes

A B C D E



$$-\frac{\eta^2 N}{m_X^2 - t}\Big\{\underbrace{\tau_1^k\tau_1^m\left(\tau_2^k\tau_2^m + \tau_2^m\tau_2^k\right)}_{\text{A} \quad \text{B}} \underbrace{-6}_{\text{C}} \underbrace{-6}_{\text{D}} \underbrace{+\tfrac{1}{2}(12)}_{\text{E}}\Big\} \Rightarrow 0$$

★ This can be generalized: see study of the large N_c limits by T. Cohen. et.al.*

---

*see, for example, Phys.Rev.C65:064008,2002.

Future

-- Other issues

- ★ Physical interpretation of angular momentum on the light cone?
- ★ Does the boost operator create particles?
  - Yes, but in a limited sense using field dynamics
  - Example: decomposition of the off-shell propagator is frame dependent
    - if $p_0 = \{P_0, \mathbf{0}\}$ then $\dfrac{1}{m - \not{p}_0} = \sum_s \left\{ \dfrac{u(\mathbf{0},s)\,\bar{u}(\mathbf{0},s)}{m - P_0} - \dfrac{v(\mathbf{0},s)\,\bar{v}(\mathbf{0},s)}{m + P_0} \right\}$

      if P_0=0, then this is an equal mixture of particle and antiparticle
    - if the propagator is boosted to $p = \left\{\sqrt{P_0^2 + \mathbf{p}^2}, \mathbf{p}\right\}$ then as $\mathbf{p} \to \infty$, the propagator contains positive energy components ONLY

$$\frac{1}{m - \not{p}} = \left(\frac{m}{\sqrt{m^2 + \mathbf{p}^2}}\right) \sum_s \left\{ \frac{u(\mathbf{p},s)\bar{u}(\mathbf{p},s)}{\sqrt{m^2 + \mathbf{p}^2} - \sqrt{P_0^2 + \mathbf{p}^2}} - \frac{v(\mathbf{p},s)\bar{v}(\mathbf{p},s)}{\sqrt{m^2 + \mathbf{p}^2} + \sqrt{P_0^2 + \mathbf{p}^2}} \right\}$$

$$\Rightarrow \frac{2m}{m^2 - P_0^2} \sum_s u(\mathbf{p},s)\bar{u}(\mathbf{p},s) \ \text{ if } \mathbf{p} \to \infty$$

  - The positive/negative energy mixture depends on the frame!

The landscape of electron scattering: Q^2 versus W

- There are two variables that characterize the photon

$$W^2 = (P_T + q)^2 = M_T^2 + Q^2\left(\frac{M_T}{mx} - 1\right)$$

$$\nu = E_\gamma = \frac{q \cdot P_T}{M_T} = \frac{Q^2}{2mx} \geq \frac{Q^2}{2M_T}$$

- For deuteron photodisintegration:

$$W^2 = (p_1 + p_2)^2 = 4m^2 + 2mE_{LAB}$$
$$= (P_T + q)^2 = M_d^2 + 2M_d\nu$$
$$\Rightarrow E_{LAB} \cong 2\nu$$

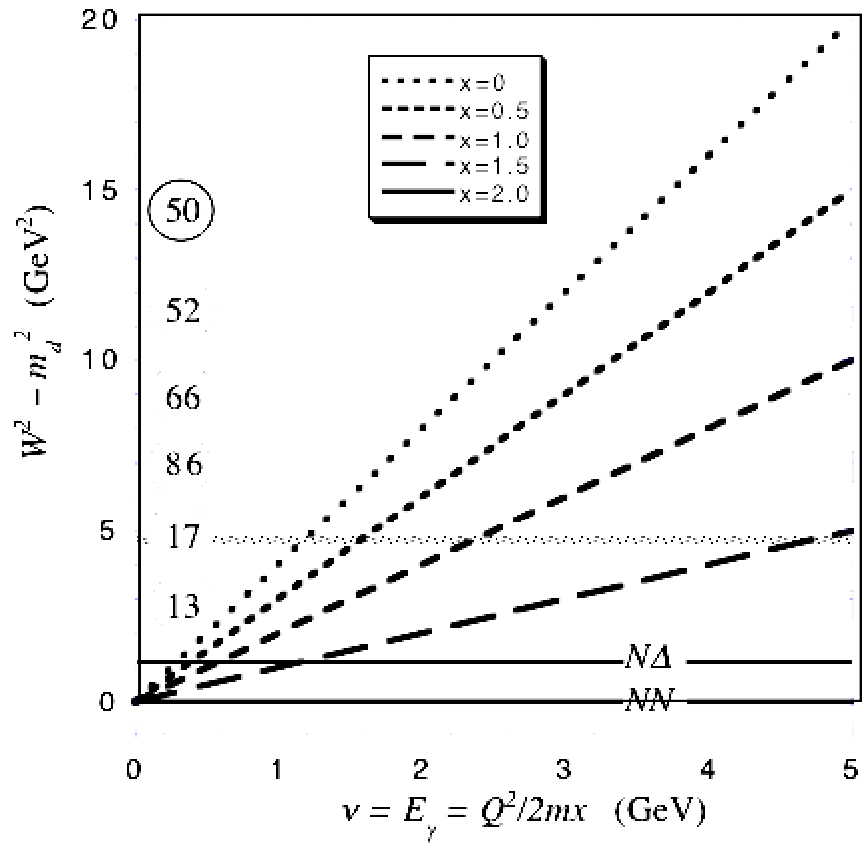
at E_γ = 12 GeV the NN system is excited to E_{LAB} = 24 GeV!!

- For deuteron elastic scattering:

$$W^2 = M_d^2 \text{ for ALL } Q^2$$

and the final state is NOT excited at all!

- These study very different regions of physics -->



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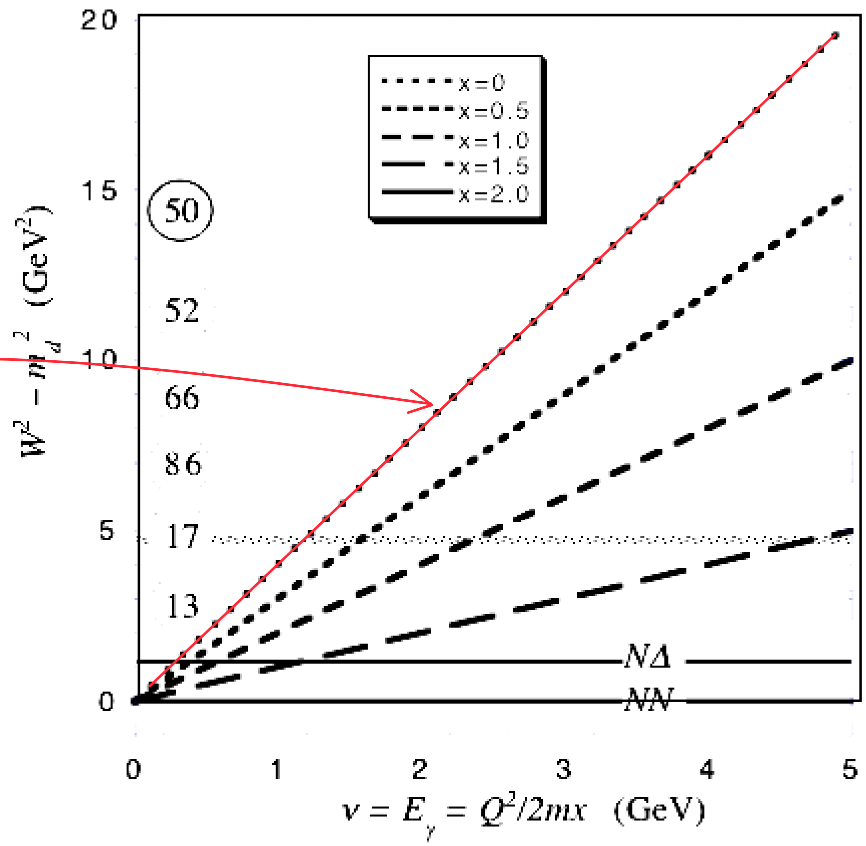

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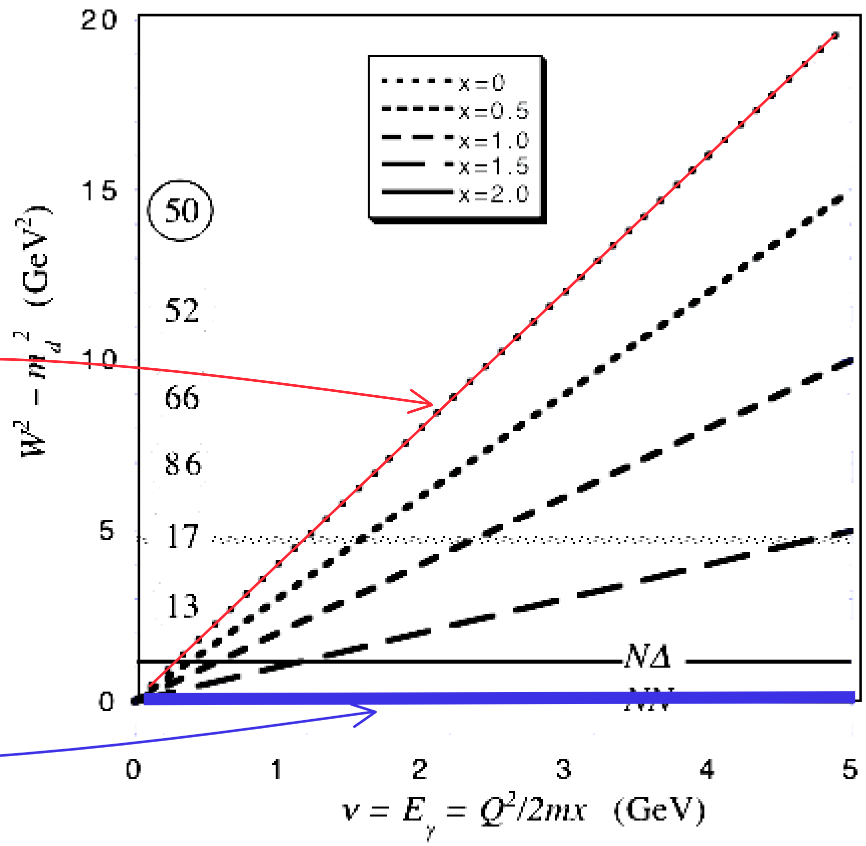

Overall Conclusions

- Jefferson Lab at 12 GeV will require relativistic calculations
  - Even for elastic scattering, recoil will be large. Is it sufficient to treat recoil correctly? Do quark degrees of freedom remain frozen out of elastic scattering observables, even as $Q^2 \to \infty$?
  - DIS explicitly uncovers the quark degrees of freedom. How deep does DIS have to be before these set in?
  - Light-front is the king for DIS, but not for weakly inelastic scattering. How does the transition from instant-forms of field dynamics (CST for example) to the light-front occur? (Or, should we use light-front dynamics at all energy scales? Not to my taste.)
- Key issues:
  - How do we renormalize? Can we completely eliminate renormalization parameters (so far, only for bubbles it seems)?
  - Can chiral perturbation theory constrain field dynamical models at low energy? How?
  - To what extent do your predictions depend on the relativistic formalism we use? If they do depend on it, can we understand the differences?

END