EMC and Polarized EMC Effects in Nuclei

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Collaborators

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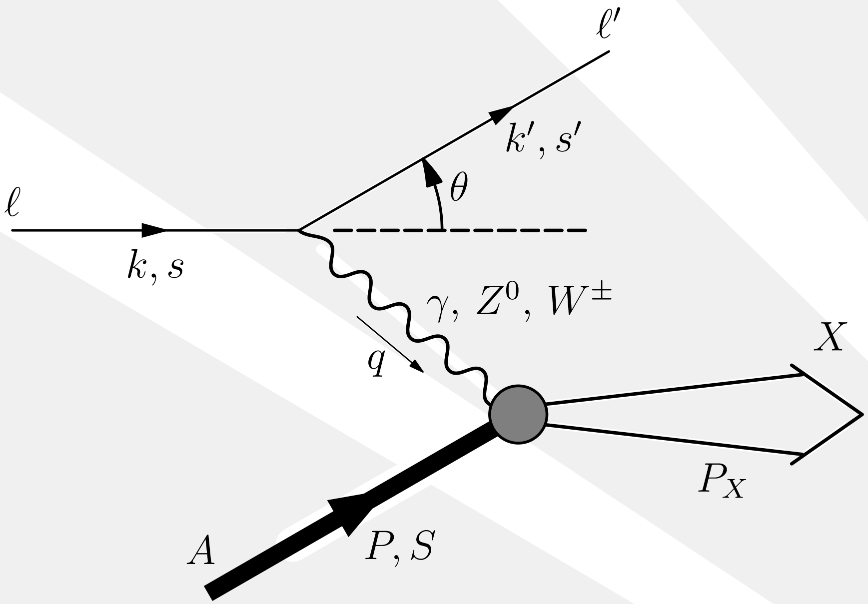
Outline

- Nuclear structure functions
  - Convolution formalism
- Nambu–Jona-Lasinio (NJL) model
  - Quark distributions
- Nucleon distributions in Nuclei
- Results
  - Quark distributions in nuclei
  - EMC effects
- Conclusion

Deep Inelastic Scattering



- Basic invariants

$$q^2 = (k - k')^2 = -Q^2 \le 0, \quad \nu = \frac{P \cdot q}{M_A},$$

$$x_A \equiv A\,\frac{Q^2}{2\,M_A\,\nu}, \quad x = \frac{\overline{M}_N}{M_N} x_A, \quad (\overline{M}_N = M_A/A)\,.$$

Hadronic Tensor

- In Bjorken limit, assuming Callen-Gross relations (e.g. $F_2 = 2x\,F_1$)
  - For $J = \frac{1}{2}$ target

$$W_{\mu\nu}=\left(g_{\mu\nu}\frac{P\cdot q}{q^2}+\frac{p_\mu p_\nu}{\nu}\right)F_2(x_A,Q^2)+i\,\frac{\varepsilon_{\mu\nu\lambda\sigma}q^\lambda s^\sigma}{\nu}\,g_1(x_A,Q^2)$$

  - For $J = 1$ target

$$W_{\mu\nu}=\left(g_{\mu\nu}\frac{P\cdot q}{q^2}+\frac{p_\mu p_\nu}{\nu}\right)F_2(x_A,Q^2)+i\,\frac{\varepsilon_{\mu\nu\lambda\sigma}q^\lambda s^\sigma}{\nu}\,g_1(x_A,Q^2)-r_{\mu\nu}\,b_1(x_A,Q^2)$$

  - For arbitrary J ($2J+1$ structure functions)

$$W^H_{\mu\nu}=\left(g_{\mu\nu}\frac{P\cdot q}{q^2}+\frac{p_\mu p_\nu}{\nu}\right)F_2^{JH}(x_A,Q^2)+i\,\frac{\varepsilon_{\mu\nu\lambda\sigma}q^\lambda s^\sigma}{\nu}\,g_1^{JH}(x_A,Q^2)$$

Quark/Probability Distributions

- $q_s^{JH}(x_A)$: probability to find a quark with momentum fraction x_A/A and spin-component s_z in nucleus with $J_z = H$.

- The familiar quark distributions are
  - $q^{JH}(x) \quad = q_+^{JH}(x) + q_-^{JH}(x)$ unpolarized
  - $\Delta q^{JH}(x) = q_+^{JH}(x) - q_-^{JH}(x)$ longitudinally polarized

Parton Model Structure Functions

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❖ DIS
❖ Hadronic Tensor
❖ Quark Dis.
❖ Parton Model . . .
❖ Multipoles
❖ New Sum Rules
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❖ Calculation
❖ NJL Model
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- Parton model expressions

$$F_{2A}^{JH}(x) = \sum_q e_q^2\, x\left[q_A^{JH}(x) + \bar{q}_A^{JH}(x)\right],$$

$$g_{1A}^{JH}(x) = \frac{1}{2}\sum_q e_q^2\left[\Delta q_A^{JH}(x) + \Delta\bar{q}_A^{JH}(x)\right],$$

$$F_{2A}(x) \equiv \frac{1}{2J+1}\sum_{H=-J}^{J} F_{2A}^{JH}(x),$$

$$F_2^{JH} = F_2^{J\,-H}, \quad g_1^{JH} = -g_1^{J\,-H}, \quad \left(q_s^{JH} = q_{-s}^{J\,-H}\right).$$

- $2J+1$ quark distributions & structure functions

Multipole Quark Distributions

- Simplify analysis of DIS for target composed of nucleons.

$$q_{jk}(x) \equiv \sum_{m=-j,\dots,j} (-1)^{j-m} \sqrt{2k+1} \begin{pmatrix} j & j & k \\ m & -m & 0 \end{pmatrix} q^{jm}(x),$$

$$\Delta q_{jk}(x) \equiv \sum_{m=-j,\dots,j} (-1)^{j-m} \sqrt{2k+1} \begin{pmatrix} j & j & k \\ m & -m & 0 \end{pmatrix} \Delta q^{jm}(x),$$

- Example: $J = 3/2$

$$q_{\frac{3}{2}\,0} = q^{\frac{3}{2}\,\frac{3}{2}} + q^{\frac{3}{2}\,\frac{1}{2}}, \qquad q_{\frac{3}{2}\,2} = q^{\frac{3}{2}\,\frac{3}{2}} - q^{\frac{3}{2}\,\frac{1}{2}},$$

$$\Delta q_{\frac{3}{2}\,1} = \frac{1}{\sqrt{5}}\left[3\,\Delta q^{\frac{3}{2}\,\frac{3}{2}} + \Delta q^{\frac{3}{2}\,\frac{1}{2}}\right], \qquad \Delta q_{\frac{3}{2}\,3} = \frac{1}{\sqrt{5}}\left[\Delta q^{\frac{3}{2}\,\frac{3}{2}} - 3\,\Delta q^{\frac{3}{2}\,\frac{1}{2}}\right].$$

New Sum Rules

- New multipole distribution sum rules

$$\int_0^A x^{n-1}\; q_{j\,k}(x)dx \;= 0, \quad k \text{ even}, \quad 2 \leqslant n < k,$$

$$\int_0^A x^{n-1}\, \Delta q_{j\,k}(x)dx = 0, \quad k \text{ odd}, \quad 1 \leqslant n < k.$$

- Example: $J = 3/2$

$$\int_0^A dx\, \Delta q_{\frac{3}{2}3}(x) = 0.$$

- R. L. Jaffe and A. Manohar, “Deep Inelastic Scattering From Arbitrary Spin Targets,” Nucl. Phys. B **321**, 343 (1989).

Why? — The EMC Effect

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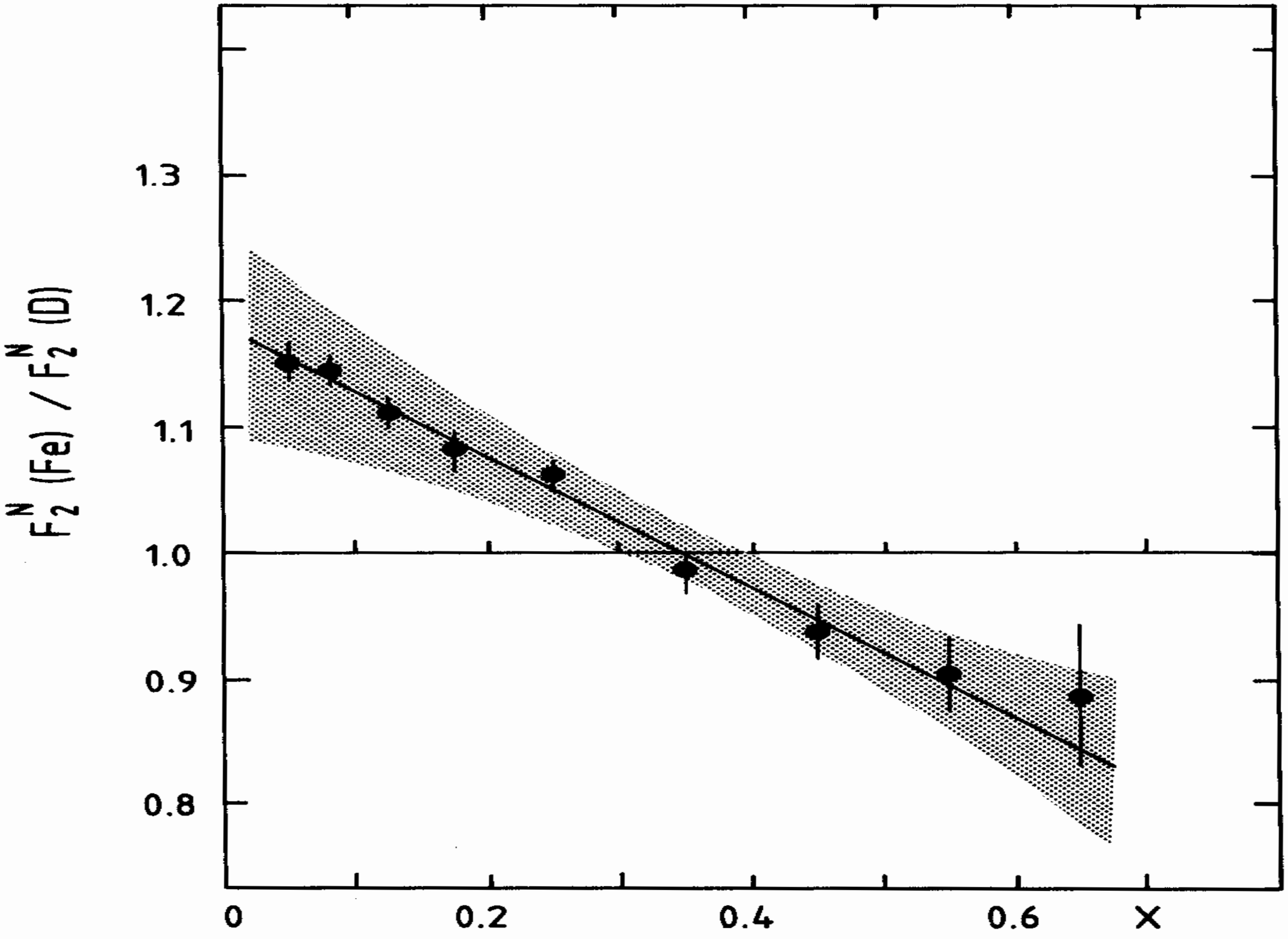


- J. J. Aubert *et al.* [European Muon Collaboration], Phys. Lett. B **123**, 275 (1983).

Calculation

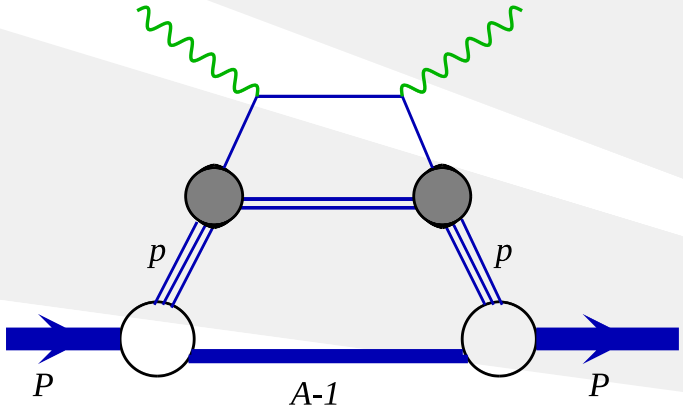
- Definition: Nuclear quark distribution functions

$$\Delta q_A^{JH}(x_A) = \frac{P^+}{A} \int \frac{d\xi^-}{2\pi} e^{iP^+ x_A \xi^- / A} \langle A, P, H | \overline{\psi}(0)\, \gamma^+ \gamma_5\, \psi(\xi^-) | A, P, H \rangle .$$

- Using Convolution formalism

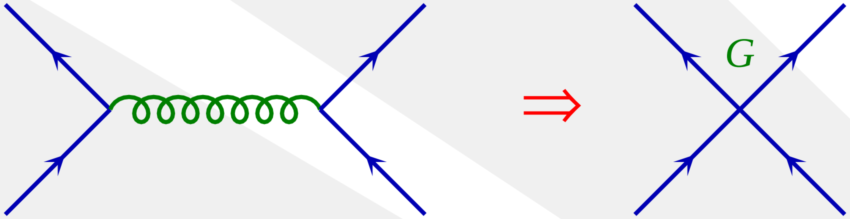
$$\Delta q_A^{JH}(x_A) = \sum_{\kappa,m} \int dy_A \int dx\ \delta(x_A - y_A\, x)\, \Delta f_{\kappa,m}^{(JH)}(y_A)\ \Delta q_\kappa(x)\, .$$

- Diagrammatically



NJL Model

- Investigate the role of quark degrees of freedom.
- Low energy effective theory

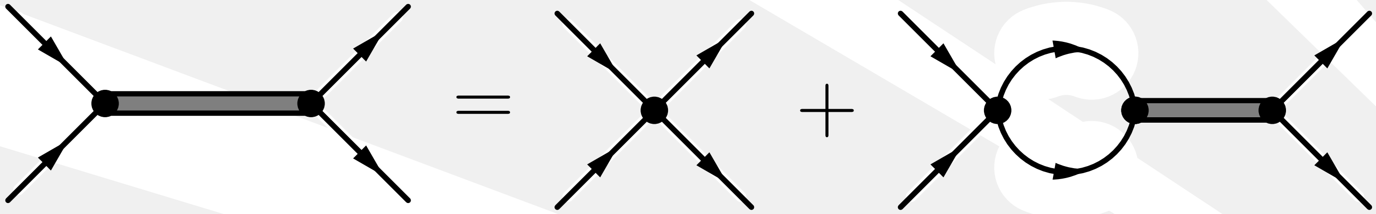


- Lagrangian has same symmetries as QCD:
  - Importantly chiral symmetry and CSB,
    $\longrightarrow$ Dynamically generated quark masses,
    $\longrightarrow$ Non-zero chiral condensate.
- Lagrangian ($\Gamma =$ Dirac, colour, isospin matrices)

$$\mathcal{L}_{NJL} = \overline{\psi}\left(i\,\partial\!\!\!/ - m\right)\psi + G\left(\overline{\psi}\Gamma\psi\right)^2,$$

Nucleon in the NJL model

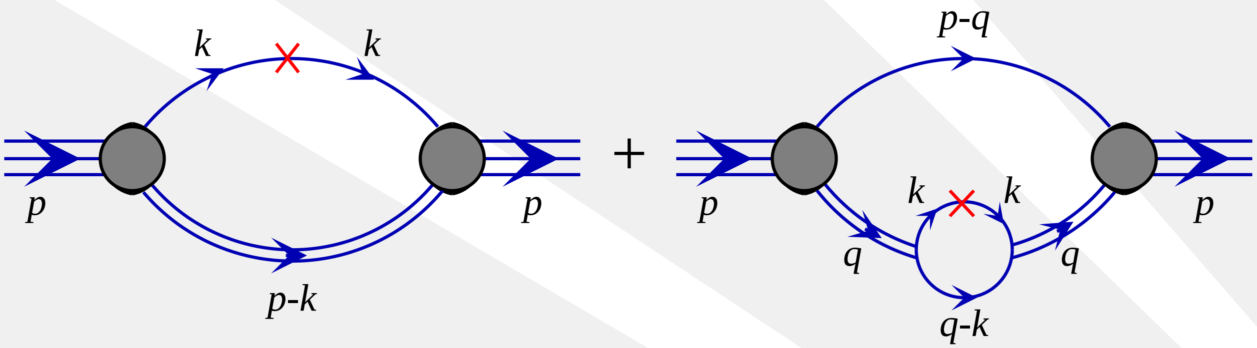
- Nucleon is approximated as a quark-diquark bound state.
- We use a relativistic Faddeev approach to describe this bound state.
- First diquark - bound state of two quarks:
- Solve Bethe-Salpeter equation for diquark.



- We include scalar and axial-vector diquarks.

Nucleon quark distributions

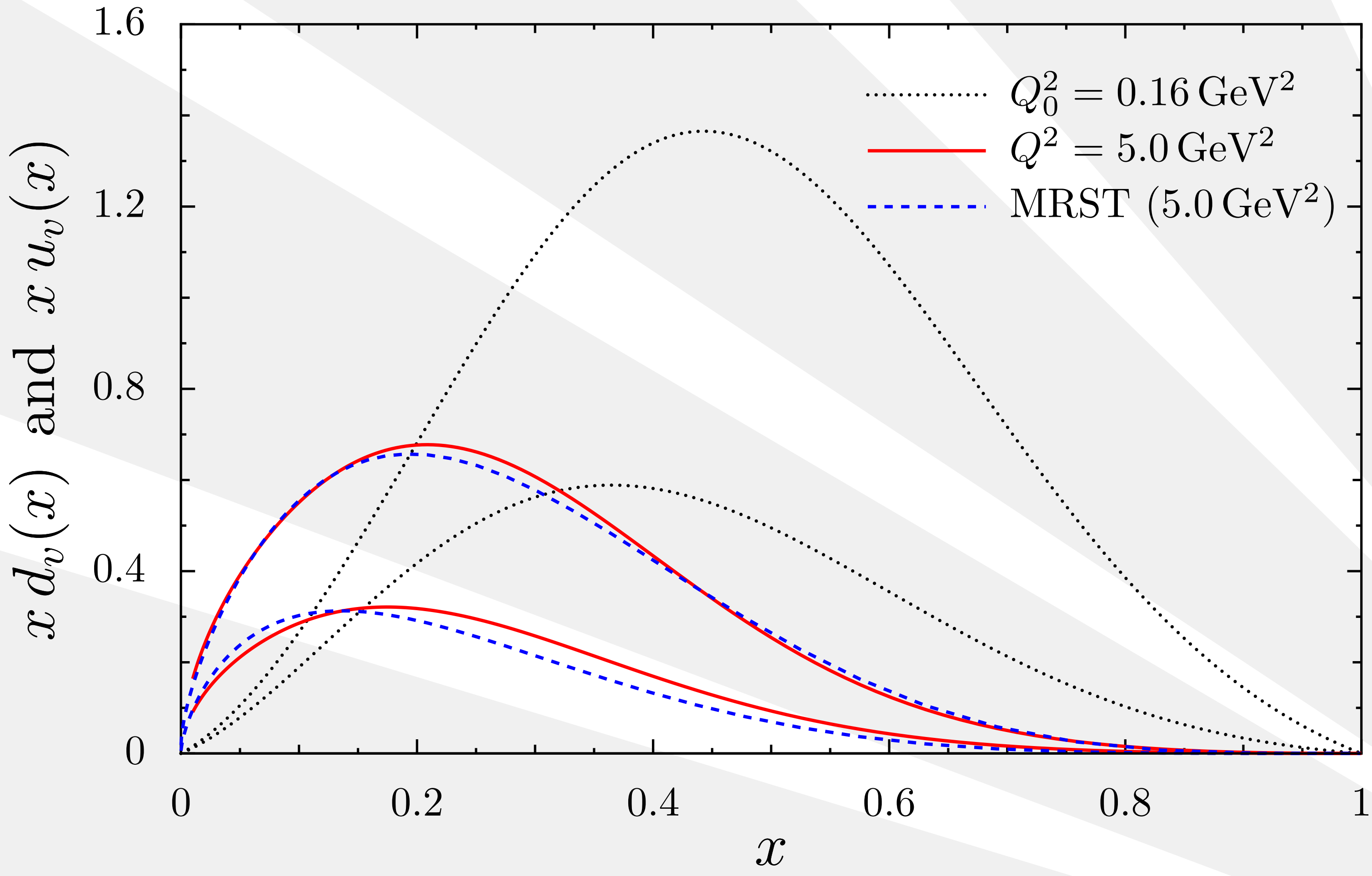
- Associated with a Feynman diagram calculation.



- $q(x) \rightarrow \mathsf{X} = \gamma^{+}\,\delta(x - \frac{k^{+}}{p^{+}})$
- $\Delta q(x) \rightarrow \mathsf{X} = \gamma^{+}\gamma_5\,\delta(x - \frac{k^{+}}{p^{+}})$

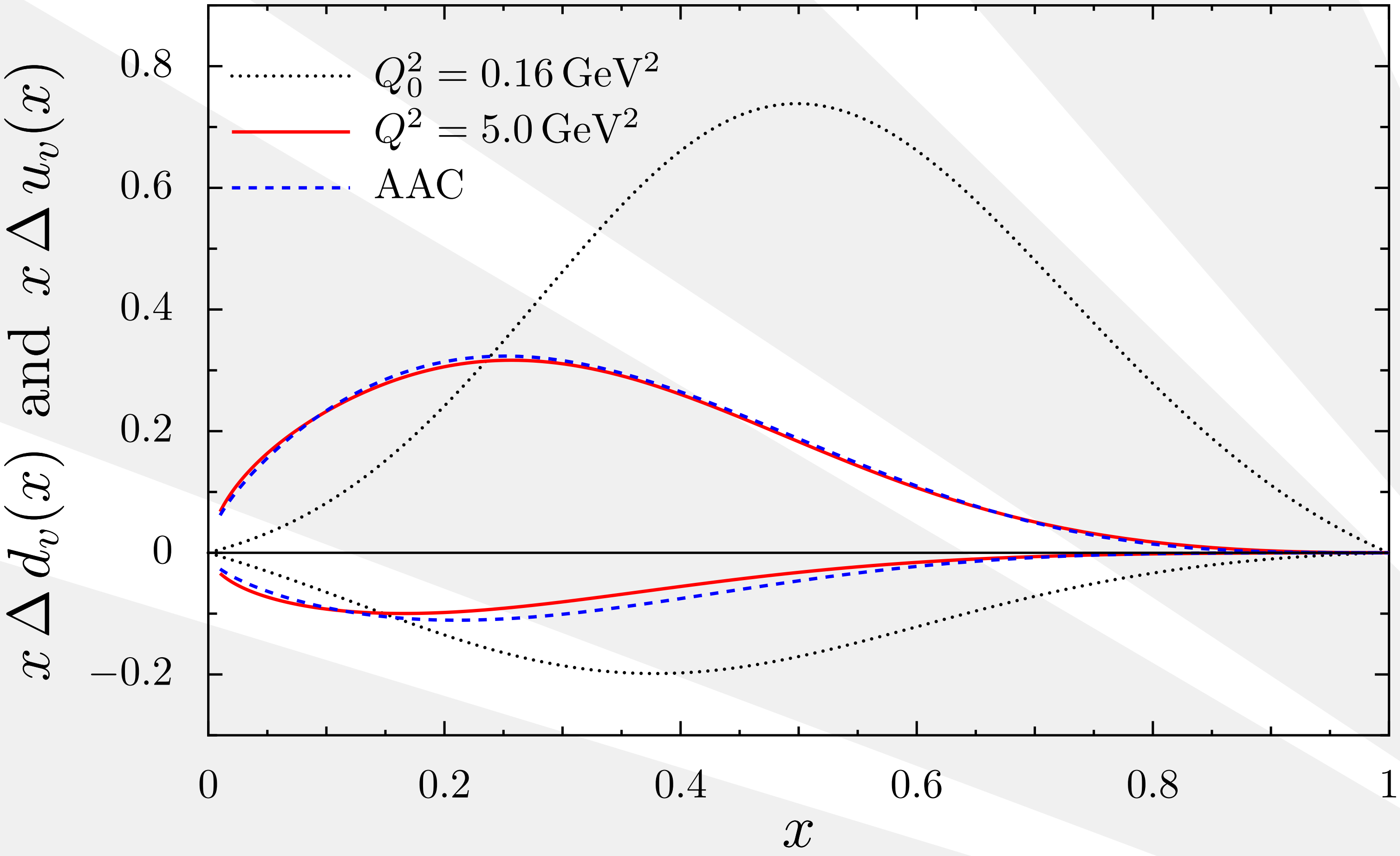
- Formalism satisfies baryon and momentum sum rules

$u_v(x)$ and $d_v(x)$ distributions



- MRST, Phys. Lett. B **531**, 216 (2002).

$\Delta u_v(x)$ *and* $\Delta d_v(x)$ *distributions*



- M. Hirai, S. Kumano and N. Saito, Phys. Rev. D **69**, 054021 (2004).

NJL Model at Finite Density

- Re-calculate diagrams

$$\mathcal{L} = \overline{\psi}\,(i\,\partial\!\!\!/ - M^{*} - V\!\!\!/\,)\,\psi + \mathcal{L}'_I$$

- Equivalent to:
  - Scalar field: via effective masses
  - Fermi motion: via convolution
  - Vector field: via scale transformation
- Nuclear Matter ($\varepsilon_F = E_F + 3V_0$)

$$q_A\,(x_A) = \frac{\varepsilon_F}{E_F}\,q_{A0}\left(\frac{\varepsilon_F}{E_F}x_A - \frac{V_0}{E_F}\right)$$

- Finite Nuclei ($\hat{M}_{N\kappa} = \overline{M}_N - 3V_\kappa$)

$$q_{A,\kappa}(x_A) = \frac{\overline{M}_N}{\hat{M}_N}\,q_{A0,\kappa}\left(\frac{\overline{M}_{N\kappa}}{\hat{M}_{N\kappa}}x_A - \frac{V_\kappa}{\hat{M}_{N\kappa}}\right)$$

Nucleon distribution functions

- Definition

$$f_{\kappa m}(y_A) = \frac{\sqrt{2}\,\overline{M}_N}{A} \int \frac{d^3p}{(2\pi)^3} \delta\left(p^3 + \varepsilon_\kappa - \overline{M}_N\, y_A\right) \overline{\Psi}_{\kappa m}(\vec{p})\; \gamma^+\, \Psi_{\kappa\, m}(\vec{p})\,,$$

- Central Potential Dirac eigenfunctions

$$\Psi_{\kappa m}(\vec{p}) = (-i)^\ell \begin{bmatrix} F_\kappa(p)\, \Omega_{\kappa m}(\theta,\phi) \\ -G_\kappa(p)\, \Omega_{-\kappa m}(\theta,\phi) \end{bmatrix},$$

  ❖ Dirac Equation

$$\left[-i\,\vec{\alpha}\cdot\vec{\nabla} + \beta\,[M(r) - V_s(r)] + V_v(r)\right] \psi_\kappa(r) = \varepsilon_\kappa\, \psi_\kappa(r)$$

Nucleon distributions: Results

- Spin-independent nucleon distribution

$$f_{\kappa,m}(y_A)=\sum_{k=0,2,\ldots,2j}(-1)^{j-m}\sqrt{2k+1}\begin{pmatrix} j & j & k \\ m & -m & 0 \end{pmatrix}$$

$$(-1)^{j+\frac{1}{2}}(2j+1)(2\ell+1)\sqrt{2k+1}\begin{pmatrix} \ell & k & \ell \\ 0 & 0 & 0 \end{pmatrix}\begin{Bmatrix} \ell & k & \ell \\ j & s & j \end{Bmatrix}$$

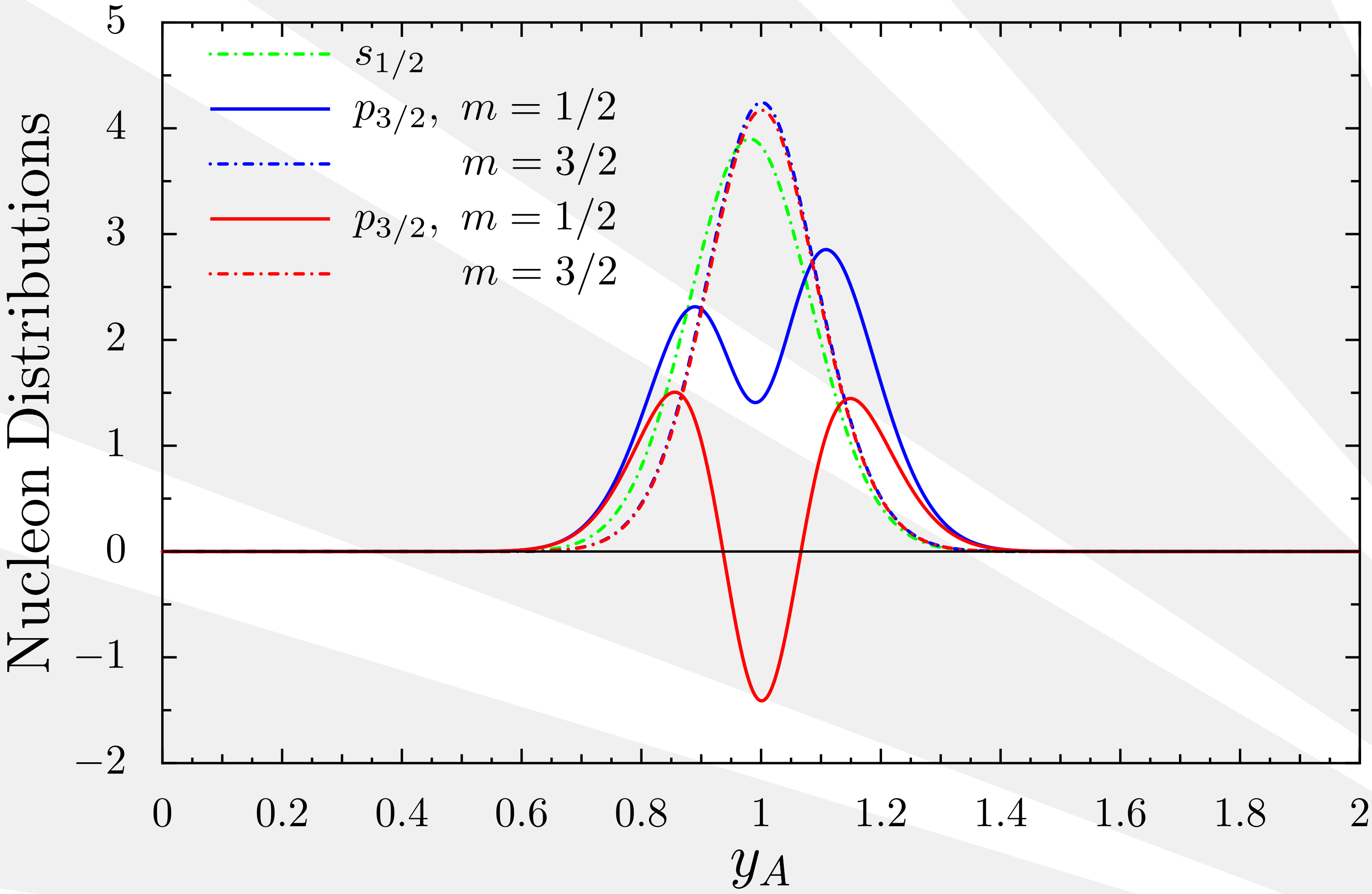
$$\frac{\overline{M}_N}{16\pi^3}\int_\Lambda^\infty dp\ p\left[F_\kappa(p)^2+G_\kappa(p)^2+\frac{2}{p}\left(\varepsilon_k-\overline{M}_N\,y_A\right)F_\kappa(p)G_\kappa(p)\right]P_k\left(\frac{\overline{M}_N\,y_A-\varepsilon_\lambda}{p}\right),$$

❖ $\Lambda = |\overline{M}_N\,y_A - \varepsilon_\kappa|$

- Infinite nuclear matter

$$f(y_A)=\frac{3}{4}\left(\frac{\varepsilon_F}{p_F}\right)^3\left[\left(\frac{p_F}{\varepsilon_F}\right)^2-(1-y_A)^2\right].$$

Nucleon distributions: ^{11}B



Nuclear Quark distributions

- Putting it all together, an example

$$u_A^{JH}(x_A) = \sum_{\kappa,m} [u_{p,\kappa}(x) \otimes f_{\kappa m}(y_A)] + \sum_{\kappa,m} [u_{n,\kappa} \otimes f_{\kappa m}(y_A)]$$

- Recall

$$q_{\frac{3}{2}\,0} = q^{\frac{3}{2}\,\frac{3}{2}} + q^{\frac{3}{2}\,\frac{1}{2}}, \qquad q_{\frac{3}{2}\,2} = q^{\frac{3}{2}\,\frac{3}{2}} - q^{\frac{3}{2}\,\frac{1}{2}},$$

$$\Delta q_{\frac{3}{2}\,1} = \frac{1}{\sqrt{5}}\left[3\,\Delta q^{\frac{3}{2}\,\frac{3}{2}} + \Delta q^{\frac{3}{2}\,\frac{1}{2}}\right], \quad \Delta q_{\frac{3}{2}\,3} = \frac{1}{\sqrt{5}}\left[\Delta q^{\frac{3}{2}\,\frac{3}{2}} - 3\,\Delta q^{\frac{3}{2}\,\frac{1}{2}}\right].$$

- $$u_A^{\frac{3}{2}\,\frac{3}{2}} \simeq u_A^{\frac{3}{2}\,\frac{1}{2}} \quad \Longrightarrow \quad u_A^{\frac{3}{2}\,0} \simeq 2u_A^{\frac{3}{2}\,\frac{3}{2}}, \quad u_A^{\frac{3}{2}\,2} \simeq 0.$$

Up distribution in ^{11}B

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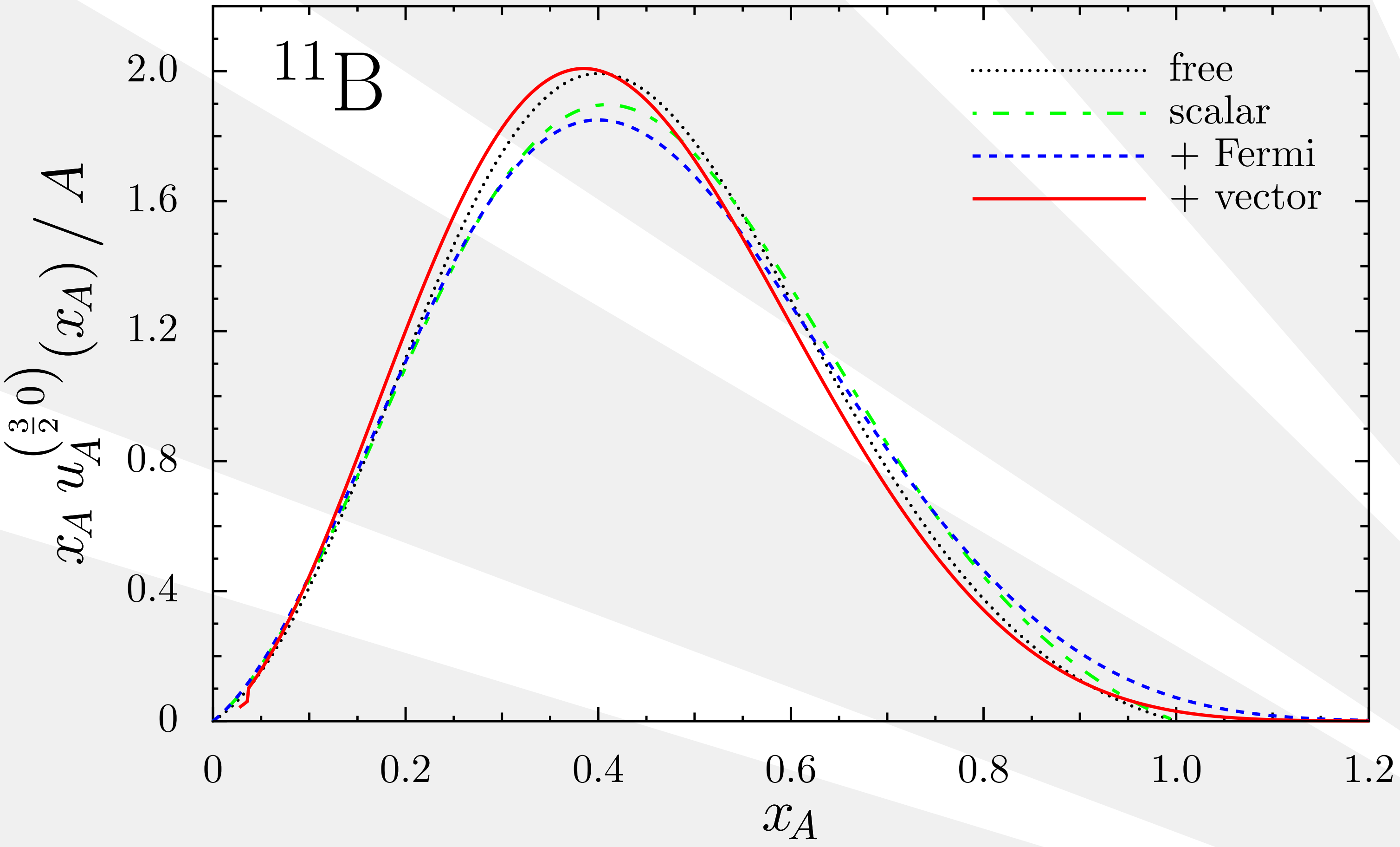
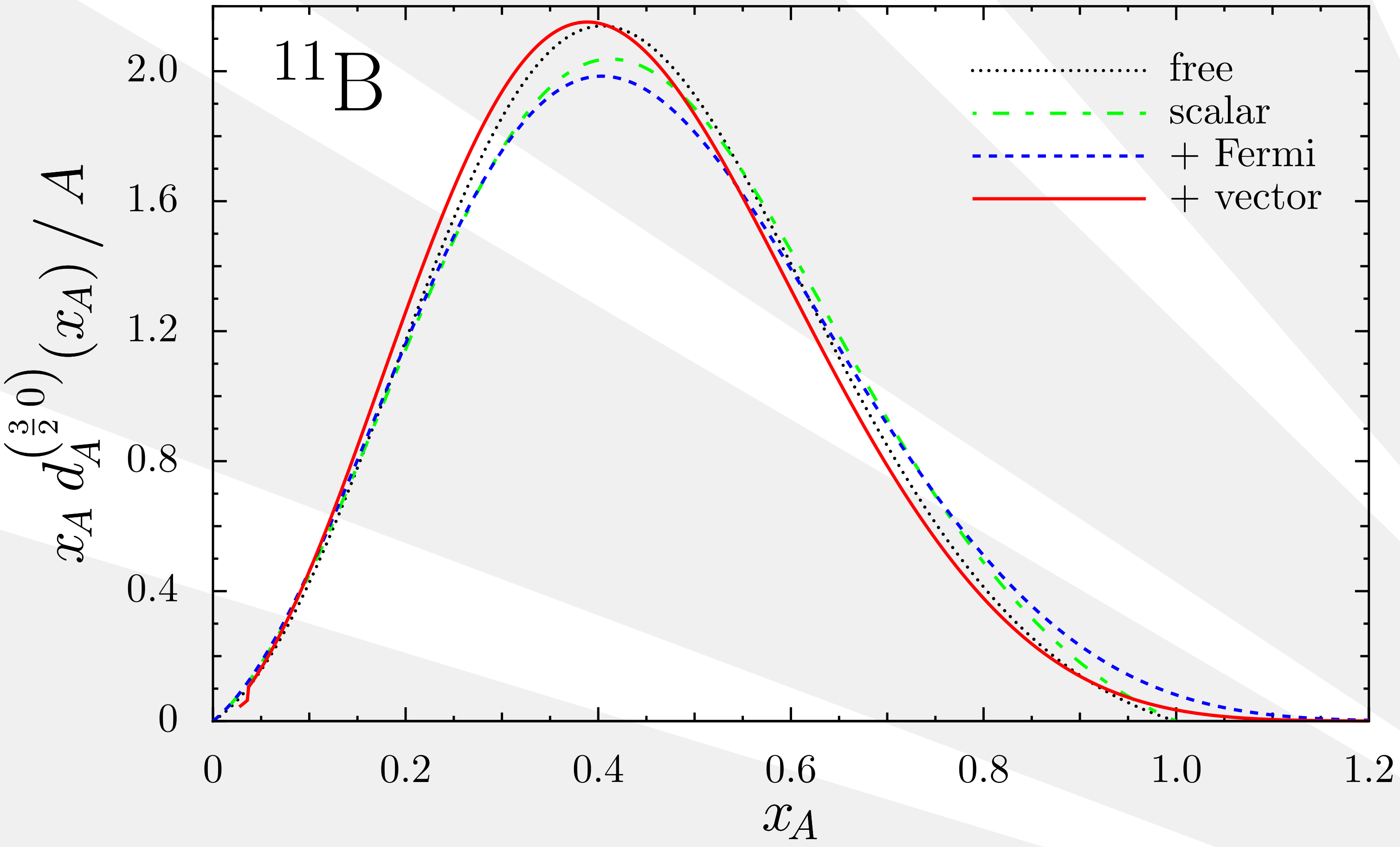
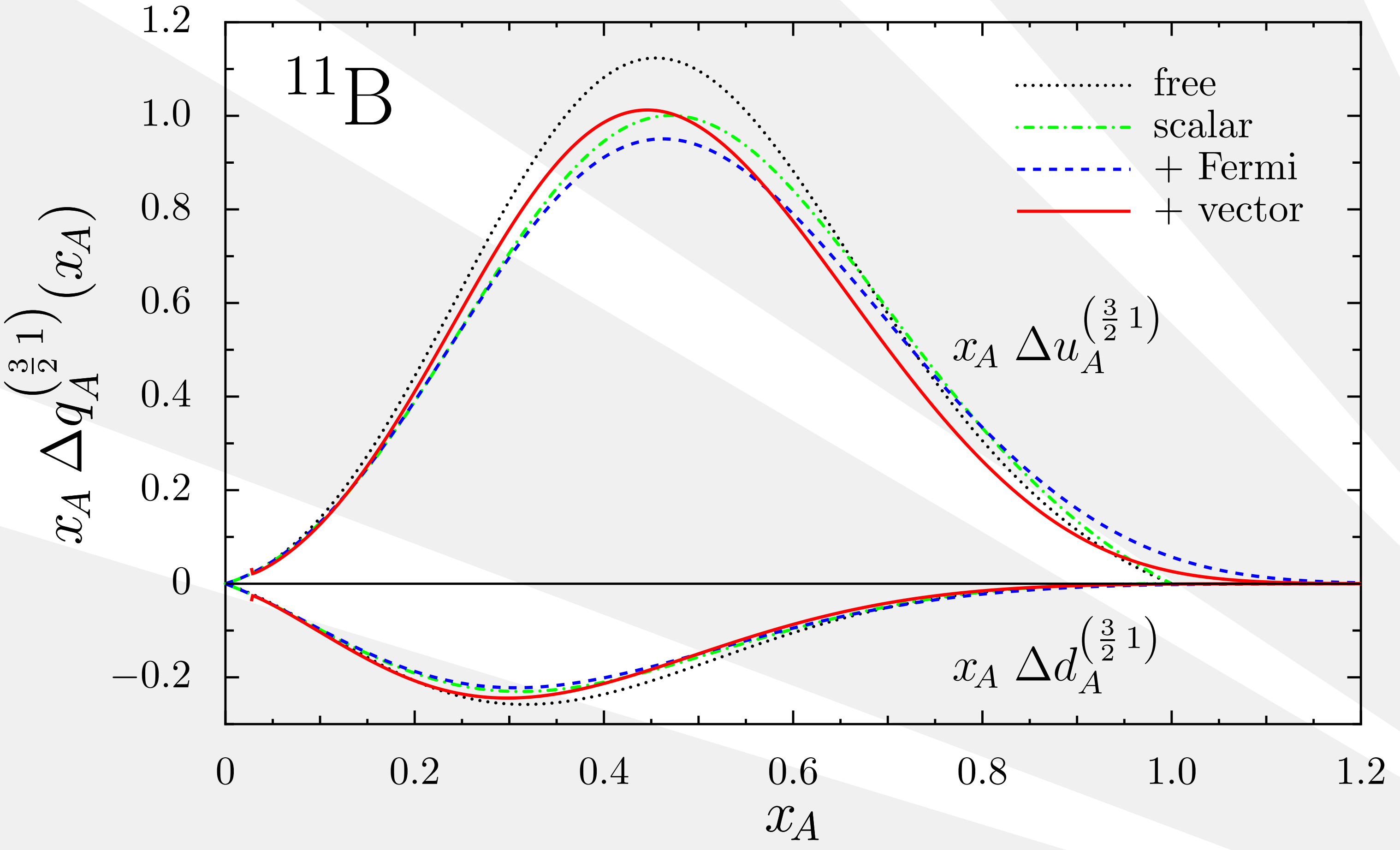


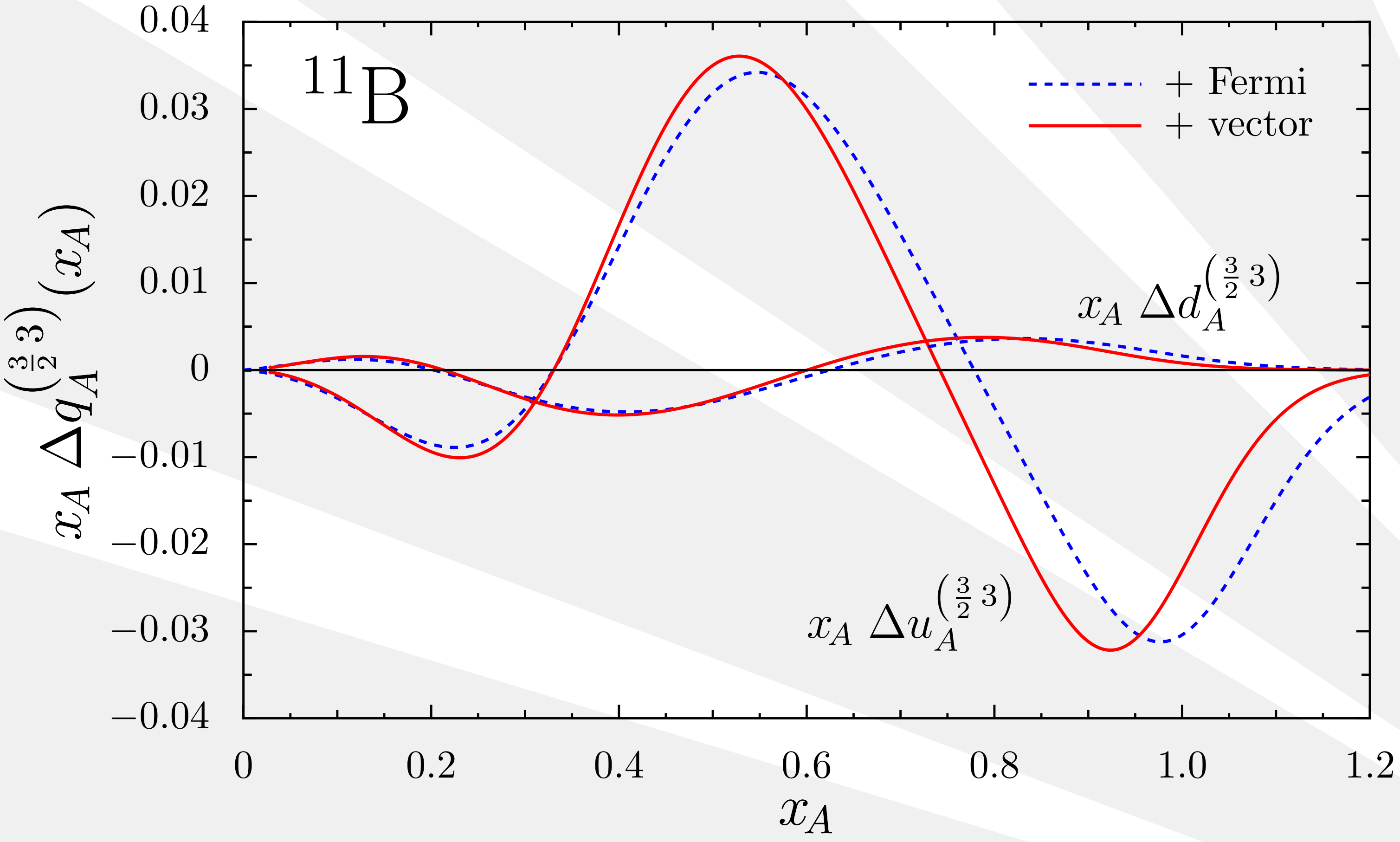
Down distribution in ^{11}B



Spin-dependent distributions in ^{11}B



Spin-dependent 2nd multipole



EMC effect

- EMC ratio

$$R_A = \frac{F_{2A}}{F_{2A}^{\text{naive}}} = \frac{F_{2A}}{Z\,F_{2p} + (A-Z)\,F_{2n}}$$

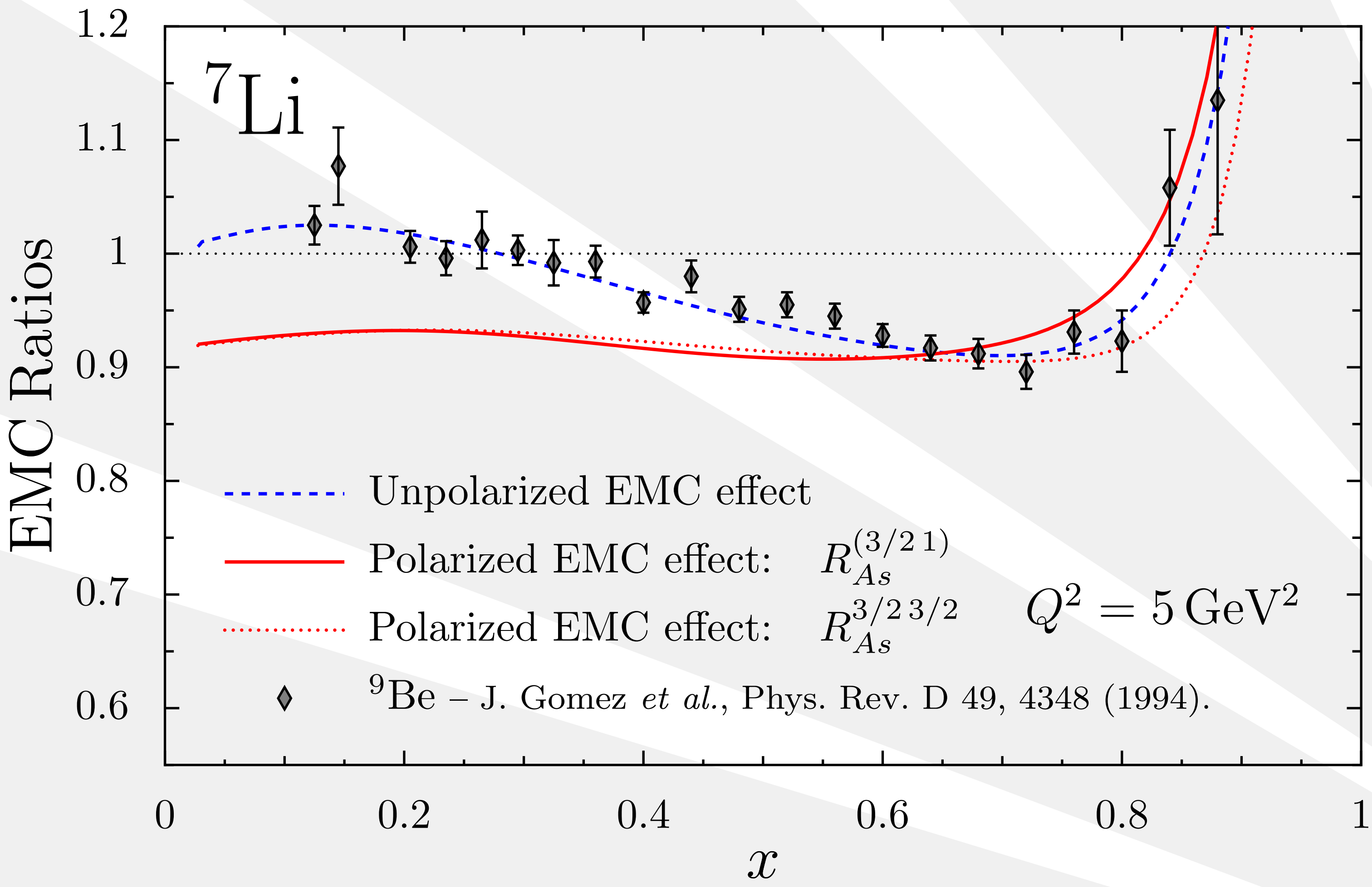
- Polarized EMC ratio

$$R_{As}^{JH} = \frac{g_{1A}^{JH}}{g_{1A,\text{naive}}^{JH}} = \frac{g_{1A}^{JH}}{P_p^{JH}\,g_{1p} + P_n^{JH}\,g_{1n}},$$

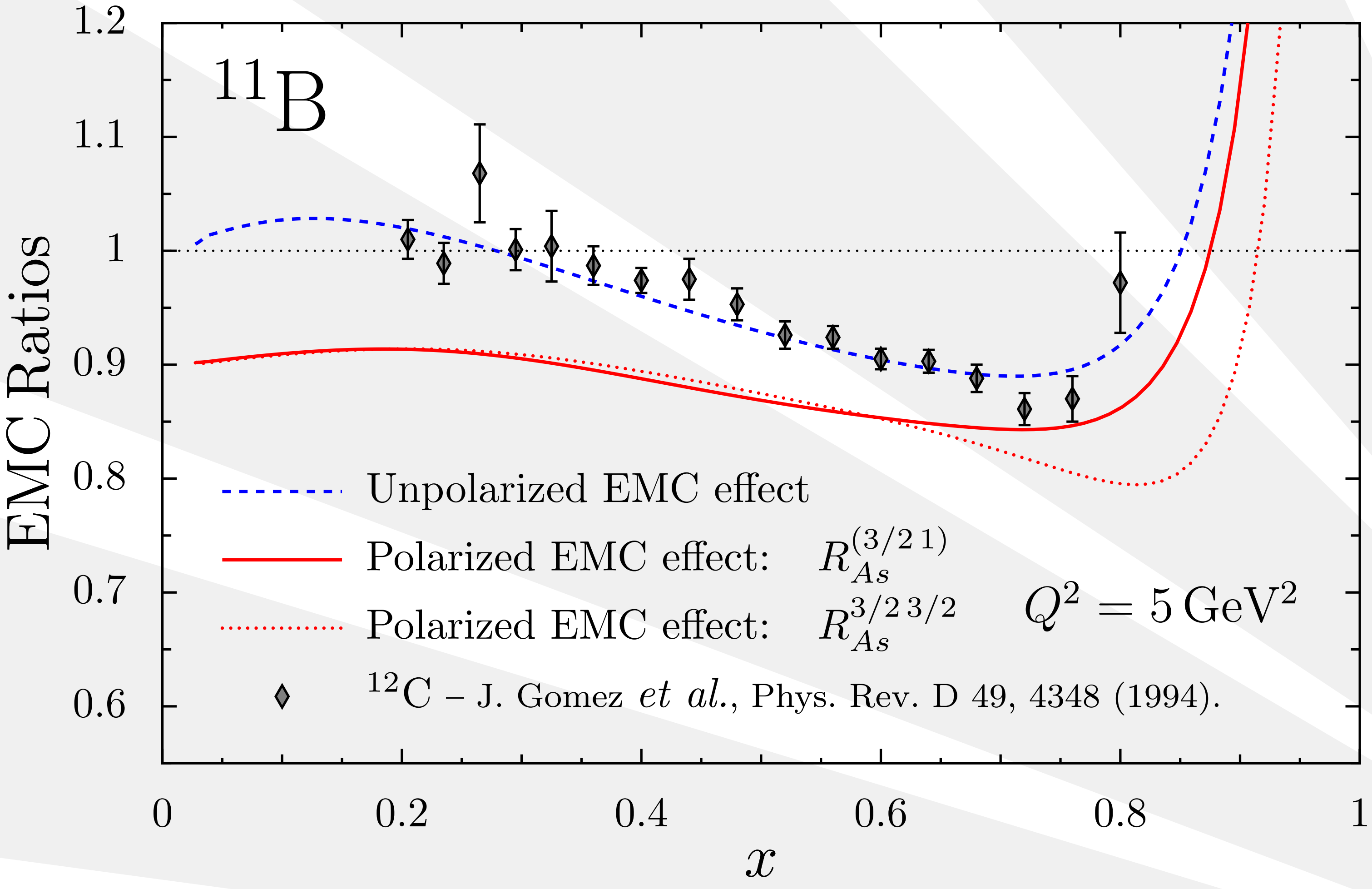
$$R_{As}^{(J1)} = \frac{g_{1A}^{(J1)}}{P_p^{(J1)}\,g_{1p} + P_n^{(J1)}\,g_{1n}}.$$

- Ratios equal 1 in non-rel. and no-medium limit.

EMC ratios ^{7}Li



EMC ratios ^{11}B



EMC ratios ^{15}N

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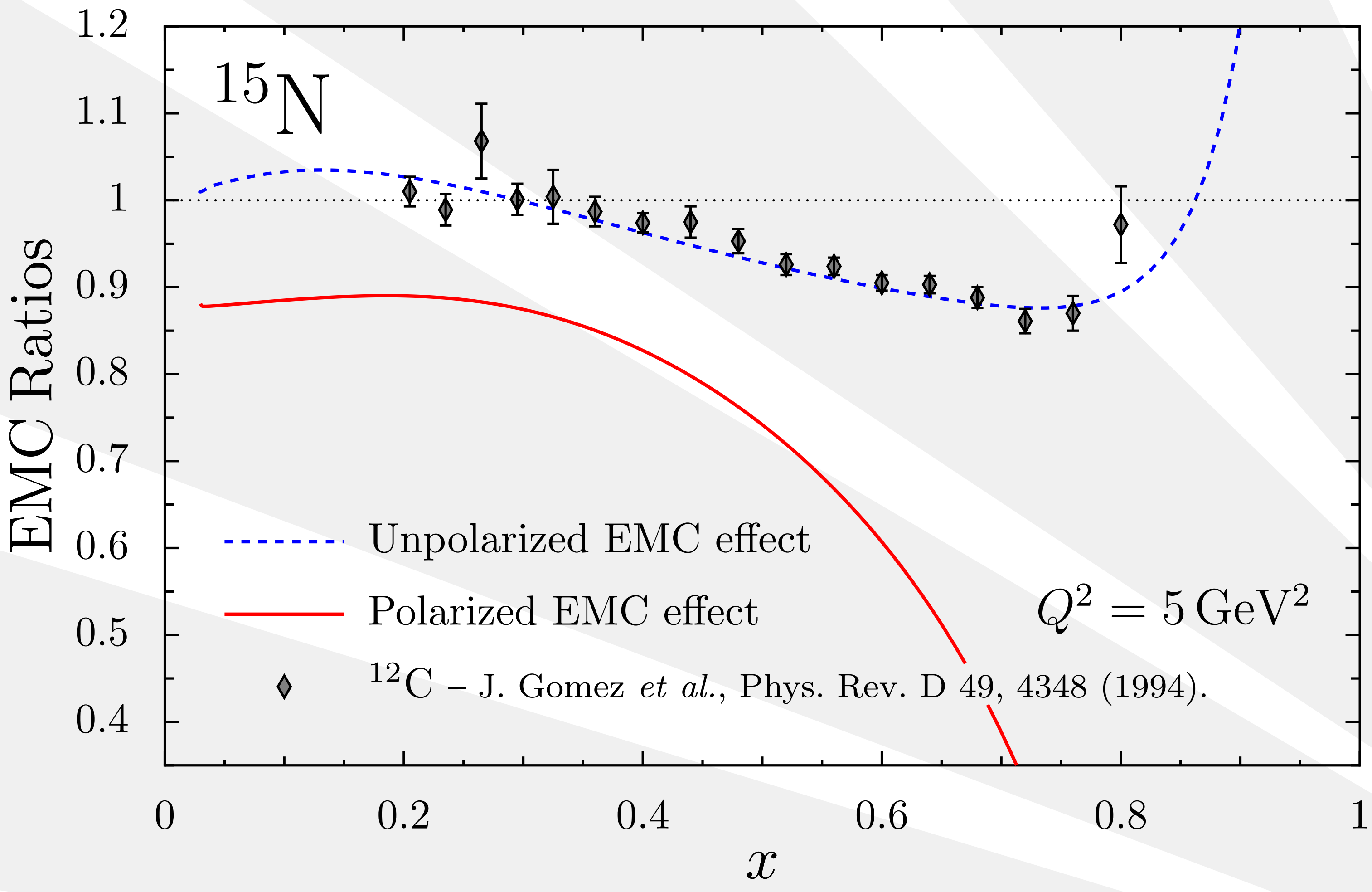
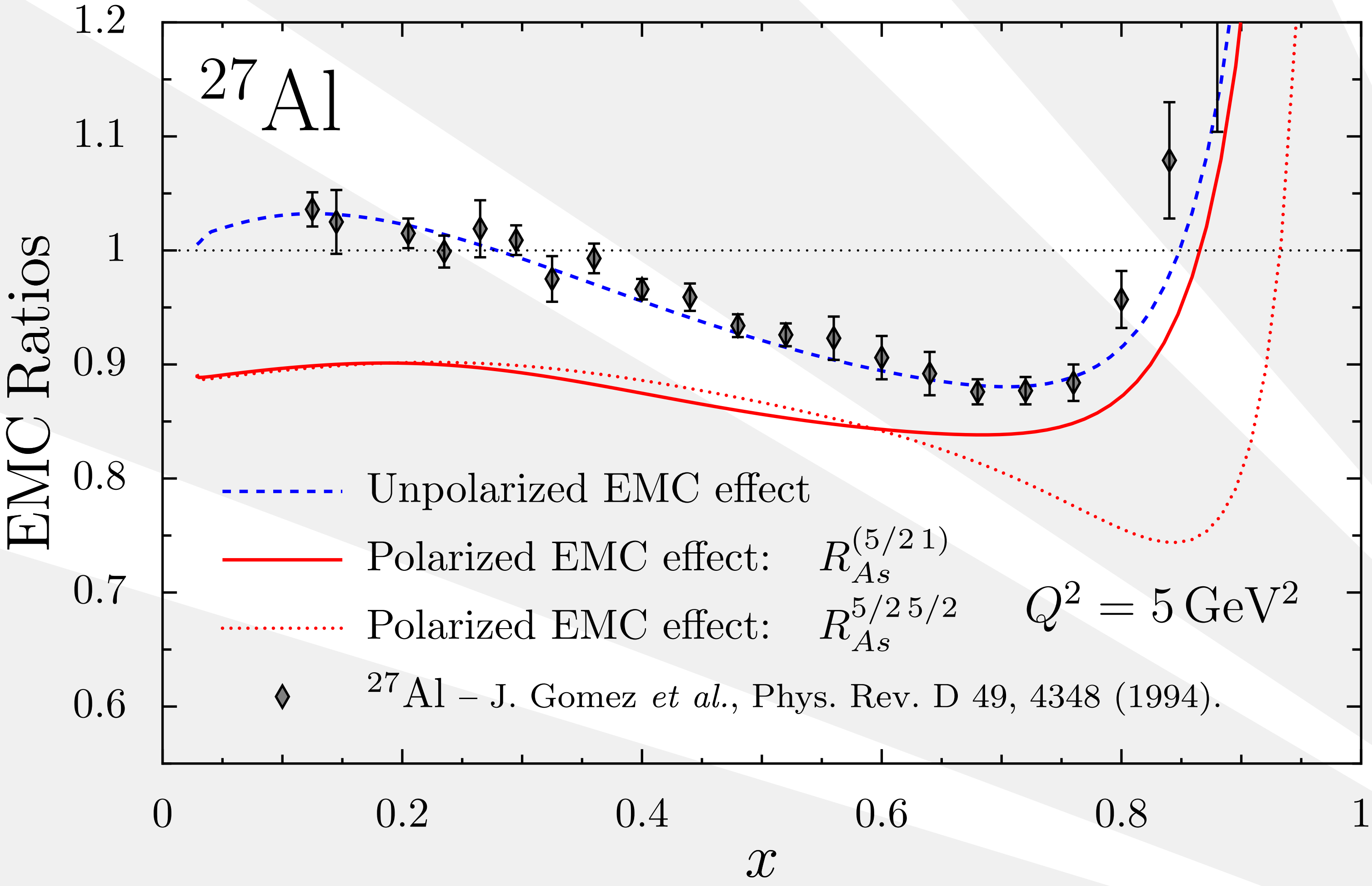
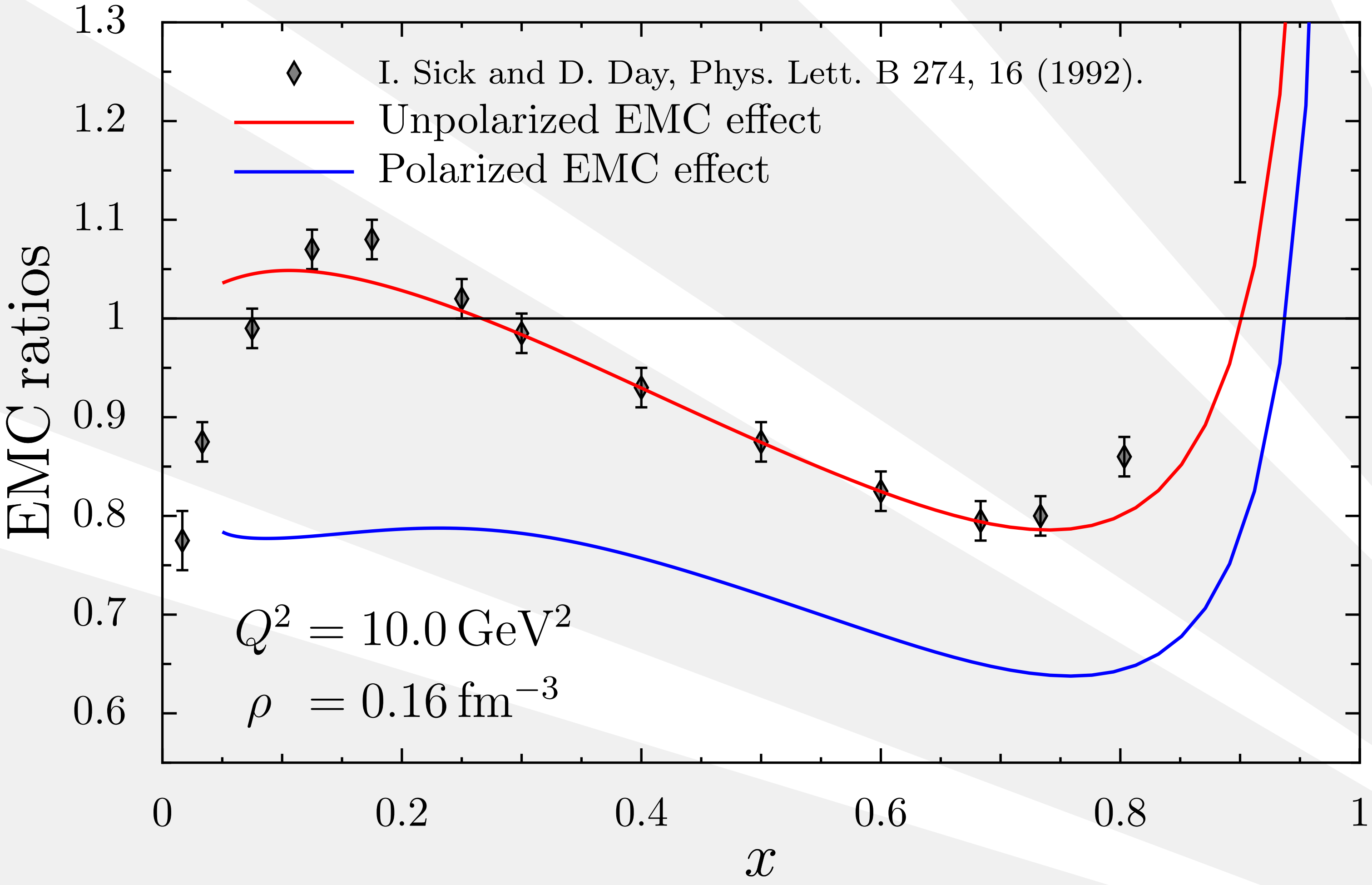


EMC ratios ^{27}Al



Nuclear Matter



Conclusions

- Effective chiral quark theories can be used to incorporate quarks into many-body physics.
- Binding of quarks to mean scalar and vector fields can largely explain the EMC effect.
- Calculated the Polarized EMC effect in nuclei.
  - pEMC effect about twice EMC effect
  - Experimental conformation would yield important insights on quark dynamics in nuclear medium.

Spin sum rules: proton states

Dis.	Δu	Δd	$\Delta\Sigma$	g_A
free	0.967	-0.300	0.667	1.267
^{7}Li	0.882	-0.280	0.602	1.162
^{11}B	0.855	-0.275	0.580	1.130
^{15}N	0.833	-0.268	0.565	1.100
^{27}Al	0.844	-0.271	0.573	1.116
NM	0.740	-0.253	0.487	0.990

Model Parameters

- Free Parameters: Λ_{IR}, Λ_{UV}, M_0, G_π, G_s and G_a.
- Constraints:
  - $f_\pi = 93$ MeV, $m_\pi = 140$ MeV and $M_N = 940$ MeV
  - $(\rho, E_B/A) = (0.16\,\text{fm}^{-3}, -15.7\text{ MeV})$
  - $\int_0^1 dx\ (\Delta u_v(x) - \Delta d_v(x)) = g_A = 1.267$
- We obtain:
  - $\Lambda_{IR} = 240$ MeV, $\Lambda_{UV} = 644$ MeV, $M_0 = 400$ MeV
  - $G_\pi = 19\text{ GeV}^{-2}$, $G_s = 7.5\text{ GeV}^{-2}$, $G_a = 2.8\text{ GeV}^{-2}$
  - $M_s = 690$ MeV, $M_a = 990$ MeV,

Quark/Probability Distributions

- $q_s^{JH}(x_A)$: probability to find a quark with momentum fraction x_A and spin-component s_z in nucleus with $J_z = H$.
- The familiar quark distributions are
  - $q^{JH}(x) \quad = q_+^{JH}(x) + q_-^{JH}(x)$ unpolarized
  - $\Delta q^{JH}(x) = q_+^{JH}(x) - q_-^{JH}(x)$ longitudinally polarized
- There are $2J + 1$ independent quark distributions

Regularization

- Proper-time regularization

$$\frac{1}{X^n} = \frac{1}{(n-1)!} \int_0^\infty d\tau\, \tau^{n-1}\, e^{-\tau X}$$

$$\longrightarrow \quad \frac{1}{(n-1)!} \int_{1/(\Lambda_{UV})^2}^{1/(\Lambda_{IR})^2} d\tau\, \tau^{n-1}\, e^{-\tau X}.$$

- IR-cutoff eliminates unphysical thresholds for hadrons decaying into quarks and mesons. ⟶ simulates confinement.
- Need this to obtain nuclear matter saturation.
  W. Bentz, A.W. Thomas, Nucl. Phys. A **696**, *138 (2001)*

Dirac Equation

- Spherically potentials: $V_s(r)$, $V^\mu(r) = \left(V_v(r), \vec{0}\right)$

$$\left[-i\,\vec{\alpha}\cdot\vec{\nabla} + \beta\left[M(r) - V_s(r)\right] + V_v(r)\right]\psi_\kappa(r) = \varepsilon_\kappa\,\psi_\kappa(r),$$

- Use Woods-Saxon potentials

$$S_N(r) = \frac{S_0}{1+\exp\left(\frac{r-R_0}{a_0}\right)}, \quad V_N(r) = \frac{V_0}{1+\exp\left(\frac{r-R_0}{a_0}\right)},$$

- Standard values: $a_0 = 1.2$ fm and $r_0 = 0.65$ fm, where $R_0 = r_0\,A^{1/3}$.
- Nuclear matter: $S_0 = -194$ MeV and $V_0 = 133$ MeV.

Quark distributions in the Proton

- Spin-independent

$$u_v(x) = f^s_{q/P}(x) + \frac{1}{2}\, f^s_{q(D)/P}(x) + \frac{1}{3}\, f^a_{q/P}(x) + \frac{5}{6}\, f^a_{q(D)/P}(x),$$

$$d_v(x) = \frac{1}{2}\, f^s_{q(D)/P}(x) + \frac{2}{3}\, f^a_{q/P}(x) + \frac{1}{6}\, f^a_{q(D)/P}(x),$$

- Spin-dependent

$$\begin{aligned}\Delta\, u_v(x) &= f^s_{q/P}(x) + \frac{1}{2}\, f^s_{q(D)/P}(x) + \frac{1}{3}\, f^a_{q/P}(x) \\ &\quad + \frac{5}{6}\, f^a_{q(D)/P}(x) + \frac{1}{2\sqrt{3}}\, f^m_{q(D)/P}(x),\end{aligned}$$

$$\begin{aligned}\Delta\, d_v(x) &= \frac{1}{2}\, f^s_{q(D)/P}(x) + \frac{2}{3}\, f^a_{q/P}(x) \\ &\quad + \frac{1}{6}\, f^a_{q(D)/P}(x) - \frac{1}{2\sqrt{3}}\, f^m_{q(D)/P}(x),\end{aligned}$$

Dirac Equation cont'd

$$\left[-i\,\vec{\alpha}\cdot\vec{\nabla}+\beta\,[M(r)-V_s(r)]+V_v(r)\right]\psi_\kappa(r)=\varepsilon_\kappa\,\psi_\kappa(r),$$

- Nucleon mass and vector potential

$$M_{N\kappa}=\int d^3r\,\psi_\kappa^\dagger(r)\,M_N(r)\psi_\kappa(r),$$
$$V_{N\kappa}=\int d^3r\,\psi_\kappa^\dagger(r)\,V_N(r)\psi_\kappa(r).$$

- Example ^{12}C (All units are in MeV.)

κ	Level	Energy	M_N	V_N
-1	$s_{1/2}$	908	793	100.8
-2	$p_{3/2}$	925	828	76.5
1	$p_{1/2}$	927	829	76.0

Interaction Lagrangians

- Using Fierz transformation can decompose $\mathcal{L}_I$ into sum of qq interaction terms.

$$\mathcal{L}_{I,s} = G_s \left(\overline{\psi}\,\gamma_5\,C\,\tau_2\,\beta^A\overline{\psi}^T\right)\left(\psi^T\,C^{-1}\,\gamma_5\,\tau_2\,\beta^A\psi\right),$$

$$\mathcal{L}_{I,a} = G_a \left(\overline{\psi}\,\gamma_\mu\,C\,\vec{\tau}\tau_2\,\beta^A\overline{\psi}^T\right)\left(\psi^T C^{-1}\gamma_\mu\,\vec{\tau}\tau_2\,\beta^A\psi\right).$$

- Solving BS equation gives

$$\tau_s(q) = \frac{4iG_s}{1 + 2\,G_s\,\Pi_s(q^2)} \quad\longrightarrow\quad 4iG_s - \frac{ig_s}{q^2 - M_s^2 + i\varepsilon}$$

$$\tau_a^{\mu\nu}(q) = 4\,i\,G_a\left[g^{\mu\nu} - \frac{2G_a\Pi_a(q^2)}{1 + 2G_a\Pi_a(q^2)}\left(g^{\mu\nu} - \frac{q^\mu\,q^\nu}{q^2}\right)\right],$$

$$\longrightarrow\quad 4\,i\,G_a - \frac{ig_a}{q^2 - M_a^2}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{M_a^2}\right)$$

The Quark Distributions $f(x)$ and $\Delta f(x)$.

- Formally

$$q(x) = p_- \int \frac{d\xi^-}{2\pi} e^{i\,x\,p^+\,\xi^-} \langle p, s|\overline{\psi}(0)\gamma^+\psi(\xi^-)|p, s\rangle_c,$$

$$\Delta q(x) = p_- \int \frac{d\xi^-}{2\pi} e^{i\,x\,p^+\,\xi^-} \langle p, s|\overline{\psi}(0)\gamma^+\gamma_5\psi(\xi^-)|p, s\rangle_c.$$

- Can show

$$f(x) = -i \int \frac{d^4k}{(2\pi)^4}\delta(x - \frac{k^+}{p^+})\ \mathrm{Tr}\left[\gamma^+ M(p, k)\right],$$

$$\Delta f(x) = -i \int \frac{d^4k}{(2\pi)^4}\delta(x - \frac{k^+}{p^+})\ \mathrm{Tr}\left[\gamma^+\gamma_5 M(p, k)\right].$$