Excited light isoscalar mesons from lattice QCD

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With Jo Dudek, Robert Edwards, Mike Peardon, Bálint Joó, David Richards and the *Hadron Spectrum Collaboration*

Jefferson Lab

Outline

- Introduction
- Method outline
- Results – isoscalar meson spectrum
- Summary and outlook

PR D83, 111502 (2011)

Motivation

Upcoming experimental efforts in light meson sector (and charmonium)

GlueX and CLAS12 (JLab), BESIII, PANDA, ...

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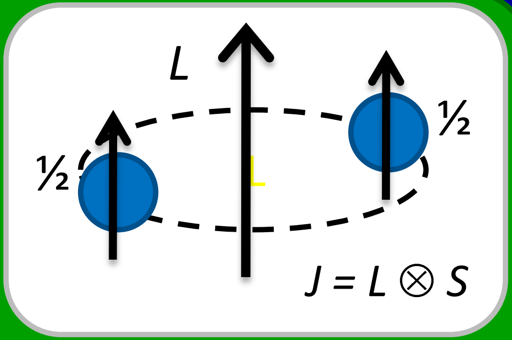
Quark-antiquark pair: $^{2S+1}L_J$

Parity: $P = (-1)^{(L+1)}$

Charge Conj Sym: $C = (-1)^{(L+S)}$



$J^{PC} = 0^{-+}, 0^{++}, 1^{--}, 1^{++}, 1^{+-}, 2^{--}, 2^{++}, 2^{-+}, \ldots$

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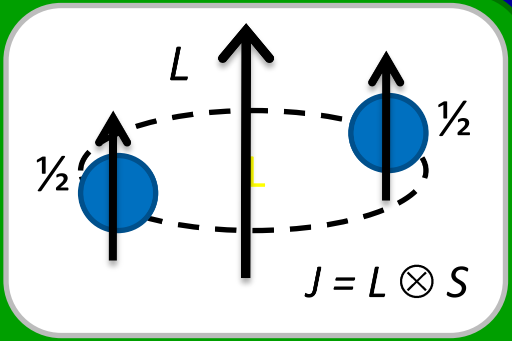

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Exotics ($J^{PC} = \mathbf{1}^{-+}, \mathbf{2}^{+-}, \ldots$)? – can't just be a $q\bar{q}$ pair

Probe low energy d.o.f. of QCD

e.g. hybrids, multi-mesons

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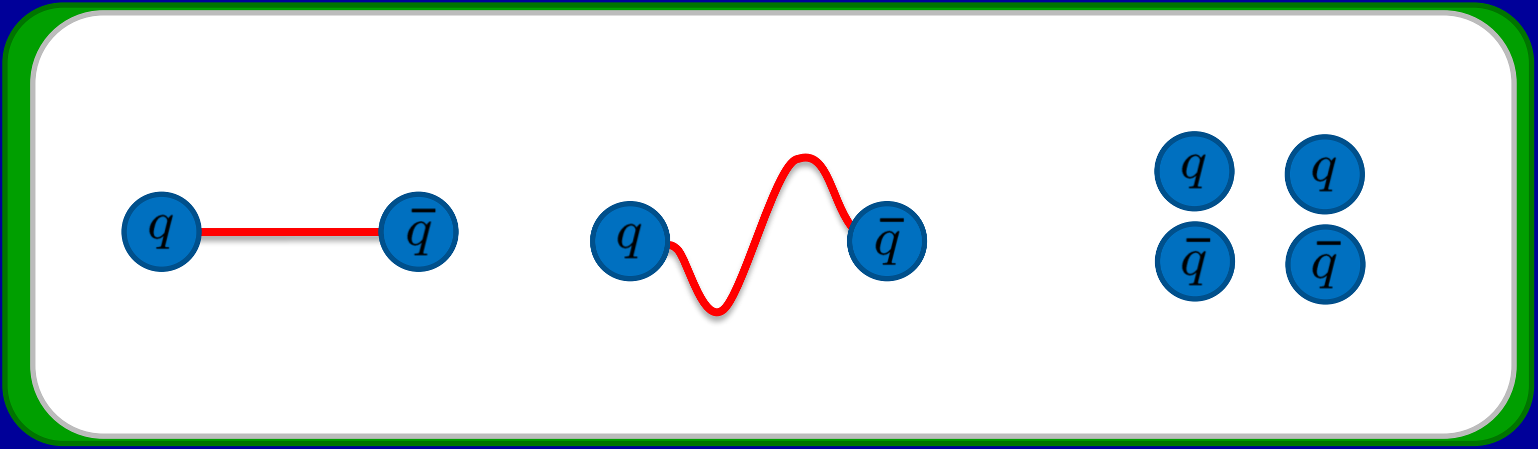

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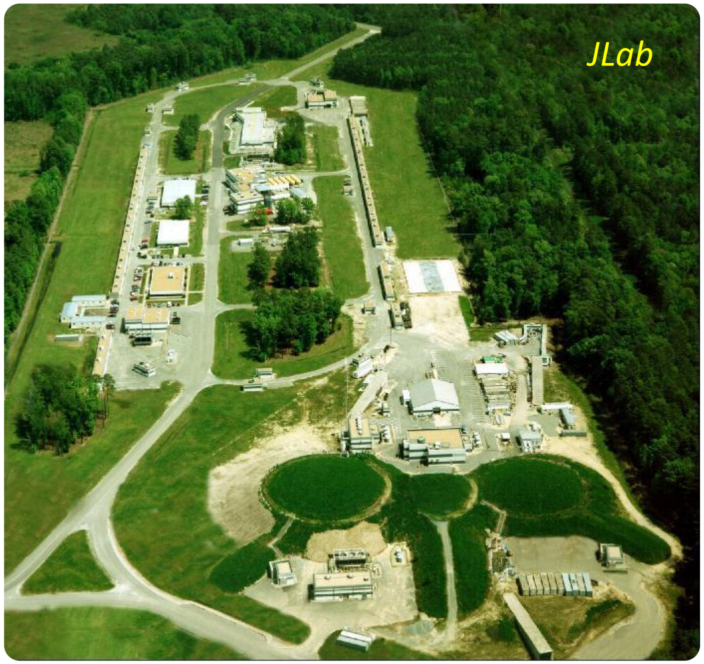
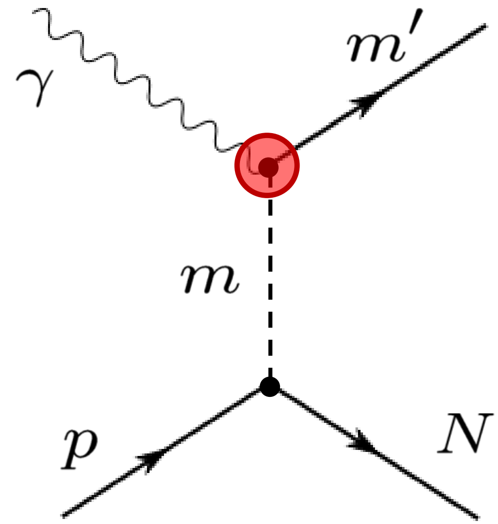
Up

...





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Photoproduction at GlueX/CLAS12
- systematic study of light mesons, particular interest in exotics

Isoscalars

Isoscalars (I = 0) e.g. η, η', ω, ϕ

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Probe annihilation dynamics in QCD

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$m_s = m_u = m_d$ [SU(3) sym]
- eigenstates are octet, singlet

$$\frac{1}{\sqrt{6}}\left(u\bar{u} + d\bar{d} - 2s\bar{s}\right)$$

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In general

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Experimentally

ω, ϕ (1^{--}) and $f_2(1270)$, $f_2'(1525)$ (2^{++}) – close to 'ideal'

η, η' (0^{-+}) – closer to octet-singlet

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Can also mix with glueballs

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Variational Method

$$C_{ij}(t) = \left\langle 0 \left| O_i(t) O_j(0) \right| 0 \right\rangle$$

definite J^{PC}

$$O(t) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \; \bar{\psi}(x) \Gamma_i \overleftrightarrow{D}_j \overleftrightarrow{D}_k \ldots \psi(x)$$

Here up to 3 derivs with **p** = **0**

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Large basis of operators → matrix of correlators

$$C_{ij}(t) v_j^{(n)} = \lambda^{(n)}(t) C_{ij}(t_0) v_j^{(n)}$$

generalised eigenvector problem

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Eigenvalues → energies

$$\lambda^{(n)}(t) \to e^{-E_n(t-t_0)}$$

($t >> t_0$)

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Var. method uses orthog of eigenvectors; don't just rely on separating energies

Isoscalars in LQCD

Use variational method with large basis of operators

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Basis doubled in size c.f. isovectors:

$$O^{\ell} \sim \frac{1}{\sqrt{2}}\left(\bar{u}\mathbf{\Gamma}u + \bar{d}\mathbf{\Gamma}d\right)$$

$$O^{s} \sim \bar{s}\mathbf{\Gamma}s$$

No glueball ops for now

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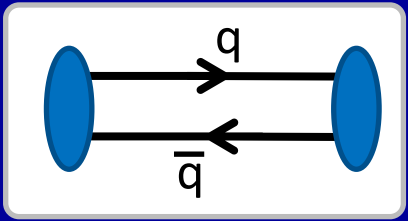
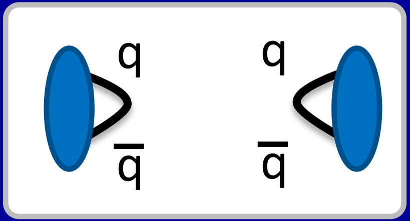
Disconnected diagrams





$$C^{q'q}_{AB}(t',t) = \langle 0|\mathcal{O}^{q'}_A(t')\mathcal{O}^{q\dagger}_B(t)|0\rangle$$

$$C = \begin{pmatrix} -\mathcal{C}^{\ell\ell} + 2\,\mathcal{D}^{\ell\ell} & \sqrt{2}\,\mathcal{D}^{\ell s} \\ \sqrt{2}\,\mathcal{D}^{s\ell} & -\mathcal{C}^{ss} + \mathcal{D}^{ss} \end{pmatrix}$$

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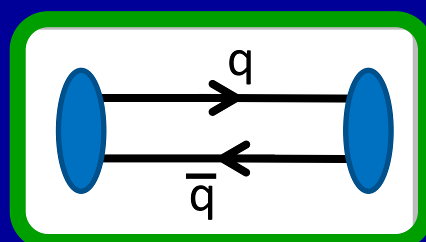

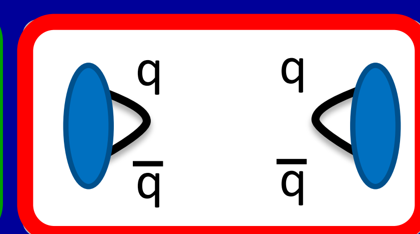

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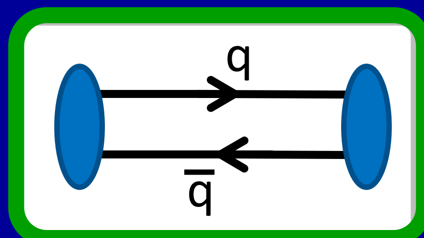

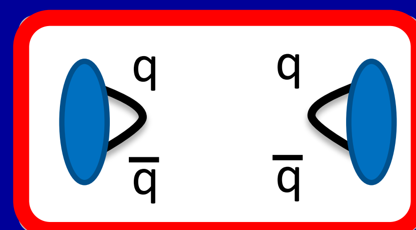

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Difficulties with traditional methods

Distillation

PR D80, 054506

Distillation

$$\Box_{xy}(t) = \sum_{k=1}^{N} v_x^{(k)}(t) v_y^{(k)\dagger}(t)$$

$$\Box(t) = V(t)V^\dagger(t)$$

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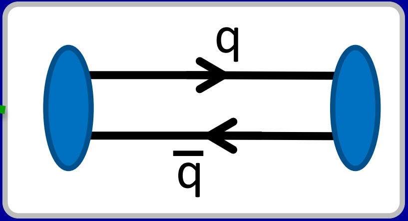
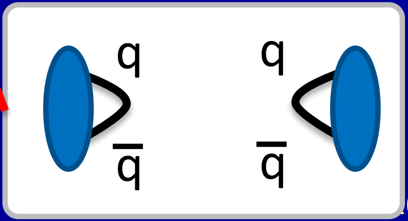
$$\mathcal{C}_{AB}^{q'q}(t',t) = \delta_{qq'} \mathrm{Tr}[\Phi^A(t') \tau_{q'}(t',t) \Phi^B(t) \tau_q(t,t')]$$



$$\mathcal{D}_{AB}^{q'q}(t',t) = \mathrm{Tr}[\Phi^A(t') \tau_{q'}(t',t')]\ \mathrm{Tr}[\Phi^B(t) \tau_q(t,t)]$$



$$\tau_q(t',t) = V_{t'}^\dagger M_q^{-1}(t',t) V_t$$

$$\Phi^A(t) = V_t^\dagger \mathbf{\Gamma}_t^A V_t$$

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PR D80, 054506

Number of inversions ~ N_{vecs} instead of (N_{colours} x L_s^3)

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Project onto **definite mom.** at source and sink

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N_t=32, L_t=128

$$\mathcal{C}^{qq}(t) = \frac{1}{N_t}\sum_{t_0}^{N_t} \mathcal{C}^{qq}(t+t_0,t_0)$$

$$\mathcal{D}^{q'q}(t) = \frac{1}{L_t}\sum_{t_0}^{L_t} \mathcal{D}^{q'q}(t+t_0,t_0)$$

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Anisotropic Clover (a_s/a_t = 3.5), a_s ~ 0.12 fm; 16^3 (2.0 fm)

N_f = 2+1, $M_\pi \approx$ 400 MeV

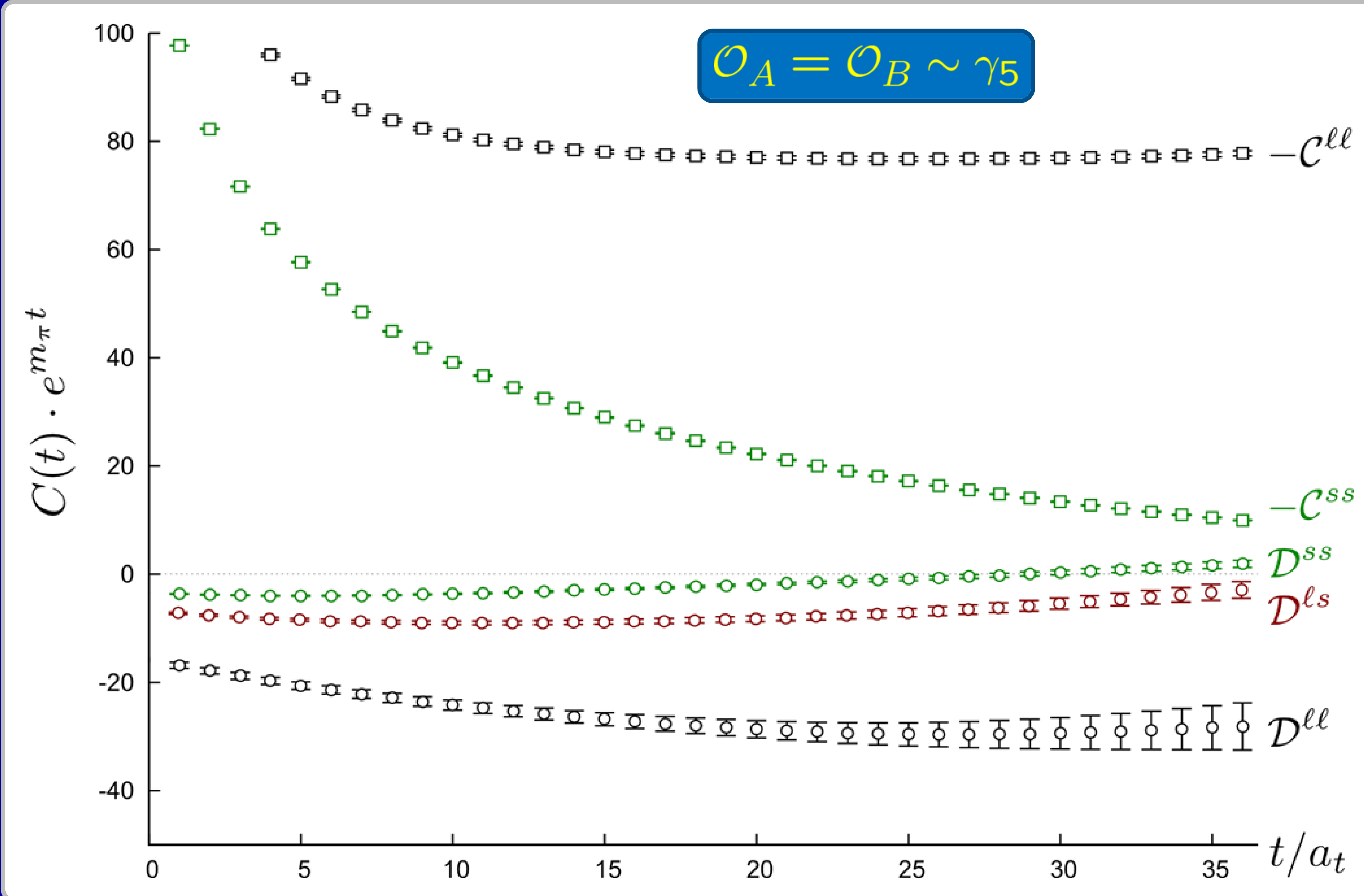
N_{vecs} = 64

479 cfgs

Lattice details in:
PR D78 054501,
PR D79 034502

Correlators

PR D83, 111502 (2011)



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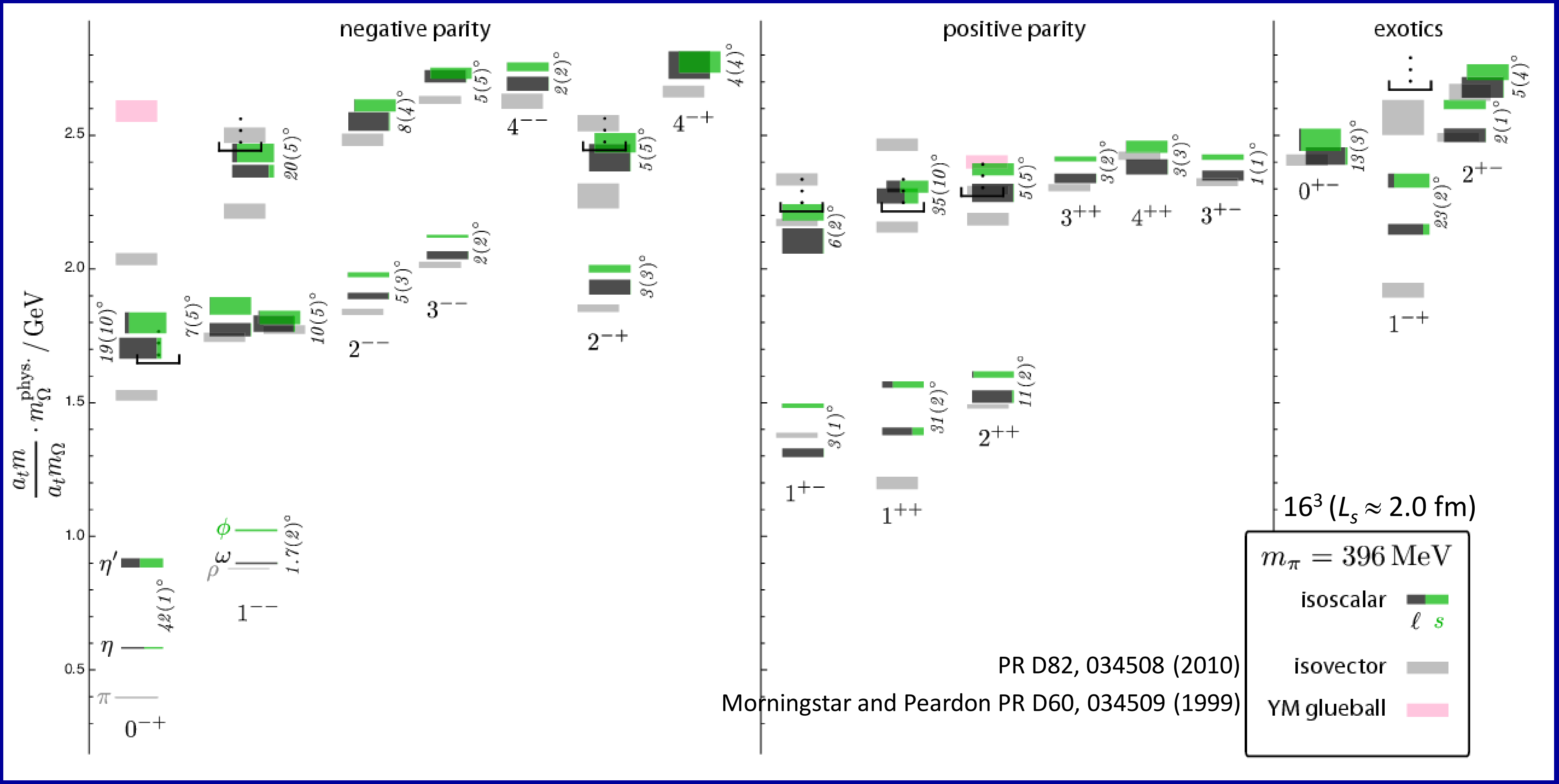

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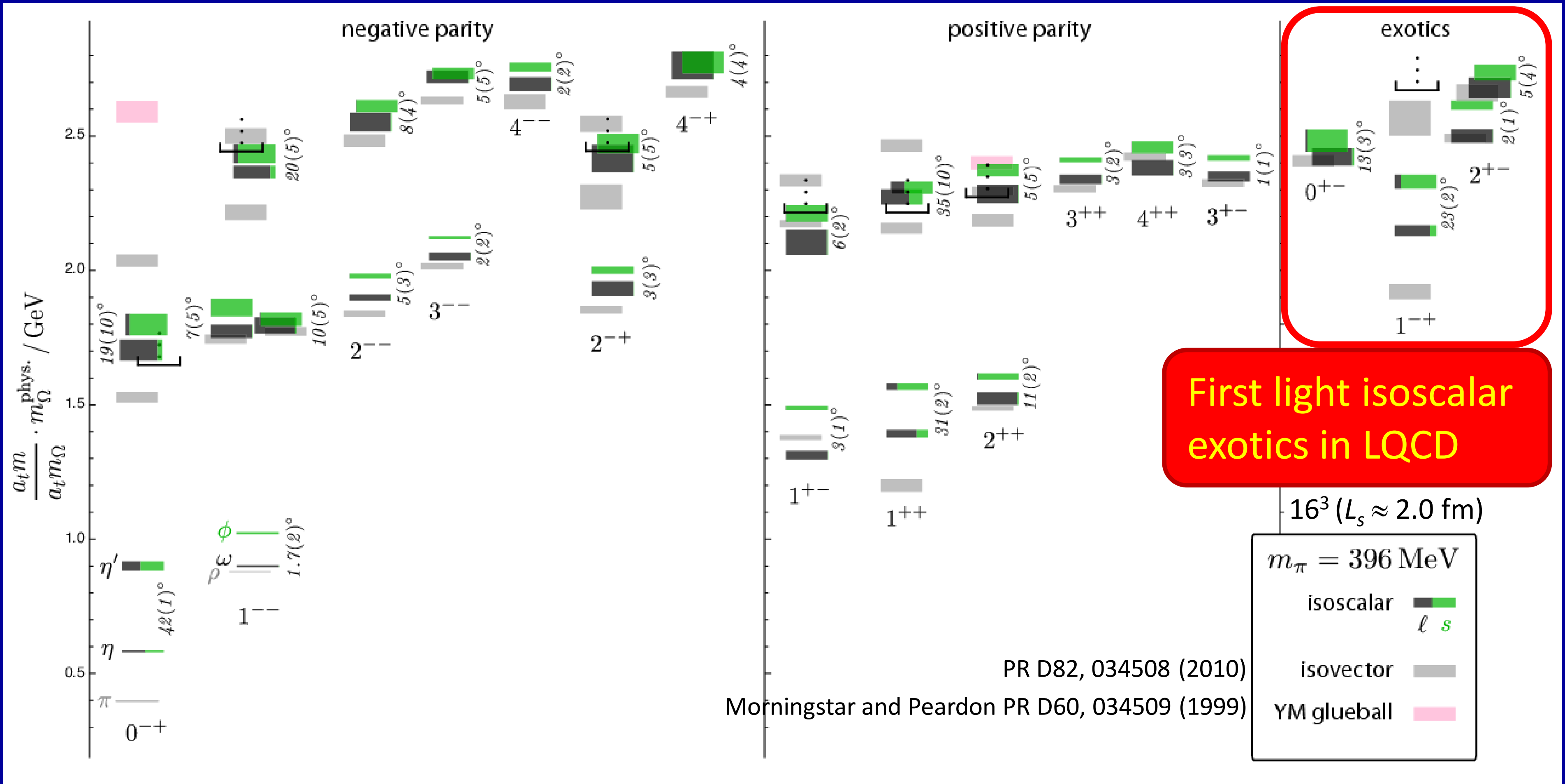

First light isoscalar exotics in LQCD

Flavour Mixings

Determine mixing of physical states by looking at overlaps

$$O^{\ell} \sim \frac{1}{\sqrt{2}}\left(\bar{u}\Gamma u + \bar{d}\Gamma d\right)$$

$$O^{s} \sim \bar{s}\Gamma s$$

$$Z_{A,\ell}^{(n)} = \left\langle 0 \left| O_A^{\ell} \right| n \right\rangle$$

$$Z_{A,s}^{(n)} = \langle 0 \left| O_A^{s} \right| n \rangle$$

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$$Z^{(n)}_{A,\ell} = \left\langle 0 \left| O^{\ell}_{A} \right| n \right\rangle$$

$$Z^{(n)}_{A,s} = \langle 0 \, | O^{s}_{A} | \, n \rangle$$

$$\alpha_A = \tan^{-1}\left[\sqrt{-\frac{Z^{(b)}_{\ell,A} \, Z^{(a)}_{s,A}}{Z^{(a)}_{\ell,A} \, Z^{(b)}_{s,A}}}\right]$$

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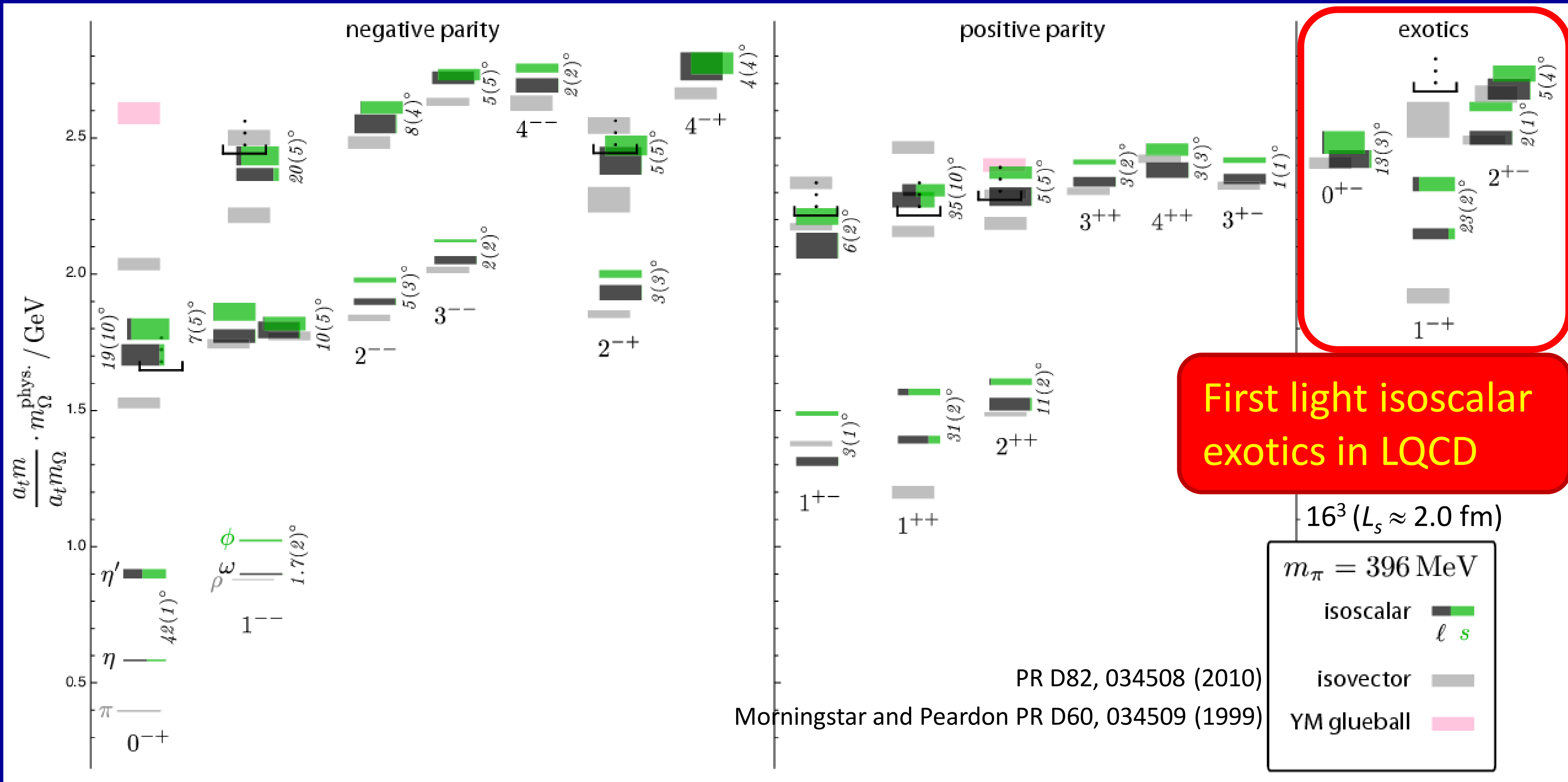

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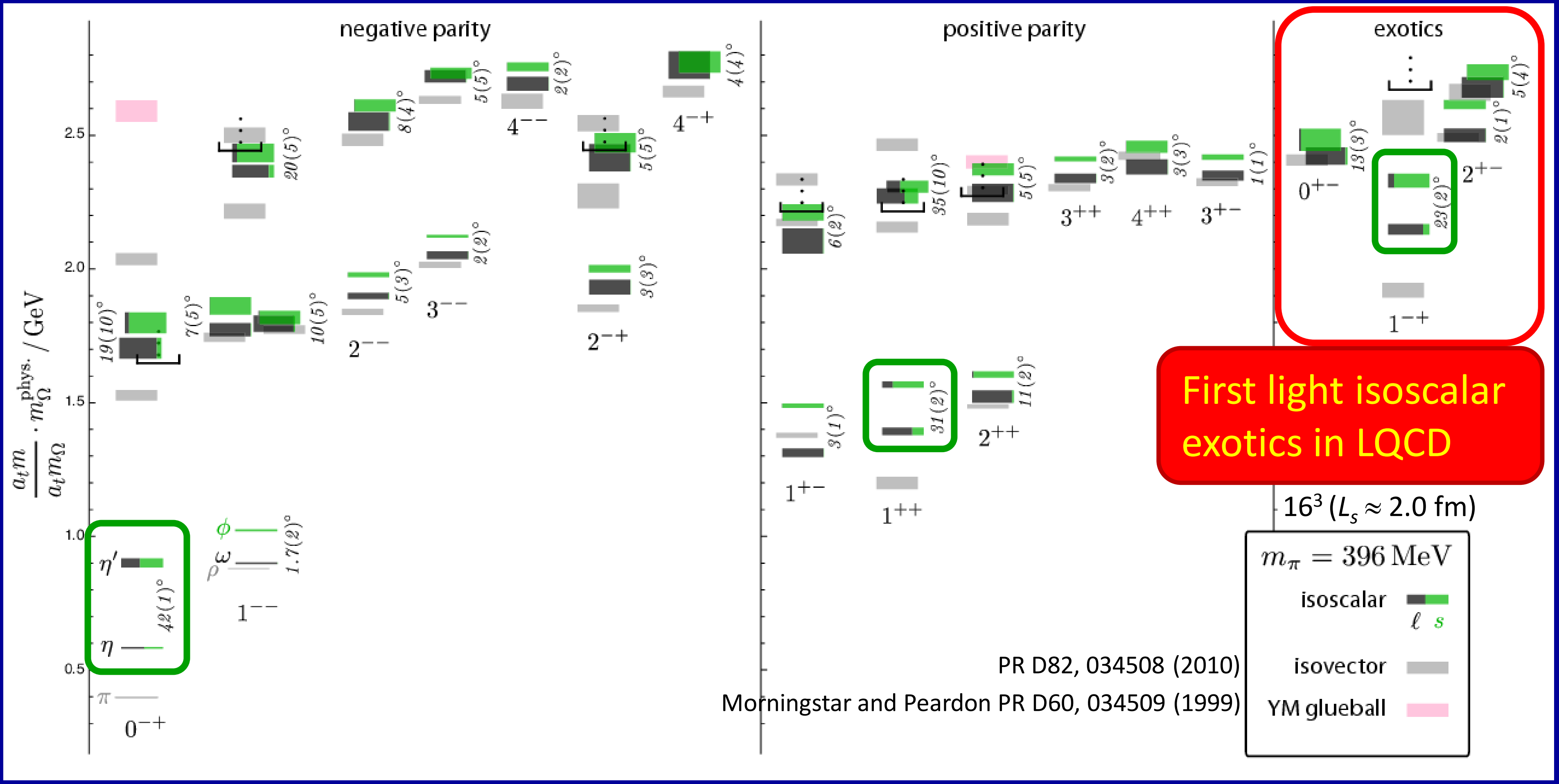
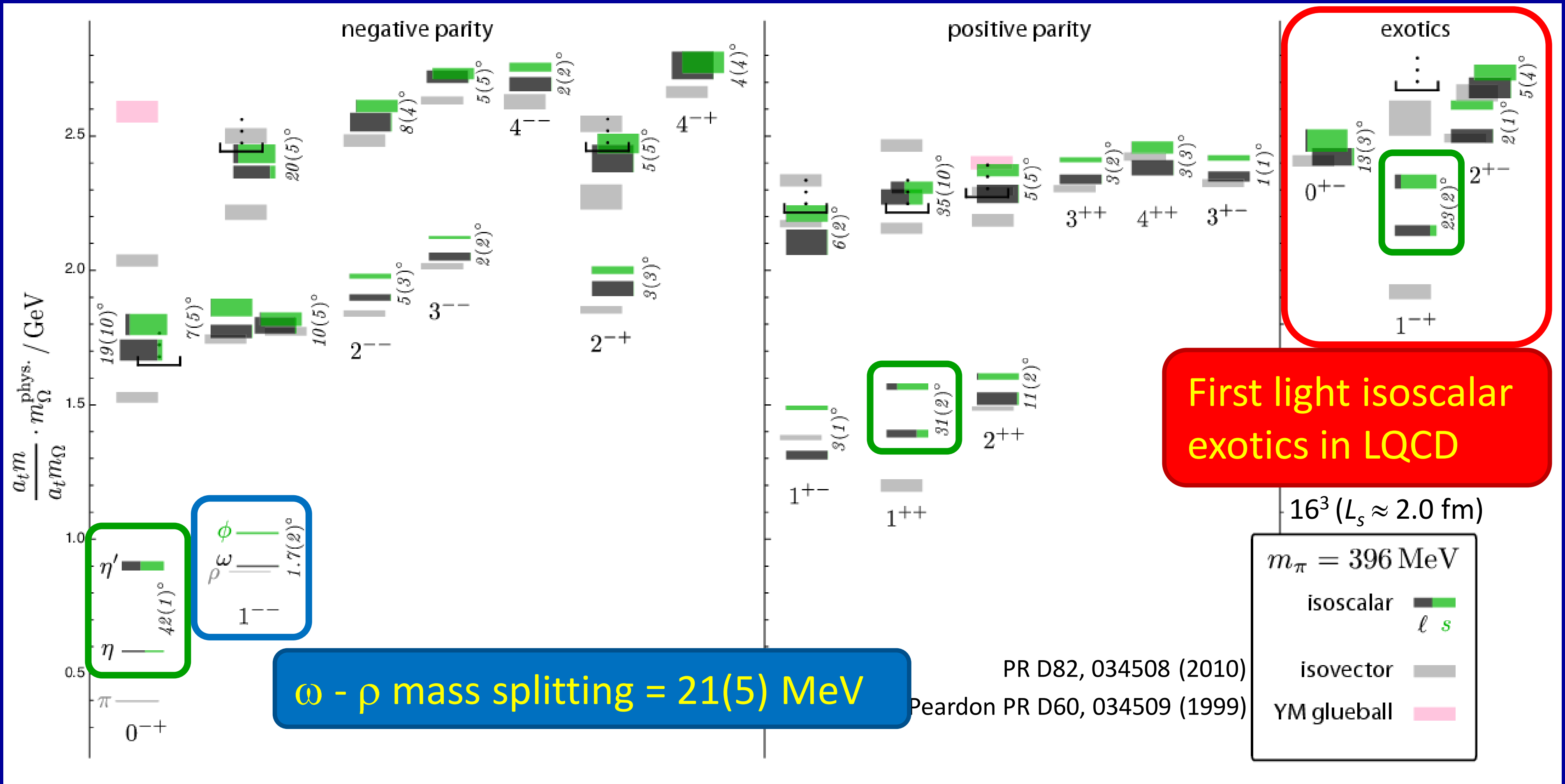


Most close to ideally mixed

But η - η' = 42(1)°, $f_1 - f_1'$ = 31(2)°, 1^{-+} exotics

Isoscalars

PR D83, 111502 (2011)



First light isoscalar exotics in LQCD

ω - ρ mass splitting = 21(5) MeV

Most close to ideally mixed

But η - η' = 42(1)° , $f_1 - f_1'$ = 31(2)°, 1^{-+} exotics

Summary and Outlook

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- **Exotics** and non-exotic **hybrids**, high spin, excited states
- Carefully constructed ops – **spin-identification**
- Mixing angles
- Using distillation, high stats, large operator basis, GPUs

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Outlook

- Include **glueball** operators
- A_1^{++} (0^{++}) channel
- Multi-meson operators – **resonances** and scattering
- Lighter pion masses, larger volumes, ...

Jefferson Lab

Extra Slides

Prin. Corr. M_{eff}

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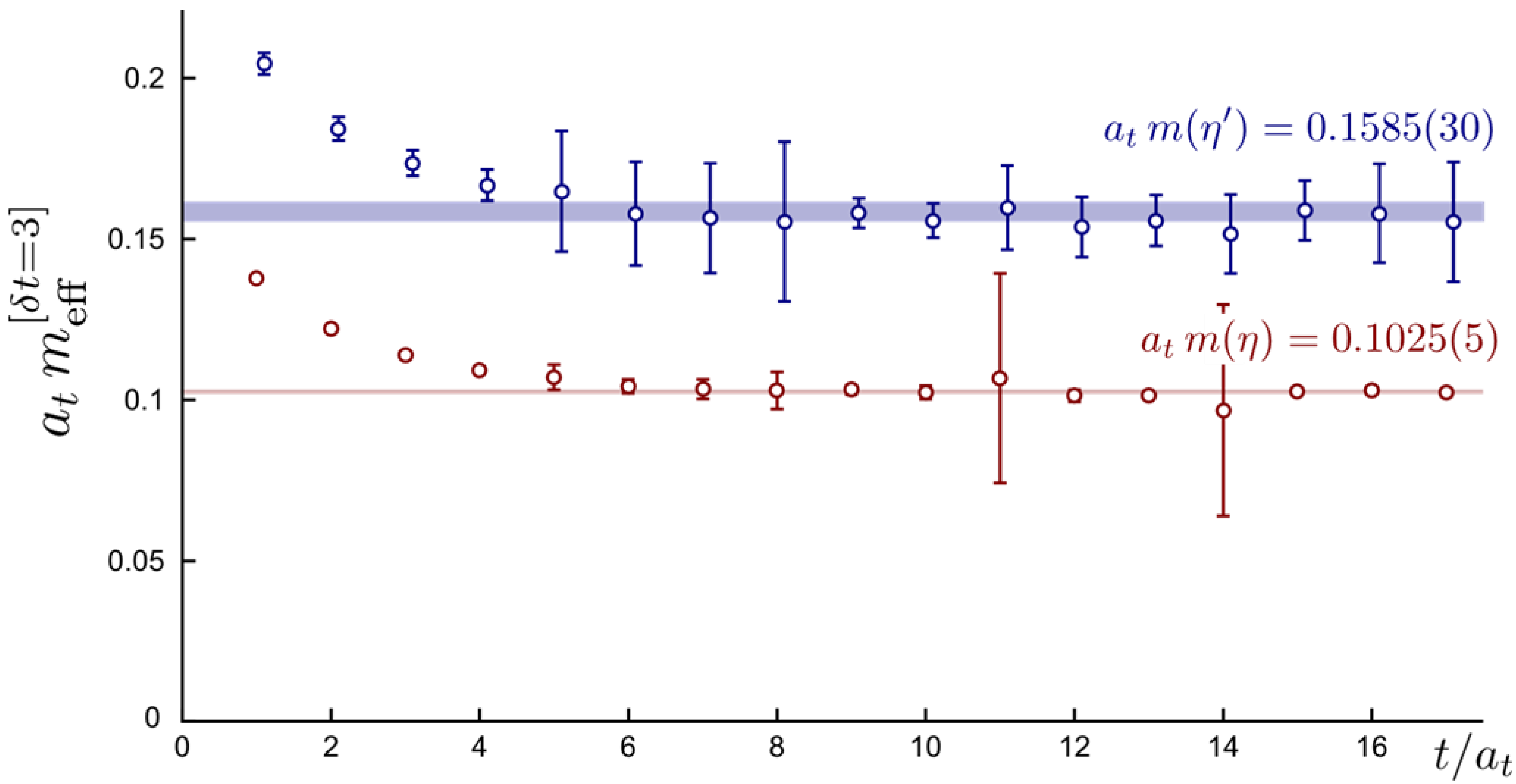

$$M_{eff}(t) = -\ln\left[\lambda(t+\delta t)/\lambda(t)\right]/dt$$

$\delta t = 3$

Mixing Angle

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