$1/N_c$ - ChPT in the one-Baryon sector

Álvaro Calle Cordón

Jefferson Lab

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In collaboration with José Goity

Chiral Symmetry & Large N_c limit

QCD fundamental theory of the strong interactions

Chiral Symmetry: $SU_L(3) \times SU_R(3)$ for $m_u = m_d = m_s = 0$

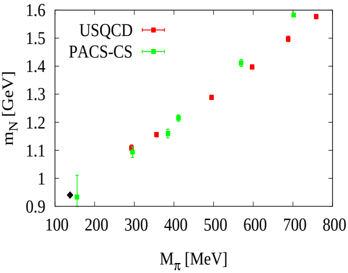
- Spontaneous Chiral Symmetry breaking: Goldstone bosons (π, K, η).
- Scale separation: mass of Goldstone bosons $\ll$ vector mesons $\sim$ 1 GeV.
- Effective Field Theory: Chiral Perturbation Theory.
- Successful description of hadron properties in the low-energy region.

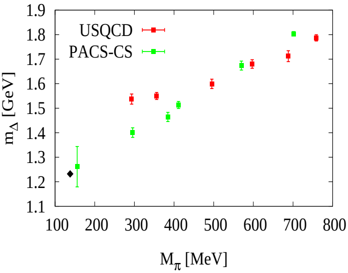
Large N_c limit: $SU(N_c)$, $N_c \to \infty$, $\lambda = g^2 N_c$ =const

- t'Hooft: planar diagrams $\mathcal{O}(N_c)$, non-planar diagrams suppressed $1/N_c$.
- Mesons are light, $m = \mathcal{O}(N_c^0)$, and stable, $\Gamma = \mathcal{O}(1/N_c)$.
- Witten: baryons color singlets with N_c valence quarks.
- Baryons are heavy particle $M = \mathcal{O}(N_c)$ with size independent of N_c.
- Meson-baryon coupling $\frac{g_A}{F_\pi} \partial_i \pi^a G^{ia} = \mathcal{O}(\sqrt{N_c})$.
- Ordering of all QCD effects in powers of $1/N_c$.

An observation on the baryon mass puzzle

- Baryon masses from lattice QCD
  USQCD Collaboration, A. Walker-Loud et al., PRD79, 054502 (2009)
  PACS-CS Collaboration, S. Aoki et al., PRD79, 034503 (2009)





- Quark mass dependence in HBχPT:
  Octet: $m_B = m_0 + \delta m_B^{(1)} + \delta m_B^{(3/2)} + \delta m_B^{(2)} + \cdots$
  Decuplet: $m_T = m_0 + \Delta_0 + \delta m_T^{(1)} + \delta m_T^{(3/2)} + \delta m_T^{(2)} + \cdots$
  m_0: baryon mass in the chiral limit.
  Δ_0: decuplet-octet (delta-nucleon) mass splitting in the chiral limit.
  $\delta m_{B,T}^{(n)}$: corrections to the baryon mass scaling as $m_q^n \sim m_\pi^{2n}$

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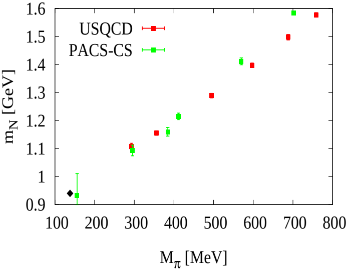
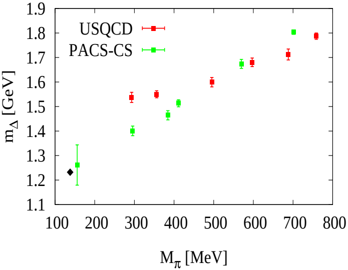
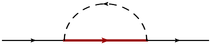




- Role of the Δ:

$$16\pi^2\, i\,\mathcal{I} \;=\; \cdots + 4(\Delta^2 - M_\pi^2)^{3/2} J(\Delta, M_\pi) + \cdots$$

$$\underset{M_\pi \gg \Delta}{\approx} \;\cdots + \frac{16}{3}\Delta^3 - 3\pi M_\pi \Delta^2 - 4M_\pi^2 \Delta + 2\pi m^3 + \cdots$$



$$\Delta = M_\Delta - M_N \to 0\,,\, N_c \to \infty$$

$$J(\Delta, M_\pi) = \begin{cases} \frac{\pi}{2} - \tanh^{-1}\left(\frac{\Delta}{\sqrt{\Delta^2 - M_\pi^2}}\right) & m < \Delta\,, \\ \frac{\pi}{2} - \tan^{-1}\left(\frac{\Delta}{\sqrt{M_\pi^2 - \Delta^2}}\right) & m > \Delta\,. \end{cases}$$

Chiral Perturbation Theory & $1/N_c$ Expansion

Meson sector (do not have spin-flavor symmetry)

- Unitary matrix of pion fields ($F_\pi = 92.4$)

$$U(x) = \exp\left(i\frac{\pi^a(x)\tau^a}{F_\pi}\right)$$

- The $\mathcal{O}(p^2)$ chiral Lagrangian reads ($\langle A\rangle \equiv \mathrm{Tr}(A)$),

$$\mathcal{L}_\pi^{(2)} = \frac{F_\pi^2}{4}\langle D_\mu U D^\mu U^\dagger + \chi^\dagger U + \chi U^\dagger\rangle$$

  with standard definitions:

$$U = u^2\,,\ \nabla_\mu U \equiv \partial_\mu U - i r_\mu U + i U \ell_\mu$$
$$r_\mu = v_\mu + a_\mu\,,\ \ell_\mu = v_\mu + a_\mu\,,\ \chi = 2B(s+ip)$$

- External fields: vector (v_μ), axial (a_μ), pseudoscalar (p) and scalar (s).

Chiral Perturbation Theory & $1/N_c$ Expansion

Baryon sector ($SU(4)$ spin-flavor symmetry)

- Consistence conditions imply $SU(2N_f)$ algebra: $\{S^i, T^a, G^{ia}\}$
  S^i spin, T^a isospin and G^{ia} axial currents at zero momentum
  `Gervais-Sakita' 84 & Dashen-Jenkins-Manohar' 93`
- Ground state baryons: spin-flavor symmetric multiplet **B** of $SU(2N_f)$ with S=I.

$$\mathbf{B} = \begin{pmatrix} N \\ \Delta_{3/2} \\ \vdots \\ \Delta_{N_c/2} \end{pmatrix} \to \mathbf{B} = \begin{pmatrix} N \\ \Delta \end{pmatrix}$$

  For $N_c > 3$, ground state baryons with $S = I = 5/2, \ldots$ appear ("ghost states").
- Matrix elements of generators acting on the multiplet

$$\begin{aligned}
\langle S', S_3', I_3' \mid G^{ia} \mid S, S_3, I_3 \rangle &= \# \langle SS_3, 1i \mid S'S_3' \rangle \langle SI_3, 1a \mid S'I_3' \rangle \sim \mathcal{O}(N_c) \\
\langle S', S_3', I_3' \mid S^{i} \mid S, S_3, I_3 \rangle &= \# \langle SS_3, 1i \mid S'S_3' \rangle \delta_{SS'} \delta_{I_3 I_3'} \sim \mathcal{O}(N_c^0) \\
\langle S', S_3', I_3' \mid T^{a} \mid S, S_3, I_3 \rangle &= \# \langle SI_3, 1a \mid S'I_3' \rangle \delta_{SS'} \delta_{S_3 S_3'} \sim \mathcal{O}(N_c^0)
\end{aligned}$$

- Key scalings with N_c: $F_\pi = \mathcal{O}(\sqrt{N_c})$, $g_A = \mathcal{O}(N_c^0)$, $m_B = \mathcal{O}(N_c)$, $M_\pi = \mathcal{O}(N_c^0)$.

Chiral Perturbation Theory & $1/N_c$ Expansion

HB chiral lagrangians

- $\mathcal{O}(p)$ chiral lagrangian

$$
\begin{aligned}
\mathcal{L}_{\pi B}^{(1)} &= i\,\mathbf{B}^\dagger D_0 \mathbf{B} + g_A\,\mathbf{B}^\dagger u_i^a G^{ia}\mathbf{B} + \mathbf{B}^\dagger \delta m_S \mathbf{B}\,,
\end{aligned}
$$

$$
\begin{aligned}
D_\mu \mathbf{B} &= \partial_\mu \mathbf{B} - i\,\Gamma_\mu^a\, T^a \mathbf{B}\,,\\
\Gamma_\mu &= \frac{1}{2}\left[u\,(\partial_\mu - i r_\mu)\,u^\dagger + u^\dagger\,(\partial_\mu - i\ell_\mu)\,u\right]\,,\\
u_\mu &= i\,\left[u^\dagger\,(\partial_\mu - i r_\mu)\,u - u\,(\partial_\mu - i\ell_\mu)\,u^\dagger\right]\,.
\end{aligned}
$$

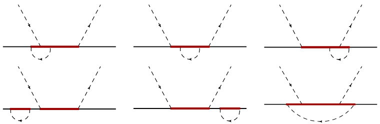
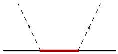
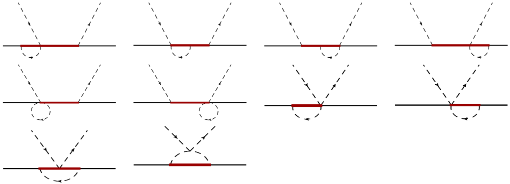
and $\Gamma_\mu^a \equiv \frac{1}{2}\,\langle \tau^a\,\Gamma_\mu \rangle$, $u_\mu^a \equiv \frac{1}{2}\,\langle \tau^a\,u_\mu \rangle$, $\delta m_S = \frac{C_{HF}}{N_c}\vec{S}^2$,

- All terms have same chiral order, they do have different $1/N_c$ orders.
  The Weinberg-Tomozawa $\sim 1/F_\pi^2$ and overall $\mathcal{O}(p/N_c)$
  The πB coupling $\sim 1/F_\pi$ and overall $\mathcal{O}(p\sqrt{N_c})$
- Higher order lagrangians (under construction):
  General form $B^\dagger\; O_\chi \otimes \mathcal{G}\;\mathbf{B}$ & EOM
  $O_\chi =$ tensor $\{u_\mu\,, D_\mu\,, \chi_\pm\}$, and $\mathcal{G} =$ tensor $\left\{1, S^i\,, T^a\,, G^{ia}\right\}$

Chiral Perturbation Theory & $1/N_c$ Expansion

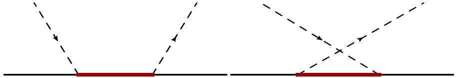
πN scattering [ACC, Goity, Long, Schat (in preparation)]

πN Feynman diagrams up to one-loop. Crossed diagrams are not shown.

	$\mathcal{O}(p)$	$\mathcal{O}(p^2)$	$\mathcal{O}(p^3)$
$\mathcal{O}(N_c^2)$			
$\mathcal{O}(N_c)$			
$\mathcal{O}(N_c^0)$			
$\mathcal{O}(1/N_c)$			

Chiral Perturbation Theory & $1/N_c$ Expansion

Combined power counting



$$\mathcal{M} = i\frac{g_A^2}{F_\pi^2}k_1^i k_2^j \sum_n G^{jb}\mathcal{P}_n G^{ia}\frac{1}{p^0 + k_1^0 - \delta m_n} + \left(a \to b\,, i \to j\,, p^0 \to p'^0\,, k_1^0 \to -k_1^0\right)$$

The HB propagator can only be expanded when p^0, p'^0, $\delta m_n \ll k_1^0, k_2^0$.

1. Strict large-N_c limit, defined by the condition,

$$\frac{1}{N_c} \sim \delta m_n \ll k^0 \sim p\,, \quad \text{i.e.}\,, \quad \frac{1}{N_c} \sim \mathcal{O}(p^2)$$

   where we can make an expansion of the HB propagator, and,

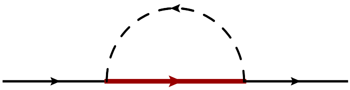
2. *Soft* limit defined by the condition,

$$\frac{1}{N_c} \sim \delta m_n \sim k^0 \sim p\,, \quad \text{i.e.}\,, \quad \frac{1}{N_c} \sim \mathcal{O}(p)$$

   in which case we cannot expand the HB propagator.

Real life we have that $M_\pi \lesssim m_\Delta - m_N$

Baryon self energy



$$\Delta_n = \delta m_n - p^0 \,,$$
$$p^0 = \delta m_{in} + \mathfrak{p}^0 \,,$$
$$\delta m = \frac{C_{HF}}{N_c} \vec{S}^2 \,.$$

$$\delta\Sigma_{(1)} = i \frac{g_A^2}{F_\pi^2} \frac{1}{d-1} \sum_n G^{ia} \mathcal{P}_n G^{ia} \, I_{(1)}(n, p^0, M_\pi) \,,$$

$$I_{(1)}(n, p^0, M_\pi) = \int \frac{d^d k}{(2\pi)^d} \frac{\vec{k}^2}{k^2 - M_\pi^2 + i\epsilon} \frac{1}{k^0 - \Delta_n + i\epsilon} \,,$$

$$= -\frac{i}{16\pi^2} \Big\{ \Delta_n (2\Delta_n^2 - 3M_\pi^2) \left(\frac{1}{\epsilon} - \gamma + \log(4\pi) - \log\left(\frac{M_\pi^2}{\mu^2} \right) \right)$$
$$- 4(\Delta_n^2 - M_\pi^2)^{3/2} \tanh^{-1}\left(\frac{\Delta_n}{\sqrt{\Delta_n^2 - M_\pi^2}} \right)$$
$$-2\pi (M_\pi^2 - \Delta_n^2)^{3/2} - \Delta_n (5M_\pi^2 - 4\Delta_n^2) \Big\} \,,$$

$$\delta m_{(1)} = \delta\Sigma_{(1)}\Big|_{\mathfrak{p}^0 \to 0} \,, \qquad \delta Z_{(1)} = \frac{\partial \delta\Sigma_{(1)}}{\partial \mathfrak{p}^0}\Big|_{\mathfrak{p}^0 \to 0}$$

Baryon self energy

- Finite + UV pieces:

$$\delta m_{(1)} = \delta m_{(1)}^{Finite} + \delta m_{(1)}^{UV} \,, \qquad \delta Z_{(1)} = \delta Z_{(1)}^{Finite} + \delta Z_{(1)}^{UV} \,.$$

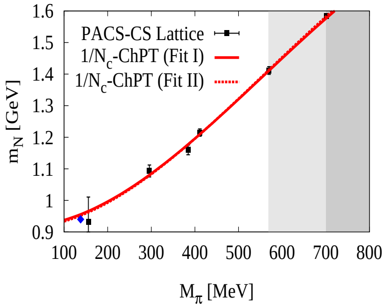
- CT lagrangian to renormalize the self energy:

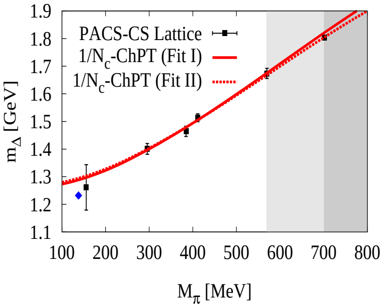
$$\begin{aligned}
\mathcal{L}^{\Sigma CT} &= -\frac{g_A^2}{F_\pi^2}\frac{1}{48\pi^2}\lambda_\epsilon \left\{ \mathbf{B}^\dagger \left[-3\,\frac{C_{HF}}{N_c} M_\pi^2 \left(-\frac{3}{8}N_c(N_c+4) + \frac{5}{2}\vec{S}^2 \right) \right.\right. \\
&+ 2\,\frac{C_{HF}^3}{N_c^3} \left. \left(-\frac{3}{2}N_c(N_c+4) + (12 - \frac{5}{2}N_c(N_c+4))\vec{S}^2 + 14\vec{S}^4 \right) \right] \mathbf{B} \\
&+ \mathbf{B}^\dagger i\tilde{D}_0 \left[3\,M_\pi^2 \,\left(\frac{3}{16}N_c(N_c+4) - \frac{1}{2}\vec{S}^2 \right) \right. \\
&- 2\,\frac{C_{HF}^2}{N_c^2} \left. \left. \left(\frac{9}{4}N_c(N_c+4) + \frac{3}{4}(N_c+6)(N_c-2)\vec{S}^2 - 6\vec{S}^4 \right) \right] \mathbf{B} \right\}
\end{aligned}$$

- Baryon mass formula $\mathcal{O}(p^3)$:

$$M_B(S) = m_0 + \underbrace{\frac{C_{HF}}{N_c} S(S+1)}_{\mathcal{L}^{(1)}} + \underbrace{c_1\, M_\pi^2 + h_1\, \frac{C_{HF}}{N_c} S(S+1)\, M_\pi^2}_{\mathcal{L}^{(2)}} + \delta m_{(1)}^{Finite}(S)$$

Baryon masses fitted to PACS-CS lattice data





	g_A	m_0 [MeV]	C_{HF} [MeV]	c_1 [GeV^{-1}]	h_1[GeV^{-2}]	M_N [MeV]	M_Δ [MeV]	χ^2/DOF
Fit I	0.5(2)	896(22)	276(31)	0.7(3)	4(3)	953(36)	1287(34)	0.66
Fit II	0.54(8)	902(19)	276(13)	0.7(1)	4(1)	952(27)	1295(25)	0.51

$$m_N^{\text{phys}} = 940(2)\text{MeV}\,, \qquad m_\Delta^{\text{phys}} = 1232(2)\text{MeV}$$

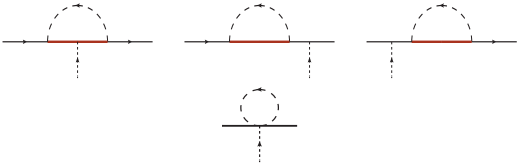
Baryon masses, expansion in $1/N_c$

Expansion of $\delta m^{Finite}_{(1)}$ in $1/N_c$ and scale $g_A \to 1$, $F_\pi \to \sqrt{N_c}$

$$\begin{aligned}\delta m^N_{(1)} &= -\frac{M_\pi^3 N_c}{128\pi} + 3\frac{C_{HF} M_\pi^2}{128\pi^2}\left(\gamma - 1 + \log M_\pi^2 - \log 4\pi\right) \\ &+ \underbrace{\frac{9 C_{HF}^2 M_\pi}{128\pi N_c}}_{\sim 0.02 M_\pi} - \frac{9 C_{HF}^3}{64\pi^2 N_c^2}\left(\gamma + \log M_\pi^2 - \log 4\pi\right) + \cdots\end{aligned}$$

$$\begin{aligned}\delta m^\Delta_{(1)} &= -\frac{M_\pi^3 N_c}{128\pi} + 3\frac{C_{HF} M_\pi^2}{128\pi^2}\left(\gamma - 1 + \log M_\pi^2 - \log 4\pi\right) \\ &+ \underbrace{\frac{21 C_{HF}^2 M_\pi}{128\pi N_c}}_{\sim 0.04 M_\pi} - \frac{29 C_{HF}^3}{64\pi^2 N_c^2}\left(\gamma + \log M_\pi^2 - \log 4\pi\right) + \cdots\end{aligned}$$

Vertex correction



$\sim \mathcal{O}(p^3 N_c^{3/2})$

$\sim \mathcal{O}(p^3 N_c^{1/2})$

$$\delta\Gamma_{(1)} = -i\left\{(1) + \frac{1}{2}((2)+(3))_{no-pole} + (4)\right\}$$

$$(1) = i\left(\frac{g_A}{F_\pi}\right)^3 \frac{q^i}{d-1}\sum_{n,n'} G^{jb}\mathcal{P}_{n'}G^{ia}\mathcal{P}_n G^{jb}\frac{\left(I_{(1)}(n,p^0,M_\pi) - I_{(1)}(n',p'^0,M_\pi)\right)}{p^0 - p'^0 - \delta m_n + \delta m_{n'}},$$

$$(2)+(3) = -\frac{g_A}{F_\pi}q^i\left\{G^{ia}\delta Z_{(1)} + \delta Z_{(1)}G^{ia} + \cdots\right\} + pole - terms,$$

$$(4) = i\frac{g_A}{3F_\pi^3}\, q_i\, \Delta(M_\pi)G^{ia}$$

Vertex correction

Cancellations in the large N_c limit

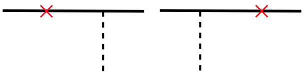
$$(1)+\frac{1}{2}((2)+(3))_{no-pole}\Big|_{N_c\to\infty} = \frac{i}{2}\left(\frac{g_A}{F_\pi}\right)^3 \frac{q^i}{d-1}\underbrace{[[G^{jb},G^{ia}],G^{jb}]}_{1/N_c^2}\frac{\partial}{\partial p^0}I_{(1)}(p^0,M_\pi)$$
$$\sim \mathcal{O}(p^3 N_c^{1/2})$$

Structure of counterterms

$$(1)^{UV} = \frac{\lambda_\epsilon}{16\pi^2}\left(\frac{g_A}{F_\pi}\right)^3\frac{q^i}{d-1} \times \left\{3M_\pi^2 G^{jb}G^{ia}G^{jb} - 2(\mathfrak{p}^{0\,2}+\mathfrak{p}'^{0\,2}+\mathfrak{p}^0\mathfrak{p}'^0)G^{jb}G^{ia}G^{jb}+\cdots\right\}$$

$$((2)+(3))^{UV} = -\frac{\lambda_\epsilon}{16\pi^2}\left(\frac{g_A}{F_\pi}\right)^3\frac{q^i}{d-1} \times \left\{3M_\pi^2\left\{G^{ia},G^2\right\} - 2(\mathfrak{p}^{0\,2}G^{ia}G^2+\mathfrak{p}'^{0\,2}G^2G^{ia})+\cdots+\text{pole-terms}\right\}$$

Pole terms cancel with diagrams of the type



Summary

Chiral Symmetry and Large N_c are fundamental features of QCD

Although chiral and large N_c limits do not commute, one can developed a combined power counting implementing them simultaneously.

In the large N_c limit, the N and Δ are degenerated. A consistent way of formulating $\Delta - \chi$EFT is by imposing large N_c constraints in the chiral lagrangians.

We have analyzed baryon masses in $1/N_c$-ChPT and preliminary results have been given for the chiral extrapolations of lattice data.

A global fit including the axial current is still needed. A systematic analysis of finite size and volume effect is also worth to consider.