Anomalous Form Factor of Pion in AdS/QCD model with Chern-Simons term

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Mar 28, 2008

Formulation of Conjecture

The AdS/CFT correspondence conjectures the equivalence of gravity theory (Type IIB string theory) on $AdS_5 \times S_5$, and strongly coupled ($N = 4$ SYM) CFT_4. *(Maldacena, 1997)*

AdS/CFT says that for $\forall \mathcal{O}(x) \in \{CFT\ operator\}$,
$\exists! \phi(x, z) \in \{5D\ bulk\ field\}$ s.t. $\phi(x, 0) = \phi_0(x)$, $x \in \partial AdS_5$.

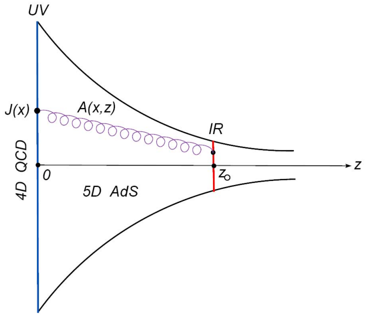
Let $S_5[\phi_0(x)]$ is the gravity or string action of $\phi(x, z)$ with $\phi(x, 0) = \phi_0(x)$, then the correspondence takes the form

$$\langle \exp(i \int d^4x \phi_0(x) \mathcal{O}(x)) \rangle_{CFT} = \exp(iS_5[\phi_0(x)]) \ ,$$

(Witten, 1998)

Addition of IR Brane

Addition of the IR brane corresponds to deformation of the CFT and to breaking of the conformal invariance.



Now, we have both particles and S-matrix elements.

In particular, KK modes in 5D are interpreted in 4D theory as resonances.

Initial Setup

We start with the (hard-wall) model proposed by *Erlich, Katz, Son & Stephanov* (EKSS), and then extend it to incorporate the QCD anomaly.

In this model, the slice of the AdS_5 space is defined as

$$ds^2 = \frac{1}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2\right), \qquad 0 < z \leq z_0 ,$$

$\eta_{\mu\nu} = \mathrm{Diag}\left(1,-1,-1,-1\right)$ and $z_0 \sim 1/\Lambda_{QCD}$ is the IR scale.

The holographic dictionary for vector sector is:

$$J^a_\mu(x) = \bar{q}\gamma_\mu t^a q(x) \leftrightarrow A^a_M(x,z) ,$$

so that $A^a_M(x,0)$ is the source for $J^a_\mu(x)$.

The 5D Gauge Action

The 5D gauge action in AdS_5 space for the vector field is:

$$S_{AdS} = -\frac{1}{4g_5^2} \int d^4x\, dz\, \sqrt{g}\, Tr\left(F_{MN}F^{MN}\right) \ ,$$

where $F_{MN} = \partial_M A_N - \partial_N A_M - i[A_M, A_N]$, $A = A^a t^a$,
($t^a \in SU(2),\ a = 1, 2, 3$) and $M, N = 0, 1, 2, 3, z$.

4D Global Chiral $SU(2) \leftrightarrow$ 5D Local Gauge $SU(2)$

N.B. We take our field to be non-Abelian, since later we are interested in calculating the 3-point function.

Equation of Motion

We work in $A_z = 0$ gauge with Fourier-transformed gauge field

$$A_\mu(q,z) = \tilde{A}_\mu(q)\frac{V(q,z)}{V(q,\epsilon)}|_{\epsilon\to 0}\ .$$

The EOM for the bulk-to-boundary propagator $V(q,z)$ is

$$z\partial_z\left(\frac{1}{z}\partial_z V(q,z)\right) + q^2 V(q,z) = 0\ .$$

From IR BC: $\partial_z V(q,z_0) = 0 \ \Rightarrow$

$$V(q,z) \propto qz(Y_0(qz_0)J_1(qz) - J_0(qz_0)Y_1(qz))\ .$$

N.B. The IR BC is a gauge invariant condition equivalent to $F_{\mu z}(x,z_0) = 0$.

Two-point Function

The 2-point function defined from the relation:

$$\int d^4x \, e^{iq\cdot x} \langle J_\mu(x) J_\nu(0) \rangle = \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \Sigma(q^2)$$

AdS/QCD predicts for the scalar part of the 2-point function:

$$\Sigma(q^2) = -\frac{1}{g_5^2} \left(\frac{1}{z} \frac{\partial_z V(q,z)}{V(q,\epsilon)} \right) \Bigg|_{z=\epsilon\to 0} \Rightarrow \sum_{n=1}^{\infty} \frac{f_n^2}{q^2 - M_n^2} \, ,$$

where $M_n = \gamma_{0n}/z_0$ and

$$f_n^2 = \frac{2M_n^2}{g_5^2 z_0^2 J_1^2(\gamma_{0,n})} \, ,$$

since: $\langle 0 | J_\mu^a | \rho_n^b \rangle = \delta^{ab} f_n \epsilon_\mu$.

Two-point Function

In the limit $qz_0 \gg 1$, AdS/QCD predicts:

$$\Sigma(q^2) = \frac{1}{2g_5^2} q^2 \ln(q^2 \epsilon^2) \ .$$

By matching this with QCD, one finds (EKSS): $g_5^2 = 12\pi^2/N_c$.

For $N_c = 3$ to get $M_1 \equiv M_\rho^{\exp} = 775.8$ MeV, we take $\frac{1}{z_0} = 323$ MeV .

As a result: $f_1 \equiv f_\rho = (392 \text{ MeV})^2$

N.B. $f_\rho^{\exp} = (401 \pm 4 \text{ MeV})^2$ (PDG, 2007)

Three-point Function of Vector Currents

For the scalar part of $\langle J_a^\alpha(p_1) J_b^\beta(-p_2) J_c^\mu(q)\rangle$ the AdS/QCD predicts

$$T(p_1^2, p_2^2, Q^2) = \sum_{n,k=1}^{\infty} \frac{f_n f_k F_{nk}(Q^2)}{\left(p_1^2 - M_n^2\right)\left(p_2^2 - M_k^2\right)} ,$$

where

$$F_{nk}(Q^2) = \int_0^{z_0} \frac{dz}{z}\, \mathcal{J}(Q,z)\, \psi_n(z)\, \psi_k(z)$$

correspond to form factors for $n \to k$ transitions, where

$$\mathcal{J}(Q,z) \equiv \frac{V(iQ,z)}{V(iQ,\epsilon)}\Big|_{\epsilon\to 0} = Qz\left[K_1(Qz) + I_1(Qz)\frac{K_0(Qz_0)}{I_0(Qz_0)}\right] ,$$

$$\psi_n(z) \equiv \frac{\sqrt{2}}{z_0 J_1(\gamma_{0,n})}\, z J_1(M_n z)$$

(HRG, Radyushkin, PLB650, 2007)

Pion in the chiral limit of AdS/QCD model

The holographic dictionary in the axial gauge ($A_z = 0$) is:

$$J^a_{A\mu}(x) = \bar{q}(x)\gamma_\mu\gamma_5 t^a q(x) \longrightarrow A^a_\mu(x,z)\ ,$$

$$\Sigma^{\alpha\beta}(x) = \langle \bar{q}^\alpha_L(x) q^\beta_R(x)\rangle \longrightarrow \frac{2}{z} X^{\alpha\beta}(x,z)\ .$$

Axial-sector of the hard-wall model is described by the action

$$S^A_{\text{AdS}} = \text{Tr}\int d^4x \int_0^{z_0} dz\,\sqrt{g}\left[|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}F_A^2\right]\ ,$$

$$DX = \partial X - iA_L X + iXA_R,\ A_{L(R)} = V \pm A,$$

$$X(x,z) = \frac{1}{2}v(z)U(x,z)\ ,\quad U(x,z) = \exp\left(2it^a\pi^a(x,z)\right)\ ,$$

$v(z) = m_q z + \sigma z^3$, m_q– quark mass and σ– quark condensate.

Setup

Expanding $U(x,z) \Rightarrow$

$$S^{A\,(2)}_{\text{AdS}} = \text{Tr} \int d^4x\, dz \left[-\frac{1}{4g_5^2 z} A^{MN} A_{MN} + \frac{v^2(z)}{2z^3} (A^a_M - \partial_M \pi^a)^2 \right]$$

In general, $A = A_\perp + A_{||}$, where $A_\perp$ and $A_{||}$ are transverse and longitudinal components of the axial-vector field.

SSB causes $A_{||}$ to be physical and associated with the GB – pion.

The $||$ component may be written as

$$A^a_{M||}(x,z) = \partial_M \psi^a(x,z) \Rightarrow \psi^a(x,z) \leftrightarrow \text{pion field}$$

Equations of Motion

Varying the action with respect to $A^a_{\perp\mu}(x,z)$ and representing the Fourier image of $A^a_{\perp\mu}(x,z)$ as $\tilde{A}^a_{\perp\mu}(p,z) = \mathcal{A}(p,z)A^a_\mu(p)$ we will get

$$\left[z^3\partial_z\left(\frac{1}{z}\partial_z\mathcal{A}\right) + p^2z^2\mathcal{A} - g_5^2v^2\mathcal{A}\right] = 0\ ,$$

with b.c. $\mathcal{A}(p,0) = 1$ and $\mathcal{A}'(p,z_0) = 0$. Remember that $v(z) = \sigma z^3$. Variation with respect to the longitudinal component $\partial_\mu\psi^a$ gives

$$z^3\partial_z\left(\frac{1}{z}\,\partial_z\psi^a\right) - g_5^2v^2\left(\psi^a - \pi^a\right) = 0\ .$$

Finally, varying with respect to A_z produces

$$p^2z^2\partial_z\psi^a - g_5^2v^2\partial_z\pi^a = 0\ ,$$

with b.c. $\partial_z\psi(z_0) = 0$, $\psi(\epsilon) = 0$ and $\pi(\epsilon) = 0$.

Equations of Motion

In the chiral limit, the equation for ψ becomes

$$z^3\partial_z\left(\frac{1}{z}\,\partial_z\Psi\right) - g_5^2 v^2\Psi = 0$$

where $\Psi \equiv \psi - \pi$ and since $\pi = -1 \Rightarrow \Psi(\epsilon) = 1$ and $\Psi'(z_0) = 0 \Rightarrow$

$$\Psi(z) = \mathcal{A}(0, z)\ .$$

It is useful to define the conjugate w.f. $\Phi(z)$ as

$$\Phi(z) = -\frac{1}{g_5^2 f_\pi^2}\left(\frac{1}{z}\,\partial_z\Psi(z)\right)\ ,$$

then $\Phi(0) = 1$ and $\Phi(z_0) = 0$.

Explicitly

$$\Phi(z) = \frac{3\,z^2}{g_5^2 f_\pi^2 z_0^4}\,\Gamma\left[2/3\right]\left(\frac{a^4}{2}\right)^{1/3}\left[I_{-2/3}\left(ax^3\right)\frac{I_{2/3}\left(a\right)}{I_{-2/3}\left(a\right)} - I_{2/3}\left(ax^3\right)\right]\;,$$

where $a = g_5\sigma z_0^3/3$ and $x = z/z_0$.

From the BC $\Phi(z_0) = 0\;\Rightarrow$

$$f_\pi^2 = 3\cdot 2^{1/3}\,\frac{\Gamma[2/3]}{\Gamma[1/3]}\,\frac{I_{2/3}\left(a\right)}{I_{-2/3}\left(a\right)}\,\frac{a^{2/3}}{g_5^2 z_0^2}$$

To get $f_\pi = 131\,\mathrm{MeV}$ for $1/z_0 = 323\,\mathrm{MeV}$, we take $a = 2.26 \equiv a_0$.

(HRG, Radyushkin, PRD76, 2007)

Electric Radius

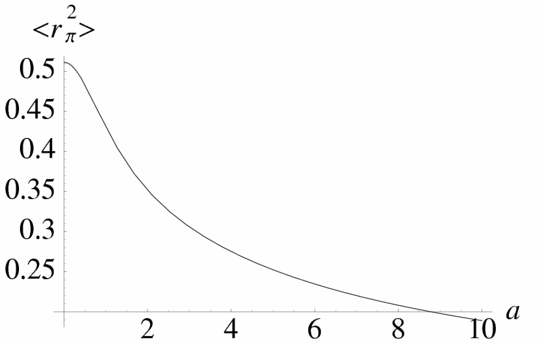


Figure: $\langle r_\pi^2 \rangle$ in fm^2 as a function of a for $z_0 = z_0^\rho \approx 0.619\,\mathrm{fm}$.

Anomalous Amplitude

- It is known from *(Witten, 1983)* that $\pi \to \gamma\gamma$ anomalous decay can be incorporated into the low energy theory by gauging the WZW term $\Gamma(U, A_L, A_R)$ with $A_L = A_R = QA$, where $Q = \mathrm{diag}\{2/3,\ -1/3\}$.
- On the other hand the WZW term naturally arises from the suitably compactified 5D theory with CS term built of YM gauge fields *(Hill, PRD73, 2006)*.
- Then it is expected that 5D holographic dual model of QCD with CS term, should naturally reproduce the anomaly.
- Since the CS (WZW) term for $SU(2)$ gauge (global) group is vanishing, the flavor symmetry is extended to $U(2)_L \times U(2)_R$. Therefore, we can write fields as: $\mathcal{B}_\mu = t^a B^a_\mu + \frac{1}{2}\hat{B}_\mu$.

Initial Setup

The 4D isovector $J_\mu^{\{I=1\},a}(x)$ and isosinglet vector $J_\mu^{\{I=0\}}(x)$ currents correspond to:

$$
\begin{aligned}
J_\mu^{\{I=1\},3} &= \frac{1}{2}\left(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\right) = \bar{q}\gamma_\mu \frac{\sigma^3}{2} q \to V_\mu^3(x,z) \ , \\
J_\mu^{\{I=0\}} &= \frac{1}{2}\left(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d\right) = \frac{1}{2}\bar{q}\gamma_\mu \mathbf{1} q \to \hat{V}_\mu(x,z) \ ,
\end{aligned}
$$

and the electromagnetic current

$$
J_\mu^{\mathrm{EM}} = J_\mu^{\{I=1\},3} + \frac{1}{3} J_\mu^{\{I=0\}}
$$

has both isovector ("ρ-type") and isosinglet ("ω-type") terms.
The matrix element $\langle 0|J^{\mathrm{EM}} J^{\mathrm{EM}}|\pi^0\rangle$ is nonzero since it contains $\langle 0|J^{\{I=1\},3} J^{\{I=0\}}|\pi^0\rangle \leftrightarrow \langle 0|J^{\{I=1\},3} J^{\{I=0\}} J_A^3|0\rangle \sim \mathrm{Tr}(\sigma^3\sigma^3)$ part.

Anomalous Form Factor

The $\pi^0\gamma^*\gamma^*$ form factor is defined by

$$\int d^4x\, e^{-iq_1x}\langle\pi,p|T\left\{J^\mu_{\rm EM}(x)\,J^\nu_{\rm EM}(0)\right\}|0\rangle$$

$$= \epsilon^{\mu\nu\alpha\beta} q_{1\,\alpha} q_{2\,\beta}\, F_{\gamma^*\gamma^*\pi^0}\left(Q_1^2, Q_2^2\right) ,$$

where $p = q_1 + q_2$ and $q^2_{1,2} = -Q^2_{1,2}$. For real photons:

$$F_{\gamma^*\gamma^*\pi^0}(0,0) = \frac{N_c}{12\pi^2 f_\pi}$$

is related to the axial anomaly of QCD.

Holographic Action with CS Term

The $\mathcal{O}(B^3)$ part of the 5D CS action, in the axial gauge $B_z = 0$ can be written as

$$S^{(3)}_{\mathrm{CS}}[\mathcal{B}] = k\frac{N_c}{48\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\int d^4x\, dz\,(\partial_z\mathcal{B}_\mu)\left[\mathcal{F}_{\nu\rho}\mathcal{B}_\sigma + \mathcal{B}_\nu\mathcal{F}_{\rho\sigma}\right],$$

where $k = 2$ to reproduce the QCD anomaly result.
Then, in the AdS/QCD model (cnf. *Domokos and Harvey, PRL99, 2007*) the CS term is:

$$S^{\mathrm{AdS}}_{\mathrm{CS}}[\mathcal{B}_L, \mathcal{B}_R] = S^{(3)}_{\mathrm{CS}}[\mathcal{B}_L] - S^{(3)}_{\mathrm{CS}}[\mathcal{B}_R]\,.$$

Taking into account that $\mathcal{B}_{L,R} = \mathcal{V} \pm \mathcal{A}$, and keeping only the longitudinal component of the axial-vector field $A = A_{\|}$ (that brings in the pion), for which $F^{A}_{\mu\nu} = 0$, we will have

$$\begin{aligned} S^{\rm AdS}_{\rm CS} &= \frac{N_c}{12\pi^2}\epsilon^{\mu\nu\rho\sigma}\int d^4x \int_0^{z_0} dz \\ &\times \left[\left(\partial_\rho V^a_\mu\right)\left(A^a_{\|\sigma}\overleftrightarrow{\partial_z}\hat{V}_\nu\right) + \left(\partial_\rho \hat{V}_\mu\right)\left(A^a_{\|\sigma}\overleftrightarrow{\partial_z} V^a_\nu\right)\right], \end{aligned}$$

where $\overleftrightarrow{\partial_z} \equiv \overrightarrow{\partial_z} - \overleftarrow{\partial_z}$. After integration by parts, taking appropriate care on the IR boundary, we get:

$$S^{\rm anom} = \frac{N_c}{4\pi^2}\epsilon^{\mu\nu\rho\sigma}\int d^4x \int_0^{z_0} dz \left(\partial_z \psi^a\right)\left(\partial_\rho V^a_\mu\right)\left(\partial_\sigma \hat{V}_\nu\right)$$

Anomalous Holographic Form Factor

Varying S^{anom} we get the 3-point function:

$$\langle J_{\alpha}^{A,3}(-p) J_{\mu}^{\text{EM}}(q_1) J_{\nu}^{\text{EM}}(q_2)\rangle = T_{\alpha\mu\nu}(p, q_1, q_2)\, i(2\pi)^4 \delta^{(4)}(q_1 + q_2 - p)\ ,$$

where

$$T_{\alpha\mu\nu}(p, q_1, q_2) = \frac{N_c}{12\pi^2} \frac{p_\alpha}{p^2}\, \epsilon_{\mu\nu\rho\sigma}\, q_1^\rho q_2^\sigma K_b(Q_1^2, Q_2^2)$$

and

$$K_b(Q_1^2, Q_2^2) = -\int_0^{z_0} \mathcal{J}(Q_1, z) \mathcal{J}(Q_2, z)\, \partial_z \psi(z)\, dz\ .$$

Here, p is the momentum of the pion and q_1, q_2 are the momenta of photons ($\psi(z)$ is the pion wave function).

(HRG, Radyushkin, 2008)

Conforming to Anomaly

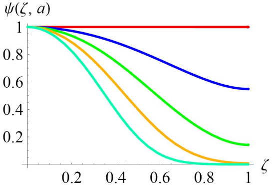
From QCD, we expect $K^{\mathrm{QCD}}(0,0) = 1$. However, AdS/QCD gives:

$$K_b(0,0) = -\int_0^{z_0} \partial_z \psi(z)\, dz = -\psi(z_0) = \Big[1 - \Psi(z_0)\Big] \,,$$

$$\Psi(z_0) = \frac{\sqrt{3}\,\Gamma\left(2/3\right)}{\pi I_{-2/3}(a)} \left(\frac{1}{2a^2}\right)^{1/3} .$$

For $a = a_0 = 2.26$, we have $\Psi(z_0) = 0.14$ (e.g., $\Psi(z_0)|_{a=4} \approx 0.02$).

N.B. Plot of $\Psi(\zeta = z/z_0, a)$ for: $a = 0$ (uppermost line), $a = 1$, $a = 2.26$, $a = 5$, $a = 10$ (lowermost line) is given below.



Conforming to Anomaly

Since $\Psi(z_0) \neq 0$, for finite a, it seems to be impossible to reproduce exactly the anomaly of QCD.

To fix this problem, we add an IR surface term, such that,

$$K(Q_1^2, Q_2^2) = \Psi(z_0)\mathcal{J}(Q_1, z_0)\mathcal{J}(Q_2, z_0)$$

$$- \int_0^{z_0} \mathcal{J}(Q_1, z)\mathcal{J}(Q_2, z)\, \partial_z \Psi(z)\, dz\ .$$

In this case, we have $K(0,0) = 1$!

Notice, that for large $Qz_0 \gg 1$, we have

$$\mathcal{J}(Q, z_0) = \frac{1}{I_0(Qz_0)} \sim e^{-Qz_0}\ .$$

Small Virtualities

For $Q_1^2 = 0$ and $Q_2^2 = Q^2 \ll 1/z_0^2$

$$K(0, Q^2) \simeq 1 - 0.66 \, \frac{Q^2 z_0^2}{4} \simeq 1 - 0.96 \, \frac{Q^2}{m_\rho^2} \; .$$

The predicted slope is $\simeq 1/m_\rho^2$ which is expected from naive VMD.

Experimentally, the slope for small timelike Q^2 is measured through the Dalitz decay $\pi^0 \to e^+ e^- \gamma$.

Predictions

The usual representation of the results is

$$a_\pi \equiv -m_\pi^2 \left(\frac{dK(0, Q^2)}{dQ^2} \right)_{Q^2 \to 0} .$$

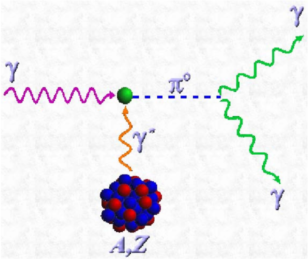
For the Q^2-slope AdS/QCD model predicts: $a_\pi \approx 0.031$.

This number is not very far from the central values of two last experiments, $a_\pi = 0.026 \pm 0.024 \pm 0.0048$ (*Farzanpay, 1992*), $a_\pi = 0.025 \pm 0.014 \pm 0.026$ (*Meijer, Drees, 1992*), but the experimental errors are rather large.

The CELLO collaboration (*Behrend, 1990*) gives the value $a_\pi = 0.0326 \pm 0.0026$ that is very close to our result.

In the spacelike region, the data are available only for the values $Q^2 \gtrsim 0.5\ \mathrm{GeV}^2$ (*CELLO, Behrend, 1990*) and $Q^2 \gtrsim 1.5\ \mathrm{GeV}^2$ (*CLEO, Gronberg, 1997*) which cannot be treated as very small.

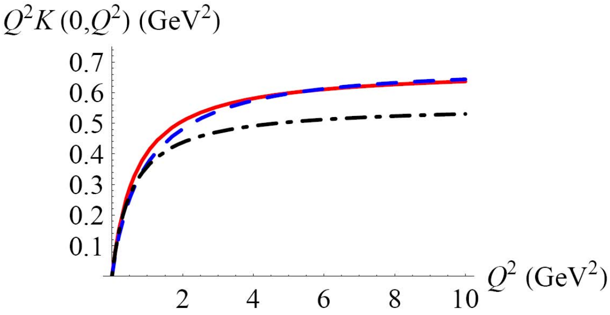
It would be interesting to have data on the slope from the spacelike region of very small Q^2, which may be obtained by modification of the PRIMEX experiment at JLab.



The aim of the Primakoff Experiment (PRIMEX) is to perform a precise measurement of π^0 lifetime from the Primakoff effect (using the small angle coherent photoproduction of the π^0 in the Coulomb field of a nucleus). The figure is taken from www.jlab.org/primex/.

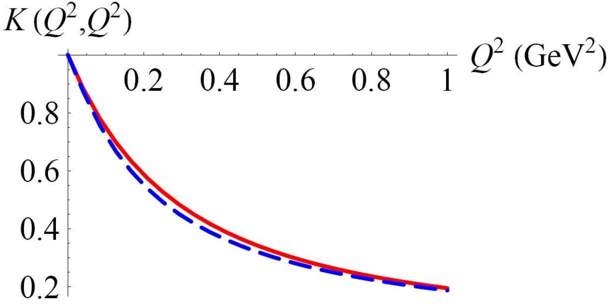
From Small to Large Virtualities

Function $Q^2 K(0, Q^2)$ in AdS/QCD model (solid curve, red online) and in local quark hadron duality model, coinciding with Brodsky-Lepage interpolation formula $1/(1 + Q^2/s_0)$, where $s_0 = 0.68$ GeV^2 (dashed curve, blue online). The monopole fit of CLEO data is shown by dash-dotted curve (black online).



From Small to Large Virtualities

Form factor $K(Q^2, Q^2)$ in AdS/QCD model (solid curve, red online) compared to the local quark-hadron duality model prediction (dashed curve, blue online).



Summary

- We showed that by including the 5D CS term into the holographic action and extending the symmetry group with appropriately defined holographic dictionary the anomalous form factor of the pion can be incorporated.
- Although we get non zero result for the anomalous amplitude, the existence of the IR wall at finite z spoils the normalization of form factor. However, when the wall is taken to infinity, the normalization is recovered. To fix the problem we add corresponding terms on the IR boundary which rapidly vanish for large virtualities.
- The slope predicted from the AdS/QCD model is in a reasonably good agreement with central values from the known experiments.
- Surprisingly, it appears that the predictions of the AdS/QCD model are almost perfect in the domain of pQCD.

THE END