Radiative Charmonium Physics

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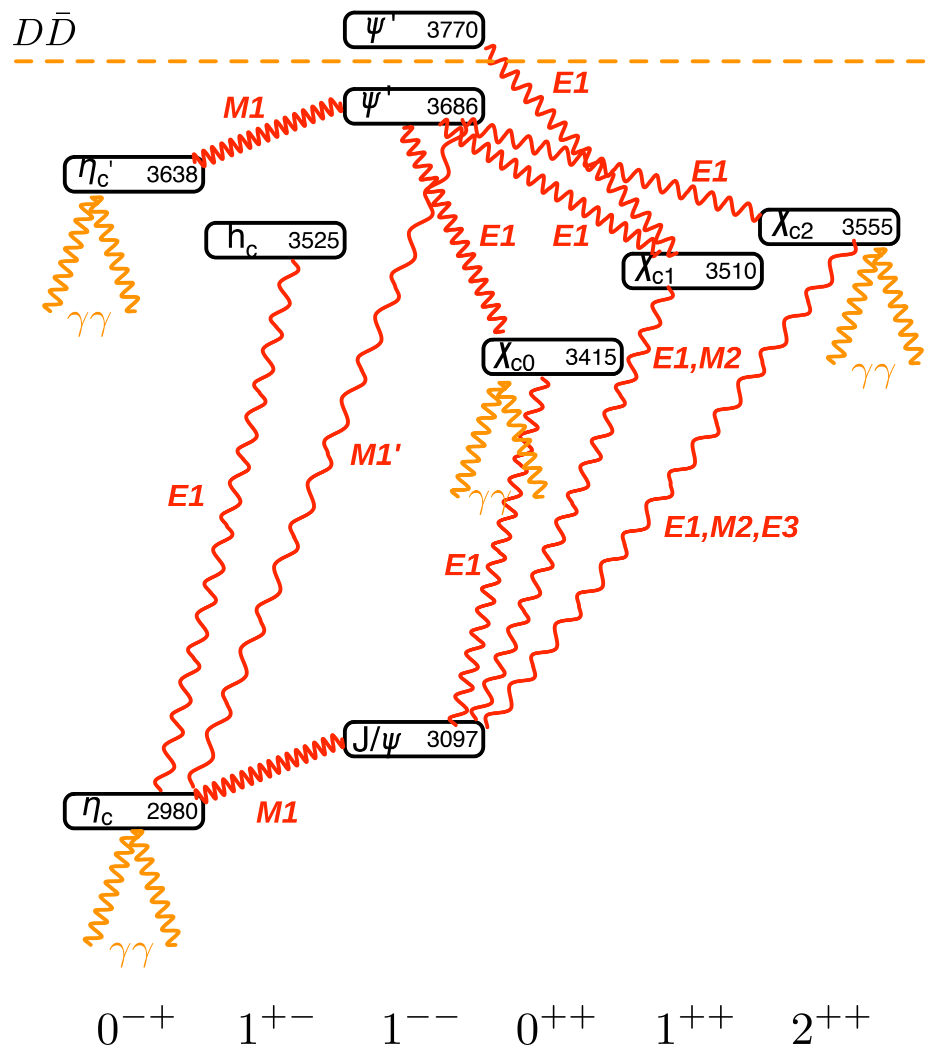
outline

- radiative transitions
  - $(c\bar{c})_1 \to (c\bar{c})_2 \ \gamma$
  - ground state → ground state
- two-photon decays
  - $(c\bar{c})_{[J^{P+}]} \to \gamma\gamma$
  - ground state decays
- excited state transitions
  - more sophisticated two-point fitting
- all quenched, so no precision, but demonstration of new methods

experimental situation

- real photon transitions ($Q^2=0$)
- good data on
  - $\chi_{cJ} \to J/\psi\ \gamma$ E1
  - $\psi(3686) \to \chi_{cJ}\ \gamma$ E1
- some data on
  - $\chi_{cJ} \to J/\psi\ \gamma$ M2, E3
  - $\psi(3686) \to \chi_{cJ}\ \gamma$ M2, E3
- one datum on $J/\psi \to \eta_c\ \gamma$
- new data on $\psi(3770) \to \chi_{cJ}\ \gamma$ E1
- part of the final program at CLEO
- & future program of PANDA @ GSI



previous calculations

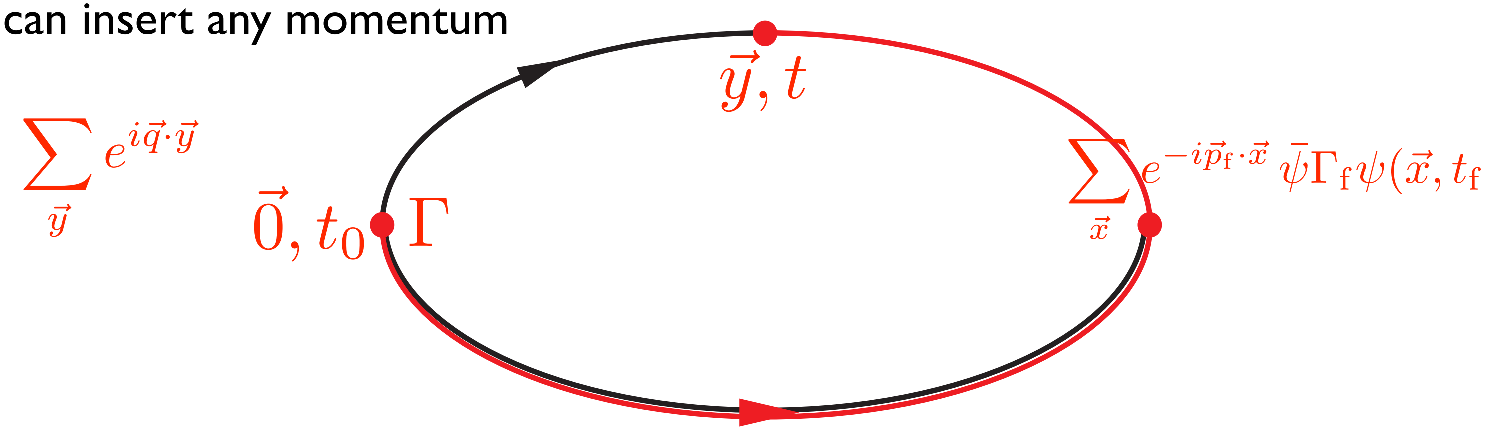
- much studied in potential models
  - solve a Schrödinger (or 'relativised') equation $$-\frac{1}{m_c}\nabla^2\psi + V(r)\psi = E_n\psi$$ $$V(r) = -\frac{\alpha}{r} + br$$
  - use the wavefunctions to compute matrix elements $$\langle\psi|V_i(0)|\chi\rangle \sim \int d^3r\, \psi_\psi^*(\vec{r})(r_i + \ldots)\psi_\chi(\vec{r})$$
  - fairly successful description of data but has limitations, e.g. explicit frame-dependence of matrix elements

- also studied in QCD sum-rules
  - with considerable sensitivity to parameters (m_c in particular)

lattice technique

- fairly straightforward application of three-point correlators
  - similar to pion, proton form-factor, N↔*Δ* ... calculations

  - compute three-point functions with sequential-source technology
    - completely specify the sink (operator & momentum)
    - can insert any momentum



  - obtain correlators at various values of photon Q^2

radiative transitions

- usually expressed in terms of multipoles
- covariant expressions can be derived
  - e.g. $\chi_{c0} \to J/\psi\ \gamma$

$$\langle \chi_{c0}(\vec{p}_\chi)|V^\mu(0)|\psi(\vec{p}_\psi,r)\rangle = \Omega^{-1}(Q^2)\Big(E_1(Q^2)\left[\Omega(Q^2)\epsilon^\mu(\vec{p}_\psi,r) - \epsilon(\vec{p}_\psi,r)\cdot p_\chi\left(p_\chi\cdot p_\psi\ p_\psi^\mu - m_\psi^2\ p_\chi^\mu\right)\right] + \frac{C_1(Q^2)}{\sqrt{Q^2}} m_\psi \epsilon(\vec{p}_\psi,r)\cdot p_\chi\left[p_\chi\cdot p_\psi(p_\chi+p_\psi)^\mu - m_\chi^2\ p_\psi^\mu - m_\psi^2\ p_\chi^\mu\right]\Big)$$

- the multipole form-factors can be obtained from the three-point functions as an overconstrained linear problem
  - need the E's and Z's from two point function fits
  - deals with all the data at a given Q^2 simultaneously - in principle can simultaneously extract excited state transitions

$$\Gamma(p_f,p_i;t) = \sum_n P(p_f,p_i;t)\cdot K_n(p_f,p_i)\cdot f_n(Q^2)$$

$$P = \frac{Z_f Z_i}{4E_f E_i} e^{-E_f t_f} e^{-(E_i-E_f)t}$$

$$\begin{bmatrix}\Gamma(a;t)\\ \Gamma(b;t)\\ \Gamma(c;t)\\ \vdots\end{bmatrix} = \begin{bmatrix} P(a;t)K_1(a) & P(a;t)K_2(a) & \dots \\ P(b;t)K_1(b) & P(b;t)K_2(b) & \\ P(c;t)K_1(c) & P(c;t)K_2(c) & \\ \vdots & & \ddots \end{bmatrix} \begin{bmatrix} f_1(Q^2)[t] \\ f_2(Q^2)[t] \\ \vdots \end{bmatrix}$$

first results

- quenched, anisotropic lattice
- $a_s = 0.1$ fm, $\xi = 3.0$, $12^3 \times 48$
- domain wall fermions ($L_5=16$)
  - *charm quark mass tuning is not perfect (5% low)*

- ground state to ground state transitions only

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Radiative transitions in charmonium from lattice QCD

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$J/\psi \to \eta_c \gamma$ *transition*

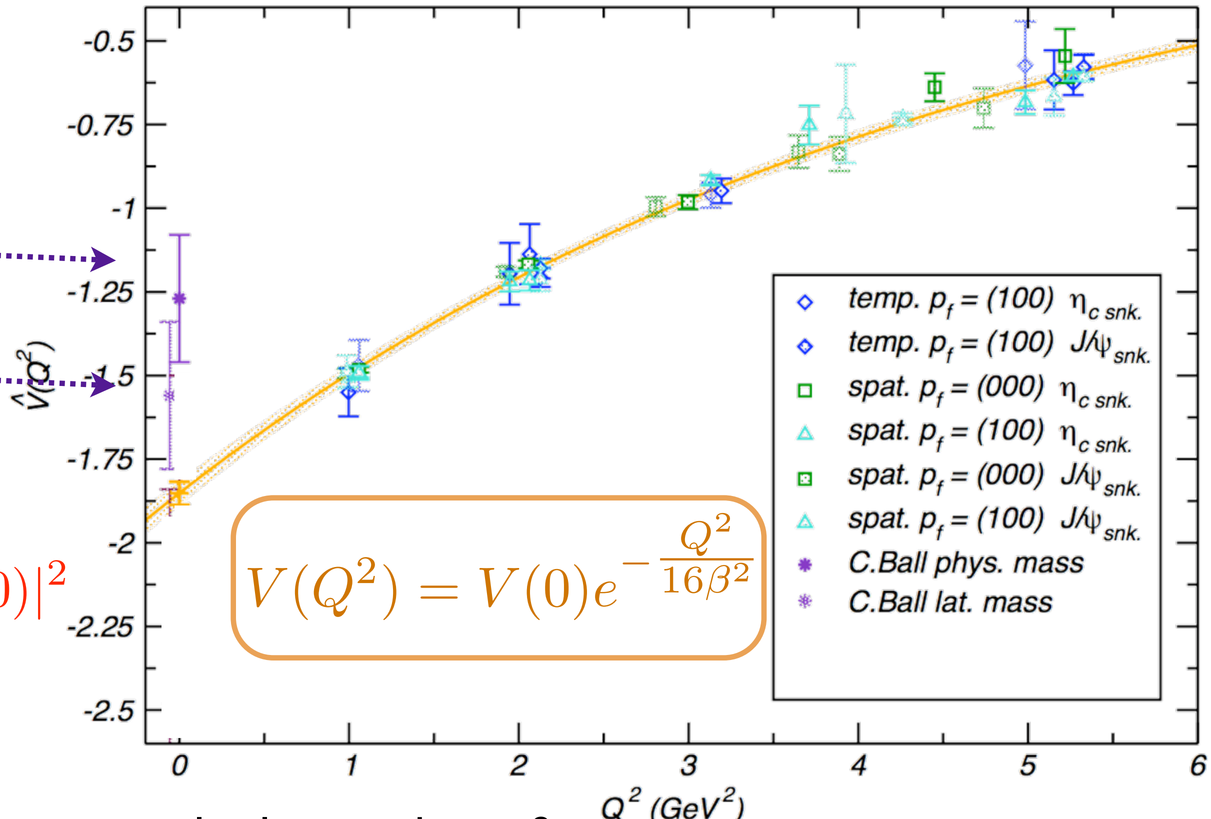
- statistically most precise channel, but very sensitive to the hyperfine splitting which is not correct on this quenched lattice ($\delta m_{lat.} \approx 80$ MeV, $\delta m_{expt.} \approx 117$ MeV)

phase space ($|q|^3$)

physical

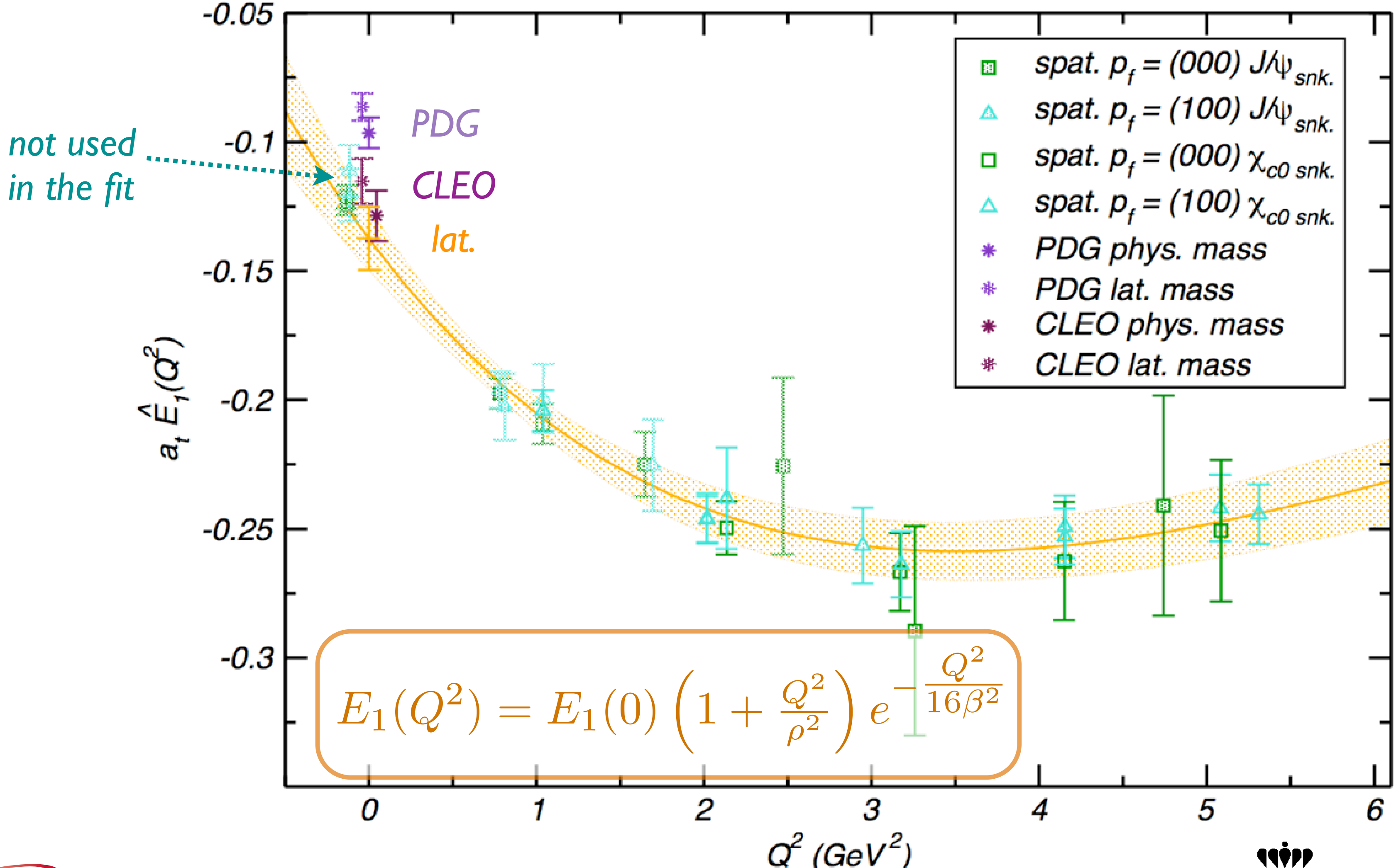
lattice

$$\Gamma(\psi \to \eta_c \gamma) = \alpha \frac{|\vec{q}|^3}{(m_\psi + m_{\eta_c})^2} \frac{64}{27} |\hat{V}(0)|^2$$



- the Crystal Ball experimental value needs confirmation

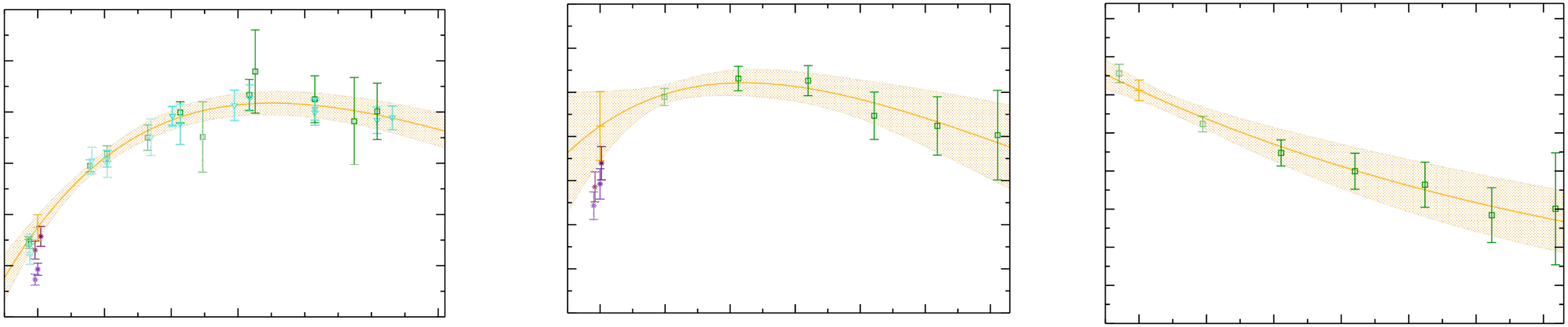
$\chi_{c0} \to J/\psi\ \gamma$ E1 transition



1P→1S transitions

- fit form inspired by potential models with spin-dependent corrections

$$E_1(Q^2) = E_1(0)\left(1 + \frac{Q^2}{\rho^2}\right) e^{-\frac{Q^2}{16\beta^2}}$$



$\chi_{c0} \to J/\psi \gamma_{E1}$	$\chi_{c1} \to J/\psi \gamma_{E1}$	$h_c \to \eta_c \gamma_{E1}$
$\beta = 542(35)$ MeV	$\beta = 555(113)$ MeV	$\beta = 689(133)$ MeV
$\rho = 1.08(13)$ GeV	$\rho = 1.65(59)$ GeV	$\rho \to \infty$

simplest quark model has all β equal and $\rho(\chi_{c0}) = 2\beta$, $\rho(\chi_{c1}) = \sqrt{2} \cdot \rho(\chi_{c0})$, $\rho(h_c) \to \infty$

two-photon decays

- e.g. $\eta_c \to \gamma\gamma$
- not obvious how to get this, no QCD 'interpolating field' for a photon
- from suggestion of *Ji & Jung PRL86, 208*, return to the LSZ reduction for the matrix element: $\langle \gamma(q_1, \lambda_1)\gamma(q_2,\lambda_2)|M(p)\rangle =$

$$- \lim_{\substack{q_1'\to q_1 \\ q_2' \to q_2}} \epsilon^*_\mu(q_1,\lambda_1)\epsilon^*_\nu(q_2,\lambda_2)\; q_1'^2 q_2'^2 \int d^4x d^4y\, e^{iq_1'.y+iq_2'.x}\langle 0|T\{A^\mu(y)A^\nu(x)\}|M(p)\rangle$$

- to first order in QED perturbation theory this is

$$(-e^2) \lim_{\substack{q_1'\to q_1 \\ q_2' \to q_2}} \epsilon^{(1)*}_\mu \epsilon^{(2)*}_\nu q_1'^2 q_2'^2 \int d^4x\, d^4w\, d^4z\, e^{iq_1'.x} D^{\mu\rho}(0,z) D^{\nu\sigma}(x,w)\langle 0|T\{j_\rho(z) j_\sigma(w)\}|M(p)\rangle$$

- using free photon propagators, most of the integrals produce momentum conserving delta functions, with one left over

$$e^2\; \epsilon^{(1)*}_\mu \epsilon^{(2)*}_\nu \int d^4y\, e^{-iq_1.y}\, \langle 0|T\{j^\mu(0) j^\nu(y)\}|M(p)\rangle$$

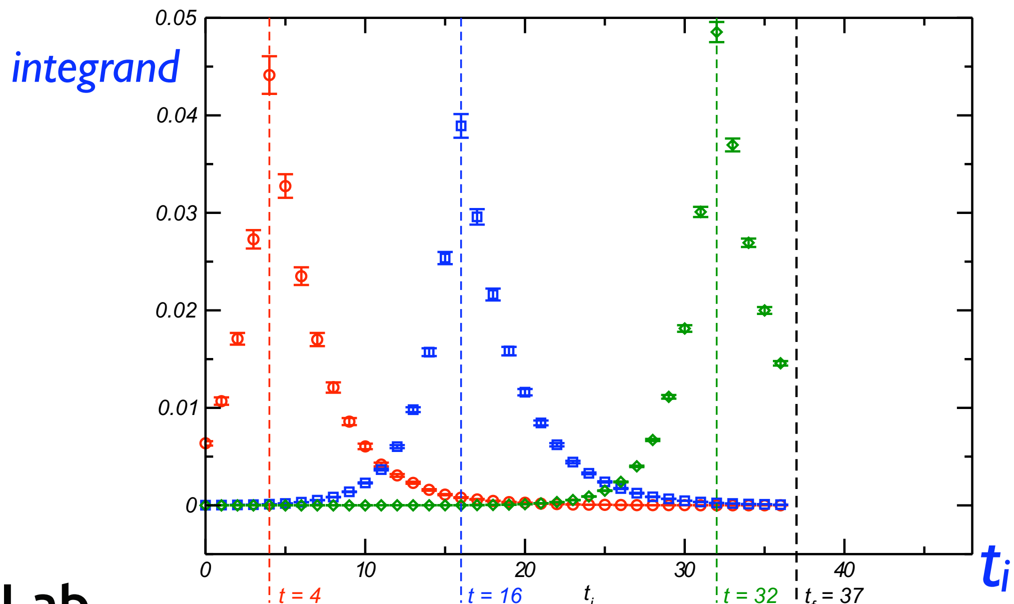
- for photons that are not 'too' time-like this can be rotated into Euclidean spacetime

two-photon decays

$$\lim_{t_f - t \to \infty} e^2 \frac{\epsilon_\mu^{(1)} \epsilon_\nu^{(2)}}{\frac{Z_M(p)}{2E_M(p)} e^{-E_M(p)(t_f - t)}}$$
$$\times \int dt_i e^{-\omega_1 (t_i - t)} \langle 0 | T\{ \int d^3\vec{x}\, e^{-i\vec{p}.\vec{x}} \varphi_M(\vec{x}, t_f) \int d^3\vec{y}\, e^{i\vec{q_2}.\vec{y}} j^\nu(\vec{y}, t) j^\mu(\vec{0}, t_i) \} | 0 \rangle$$

- blue piece is exactly the *V-M* vector current three-point function *(VVM)*
- ‘extra’ time integral sums the QCD vector states in the right way to make a photon of energy ω_1

- we can get the matrix element by computing $VVM(t, t_i)$ for multiple source positions t_i and summing them with an exponential prefactor

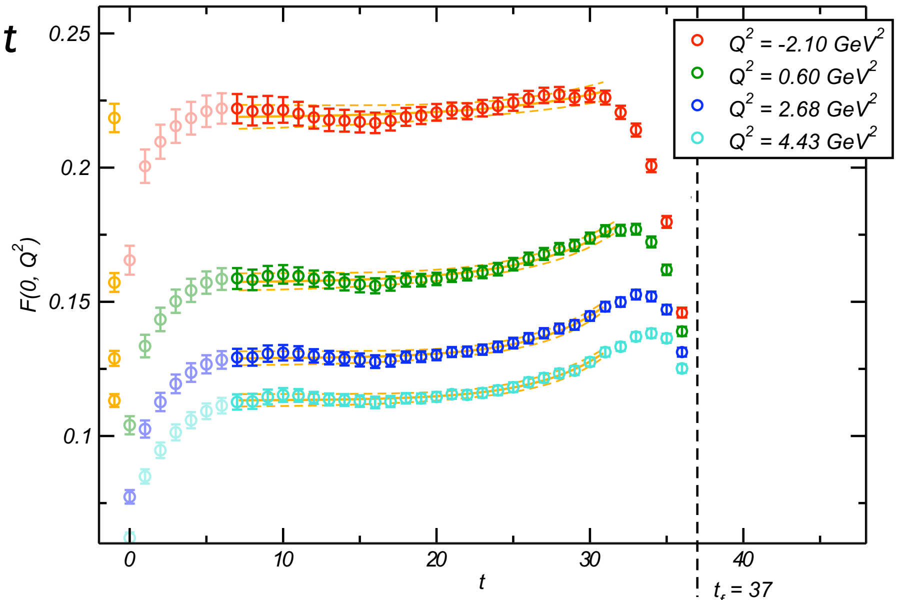


appears to be possible to capture the whole integral for *t* away from the lattice walls

two-photon decays

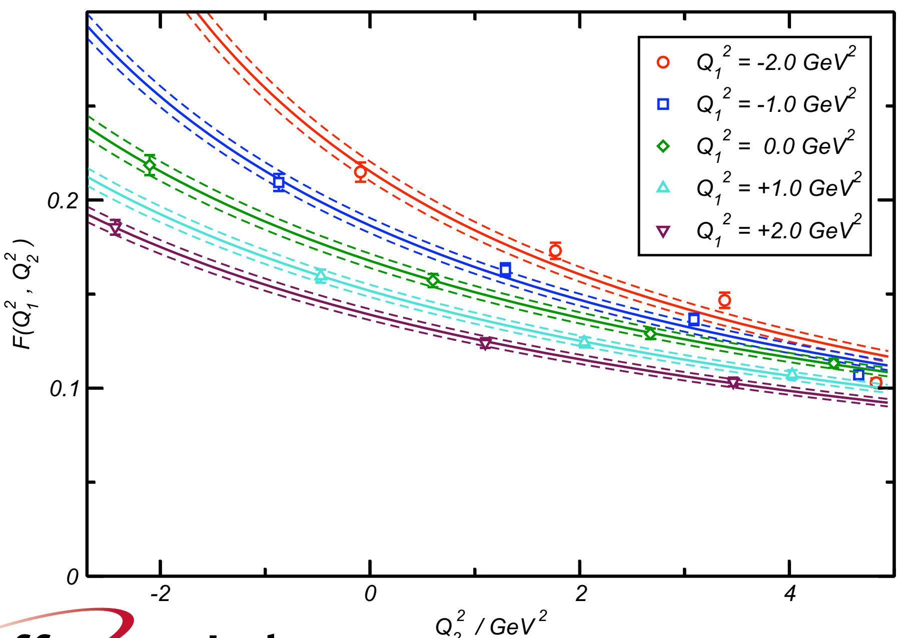
$$\lim_{t_f - t\to\infty} e^2 \frac{\epsilon_\mu^{(1)}\epsilon_\nu^{(2)}}{\frac{Z_M(p)}{2E_M(p)} e^{-E_M(p)(t_f-t)}} \times \int dt_i\, e^{-\omega_1(t_i-t)} \langle 0|T\{ \int d^3\vec{x}\, e^{-i\vec{p}.\vec{x}} \varphi_M(\vec{x},t_f) \int d^3\vec{y}\, e^{i\vec{q}_2.\vec{y}} j^\nu(\vec{y},t) j^\mu(\vec{0},t_i)\}|0\rangle$$

- summing the integrand gives plateaus in t
- done this way one is computing for all possible ω_1 & hence Q_1^2 at once
  - but it is expensive - L_t times the cost of a three-point function



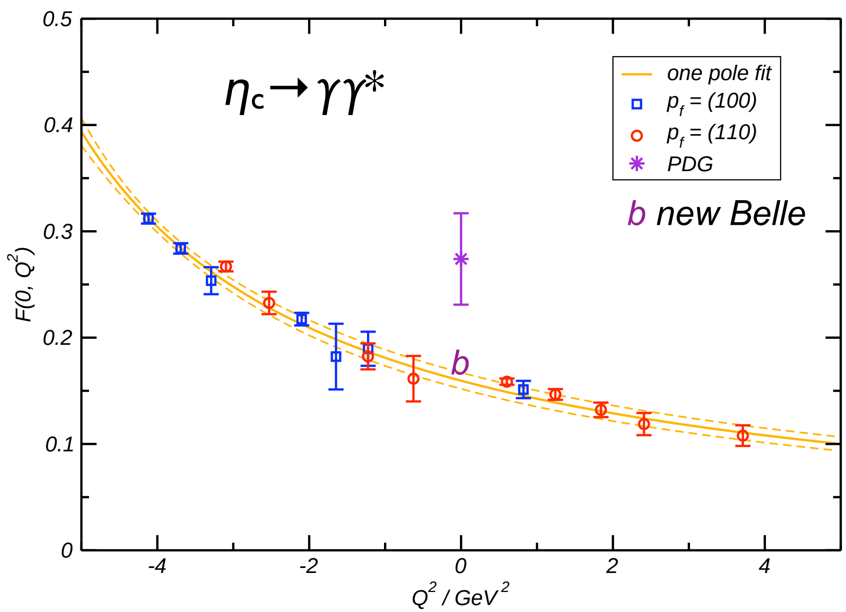
vector current insertion time

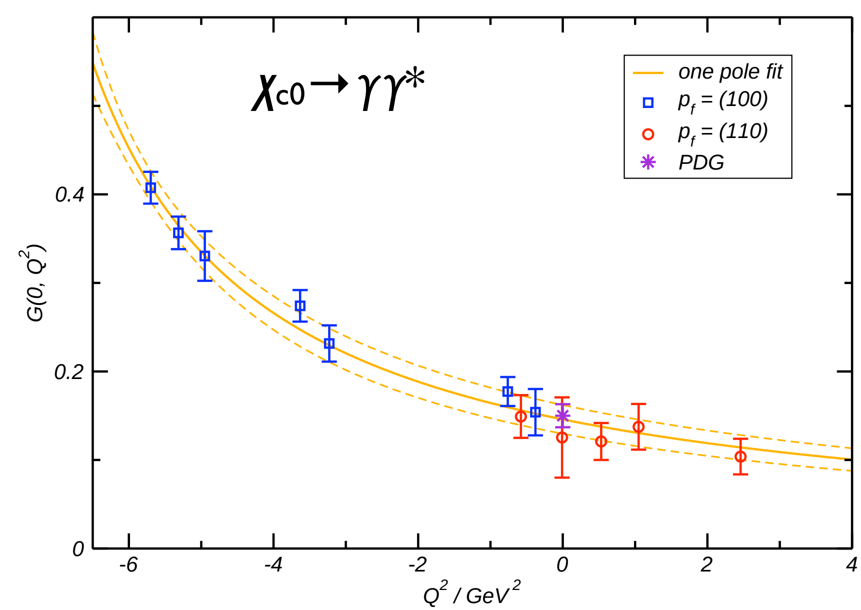
meson source



cheap two-photon decays

- cheaper method puts the timeslice sum in the sequential source
  - but must specify ω_1 for each calculation & can't view integrand





in this case the calculation done with clover quarks on quenched isotropic lattices, a=0.047 fm, 24^3x48

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Two-Photon Decays of Charmonia from Lattice QCD

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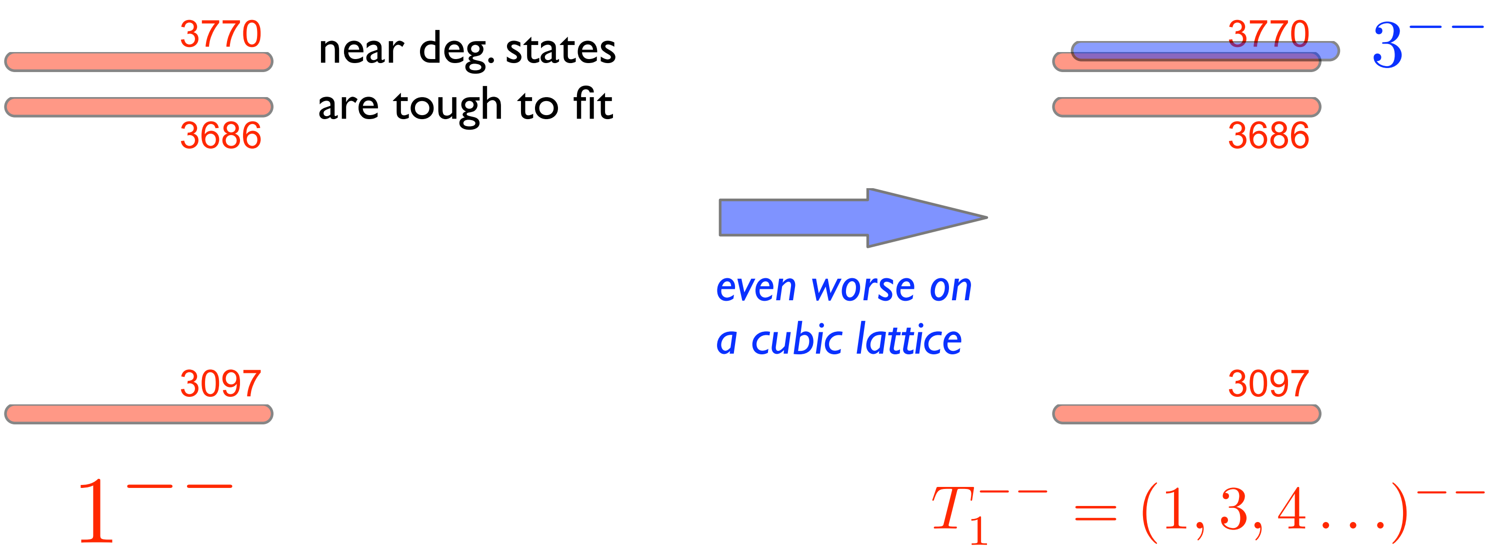
excited states

- recall there is good experimental data on $\psi(3686) \rightarrow \chi_{cJ}\,\gamma$
- the $\psi(3686)$ is an excited state in the vector channel:



- need a reliable excited state extraction procedure
  - variational method in a large operator basis?

excited states

- first results are promising

$\bar{\psi}\,\Gamma\,\psi$

$\bar{\psi}\,\Gamma\overleftrightarrow{D}_k\,\psi$

$\bar{\psi}\,\Gamma\overleftrightarrow{D}_j\overleftrightarrow{D}_k\,\psi$

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Charmonium excited state spectrum in lattice QCD

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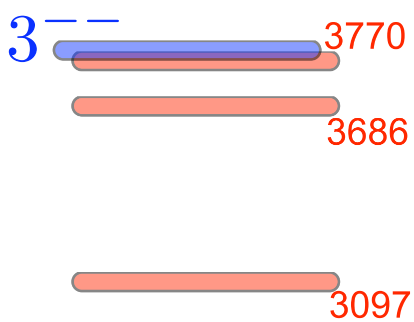
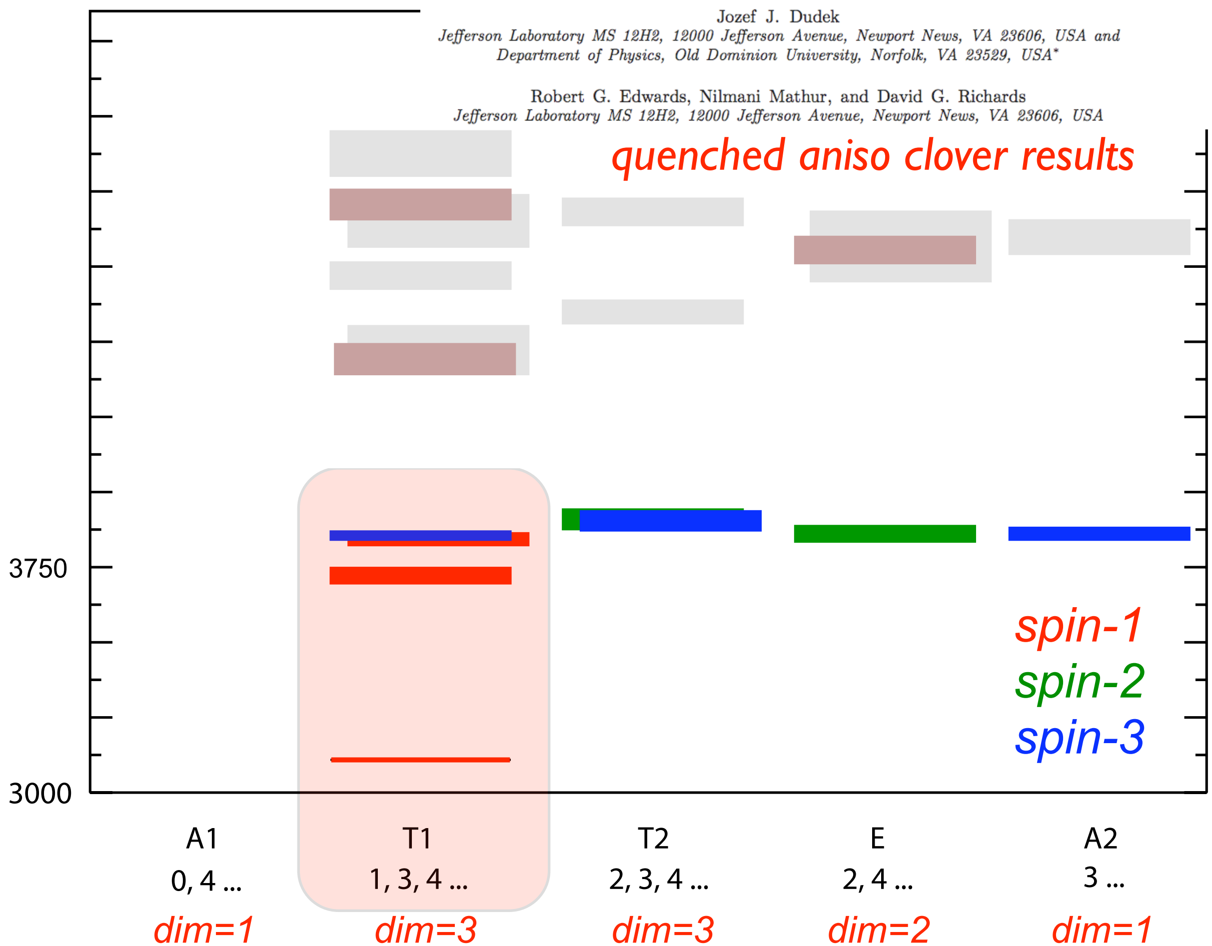




$a \to 0$

$\langle 0|(\bar{\psi}\,\Gamma\overleftrightarrow{D}_k\,\psi)_{T_2}|J\rangle = Z_J \cdot K^J_{T_2}$

$\langle 0|(\bar{\psi}\,\Gamma\overleftrightarrow{D}_k\,\psi)_{E}|J\rangle = Z_J \cdot K^J_{E}$

charmonium radiative physics

- first calculations of transitions and two-photon decays are promising
  - methods seem to be sound...
- ... but
  - small volumes *(may not be important)*
  - quenched
  - limited *ma* improvement

- relevant to final CLEO analyses
  - new $J/\psi \to \eta_c \gamma$ number coming soon
  - angular analysis of $\chi_{cJ} \to J/\psi \gamma$ *(M2, E3)*

- methods for excited state transitions being investigated
  - variational methods seem to be well constrained in spectrum case

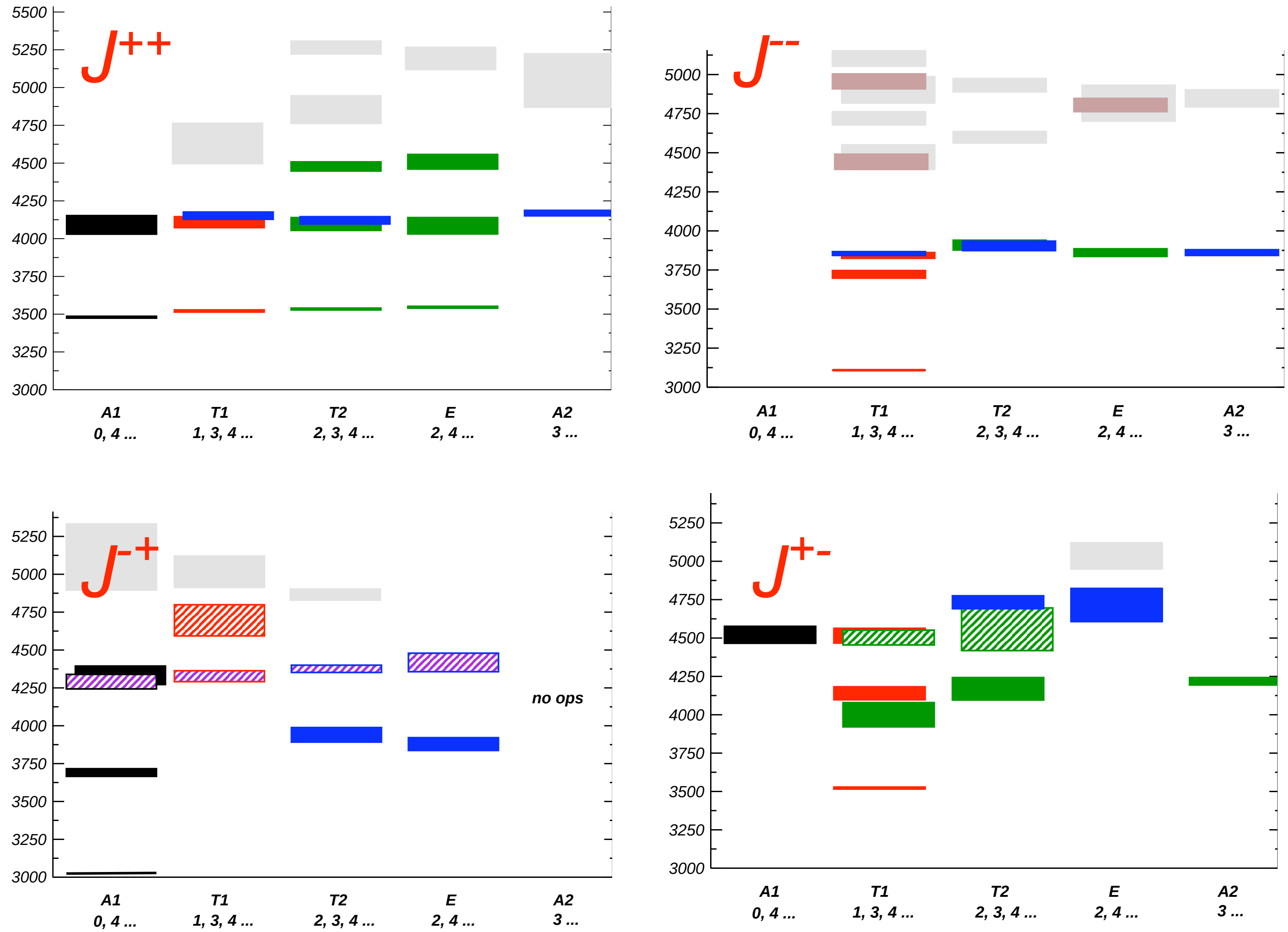
- anisotropy is a powerful tool for excited state extraction
  - anisotropic dynamical $N_F=2+1$ clover lattices being generated under USQCD

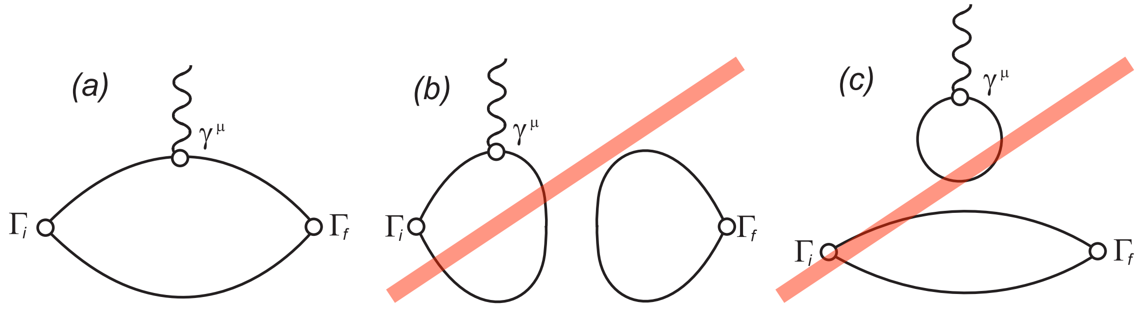
variational method results

$\bar{\psi}\,\Gamma\psi$

$\bar{\psi}\,\Gamma\overleftrightarrow{D}_k\,\psi$

$\bar{\psi}\,\Gamma\overleftrightarrow{D}_j\overleftrightarrow{D}_k\,\psi$




(a)
(b)
(c)

$\chi_{c1} \to J/\psi\, \gamma$ *transition*

- derived the covariant multipole decomposition

$$\langle A(\vec{p}_A, r_A)|j^\mu(0)|V(\vec{p}_V, r_V)\rangle = \frac{i}{4\sqrt{2}\Omega(Q^2)} \epsilon^{\mu\nu\rho\sigma}(p_A - p_V)_\sigma \times$$

$$\times \Bigg[E_1(Q^2)(p_A + p_V)_\rho \Big(2m_A[\epsilon^*(\vec{p_A}, r_A).p_V]\epsilon_\nu(\vec{p_V}, r_V) + 2m_V[\epsilon(\vec{p}_V, r_V).p_A]\epsilon^*_\nu(\vec{p}_A, r_A) \Big)$$

$$+ M_2(Q^2)(p_A + p_V)_\rho \Big(2m_A[\epsilon^*(\vec{p_A}, r_A).p_V]\epsilon_\nu(\vec{p_V}, r_V) - 2m_V[\epsilon(\vec{p}_V, r_V).p_A]\epsilon^*_\nu(\vec{p}_A, r_A) \Big)$$

$$+ \frac{C1(Q^2)}{\sqrt{q^2}} \Big(-4\Omega(Q^2)\epsilon^*_\nu(\vec{p}_A, r_A)\epsilon_\rho(\vec{p}_V, r_V)$$

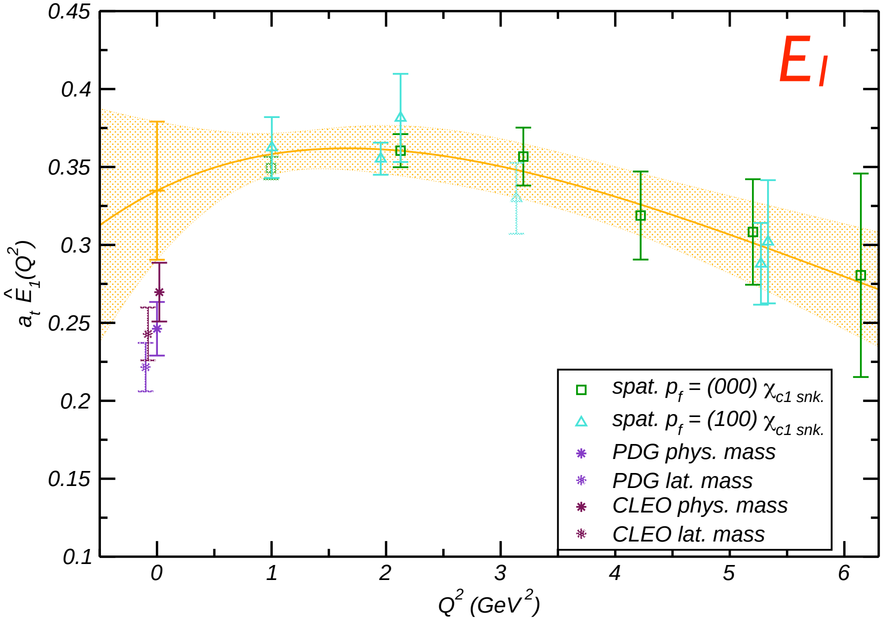
$$+ (p_A + p_V)_\rho \Big[(m_A^2 - m_V^2 + q^2)[\epsilon^*(\vec{p}_A, r_A).p_V]\,\epsilon_\nu(\vec{p}_V, r_V) + (m_A^2 - m_V^2 - q^2)[\epsilon(\vec{p}_V, r_V).p_A]\,\epsilon^*_\nu(\vec{p_A}, r_A) \Big] \Big) \Bigg].$$

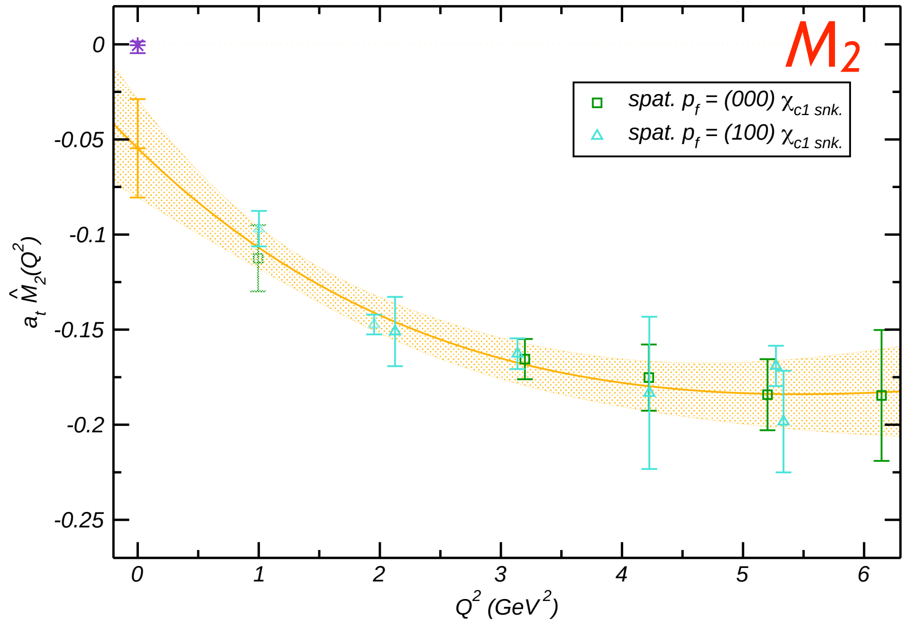
- $E_1(Q^2)$ - *electric dipole* - experimentally measured at $Q^2 = 0$
- $M_2(Q^2)$ - *magnetic quadrupole* - experimentally measured (via photon angular dependence) at $Q^2 = 0$
- $C_1(Q^2)$ - *longitudinal* - goes to zero at $Q^2 = 0$

- this lattice $\delta m(\chi_{c1} - J/\psi)$ close to experiment, so small phase-space ambiguity

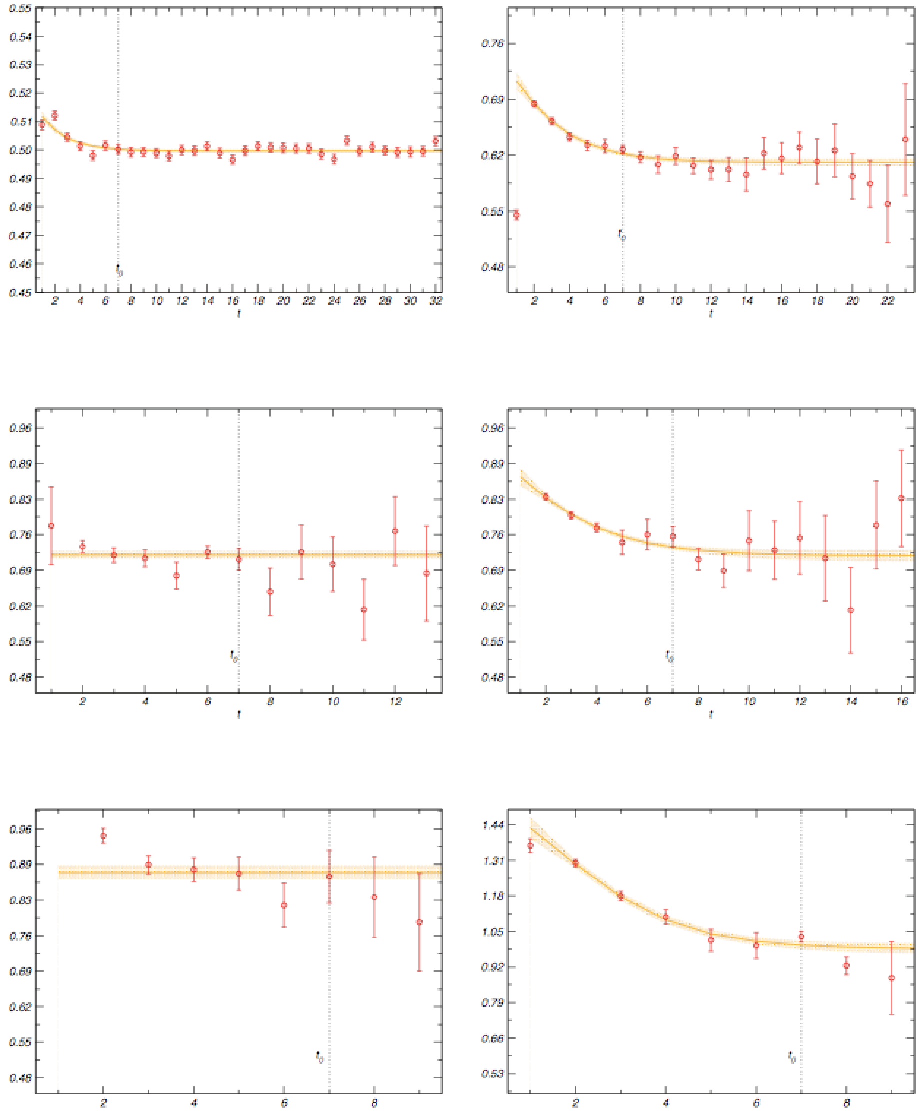
$\chi_{c1} \to J/\psi\,\gamma$ transition

- no $Q^2 < 0$ points owing to kinematical structure of matrix element





principal effective masses



reconstructed diagonal correlators