Covariant calculation of the N and $N\to\Delta$ form factors*

XXth European Conference on Few-Body Problems in Physics
Pisa, Italy; September 10, 2007

Franz Gross, JLab and W&M

Outline:

I. Background; definition of "shape"

II. "Fixed-axis" polarization states

III. The S-wave model: parameters and results

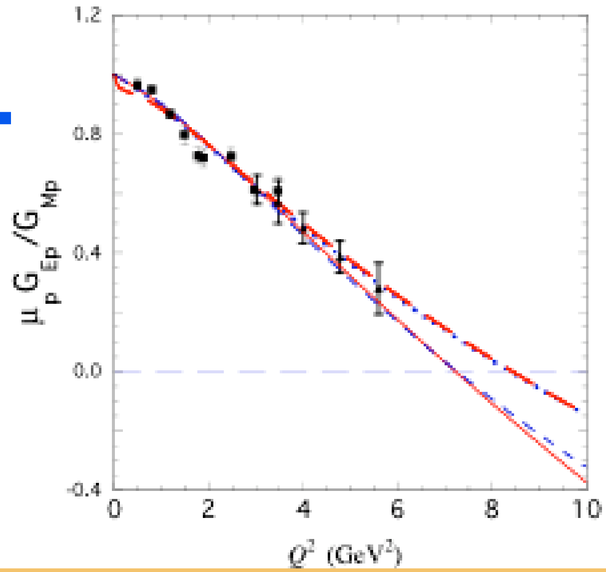
IV. Brief comparison with light-front wave function

---

*Work done jointly with Gilberto Ramalho and Teresa Peña

Background for this talk (1)

- ★ Beautiful data showing decrease in the ratio $R_p = G_{Ep}/G_{Mp}$ with Q^2



- ★ JLab workshop of May 2002 -- the proton is deformed!
- ★ Sept. 2002: article in USA today. ⇒

Physicists thrown for a loop ; New insight on protons changes the shape of things; [FINAL Edition]
Dan Vergano. **USA TODAY.** McLean, Va.: Sep 23, 2002. pg. D.07
Copyright USA Today Information Network Sep 23, 2002
Science and education

The humble proton, an atomic particle with mysteries long thought solved, turns out to have a hidden secret, scientists report.

Experimental results released this year by the Department of Energy's Jefferson Lab in Newport News, Va., ... suggestion that protons...aren't round. Instead, they seem somewhat elliptical.

"The proton is the simplest thing around, and it is not spherical," says physicist Charles Glashausser ... .

Background for this talk (2)

- ★ My paper with Peter Agbakpe [Phys. Rev. C **73**, 015203 (2006)] showing that the data can be fit with a spherical proton.
- ★ Email correspondence from Miller saying that our proton was "not round" and paper by Kvinikhidze and Miller [Phys. Rev. C **73**, 065203 (2006)]

  "This behavior indicates that the sum of the orbital angular momentum of the quarks in the proton is nonvanishing"

  "The nucleon is far from round in each of the models considered, and this arises from the relativistic nature of each."
- ★ Work with Gilberto Ramalho and Teresa Pena on N -> Δ transition and realization that a simpler model of the proton, with *ONLY* S-wave components, was possible.

  Recent Papers and controversy:

  1. "FG, G. Ramalho and M.T. Pena, *A pure S-wave covariant model for the nucleon* arXiv:nucl-th/0606029 [rejected by PRC because "too many parameters" -- relpaced by Ref. 4 below.].
  2. A. Kvinikhidze and G.A. Miller *Subtleties of Lorentz invariance and shapes of the nucleon*, Phys. Rev. *C***76**:025203,2007.
  3. FG, G. Ramalho and M.T. Pena *Comment on the "Subtleties of Lorentz invariance and Shapes of the Nucleon"* arXiv:0708.0995 [nucl-th]. Retitled: *Fixed-axis polarizations states: covariance and comparison*
  4. FG, G. Ramalho and M.T. Pena, *A pure S-wave covariant model for the nucleon,* (submitted to PRC) [replaces Ref. 1; much expanded with lots of new physics],
  5. G. Ramalho, M.T. Pena, and FG, *A covariant model for the nucleon and the Δ,* (to be submitted)

What are the issues?

- What do we mean when we say the proton is not "round"?

  I will not discuss this here any more. See our papers

- Is it possible to construct a pure *S*-wave model of the nucleon and the Δ ?

  We say yes. This is easily done using new "fixed-axis" polarization states. This is the subject of this talk.

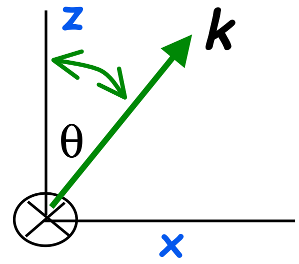
- What are "fixed-axis" polarization states and are they covariant?

  For discussion of this controversy, see Refs. 2 and 3

"Fixed-axis" polarization states: one of 2 choices

1. Helicity states are usually used to definite polarization. In the x-z plane, with $k=\{E_k, k\sin\theta, 0, k\cos\theta\}$,

$$\xi(0)=\frac{1}{m}\begin{bmatrix} k \\ E_k\sin\theta \\ 0 \\ E_k\cos\theta \end{bmatrix},\quad \xi(\pm)=\mp\frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ \cos\theta \\ \pm i \\ -\sin\theta \end{bmatrix}$$



note the dependence on θ, and the constraint $k\cdot\xi=0$

2. Alternatively, *if a vector particle is bound to a system* with 4-momentum $P=\{P_o, 0, 0, P\}$, then we can use "fixed-axis" states

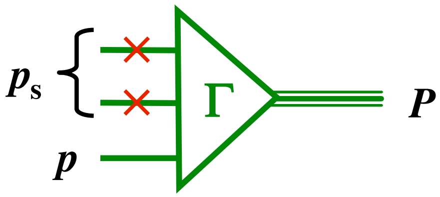
$$\varepsilon(0)=\frac{1}{M}\begin{bmatrix} \mathrm{P} \\ 0 \\ 0 \\ P_0 \end{bmatrix},\quad \varepsilon(\pm)=\mp\frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ \pm i \\ 0 \end{bmatrix}$$

these have no angular dependence, and describe only three degrees of freedom because of the constraint $P\cdot\varepsilon=0$

Wave functions using "fixed-axis" states: assumptions

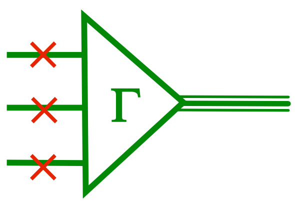
- ★ The nucleon consists of 3 constituent quarks (CQ) with a size, mass, and form factor given by the dressing of the quark in the sea of gluons and $q\bar{q}$ pairs. The sea quarks can be neglected.

- ★ Using the Covariant Spectator© theory, the nucleon and Δ is described by a 3-CQ vertex function with two of the CQ on shell



$$\Psi_\alpha = \left(\frac{1}{m - \not{p} - i\varepsilon} \right)_{\alpha\beta} \Gamma_\beta (P, p_s)$$

- ★ Confinement insures that this vertex function is zero when all three quarks are on shell (i.e. there is no 3q scattering)



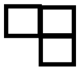
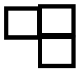
= 0 **Hence, model Ψ directly**

- ★ How should the wave function be modeled? Ockham's razor: *Start with the simplest case -- pure S-state with same spin-isospin structure as the nonrelativistic wave function.* See if it works!

Spin-isospin structure (1): nonrelativistic (NR) wave function

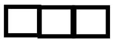
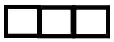
- ★ Fermion antisymmetry comes from the color factor $\varepsilon_{\alpha\beta\gamma}$
- ★ Assume a simple, fully symmetric S state spatial wave function
- ★ Then spin-isospin structure of the NR wave function must be symmetric
- ★ Spin-isospin 1/2 requires superposition of mixed symmetry states

$$\left|p\uparrow\right\rangle = \frac{1}{\sqrt{2}}\left\{\Phi_F^0\Phi_S^0 + \Phi_F^1\Phi_S^1\right\}$$

- ★ Spin-isospin 3/2 requires pure symmetric states

$$\left|\Delta\uparrow\right\rangle = \bar{\Phi}_F^1\bar{\Phi}_S^1$$

Spin-isospin structure (2): nucleon

- Spin-isospin structure of the NR nucleon wave function (cont'd)

  introduce a mathematically compact form; suppress name of scalar diquark

$$|sf\rangle = \frac{1}{\sqrt{2}}\left\{\chi^s\chi^f \quad - \quad \frac{1}{3}\left[\sigma\cdot\xi_m^*\,\chi^s\right]\left[\tau\cdot\xi_F^*\,\chi^f\right]\right\}$$

(0,0) diquark
scalar spin
scalar flavor

(1,1) diquark
axial vector spin
vector flavor

$$\chi^{\frac{1}{2}} = \begin{pmatrix}1\\0\end{pmatrix} \qquad \chi^{-\frac{1}{2}} = \begin{pmatrix}0\\1\end{pmatrix}$$

- Relativistic wave function, including spin-flavor structure

$$\Psi_N(sf) = \frac{1}{\sqrt{2}}\left\{u(P,s)\chi^f + \frac{1}{3}\left(\gamma^5 \not{\xi}_m^*\right)u(P,s)\left[\xi_F^*\cdot\tau\,\chi^f\right]\right\}\phi(P,p_s)$$

$$u(P,s) = N\begin{bmatrix}1\\ \dfrac{\sigma\cdot\mathbf{P}}{E_P+M}\end{bmatrix}\chi^s$$

- When P = 0, the lower component is 0 and this reduces *exactly* to the nonrelativistic form

This is a fixed-axis state

Spin-isospin structure (3): Delta

★ Spin-isospin structure of the Delta wave function is pure (1,1); diquark with axial-vector spin and vector flavor

$$\Psi_\Delta(sf) = -\left[\xi_F^* \cdot T\, \tilde{\chi}^f\right] \varepsilon_m^{\mu *} w_\mu(P,s)\, \phi(P,p_s)$$

- where $\varepsilon_m^{\mu *}$ is a fixed-axis axial-vector polarization
- $w_\mu(P,s)$ is a Rarita-Schwinger wave function satisfying

$$\gamma^\mu w_\mu = 0; \quad P^\mu w_\mu = 0$$

- T^i is an isospin 3/2 -> 1/2 transition operator
- $\tilde{\chi}^f$ is the isospin state of the Δ

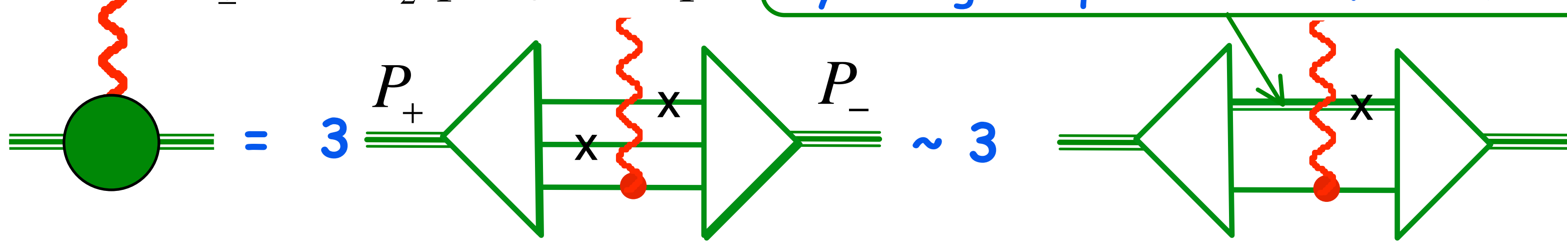
★ when P = 0, the lower component is zero and this reduces *exactly* to the nonrelativistic form

Relativistic impulse approximation for the N form factor

In the spectator theory, the photon couples to the off-shell quark, and because of the symmetry, the coupling to all three quarks is 3 times the coupling to one

$$P_\pm = P \pm \tfrac{1}{2} q \qquad Q^2 = -q^2$$

approximate the two spectator quarks by a single diquark with a fixed mass



$$J_I^\mu = \overline{u}(P_+,\lambda')\,\frac{3}{2}\int \frac{d^3p_s}{(2\pi)^3 2E_s(p_s)}\phi(P_+,p_s)\phi(P_-,p_s)\left\{ j_I^\mu - \frac{1}{9}\gamma^\nu\gamma^5\tau_j j_I^\mu \tau_j\gamma^5\gamma^{\nu'} D_{\nu\nu'}\right\} u(P_-,\lambda)$$

integrate over the (on-shell) spectator three momentum

quark currents with form factors

sum over the vector diquark polarization $D_{\nu\nu'} = \sum_\lambda \varepsilon_\nu \varepsilon^*_{\nu'}$

$$\phi(P\cdot p) = \frac{N_0}{\left(\alpha_1 + \chi(P\cdot p)\right)\left(\alpha_2 + \chi(P\cdot p)\right)}$$

Quark form factors (based on vector meson dominance)

★ The quark currents are

$$j_I^{\mu} = j_1\left(\gamma^{\mu} - \frac{\not{q}\, q^{\mu}}{q^2}\right) + j_2\,\frac{i\sigma^{\mu\nu}q_{\nu}}{2M},$$

$+$ is isoscalar

$-$ is isovector

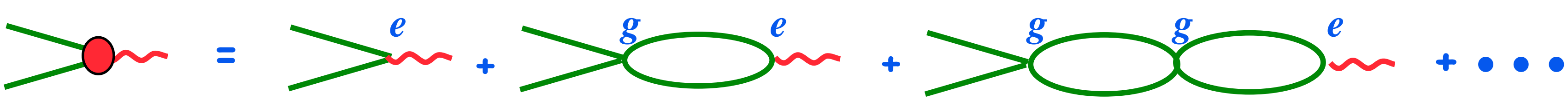
with 4 form factors:

$$j_i = \tfrac{1}{6} f_{i+}(Q^2) + \tfrac{1}{2}\tau_3 f_{i-}(Q^2)$$

$f_{1\pm}(0) = 1$

$f_{2\pm}(0) = \kappa_{\pm}$ quark anomalous moment

★ The quark form factors come from vector dominance



in a simple bubble model

$$f\left(Q^2\right) = e + gB\left(Q^2\right)e + gB\left(Q^2\right)gB\left(Q^2\right)e + \cdots = e + \frac{gB\left(Q^2\right)e}{1 - gB\left(Q^2\right)}$$

if $gB\left(Q^2\right) = \dfrac{\lambda^2}{\Lambda^2 + Q^2}$, then $f\left(Q^2\right) = e + \dfrac{\lambda^2 e}{\Lambda^2 - \lambda^2 + Q^2}$

★ We use

$$f_{1\pm} = \lambda + \frac{1-\lambda}{1 + Q_0^2/m_v^2} + \frac{c_{\pm}\, Q_0^2/M_h^2}{\left(1 + Q_0^2/M_h^2\right)^2}$$

3 parameters: λ fixed by DIS; $c_{\pm}$ fit

$$f_{2\pm} = \kappa_{\pm}\left(\frac{d_{\pm}}{1 + Q_0^2/m_v^2} + \frac{(1 - d_{\pm})}{1 + Q_0^2/M_h^2}\right)$$

4 parameters: $\kappa_{\pm}$ fixed by moments; $d_{\pm}$ fit

The sum over the diquark polarization is tricky

- First, must be in a collinear frame so that the fixed-axes (tied to the nucleon and Δ momentum) are identical
- Then, the most general form is

$$D^{\mu\nu} = \sum_{\lambda} \varepsilon_{+}^{\mu} \varepsilon_{-}^{\nu *} = \left(-g^{\mu\nu} + \frac{P_{-}^{\mu} P_{+}^{\nu}}{} \right) + a_1 \left(P_{-} - \frac{bP_{+}}{M_{+}^{2}} \right)^{\mu} \left(P_{+} - \frac{bP_{-}}{M_{-}^{2}} \right)^{\nu}$$

with $b = (P_{+} \cdot P_{-})$ **and** $a = \dfrac{M_{+} M_{-}}{b(M_{+} M_{-} + b)}$

- For equal masses this becomes

$$D^{\mu\nu} = -g^{\mu\nu} - \frac{P_{+}^{\mu} P_{-}^{\nu}}{M^{2}} + 2 \frac{P^{\mu} P^{\nu}}{P^{2}}$$ **where** $P = \frac{1}{2}(P_{+} + P_{-})$

Results: Nucleon form factors

- ★ Nucleon Form factors
- ★ Four models χ^2/N

I (4 parameters)

$\alpha_1 \quad \alpha_2$

$c_+ = c_-$

$d_+ = d_-$

quarks with isospin symmetry

(dotted line) **9.26**

II (5 parameters)

$\alpha_1 \quad \alpha_2$

$c_+ \neq c_-$

$d_+ = d_-$

break charge symmetry

(dashed line) **1.36**

Best phenomenology!

III (6 parameters)

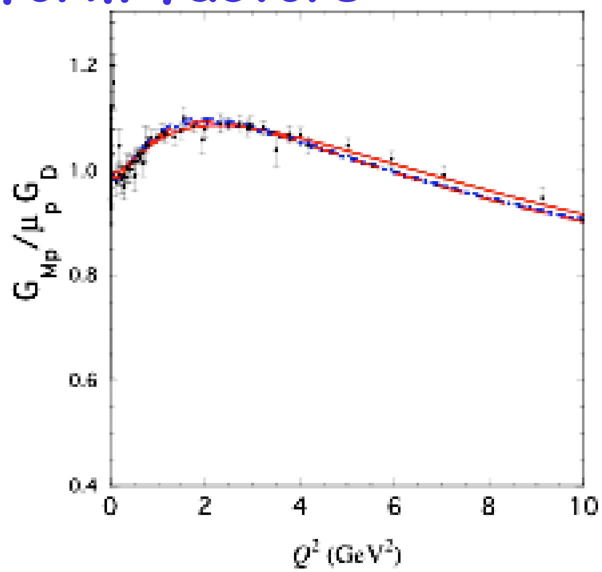
isospin symmetry with pion clouds

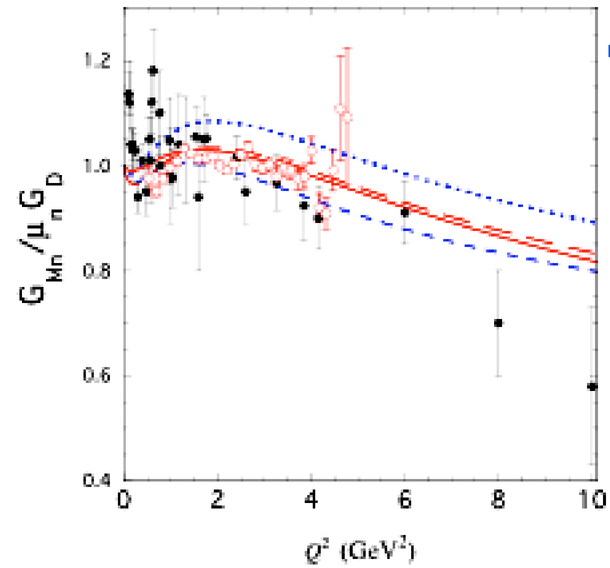
(long-dashed line) **1.85**

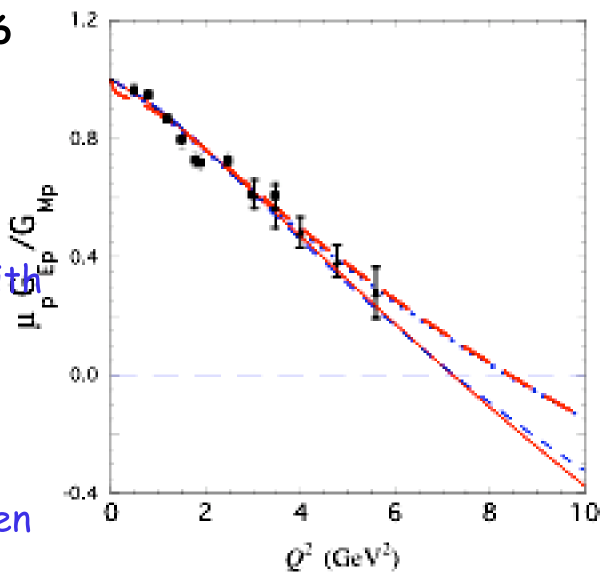
IV (9 parameters)

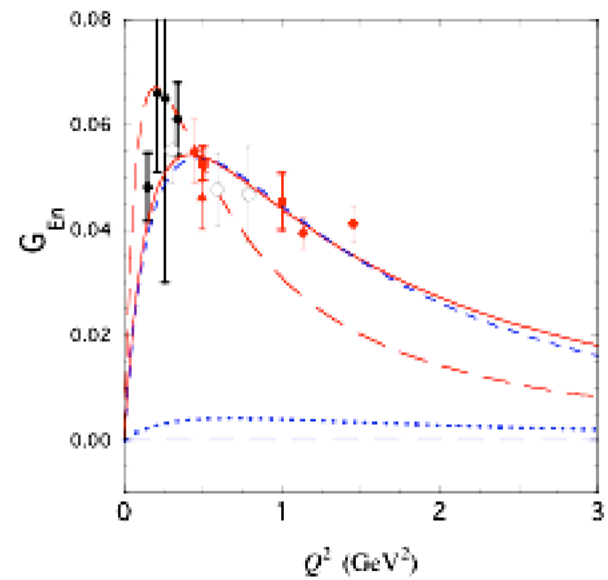
pion cloud and broken isospin symmetry

(solid line) **1.03**









Quark distribution function from DIS

- ★ Our model predicts the quark distribution amplitude measured in DIS
- ★ Our normalization gives

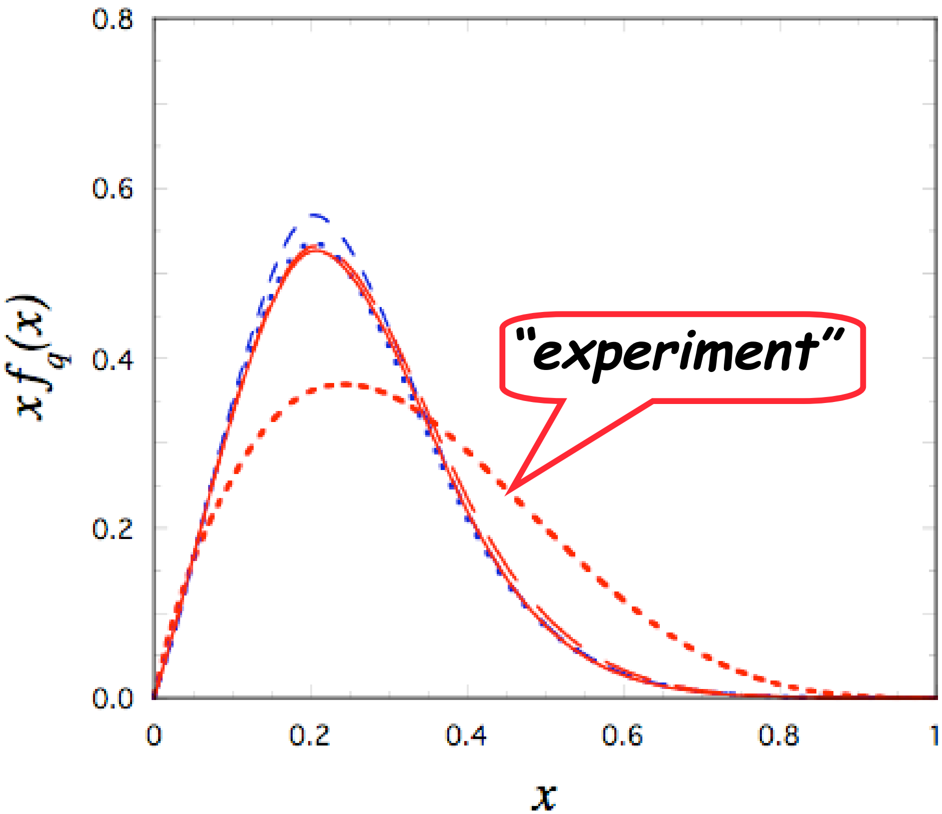
$$1 = \int_{-\infty}^{1} dx f(x) \quad \textbf{instead of} \quad 1 = \int_{0}^{1} dx f(x)$$

- ★ Choose quark charge at $Q^2=\infty$ to be $\lambda>1$, where

$$\int_{0}^{1} dx f(x) = \frac{1}{\lambda^2} < 1$$

- ★ Choose diquark mass to give experimental momentum fraction

$$\frac{\langle xf \rangle}{\langle f \rangle} = \frac{\int_{0}^{1} dx\, xf(x)}{\int_{0}^{1} dx f(x)} = 0.171$$



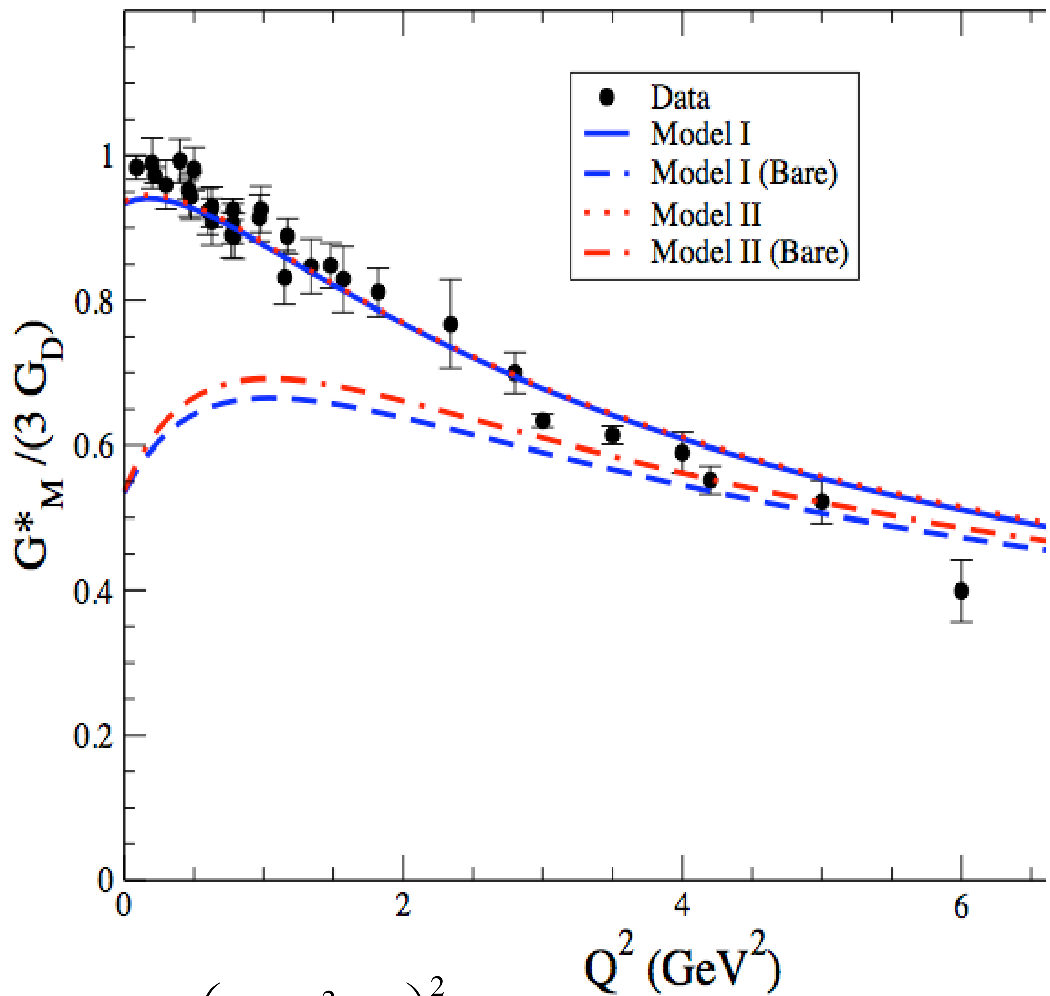
Results: N -> Δ transition with PURE *S*-wave states

- Three form factors, but ONLY $G^*_M \neq 0$ if BOTH the N and Δ wave functions are pure S-wave.
- The value $G^*_M(0)$ cannot equal the correct value unless a separate pion cloud term is added, because of the Schwartz inequality

$$G^*_M(0) = \frac{8}{3\sqrt{3}}\left(\frac{m}{M+m}\right) j_- \int \psi_\Delta \psi_N$$

$$= 2.07 \int \psi_\Delta \psi_N, \quad \text{and}$$

$$\int \psi_\Delta \psi_N \le \sqrt{\int |\psi_N|^2} \sqrt{\int |\psi_\Delta|^2} \le 1$$



- Fit done with an empirical pion cloud term of the form

$$\frac{G^\pi_M}{3G_D} = \lambda_\pi \left(\frac{\Lambda_\pi^2}{\Lambda_\pi^2 + Q^2}\right)^2$$

Discussion and Implications

- ★ The data do not require the proton to be deformed

  This is forbidden by quantum mechanics (unless there are rotational bands),

- ★ or even that it contain components with $\ell > 0$!

  This model is a counter example to claims that the data cannot be explained by a spherical proton

- ★ The data do give interesting new information about the CQ form factors, and tell us about the proton wave function.

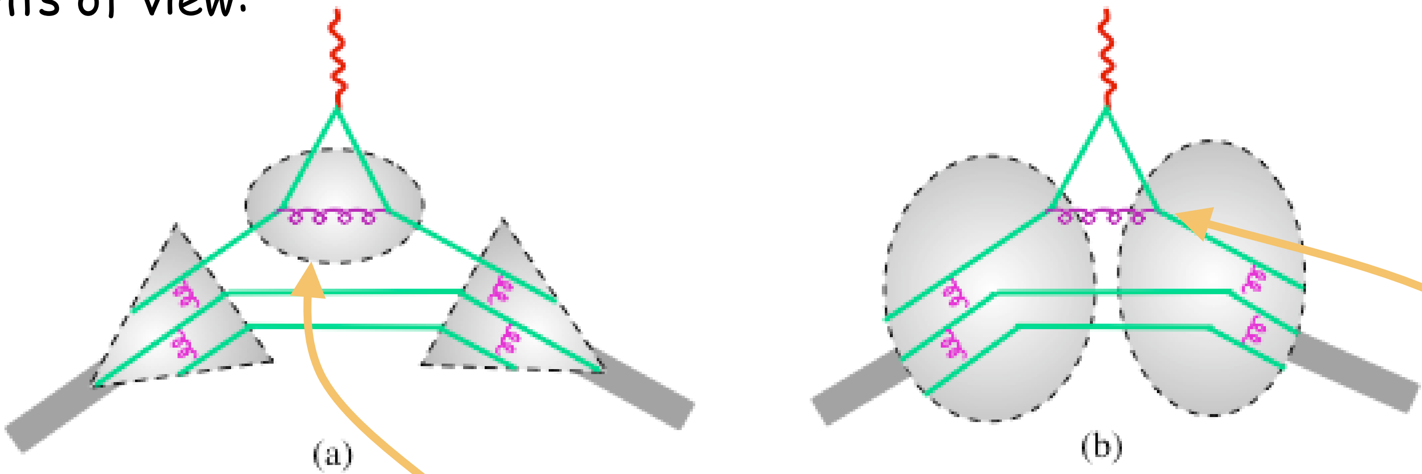
- ★ The N->Δ form factors *require* $\ell > 0$ components. A spherical Δ and a spherical N gives $G_C = G_E = 0$

- ★ The model is so simple that it can be used to study many phenomena near the quark-hadron transition

- ★ Predictions
  - G_{Ep} will change sign near $Q^2 \sim 8$ GeV^2
  - $\ell > 0$ components are required in either the N or Δ. This is currently under study

Does $F_2 > 0$ require $\ell > 0$? [Angular momentum theorem]

★ Answer to this question depends on the formalism. There are two points of view:



CS view: All interactions involving gluon exchange between the $q\bar{q}$ pair coupled to the photon are included in quark form factors; including the quark anomalous moments

Light-front view: nucleon wave function is a sum over Fock components; quark "structure" comes from higher Fock components.

The light-front view requires $\ell > 0$ components just to give $\kappa_\pm \neq 0$

END