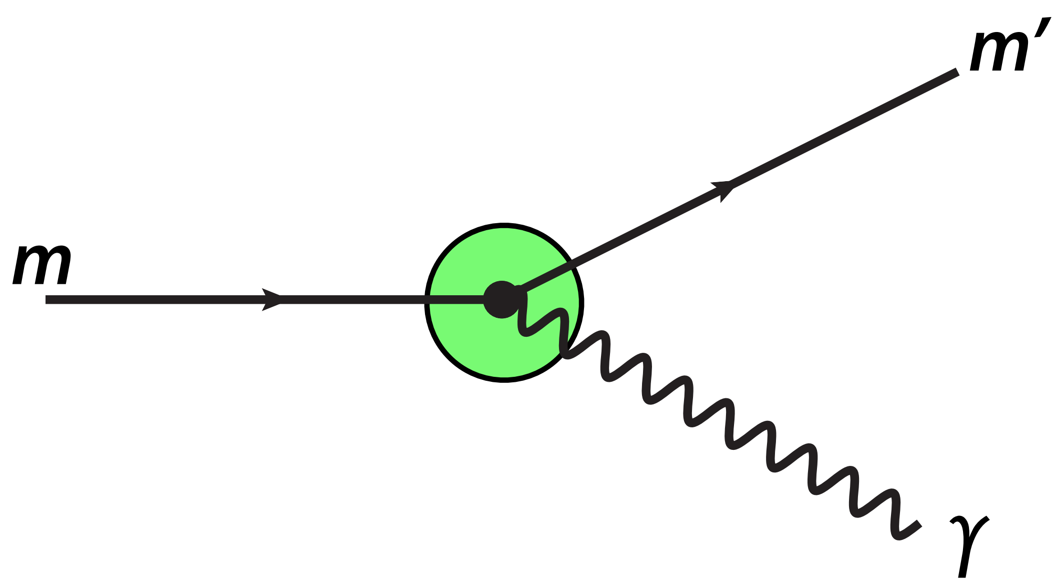
Meson spectrum and coupling to photons from Lattice QCD

applications to charmonium

Jo Dudek
Old Dominion University
&
Theory Center, Jefferson Lab

collaborations with:
Robert Edwards (JLab)
Nilmani Mathur (Tata)
David Richards (JLab)
Ermal Rrapaj (ODU u.grad)
Christopher Thomas (JLab)

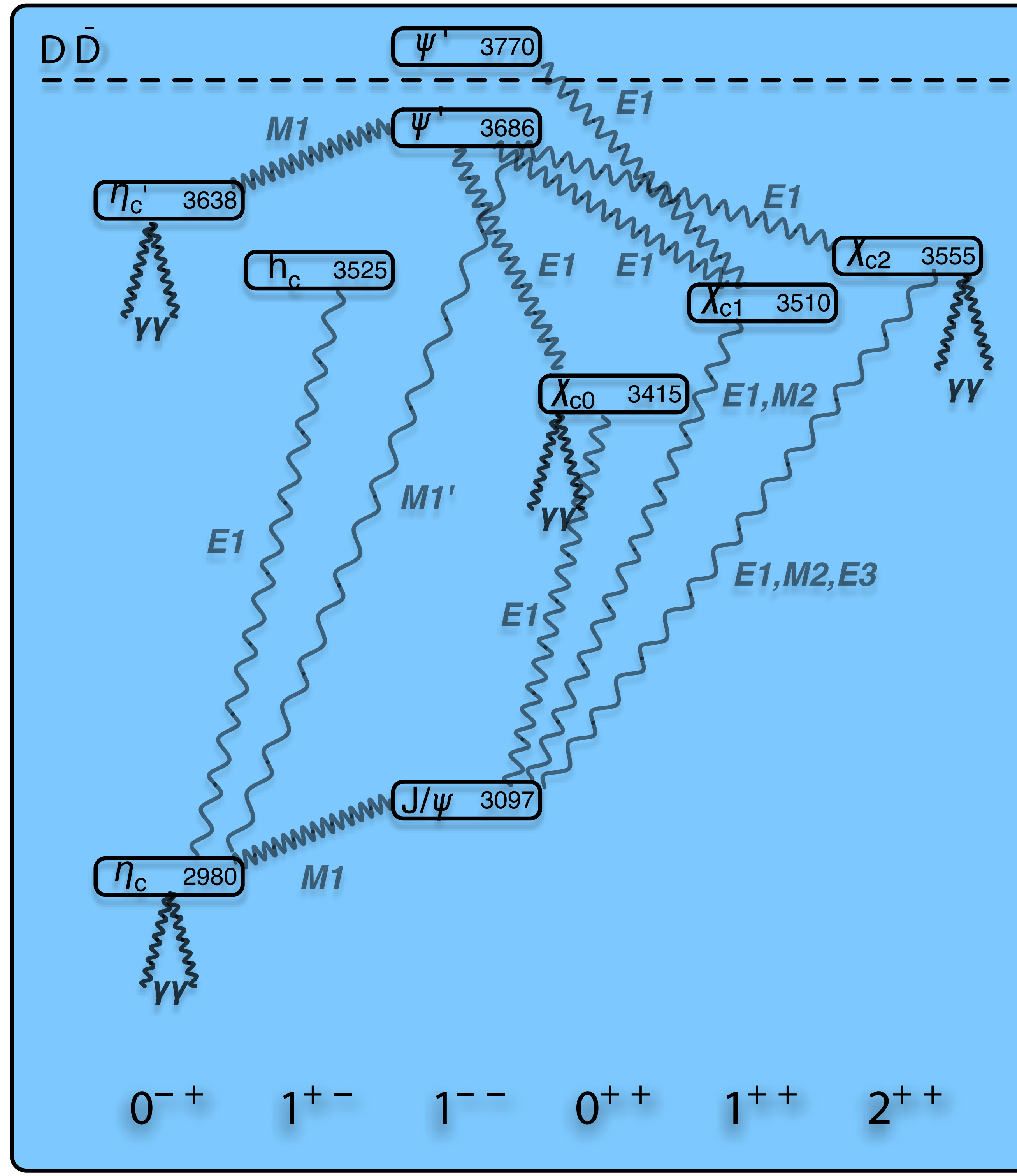
meson photocouplings



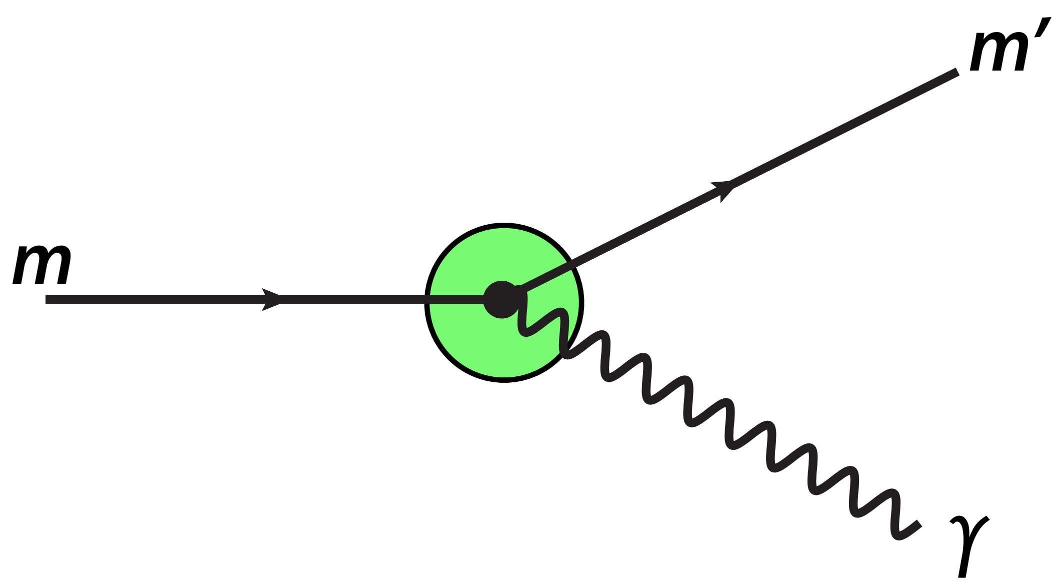
radiative transitions

CLEO-c, BES III

basic object: $\langle \gamma\, m' | m \rangle$



meson photocouplings

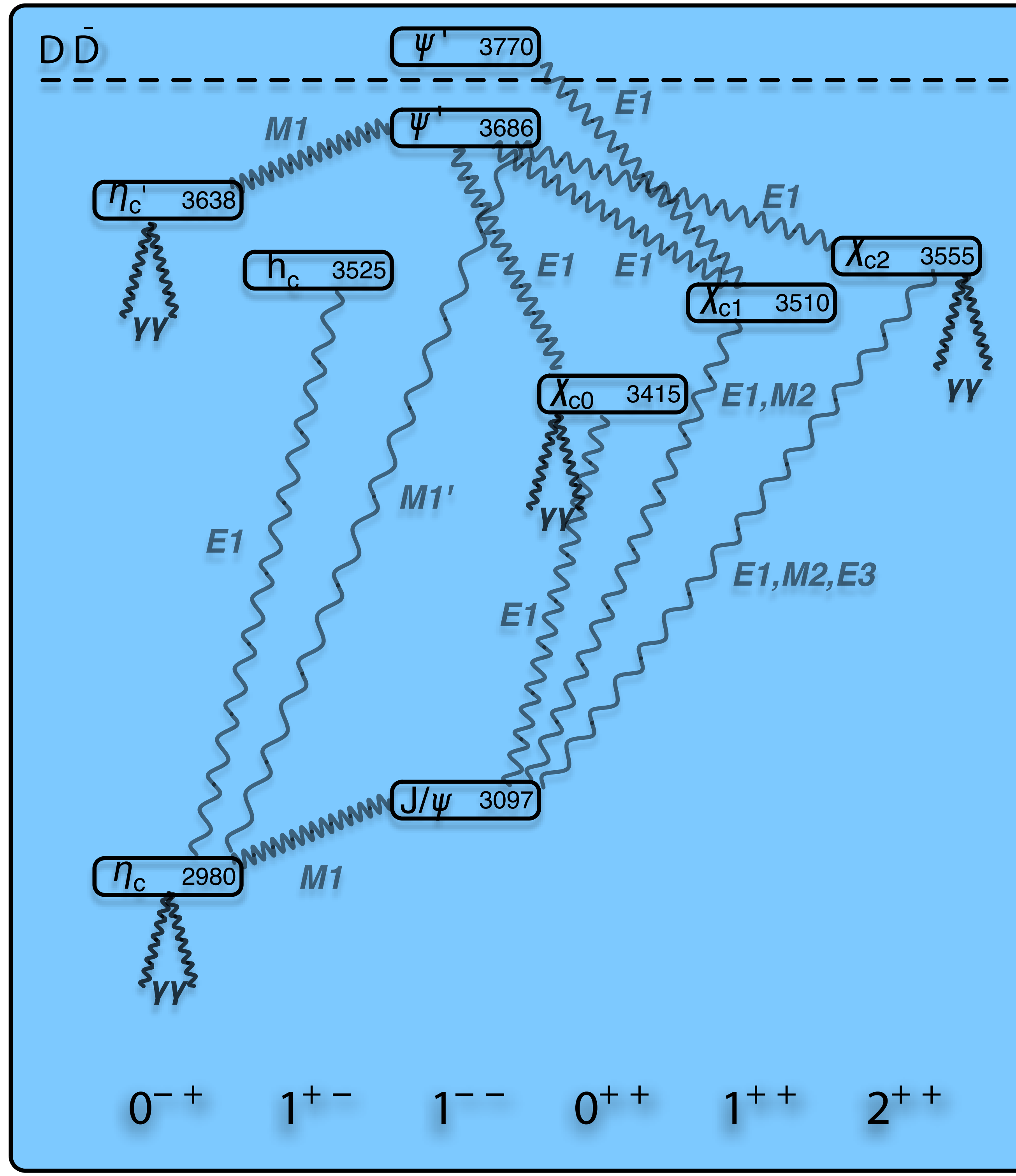


radiative transitions

CLEO-c, BES III

basic object: $\langle \gamma\, m' | m \rangle$

$$\langle m' | \bar{\psi} \gamma^{\mu} \psi | m \rangle \, \langle \gamma | A_{\mu} | 0 \rangle$$



meson photocouplings

in Euclidean time (t → -i t)

extract from three-point correlators

$$C(t_f, t, t_i) = \langle 0 | \Phi'(t_f)\, \left[\bar{\psi}\gamma^\mu\psi\right](t)\, \Phi(t_i) | 0 \rangle$$

$$C(t_f, t, t_i) = \sum_{\mathfrak{n},\mathfrak{m}} \langle 0 | \Phi'(0) | \mathfrak{n} \rangle e^{-E_{\mathfrak{n}}(t_f - t)} \langle \mathfrak{n} | \left[\bar{\psi}\gamma^\mu\psi\right](0) | \mathfrak{m} \rangle e^{-E_{\mathfrak{m}}(t - t_i)} \langle \mathfrak{m} | \Phi(0) | 0 \rangle$$

meson photocouplings

in Euclidean time (t → -i t)

extract from three-point correlators

$$C(t_f, t, t_i) = \langle 0 | \Phi'(t_f)\ [\bar{\psi}\gamma^\mu\psi](t)\ \Phi(t_i) | 0 \rangle$$

$$C(t_f, t, t_i) = \sum_{\mathfrak{n},\mathfrak{m}} \langle 0 | \Phi'(0) | \mathfrak{n} \rangle e^{-E_\mathfrak{n}(t_f - t)} \langle \mathfrak{n} | [\bar{\psi}\gamma^\mu\psi](0) | \mathfrak{m} \rangle e^{-E_\mathfrak{m}(t - t_i)} \langle \mathfrak{m} | \Phi(0) | 0 \rangle$$

spectrum of eigenstates of H_{QCD}
- i.e. the meson spectrum

meson photocouplings

in Euclidean time (t → -i t)

extract from three-point correlators

$$C(t_f, t, t_i) = \langle 0 | \Phi'(t_f)\, \left[\bar{\psi}\gamma^\mu\psi\right](t)\, \Phi(t_i) | 0 \rangle$$

$$C(t_f, t, t_i) = \sum_{\mathfrak{n},\mathfrak{m}} \langle 0 | \Phi'(0) | \mathfrak{n} \rangle e^{-E_{\mathfrak{n}}(t_f - t)} \langle \mathfrak{n} | \left[\bar{\psi}\gamma^\mu\psi\right](0) | \mathfrak{m} \rangle e^{-E_{\mathfrak{m}}(t - t_i)} \langle \mathfrak{m} | \Phi(0) | 0 \rangle$$

spectrum of eigenstates of H_{QCD}
- i.e. the meson spectrum

composite ***QCD*** operators with meson quantum numbers

meson photocouplings

in Euclidean time ($t \to -it$)

extract from three-point correlators

$$C(t_f, t, t_i) = \langle 0 | \Phi'(t_f) \, [\bar{\psi}\gamma^\mu\psi](t) \, \Phi(t_i) | 0 \rangle$$

$$C(t_f, t, t_i) = \sum_{\mathfrak{n},\mathfrak{m}} \langle 0 | \Phi'(0) | \mathfrak{n} \rangle e^{-E_\mathfrak{n}(t_f - t)} \langle \mathfrak{n} | [\bar{\psi}\gamma^\mu\psi](0) | \mathfrak{m} \rangle e^{-E_\mathfrak{m}(t - t_i)} \langle \mathfrak{m} | \Phi(0) | 0 \rangle$$

spectrum of eigenstates of H_{QCD}
- i.e. the meson spectrum

composite ***QCD*** operators with meson quantum numbers

e.g. pseudoscalars can be 'made' with

$$\bar{\psi}\gamma^5\psi$$
$$\epsilon_{ijk} \, \bar{\psi}\gamma^j\gamma^k(\partial^i - A^i)\psi$$
$$\epsilon_{ijk} \, \bar{\psi}\gamma^i\psi \, F^{jk}$$
$$\vdots$$

so we need to know the spectrum & vacuum matrix elements of operators first

meson photocouplings

in Euclidean time ($t \to -i\,t$)

extract from three-point correlators

$$C(t_f, t, t_i) = \langle 0 | \Phi'(t_f)\, [\bar{\psi}\gamma^\mu\psi](t)\, \Phi(t_i) | 0 \rangle$$

$$C(t_f, t, t_i) = \sum_{\mathfrak{n},\mathfrak{m}} \langle 0 | \Phi'(0) | \mathfrak{n} \rangle e^{-E_{\mathfrak{n}}(t_f - t)} \langle \mathfrak{n} | [\bar{\psi}\gamma^\mu\psi](0) | \mathfrak{m} \rangle e^{-E_{\mathfrak{m}}(t - t_i)} \langle \mathfrak{m} | \Phi(0) | 0 \rangle$$

spectrum of eigenstates of H_{QCD}
- i.e. the meson spectrum

composite ***QCD*** operators with meson quantum numbers

e.g. pseudoscalars can be 'made' with

$$\bar{\psi}\gamma^5\psi$$
$$\epsilon_{ijk}\, \bar{\psi}\gamma^j\gamma^k(\partial^i - A^i)\psi$$
$$\epsilon_{ijk}\, \bar{\psi}\gamma^i\psi\, F^{jk}$$
$$\vdots$$

so we need to know the spectrum & vacuum matrix elements of operators first

meson spectrum

basic object is two-point correlator

$$C(t) = \langle 0 \big| \Phi'(t)\, \Phi(0) \big| 0 \rangle$$

$$C(t) = \sum_{\mathfrak{n}} e^{-E_{\mathfrak{n}} t} \langle 0 \big| \Phi'(0) \big| \mathfrak{n} \rangle \langle \mathfrak{n} \big| \Phi(0) \big| 0 \rangle$$

e.g. pseudoscalars can be 'made' with

$$\bar{\psi}\gamma^5\psi$$

$$\epsilon_{ijk}\, \bar{\psi}\gamma^j\gamma^k(\partial^i - A^i)\psi$$

$$\epsilon_{ijk}\, \bar{\psi}\gamma^i\psi\, F^{jk}$$

$$\vdots$$

meson spectrum

basic object is two-point correlator

$$C(t) = \langle 0 \big| \Phi'(t)\, \Phi(0) \big| 0 \rangle$$

$$C(t) = \sum_{\mathfrak{n}} e^{-E_{\mathfrak{n}} t} \langle 0 \big| \Phi'(0) \big| \mathfrak{n} \rangle \langle \mathfrak{n} \big| \Phi(0) \big| 0 \rangle$$

e.g. pseudoscalars can be ‘made’ with

$$\bar{\psi}\gamma^5\psi$$

$$\epsilon_{ijk}\, \bar{\psi}\gamma^j\gamma^k(\partial^i - A^i)\psi$$

$$\epsilon_{ijk}\, \bar{\psi}\gamma^i\psi\, F^{jk}$$

$$\vdots$$

each operator will have different ‘overlap’ on to the tower of pseudoscalar states

sampling the ‘wavefunction’ of the states

meson spectrum

basic object is two-point correlator

$$C(t) = \big\langle 0 \big| \Phi'(t)\, \Phi(0) \big| 0 \big\rangle$$

$$C(t) = \sum_{\mathfrak{n}} e^{-E_{\mathfrak{n}} t} \big\langle 0 \big| \Phi'(0) \big| \mathfrak{n} \big\rangle \big\langle \mathfrak{n} \big| \Phi(0) \big| 0 \big\rangle$$

e.g. pseudoscalars can be 'made' with

$$\bar{\psi}\gamma^5\psi$$
$$\epsilon_{ijk}\, \bar{\psi}\gamma^j\gamma^k(\partial^i - A^i)\psi$$
$$\epsilon_{ijk}\, \bar{\psi}\gamma^i\psi\, F^{jk}$$
$$\vdots$$

each operator will have different 'overlap' on to the tower of pseudoscalar states

sampling the 'wavefunction' of the states

some linear combination of the operators is optimal for a certain state

$$\Omega_{\mathfrak{n}} = v_1^{\mathfrak{n}}\Phi_1 + v_2^{\mathfrak{n}}\Phi_2 + \ldots$$

meson spectrum

basic object is two-point correlator

$$C(t) = \langle 0 \big| \Phi'(t)\, \Phi(0) \big| 0 \rangle$$

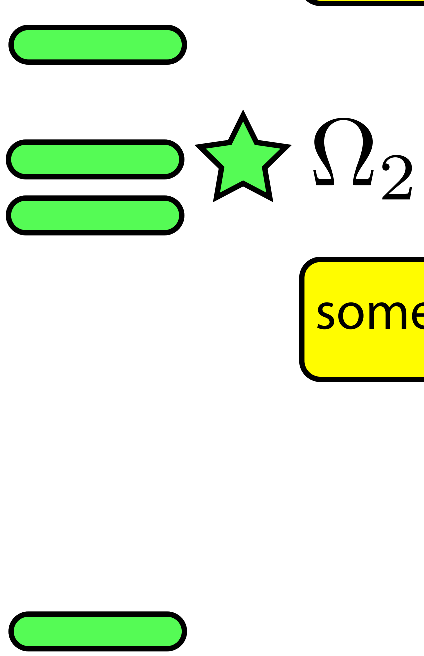
$$C(t) = \sum_{\mathfrak{n}} e^{-E_{\mathfrak{n}} t} \langle 0 \big| \Phi'(0) \big| \mathfrak{n} \rangle \langle \mathfrak{n} \big| \Phi(0) \big| 0 \rangle$$

e.g. pseudoscalars can be 'made' with

$$\bar{\psi}\gamma^5\psi$$
$$\epsilon_{ijk}\, \bar{\psi}\gamma^j\gamma^k(\partial^i - A^i)\psi$$
$$\epsilon_{ijk}\, \bar{\psi}\gamma^i\psi\, F^{jk}$$
$$\vdots$$

each operator will have different 'overlap' on to the tower of pseudoscalar states

sampling the 'wavefunction' of the states



some linear combination of the operators is optimal for a certain state

$$\Omega_{\mathfrak{n}} = v_1^{\mathfrak{n}}\Phi_1 + v_2^{\mathfrak{n}}\Phi_2 + \ldots$$

meson spectrum

basic object is two-point correlator

$$C(t) = \langle 0 \big| \Phi'(t)\, \Phi(0) \big| 0 \rangle$$

$$C(t) = \sum_{\mathfrak{n}} e^{-E_{\mathfrak{n}} t} \langle 0 \big| \Phi'(0) \big| \mathfrak{n} \rangle \langle \mathfrak{n} \big| \Phi(0) \big| 0 \rangle$$

e.g. pseudoscalars can be 'made' with

$$\bar{\psi}\gamma^5\psi$$

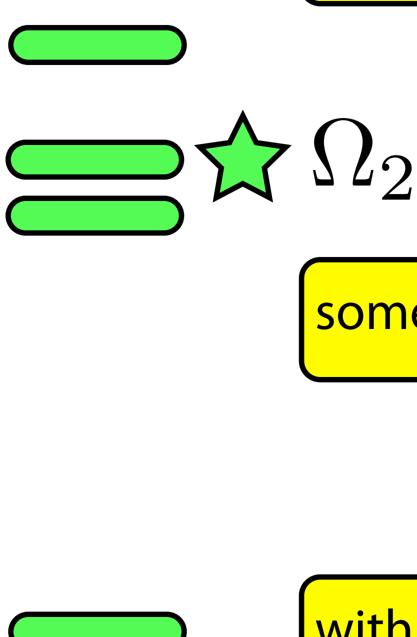
$$\epsilon_{ijk}\, \bar{\psi}\gamma^j\gamma^k(\partial^i - A^i)\psi$$

$$\epsilon_{ijk}\, \bar{\psi}\gamma^i\psi\, F^{jk}$$

$$\vdots$$

each operator will have different 'overlap' on to the tower of pseudoscalar states

sampling the 'wavefunction' of the states



some linear combination of the operators is optimal for a certain state

$$\Omega_{\mathfrak{n}} = v_1^{\mathfrak{n}} \Phi_1 + v_2^{\mathfrak{n}} \Phi_2 + \ldots$$

within a finite basis of operators, our best estimate is from a ***variational solution***

variational analysis

matrix of correlators

$$C(t) = \begin{bmatrix} \langle 0|\Phi_1(t)\Phi_1(0)|0\rangle & \langle 0|\Phi_1(t)\Phi_2(0)|0\rangle & \dots \\ \langle 0|\Phi_2(t)\Phi_1(0)|0\rangle & \langle 0|\Phi_2(t)\Phi_2(0)|0\rangle & \dots \\ \vdots & & \ddots \end{bmatrix}$$

e.g. $\bar{\psi}\gamma^5\psi$

$$\epsilon_{ijk}\,\bar{\psi}\gamma^j\gamma^k(\partial^i - A^i)\psi$$

$$\epsilon_{ijk}\,\bar{\psi}\gamma^i\psi\,F^{jk}$$

$$\vdots$$

variational analysis

matrix of correlators

$$C(t) = \begin{bmatrix} \langle 0|\Phi_1(t)\Phi_1(0)|0\rangle & \langle 0|\Phi_1(t)\Phi_2(0)|0\rangle & \ldots \\ \langle 0|\Phi_2(t)\Phi_1(0)|0\rangle & \langle 0|\Phi_2(t)\Phi_2(0)|0\rangle & \ldots \\ \vdots & & \ddots \end{bmatrix}$$

e.g. $\bar{\psi}\gamma^5\psi$

$$\epsilon_{ijk}\,\bar{\psi}\gamma^j\gamma^k(\partial^i - A^i)\psi$$

$$\epsilon_{ijk}\,\bar{\psi}\gamma^i\psi\,F^{jk}$$

$$\vdots$$

variational solution = generalised eigenvalue problem

$$C(t)v^{\mathfrak{n}} = \lambda_{\mathfrak{n}}(t)C(t_0)v^{\mathfrak{n}}$$

variational analysis

matrix of correlators

$$C(t) = \begin{bmatrix} \langle 0|\Phi_1(t)\Phi_1(0)|0\rangle & \langle 0|\Phi_1(t)\Phi_2(0)|0\rangle & \dots \\ \langle 0|\Phi_2(t)\Phi_1(0)|0\rangle & \langle 0|\Phi_2(t)\Phi_2(0)|0\rangle & \dots \\ \vdots & & \ddots \end{bmatrix}$$

e.g. $\bar{\psi}\gamma^5\psi$

$$\epsilon_{ijk}\,\bar{\psi}\gamma^j\gamma^k(\partial^i - A^i)\psi$$

$$\epsilon_{ijk}\,\bar{\psi}\gamma^i\psi\,F^{jk}$$

$$\vdots$$

variational solution = generalised eigenvalue problem

$$C(t)v^{\mathfrak{n}} = \lambda_{\mathfrak{n}}(t)C(t_0)v^{\mathfrak{n}}$$

eigenvalues give spectrum

$$\lambda_{\mathfrak{n}}(t) \to e^{-E_{\mathfrak{n}}(t-t_0)}$$

variational analysis

matrix of correlators

$$C(t) = \begin{bmatrix} \langle 0|\Phi_1(t)\Phi_1(0)|0\rangle & \langle 0|\Phi_1(t)\Phi_2(0)|0\rangle & \dots \\ \langle 0|\Phi_2(t)\Phi_1(0)|0\rangle & \langle 0|\Phi_2(t)\Phi_2(0)|0\rangle & \dots \\ \vdots & & \ddots \end{bmatrix}$$

e.g. $\bar{\psi}\gamma^5\psi$

$$\epsilon_{ijk}\,\bar{\psi}\gamma^j\gamma^k(\partial^i - A^i)\psi$$

$$\epsilon_{ijk}\,\bar{\psi}\gamma^i\psi\,F^{jk}$$

$$\vdots$$

variational solution = generalised eigenvalue problem

$$C(t)v^{\mathfrak{n}} = \lambda_{\mathfrak{n}}(t)C(t_0)v^{\mathfrak{n}}$$

eigenvalues give spectrum

$$\lambda_{\mathfrak{n}}(t) \to e^{-E_{\mathfrak{n}}(t-t_0)}$$

eigenvectors give the ‘optimal’ operators

$$\Omega_{\mathfrak{n}} = v_1^{\mathfrak{n}}\Phi_1 + v_2^{\mathfrak{n}}\Phi_2 + \dots$$

variational analysis

matrix of correlators

$$C(t) = \begin{bmatrix} \langle 0|\Phi_1(t)\Phi_1(0)|0\rangle & \langle 0|\Phi_1(t)\Phi_2(0)|0\rangle & \dots \\ \langle 0|\Phi_2(t)\Phi_1(0)|0\rangle & \langle 0|\Phi_2(t)\Phi_2(0)|0\rangle & \dots \\ \vdots & & \ddots \end{bmatrix}$$

e.g. $\bar{\psi}\gamma^5\psi$

$$\epsilon_{ijk}\,\bar{\psi}\gamma^j\gamma^k(\partial^i - A^i)\psi$$

$$\epsilon_{ijk}\,\bar{\psi}\gamma^i\psi\,F^{jk}$$

$$\vdots$$

variational solution = generalised eigenvalue problem

$$C(t)v^{\mathfrak{n}} = \lambda_{\mathfrak{n}}(t)C(t_0)v^{\mathfrak{n}}$$

eigenvalues give spectrum

$$\lambda_{\mathfrak{n}}(t) \rightarrow e^{-E_{\mathfrak{n}}(t-t_0)}$$

eigenvectors give the 'optimal' operators

$$\Omega_{\mathfrak{n}} = v_1^{\mathfrak{n}}\Phi_1 + v_2^{\mathfrak{n}}\Phi_2 + \dots$$

orthogonality of eigenvectors - required to extract near degenerate states

variational analysis

matrix of correlators

$$C(t) = \begin{bmatrix} \langle 0|\Phi_1(t)\Phi_1(0)|0\rangle & \langle 0|\Phi_1(t)\Phi_2(0)|0\rangle & \dots \\ \langle 0|\Phi_2(t)\Phi_1(0)|0\rangle & \langle 0|\Phi_2(t)\Phi_2(0)|0\rangle & \dots \\ \vdots & & \ddots \end{bmatrix}$$

e.g. $\bar{\psi}\gamma^5\psi$

$$\epsilon_{ijk}\,\bar{\psi}\gamma^j\gamma^k(\partial^i - A^i)\psi$$

$$\epsilon_{ijk}\,\bar{\psi}\gamma^i\psi\,F^{jk}$$

$$\vdots$$

variational solution = generalised eigenvalue problem

$$C(t)v^{\mathfrak{n}} = \lambda_{\mathfrak{n}}(t)C(t_0)v^{\mathfrak{n}}$$

eigenvalues give spectrum

$$\lambda_{\mathfrak{n}}(t) \to e^{-E_{\mathfrak{n}}(t-t_0)}$$

eigenvectors give the 'optimal' operators

$$\Omega_{\mathfrak{n}} = v_1^{\mathfrak{n}}\Phi_1 + v_2^{\mathfrak{n}}\Phi_2 + \dots$$

*

orthogonality of eigenvectors - required to extract near degenerate states

* how big does the basis need to be ?

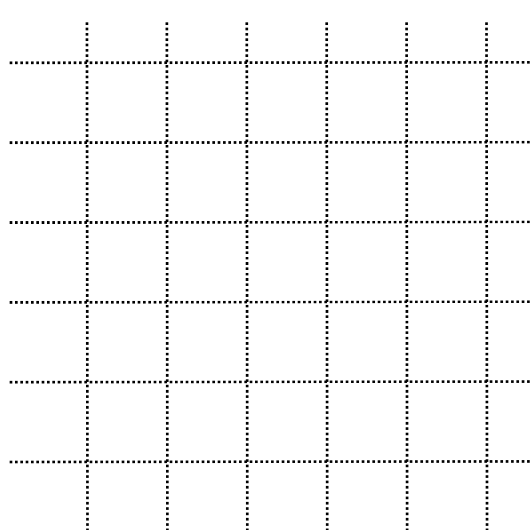
vector spectrum

e.g. charmonium vector spectrum

near deg. states are tough to fit

using multi-exponential fits

$$C(t) = \sum_{\mathfrak{n}} e^{-E_{\mathfrak{n}} t} \langle 0 | \Phi'(0) | \mathfrak{n} \rangle \langle \mathfrak{n} | \Phi(0) | 0 \rangle$$

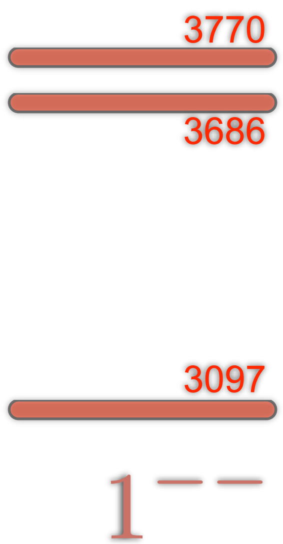


e.g. in two dimensions: $\psi_J(\theta) = e^{iJ\theta}$

so under the allowed π/2 rotations, *J*=0,4,8... indistinguishable

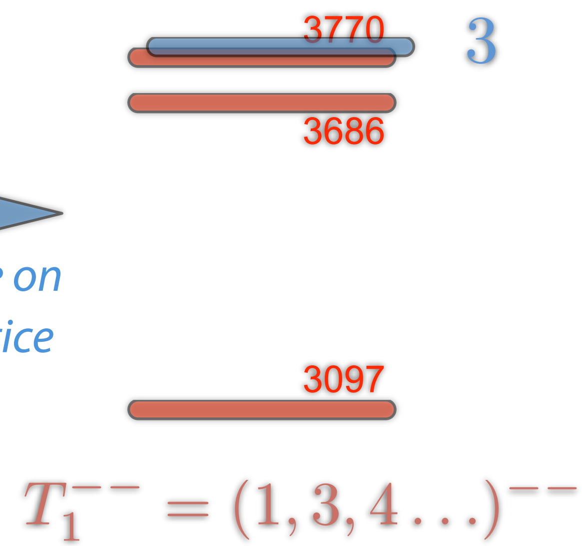
vector spectrum

e.g. charmonium vector spectrum



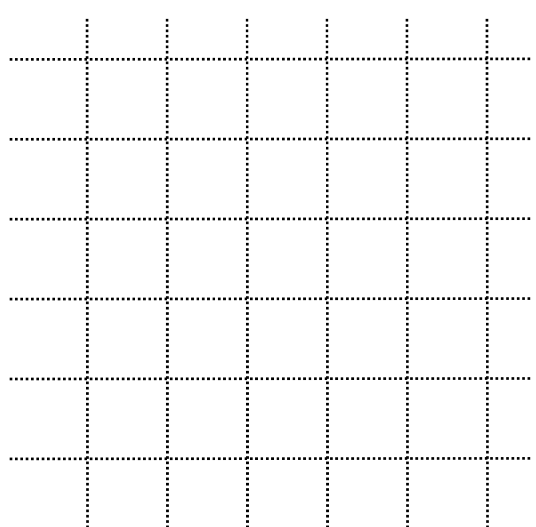
near deg. states are tough to fit

even worse on a cubic lattice



using multi-exponential fits

$$C(t) = \sum_{\mathfrak{n}} e^{-E_{\mathfrak{n}} t} \langle 0 | \Phi'(0) | \mathfrak{n} \rangle \langle \mathfrak{n} | \Phi(0) | 0 \rangle$$



e.g. in two dimensions: $\psi_J(\theta) = e^{iJ\theta}$

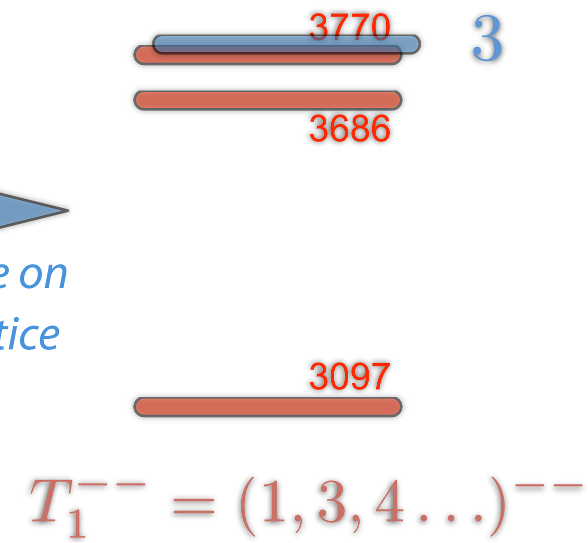
so under the allowed $\pi/2$ rotations, J=0,4,8... indistinguishable

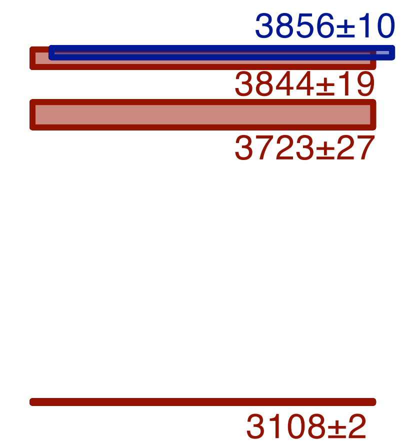
vector spectrum

e.g. charmonium vector spectrum

near deg. states are tough to fit

even worse on a cubic lattice

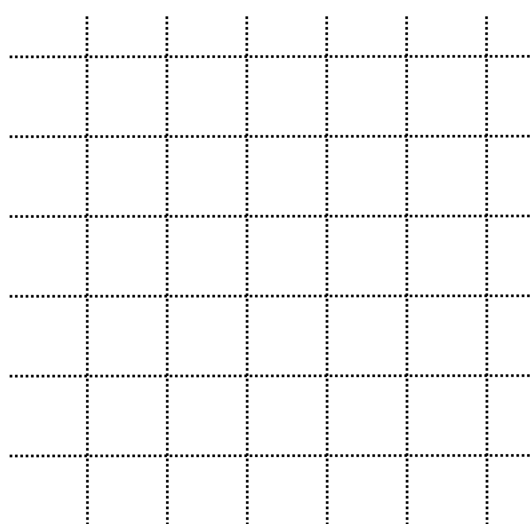




variational solution

using multi-exponential fits

$$C(t) = \sum_{\mathfrak{n}} e^{-E_{\mathfrak{n}} t} \langle 0 | \Phi'(0) | \mathfrak{n} \rangle \langle \mathfrak{n} | \Phi(0) | 0 \rangle$$



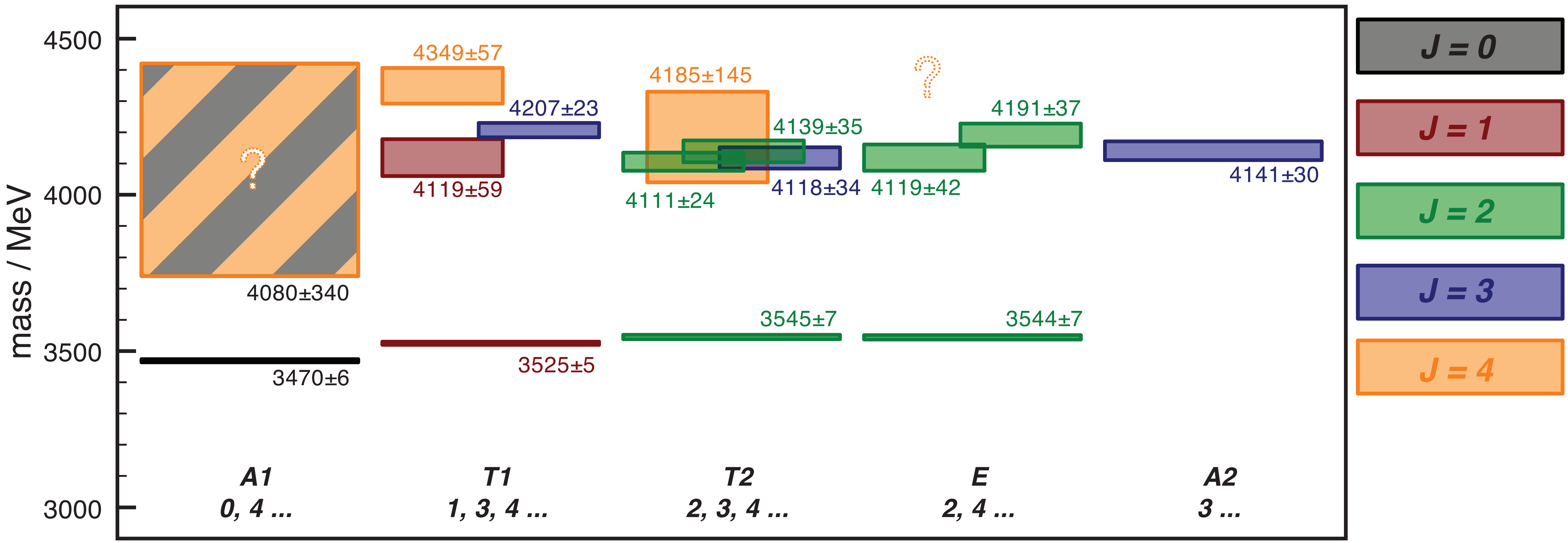
e.g. in two dimensions: $\psi_J(\theta) = e^{iJ\theta}$

so under the allowed π/2 rotations, *J*=0,4,8... indistinguishable

more spectrum

e.g. J^{++}

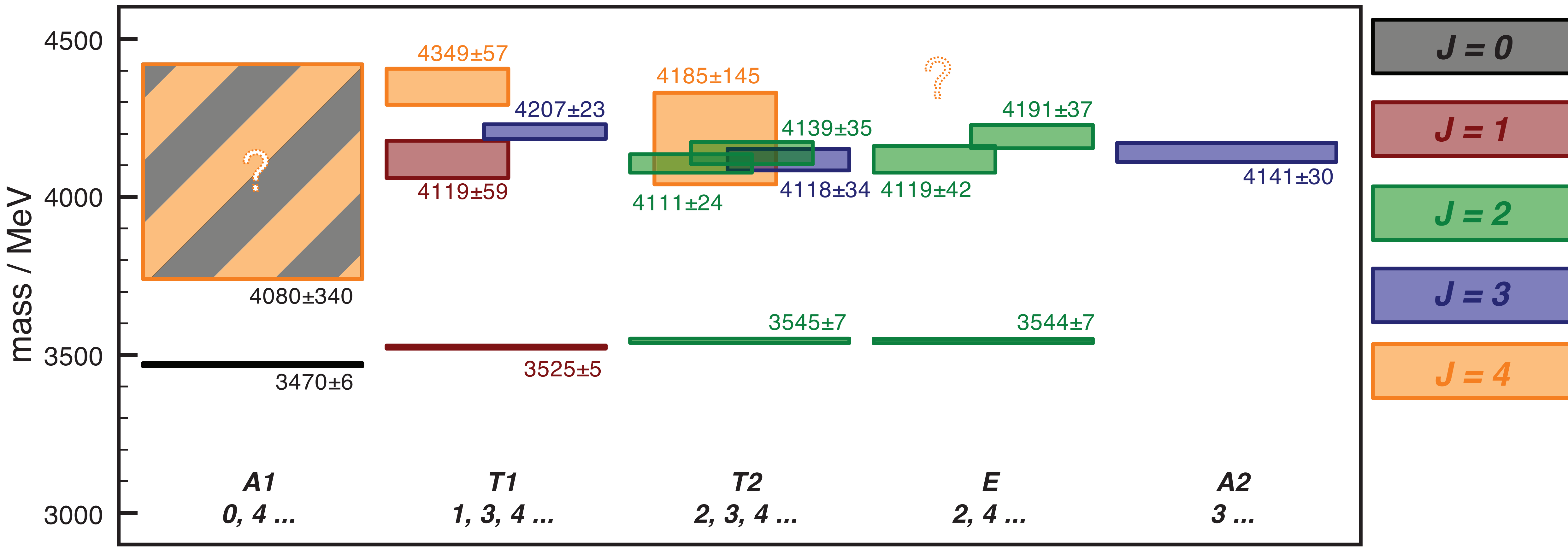
$a_s = 0.1$ fm



more spectrum

e.g. J^{++}

$a_s = 0.1$ fm



$\chi_{c(0,1,2)}\ [\,2^3P_J\,]$

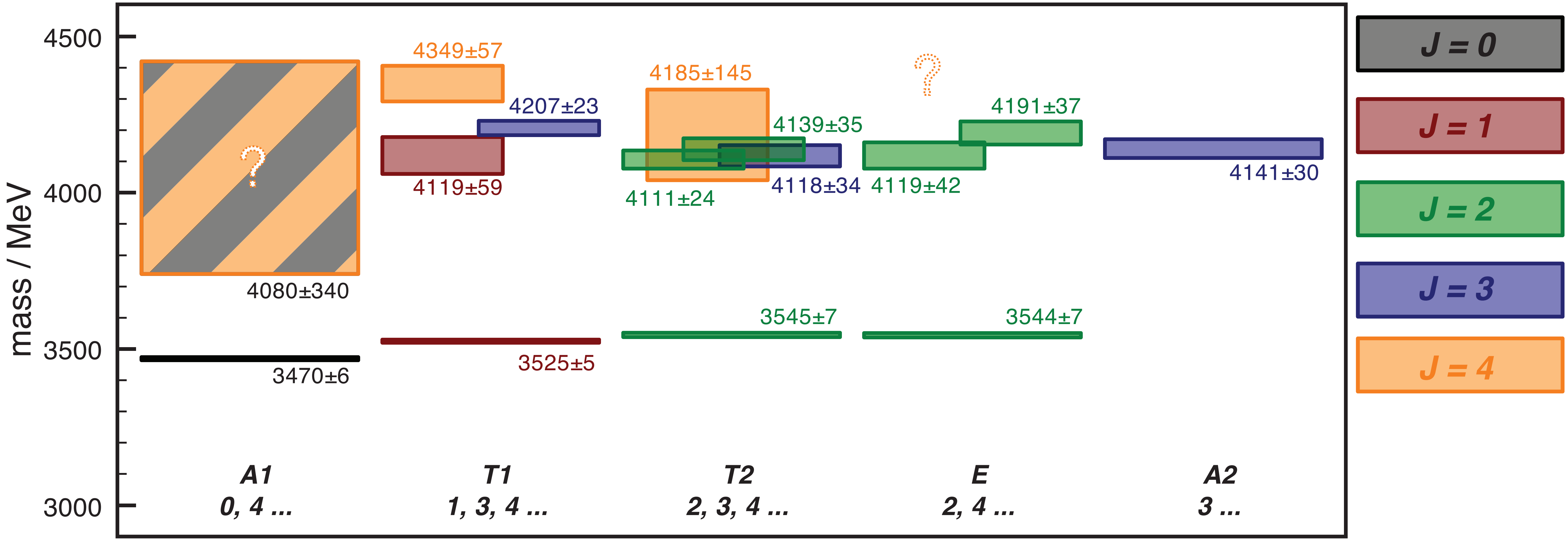
$\chi_{c(2,3,4)}\ [\,^3F_J\,]$

$\chi_{c(0,1,2)}\ [\,^3P_J\,]$

more spectrum

e.g. J^{++}

$a_s = 0.1$ fm



$\chi_{c(2,3,4)}$ [^3F_J]

$\chi_{c(0,1,2)}$ [2^3P_J]

$\chi_{c(0,1,2)}$ [^3P_J]

somewhat limited by the size of operator basis - have subsequently expanded

the calculation

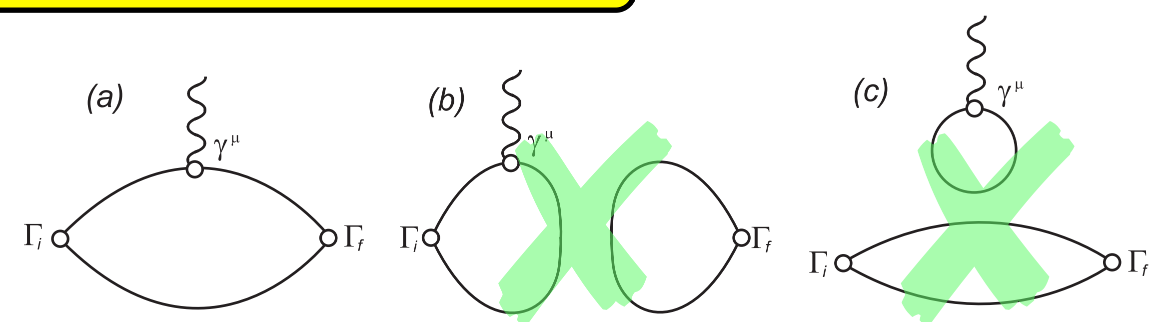
first attempt - little systematic control

quenched - no light quarks at all (like models)

one lattice spacing ***a = 0.1 fm*** (anisotropic, ***a_t = 0.033 fm***)

box possibly too small for highly excited states (***1.2 fm***)

only connected diagrams



OZI justification ?

allowed us to get ***high statistics*** (1000 gauge field configs) and most importantly to 'try things out'

Monte-Carlo - statistical error from finite number of samples

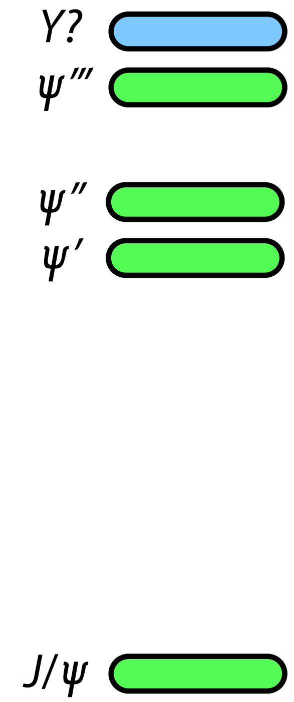
all of these 'lattice issues' are systematically improvable: see papers by ***Fermilab/MILC*** & ***HPQCD***

vector states

Level	Mass/MeV	Suggested state	Model assignment
0	3106(2)	J/ψ	1^3S_1
1	3746(18)	$\psi'(3686)$	2^3S_1
2	3846(12)	ψ_3	Lattice artifact
3	3864(19)	$\psi''(3770)$	1^3D_1
4	4283(77)	ψ(“4040”)	3^3S_1
5	4400(60)	Y?	Hybrid

masses systematically high:
quenched?
finite volume?

vacuum matrix elements compared to potential model : ***PRD78:094504 (2008)***



meson photocouplings

in Euclidean time (t → -i t)

extract from three-point correlators

$$C(t_f, t, t_i) = \big\langle 0 \big| \Phi'(t_f)\, \big[\bar{\psi}\gamma^\mu\psi\big](t)\, \Phi(t_i) \big| 0 \big\rangle$$

$$C(t_f, t, t_i) = \sum_{\mathfrak{n},\mathfrak{m}} \big\langle 0 \big| \Phi'(0) \big| \mathfrak{n} \big\rangle e^{-E_\mathfrak{n}(t_f - t)} \big\langle \mathfrak{n} \big| \big[\bar{\psi}\gamma^\mu\psi\big](0) \big| \mathfrak{m} \big\rangle e^{-E_\mathfrak{m}(t - t_i)} \big\langle \mathfrak{m} \big| \Phi(0) \big| 0 \big\rangle$$

meson photocouplings

in Euclidean time (t → -i t)

extract from three-point correlators

$$C(t_f, t, t_i) = \langle 0 | \Phi'(t_f)\, \left[\bar{\psi}\gamma^\mu \psi\right](t)\, \Phi(t_i) | 0 \rangle$$

$$C(t_f, t, t_i) = \sum_{\mathfrak{n},\mathfrak{m}} \langle 0 | \Phi'(0) | \mathfrak{n} \rangle e^{-E_\mathfrak{n}(t_f - t)} \langle \mathfrak{n} | \left[\bar{\psi}\gamma^\mu \psi\right](0) | \mathfrak{m} \rangle e^{-E_\mathfrak{m}(t - t_i)} \langle \mathfrak{m} | \Phi(0) | 0 \rangle$$

eigenvectors give the 'optimal' operators

$$\Omega_\mathfrak{n} = v_1^\mathfrak{n} \Phi_1 + v_2^\mathfrak{n} \Phi_2 + \ldots$$

meson photocouplings

in Euclidean time ($t \to -i\,t$)

extract from three-point correlators

$$C(t_f, t, t_i) = \langle 0 | \Phi'(t_f)\, \big[\bar{\psi}\gamma^\mu\psi\big](t)\, \Phi(t_i) | 0 \rangle$$

$$C(t_f, t, t_i) = \sum_{\mathfrak{n},\mathfrak{m}} \langle 0 | \Phi'(0) | \mathfrak{n} \rangle e^{-E_\mathfrak{n}(t_f - t)} \langle \mathfrak{n} | \big[\bar{\psi}\gamma^\mu\psi\big](0) | \mathfrak{m} \rangle e^{-E_\mathfrak{m}(t - t_i)} \langle \mathfrak{m} | \Phi(0) | 0 \rangle$$

eigenvectors give the 'optimal' operators

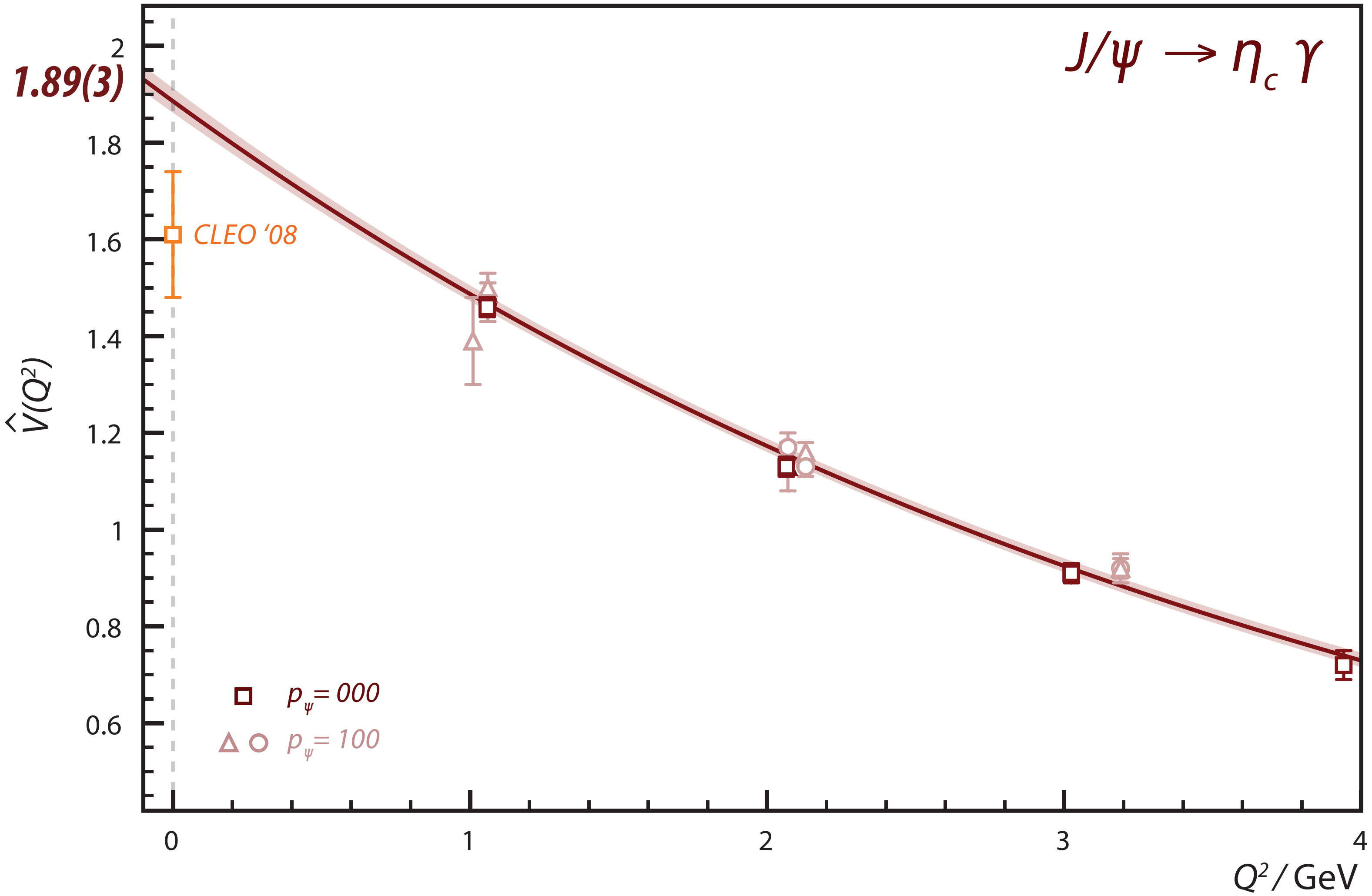
$$\Omega_\mathfrak{n} = v_1^\mathfrak{n} \Phi_1 + v_2^\mathfrak{n} \Phi_2 + \ldots$$

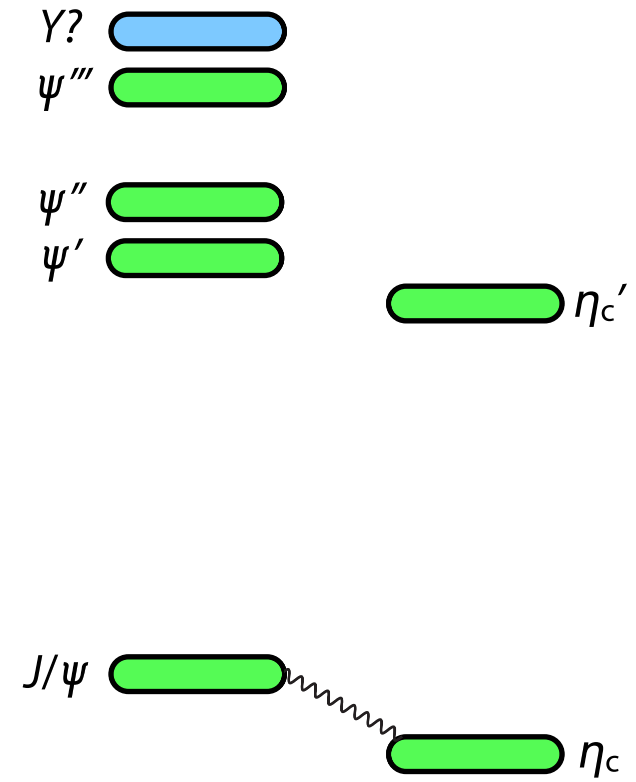
compute for multiple operators & project with eigenvectors

$$v^\mathfrak{p} C(t_f, t, t_i) = \sum_{\mathfrak{m}} \langle 0 | \Omega^\mathfrak{p}(0) | \mathfrak{p} \rangle e^{-E_\mathfrak{p}(t_f - t)} \langle \mathfrak{p} | \big[\bar{\psi}\gamma^\mu\psi\big](0) | \mathfrak{m} \rangle e^{-E_\mathfrak{m}(t - t_i)} \langle \mathfrak{m} | \Phi(0) | 0 \rangle$$

now just a single state ***p*** contributing - can be an excited state

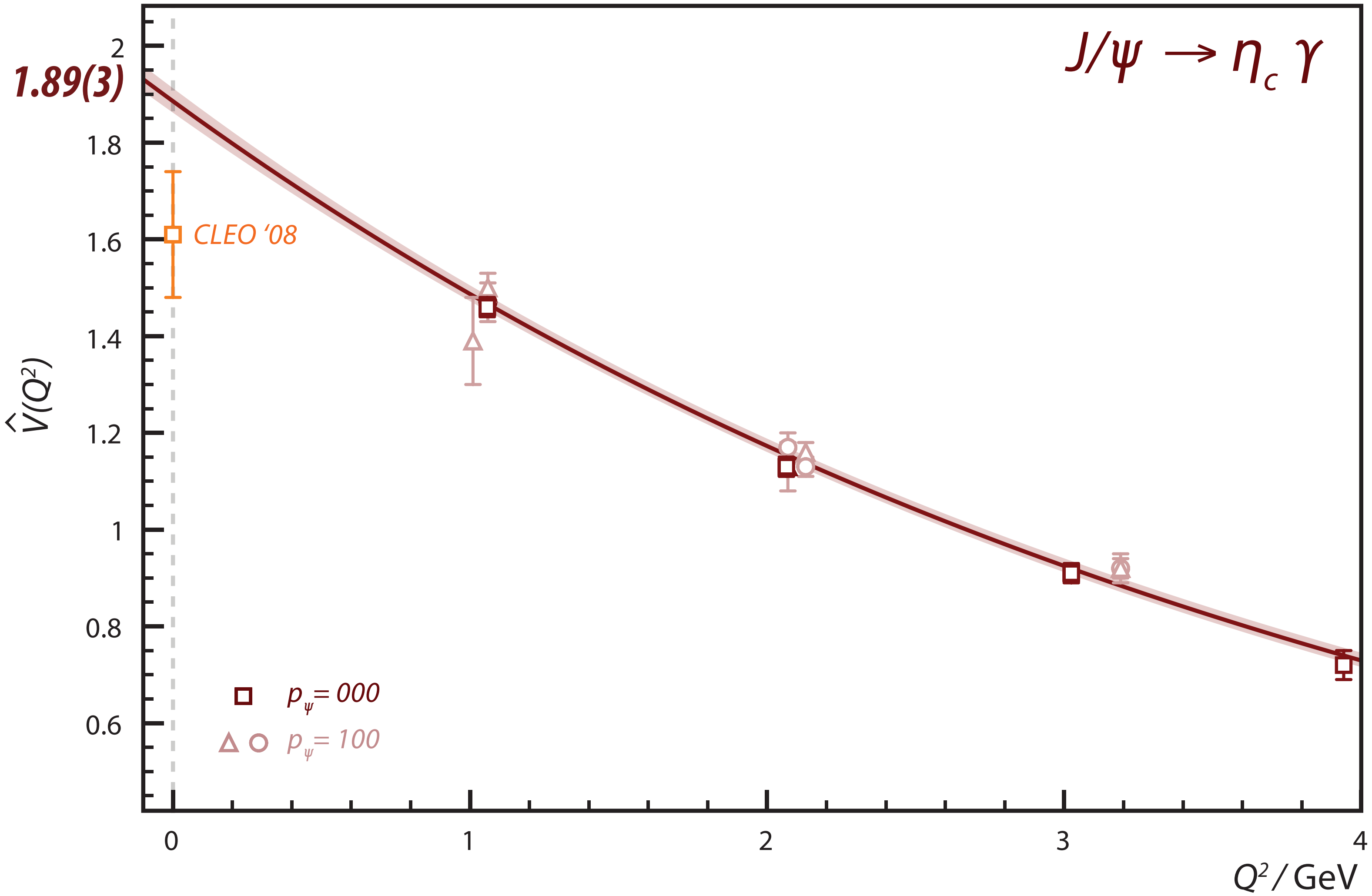
vector - pseudoscalar (M1)

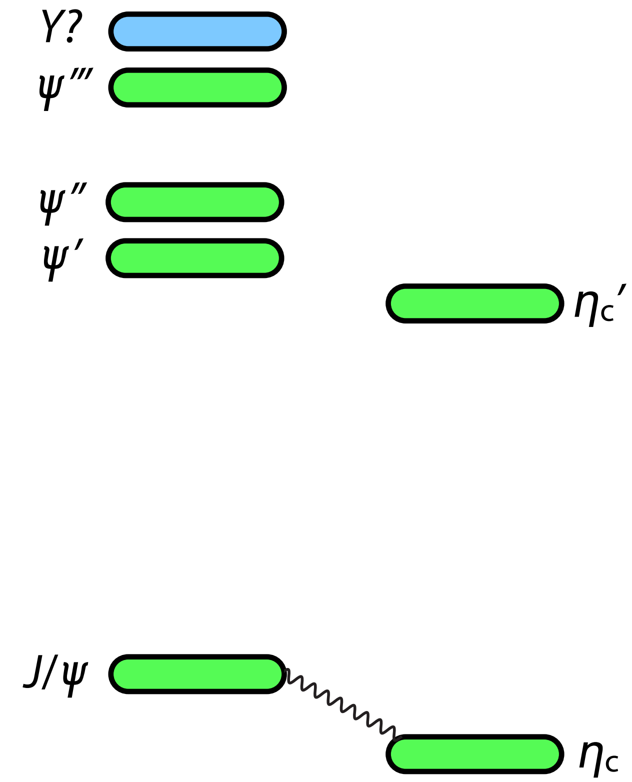




$$\hat{V} \propto \left\langle J/\psi \middle| \bar{\psi}\gamma^\mu\psi \middle| \eta_c \right\rangle \propto \Gamma^{1/2}_{J/\psi\to\eta_c\gamma}$$

vector - pseudoscalar (M1)





$$\hat{V} \propto \langle J/\psi | \bar{\psi}\gamma^\mu\psi | \eta_c \rangle \propto \Gamma^{1/2}_{J/\psi \to \eta_c\gamma}$$

in quark-potential models:

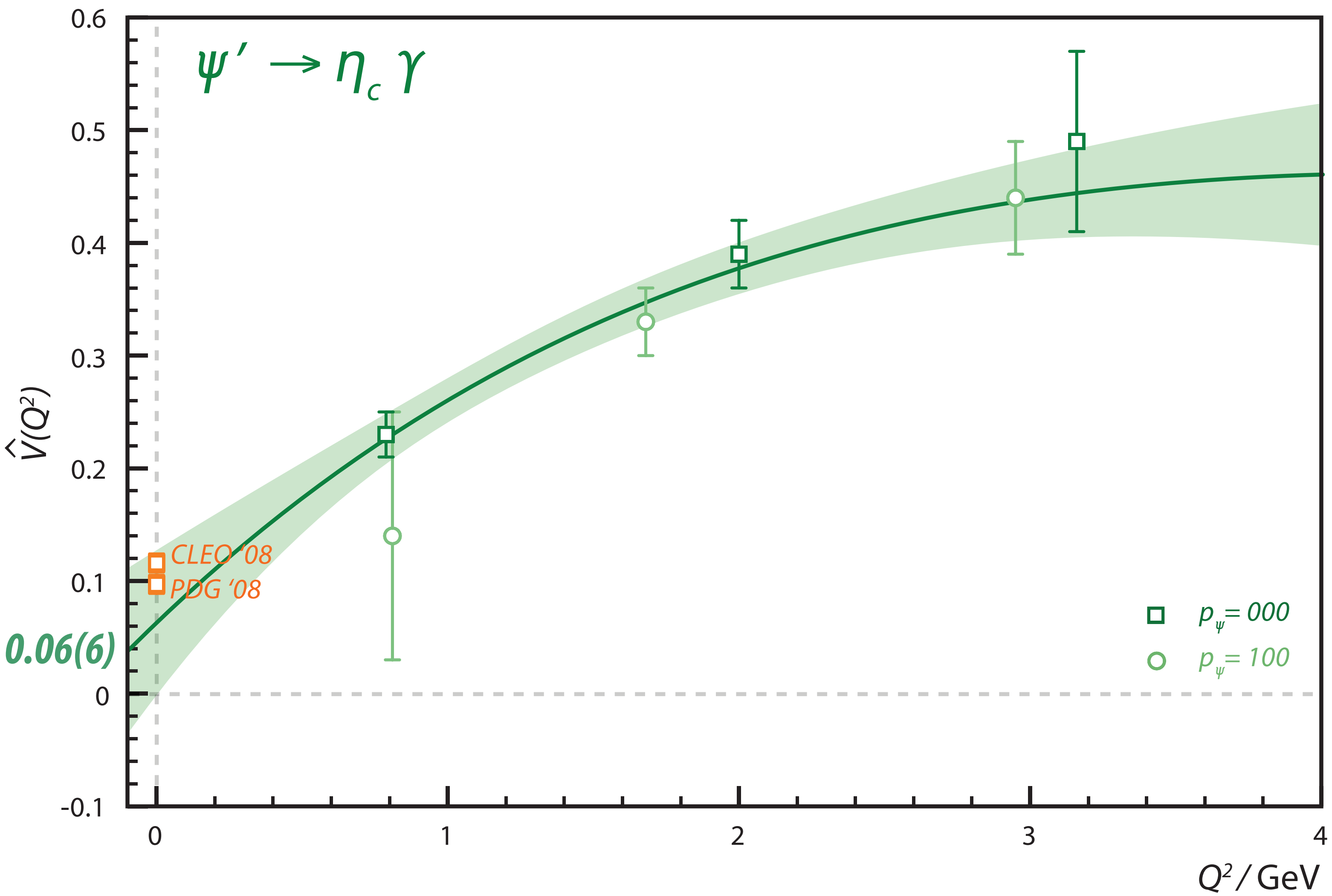
quark spin flip ~ $\dfrac{\sigma}{m_c}$

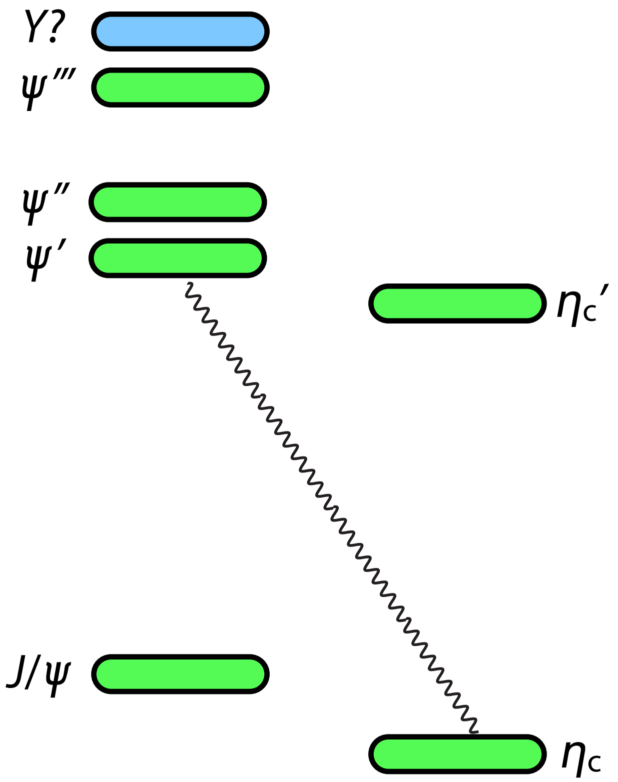
$$V \sim \frac{1}{m_c} \int r^2 dr\, R_f(r) j_0(|\vec{q}|r) R_i(r)$$

'Higher Charmonia'
(Barnes, Godfrey, Swanson)

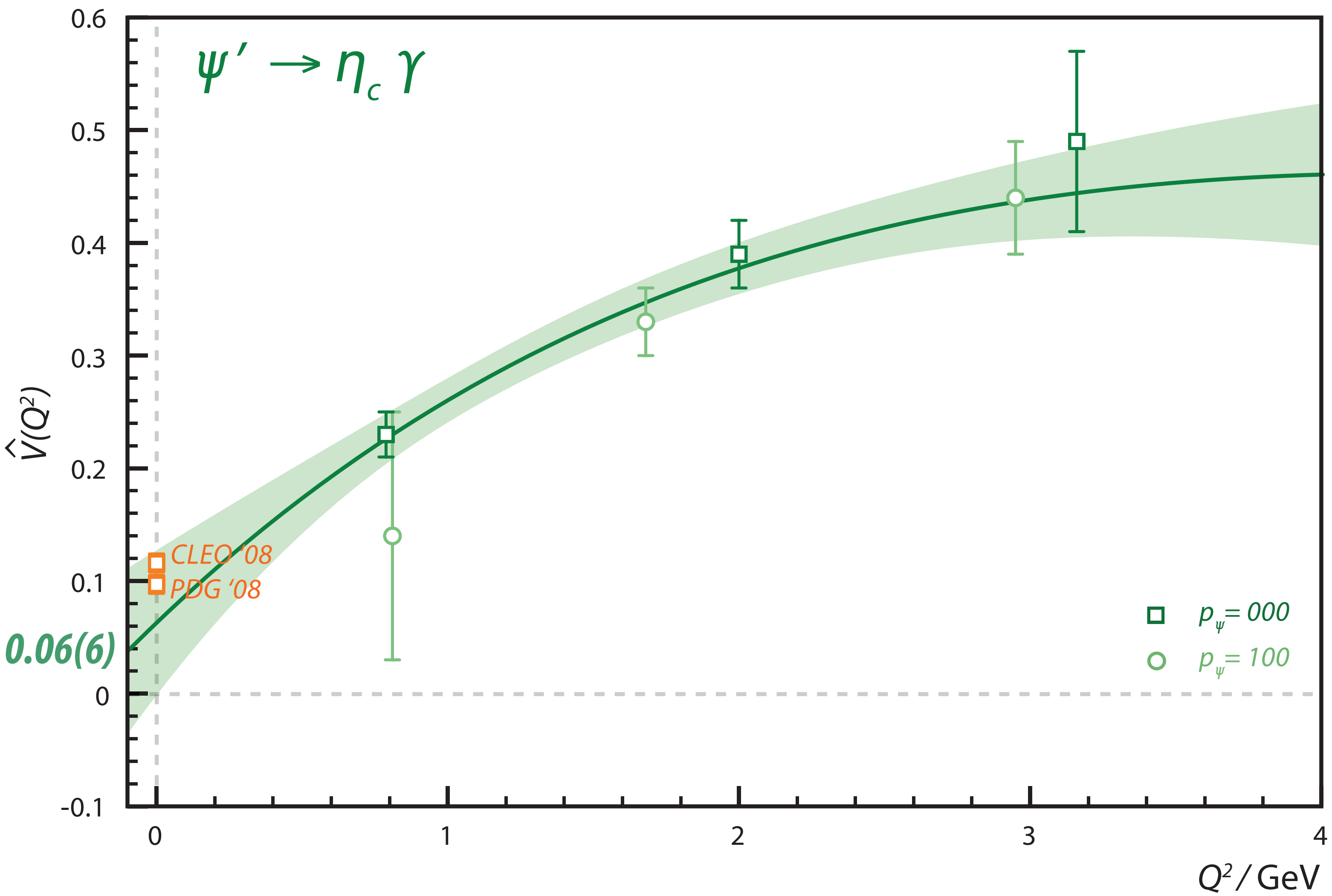
Γ ~ 2.4 - 2.9 keV vs. expt^al (***CLEO-c***) = 1.85(30) keV

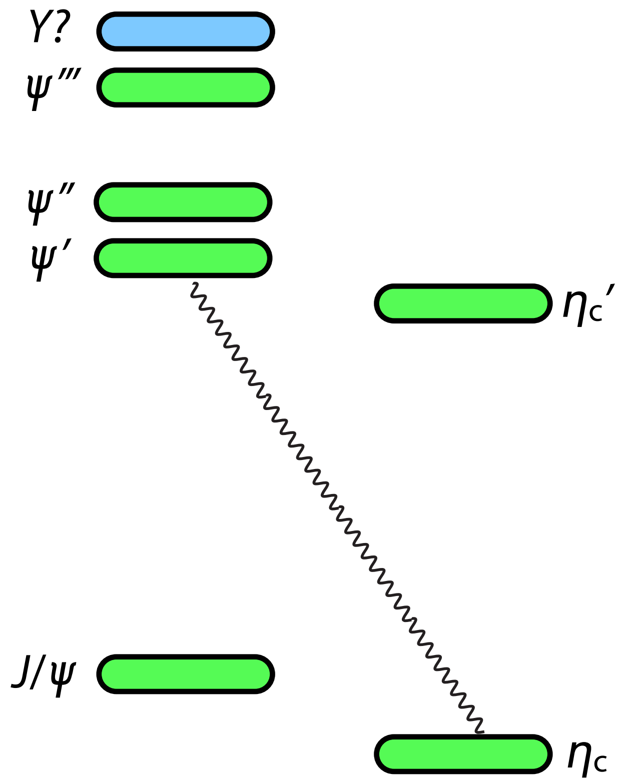
vector - pseudoscalar (M1)





vector - pseudoscalar (M1)





in quark-potential models:

'hindered' : orthogonal (***2S***, ***1S***) wavefunctions

$$V \sim \frac{1}{m_c} \int r^2 dr\, R_f(r) \left(1 + \mathcal{O}(|\vec{q}|^2 r^2)\right) R_i(r)$$

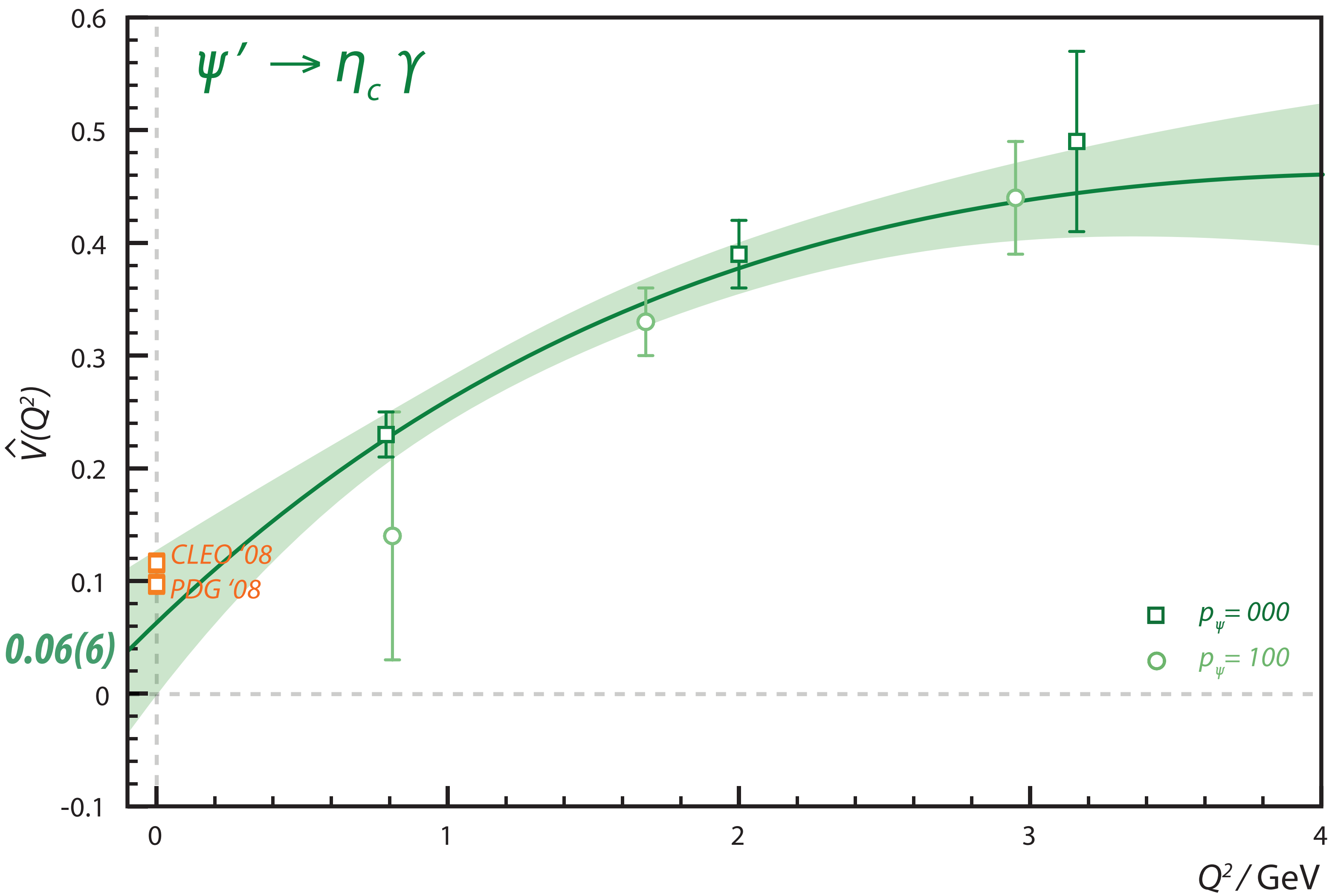
relativistic corrections at the same order

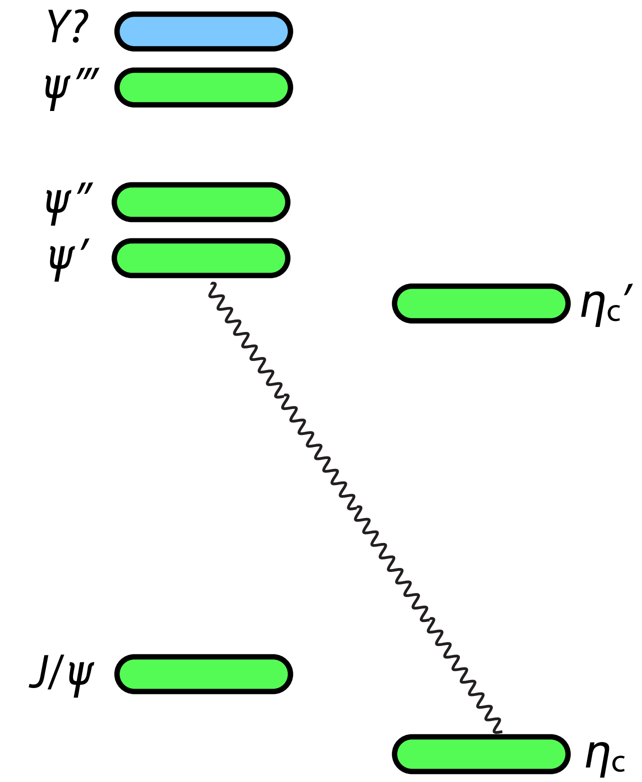
frame (in)dependence of non-rel wavefunctions

'Higher Charmonia'
(Barnes, Godfrey, Swanson)

$\Gamma \sim$ 4 - 10 keV vs. exptal (***CLEO-c***) = 1.37(20) keV

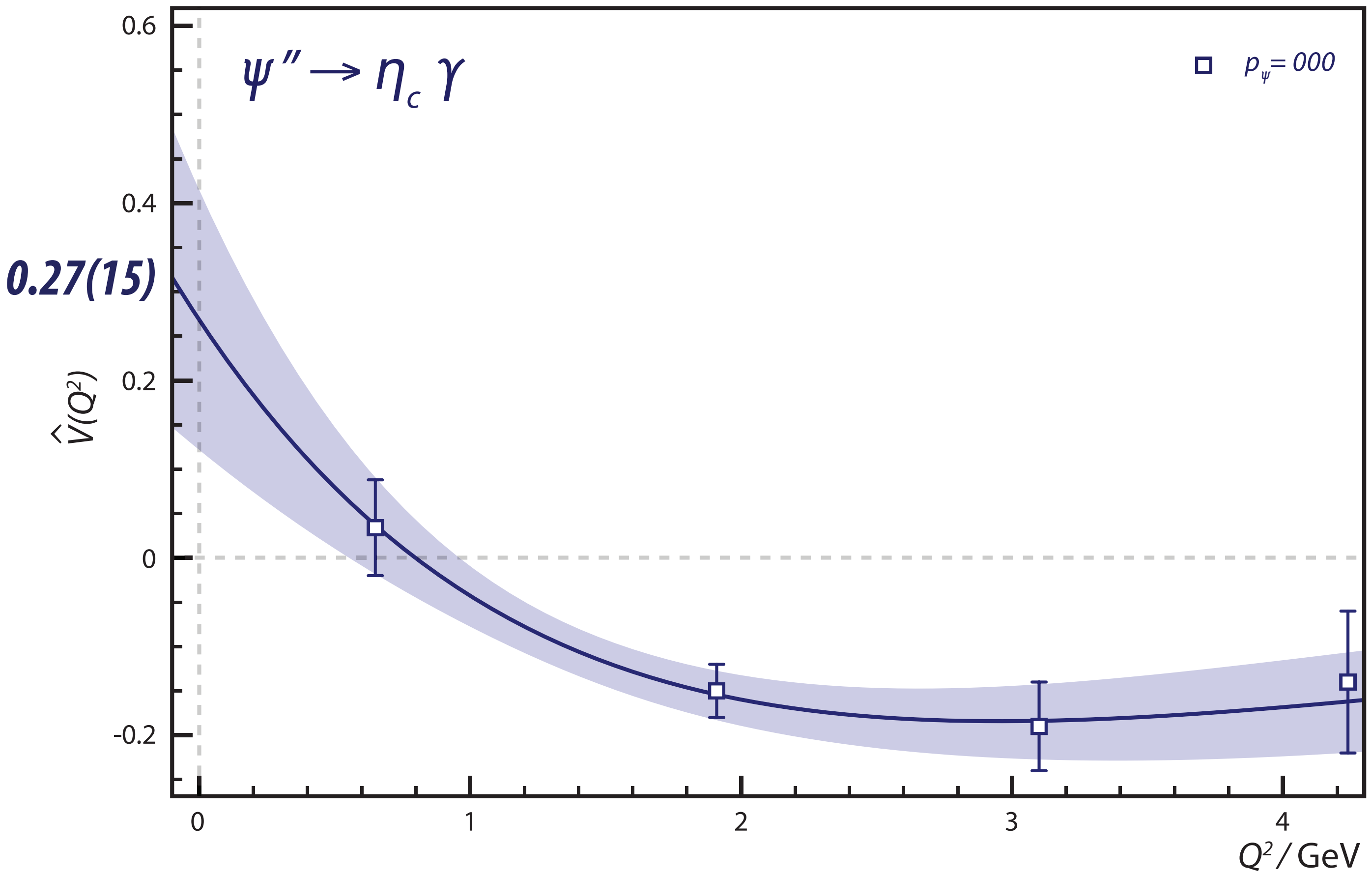
vector - pseudoscalar (M1)

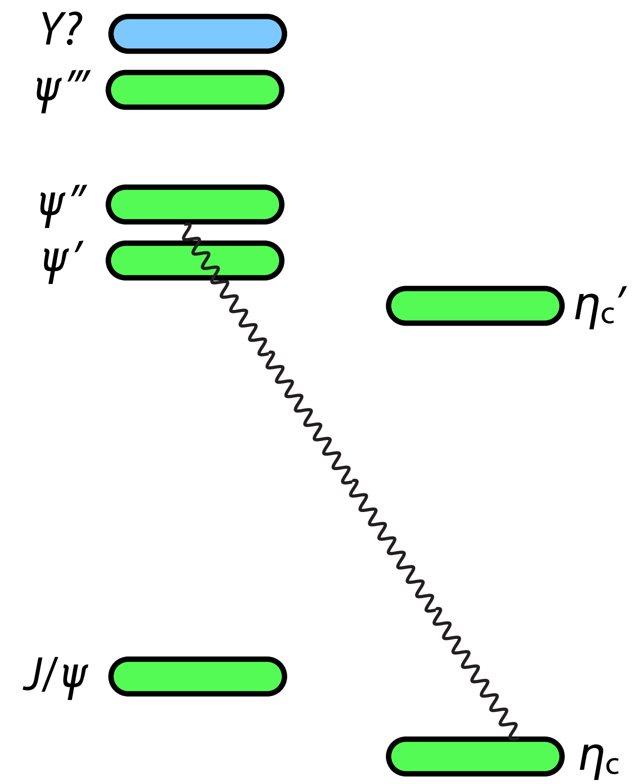




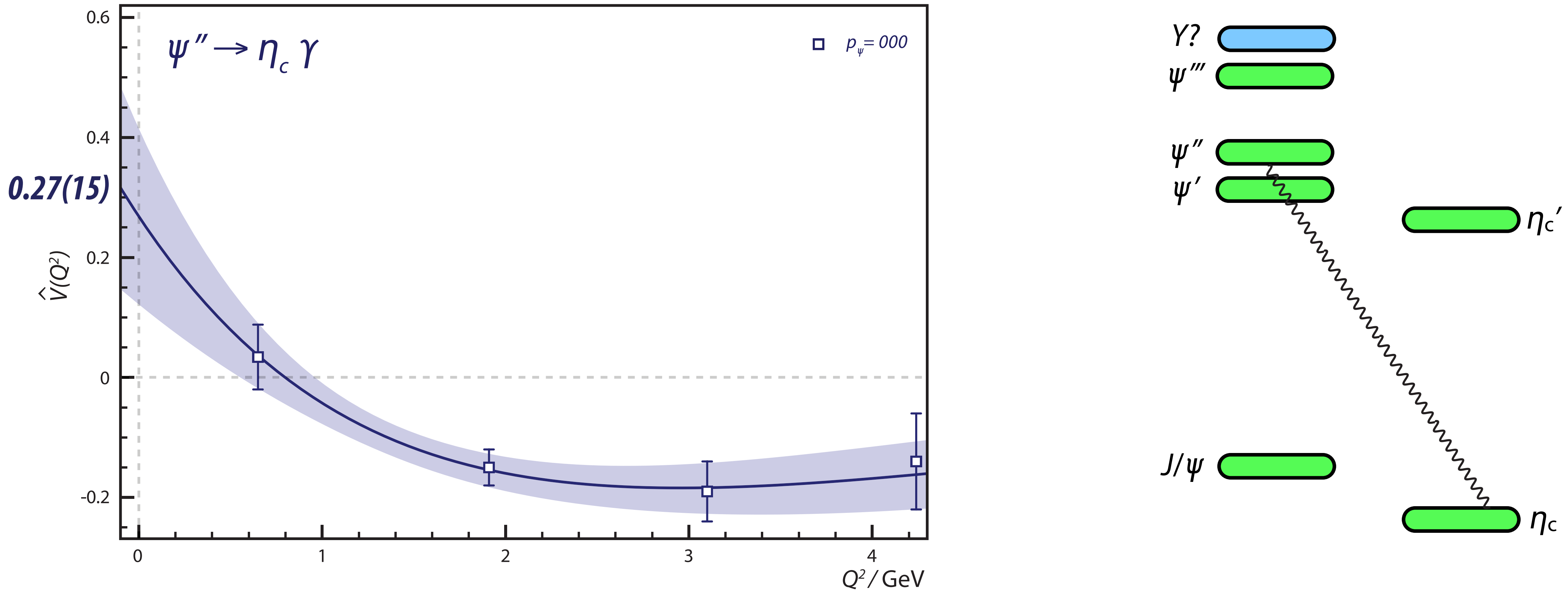
first lattice QCD extraction of a radiative transition involving an excited meson

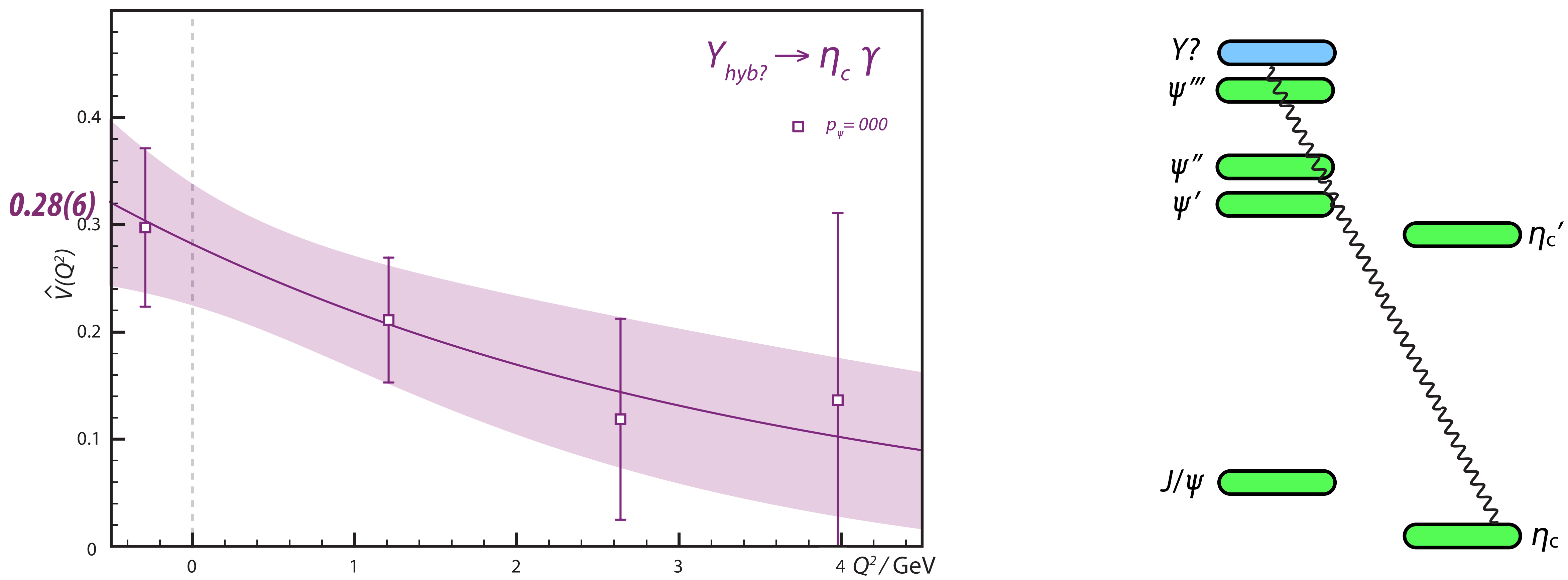
vector - pseudoscalar (M1)



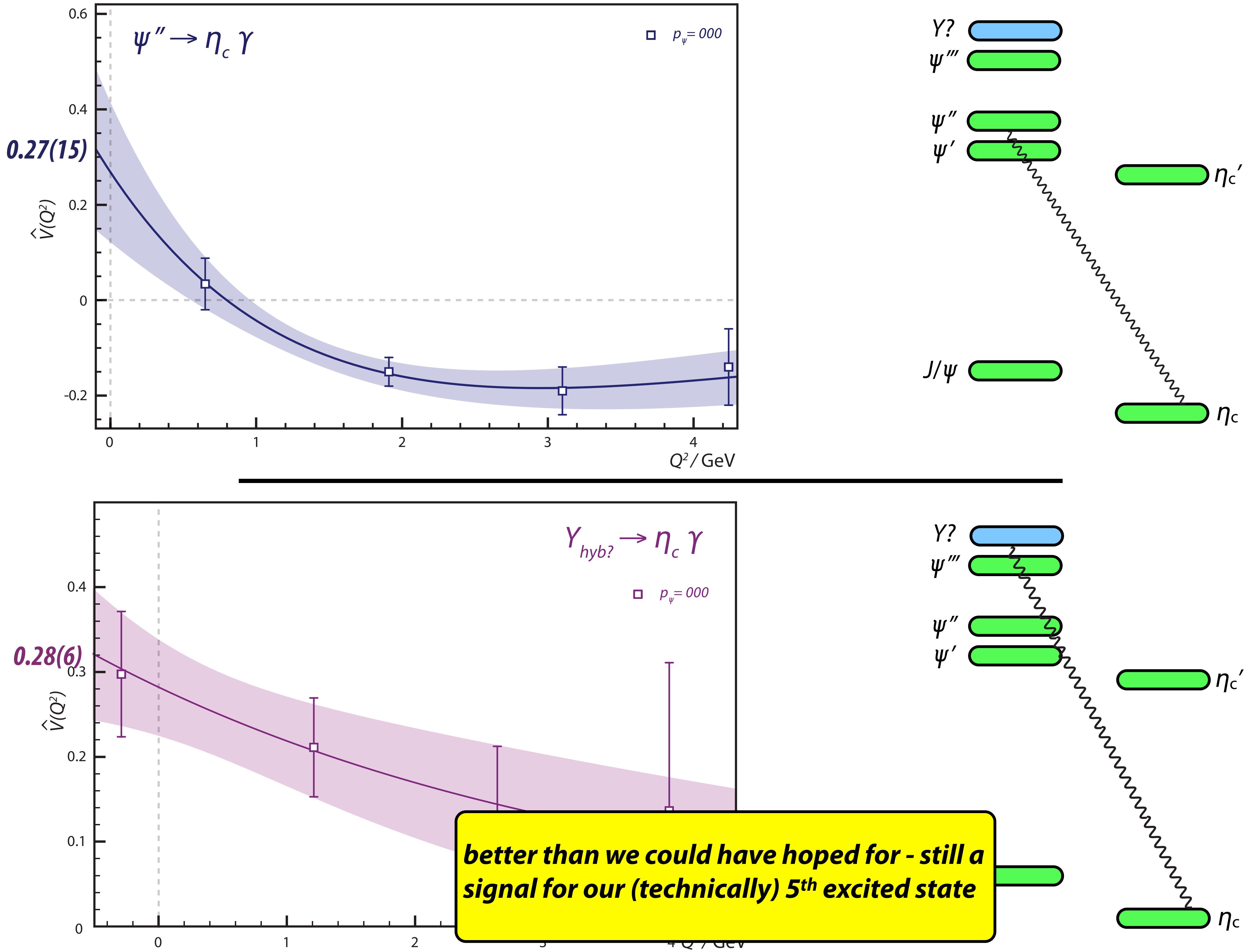


vector - pseudoscalar (M1)



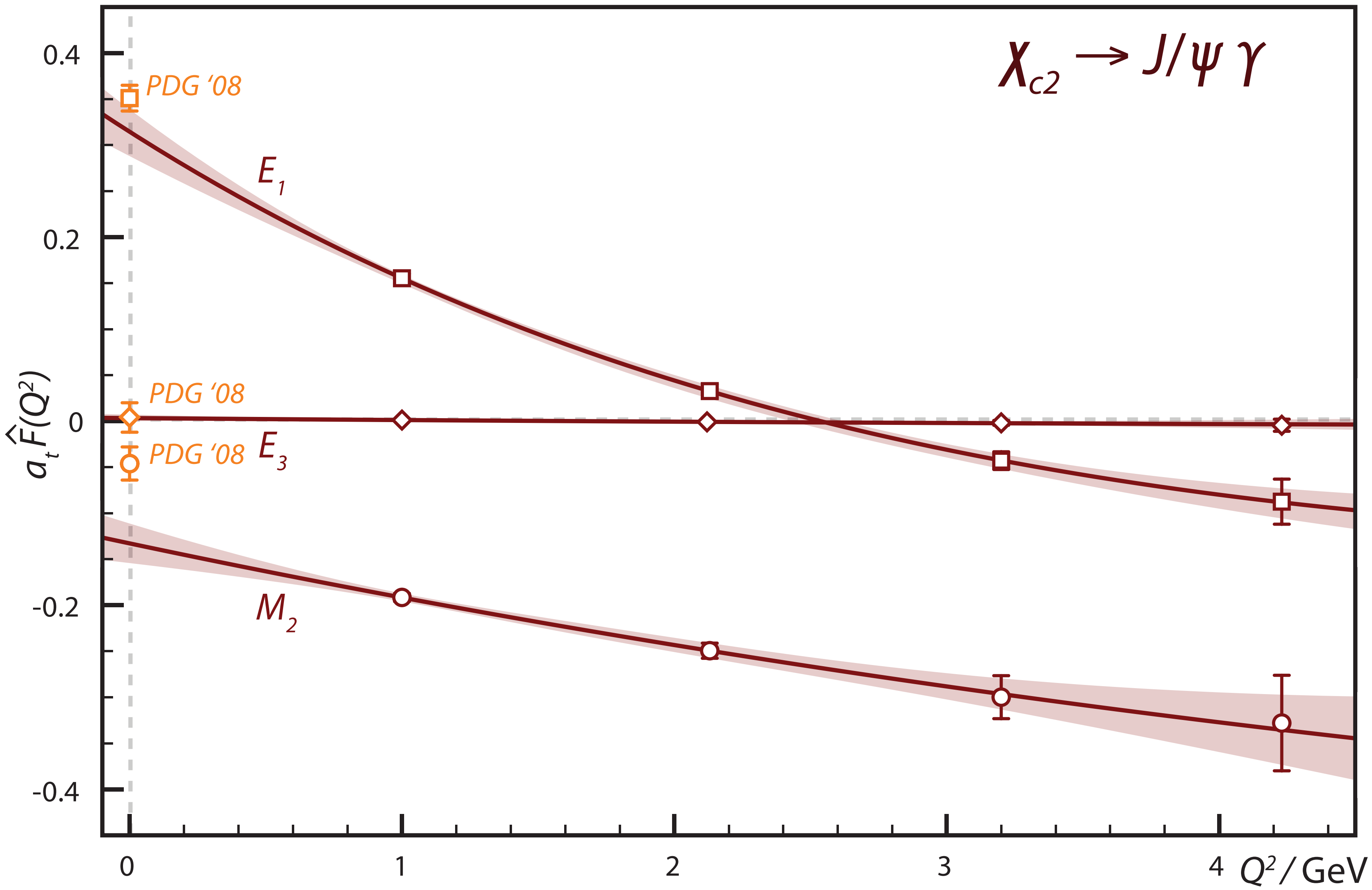


vector - pseudoscalar (M1)



better than we could have hoped for - still a signal for our (technically) 5th excited state

tensor - vector (E1,M2,E3)



experimental results from angular dependence of radiative decay events

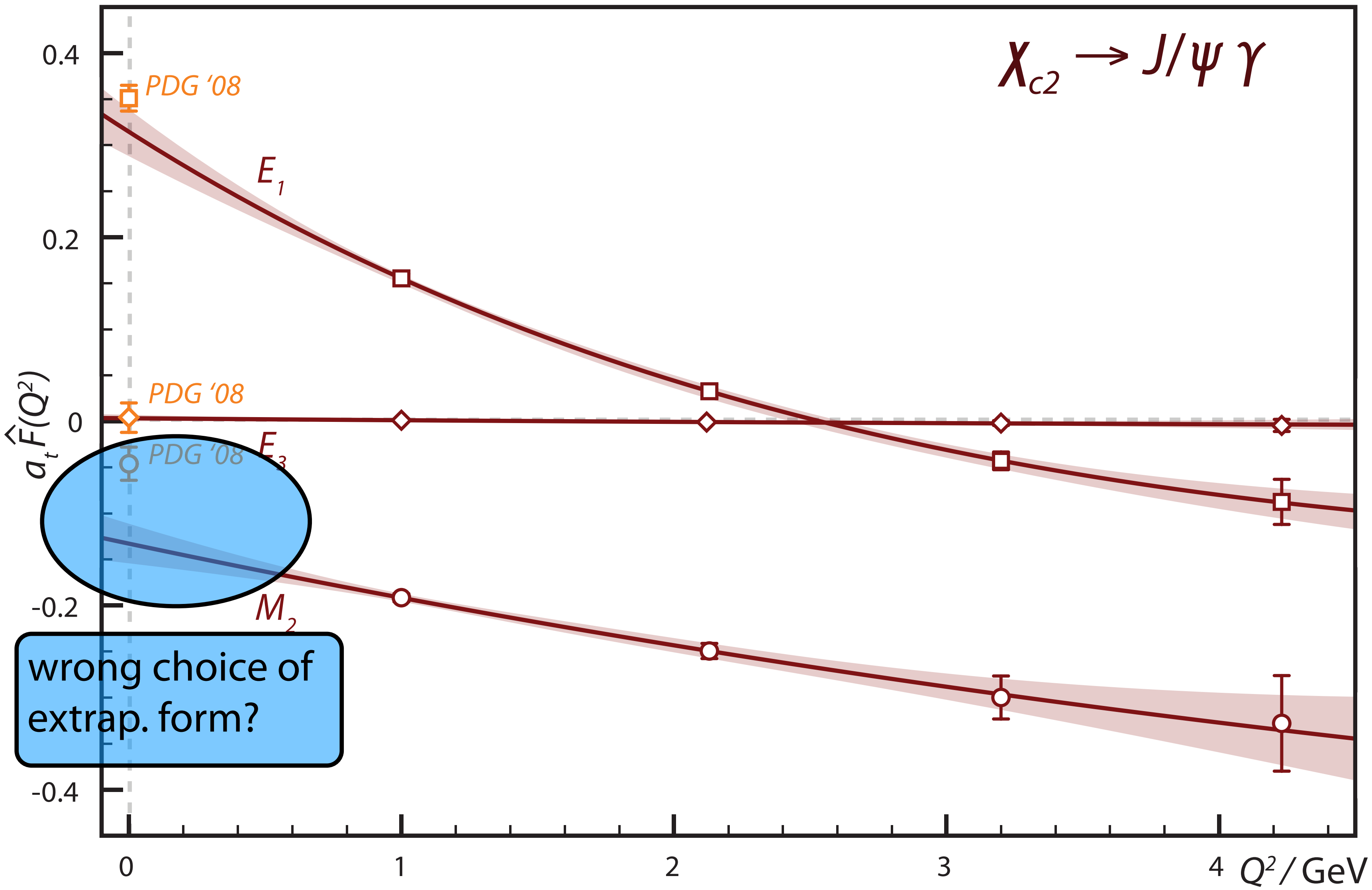
suppressed magnetic quadrupole of right sign, but too large in magnitude

electric octopole consistent with zero

relativistic correction in quark models
- rather model dependent

has quite simple explanation

tensor - vector (E1,M2,E3)



experimental results from angular dependence of radiative decay events

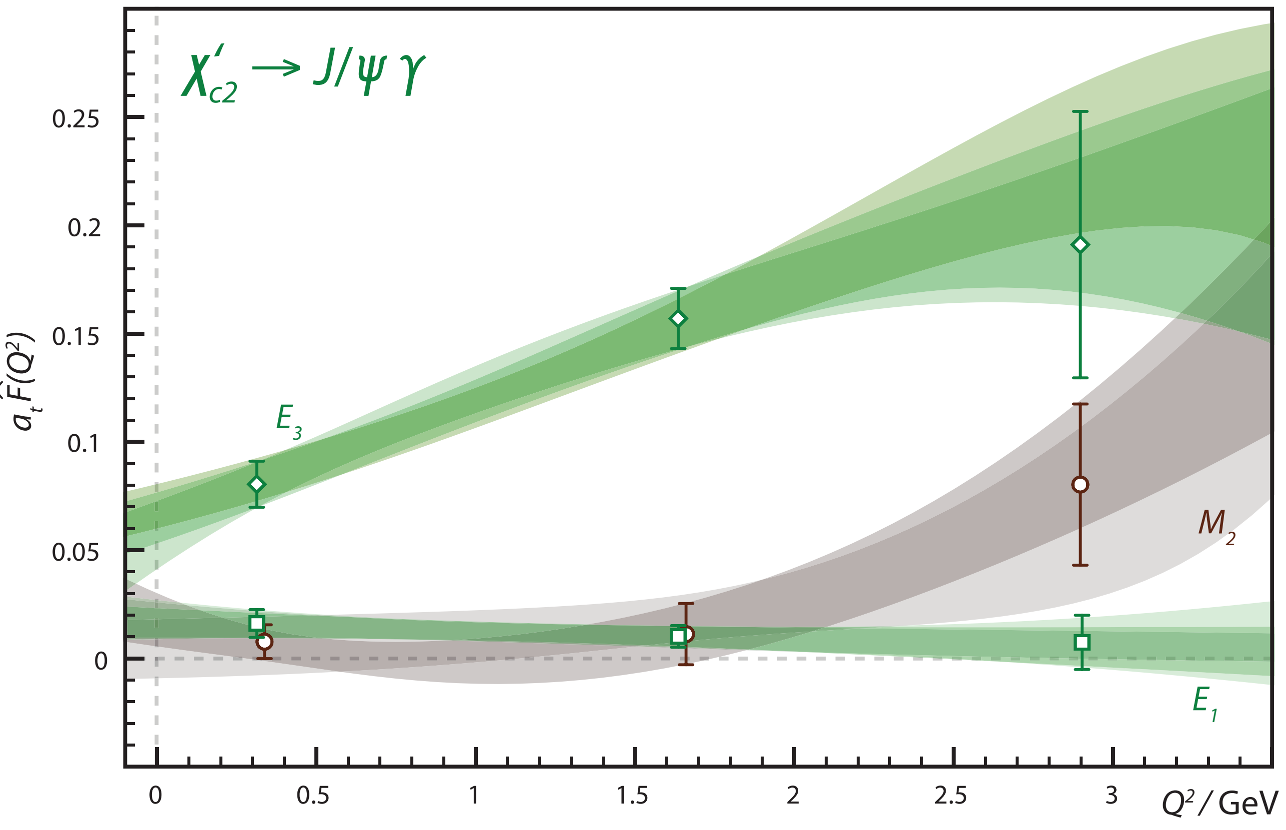
suppressed magnetic quadrupole of right sign, but too large in magnitude

relativistic correction in quark models - rather model dependent

electric octopole consistent with zero

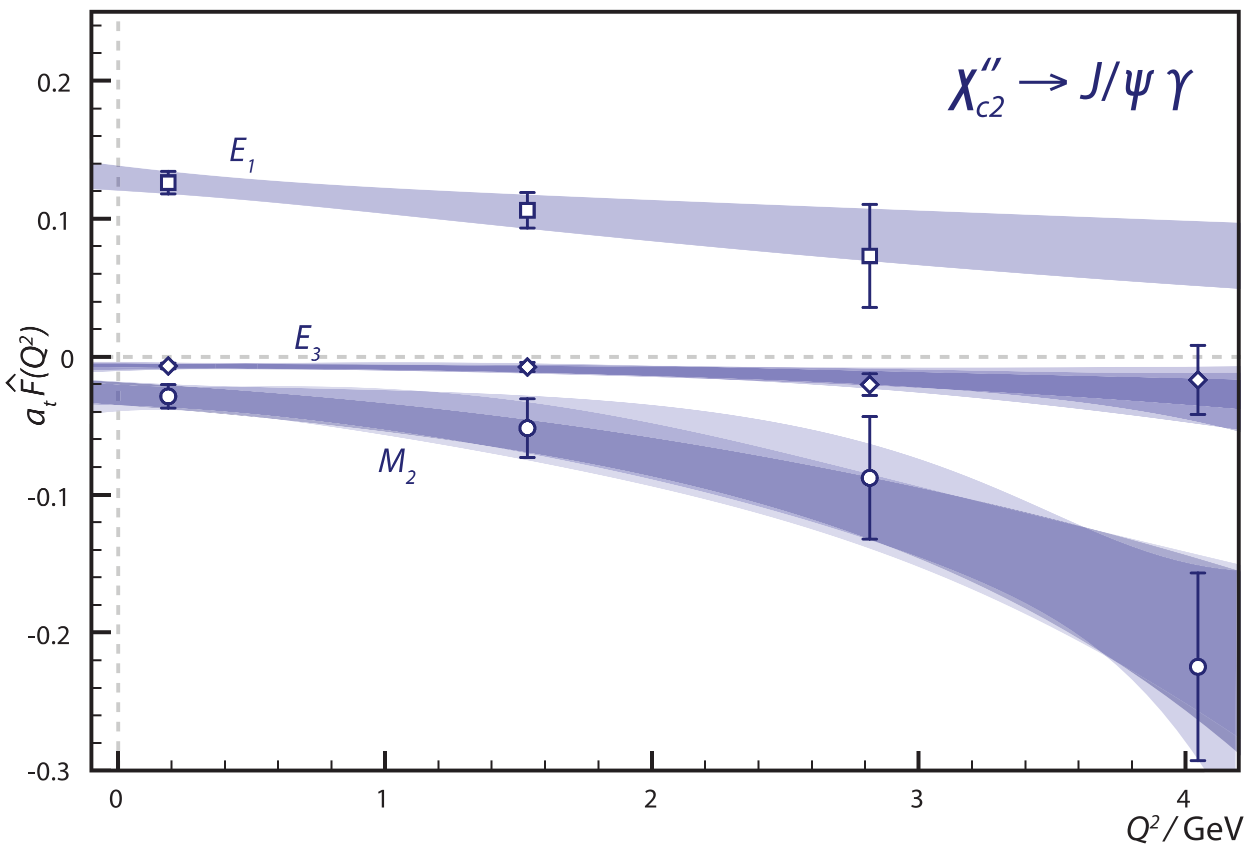
has quite simple explanation

tensor - vector (E1,M2,E3)



E1≈0, E3 ≠ 0

m = 4115(28) MeV



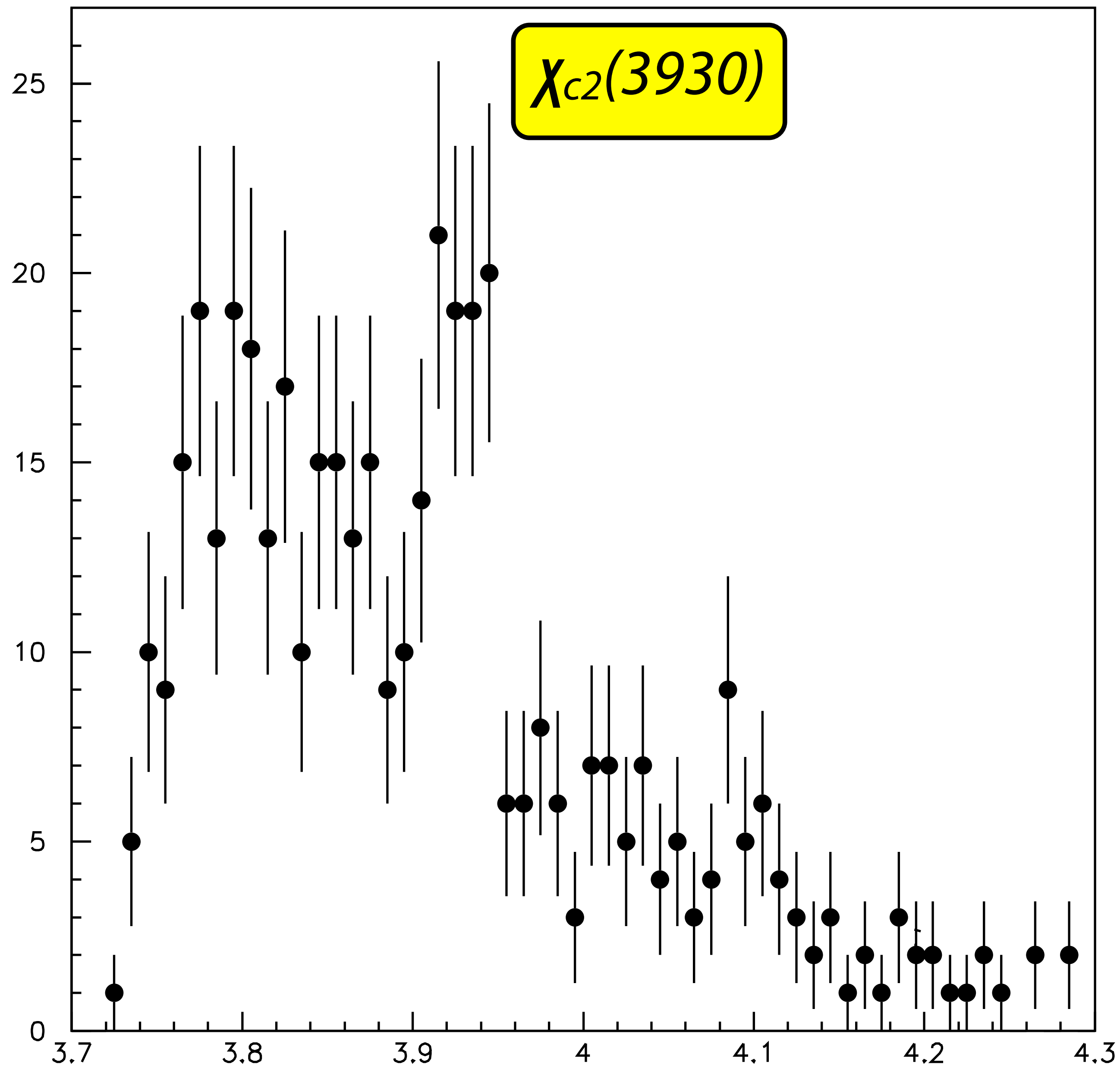
E3≈0, E1 ≠ 0

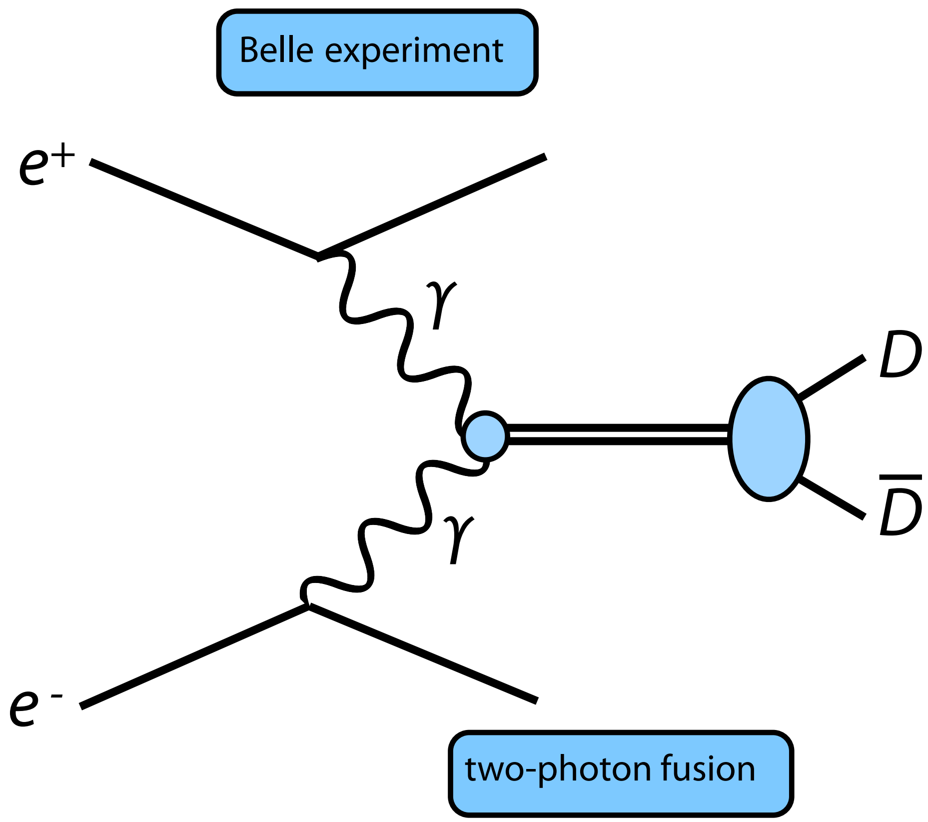
m = 4165(30) MeV

see Christopher Thomas's talk for explanation

excited tensor states?

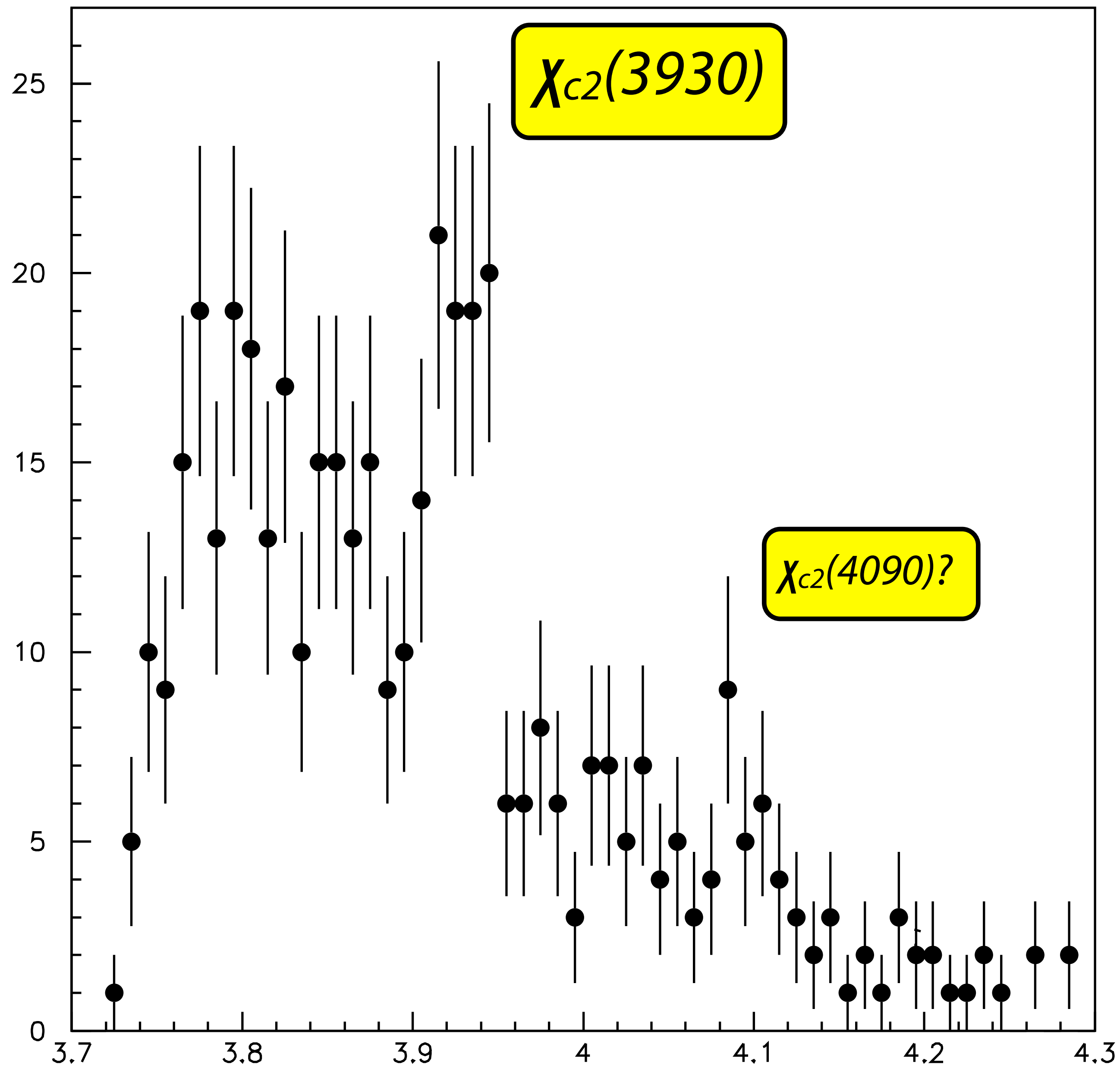
Belle $\gamma\gamma \to D\bar{D}$

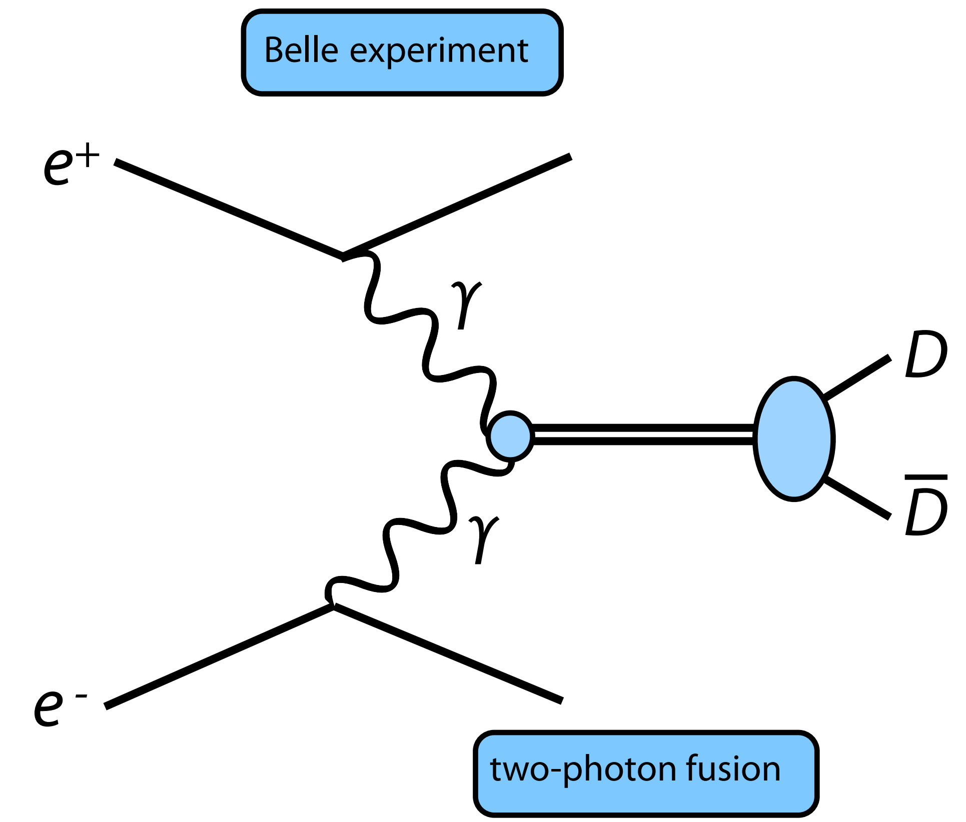




excited tensor states?

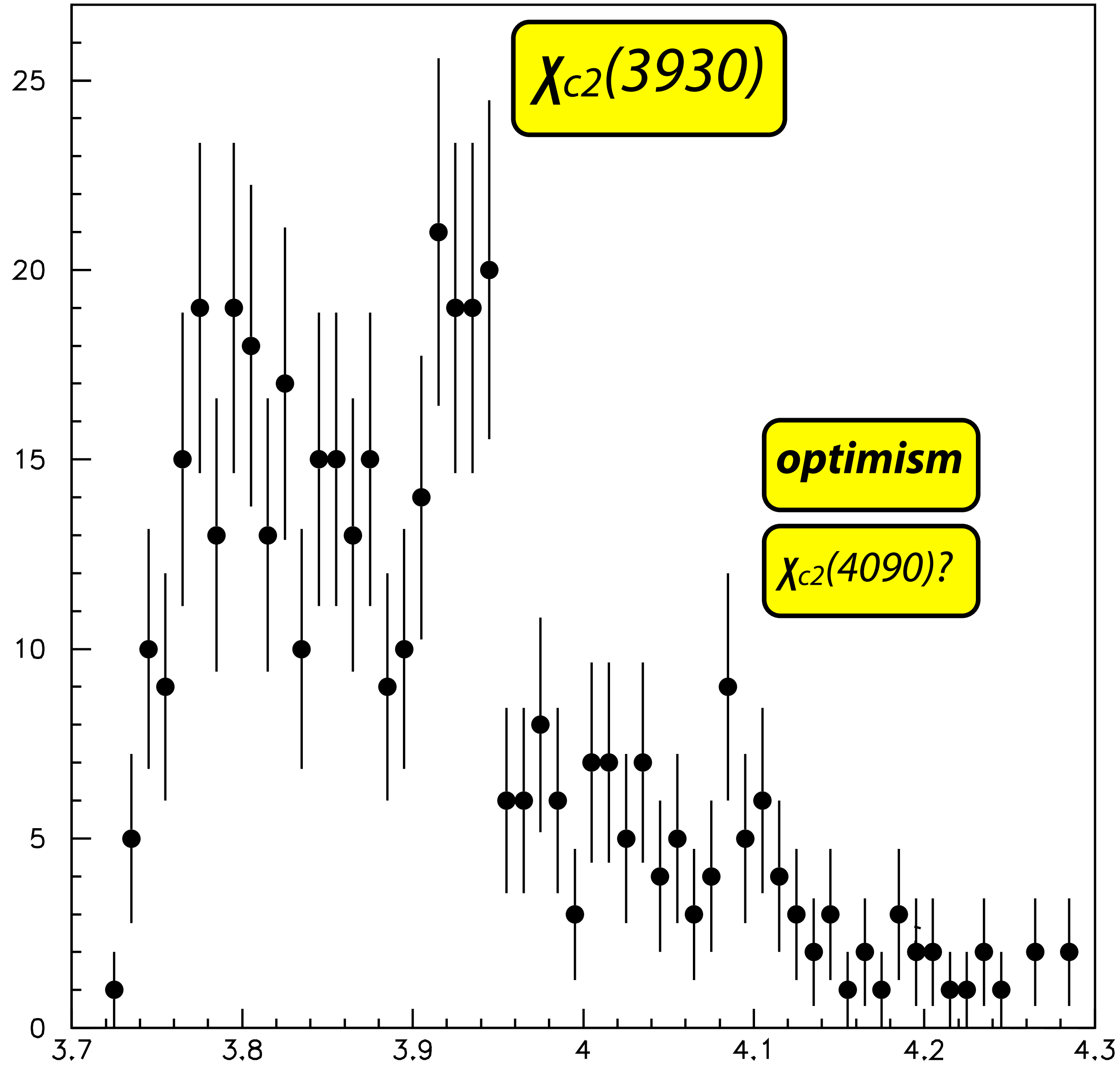
Belle $\gamma\gamma \rightarrow D\bar{D}$

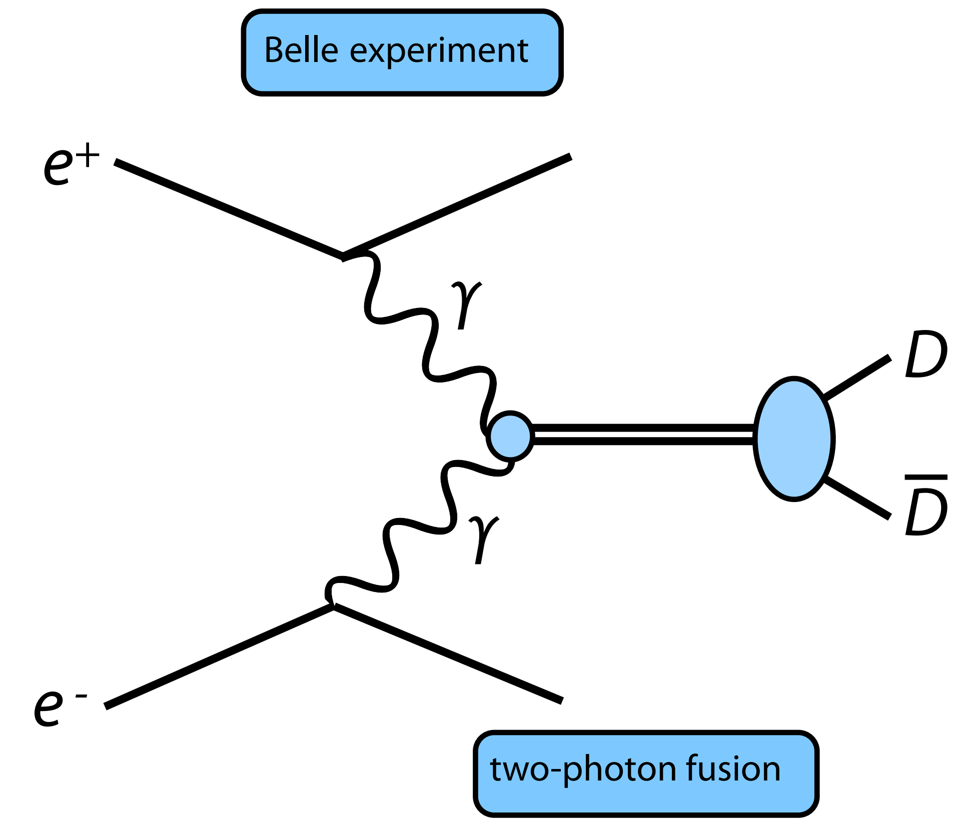




excited tensor states?

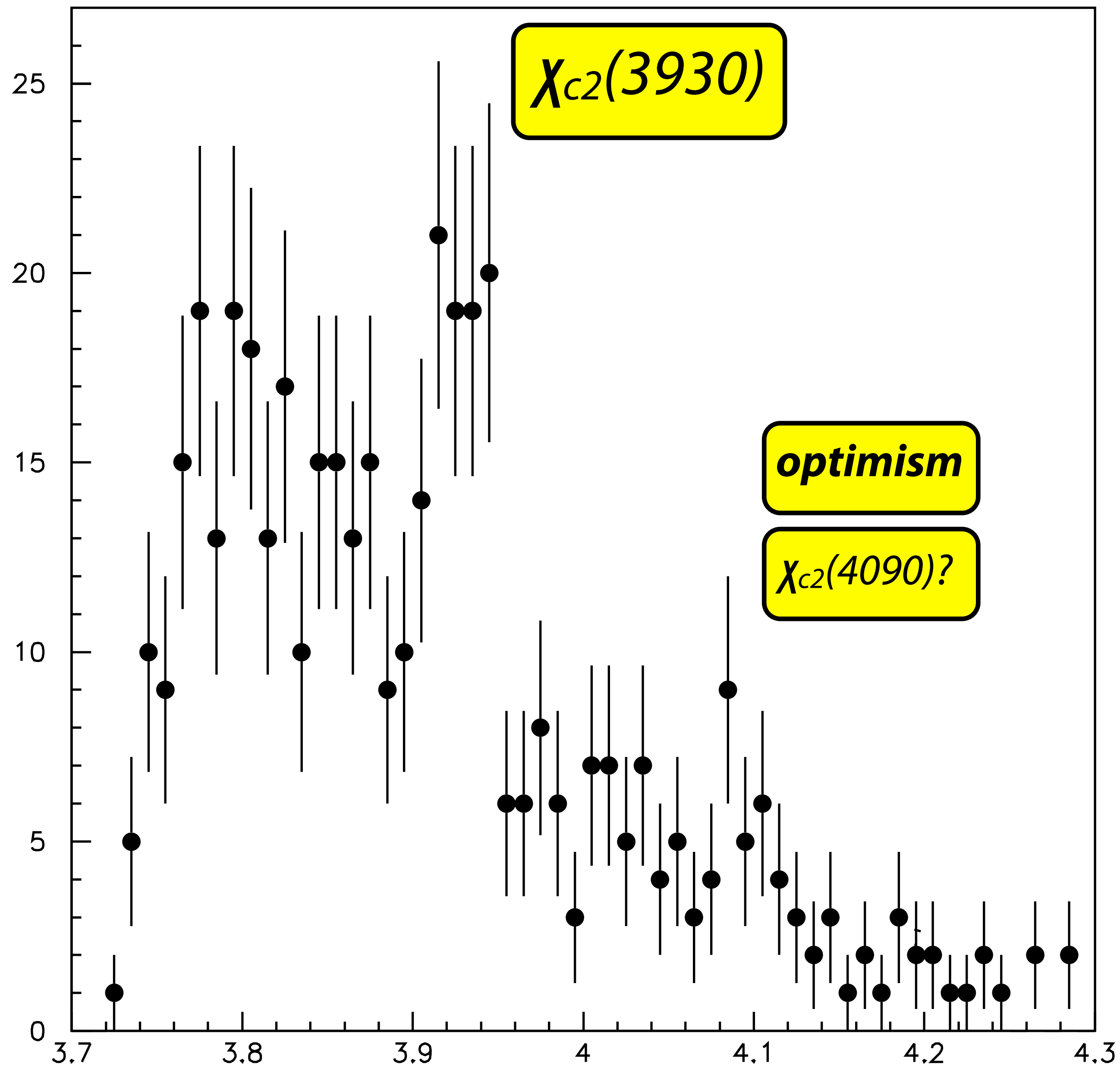
Belle $\gamma\gamma \to D\bar{D}$



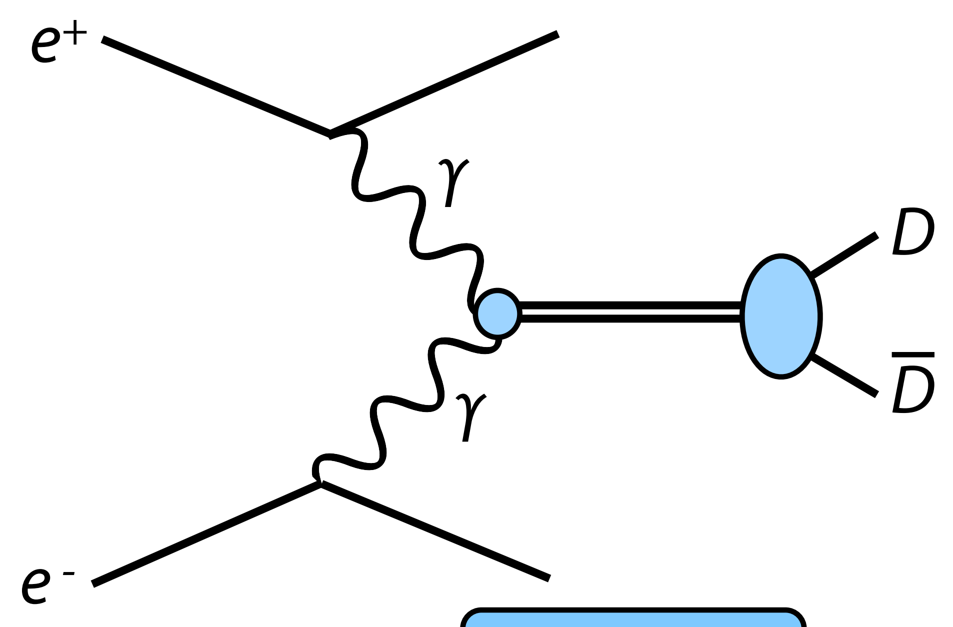


excited tensor states?

Belle $\gamma\gamma \to D\bar{D}$



Belle experiment



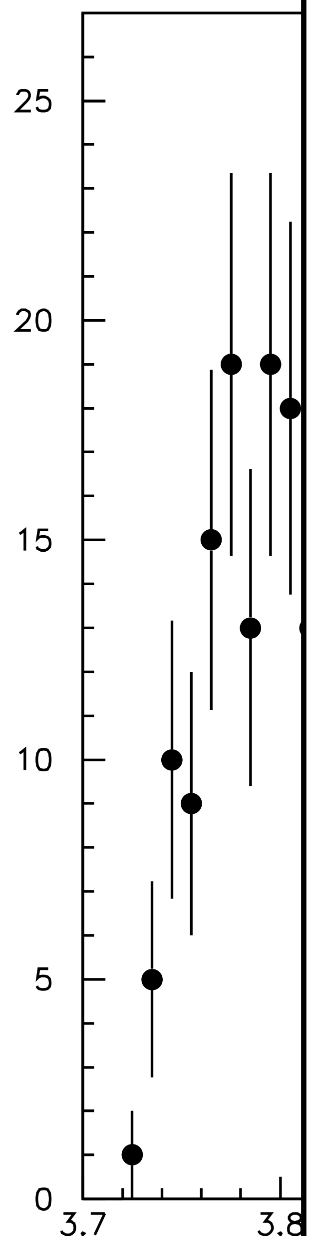
two-photon fusion

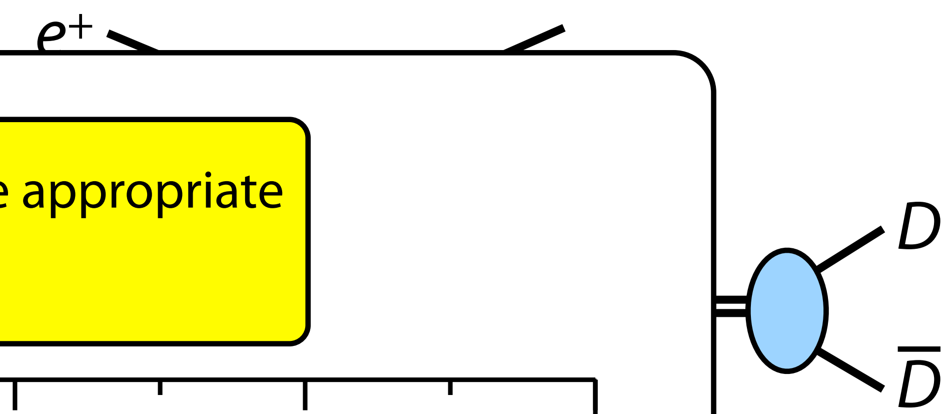
our calculation finds the *F* -wave lighter - may be artifact of small box 'squeezing'

excited tensor states?

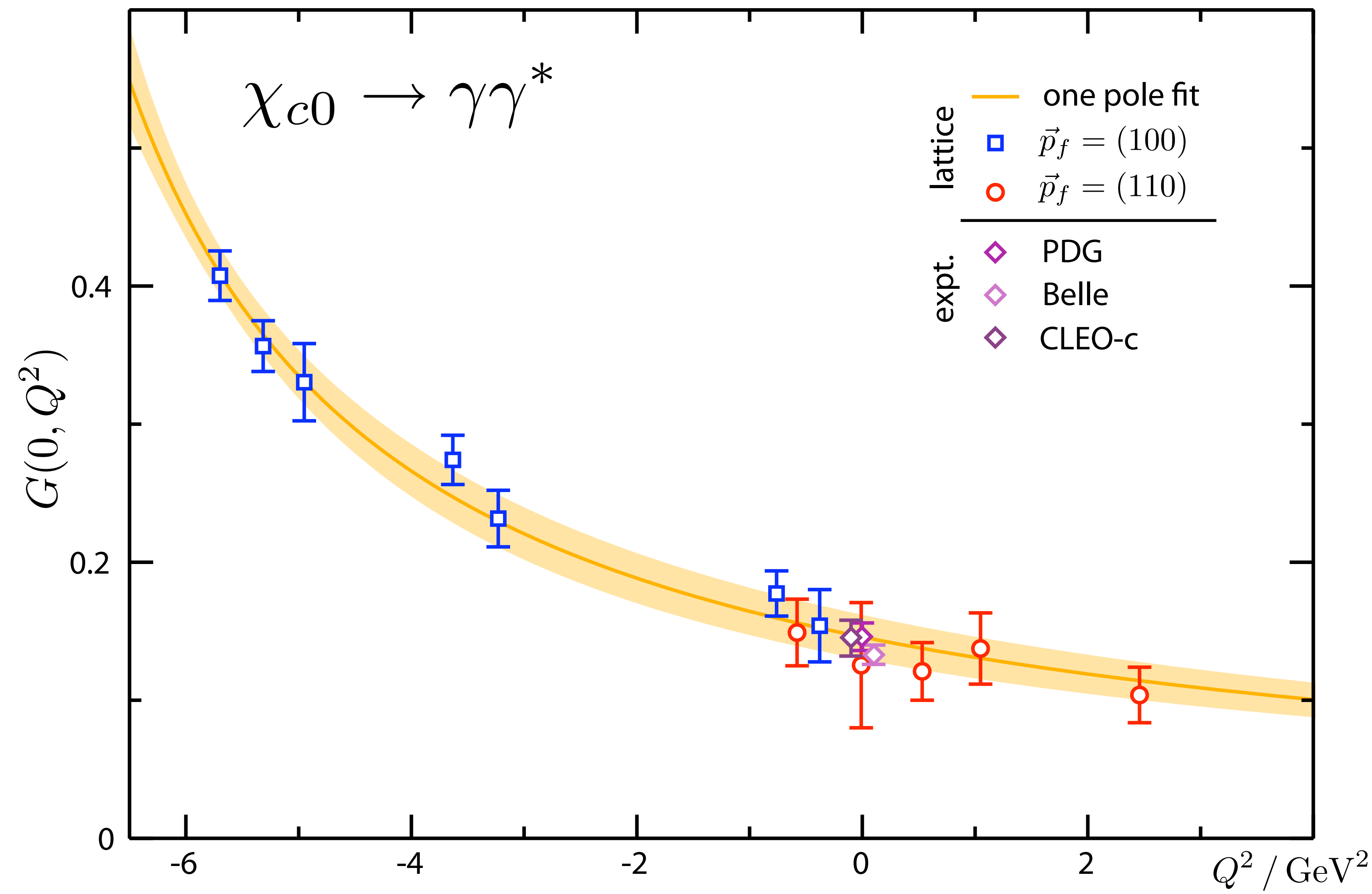
Belle $\gamma\gamma \to D\bar{D}$

Belle experiment





we have the technology to do the appropriate two-photon coupling calculation



...sion

...e lighter -
...ueezing'

exotics

***J**^{PC}=1^{-+}* not accessible to $c\bar{c}$ pair

we find state at about 4.3 GeV

HYBRID MESON:
excited gluonic field

exotics

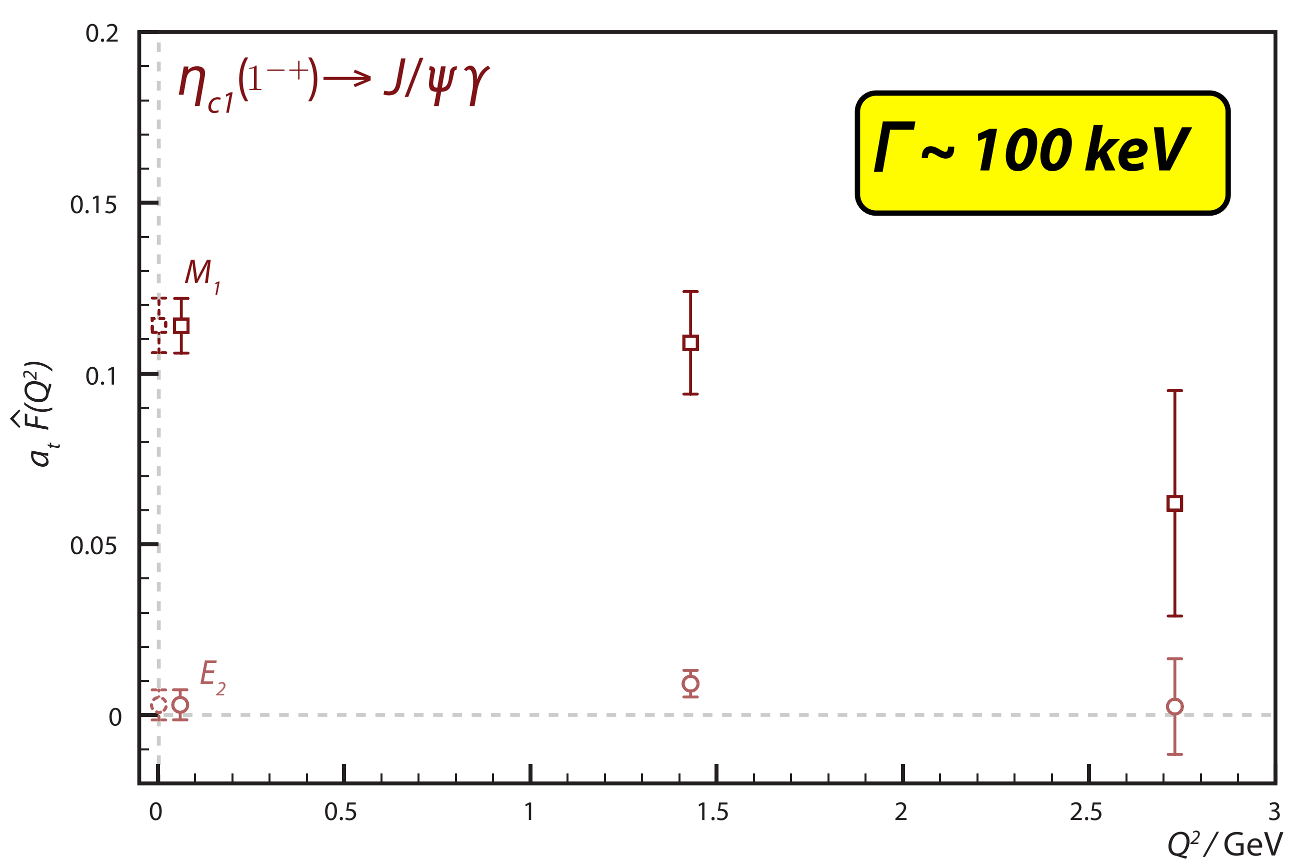
$J^{PC}=1^{-+}$ not accessible to $c\bar{c}$ pair

we find state at about 4.3 GeV

HYBRID MESON:
excited gluonic field

$\eta_{c1} \rightarrow J/\psi\ \gamma$

magnetic dipole transition



exotics

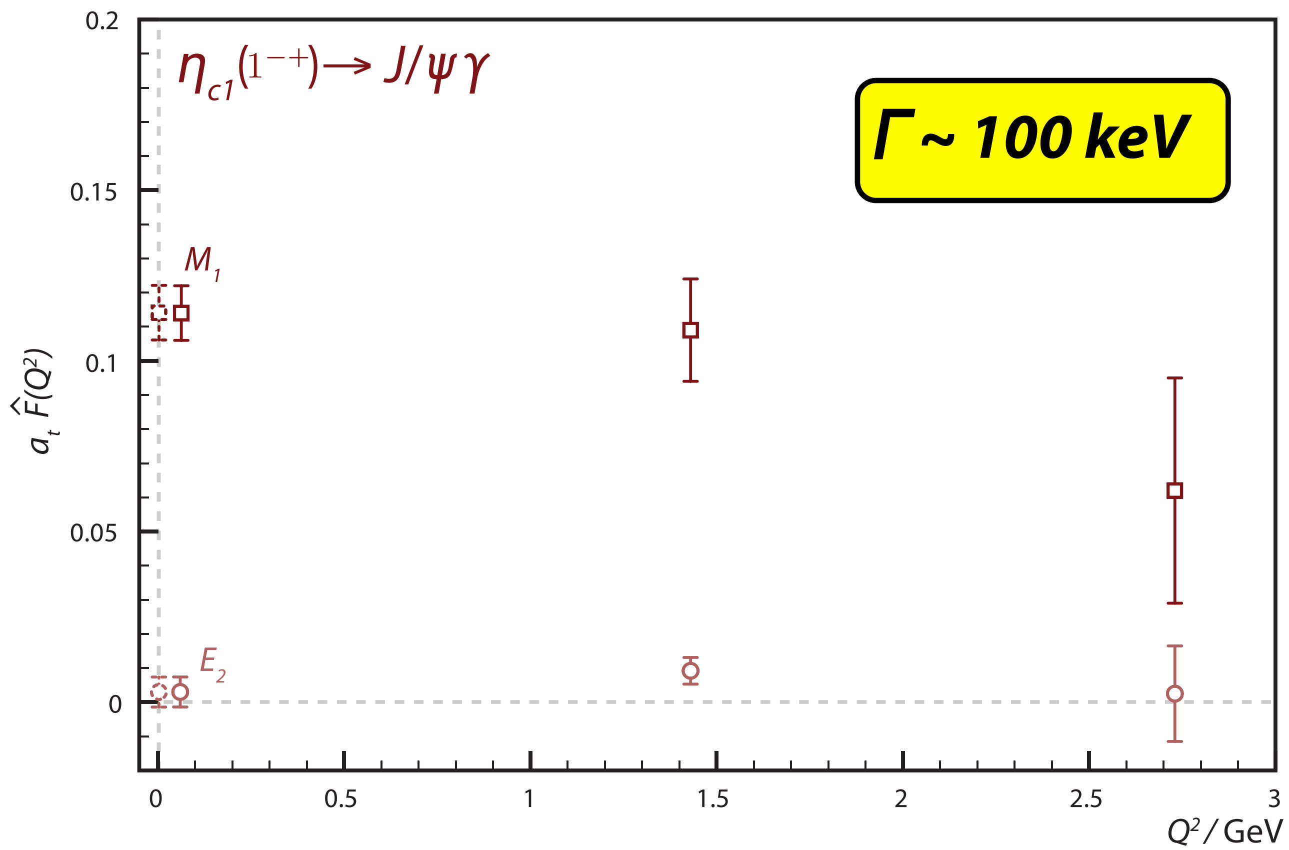
$J^{PC}=1^{-+}$ not accessible to $c\bar{c}$ pair

we find state at about 4.3 GeV

HYBRID MESON:
excited gluonic field

$\eta_{c1} \rightarrow J/\psi\ \gamma$

magnetic dipole transition



$\Gamma \sim 100$ keV

compare with $J/\psi \rightarrow \eta_c \gamma$ ~ 1 keV

quark spin flip ~ $\dfrac{\sigma}{m_c}$

exotics

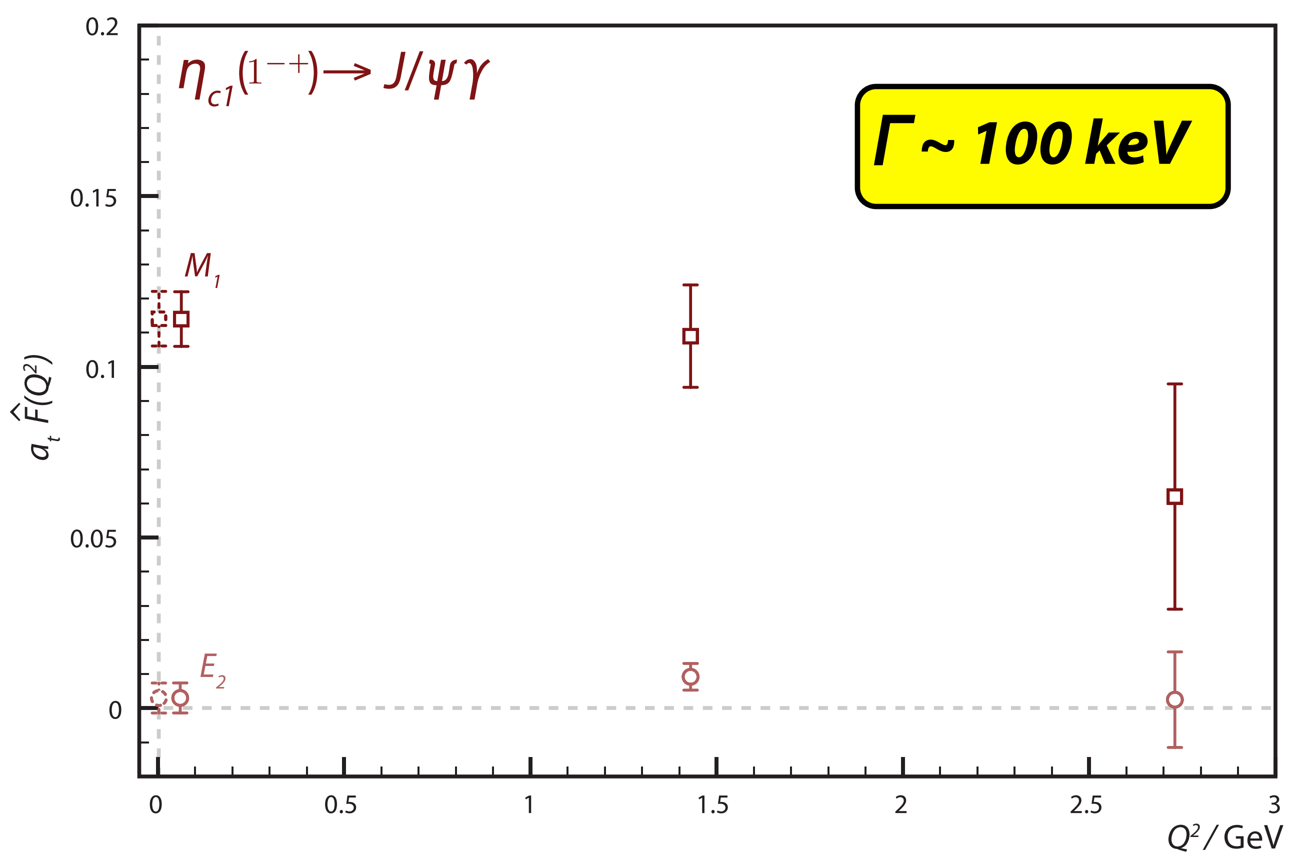
$J^{PC}=1^{-+}$ not accessible to $c\bar{c}$ pair

we find state at about 4.3 GeV

HYBRID MESON:
excited gluonic field

$\eta_{c1} \rightarrow J/\psi\ \gamma$

magnetic dipole transition



Γ ~ 100 keV

compare with $J/\psi \rightarrow \eta_c\gamma$ ~ 1 keV

quark spin flip ~ $\dfrac{\sigma}{m_c}$

perhaps this is not spin-flip?

exotics

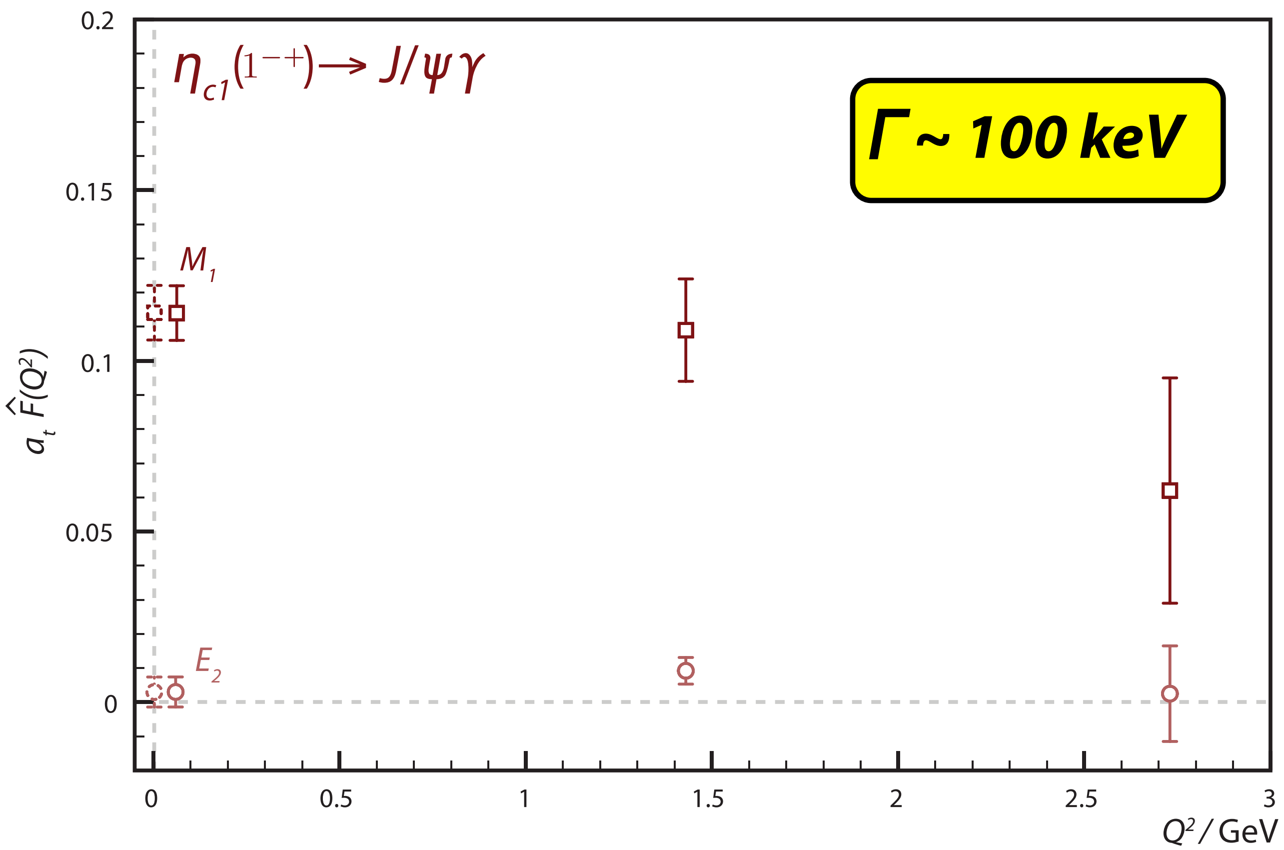
$J^{PC}=1^{-+}$ not accessible to $c\bar{c}$ pair

we find state at about 4.3 GeV

HYBRID MESON:
excited gluonic field

$\eta_{c1} \to J/\psi\ \gamma$

magnetic dipole transition



Γ ~ 100 keV

compare with $J/\psi \to \eta_c\gamma$ ~ 1 keV

quark spin flip ~ $\dfrac{\sigma}{m_c}$

perhaps this is not spin-flip?

$^3\mathcal{H}_1 \to {}^3S_1\gamma$

exotics

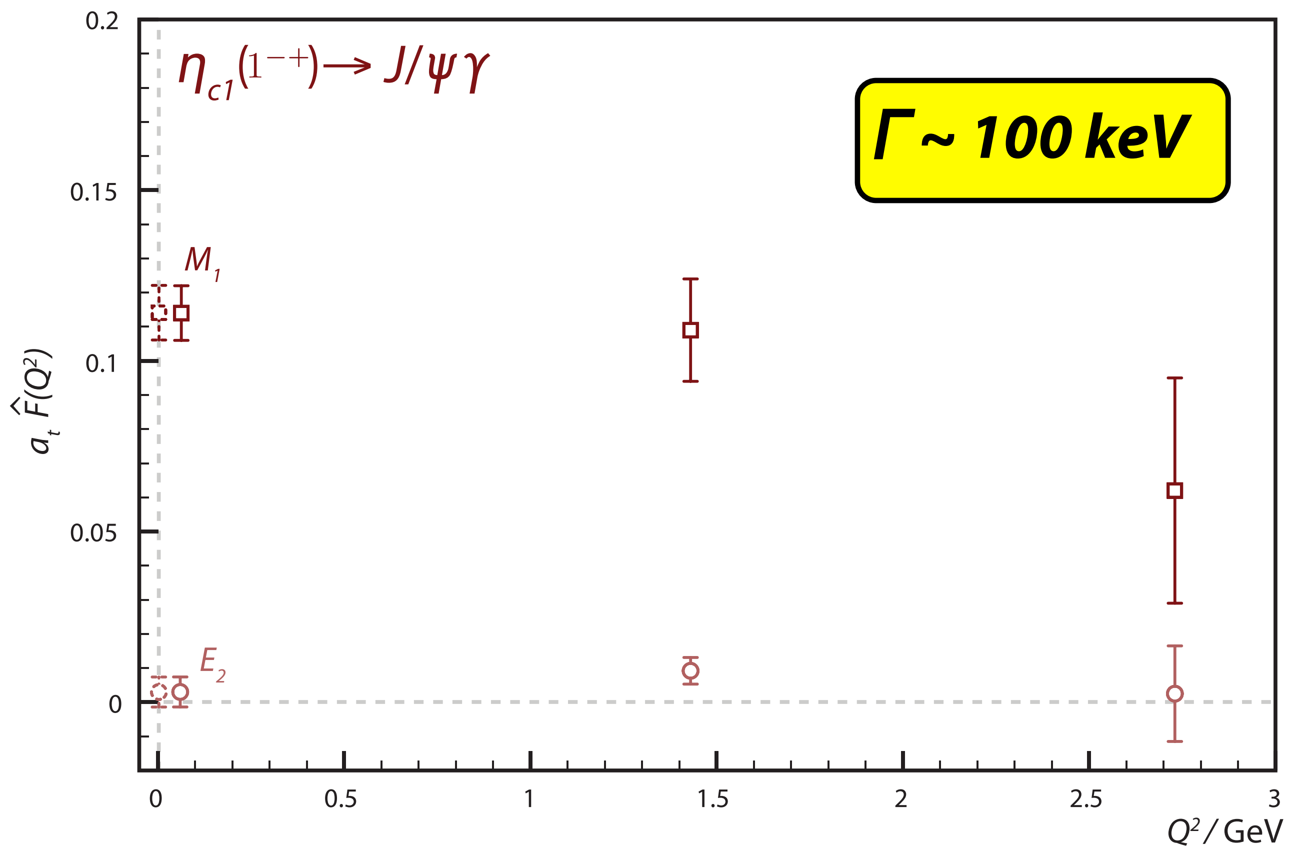
$J^{PC}=1^{-+}$ not accessible to $c\bar{c}$ pair

we find state at about 4.3 GeV

HYBRID MESON:
excited gluonic field

$\eta_{c1} \rightarrow J/\psi\ \gamma$

magnetic dipole transition



Γ ~ 100 keV

compare with $J/\psi \rightarrow \eta_c\gamma$ ~ 1 keV

quark spin flip ~ $\dfrac{\sigma}{m_c}$

perhaps this is not spin-flip?

$^3\mathcal{H}_1 \rightarrow {}^3S_1\gamma$

supports models in which the exotic has $S_{q\bar{q}} = 1$

exotics

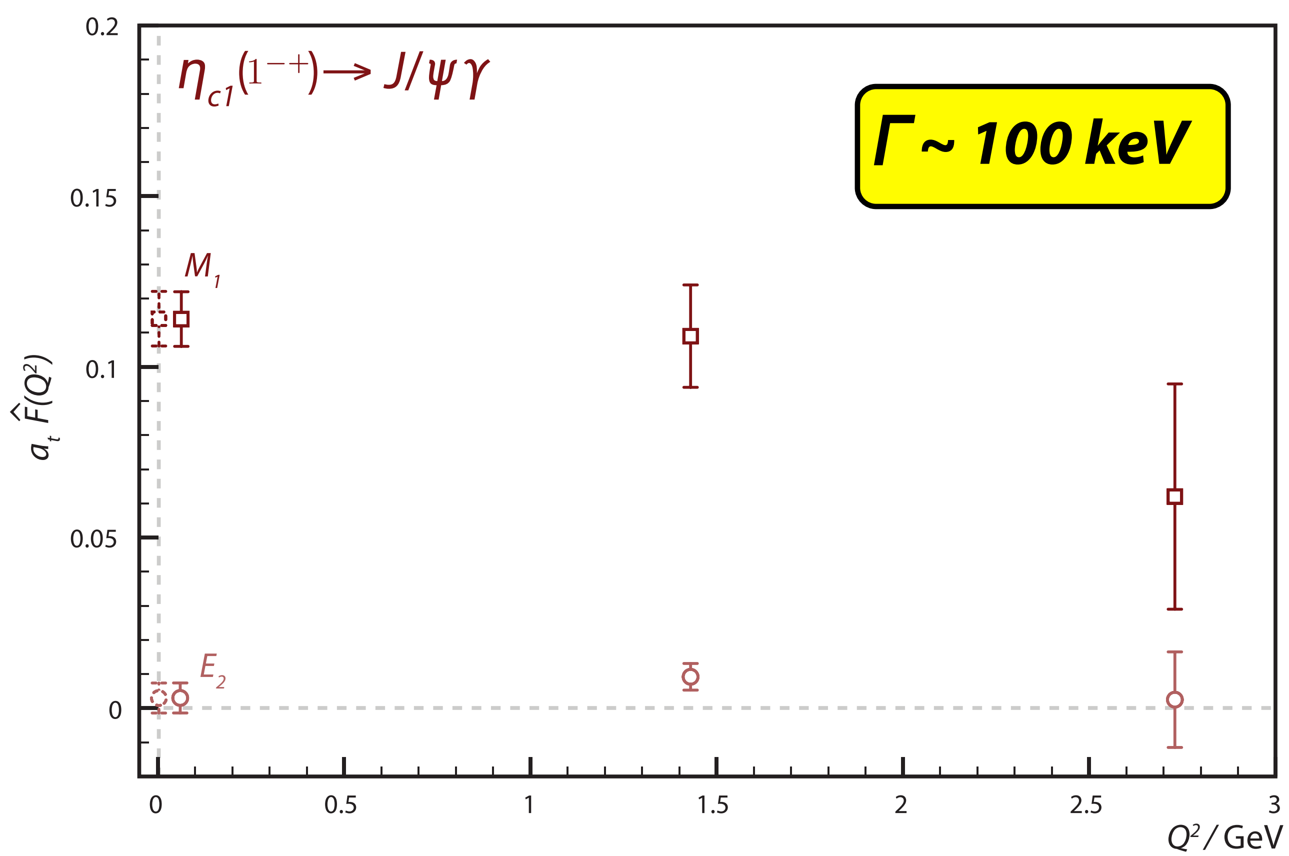
$J^{PC}=1^{-+}$ not accessible to $c\bar{c}$ pair

we find state at about 4.3 GeV

HYBRID MESON:
excited gluonic field

$\eta_{c1} \to J/\psi\ \gamma$

magnetic dipole transition



compare with $J/\psi \to \eta_c \gamma$ ~ 1 keV

quark spin flip ~ $\dfrac{\sigma}{m_c}$

perhaps this is not spin-flip?

$^3\mathcal{H}_1 \to\ ^3S_1\gamma$

supports models in which the exotic has $S_{q\bar{q}} = 1$

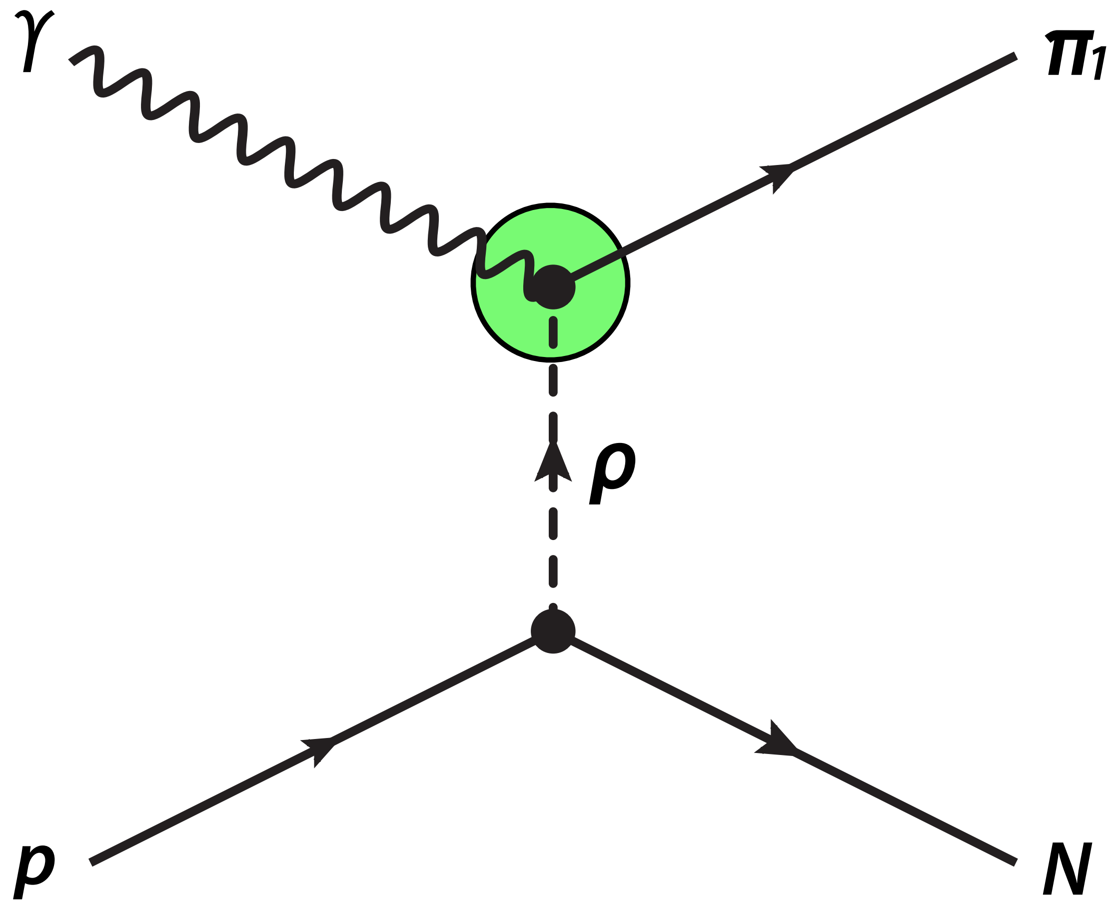
e.g. flux-tube model, Coulomb gauge ...

exciting?

charmonium exotic (probably) has negligible radiative relative to hadronic width exciting?

charmonium exotic (probably) has negligible radiative relative to hadronic width

but ... if this large number is duplicated in the light meson case:



peripheral photoproduction

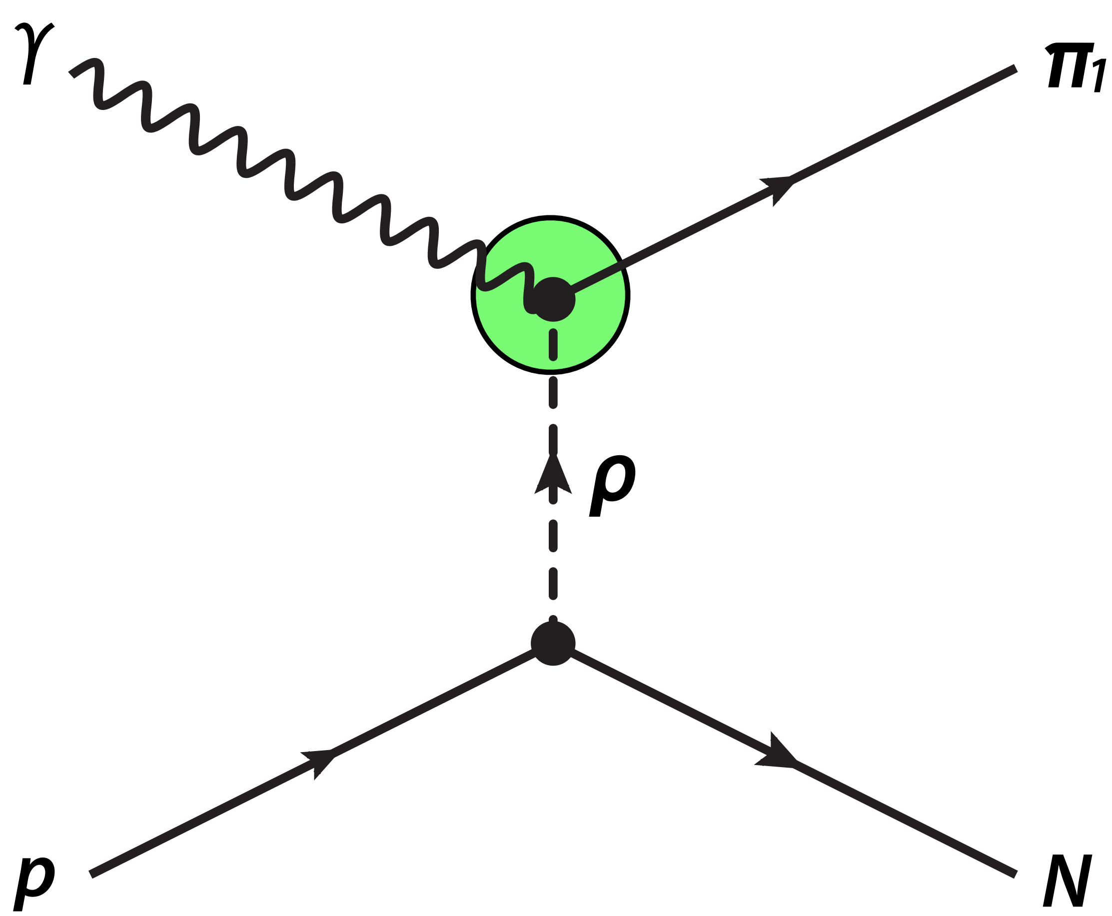
GlueX

plenty of exotic photoproduction ?

exciting?

charmonium exotic (probably) has negligible radiative relative to hadronic width

but ... if this large number is duplicated in the light meson case:



peripheral photoproduction

GlueX

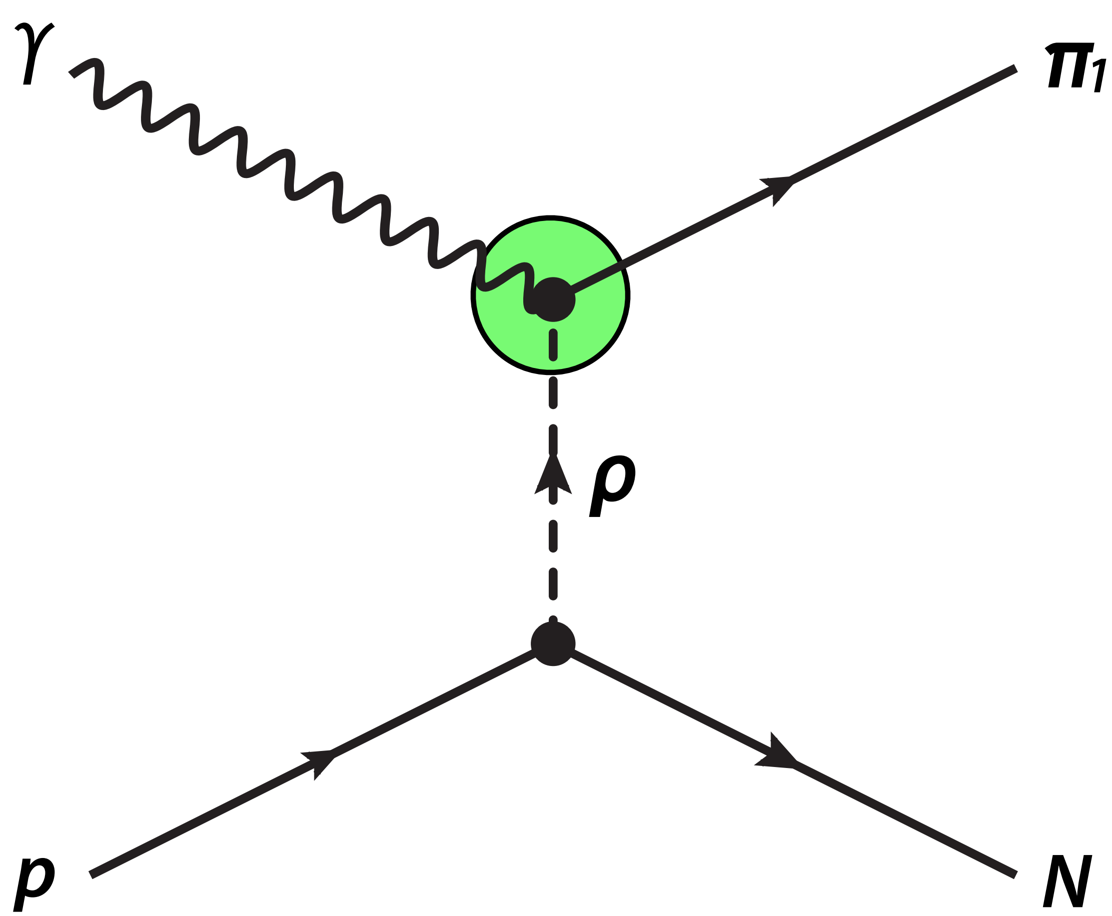
plenty of exotic photoproduction ?

our group's current aim :
perform similar calculations with much lighter quarks - say something useful for GlueX

exciting?

charmonium exotic (probably) has negligible radiative relative to hadronic width

but ... if this large number is duplicated in the light meson case:



peripheral photoproduction

GlueX

plenty of exotic photoproduction ?

our group's current aim :
perform similar calculations with much lighter quarks - say something useful for GlueX

as part of larger ***HadSpec*** collaboration

summary

reliable techniques for extraction of excited states in lattice field theory

now applied to radiative matrix element calculations

initial trials with quenched studies of charmonium - compare with potential models

I have emphasized the exceptions - but actually potential models agree rather well with many results I have not presented

exotic (hybrid?) to conventional meson radiative transitions are large

same techniques can be used in baryon sector - applications to ***CLAS*** electroproduction program (some first attempts from Huey-Wen Lin et.al.)

summary

reliable techniques for extraction of excited states in lattice field theory

now applied to radiative matrix element calculations

initial trials with quenched studies of charmonium - compare with potential models

I have emphasized the exceptions - but actually potential models agree rather well with many results I have not presented

exotic (hybrid?) to conventional meson radiative transitions are large

same techniques can be used in baryon sector - applications to ***CLAS*** electroproduction program (some first attempts from Huey-Wen Lin et.al.)

for more information attend ***Christopher Thomas's*** talk at 4.30pm in the 'Charm Spectroscopy' parallel session