Relativistic Description of Few-Nucleon Systems

LECTURES: 2007 PRAGUE SUMMER SCHOOL

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Lecture III

Results: Energies below the pion production threshold

- ***New, high precision fits to np data below 350 MeV lab energy, and the relativistic properties of the deuteron and triton.***
- ***What do these new results tell us about the nature of nuclear forces?***

Jefferson Lab

Outline

- CS theory with spin
- Structure of the deuteron wave functions
- Antisymmetrize the kernel
- Removal of spurious singularities
- One Boson Exchange (OBE) interaction with off-shell coupling
- Implications of off-shell couplings
- New model fits to the NN data
- Three-body binding energy and the role of off-shell couplings
- Changes in the phase shifts
- Scaling and rejecting certain data sets
- Conclusions

References

- Original spectator paper
  - FG, Phys. Rev. **186**, 1448 (1969)
- Effective NN potential and OBE models
  - FG, Phys. Rev. D **10**, 223 (1974)
  - J. W. Van Orden, FG, and K. Holinde, Phys. Rev. C **45**, 2094 (1992)
  - M. T. Peña, FG, and Yohanes Surya, Phys. Rev. C **54**, 2235 (1996); [Two-pion exchange potential and the π N amplitude]
  - FG and Alfred Stadler, arXiv:0704.1229 [nucl-th] (2007)
- Cancellations for scalar and chiral cases
  - FG, Phys. Rev. C **26**, 2203 (1982).
- Three body CS equations, and three body binding energy
  - FG, Phys. Rev. C **26**, 2226 (1982).
  - Alfred Stadler, FG, and Michael Frank, Phys. Rev. C **56**, 2396 (1997)
  - Alfred Stadler and FG, Phys. Rev. Letters **78**, 26 (1997)
- Normalization of three-body bound state vertices
  - J. Adam, Jr., FG, Cetin Savkli, J. W. Van Orden, Phys. Rev. C **56**, 641 (1997)
- Charge conjugation invariance of the spectator equations
  - FG, Few Body Syst. **30**, 21 (2001)

The Covariant Spectator© (CS) theory with spin

- In the *n* nucleon problem, make the following substitution for *n* – 1 nucleon propagators (with $\bar{u}_\alpha(\mathbf{p},s)\,u_\alpha(\mathbf{p},s') = 2m\,\delta_{ss'}$)

$$S_{\alpha\beta}(p) = \frac{(m+\not{p})_{\alpha\beta}}{m^2 - p^2 - i\varepsilon} \Rightarrow 2\pi i\,\delta_+(m^2 - p^2)\sum_s u_\alpha(\mathbf{p},s)\bar{u}_\beta(\mathbf{p},s)$$

- The off-shell propagator is (in the CM with $\boldsymbol{k} = \boldsymbol{P} - \boldsymbol{p}$)

$$S_{\alpha\beta}(k) = \frac{(m+\not{k})_{\alpha\beta}}{m^2 - k^2 - i\varepsilon} \Rightarrow \left(\frac{1}{2E(p)}\right)\sum_s \left\{ \overset{+}{\boxed{\frac{u_\alpha(-\mathbf{p},s)\bar{u}_\beta(-\mathbf{p},s)}{(2E(p)-W)}}} - \overset{-}{\boxed{\frac{v_\alpha(\mathbf{p},s)\bar{v}_\beta(\mathbf{p},s)}{W}}} \right\}$$

- Integration over all internal p_0's places *n* – 1 particles on their *positive energy* mass-shell. All 4-d integrations reduce to 3-d integrations.
- Antisymmetrize for identical fermions; remove spurious singularities(!)
- Mass of off-shell particle is $k^2 - m^2 = (P - p)^2 - m^2 = W^2 - 2WE(p) < W(W - 2m)$. If $W < m + m_{res}$, then $k^2 < (m_{res})^2$ and nucleon resonances are frozen out (see last lecture).

Coupled equations with spin (1)

- The positive and negative energies give separate coupled channels

$$M^{++}=V^{++}-\int\left\{V^{++}G^{+}M^{++}+V^{+-}G^{-}M^{-+}\right\}$$
$$M^{-+}=V^{-+}-\int\left\{V^{-+}G^{+}M^{++}+V^{--}G^{-}M^{-+}\right\}$$

where + and - refer to the *u* or *v* spinor matrix element of the off-shell particle 2

$$V^{++}(k,p;P)=\bar{u}_{\alpha'}(\mathbf{k},\lambda_1')\,\bar{u}_{\beta'}(-\mathbf{k},\lambda_2')\mathcal{V}_{\alpha'\alpha,\beta'\beta}(k,p;P)u_\alpha(\mathbf{p},\lambda_1)\,u_\beta(-\mathbf{p},\lambda_2)$$
$$V^{+-}(k,p;P)=\bar{u}_{\alpha'}(\mathbf{k},\lambda_1')\,\bar{u}_{\beta'}(-\mathbf{k},\lambda_2')\mathcal{V}_{\alpha'\alpha,\beta'\beta}(k,p;P)u_\alpha(\mathbf{p},\lambda_1)\,v_\beta(\mathbf{p},\lambda_2)$$
$$V^{-+}(k,p;P)=\bar{u}_{\alpha'}(\mathbf{k},\lambda_1')\,\bar{v}_{\beta'}(\mathbf{k},\lambda_2')\mathcal{V}_{\alpha'\alpha,\beta'\beta}(k,p;P)u_\alpha(\mathbf{p},\lambda_1)\,u_\beta(-\mathbf{p},\lambda_2)$$
$$V^{--}(k,p;P)=\bar{u}_{\alpha'}(\mathbf{k},\lambda_1')\,\bar{v}_{\beta'}(\mathbf{k},\lambda_2')\mathcal{V}_{\alpha'\alpha,\beta'\beta}(k,p;P)u_\alpha(\mathbf{p},\lambda_1)\,v_\beta(\mathbf{p},\lambda_2)$$

Coupled equations with spin (2)

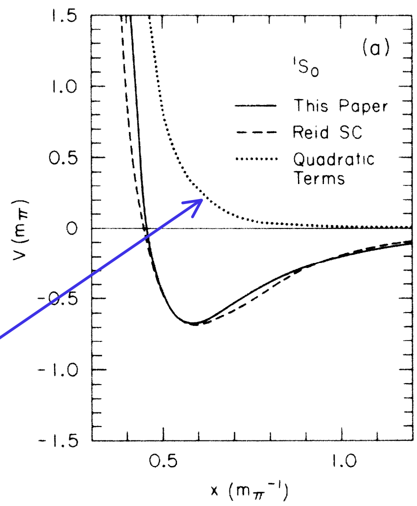
- In the nonrelativistic limit, the equations reduce in coordinate space to

$$\left(\frac{\nabla^2}{m}+\varepsilon\right)\Psi^+(r)=V^{++}(r)\Psi^+(r)+V^{+-}(r)\Psi^-(r)$$
$$2m\,\Psi^-(r)=V^{-+}(r)\Psi^+(r)+V^{--}(r)\Psi^-(r)$$

- These can be solved (even when dependent on spin operators-see Ref.~1*). For scalar V^{--}, we have ($V^{-+}=\left(V^{+-}\right)^\dagger$)

$$\Psi^-(r)=\frac{V^{-+}(r)}{2m-V^{--}(r)}\Psi^+(r)$$
$$\left(\frac{\nabla^2}{m}+\varepsilon\right)\Psi^+(r)=\left\{V^{++}(r)+\frac{V^{+-}(r)\left(V^{+-}(r)\right)^\dagger}{2m-V^{--}(r)}\right\}\Psi^+(r)$$

if V^{--} < 2m, this is a positive definite repulsive core



*FG, *Phys. Rev. D* **10**, 223 (1974)

CS deuteron wave function fixed by Poincaré covariance

★ from translational invariance:

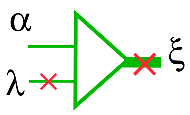
$$\int d^4x\, e^{-ip\cdot x}\langle n|\psi(x)|d\rangle = (2\pi)^4\,\delta(p+n-d)\langle n|\psi(0)|d\rangle$$

conservation of momentum *and energy* at the vertex

★ from rotational invariance

$$\langle n,\lambda|\psi_\alpha(0)|d,\xi\rangle = \frac{1}{\sqrt{2M_d(2\pi)^3}}\left[S(p)\Gamma^\mu(p)\,C\right]_{\alpha\beta}\bar{u}^T_\beta(\mathbf{n},\lambda)\xi_\mu$$

$$= \left\{\psi^{+\mu}_{\lambda'\lambda}(\mathbf{p})\; u_\alpha(\mathbf{p},\lambda') + \psi^{-\mu}_{\lambda'\lambda}(\mathbf{p})\; v_\alpha(-\mathbf{p},\lambda')\right\}\xi_\mu$$



positive energy spinor

negative energy spinor

exact
the most general form possible for the coupling of a spin 1 particle to two spin 1/2 particles, one off-shell

★ from transformations under boosts

$$B(\Lambda)\langle n,\lambda|\psi_\alpha(0)|d,\xi\rangle = B_{\alpha\alpha'}\langle\Lambda n,\lambda'|\psi_{\alpha'}(0)|\Lambda d,\Lambda\xi\rangle D^{(1/2)}_{\lambda'\lambda}(\omega)$$

boost matrix for off-shell particle in Dirac space

Wigner rotation of the spin of the on-shell particle

exact
obtained from Wigner rotations and Dirac boost matrix

Deuteron wave functions (1)

★ The relativistic deuteron wave function has one nucleon off-shell. This off-shell nucleon has both a positive energy spinor part (*u*) and a negative energy spinor part (*v*)

$$\Psi_{\alpha,\lambda}(P,p) = u_\alpha(\mathbf{p}_2,\lambda')\;\psi^{+}_{\lambda'\lambda}(\mathbf{P},\mathbf{p}) + v_\alpha(-\mathbf{p}_2,\lambda')\;\psi^{-}_{\lambda'\lambda}(\mathbf{P},\mathbf{p})$$

positive energy part

negative energy part

★ Four scalar wave functions are needed, 2 for each part

$$\psi^{+}_{\lambda'\lambda}(\mathbf{P},\mathbf{p}) = \tfrac{1}{\sqrt{4\pi}}\left[u(p)\,\sigma_1\cdot\sigma_2 - \tfrac{1}{\sqrt{8}}w(p)\left(3\sigma_1\cdot\hat{\mathbf{p}}\,\sigma_2\cdot\hat{\mathbf{p}} - \sigma_1\cdot\sigma_2\right)\right]$$

$$\psi^{-}_{\lambda'\lambda}(\mathbf{P},\mathbf{p}) = -\sqrt{\tfrac{3}{16\pi}}\left[v_s(p)\left(\sigma_1-\sigma_2\right)\cdot\hat{\mathbf{p}} + \tfrac{1}{\sqrt{2}}v_t(p)\left(\sigma_1+\sigma_2\right)\cdot\hat{\mathbf{p}}\right]$$

same as non relativistic

★ The normalization condition becomes

$$1 = \int_0^\infty p^2\,dp\left\{u^2 + w^2 + v_t^2 + v_s^2\right\} + \left\langle\frac{\partial V}{\partial M_d^2}\right\rangle$$

P-state probabilities

Deuteron wave functions (2)

- ★ AV18: Argonne AV18 nonrelativistic model
- ★ Model IIB: earlier model* used to predict the deuteron form factors

  *J.W. Van Orden, FG, and K. Holinde, *Phys. Rev.* **C 45**, 2094 (1992).

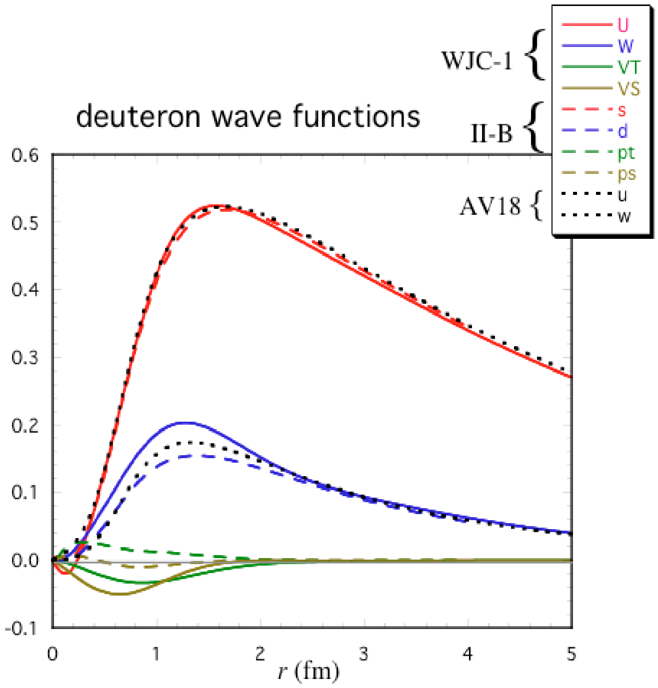
- ★ WJC-1: new high precision fit described here.

$P_s = 92.35\%$ $\quad D/S = 0.0256$

$P_d = \ 7.32\%$ $\quad$ ***agrees with experimental value***

$P_{vt} = \ 0.11\%$

$P_{vs} = \ 0.22\%$



Antisymmetrize the Kernel

- ★ The kernel must be explicitly antisymmetrized

$$\mathcal{V}_{\alpha'\alpha,\beta'\beta}(k,p;P) \to \bar{\mathcal{V}}_{\alpha'\alpha,\beta'\beta}(k,p;P)$$

$$= \frac{1}{2}\left\{\mathcal{V}_{\alpha'\alpha,\beta'\beta}(k,p;P) + (-1)^I \mathcal{V}_{\beta'\alpha,\alpha'\beta}(-k,p;P)\right\}$$

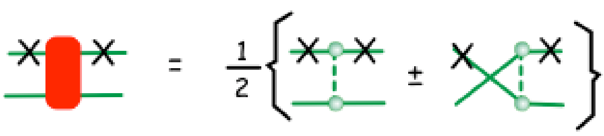
interchange

- ★ Under interchange of Dirac and momentum indices,

$$\bar{\mathcal{V}}_{\alpha'\alpha,\beta'\beta}(k,p;P) = (-1)^I \bar{\mathcal{V}}_{\beta'\alpha,\alpha'\beta}(-k,p;P)$$

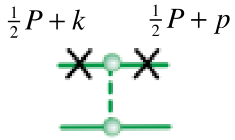
corresponding to antisymmetry of I=1 states and symmetry of I=0 states, corresponding to full antisymmetry.

- ★ Diagrammatically



Implications of the antisymmetrization

- The direct OBE has no singularities



$$\frac{1}{m_b^2+(\mathbf{k}-\mathbf{p})^2-\left(E_k-E_p\right)^2}=\frac{1}{m_b^2+2\left(E_pE_k-m^2\right)-2\mathbf{k}\cdot\mathbf{p}}\geq 0$$

- However, the exchange OBE has singularities

$$\frac{1}{m_b^2+(\mathbf{k}+\mathbf{p})^2-\left(W-E_k-E_p\right)^2}=\frac{1}{m_b^2+2\left(E_pE_k-m^2\right)-\underbrace{\left(W-2E_K\right)\left(W-2E_p\right)}+2\mathbf{k}\cdot\mathbf{p}}$$

this =0 if either initial or final state is on-shell

- If $W < 2m + m_b$, these singularities are *spurious*, because they are cancelled if the kernel is calculated to ALL orders.
- So, imaginary part may be dropped (calculate the principal value), but how to handle the real part?

Removal of spurious singularities

- Exploit a great freedom: the kernel may be defined in any convenient way, with "corrections" included in higher order
- An elegant way to remove the singularities is to replace

$$\frac{1}{m_b^2+(\mathbf{k}+\mathbf{p})^2-\left(W-E_k-E_p\right)^2}\Rightarrow\frac{1}{m_b^2+\left|(\mathbf{k}+\mathbf{p})^2-\left(W-E_k-E_p\right)^2\right|}=\frac{1}{m_b^2+\left|q^2\right|}$$

  - preserves exchange symmetry exactly
  - removes all singularities (does NOT work for coulomb scattering)
  - does not change the direct term, or any results if either the initial or final state is on shell

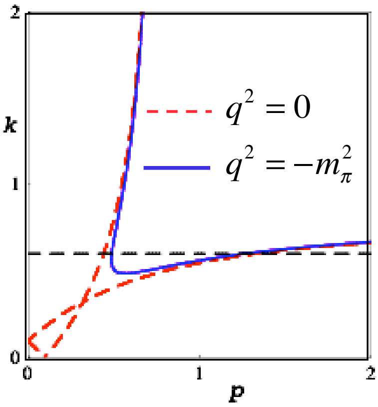
research exercise: calculate the 4th order kernel in ϕ^3 theory using this prescription, and study the cancellations

Look at the details for *S*-wave scattering

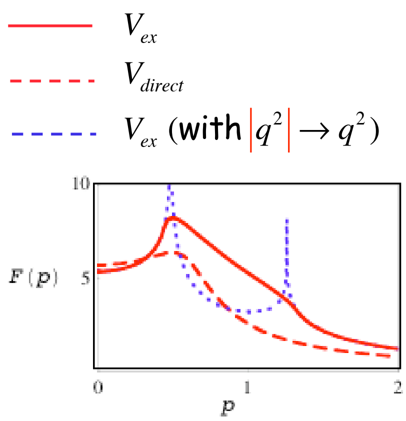
- ★ The angular integral is

$$V_{ex}(k,p)=\int_{-1}^{1} dz\frac{1}{m_b^2+\left|q^2\right|}=\int_{-1}^{1} dz\frac{1}{m_b^2+\left|k^2+p^2+kpz-q_0^2\right|}$$

- ★ Locus of singularities and (in units of *m*)



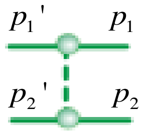
$F(p)$
$=V(0.6,p)$



The prescription smooths out the singularities and interpolates between them

CS Dynamics: OBE with off-shell couplings

- ★ Kernel is a sum of One Boson Exchange diagrams

$V=\sum$ 

$=\sum \varepsilon_b\delta\frac{\Lambda_{\alpha'\alpha}(k_1,p_1)\otimes\Lambda_{\beta'\beta}(k_2,p_2)}{m_m^2+\left|q^2\right|}f^4(\Delta_b,q)$

$$\delta=\begin{cases}1 & \text{isoscalar}\\ \tau_1\cdot\tau_2 & \text{isovector}\end{cases}$$

$$f^4(\Lambda_b,q)=\left[\frac{\Lambda_b^2}{\Lambda_b^2+\left|q^2\right|}\right]^4$$

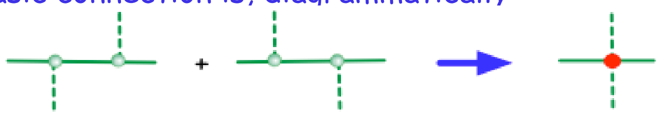
boson	$J^P(b)$	ε_b	$\Lambda_1\otimes\Lambda_2$	$\Lambda(k,p)$ or $\Lambda^\mu(k,p)$
σ_0 σ_1	0^+(s)	–	$\Lambda_1\Lambda_2$	$\Lambda(k,p)=g_s+\nu_s\left[\theta(k)+\theta(p)\right]$
π η	0^-(p)	+	$\Lambda_1\Lambda_2$	$\Lambda(k,p)=g_p\left\{\gamma^5-(1-\nu_p)\left[\theta(k)\gamma^5+\gamma^5\theta(p)\right]\right\}$
ρ ω	1^-(v)	+	$\Lambda_1^\mu\Lambda_2^\nu\Delta_{\mu\nu}$	$\Lambda^\mu(k,p)=g_v\left\{\gamma^\mu+\frac{\kappa_v}{2m}i\sigma^{\mu\nu}(k-p)_\nu+\nu_v\left[\theta(k)\gamma^\mu+\gamma^\mu\theta(p)\right]\right\}$
a_1 h_1	1^+(a)	+	$\Lambda_1^\mu\Lambda_2^\nu g_{\mu\nu}$	$\Lambda^\mu(k,p)=g_a\gamma^5\gamma^\mu$

$\theta(p)=\frac{m-\not{p}}{2m}$ ***vanishes on-shell*** $\qquad \Delta_{\mu\nu}=g_{\mu\nu}-q_\mu q_\nu/m_v^2$

Implications of off-shell couplings

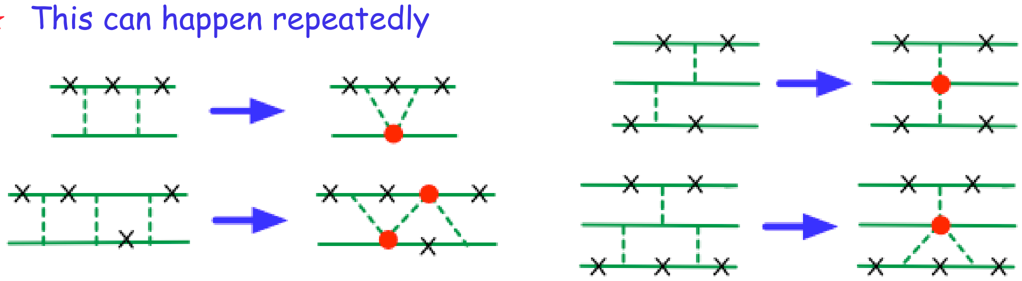
OBE with off-shell couplings ⟷ *OBE without off-shell couplings PLUS a specific set of N-meson exchange and N-body forces*

- In basic connection is, diagrammatically



$$v_\sigma \frac{m-\not{k}_1}{2m}\left(\frac{1}{m-\not{k}_1}\right) g_\sigma + g_\sigma \left(\frac{1}{m-\not{k}_2}\right)\frac{m-\not{k}_2}{2m} v_\sigma = \frac{g_\sigma v_\sigma}{m}$$

- This can happen repeatedly



Two precision fits to the 2007 data base

- Comparison with other precision fits

models			Data set [χ^2/N_{data} (N_{data})]		
Ref	year	# para	1993	2000	2007
PWA93	1993	39	0.99 (2514)		
			1.09 (3010)	1.11 (3336)	1.12 (3788)
Nijm I	1993	41	1.03 (2514)		
AV18	1995	40	1.06 (2526)		
CD-Bonn	2000	43		1.02 (3058)	
WJC-1	2007	27	**1.03** (3010)	**1.05** (3336)	**1.06** (3788)
WJC-2	2007	15	**1.09** (3010)	**1.11** (3336)	**1.12** (3788)

#'s in green are for fits to BOTH np and pp data

OBE parameters obtained from the fits

b	I	$G_b = \frac{g_b^2}{4\pi}$		m_b		λ_b or ν_b		κ_v		Λ_b	
π^0	1	14.608	14.038	134.9766		0.153	0.0	---		4400	3661
$\pi^\pm$	1	13.703	14.038*	139.5702		-0.312	0.0	---		4400*	3661*
η	0	10.684	4.386	604	547.51	0.622	0.0	---		4400*	3661*
σ_0	0	2.307	4.486	429	478	-6.500	-1.550	---		1435	3661*
σ_1	1	0.539	0.477	515	454	0.987	1.924	---		1435*	3661*
ω	0	3.456	8.711	657	782.65	0.843	0.0	0.048	0.0	1376	1591
ρ	1	0.327	0.626	787	775.50	-1.263	-2.787	6.536	5.099	1376*	1591*
h_1	0	0.0026	0.0							1376*	
a_1	1	-0.436	0.0							1376*	
									Λ_N	1656	1739

left column: WJC-1 27 parameters
right column: WJC-2 15 parameters

Conclusions from the fits

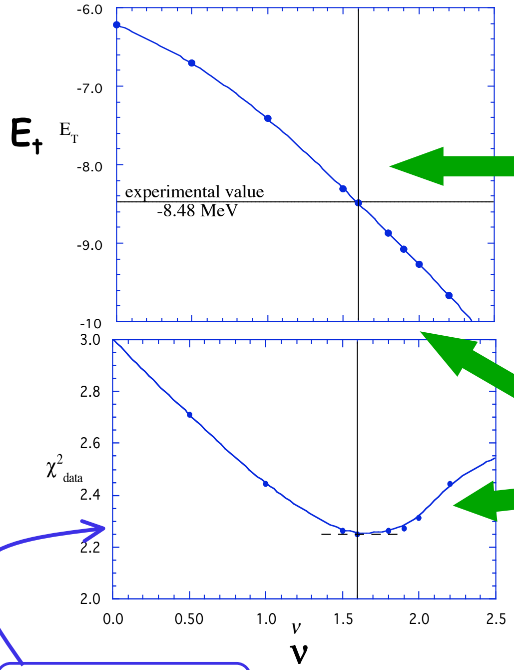
★ Model WJC-1: 27 parameters:

- As good as any phase shift analysis or any fit to date; truly QUANTATIVE
- 27 parameters is less than other fits (but only *np* data fit so far)
- OBE parameters are reasonable:
  - masses close to observed masses of actual mesons (within 50 MeV except for the ω, which is 126 MeV lower); π masses fixed at observed values
  - σ (0 and 1) have masses near the peak of the 2 pion continuum
  - π couplings are close to expected values; BUT $g_0 > g_+$ (!)
  - ω and ρ are week; η is strong (compared to WJC-2)

★ Model WJC-2: ONLY 15 parameters

- EXCELLENT; as good as the Nijmegen phases
- OBE parameters are reasonable, and SATISFY constraints:
  - masses of ω, ρ, and η (and π) fixed at observed values; σ (0 and 1) masses still near the peak of the 2 pion continuum
  - π^0 and $\pi^\pm$ couplings equal; PURE pv coupling as required by chiral symmetry
  - No novel features (i.e. κ_ω=0, η pure pv, no off=shell coupling for ω) EXCEPT off-shell couplings for σ (0 and 1) and ρ

Relativistic effects in ^3H binding (1997 results)*



It turns out that the relativistic calculation of the three body binding energy is sensitive to a new, relativistic off-shell coupling (described by the parameter ν). Non-zero ν is equivalent to effective three-body (and n-body forces).

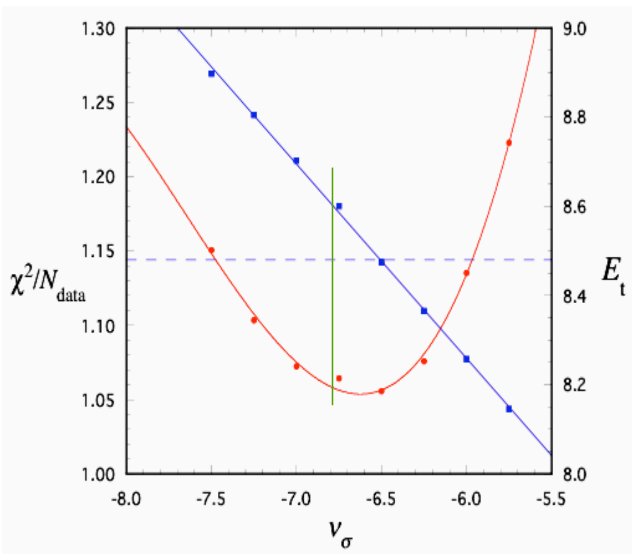
The value of ν that gives the correct binding energy is close to the value that gives the best fit to the two-body data!

note poor χ^2

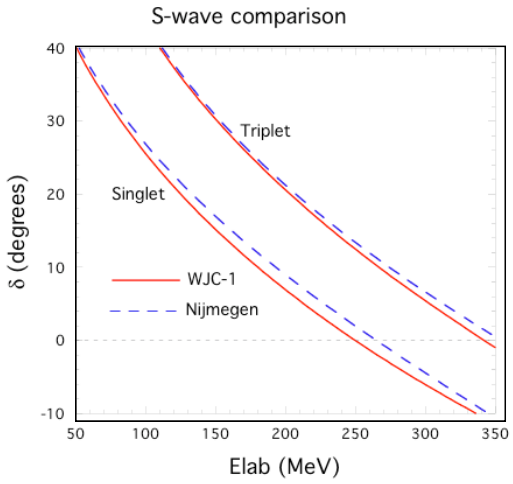
*three body calculations FG and Alfred Stadler, Phys. Rev. Letters **78**, 26 (1997)

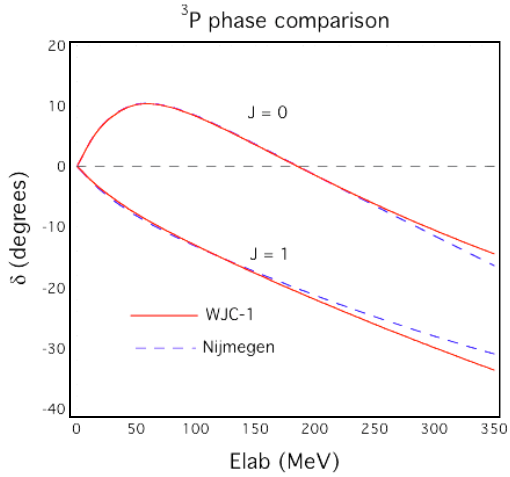
New results confirm the 1997 findings

★ Minimum $\chi^2/N_{\rm data}$ for Model WJC-1 coincides with experimental triton binding energy of -8.48 MeV



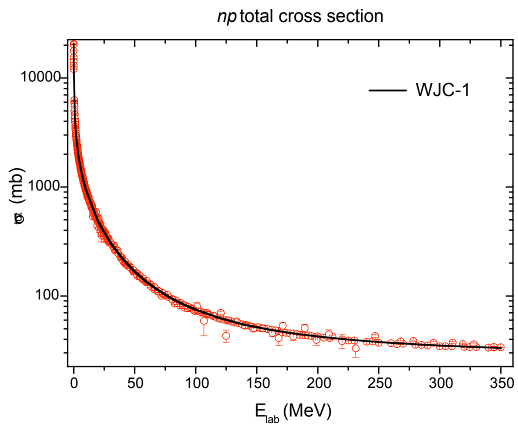
Changes in the phase shifts





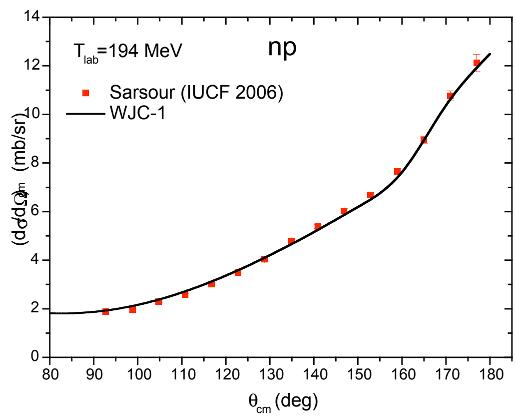
★ Nijmegen phases differ by several degrees from the WJC-1 phases. (Explains earlier problem fitting the data.)

Low χ^2 implies excellent fits to data (of course)



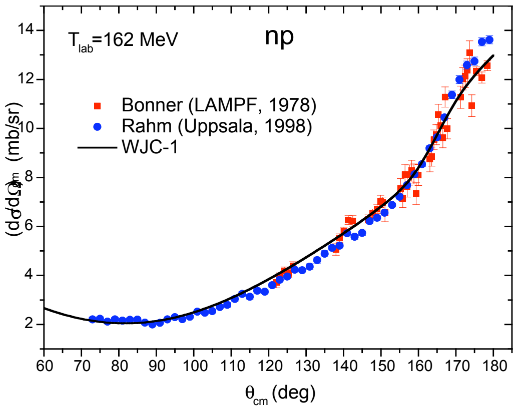
Total cross sections fit over the entire energy range

New accurate differential cross sections

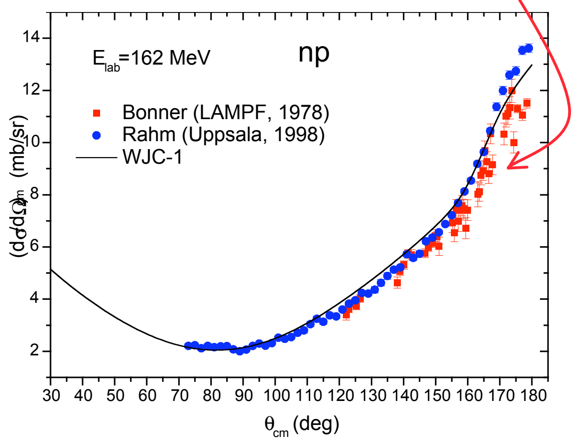


Scaling and rejection of data sets

Experimentalists may specify that data has a systematic error; it may be scaled (within the error) to agree with theory. The red Bonner data has been scaled (below).



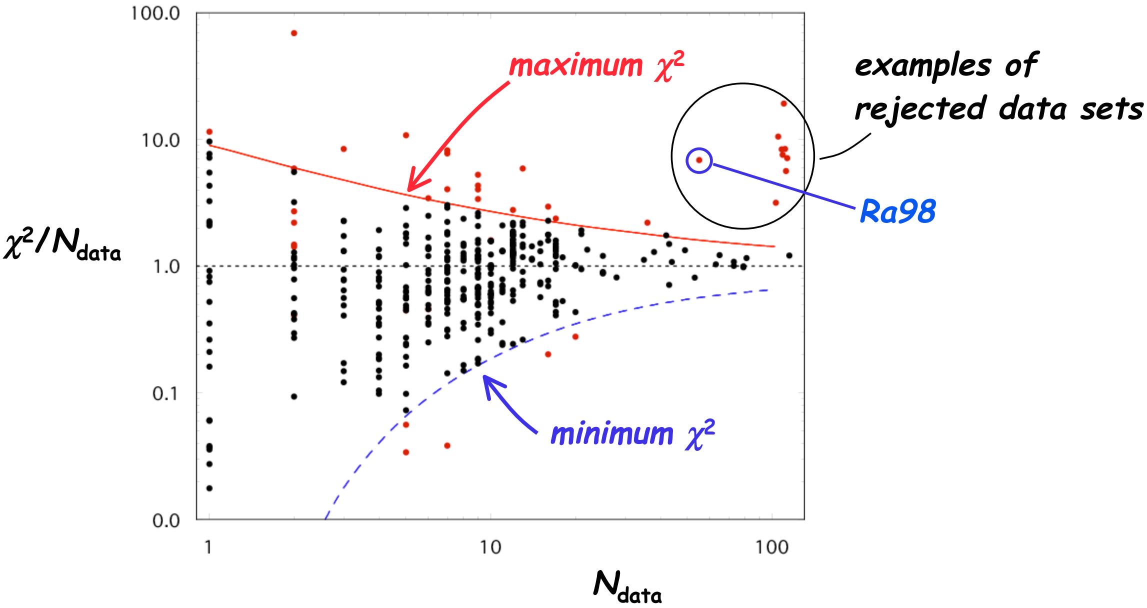
This data initially unscaled



However, the Uppsala data (blue) is rejected; no scaling can change its incorrect shape.

Rejected data sets can be identified

- ★ Nijmegen identifies a 3σ criterion. Data sets with χ^2 too large or too small are rejected.



Conclusions

- ★ We have a simple (comparatively) covariant model of the *NN* kernel based on OBE that gives a quantitatively EXCELLENT description of the low energy *NN* data.
- ★ The OBE mechanism works very well, with only a few parameters needed.
- ★ ALL Poincaré transformations are kinematic -- i.e. exact.
- ★ Three body forces are incorporated as off-shell effects arising from two body interactions.
- ★ These models can be used for precision calculations of few body interactions
- ★ The kernel provides a "bridge" between hadronic physics and QCD -- in the sense that the task of QCD is now to understand the kernel we have found

END