Relativistic Description of Few-Nucleon Systems

LECTURES: 2007 PRAGUE SUMMER SCHOOL

Franz Gross JLab and W&M

Lecture IV

Electromagnetic Interactions: gauge invariance and effective current operators

- ***General method for doing gauge invariant calculations in systems composed of composite particles.***
- ***What can be learned from high energy electron scattering experiments?***

Jefferson Lab

Outline

- General method for obtaining a gauge invariant current
- Gauge invariant two-body current operator in CS theory
- Gauge invariant three-body current operator in CS theory
- Implications of energy dependent interactions
- Construction of off-shell single nucleon current
- Applications: deuteron form factor
- Applications: deuteron photodisintegration
- Conclusions

References

- Current conservation and interaction currents for relativistic theories with composite particles

  F. G, and D. O. Riska, PRC 36, 1928 (1987)

- Isoscalar Meson Exchange Currents and the Deuteron Form Factors.

  FG and H. Ito, Phys. Rev. Letters **71**, 2555 (1993)

  J. W. Van Orden, N. Devine, and FG, Phys. Rev. Letters **75**, 4369 (1995)

- Electromagnetic interactions for the two-body CS equations

  J. Adam, Jr., J. W. Van Orden, and FG, Nucl. Phys. **A640**, 391 (1998)

- Electromagnetic structure of the deuteron (review paper)

  R. Gilman and FG, J. Phys. G: Nucl. Part. Phys. **28**, R37-R116 (2002)

- Inelastic ed scattering: predictions of the RIA

  J.Adam, Jr., FG, Sabine Jeschonnek, Paul Ulmer, and J. W. Van Orden, Phys. Rev. C **66**: 044003 (2002)

- Electromagnetic interactions for three-body CS equations

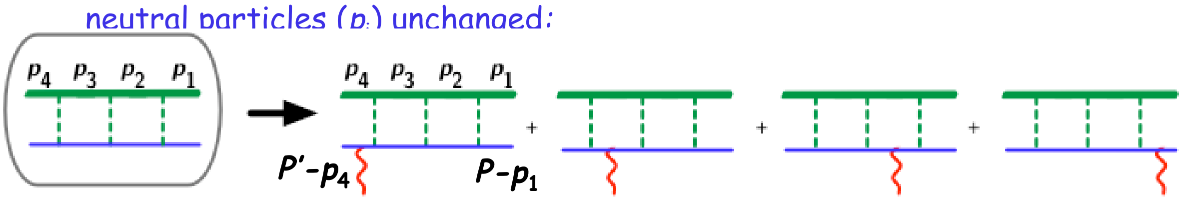
  S. Kvinikhidze and R. Blankleider PRC 56, 2973 (1997)

  FG, Alfred Stadler, and Teresa Pena, Phys. Rev. C 69: 034007 (2004)

  J. Adam, Jr. & Van Orden PRC 71: 034003 (2005)

Gauge invariance with point particles

- Proof that the ladder sum is gauge invariant, order by order (Feynman's proof). First, couple the photon to ALL places where there is a charged particle (only the blue particle below) and keep the labeling of the neutral particles (p_i) unchanged:



$$J^\mu = \iint dp_2 dp_3 \mathbb{C}(p_1 p_2 p_3 p_3)\left\{\gamma^\mu S_4 S_3 S_2 + \bar{S}_3 \gamma^\mu S_3 S_2 + \bar{S}_3 \bar{S}_2 \gamma^\mu S_2 + \bar{S}_3 \bar{S}_2 \bar{S}_1 \gamma^\mu\right\}$$

where $S_i = S(P-p_i)$; $\bar{S}_i = S(P'-p_i)$; $P' = P+q$

- Use the Ward-Takashi (WT) identity

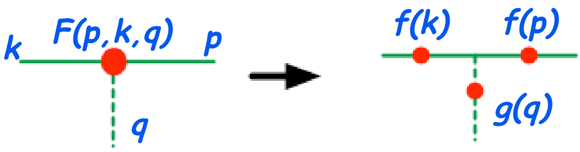
$$q_\mu \gamma^\mu = \left((P'-p_i)-(P-p_i)\right)_\mu \gamma^\mu = \left(m-(\not{P}-\not{p}_i)\right)-\left(m-(\not{P}'-\not{p}_i)\right) = S_i^{-1} - \bar{S}_i^{-1}$$

to reduce the result

$$q_\mu J^\mu = \iint dp_2 dp_3 \mathbb{C}(p_1 p_2 p_3 p_3)\left\{\cancel{S_3 S_2} + \left(\cancel{\bar{S}_3 S_2} - \cancel{S_3 S_2}\right) + \left(\cancel{\bar{S}_3 \bar{S}_2} - \cancel{\bar{S}_3 S_2}\right) - \cancel{\bar{S}_3 \bar{S}_2}\right\} = 0$$

Generalization to particles with form factors*

- Require that form factors at vertices have the factorized form $F(k,p,q)=f(k)f(p)g(q)$ with *universal* form factors for each particle



- Three steps
  - Reinterpret vertex form factors as dressings of the particle propagators (can be done if they are universal)
  - Construct one body currents satisfying a WT identity for the dressed propagators. Construct contact interactions satisfying WT identities.
  - Using relativistic equations, derive current operator (that effectively couples photon to all charged particles and all "charged" contact interactions)
- Off-shell nucleon current operator is the solution of a WT identity with a nucleon propagator "dressed" by the form factor *f*

$$S_N(p)=\frac{\left(m+\not{p}\right)f^2(p)}{m^2-p^2-i\varepsilon}$$

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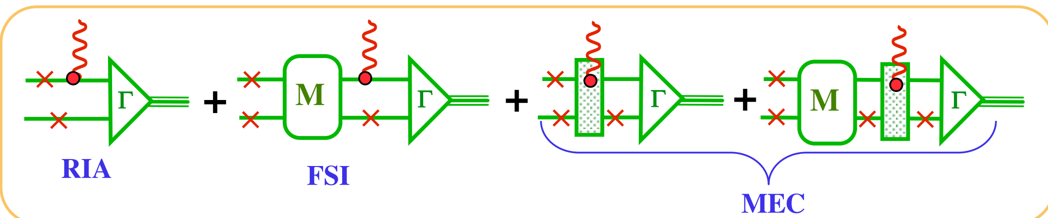
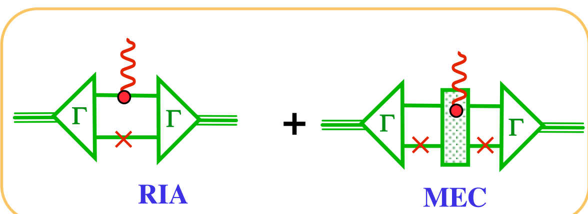
*F. G, and D. O. Riska, PRC 36, 1928 (1987)

Gauge invariant two body current operator in the CS© theory

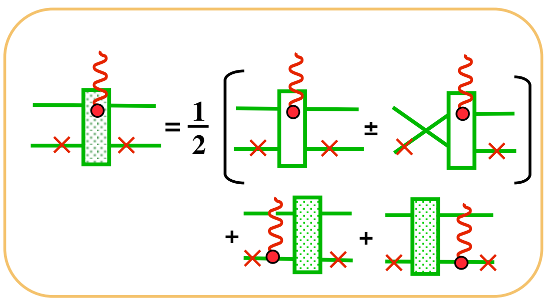
- Inelastic Scattering**



- Elastic Scattering



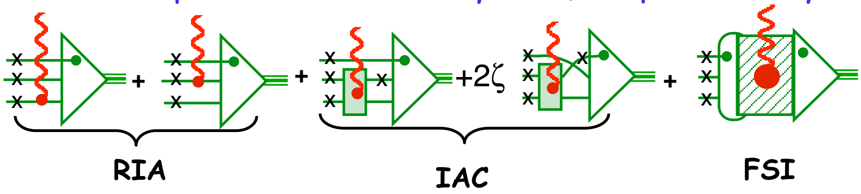
- Interaction current



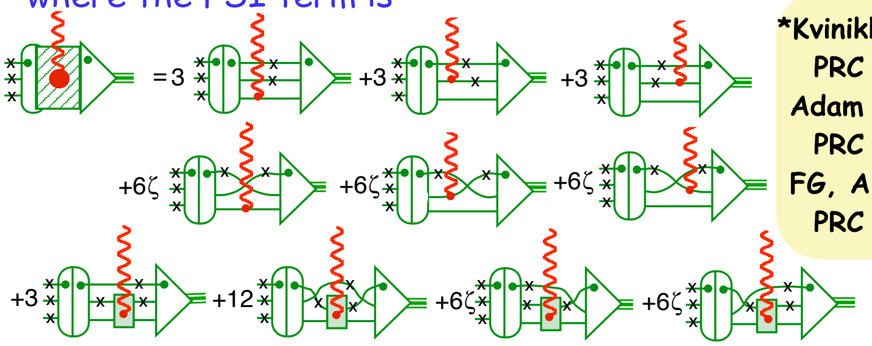
**J. Adam Jr., FG,S. Jeschonnek, P. Ulmer, and J. W. Van Orden, PRC 66: 044003 (2002).

Gauge invariant 3-body current operator in the CS© theory*

★ The gauge invariant* three-body breakup current in the spectator theory (with on-shell particles labeled by an x) requires many diagrams



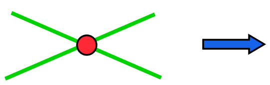
where the FSI term is



*Kvinikhidze & Blankleider, PRC 56, 2973 (1997)
Adam & Van Orden PRC 71: 034003 (2005)
FG, A. Stadler, & T. Pena PRC 69: 034007 (2004)

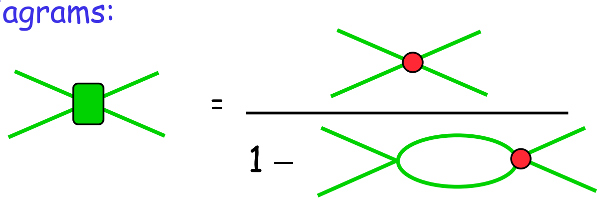
Implications of energy dependent interactions

★ **Lessons from the bubble sum (in 1+1 d for simplicity)** suppose the *NN* interaction is an energy dependent four-point coupling:



$a + \lambda\,(s-M_d^2)$

★ then the scattering amplitude is a geometric sum of bubble diagrams:



$$M = \frac{a+\lambda\left(s-M_d^2\right)}{1-B(s)\left[a+\lambda\left(s-M_d^2\right)\right]}$$

★ the bound state condition fixes a, but the energy dependent parameter λ is undetermined

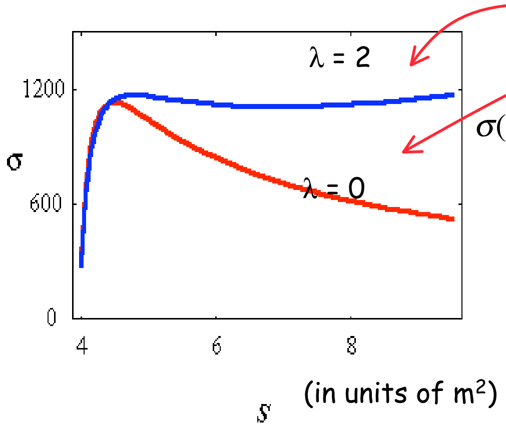
$$a\,B\left(M_d^2\right) = 1$$

Lessons from the bubble sum (2)

★ the deuteron wave function is independent of λ,

$$\Psi(p,M_d)=\frac{N}{\left(m^2-\left(\tfrac{1}{2}P+p\right)^2\right)\left(m^2-\left(\tfrac{1}{2}P-p\right)^2\right)};\quad P^2=M_d^2$$

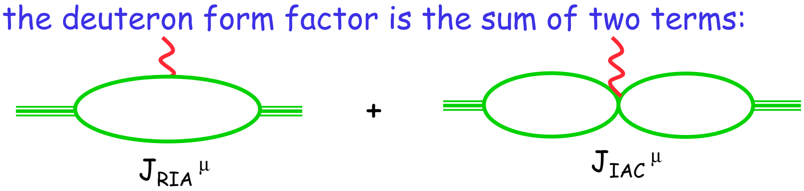
★ but the NN cross section is not:



$$\sigma(s)=\frac{1}{\sqrt{s}}\left|M(s)\right|^2;\qquad M=\frac{a+\lambda\left(s-M_d^2\right)}{1-B(s)\left[a+\lambda\left(s-M_d^2\right)\right]}$$

Lessons from the bubble sum (3)
"energy dependence comes with a price"

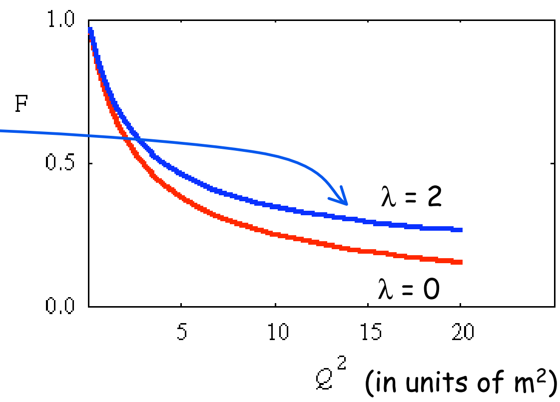
★ the deuteron form factor is the sum of two terms:



★ $\boxed{\text{the energy dependence of the interaction generates an interaction current (IAC) which depends on } \lambda}$

★ the IAC required by the interaction is unique and separately gauge invariant

$$J^{\mu}_{IAC}(Q^2)=\lambda\, N^2(P+P')^{\mu}B^2(Q^2)$$



★ FSI and IAC must be consistent with the dynamics! Calculations must be consistent.

The current is constrained by the WT identity

- ★ To conserve current, the current operator must satisfy the WT identity

$$\boxed{q_\mu j_N^\mu(p',p) = S^{-1}(p) - S^{-1}(p')} \longrightarrow S(p) = \frac{f^2(p)}{m-p} = \frac{f^2(p)}{\Delta_-(p)}$$

- ★ The spectator models use a *nucleon form factor, f(p).* This means that the nucleon propagator can be considered to be dressed. One solution (the simplest) is

off-shell effects

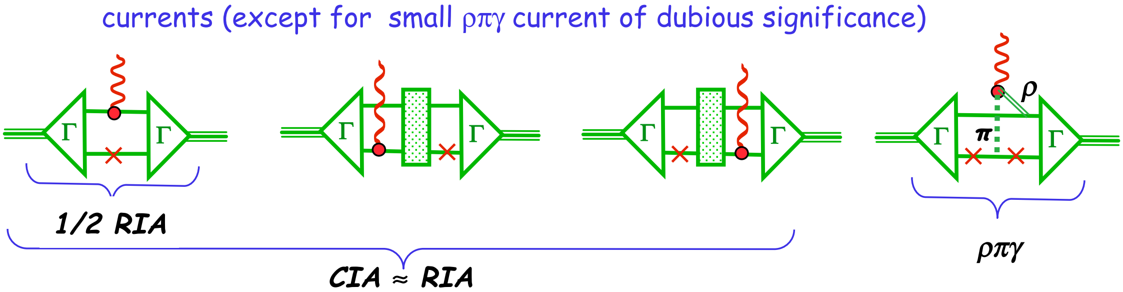
$$\boxed{j^\mu(p',p) = \boxed{F_0}\left\{F_1\,\gamma^\mu + F_2\,\frac{i\sigma^{\mu\nu}q_\nu}{2m}\right\} + \boxed{G_0\,F_3\,\Lambda_-(p')\,\gamma^\mu\,\Lambda_-(p)}}$$

$$F_0 = \frac{f(p)}{f(p')}\left(\frac{m^2-p'^2}{p^2-p'^2}\right) - \frac{f(p')}{f(p)}\left(\frac{m^2-p^2}{p^2-p'^2}\right) \qquad G_0 = \left(\frac{f(p')}{f(p)} - \frac{f(p)}{f(p')}\right)\frac{4m^2}{p^2-p'^2}$$

- ★ $F_3(Q^2)$ is unknown, except $F_3(0)=1$. EXPLOIT THIS FREEDOM

Applications

- ★ Deuteron form factors
  - Last calculation using the CS model was done in 1995*using an old model (called IIB) with NO off-shell couplings
  - This is a pure isoscalar transition process, with no isoscalar interaction currents (except for small $\rho\pi\gamma$ current of dubious significance)



  - New calculation using WJC-1 and WJC-2 with large interaction currents coming from ν dependent off-shell sigma couplings are needed.
- ★ Photodisintegration (a very different story)

*J. W. Van Orden, N. Devine, and FG, Phys. Rev. Letters 75, 4369 (1995)

Deuteron form factors definitions and observables

★ There are three form factors that enter the relativistic current:

$$-\langle d'|J^\mu|d\rangle = \left\{ G_1(Q^2)(\xi'^*\cdot\xi) - G_3(Q^2)\frac{(\xi'^*\cdot q)(\xi\cdot q)}{2M_d^2} \right\}(d+d')^\mu$$
$$+G_M(Q^2)\left[\xi^\mu(\xi'^*\cdot q) - \xi'^{*\mu}(\xi\cdot q)\right]$$

★ The physical combinations are:

$$G_C = G_1 + \tfrac{2}{3}\eta G_Q$$
$$G_Q = G_1 - G_M + (1+\eta)G_3$$

$G_C(0) = 1$ in units of e

$G_M(0) = \mu_d$ in units of $e/2M_d$

$G_Q(0) = Q_d$ in units of e/M_d^2

with $\eta = \dfrac{Q^2}{4M_d^2}$

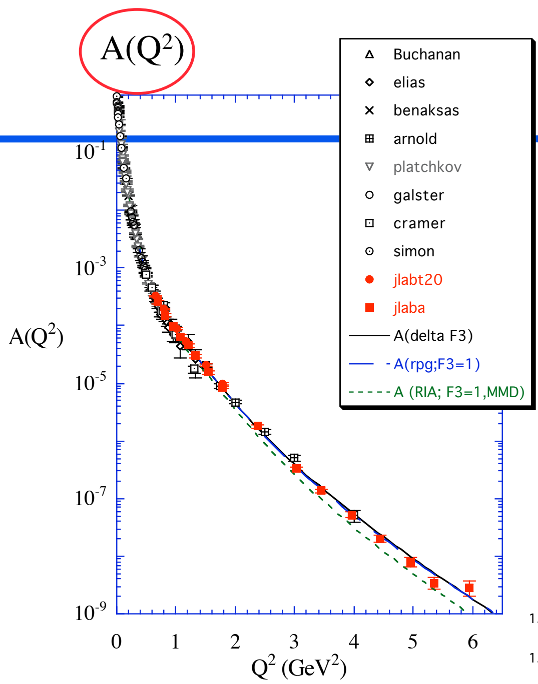
★ The observables are:

$$A(Q^2) = G_C^2(Q^2) + \frac{8}{9}\eta^2 G_Q^2 + \frac{2}{3}\eta G_M^2$$

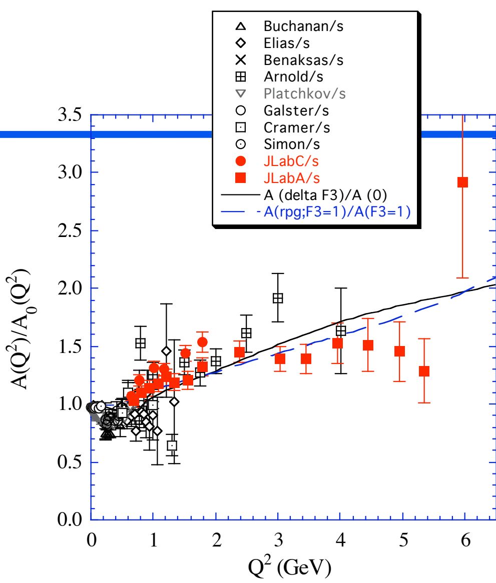
$$B(Q^2) = \frac{4}{3}\eta(1+\eta)G_M^2$$

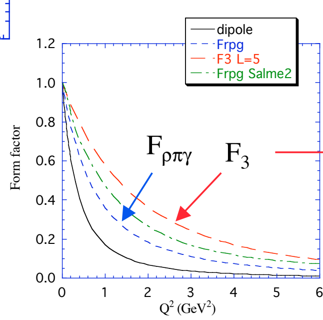
$$\tilde{T}_{20} = -\sqrt{2}\,\frac{y(2+y)}{1+2y^2}$$

$$y = \frac{2\eta G_Q}{3\,G_C}$$



Choice of a "hard" F_3 is sufficient for an excellent fit!



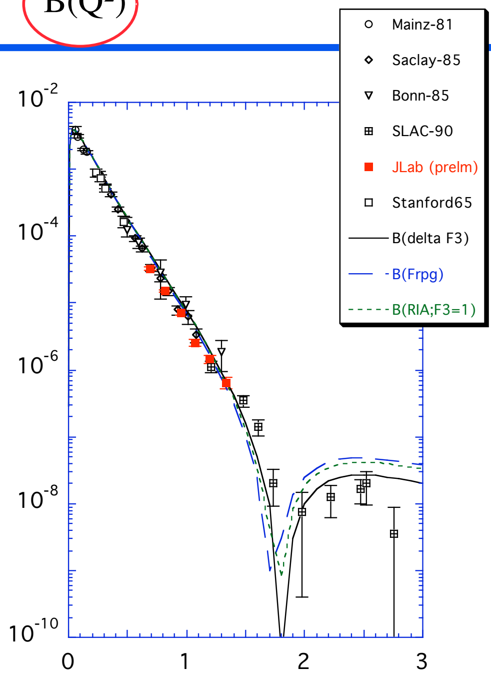


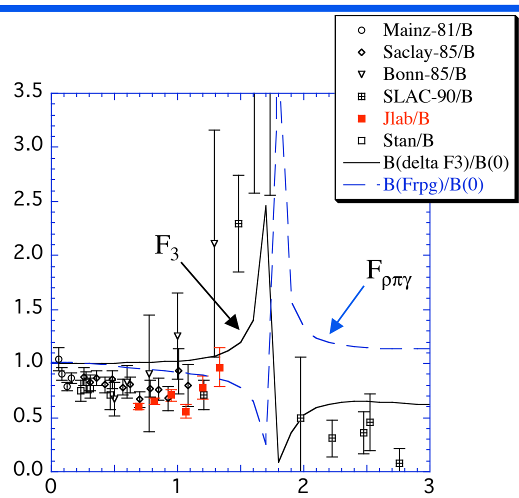
$F_{\rho\pi\gamma}$ F_3

$$F_3(Q^2) = \frac{1}{(1+Q^2/5)^3}$$

See R. Gilman & FG JPG, 28, R37-R116 (2002)

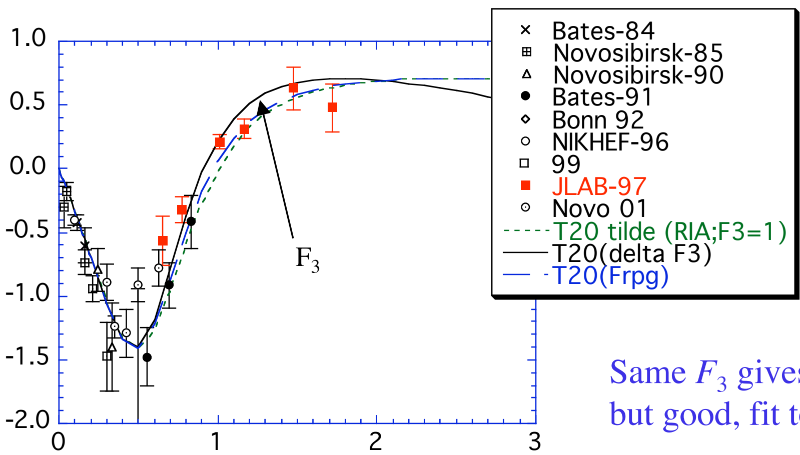
$B(Q^2)$





Same F_3 also works for $B(Q^2)$

$T_{20}(Q^2)$

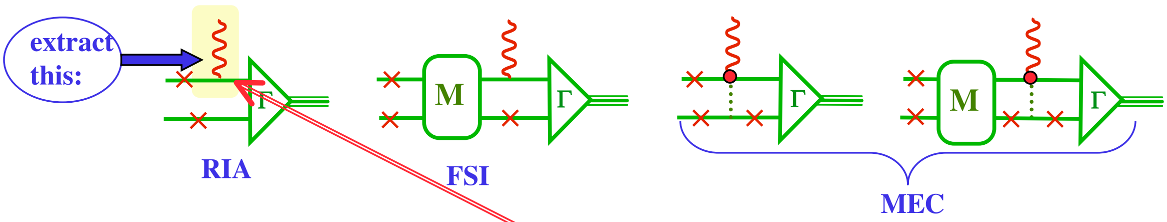


Same F_3 gives a different, but good, fit to T_{12}!

The Spectator theory, with a suitable F_3 , can explain the elastic electron deuteron scattering data!

Extracting free neutron properties

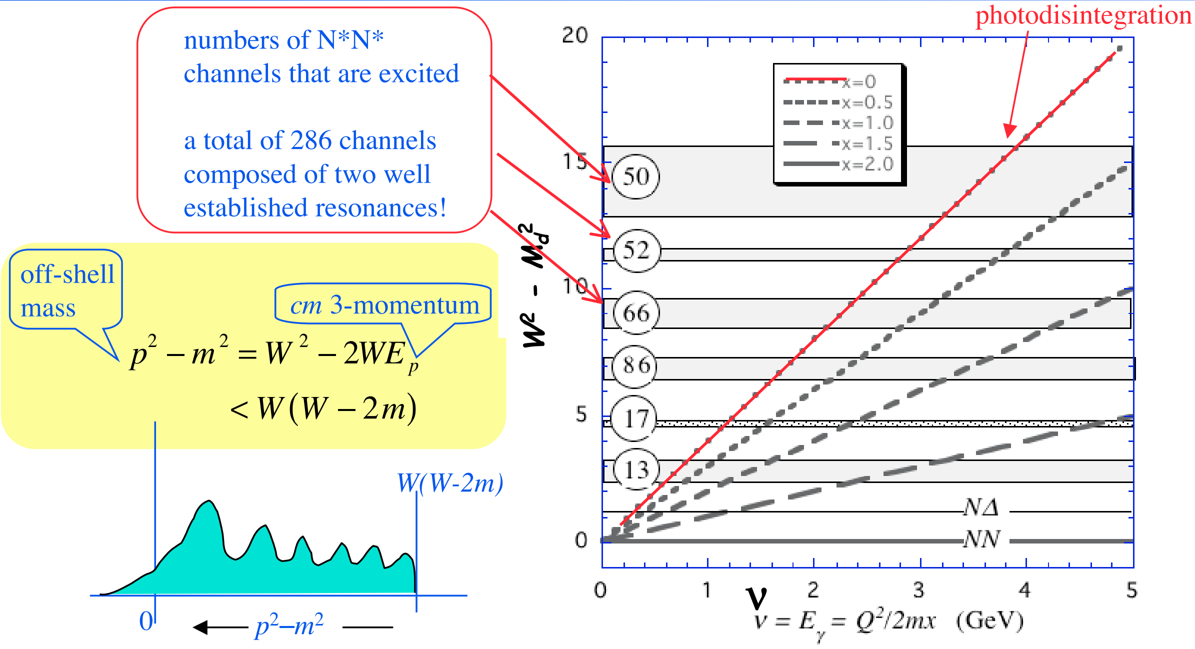
- ★ Spectator formalism ideal; but still many problems:
  - The impulse term is *well defined*, and distinct from the other contributions (FSI and MEC)



  - What kinematics will separating out the FSI, the proton contributions, or the contributions for *np* overlap (quark exchange)?
  - If the RIA diagram dominates, how are off-shell effects controlled?
- ★ How does this work in other formalisms?

Deuteron Photodisintegration

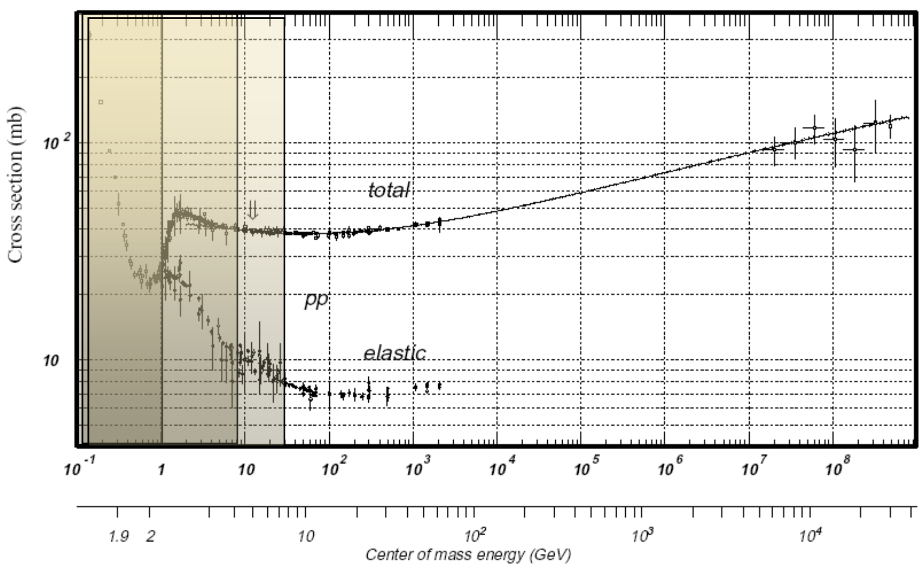
100's of channels excited in photodisintegration at 4 GeV*



IN DEUTERON PHOTODISINTEGRATION, THE "RIGHT-HAND" RESONANCES ARE EXPOSED

*R. Gilman & FG, JPG, 28, R37-R116 (2002)

total *NN* cross sections



12 Gev photons

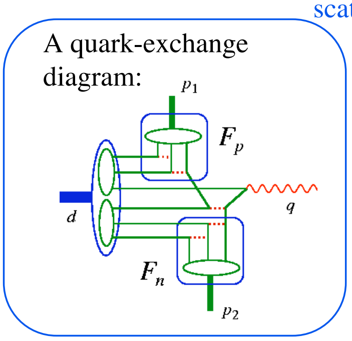
High energy photodisintegration probes deep into the inelastic region

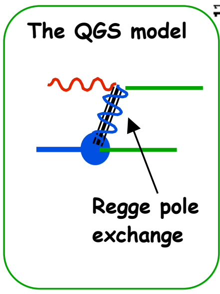
Smooth, scaling-like behavior at high energies

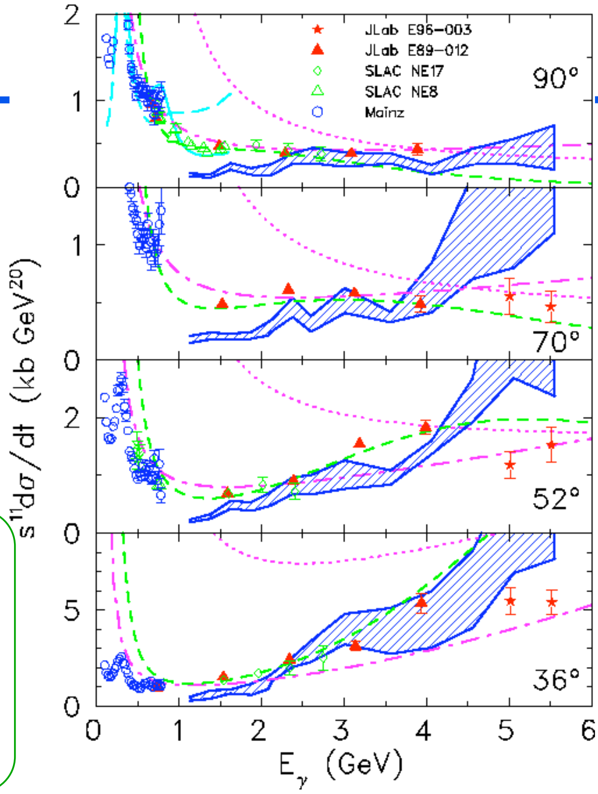
Conventional models fail (so far)

Quark-interaction models:

- Radyushkin
- Brodsky and Hiller (RNA)
- Kondratyuk, *et.al.* quark-gluon string model
- Frankfurt, Miller, Strikman, and Sargsian (final state *NN* scattering)







Conclusions: (form factors vs. photodisintegration)

- The deuteron form factors and deuteron photodisintegration probe completely different physics even though both are done with electrons of a few GeV.
- Form factors:
  - one nucleon is off-shell, but total mass of the final state is M_d
  - all resonances are frozen out
- Photodisintegration (or electrodisintegration)
  - both nucleons can be on-shell and mass of final state varies up to 8 GeV!
  - resonances and resonance channels are explicitly excited
  - proper description of high energy *NN* interaction (Eikonal or GEA) is essential

Conclusions

- ★ The CS theory works well for the deuteron form factors, showing that hadronic degrees of freedom are sufficient for the description of *elastic* scattering up to the highest Q^2 measured.
- ★ There is no evidence for the appearance of explicit quark degrees of freedom
- ★ BUT: these results must be confirmed using the new off-shell OBE models, and calculations of other few-body reactions are needed.
- ★ Photodisintegration of the deuteron *does* show the appearance of quark degrees of freedom.

END