Hadron Spectroscopy

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hadrons

- defined to be those particles which experience the 'strong nuclear force'
- these days we have a strong suspicion that this force is QCD, and that hadrons are made up of dynamically confined quarks and gluons
  - QCD is a gauge field theory

$$\mathcal{L} = \bar{q}(i\gamma^\mu \partial_\mu - m)q + g\,\bar{q}\gamma^\mu t_a q\, A^a_\mu - \frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu}$$

  - just one parameter of its own - the coupling 'constant', g
  - quark masses also appear but they don't really 'belong' to QCD
  - the t_a are the generator matrices of the group SU(3) $[t_a, t_b] = i f_{abc} t_c$

- this doesn't look too bad - quite like QED which we have few problems with
- in fact it is an enormously challenging problem to find solutions

- for now I will just point out that g is not a small "number" so probably the perturbation theory (expansion in g) so useful in QED won't work here

  - there are small numbers though - the quark masses ($m_{u,d} \sim \mathcal{O}(1)$ MeV)

hadrons

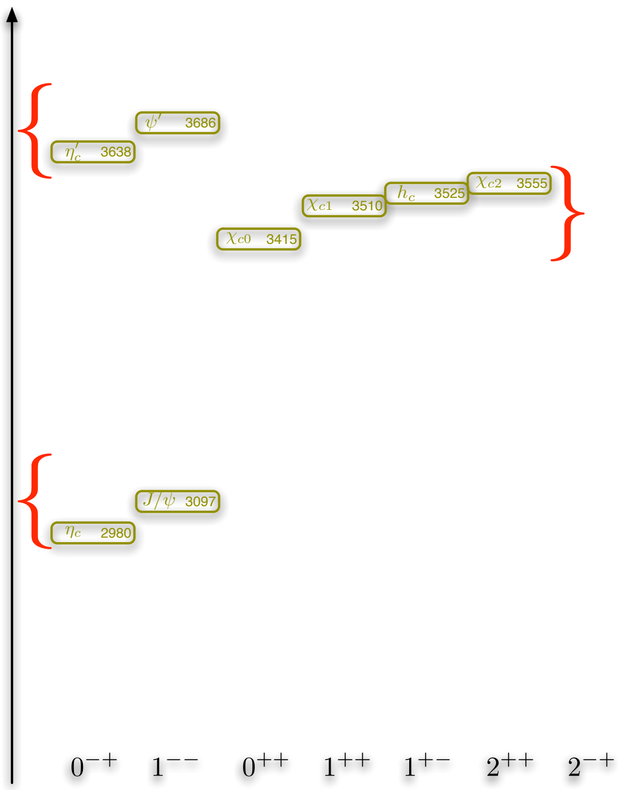
- fall into two categories based upon spin
  - fermionic baryons $J = \frac{1}{2}, \frac{3}{2}, \frac{5}{2} \ldots$
    - most famously the stable proton and the long-lived neutron
  - bosonic mesons $J = 0, 1, 2 \ldots$
    - none are stable, but the lightest, the pion plays a fundamental role in nuclei

charmonium - the 'easy' case

- one set of hadrons that are particularly simple are the charmonium mesons
  - each box represents an observed particle
  - particles fall in groups - 'gross structure'
  - splitting within a group - 'fine structure'
  - reminds us of quantum mechanics of atoms
  - a reasonable description of the spectrum of charmonium comes from solving a Schrödinger equation assuming a potential between a charm quark and an anti-charm quark

$$m_n = 2m_c + E_n$$

$$-\frac{1}{m_c}\nabla^2\psi + V(r)\psi = E_n\psi$$



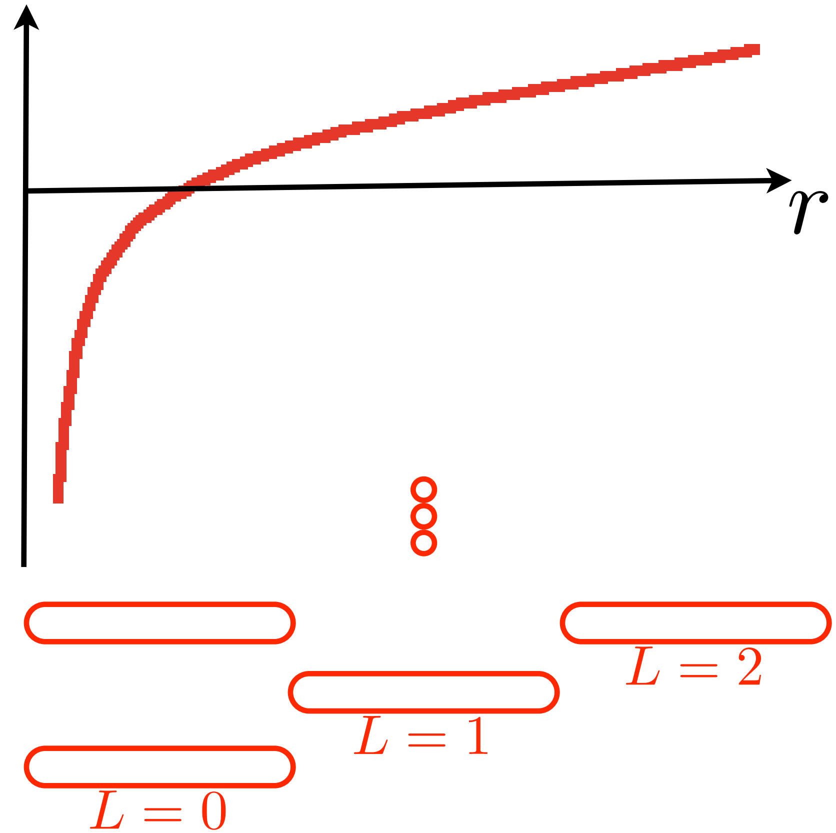
charmonium potential model

- a common 'guess' for the potential is

$$V(r) = -\frac{\alpha}{r} + br$$

short distance
one gluon exchange ($-\frac{\alpha}{r}$)

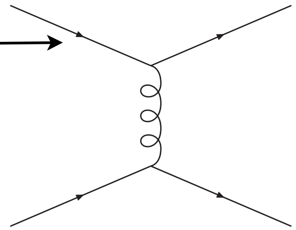
long distance
confinement (br)



solve the Schrödinger equation with this potential giving the gross energy level structure

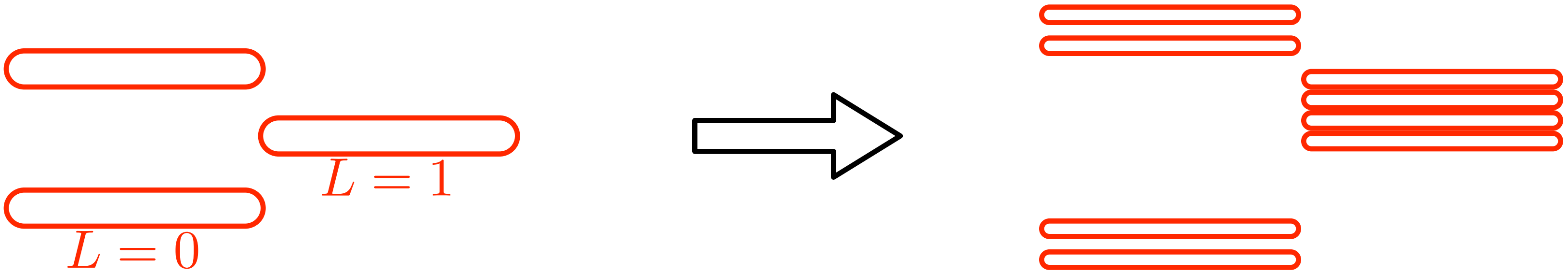
charmonium potential model

- non-relativistically reducing diagrams like this give rise to fine-structure producing terms in the hamiltonian that are suppressed by inverse powers of m_c



$$\vec{\sigma}_q \cdot \vec{\sigma}_{\bar{q}}$$ *'hyperfine interaction'*

$$\vec{\Sigma} \cdot \vec{L}$$ *'spin-orbit interaction'*



- splits up the levels just as in the experimental spectrum

charmonium

- charmonium (and the heavier bottomonium) seem to be well enough described as quantum mechanical problems
- we seem to have avoided much of the complexity of field theory *(suspicious?)*
- hadron spectroscopy might be an easy subject ?

- let's examine the lighter meson spectrum...

  - start with things that appear to be generally true - *symmetries*

symmetries of light hadrons

- experimentally it is found that the strong interaction is invariant under the parity operation (sends $\vec{r} \to -\vec{r}$)
  - provided one assigns an intrinsic parity to hadron states
    - e.g. $\mathcal{P}|p\rangle = +|p\rangle$
      $\mathcal{P}|\pi\rangle = -|\pi\rangle$

symmetries of light hadrons

- certain light hadrons, through their masses (and couplings to other states), appear to sit in definite representations of *SU(2) - 'isospin'* $|I, I_z\rangle$
  - e.g. the proton and the neutron have approximately the same mass, with no other baryon having a similar mass ⇒ form an isospin doublet $\left|{p \atop n}\right\rangle = \left|\frac{1}{2}, \pm\frac{1}{2}\right\rangle$
  - we observe three different charged pions, all with roughly the same mass ⇒ form an isospin triplet $|\pi^\pm\rangle = |1, \pm 1\rangle$, $|\pi^0\rangle = |1, 0\rangle$
  - there is a single isolated meson state with mass~550 MeV, which we call the η

    ⇒ this is an isospin singlet $|\eta\rangle = |0, 0\rangle$

- experimentally it is found that the strong interaction is to an excellent approximation isospin invariant, so that for example, an isospin 1 meson cannot decay into a set of mesons having total isospin 0 through the strong interaction
  - (the electromagnetic interaction is not isospin invariant)

- e.g. the strong interaction cross-section for $\pi^+ p$ scattering is the same as that for $\pi^- n$

$$|\pi^+ p\rangle = |1, +1\rangle \otimes |\tfrac{1}{2}, +\tfrac{1}{2}\rangle = |\tfrac{3}{2}, +\tfrac{3}{2}\rangle$$

$$|\pi^- n\rangle = |1, -1\rangle \otimes |\tfrac{1}{2}, -\tfrac{1}{2}\rangle = |\tfrac{3}{2}, -\tfrac{3}{2}\rangle$$

symmetries of light hadrons

- an operation known as charge conjugation exists, which turns particle states into antiparticle states, up to a phase
- it is possible for neutral bosons to be eigenstates of this operation
  - e.g. $\mathcal{C}|\gamma\rangle = -|\gamma\rangle$ *, and we say the photon has negative 'charge parity'*
  - e.g. $\mathcal{C}|\pi^0\rangle = +|\pi^0\rangle$ *where we can determine the 'charge parity' from the expt[al] observation of* $\pi^0 \to \gamma\gamma$

- experimentally we find that the strong interactions are invariant under the charge conjugation operation

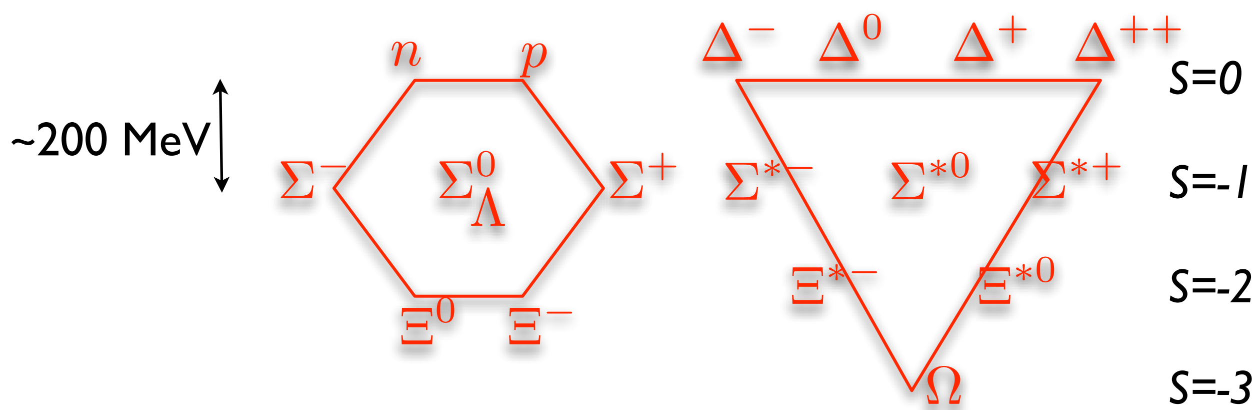
- by merging an isospin transformation and charge conjugation one finds an (invariant) operation on charged & neutral boson states

$$\mathcal{G} \equiv \mathcal{C}\, e^{-i\pi \mathcal{I}_y}$$

  - without going into details this defines a conserved '*G-parity*' for meson states
    - e.g. $\mathcal{G}|\pi^{\pm}\rangle = -|\pi^{\pm}\rangle$
    - for a neutral boson $G = C\,(-1)^I$

approximate symmetries

- furthermore, there appears to be an approximate *SU(3)* symmetry if we look at a broader selection of hadron states
  - extra conserved quantum number: strangeness
    - e.g. $K^* \rightarrow K\,\pi$ has strong interaction decay ($\tau \sim 10^{-23}$ s)
      - conserved strangeness process: $K^*(S=1) \rightarrow K(S=1)\ \pi(S=0)$
    - e.g. $K \rightarrow \pi\,\pi$ has weak interaction decay ($\tau \sim 10^{-10}$ s)
      - strangeness not conserved: $K(S=1) \rightarrow \pi(S=0)\ \pi(S=0)$
  - (broken) symmetry clearly seen in baryon masses:
    - representations of SU(3) include *singlets, octets, decuplets* ...



labelling a meson

- so then a neutral non-strange meson state can be labelled by the (strong-interaction conserved) quantum numbers $I^G \; J^{PC}$
- electrically charged non-strange mesons are not eigenstates of *C*

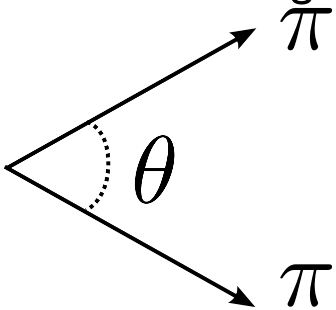
experimental hadron spectrum

- there are a small number of hadrons that cannot decay through the strong interaction
  - they instead decay electromagnetically or weakly with a relatively long lifetime
    - e.g. $\pi^\pm$ has $c\tau \sim 8$ m, $K^\pm$ has $c\tau \sim 4$ m, $\pi^0 \to \gamma\gamma$
    - charged particles and photons ionise matter and so are 'easy' to detect

- the other hadrons are short-lived resonances and are detected via their 'stable' decay products
  - e.g. $\rho^\pm \to \pi^\pm\pi^0$ with $c\tau \sim \mathcal{O}(\mathrm{fm})$

resonances in $\pi\pi$

- say we've got a beam of pions that we fire at a proton target
- one possible reaction is $\pi\, p \to \pi\pi\, p$
- observe the angular and invariant-mass distributions of the two pions

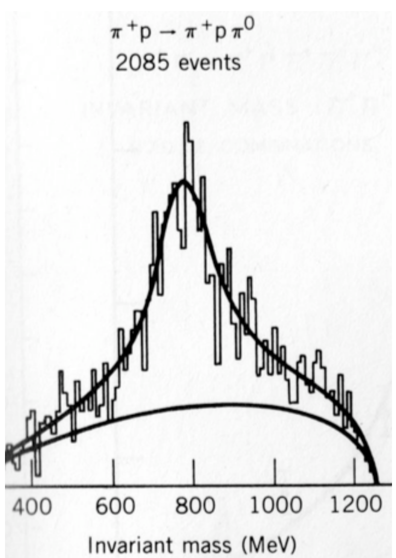


$$m^2$$

$$m^2 = (p_1 + p_2)^\mu (p_1 + p_2)_\mu$$

$$\left|f(\theta, m^2)\right|^2 = \left|\sum_L f_L(m^2) P_L(\cos\theta)\right|^2$$

- e.g. say the pion state is $\pi^+\pi^0$ - the reconstructed invariant mass might look like



two possible isospins contribute

$$|\pi^+\pi^0\rangle = |1,+1\rangle \otimes |1,0\rangle$$
$$= a|1,+1\rangle + b|2,+1\rangle$$

resonances in $\pi\pi$

- there isn't a peak at the same position in $\pi^+\pi^+$ - since the strong interactions are isospin invariant, we can eliminate the isospin 2 possibility
  - $\Rightarrow$ we have an isospin 1 resonance, X ($X^\pm$, X^0)

- the G-parity of this resonance can be inferred immediately
  - $G_X = G_\pi G_\pi = (-1)(-1) = +1$
  - hence the neutral member X^0 has $C = -1$

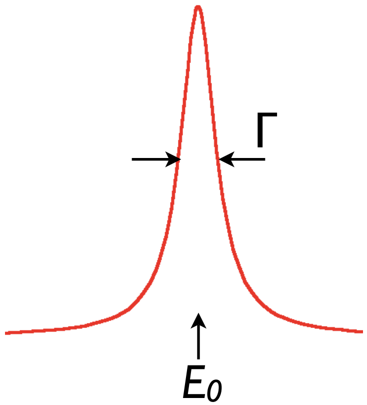
- information on the spin of the resonance comes from the angular distribution of pions
  - experimentally this is found to behave like $\cos^2\theta$ when $m_{\pi\pi} \sim 770$ MeV

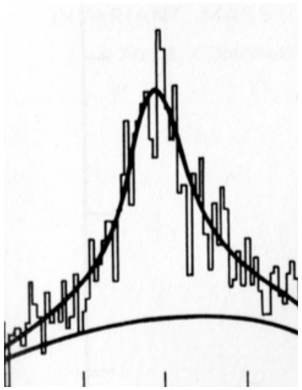
    $\cos^2\theta = |P_{L=1}(\cos\theta)|^2 \Rightarrow J=1$
  - the parity of two particles in a relative L-wave is $P_1 P_2 (-1)^L$, so that with $P_\pi = -1$ & using the parity invariance of strong interactions we have $P_X = -1$
  - this is the *rho* meson $I^G J^{PC} = 1^+ \; 1^{--}$

invariant mass dependence of a resonance

- the rho meson appeared as a bump-like structure in the two-pion invariant mass
- in many cases resonant bumps can be described by some variant of the *Breit-Wigner* formula:

$$\sim \frac{1}{(E-E_0)^2 + \frac{1}{4}\Gamma^2}$$





- at the quantum mechanical amplitude level $A(E) = \left(E - E_0 + i\frac{1}{2}\Gamma\right)^{-1}$

- admits a simple non-relativistic interpretation:

$$A(t) = \int dE \frac{e^{-iEt}}{E - E_0 + i\frac{1}{2}\Gamma} \sim e^{-iE_0 t} e^{-\frac{1}{2}\Gamma t}$$

$$P(t) \sim e^{-\Gamma t}$$

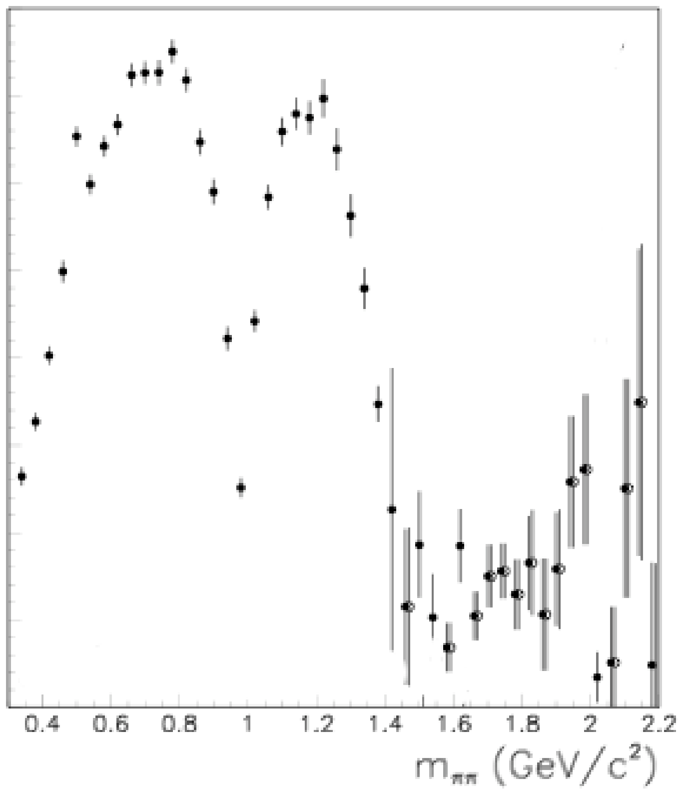
- 'relativistic' version corresponds to a simple pole of the *S*-matrix

$$T^{\mathrm{elastic}}_{\mathrm{BW(rel)}}(s) = \frac{-m_0\Gamma}{s - m_0^2 + i m_0 \Gamma}$$

not always so simple - ππ isospin 0

- angular independent piece ($P_{L=0}(\cos\theta) \Rightarrow J=0$)
- scalar, isoscalar channel $I^G J^{PC} = 0^+ 0^{++}$

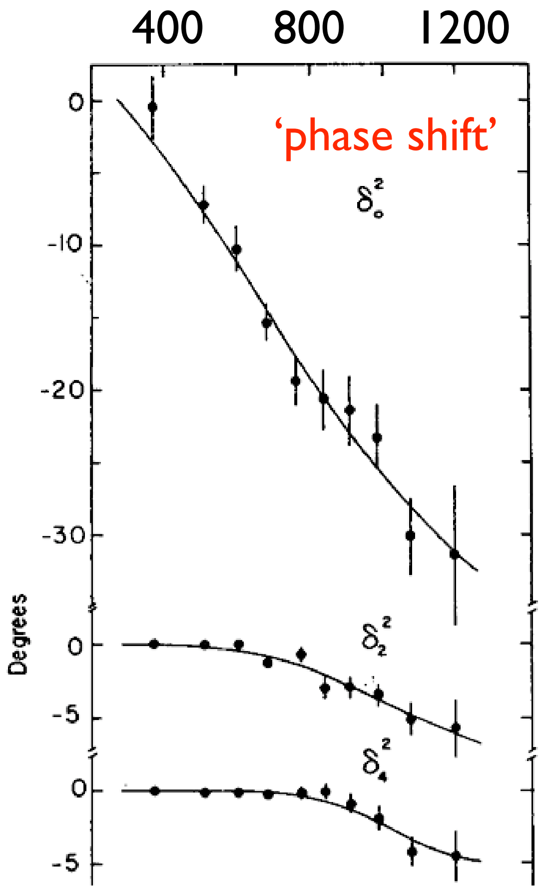
- not clear what is going on here
  - tempting (and many are tempted) to fit the data as a simple sum of *Breit-Wigner* fns
  - ¡ this is not allowed !
  - *unitarity* is a strong constraint on elastic scattering $\mathrm{Im}T = |T|^2$
  - $T^{\mathrm{elastic}}_{\mathrm{BW(rel)}}(s = m_0^2) = i$ - already saturates unitarity



- there are ways around this to deal with this case where *multiple resonances overlap*
  - method is rarely unique & hence
  - *analyses of this type can be rather controversial even with very high quality data*

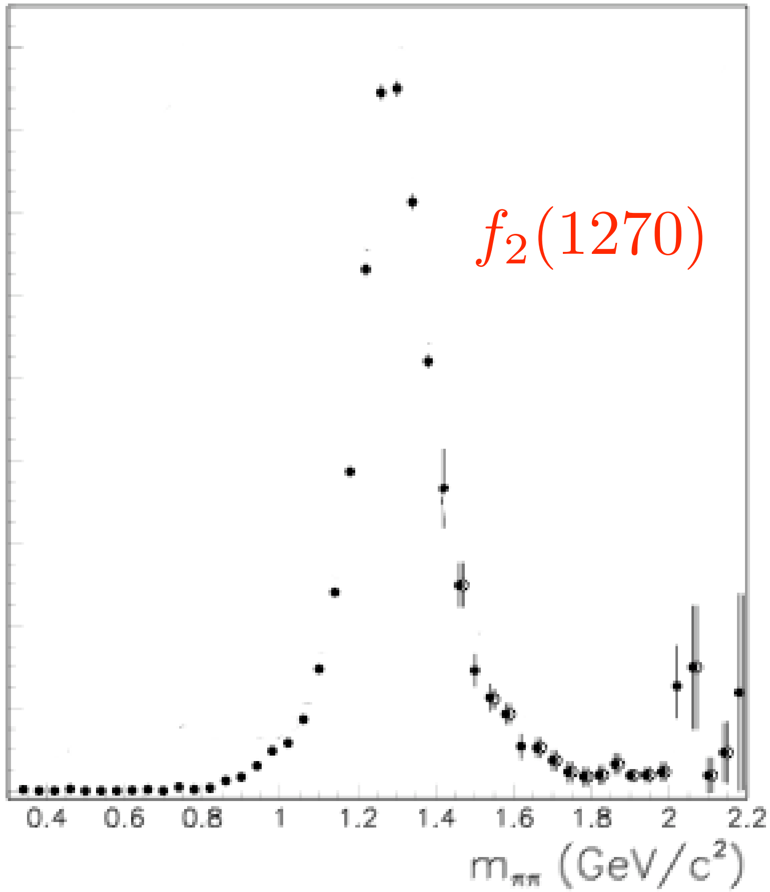
sometimes very simple - ππ isospin 2

- for elastic scattering can express the *T*-matrix via a single phase δ $T(s) = e^{i\delta(s)} \sin\delta(s)$
- resonance peak when $T = i \Rightarrow \delta = \pi/2$
- clearly ***no resonances*** with isospin 2 and $J=(0,2,4)^{++}$



higher mass resonances

- $\pi\pi$ L = 2 has $J^{PC} = 2^{++}$ "tensor meson"



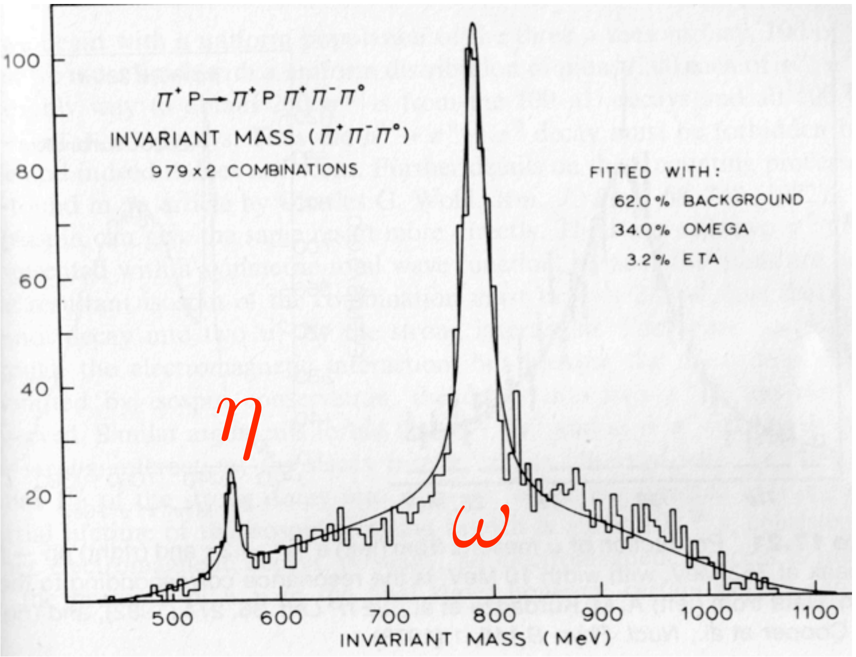
also has other decay channels

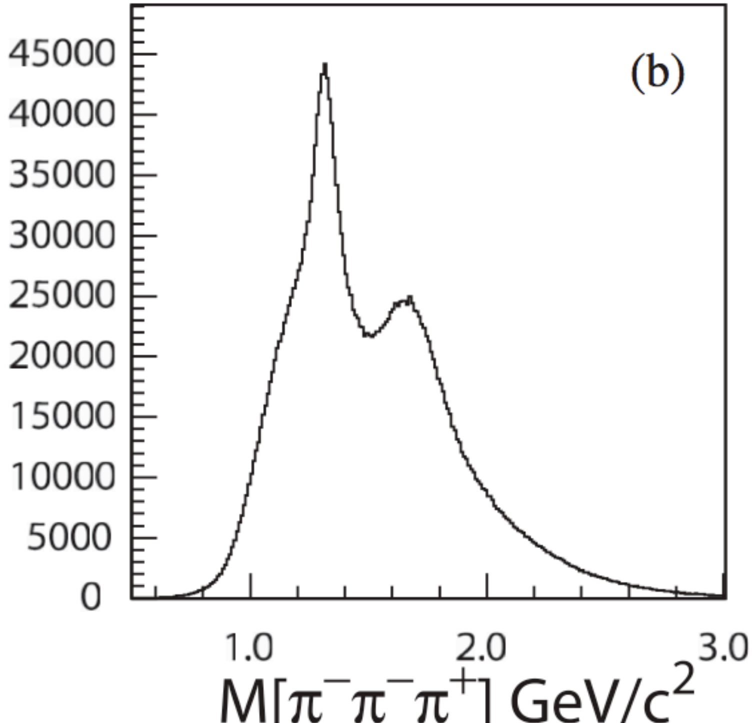
branching fractions

$\pi^+\pi^-2\pi^0$	$(7.1\ {}^{+1.4}_{-2.7})$ %
$K\overline{K}$	$(4.6\ \pm 0.4)$ %
$2\pi^+2\pi^-$	$(2.8\ \pm 0.4)$ %
$\eta\eta$	$(4.0\ \pm 0.8) \times 10^{-3}$
$4\pi^0$	$(3.0\ \pm 1.0) \times 10^{-3}$
$\gamma\gamma$	$(1.41 \pm 0.13) \times 10^{-5}$

$\pi\pi\pi$

- to access negative *G*-parity states we'll need at least three pions
  - in the $\pi^+ \pi^- \pi^0$ channel the invariant mass shows two resonances below 1GeV
    - in the charged $\pi^+ \pi^- \pi^-$ channel, there is quite a lot going on

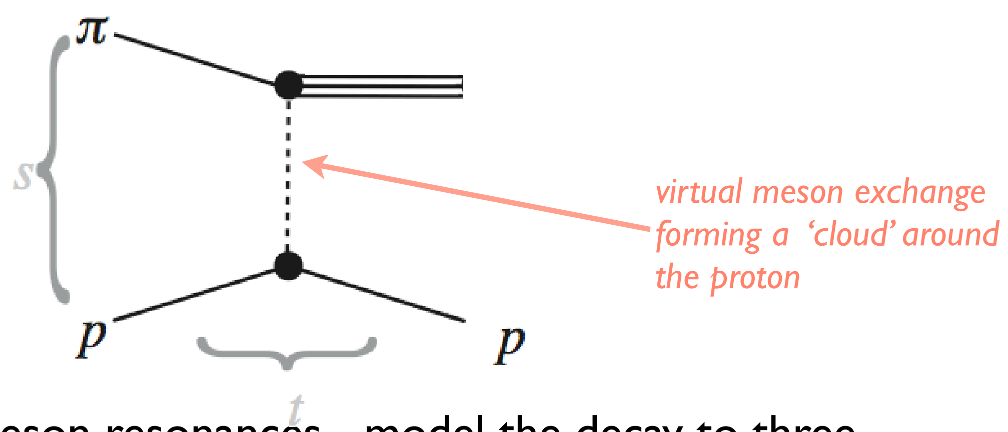




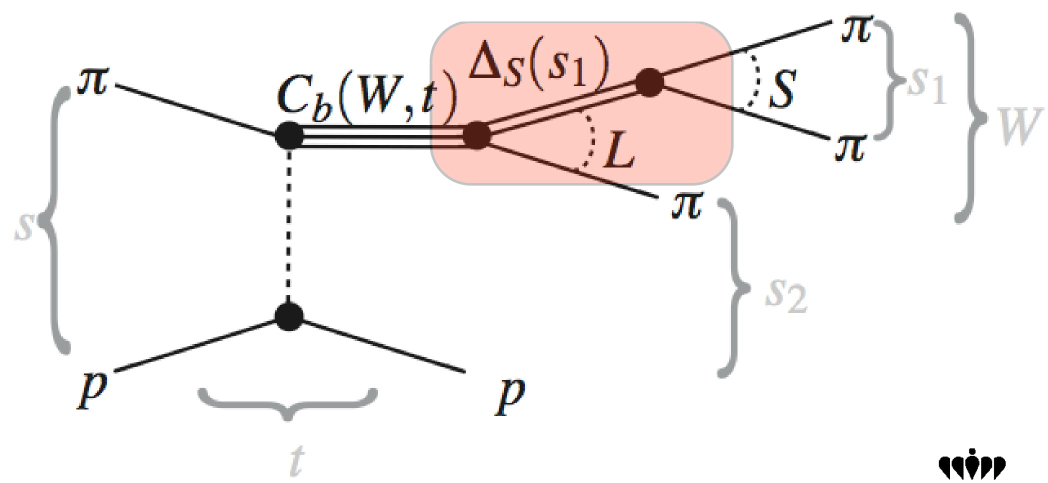
- getting spin information here is a non-trivial task - leads into a model of hadron production
  - *isobar model of partial wave analysis*

PWA in isobar approximation

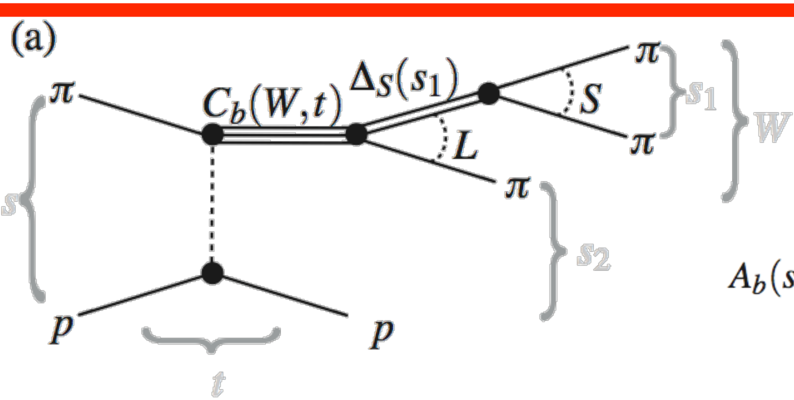
- in high energy scattering, meson production is dominantly peripheral $A \sim e^{bt}$
  - peaked in the forward direction



- coherent production of many meson resonances - model the decay to three pions as going through a two body state



PWA in isobar approximation

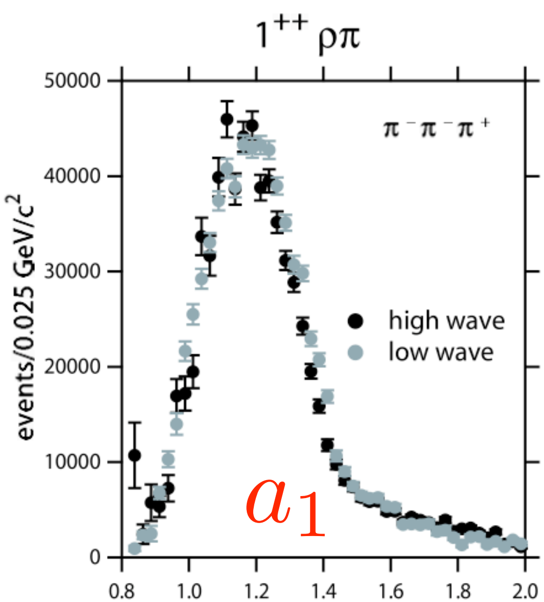


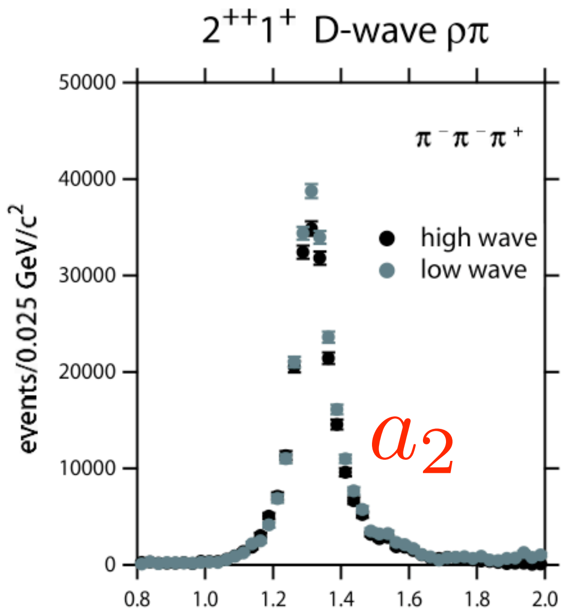
$$I(W,t,s_1,\text{angles}) = \left|\sum_b C_b(W,t)A_b(s_1,\text{angles})\right|^2$$

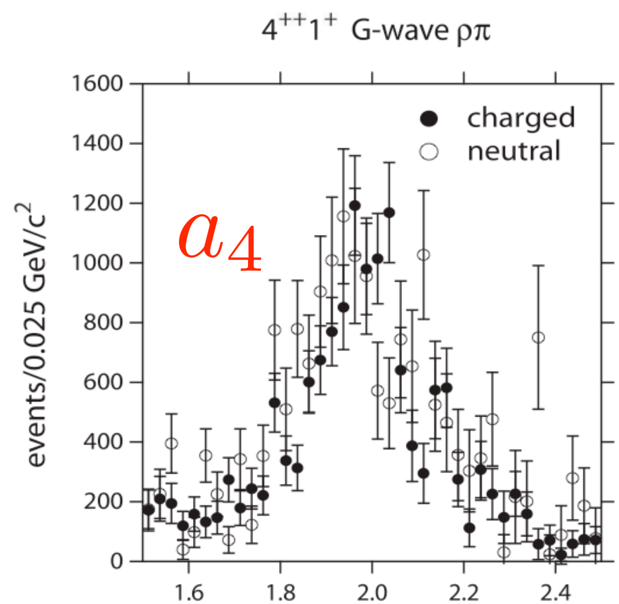
$$b = J^P L M^\eta[\text{isobar}].$$

$$A_b(s_1,\text{angles}) = \sum_\lambda D^{J*}_{M\lambda}(\Omega_1)D^{S*}_{\lambda 0}(\Omega_3)\langle L0;S\lambda|J\lambda\rangle F_L(p_1)F_S(p_3)\Delta^{\text{isobar}}_S(s_1)$$

- isobar propagator ($\Delta_S(s_1)$) is supplied in advance, fitting returns $C_b(W,t)$







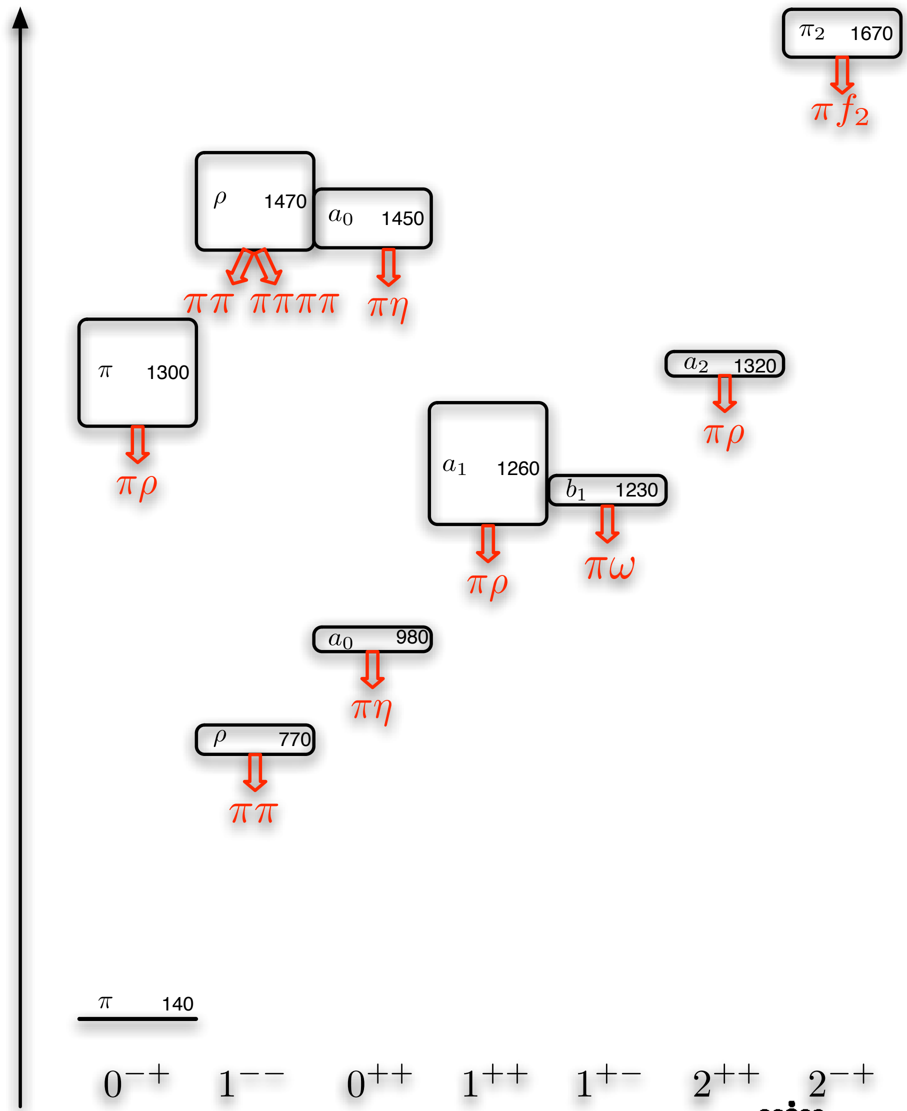
patterns in the meson spectrum

- no clear resonances with $I \geq 2$
- no clear resonances with $|S| \geq 2$
- no unambiguous resonances with $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}$

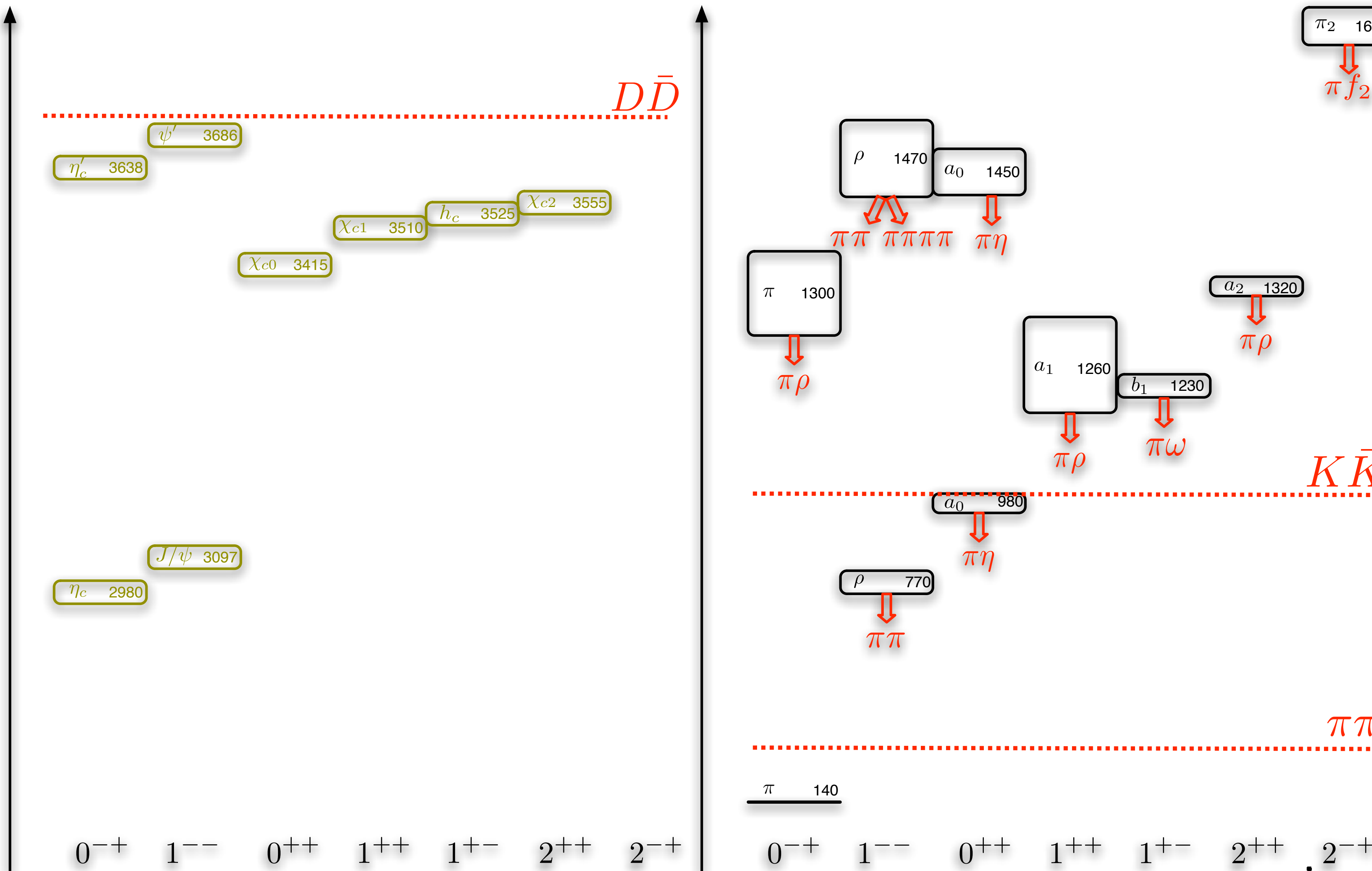
(non-dynamical) quark model

- we can explain the presence of *I*=0,1 & |*S*|=0,1 mesons and the absence of others by a simple proposal
  - all mesons are made from a quark and an anti-quark $q\bar{q}$
    - up quark $u \sim \left|I = \frac{1}{2}, I_3 = +\frac{1}{2}\right\rangle$
    - up anti-quark $\bar{u} \sim \left|I = \frac{1}{2}, I_3 = -\frac{1}{2}\right\rangle$
    - down quark $d \sim \left|I = \frac{1}{2}, I_3 = -\frac{1}{2}\right\rangle$
    - down anti-quark $\bar{d} \sim \left|I = \frac{1}{2}, I_3 = +\frac{1}{2}\right\rangle$
    - strange quark $s \sim |S = -1\rangle$
    - strange anti-quark $\bar{s} \sim |S = +1\rangle$
  - we can't make *I* ≥ 2, |*S*| ≥ 2 in this way (would require at least $q\bar{q}q\bar{q}$)

isospin 1 spectrum



isospin 1 spectrum vs. charmonium



dynamical quark model

- goes beyond the 'group theory' exercise and assigns physical meaning to the quarks
  - they are degrees-of-freedom with spin-½ moving with relative orbital angular momentum, L .
    - then total quark-antiquark spin, $(\Sigma=0,1) = (\sigma=\frac{1}{2})\otimes(\sigma=\frac{1}{2})$
    - so that the meson spin is $\Sigma\otimes L = J$
    - using atomic physics style notation, state defined by $^{2\Sigma+1} L_J$
    - fermion-antifermion pair has $P = P_f\,(-P_f)(-1)^L = (-1)^{L+1}$ and $C=(-1)^{L+\Sigma}$

$$L=0 \left\{ \begin{array}{ll} \Sigma=0 & {}^1S_0 = 0^{-+} \\ \Sigma=1 & {}^3S_1 = 1^{--} \end{array} \right.$$

pseudoscalar π

vector ρ

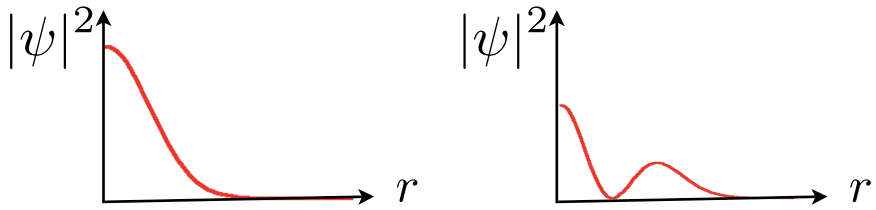
$$L=1 \left\{ \begin{array}{ll} \Sigma=0 & {}^1P_1 = 1^{+-} \\ \Sigma=1 & {}^3P_{0,1,2} = (0,1,2)^{++} \end{array} \right.$$

b_1

scalar, axial, tensor $a_{0,1,2}$

dynamical quark model cont...

- also expect radial excitations with same ${}^{2\Sigma+1}L_J$ but with a node in the radial wavefunction,



e.g. π(1300) as radial excitation of π(140) ?

- another interesting feature is that we can't make $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}$ in this model
  - there seems to be remarkable qualitative agreement between the model and the experimental spectrum, *except the pion looks unnaturally light*
  - obtaining a quantitative description of the spectrum requires a further set of dynamical assumptions like those in the potential model of charmonium

‘constituent’ quarks

- these potential-based quark models usually require the light quarks to have masses in the Schrödinger equation, $m_{u,d} \sim \mathcal{O}(350)$ MeV and the strange quark to have a mass $m_s \sim \mathcal{O}(550)$ MeV
  - *then we have* $m_\rho \sim 2m_{u,d}$ & $m_p \sim 3m_{u,d}$ & $m_\Sigma \sim 2m_{u,d} + m_s$
- these degrees-of-freedom are often called ‘constituent quarks’ to distinguish them from the ‘fundamental’ quarks that appear in the QCD lagrangian which have $m_{u,d} \sim \mathcal{O}(1)$ MeV

- it is *not understood* from first principles why these appear to be appropriate degrees-of-freedom for describing the light meson resonance spectrum
- furthermore we seem to be lacking within the quark model a good explanation for the lightness of the pion: $m_\pi \ll 2m_{u,d}$

- we can go some way toward answering both of these questions by considering an important symmetry of the QCD lagrangian

spontaneous breaking
of chiral symmetry,
the light pion,
constituent quarks
& other strong coupling phenomena

chiral symmetry of QCD

- consider the QCD lagrangian, excluding explicit quark masses

$$\mathcal{L} = \bar{q} i\gamma^\mu \partial_\mu q + g\,\bar{q}\gamma^\mu t_a q\, A^a_\mu - \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a$$

- spin-½ fermions have two helicity states, we'll call them *L* & *R*

$$q_L \equiv \tfrac{1}{2}(1-\gamma_5)q$$
$$q_R \equiv \tfrac{1}{2}(1+\gamma_5)q$$

- the lagrangian does not couple them

$$\mathcal{L} = \mathcal{L}_L + \mathcal{L}_R + \mathcal{L}_G$$

$$\mathcal{L}_L = \bar{q}_L i\gamma^\mu \partial_\mu q_L + g\,\bar{q}_L \gamma^\mu t_a q_L\, A^a_\mu$$

- this is the *chiral symmetry* of massless QCD
- 'real' QCD has two flavours of quark that are believed to be nearly massless - consider these to form a doublet field $q = \begin{pmatrix} u \\ d \end{pmatrix}$
- then we have a global *U(2)* symmetry in flavour space $q^\dagger q = q'^\dagger U^\dagger U q' = q'^\dagger q'$
- *U(2)* matrices can be expressed as exponentials of *hermitian generator matrices*

$$U = e^{-i\alpha_a T_a} \qquad [T_a, T_b] = i\epsilon_{abc} T_c \quad SU(2)$$

will focus on the SU(2) piece

chiral U(2) symmetries

- notice that since q_L and q_R are totally decoupled, we have a separate U(2) symmetry for each $$q_L = e^{-i\alpha_L^a T_a} q'_L \qquad q_R = e^{-i\alpha_R^a T_a} q'_R$$
- alternatively we can take orthogonal combinations applied to the q fields

$$q = e^{-i\alpha_V^a T_a} q' \qquad e^{-i\alpha_V^a T_a} = e^{-i\alpha_L^a T_a \frac{1}{2}(1-\gamma^5)} e^{-i\alpha_R^a T_a \frac{1}{2}(1+\gamma^5)} \quad \alpha_L^a = \alpha_R^a$$

$$q = e^{-i\alpha_A^a T_a \gamma^5} q' \qquad e^{-i\alpha_A^a T_a \gamma^5} = e^{-i\alpha_L^a T_a \frac{1}{2}(1-\gamma^5)} e^{-i\alpha_R^a T_a \frac{1}{2}(1+\gamma^5)} \quad \alpha_L^a = -\alpha_R^a$$

- the lagrangian is invariant under either of these - we say that there are separate
  - *vector U(2)*
  - *axial U(2)*
- Noether's theorem tells us that where there's a symmetry, there's a conserved current
  - vector case: $V_\mu^a = \dfrac{\partial \mathcal{L}}{\partial(\partial^\mu q_i)}(-iT_{ij}^a q_j) = \bar{q} T^a \gamma_\mu q$
  - axial case: $A_\mu^a = \dfrac{\partial \mathcal{L}}{\partial(\partial^\mu q_i)}(-iT_{ij}^a \gamma_5 q_j) = \bar{q} T^a \gamma_\mu \gamma_5 q$

$$\partial^\mu V_\mu^a = \partial^\mu A_\mu^a = 0$$

chiral U(2) symmetries

- can also define conserved 'charges', which act like generators of the U(2) group at the level of fields:

$$Q_V^a \equiv \int d^3x\, V_0^a(x)$$

$$Q_A^a \equiv \int d^3x\, A_0^a(x)$$

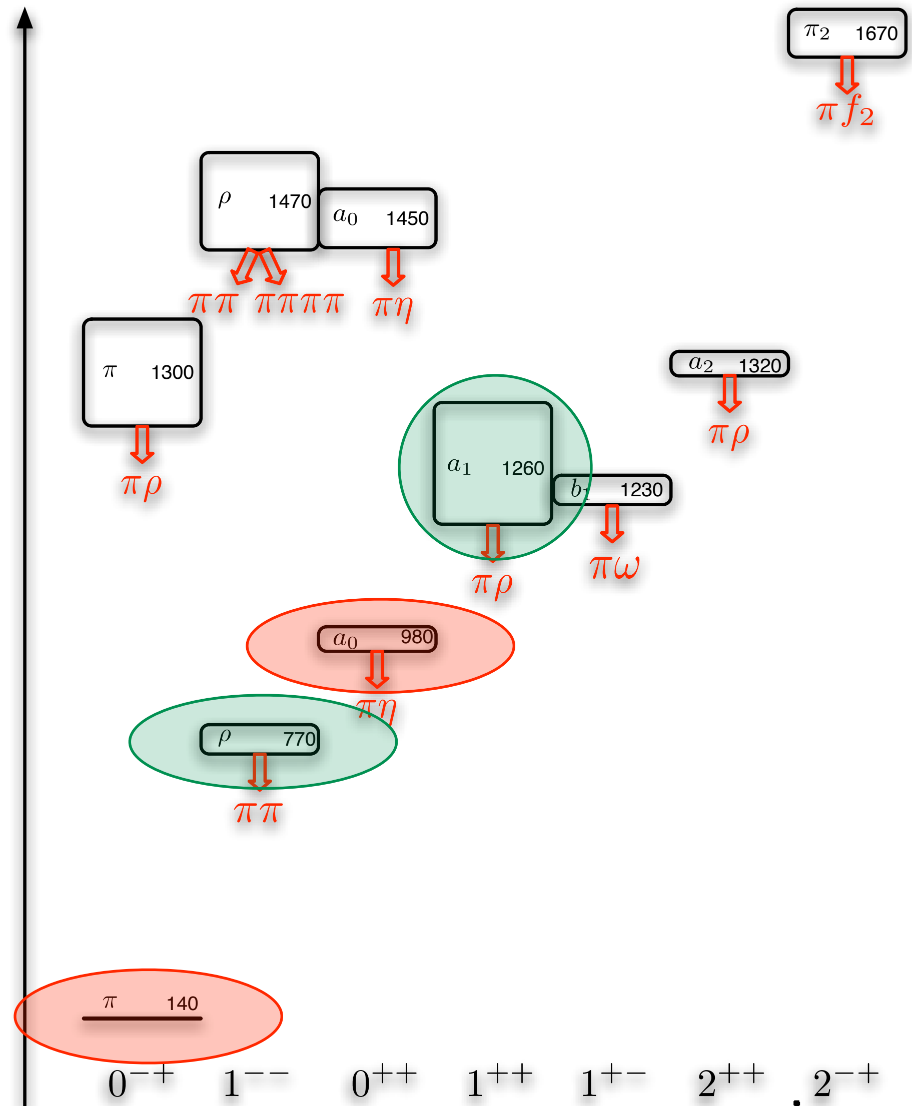
$$\left([Q_V^a, q(x)] = -T^a q(x) \right)$$

- under parity transformation: $\mathcal{P} Q_A^a \mathcal{P}^{-1} = -Q_A^a$ $\quad \mathcal{P} Q_V^a \mathcal{P}^{-1} = Q_V^a$

- does this symmetry have any consequences for the meson spectrum?
  - *consider a positive parity meson state X of energy E* $\quad \mathcal{H}|X\rangle = E|X\rangle$, $\mathcal{P}|X\rangle = +|X\rangle$
  - *our theory has an axial (chiral) symmetry* so $\frac{d}{dt} Q_A^a = i[\mathcal{H}, Q_A^a] = 0$
  - *and hence* $\mathcal{H} Q_A^a |X\rangle = E Q_A^a |X\rangle$ *, so there's a state* $Q_A^a|X\rangle$ *degenerate with* $|X\rangle$
  - *this state has negative parity* $\mathcal{P}(Q_A^a|X\rangle) = \mathcal{P} Q_A^a \mathcal{P}^{-1} \mathcal{P}|X\rangle = -(Q_A^a|X\rangle)$
  - so the axial symmetry predicts parity partners in the meson spectrum

parity partners?

- experimental spectrum doesn't seem to show parity doubling!



a loophole

- we must have overlooked something
- one possibility is that although the lagrangian has the chiral symmetry, the vacuum state does not
- we know from other bits of physics that this can happen - consider the ferromagnet
  - there is an attractive spin-spin interaction between neighbouring spins - this is rotationally symmetric
  - any small perturbation causes the spins to all align in one direction - this lowest energy state is not rotationally symmetric
  - we call this a spontaneous breaking of a symmetry
- we will propose that this happens in QCD & see what consequences it would have
  - we'll guess that
$$Q_A^a|0\rangle \neq 0$$
$$Q_V^a|0\rangle = 0$$
  - so that the *axial SU(2) symmetry is broken* while the *vector U(2) is unbroken*
  - *turns out that this retains the isospin symmetry we observe in the spectrum*

spontaneously broken axial SU(2)

- *in general* an axial current can produce a pseudoscalar (*0^-*) from the vacuum

$$\langle 0|A^a_\mu(x)|\pi^b(q)\rangle \equiv i f_\pi q_\mu \delta^{ab} e^{-iq\cdot x}$$

- the axial current is still conserved in the Noether sense $\partial^\mu A^a_\mu = 0$

$$\langle 0|\partial^\mu A^a_\mu(x)|\pi^b(q)\rangle \equiv f_\pi m_\pi^2 \delta^{ab} e^{-iq\cdot x} = 0$$

- a consequence of spontaneous breaking of the axial SU(2) is that $f_\pi \neq 0$
  - *hence we must have $m_\pi = 0$*

- we get massless pions as a consequence of spontaneously broken chiral symmetry
  - *this is a specific case of Goldstone's theorem - for each spontaneously broken generator of a global symmetry there will be a massless boson with those quantum numbers*

a nucleon consequence

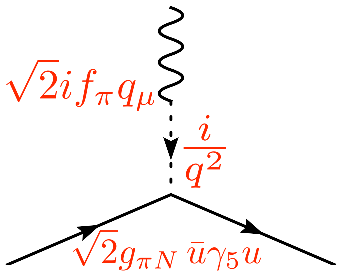
- consider the nucleon matrix element of the axial current (occurs in e.g. neutron beta decay) $\langle n(p')|A^-_\mu(x)|p(p)\rangle \equiv e^{iq\cdot x}\,\bar{u}(p')\big[\gamma_\mu\gamma_5 g_A(q^2) + q_\mu\gamma_5 h(q^2)\big]u(p)$
- and since the axial current is conserved
$$0 = \bar{u}(p')\big[\not{q}\gamma_5 g_A(q^2) + q^2\gamma_5 h(q^2)\big]u(p)$$
- using the Dirac equation for the free nucleons
$$0 = \bar{u}(p')\gamma^5 u(p)\,\big[2m_N g_A(q^2) + q^2 h(q^2)\big]$$
- at $q^2=0$ there are two possible solutions:
  - $g_A(0)=0,\ h(0)=const.$
  - $h(q^2\to 0)\ \to\ -2m_N\, g_A(0)/q^2$
- experimentally $g_A(0)\neq 0$, so only the second solution is acceptable
  - what causes a pole at $q^2\to 0$?

a nucleon consequence

- consider an effective interaction between a nucleon an a pion

$$g_{\pi N} \bar{N} i\vec{\tau}\cdot\vec{\pi}\gamma_5 N$$

- in such an effective theory there will be a tree-level diagram contributing to the matrix element of the axial current



$$(\sqrt{2} i f_\pi q_\mu)\cdot\left(\frac{i}{q^2}\right)\cdot(\sqrt{2} g_{\pi N}\bar{u}\gamma_5 u)\, e^{iq\cdot x}$$

$$\langle n(p')|A^-_\mu(x)|p(p)\rangle \equiv e^{iq\cdot x}\ \bar{u}(p')\left[\gamma_\mu\gamma_5 g_A(q^2) + q_\mu\gamma_5 h(q^2)\right]u(p)$$

- this diagram contributes to the second term

$$h(q^2\to 0) \to -\frac{2 f_\pi g_{\pi N}}{q^2}$$

- and from the conservation of the axial current $h(q^2\to 0) \to -2m_N\, g_A(0)/q^2$
  - *hence* $f_\pi g_{\pi N} = m_N g_A(0)$ *- the Goldberger-Treiman relation*
  - *experimentally works rather well (better than 10%)*

vacuum condensates

- what is happening to the vacuum that is causing it to be non-invariant under axial *SU(2)* transformations?
- at low energies (or long distances), the QCD interactions are really strong, we believe strong enough that the *vacuum fills up with quark-antiquark pairs*
- we know that Lorentz symmetry and parity remain good symmetries so the vacuum should be invariant w.r.t. these
  - a possibility is $\langle 0|\bar{q}q|0\rangle = \langle 0|\bar{q}_L q_R + \bar{q}_R q_L|0\rangle$
  - since it couples *L* & *R*, it breaks the chiral symmetry
  - it remains symmetric under the $\alpha_L = \alpha_R$ vector transforms though

chiral condensate & the pion

- we suggested that spontaneous chiral symmetry breaking manifested itself as $f_\pi \neq 0, m_\pi = 0$
- we can demonstrate a connection to the chiral condensate
- begin with a Ward identity (an expression of the chiral symmetry of the lagrangian) $\partial^\mu_{\vec{x}} \langle 0 | T\{ A^a_\mu(x), i\bar{q}T^b\gamma_5 q(y)\} |0\rangle$
$$= -i\delta^4(x-y)\langle 0|\mathrm{Tr}\big(\{T^a, \bar{q}q\}T^b\big)|0\rangle$$
- let's propose that the chiral symmetry breaking causes only an isosinglet condensate $\langle 0|\bar{u}u + \bar{d}d|0\rangle$ $\quad \langle 0|\bar{q}_i q_j|0\rangle = v\delta_{ij}$ $\quad v = \langle \bar{u}u\rangle = \langle \bar{d}d\rangle$
- then the trace is easy to compute, giving overall $= -i\delta^4(x-y)v\delta^{ab}$
- by fourier transforming we get a momentum space relation $p^\mu \widetilde{G}^{ab}_\mu(p) = v\delta^{ab}$
- by Lorentz symmetry $\widetilde{G}^{ab}_\mu(p) = p_\mu F(p^2)$, so $p^2 F(p^2) = v$ and ***F*** must have a pole at $p^2 \to 0$
  - $\widetilde{G}^{ab}_\mu(p) = p_\mu \frac{v}{p^2}\delta^{ab}$ - looks like there's going to be a massless boson here

chiral condensate & the pion

- the contribution of a single particle state of mass *m* to the pseudoscalar correlation function takes the form

$$\widetilde{H}^{ab}(p) = \int d^4x\, e^{-ip\cdot x} \langle 0|T\{i\bar{q}T^a\gamma_5 q(x) \;\cdot\; i\bar{q}T^b\gamma_5 q(0)\}|0\rangle$$
$$= \frac{iZ^2}{p^2 - m^2}\delta^{ab} + \ldots$$

this stuff is technical but obvious

- the matrix element $\langle 0|A^a_\mu(0)|\pi^b(p)\rangle$ is related to *H* and *G* via the *LSZ* reduction formula:

$$\langle 0|A^a_\mu(0)|\pi^b(p)\rangle = Z\widetilde{H}^{ac}(p)^{-1}\widetilde{G}^{cb}_\mu(p) = -\tfrac{i}{Z}(p^2 - m^2)\cdot p_\mu \frac{v}{p^2}\delta^{ab}$$

$$if_\pi p_\mu \delta^{ab} = -\tfrac{i}{Z}\cdot p_\mu v \delta^{ab}$$

$$m_\pi = 0$$

$$f_\pi = -\frac{\langle \bar{u}u + \bar{d}d\rangle}{2Z}$$

- and we've proven what we assumed before, that there is a massless pion with a non-zero decay constant.

explicit symmetry breaking

- actually the quarks have finite, but small, masses - they enter via a term in the lagrangian $\mathcal{L} = -\bar{q}_i m_{ij} q_j$ $\quad m = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}$
- we can include the effect of this as a perturbation to the chiral limit results
  - easy analysis uses the quark equations of motion

$$f_\pi m_\pi^2 \delta^{ab} e^{-ip\cdot x} = \langle 0|\partial^\mu A^a_\mu(x)|\pi^b(p)\rangle = \langle 0|i\bar{q}\{m, T^a\}\gamma_5 q(x)|\pi^b(p)\rangle$$

$$m = \tfrac{1}{2}(m_u + m_d)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \tfrac{1}{2}(m_u - m_d)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \tfrac{1}{2}(m_u + m_d)1 + (m_u - m_d)T^3$$

$$f_\pi m_\pi^2 \delta^{ab} e^{-ip\cdot x} = (m_u + m_d)\langle 0|i\bar{q}T^a\gamma_5 q(x)|\pi^b(p)\rangle + 0 = (m_u + m_d)\delta^{ab} Z e^{-ip\cdot x}$$

$$f_\pi m_\pi^2 = (m_u + m_d) Z$$

  - since this is a perturbative treatment of the quark mass, we'll use the zeroth-order result for the condensate $\quad f_\pi = -\dfrac{\langle \bar{u}u + \bar{d}d\rangle}{2Z}$

$$m_\pi^2 = (m_u + m_d)\frac{\langle \bar{u}u + \bar{d}d\rangle}{2 f_\pi^2}$$

pion gets a mass, but one proportional to the square root of the quark mass - not at all like a quark model

effective theories of pseudo-Goldstone bosons

- another consequence of spontaneous chiral symmetry breaking:
  - Goldstone bosons couple to each other by powers of the momentum
  - consider the following function of pion fields $U(\pi) = \exp[\frac{2i}{f_\pi} T_a \pi_a]$
  - since the generators are Hermitian, U is unitary ⇒ $U^\dagger U = I$
  - then if we try to write a lagrangian featuring this field, the lowest dimension term will be proportional to $\mathrm{tr}\, \partial_\mu \mathrm{U}^\dagger \partial^\mu \mathrm{U}$
  - expanding in powers of the pion field $\frac{2}{f_\pi^2}\, \partial_\mu \pi_a \partial^\mu \pi_a + \ldots$
  - so a conventionally normalised kinetic term is obtained if $\mathcal{L} = \frac{f_\pi^2}{4} \mathrm{tr} \partial_\mu \mathrm{U}^\dagger \partial^\mu \mathrm{U}$
  - higher powers of the pion field give interactions: $\mathcal{L}_{\mathrm{int}} = \frac{2}{f_\pi^2}\, \pi_b \pi_b\, \partial_\mu \pi_a \partial^\mu \pi_a + \ldots$
    - pion four-point interaction, with 'coupling' $\frac{p^2}{f_\pi^2}$
  - extending this can develop a perturbation theory in small momenta of pions
    - ***chiral perturbation theory***

chiral symmetry breaking and constituent quarks

- don't have a model-independent formalism here, best we can do is try to write down a toy theory that has the right ingredients (fields and symmetries)
  - *chiral symmetry of fermion fields*
  - *strong coupling between fermions*
- NJL (Nambu–Jona-Lasinio) model is a good example

$$\mathcal{L}_{\mathrm{NJL}} = \bar{q}(i\gamma^\mu\partial_\mu - m_0)q - G\bar{q}\gamma_\mu t^a q \cdot \bar{q}\gamma^\mu t^a q + \dots$$

  - if $m_0=0$ this has chiral symmetry
  - the interaction term is local between two "colour" vector currents
  - we can increase G to make the model strongly coupled
  - model isn't renormalisable so we require a cutoff, Λ - in terms of this we can define a dimensionless coupling $g = \Lambda\sqrt{G}$
  - an approximate, non-perturbative, self-consistent solution for a fermion condensate and an 'effective quark mass' can be defined

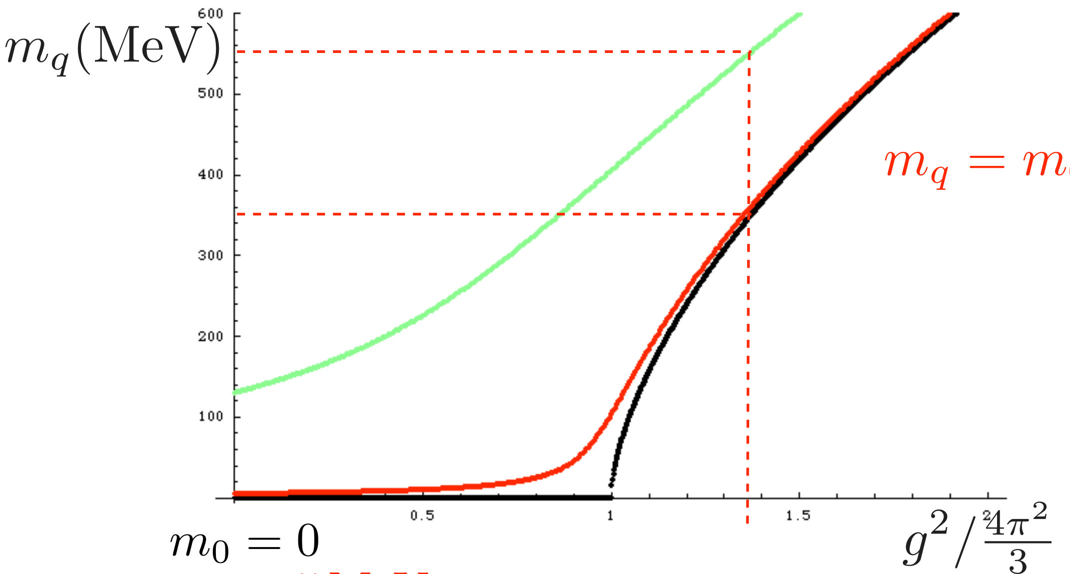
$$\langle\bar{q}q\rangle = -4iN_c \int^\Lambda \frac{d^4p}{(2\pi)^4}\frac{m_q}{p^2 - m_q^2 + i\epsilon} \qquad m_q = m_{0q} - \tfrac{8}{9}G\langle\bar{q}q\rangle$$

  - evaluating the integral we get

$$m_q = m_{0q} + \frac{3g^2}{4\pi^2}m_q\left[1 - \frac{m_q^2}{\Lambda^2}\ln\left(1+\frac{\Lambda^2}{m_q^2}\right)\right]$$

chiral symmetry breaking and constituent quarks

- this *effective fermion mass* appears in fermion propagators (within the approximate solution) - it takes into account the condensate of fermion pairs through which fermions must force themselves
- how does this effective mass depend upon the coupling?

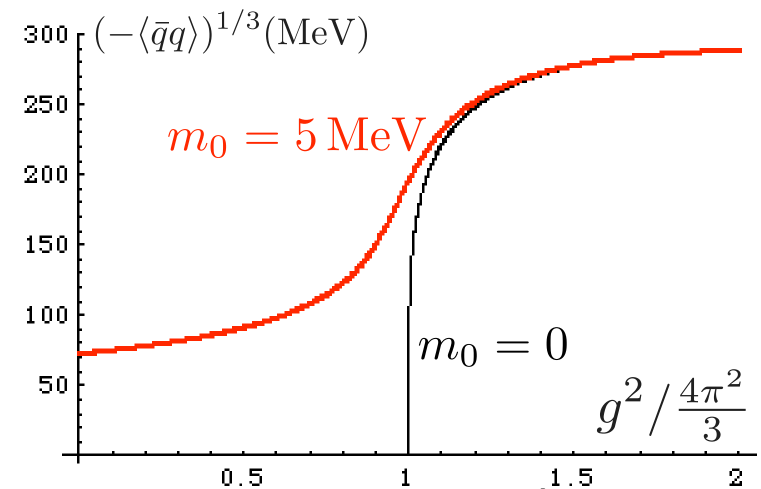


$$m_q = m_{0q} + \frac{3g^2}{4\pi^2} m_q \left[1 - \frac{m_q^2}{\Lambda^2}\ln\left(1+\frac{\Lambda^2}{m_q^2}\right)\right]$$

$m_0 = 0$
$m_0 = 5\,\mathrm{MeV}$
$m_0 = 130\,\mathrm{MeV}$

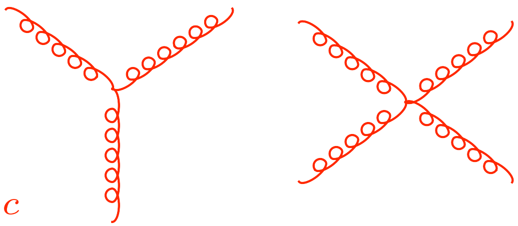
$\Lambda = 1\,\mathrm{GeV}$

appears reasonable that chiral symmetry breaking might give rise to effective "constituent" quark masses



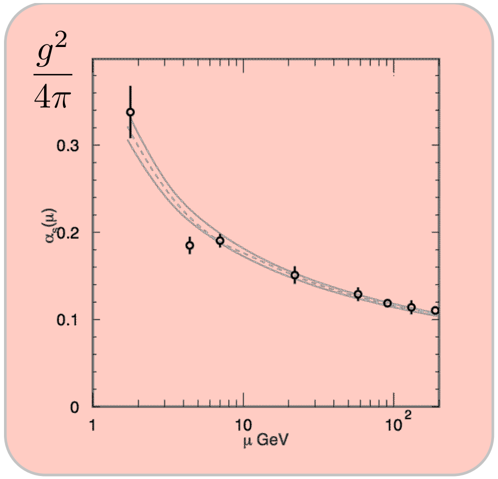
strongly coupled gluonic field

- so we've seen one result of the strong-coupling or non-perturbative nature of QCD in the formation of quark condensates
- we expect there to be others,
  - e.g. gluons couple strongly to each other



$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu}$$

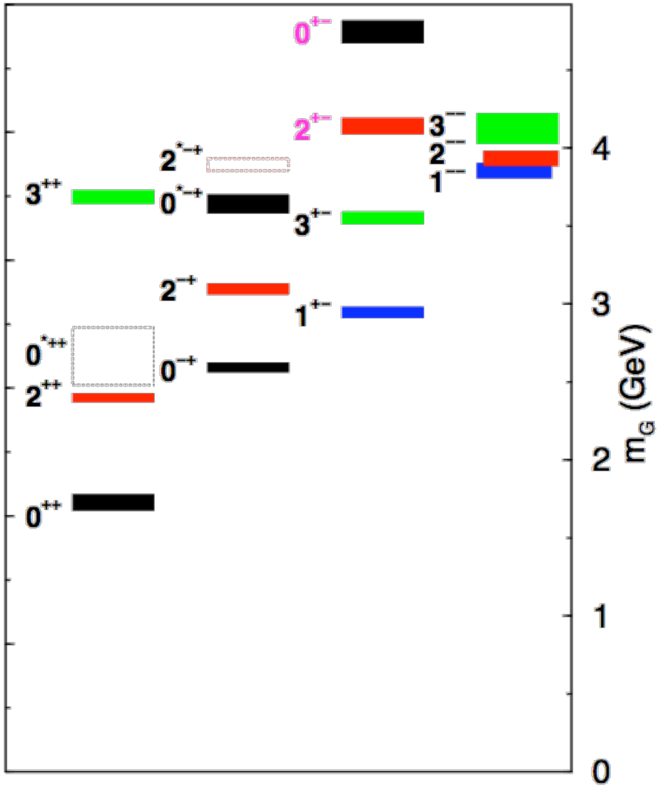
$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$



  - then we might expect there to be a spectrum of collective gluonic excitations
  - possible even in a theory without quarks
    - 'gluodynamics' or 'pure Yang-Mills'
    - particles are called *'glueballs'*

glueballs

- bosons made only from the gluonic field
- spectrum of pure *SU(3)* Yang-Mills has been extracted using computerised lattice calculations



- glueballs in full QCD are significantly more complicated
  - they have the same quantum numbers as isospin 0 mesons
  - in a strongly coupled theory there is nothing to stop them mixing with the 'quark-based' states

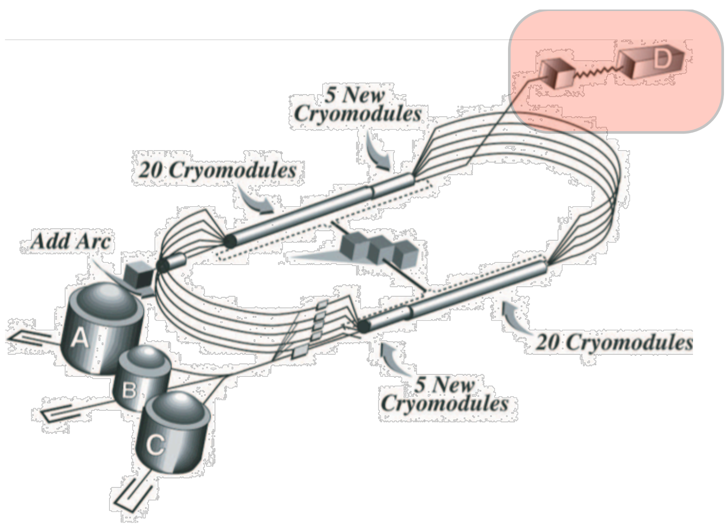
- some people have suggested that there is one more isoscalar scalar meson between 1 & 2 GeV than there 'should be'
  - quark model expects two:

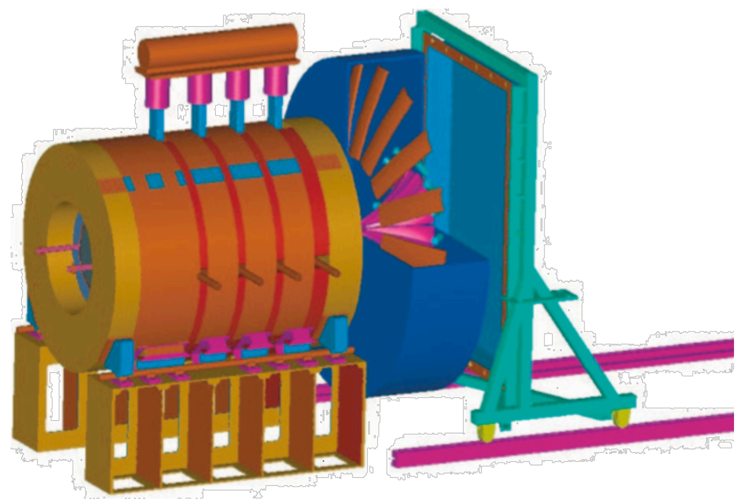
$$\cos\theta \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d) + \sin\theta\, \bar{s}s$$
$$-\sin\theta \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d) + \cos\theta\, \bar{s}s$$

  - some claims that there are three: $f_0(1370),\ f_0(1500),\ f_0(1710)$

hybrid mesons

- possibly a better chance to see gluonic excitations in experiment comes from 'hybrid mesons' - states that have both quarks & excited gluonic field
  - signal is exotic J^{PC}
    - with $J^{PC}_{\rm glue} \neq 0^{++}$ we can have $J^{PC}_{q\bar{q}} \otimes J^{PC}_{\rm glue}$ like 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}...
  - we've got no model-independent theoretical knowledge of these states
  - major new experimental effort forthcoming at Jefferson Lab
    - ***GlueX***





looking for new experimental and theoretical members

Lattice QCD as a tool for hadron spectroscopy

Lattice QCD & the path integral

- a quantum field theory can be expressed in terms of a mathematical object called a 'path-integral'

$$Z = \underbrace{\int \mathcal{D}\varphi(x)}_{\text{'functional' integral over all possible field configurations}} \, e^{i \int d^4x \, \overbrace{\mathcal{L}[\varphi(x)]}}$$

e.g. $\mathcal{L}[\varphi] = \frac{1}{2}\partial^\mu\varphi\partial_\mu\varphi - \frac{1}{2}m^2\varphi^2$

'functional' integral over all possible field configurations

- familiar quantities can be expressed in terms of path-integrals, e.g. the propagator in the free scalar field theory

$$\langle\varphi(y)\varphi(x)\rangle = Z^{-1} \int \mathcal{D}\varphi \, \varphi(y)\varphi(x) \, e^{i \int d^4x \, \mathcal{L}[\varphi]}$$

- in rare cases like this one we can perform the (Gaussian) functional integral exactly
- more generally this method lets us write down a functional integral for any *N*-point function that is true non-perturbatively, although we can't necessarily perform the integral exactly

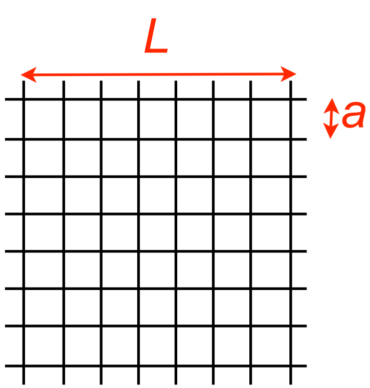
Lattice QCD & the path integral

- for QCD we have

$$Z_{\rm QCD} = \int \underbrace{\mathcal{D}\bar{q}\mathcal{D}q\mathcal{D}A_\mu}_{} \, e^{i\int d^4x\, \bar{q}(i\gamma^\mu\partial_\mu - m)q + g\,\bar{q}\gamma^\mu t_a q\, A^a_\mu - \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a}$$

infinite number of degrees of freedom -
field strength at every point in a
continuous, infinite spacetime

- what if we could make the number of degrees of freedom finite?
- then we could try to compute the path integral numerically
  - this is the tactic followed in *lattice field theory*
  - consider spacetime to be a grid of points of finite extent separated by a finite spacing



in the limit *$a \rightarrow 0$, $L \rightarrow \infty$* we should recover QCD

Lattice QCD practicalities

- how can we do this practically?
  - write a discretised version of the action that in the limit *a→0* becomes the QCD action $\bar{q}_i Q_{ij} q_j$ *+ discretisation of gauge part*
  - there are a very large number of possible constructions of the 'Dirac matrix'
  - as a simpler example consider the discretisation of the simple derivative in one-dimension

$$\frac{df}{dx} = \frac{f(x+a) - f(x-a)}{2a} + \mathcal{O}(a^2)$$

$$\text{or } \frac{df}{dx} = \frac{-f(x+3a) + 27f(x+a) - 27f(x-a) + f(x-3a)}{48a} + \mathcal{O}(a^4)$$

  - different discretisations of the fermion action lead to many of the jargon terms you'll hear
    - Wilson, Clover
    - Staggered, Kogut-Susskind, asqtad
    - Domain Wall, Overlap

Lattice QCD practicalities

- an important computational simplification comes if we can make $e^{i\int d^4x\,\mathcal{L}}$ real
  - then we can treat it like a probability distribution function
  - this can be achieved by moving to Euclidean spacetime

$$t \to i\tilde{t}; \quad i\int d^4x\,\mathcal{L} \to -\int d^4\tilde{x}\,\tilde{\mathcal{L}}$$

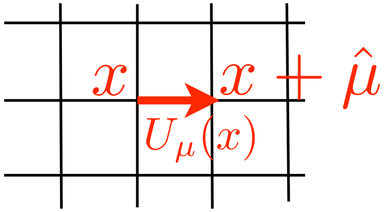
  - then e.g. $\langle \bar{q}_x q_x\, \bar{q}_y q_y \rangle = \int \mathcal{D}\bar{q}\mathcal{D}q\mathcal{D}A_\mu\, \bar{q}_x q_x\, \bar{q}_y q_y\, e^{-\tilde{S}}$

average of $\bar{q}_x q_x\, \bar{q}_y q_y$ over all field configurations with weight $e^{-\tilde{S}[q,\bar{q},A_\mu]}$

  - since the action is bilinear in the fermion fields we can integrate them out exactly so that we don't need to include them directly in the computation

$$\int \mathcal{D}\bar{q}\mathcal{D}q\, e^{-\bar{q}_i Q_{ij} q_j} = \det Q$$

  - a natural way to include the gluon fields is to make *SU(3)* group elements $U_\mu(x) = e^{-aA_\mu(x)}$ these act like parallel transporters of colour between neighbouring sites - *"the gluons live on the links of the lattice"*



Lattice QCD practicalities

- physically interesting quantities for spectroscopy include things like

$$\langle \bar{q}(\vec{x},t')\Gamma' q(\vec{x},t') \cdot \bar{q}(\vec{y},t)\Gamma q(\vec{y},t)\rangle = \int \mathcal{D}U\, Q^{-1}_{x,y}[U]\Gamma' Q^{-1}_{y,x}[U]\Gamma \; \det Q[U] e^{-\tilde{S}_{\text{gauge}}[U]}$$

- now $\det Q[U] e^{-\tilde{S}_{\text{gauge}}[U]}$ is like a probability weight ⇒ why not generate gauge-field configurations according to this weight & save them (*an ensemble*), then

$$\langle \bar{q}(\vec{x},t')\Gamma' q(\vec{x},t') \cdot \bar{q}(\vec{y},t)\Gamma q(\vec{y},t)\rangle = \sum_{\{U\}} Q^{-1}_{x,y}[U]\Gamma' Q^{-1}_{y,x}[U]\Gamma$$

- once the gauge-field configurations are saved, we have only to perform inversions of the Dirac matrix, *Q[U]*, to give the fermion propagators
  - how big is this matrix?
    - lattice size might be 24x24x24x48 ≈ 83,000
    - a fermion has 4 Dirac components
    - there are 3 colours in SU(3)
      - ⇒ *Q* could easily be 1million x 1million - **HUGE**
    - can reduce this down to 1million x 12 by fixing $\vec{y} = \vec{0},\; t = 0$
    - *will need a big computer*

$$Q^{y,\beta,b}_{x,\alpha,a}$$

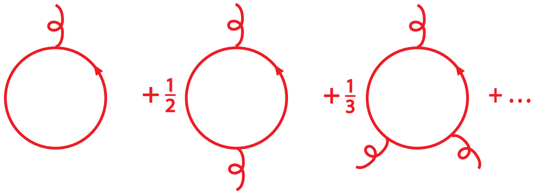
"point to all propagator"

quenched approximation

- one way to reduce the computational cost is to set the determinant to 1

$$Z = \int \mathcal{D}U \det Q[U] e^{-\tilde{S}_{\text{gauge}}[U]} \to \int \mathcal{D}U e^{-\tilde{S}_{\text{gauge}}[U]}$$

- what does this do to the theory?
  - consider the Dirac matrix to be a sum of a free part and an interaction
  $$Q = Q^{(0)} - V[U]$$
  - then we can write $Q = Q^{(0)}(1 - \Delta \cdot V)$, where $\Delta = (Q^{(0)})^{-1}$ is the free fermion propagator
  - hence
  $$\det Q = \det Q^{(0)} \cdot \det[1 - \Delta \cdot V] = \det Q^{(0)} \exp[\operatorname{tr}\log(1 - \Delta \cdot V)] = \det Q^{(0)} \exp\left[\sum_j j^{-1} \operatorname{tr}(\Delta \cdot V)^j\right]$$
  - the terms in the exponential can be expressed graphically as



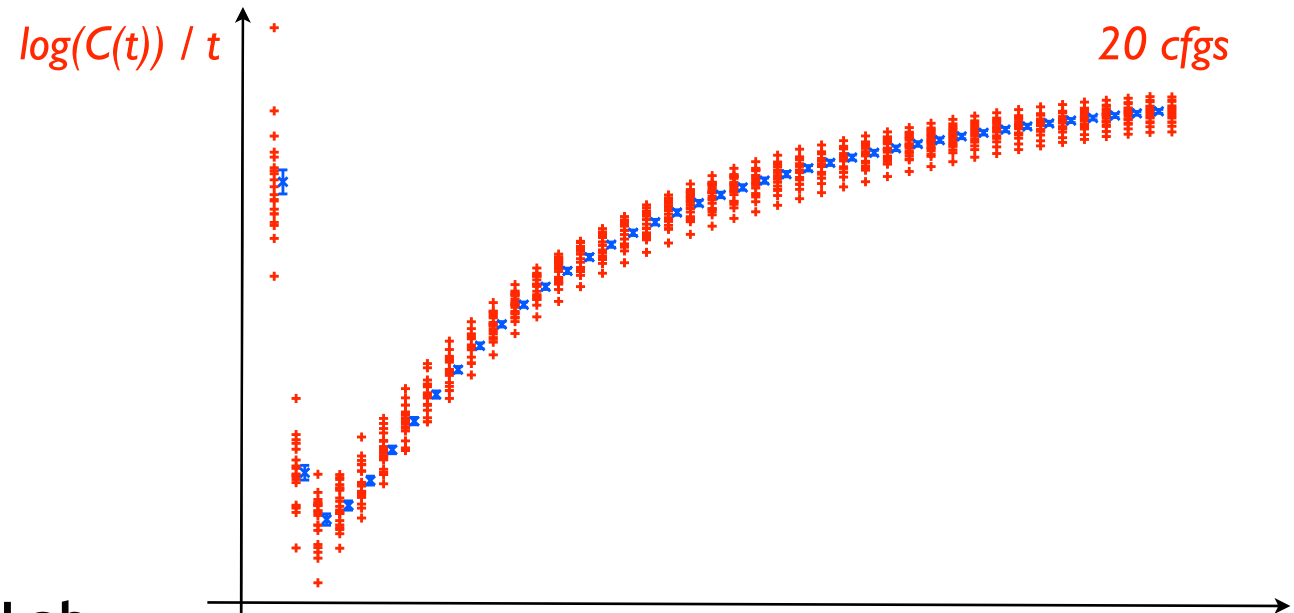
  - so the fermion determinant can be considered as a set of gauge field interactions generated by *closed fermion loops*
  - *the quenched approximation corresponds to neglecting closed fermion loops*

meson two-point correlator

- for example

$$C(\vec{p},t) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \langle \bar{q}(\vec{x},t)\Gamma q(\vec{x},t) \cdot \bar{q}(\vec{0},0)\Gamma q(\vec{0},0)\rangle = \sum_{\{U\}} \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}}\, Q^{-1}_{x,0}[U]\Gamma Q^{-1}_{0,x}[U]\Gamma$$

- where the fermion propagators come from the connected Wick contraction of the fermion fields
- once we've computed this quantity on all configurations of our ensemble, we have an ensemble of values of *C(p,t)*, the average gives our estimate of the quantity and we can quote a statistical error (from the variance) due to the finite number of configurations in our ensemble



meson two-point correlator

- for example

$$C(\vec{p},t) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \langle \bar{q}(\vec{x},t)\Gamma q(\vec{x},t) \cdot \bar{q}(\vec{0},0)\Gamma q(\vec{0},0)\rangle = \sum_{\{U\}} \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}}\, Q^{-1}_{x,0}[U]\Gamma Q^{-1}_{0,x}[U]\Gamma$$

- where the fermion propagators come from the connected Wick contraction of the fermion fields
- once we've computed this quantity on all configurations of our ensemble, we have an ensemble of values of *C(p,t)*, the average gives our estimate of the quantity and we can quote a statistical error (from the variance) due to the finite number of configurations in our ensemble
- *C(p,t)* contains information about the spectrum of mesons with the quantum numbers of $\bar{q}\Gamma q$:
  - insert a complete set of states $1 = \sum_{N,\vec{q}} \frac{|N(\vec{q})\rangle\langle N(\vec{q})|}{2E_N(\vec{q})}$

$$C(\vec{p},t) = \sum_N \frac{e^{-E_N t}}{2E_N} \langle 0|\bar{q}(0)\Gamma q(0)|N(\vec{p})\rangle\langle N(\vec{p})|\bar{q}(0)\Gamma q(0)|0\rangle$$

  - in particular at zero three-momentum $C(\vec{0},t) = \sum_N \frac{|Z_N|^2}{2m_N} e^{-m_N t}$

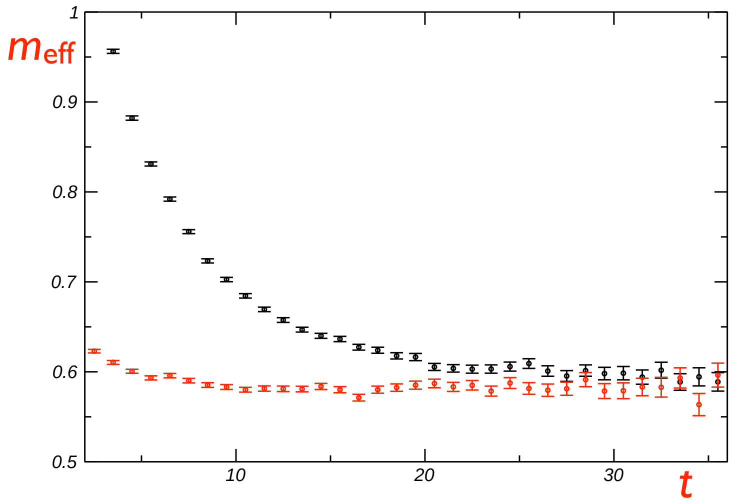
meson two-point correlator

$$C(\vec{0},t) = \sum_N \frac{|Z_N|^2}{2m_N} e^{-m_N t}$$

- the presence of a decaying exponential and not an oscillating exponential is because we're working in Euclidean space-time
- clearly as $t \to \infty$ only the lightest state will contribute
- a handy quantity for visualisation is the *effective mass*

$$m_{\text{eff}} = -\frac{d}{dt} \log C(\vec{0},t) \xrightarrow[t\to\infty]{} m_0$$

- heading toward a plateau at large times?



the red data is flat even from short times

not clear that the black data gets there?

black data uses simple local operator:

$$\bar{q}(0)\gamma_i\gamma_5 q(0)$$

gets contributions from many excited states

meson two-point correlator

- how did we get the red data, which increased the overlap on to the ground state?
- we smeared the operator over space: $\sum_{\vec{x}} F(|\vec{x}|)\, \bar{q}_{\vec{0}-\vec{x},0} \gamma_i \gamma_5 q_{\vec{0}+\vec{x},0}$
  - *F(x)* is a gauge-invariant approximation to a rotationally symmetric gaussian
  - idea is that the ground state wavefunction (at least with heavy quarks) looks something like a gaussian - we're maximising the overlap
  - the excited state wavefunctions have nodes so there'll be cancellations, reducing their overlap

setting the scale

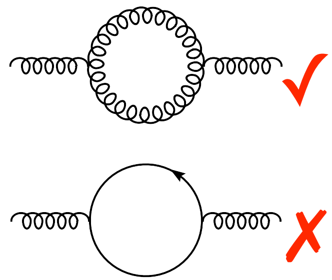
- (excluding quark masses) QCD has one scale which is dynamically generated
  - it appears, e.g. in the running coupling $g^2(k) = \dfrac{g^2}{1 + \frac{g^2}{3(4\pi)^2}(33 - 2N_f)\log\frac{k^2}{\Lambda^2}}$
  - in a lattice simulation, setting its value is equivalent to setting the value of the lattice spacing, *a*
  - usual to do this by comparing a lattice computed value to a dimensionful experimental quantity
    - e.g. we could compare $\tilde{m}_\rho = m_\rho a$ with the experimental mass $m_\rho^{\mathrm{expt.}} = 770\,\mathrm{MeV}$

$$a = \frac{\tilde{m}_\rho}{m_\rho^{\mathrm{expt.}}}$$

    - this isn't so wise since the rho mass depends strongly on the quark mass and we're unlikely to have this low enough
  - more usual these days to use some property of charmonium or bottomonium since they're expected to be less sensitive to details of the light quarks - *typically 'long-distance' dominated quantities*

quenched approximation

- one aspect of the quenched approximation is that the running of the QCD coupling isn't correct
  - so scale setting via a long-distance quantity runs to an incorrect short-distance g



$$g^2(k) = \frac{g^2}{1 + \frac{g^2}{3(4\pi)^2}(33 - 2N_f)\log\frac{k^2}{\Lambda^2}}$$

TABLE I. The QCD coupling $\alpha_V(6.3\text{ GeV})$ from 1×1 Wilson loops in simulations with different u/d and s sea-quark masses (in units of the physical s mass), and using two different tunings for the lattice spacing. The first error shown is statistical, and the second is truncation error which we take to be $\mathcal{O}(1\alpha_V^3)$ [11].

a (fm)	$m_{u,d}$	m_s	$1P - 1S$	$2S - 1S$	
1/8	∞	∞	0.177 (1)(5)	0.168 (0)(4)	*quenched*
1/8	0.5	∞	0.211 (1)(9)	0.206 (1)(8)	
1/8	1.3	1.3	0.231 (2)(12)	0.226 (2)(11)	
1/8	0.5	1.3	0.234 (2)(12)	0.233 (1)(12)	
1/8	0.2	1.3	0.234 (1)(12)	0.234 (1)(12)	
1/11	0.2	1.1	0.238 (1)(13)	0.236 (1)(13)	*light dynamical*

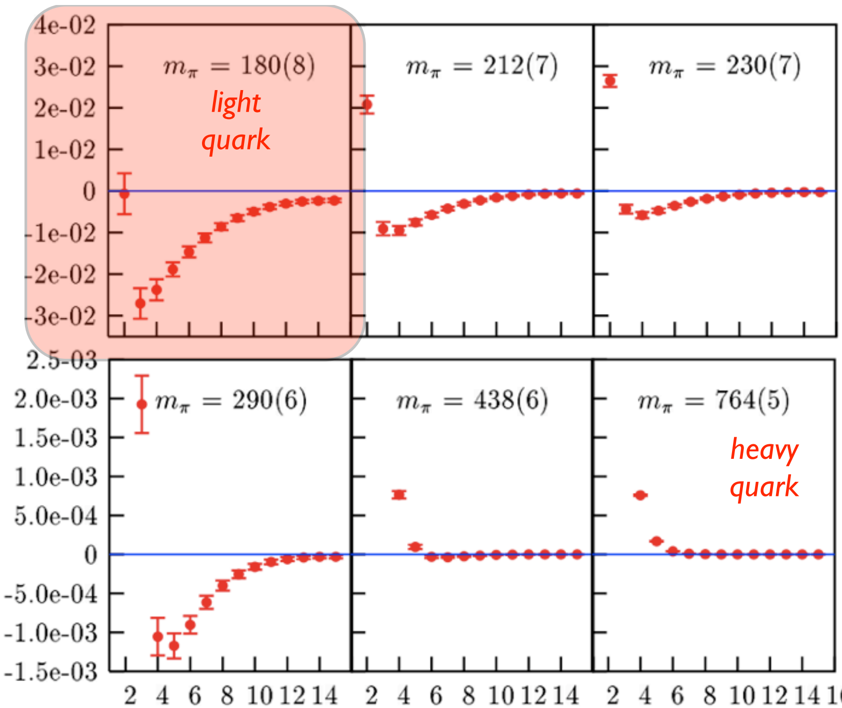
scale set by: Υ(1P-1S) Υ(2S-1S)

quenched non-unitarity

- a more serious problem with the quenched approximation is that it does not correspond to a unitary (probability conserving) field theory
- recall the meson two-point function, we found that it could be expressed as

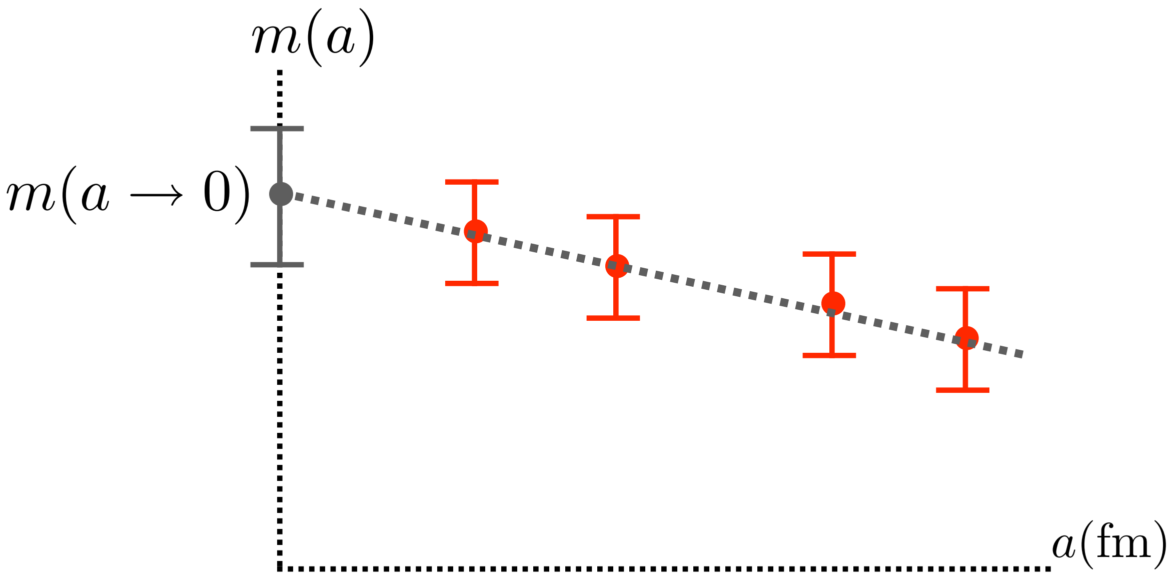
$$C(\vec{0},t) = \sum_N \frac{|Z_N|^2}{2m_N} e^{-m_N t}$$ which is positive definite

- consider the following quenched results:
  - correlator is clearly negative at lightest quark masses
  - violating unitarity



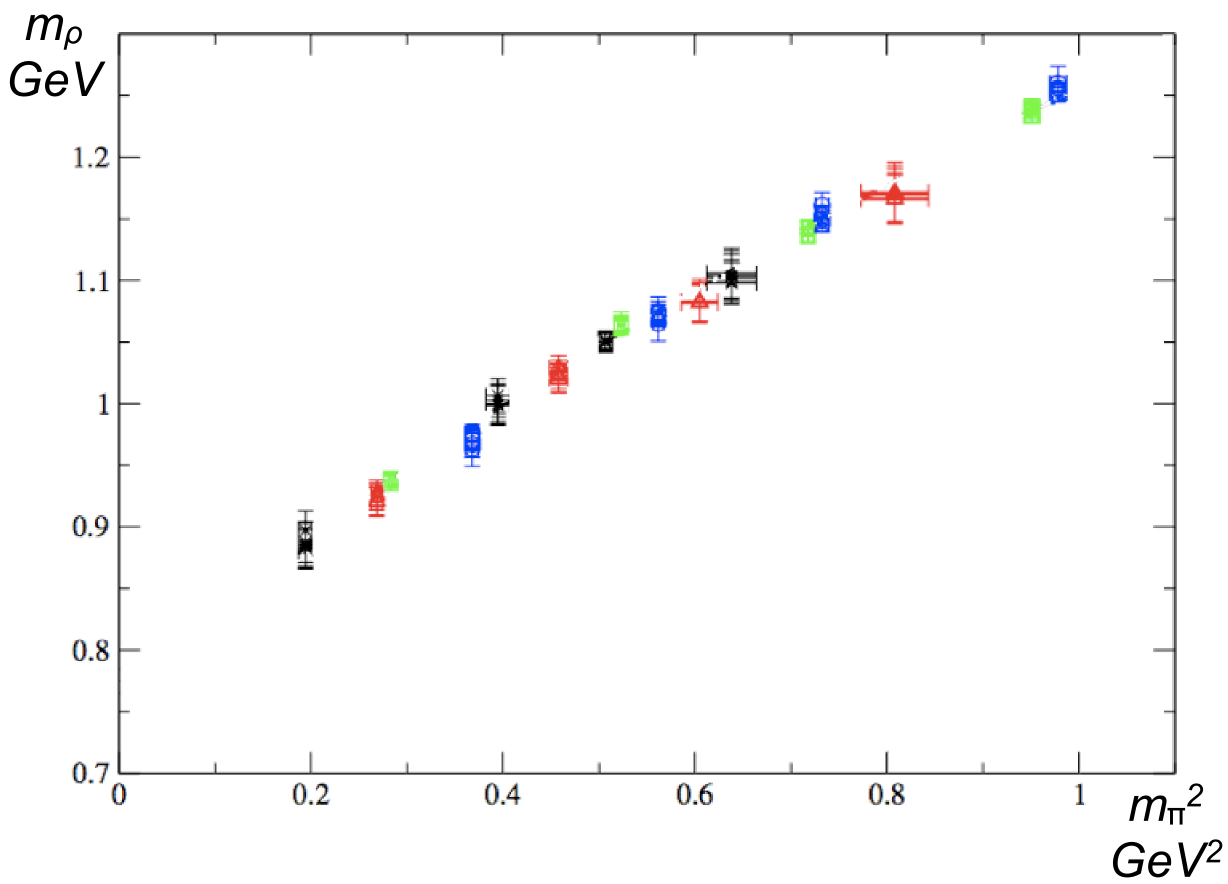
meson spectrum

- computing limitations ultimately prevent calculation with realistically light quarks
  - time to compute *det Q* and to invert *Q* grows very rapidly as we reduce the quark mass
  - instead calculate with a range of quark values and try to extrapolate
  - quark mass not usually quoted, instead use *pion mass* at this quark mass
- there's also the $a \to 0$ and $L \to \infty$ limits - assume we can take the $a \to 0$ limit with multiple simulations and simple extrapolation



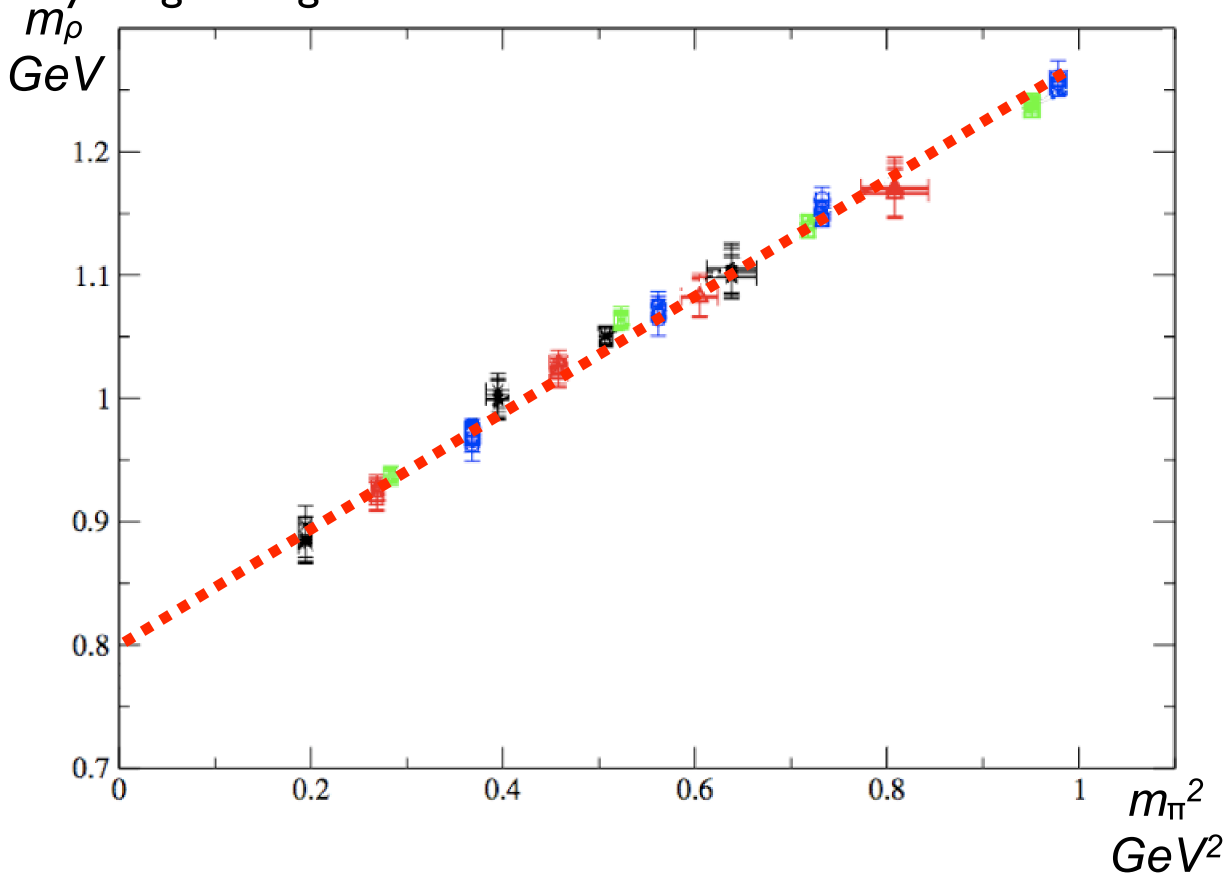
light meson spectrum (idealised)

- do the *a→0* extrapolation for each quark mass simulation and plot the masses versus m_π^2 . (in *χSB* picture $m_\pi^2 \propto m_q$)



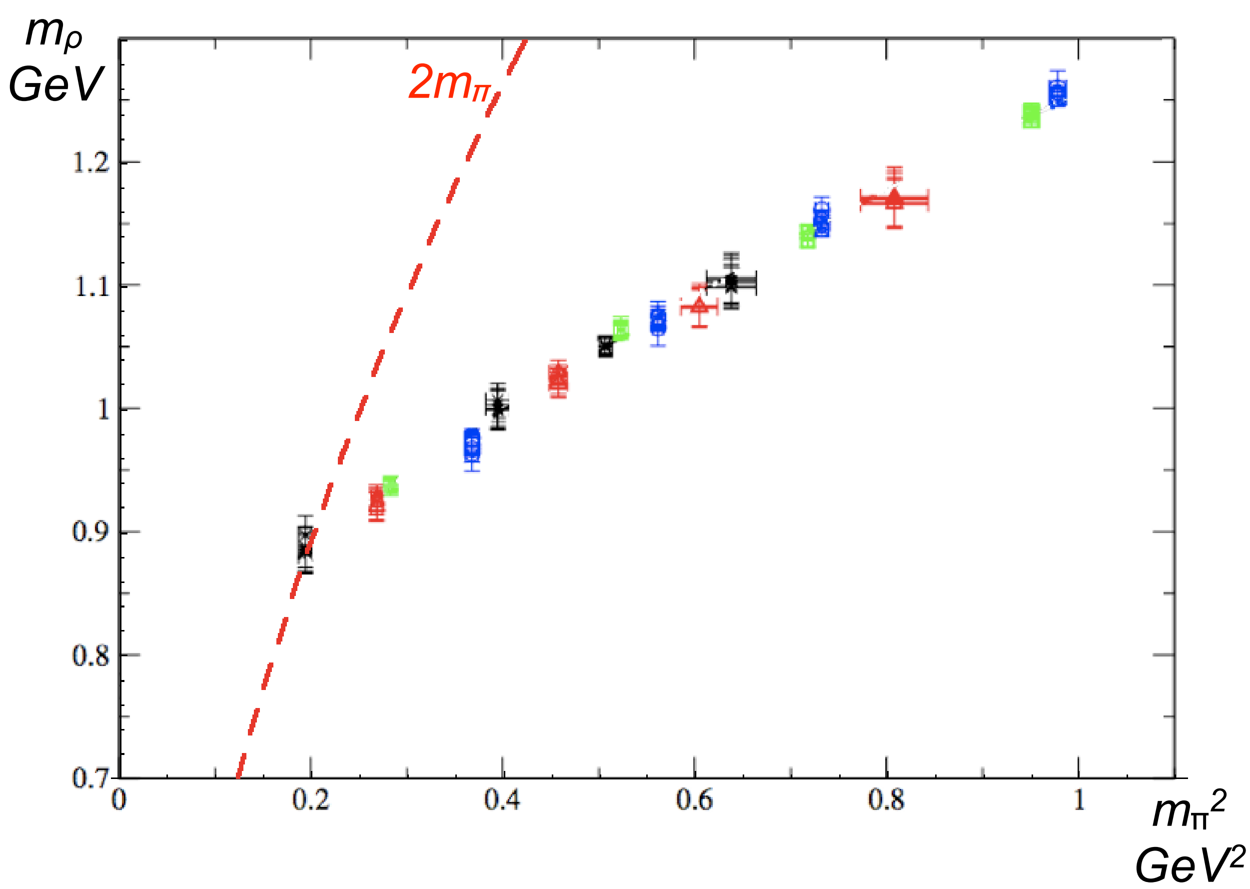
light meson spectrum (idealised)

- looking at the data there's the temptation (followed by many) to extrapolate linearly in m_π^2 *(or a power series in m_π^2)*
  - anything wrong with that?



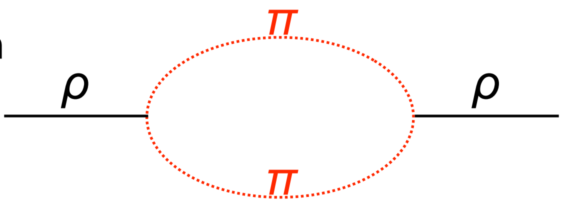
light meson spectrum (idealised)

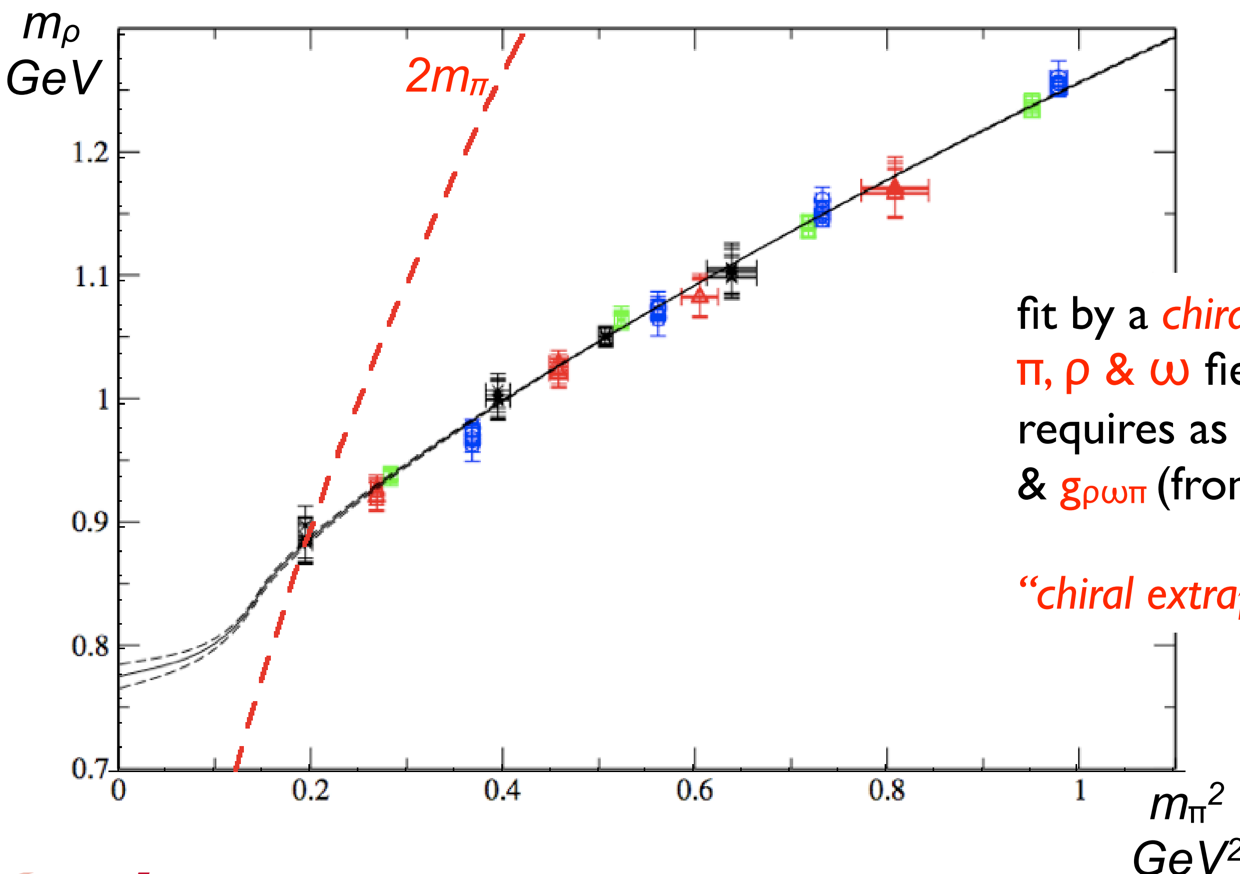
- Yes! It ignores some important physics.
  - consider the mass of a two-pion state = 2 m_π



light meson spectrum (idealised)

- at light quark masses the rho can decay into two pions
  - occurs through the imaginary part of the diagram
  - the real part contributes to the mass





fit by a *chiral effective theory* featuring π, ρ & ω fields

requires as input the couplings $g_{\rho\pi\pi}$ & $g_{\rho\omega\pi}$ (from expt.)

"chiral extrapolation"

excited states?

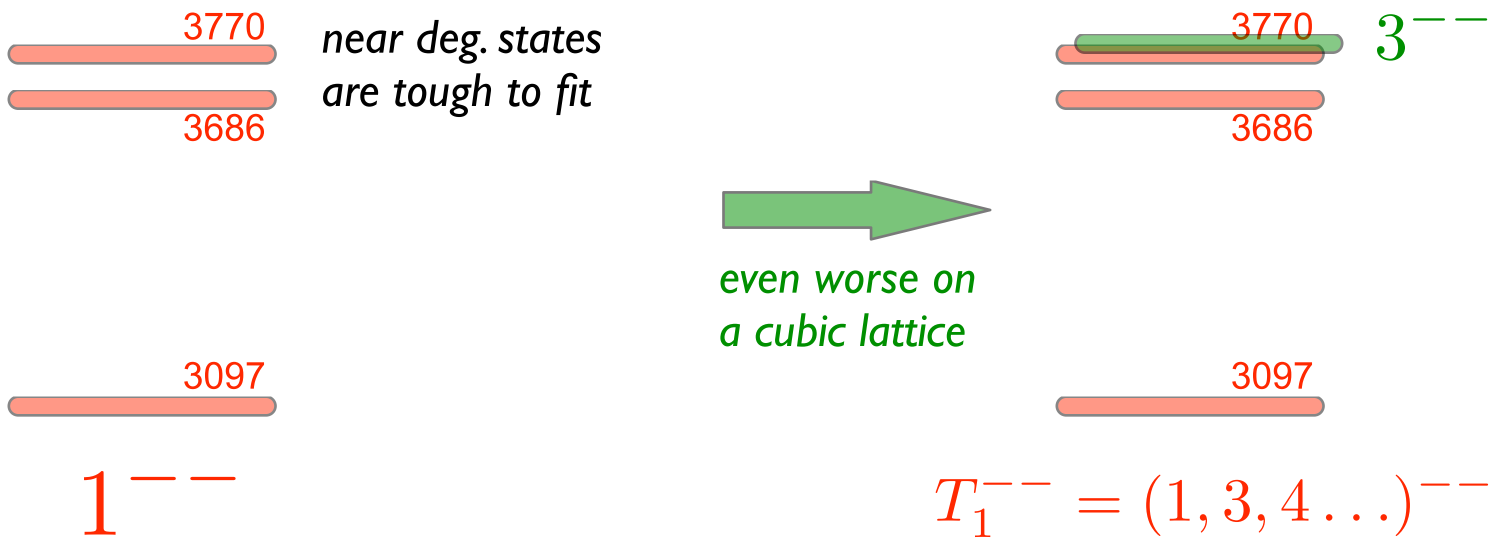
- recall that a two-point correlator is related to the spectrum by

$$C(\vec{0},t) = \sum_N \frac{|Z_N|^2}{2m_N} e^{-m_N t}$$

- hence in principal one can extract information about excited states by fitting *C(t)* as a sum of exponentials
  - this tends to not be very stable, especially on noisy data
  - particularly bad if the state masses are not widely spaced

e.g. excited vector states in charmonium

- very difficult case



- cubic lattice states are not labeled by a spin - instead they take the label of the irreducible representation of the cubic rotation group
  - these contain multiple continuum spins
    - e.g. in two spatial dimensions $\psi_J(\theta) = e^{iJ\theta}$
      - so under the allowed *π/2* rotations, spin *0* & *4* are indistinguishable

variational method

- powerful technique to extract excited states:
  - use a basis of interpolating fields with the same quantum numbers
  - form a matrix of correlators & 'diagonalise'