Semi-inclusive processes at low and high transverse momentum

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Jefferson Lab
N. Isgur Fellowship

Based on:

AB, Daniel Boer, Markus Diehl, Piet J. Mulders

arXiv:0803.0227 [hep-ph]

Outline

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- High and low transverse momentum

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- Matches, expected mismatches, unexpected mismatches

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- High and low transverse momentum
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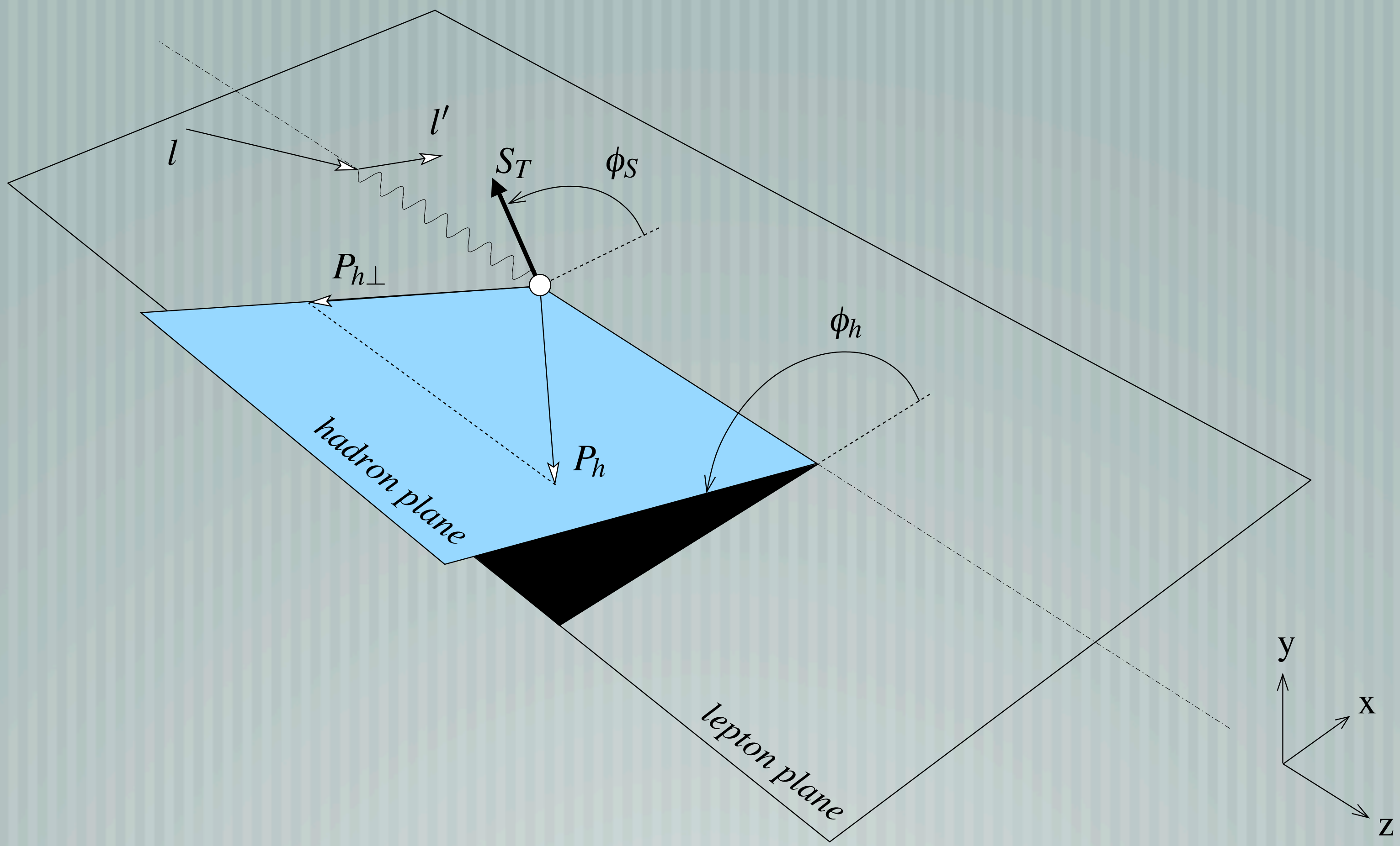
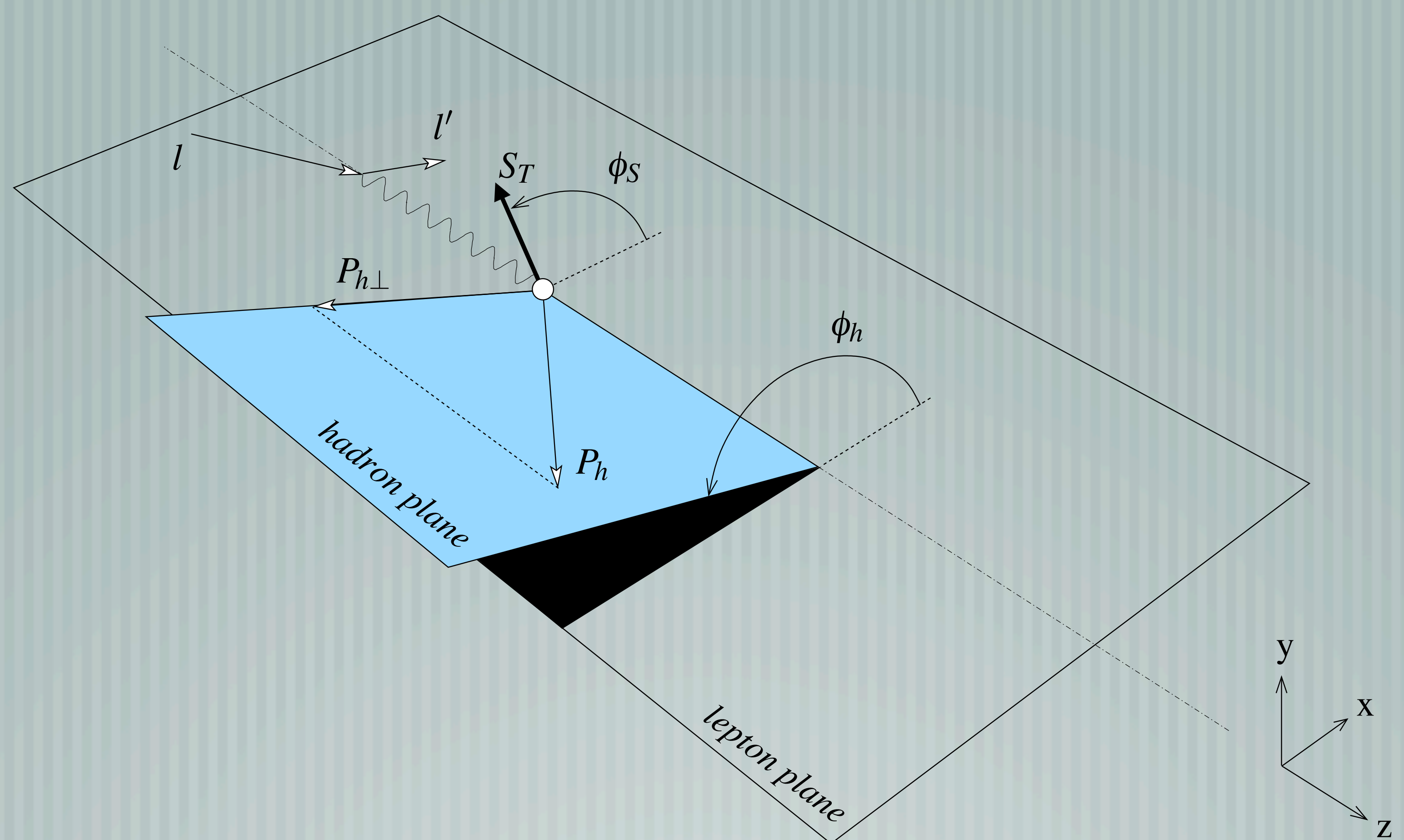
- Calculations at high and low transverse momentum
- Selected examples of matching and mismatching structure functions
- Some consequences relevant to phenomenology
- Summary of all structure functions

Semi-inclusive DIS



Semi-inclusive DIS



Q = photon virtuality

M = hadron mass

$P_{h\perp}$ = hadron transverse momentum

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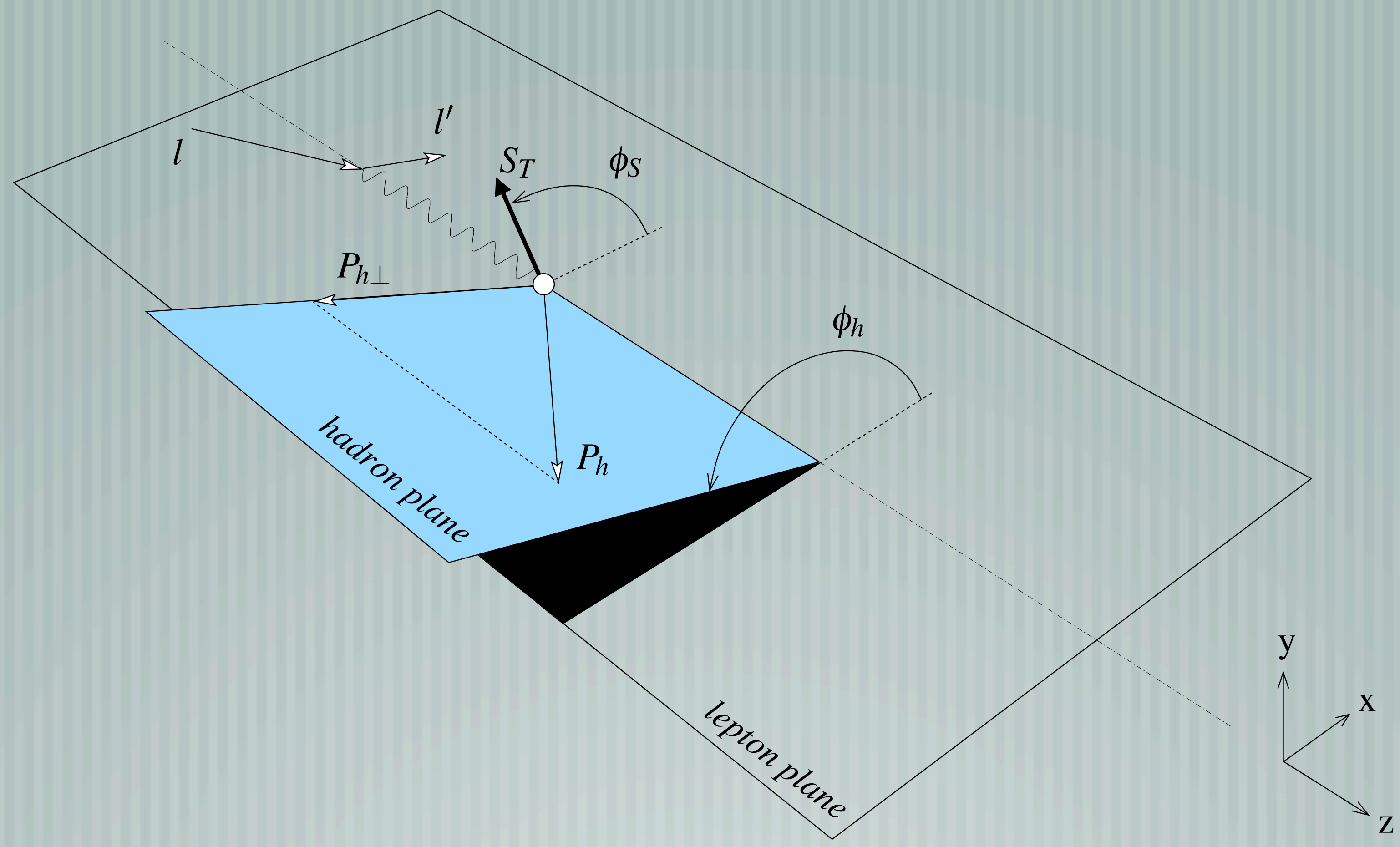


Q = photon virtuality

M = hadron mass

$P_{h\perp}$ = hadron transverse momentum

$$q_T^2 \approx P_{h\perp}^2 / z^2$$

SIDIS structure functions

$$\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2}$$

$$= \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\biggl\{F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}$$

$$+ \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} + S_L\biggl[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\biggr]$$

$$+ S_L\,\lambda_e\biggl[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\biggr]$$

$$+ S_T\biggl[\sin(\phi_h-\phi_S)\Bigl(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\Bigr) + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}$$

$$+ \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}$$

$$+ \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\biggr] + S_T\lambda_e\biggl[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}$$

$$+ \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\biggr]\biggr\}$$

see e.g. AB, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP093 (07)

SIDIS structure functions

$$F_{UU,T}(x, z, P_{h\perp}^2, Q^2)$$

$$\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2}$$

$$= \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left\{F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}\right.$$

$$+ \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} + S_L\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right]$$

$$+ S_L\,\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right]$$

$$+ S_T\left[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}\right.$$

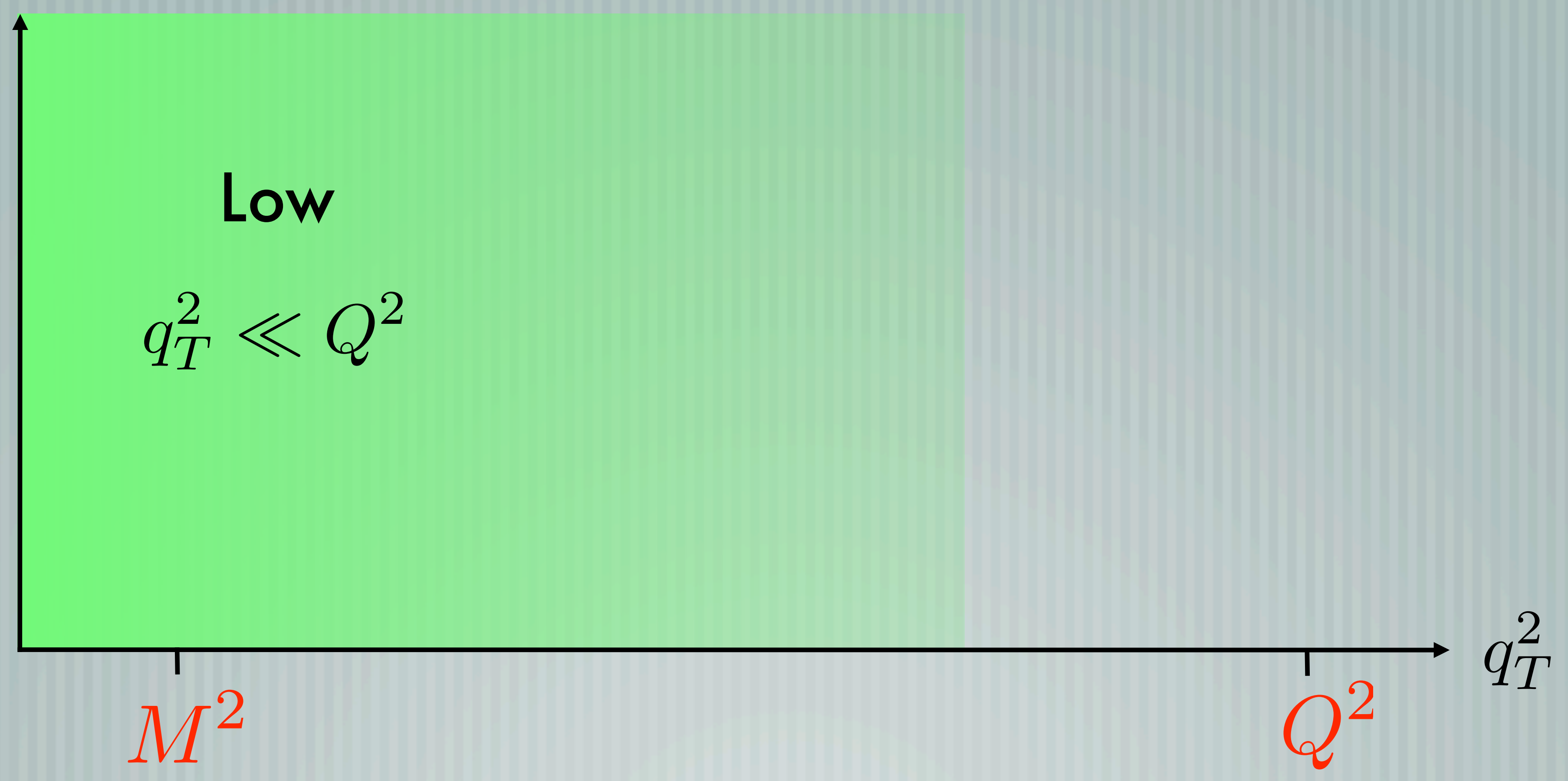
$$+ \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}$$

$$\left.+ \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\right] + S_T\lambda_e\left[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}\right.$$

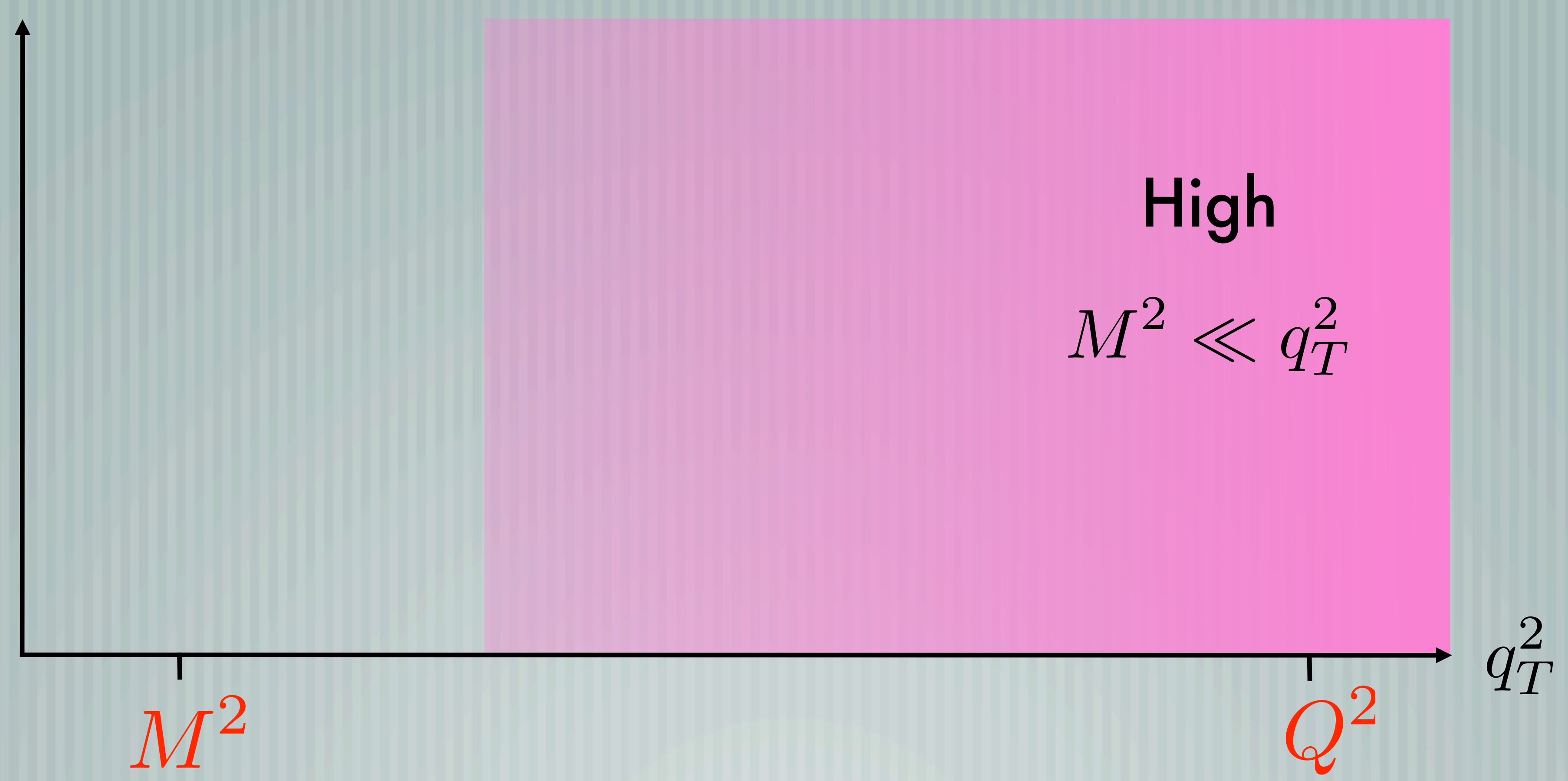
$$\left.\left.+ \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\right]\right\}$$

see e.g. AB, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP093 (07)

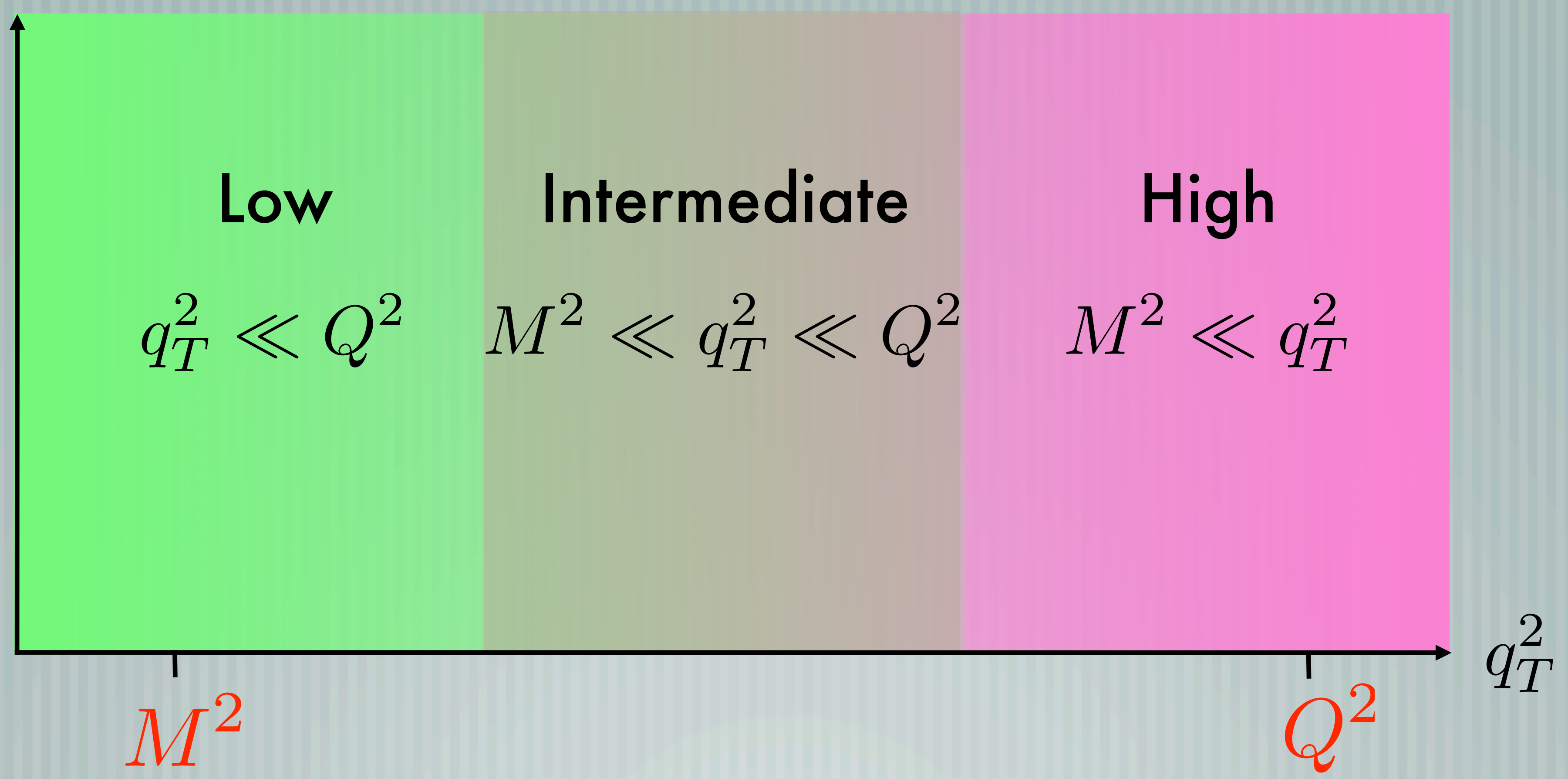
High and low transverse momentum



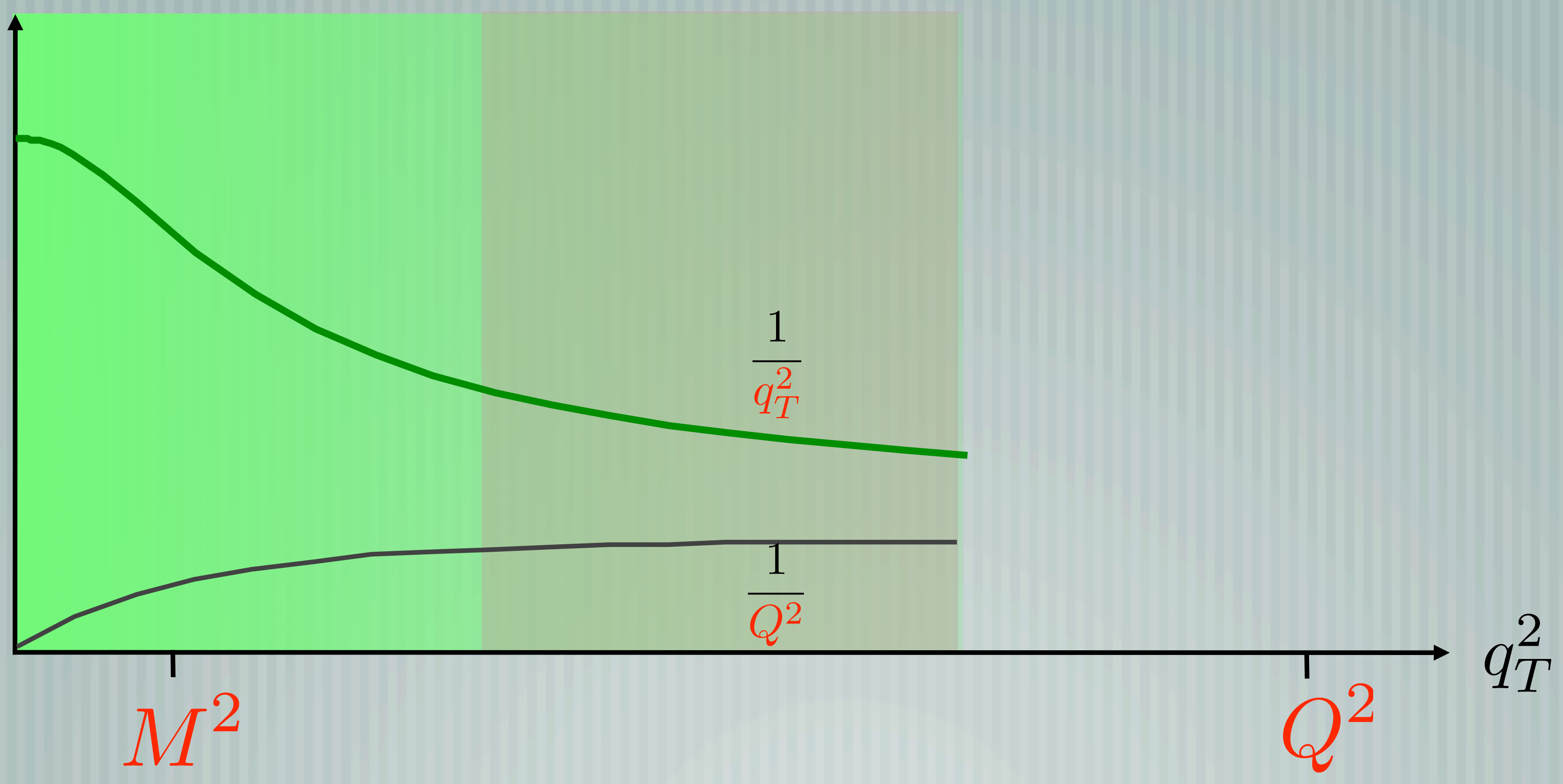
High and low transverse momentum



High and low transverse momentum

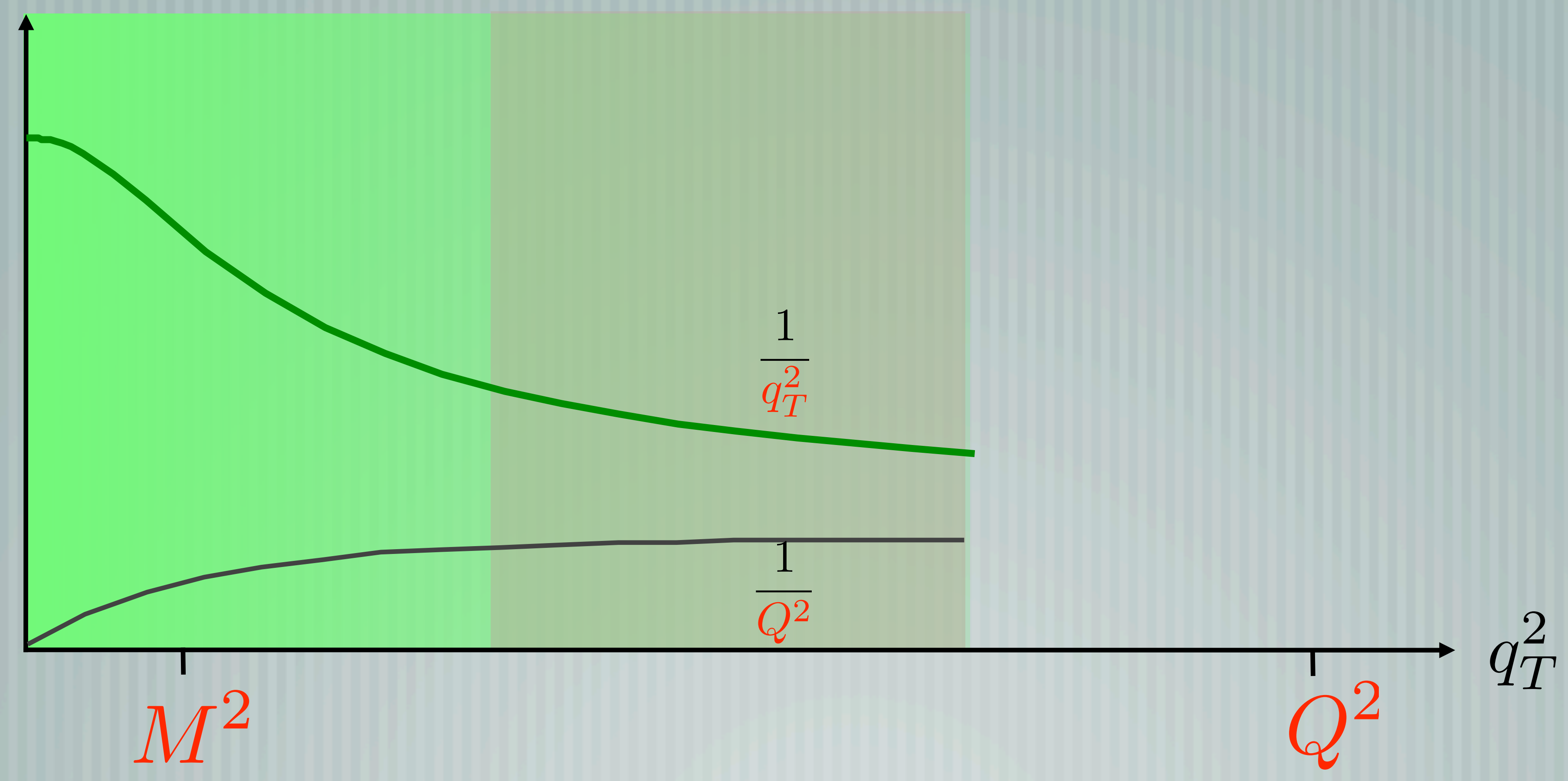


Expansion at low trans. momentum



Expansion at low trans. momentum

First do the $1/Q$ expansion



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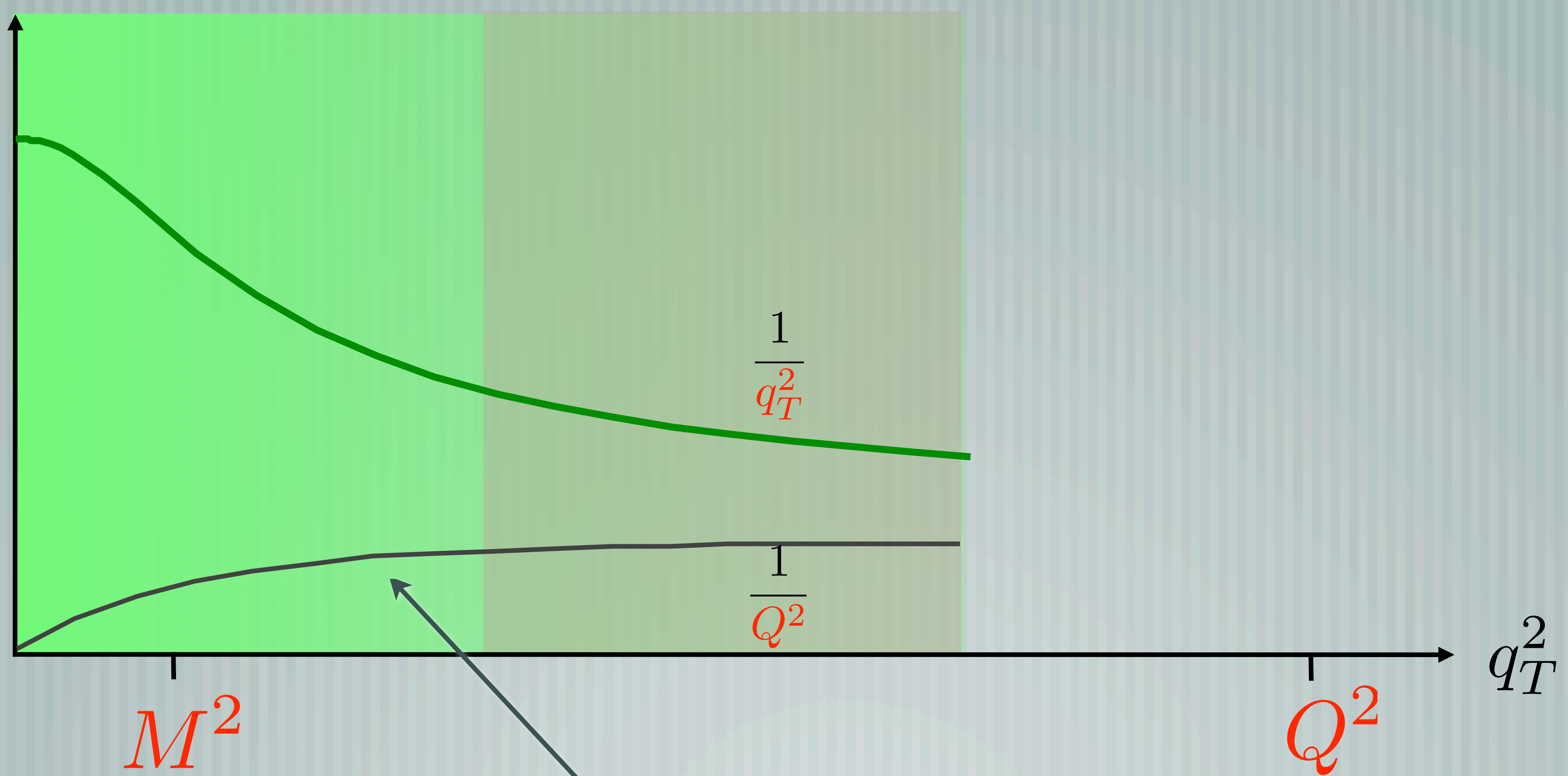
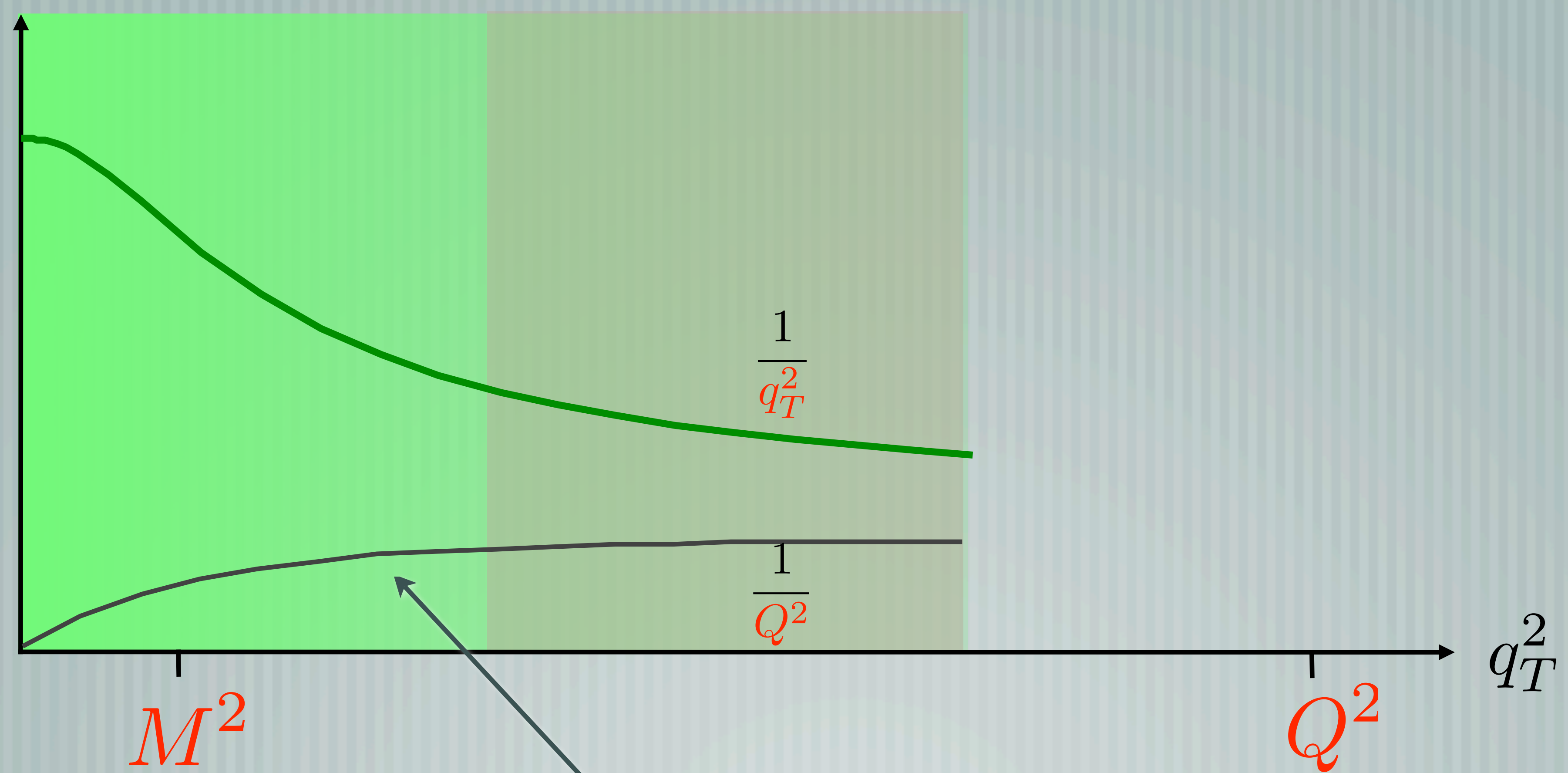


This term is suppressed (twist-4)

Expansion at low trans. momentum

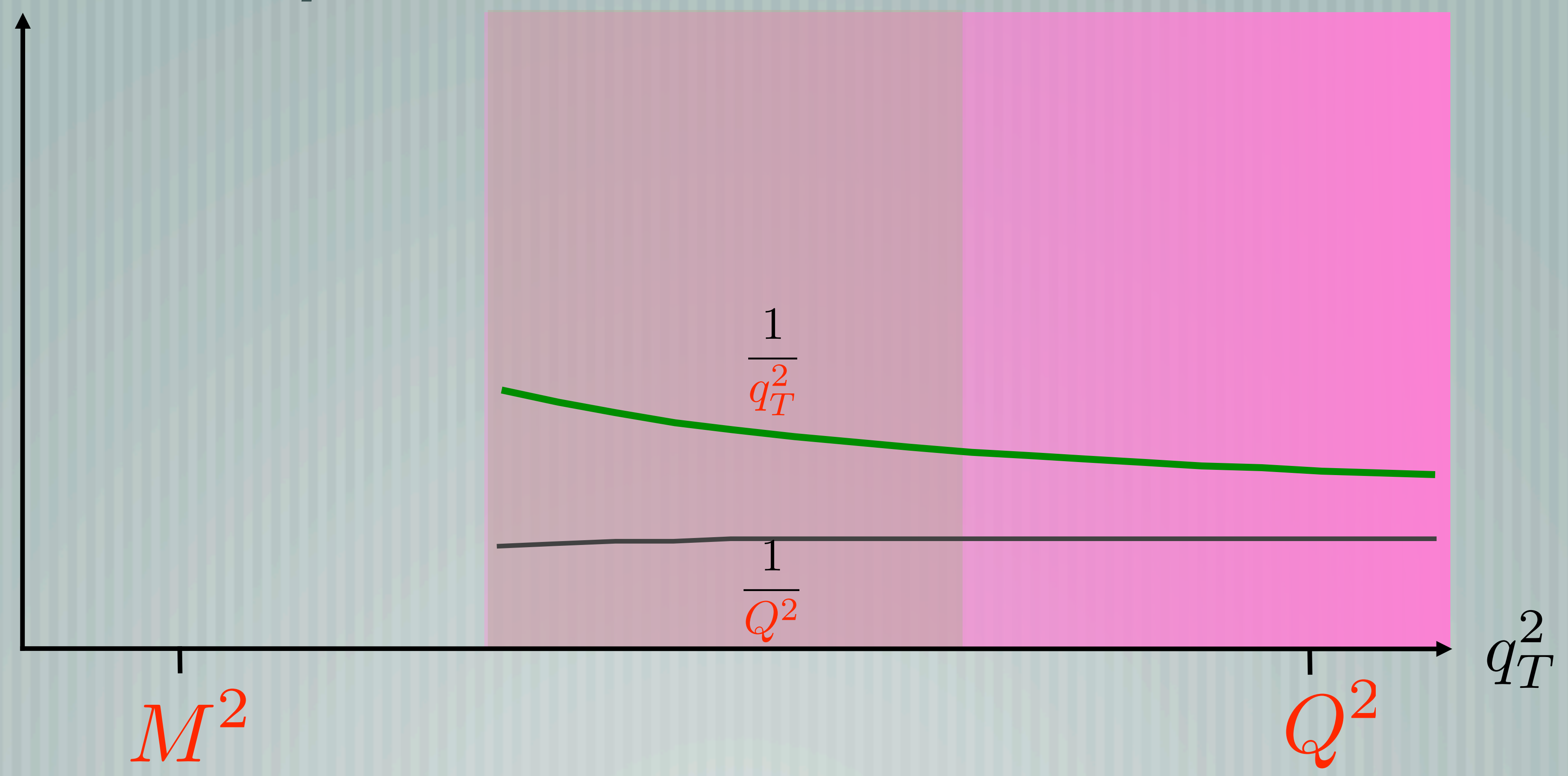
Then study the behavior at intermediate q_T



This term is still suppressed because $q_T << Q$

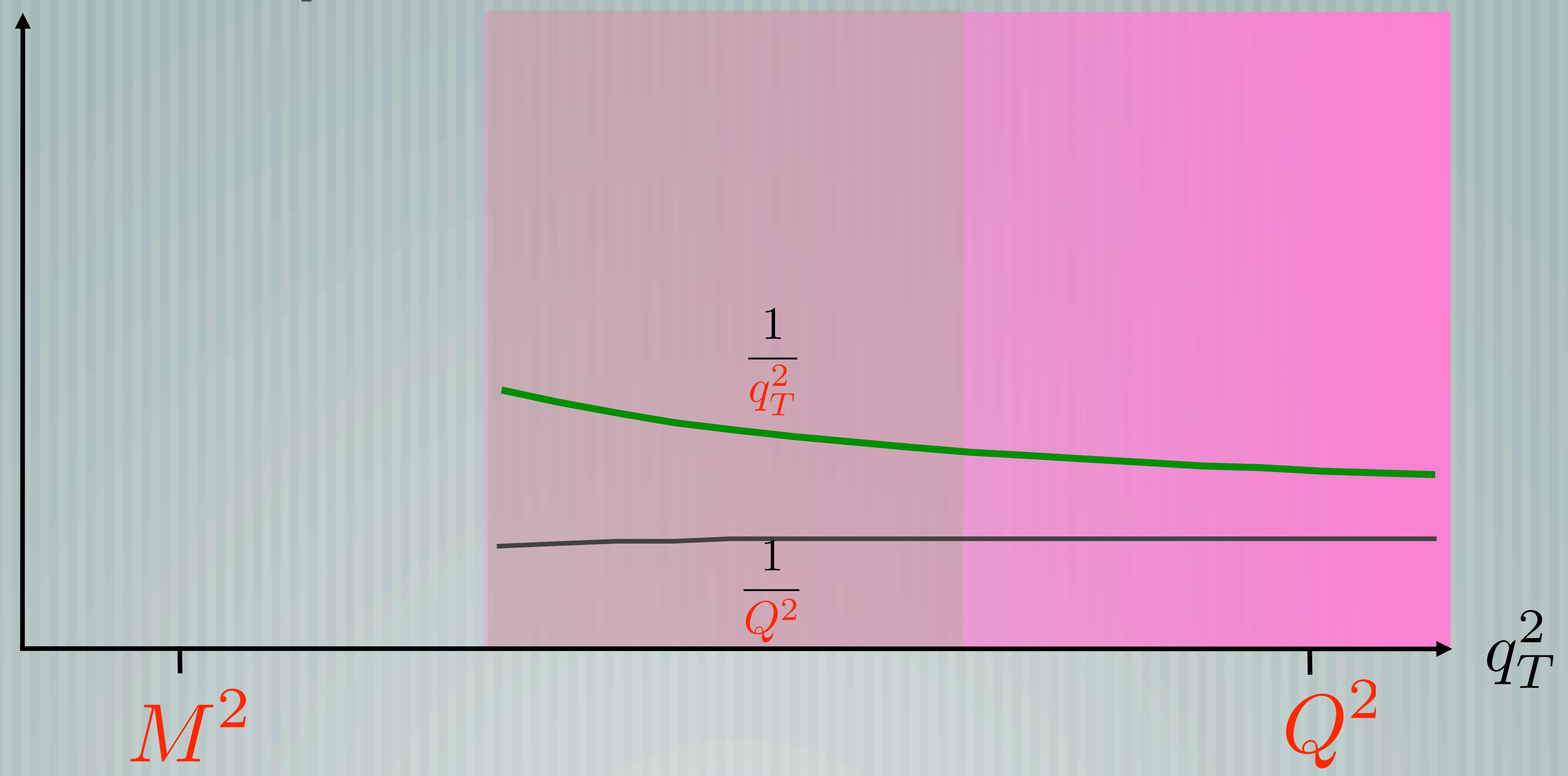
Expansion at high trans. momentum

At high q_T the two terms are equally important



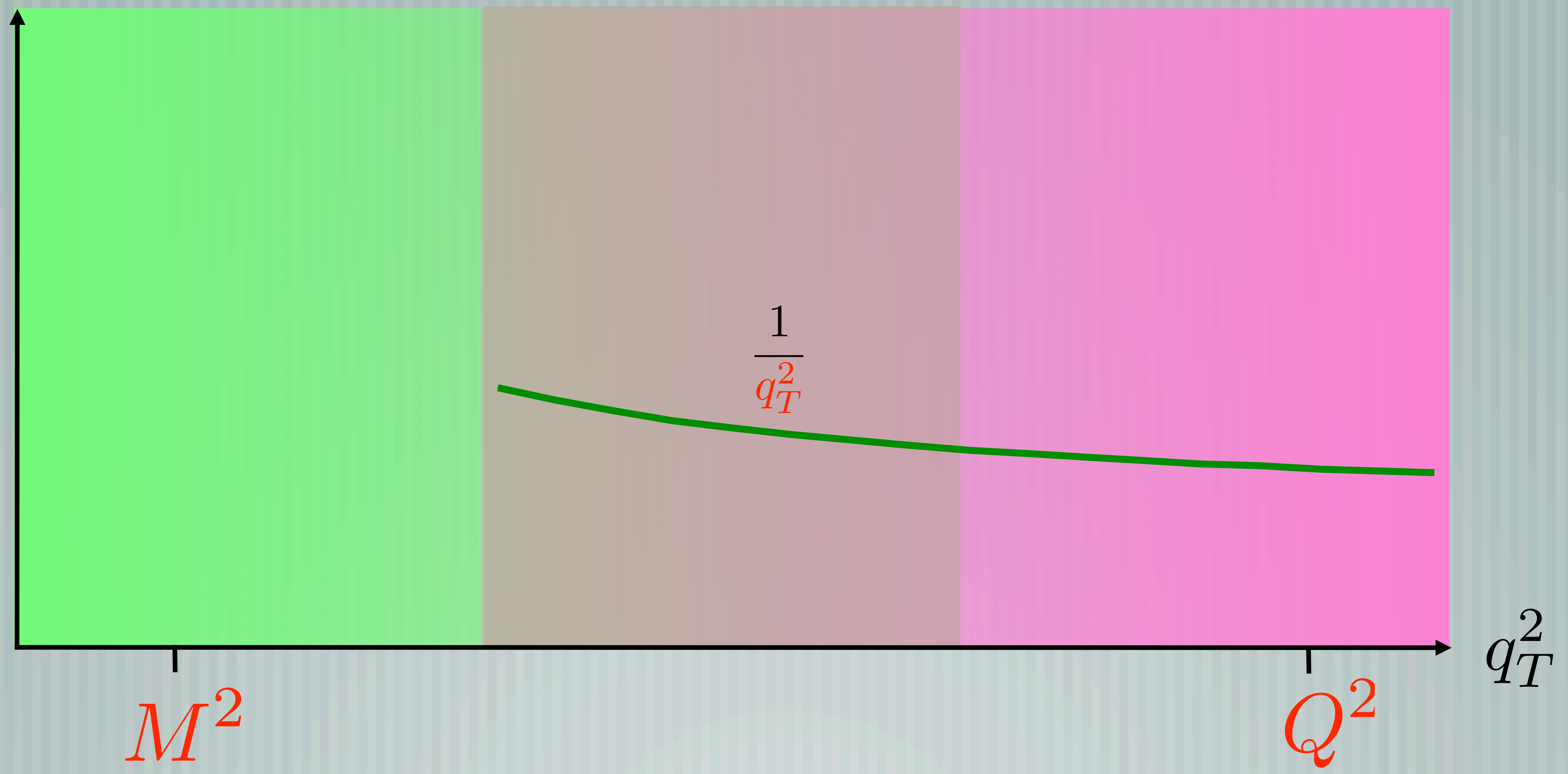
Expansion at high trans. momentum

At high q_T the two terms are equally important

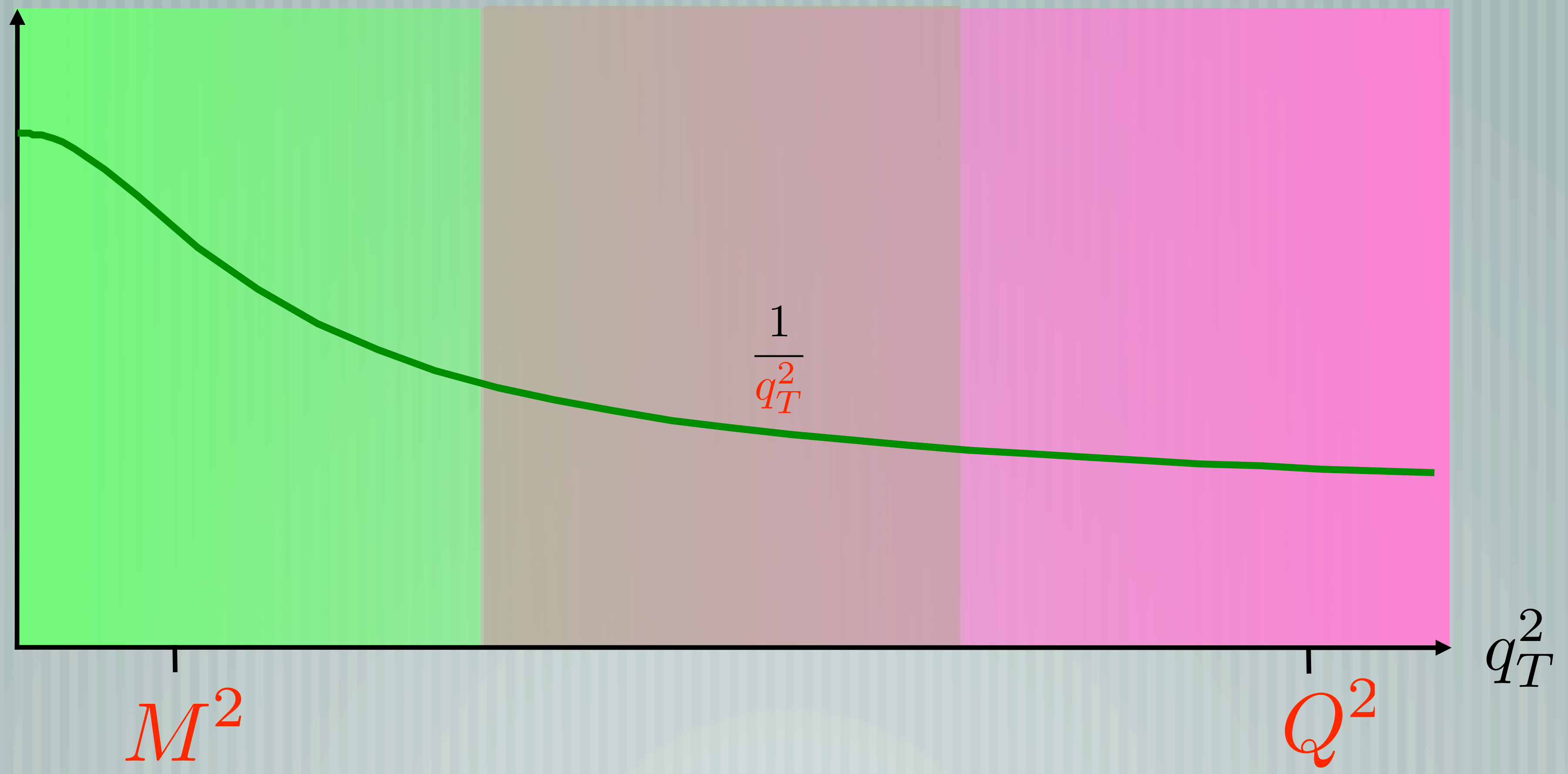


Going to intermediate q_T the upper term becomes dominant

Matching



Matching



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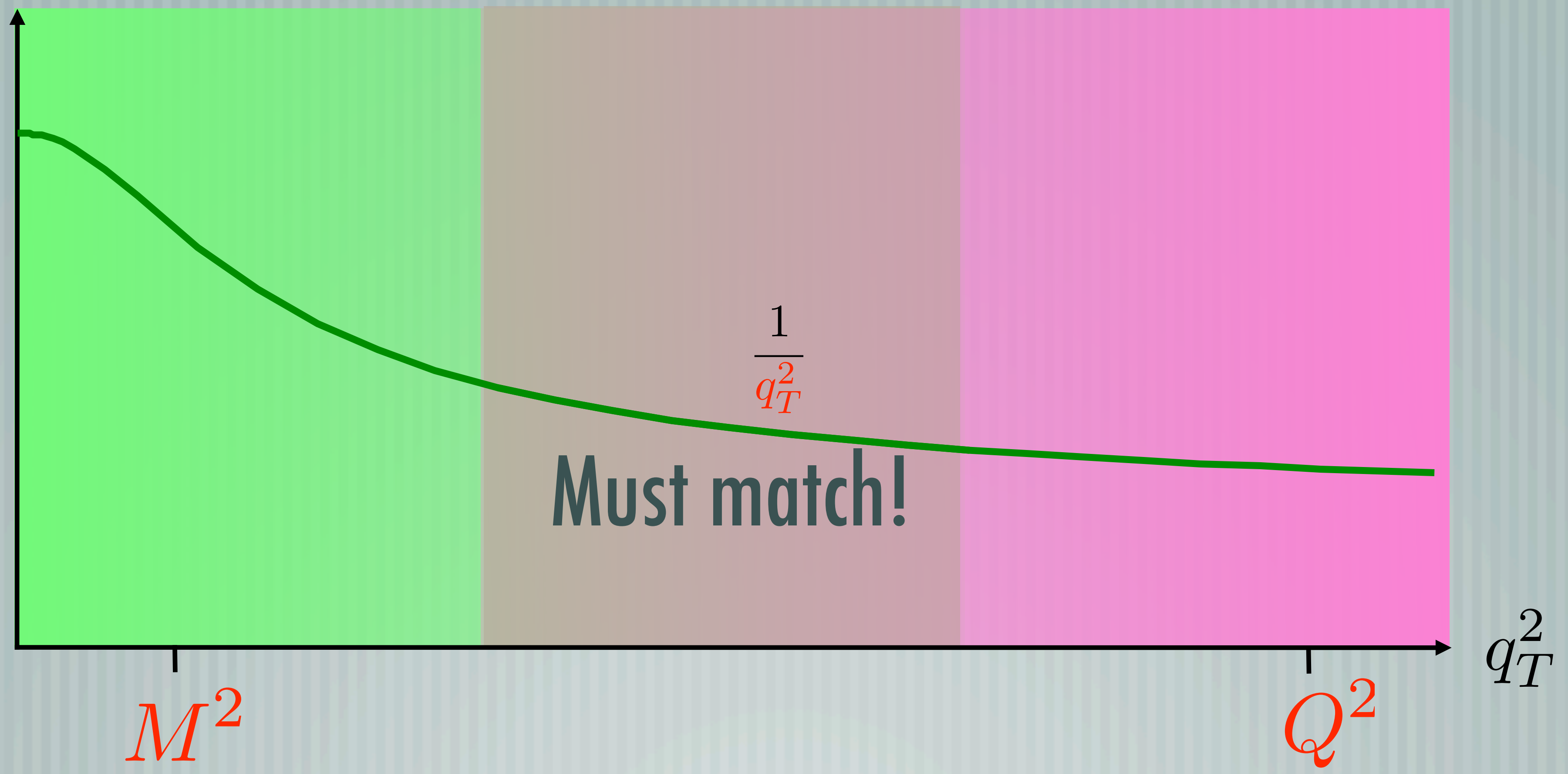

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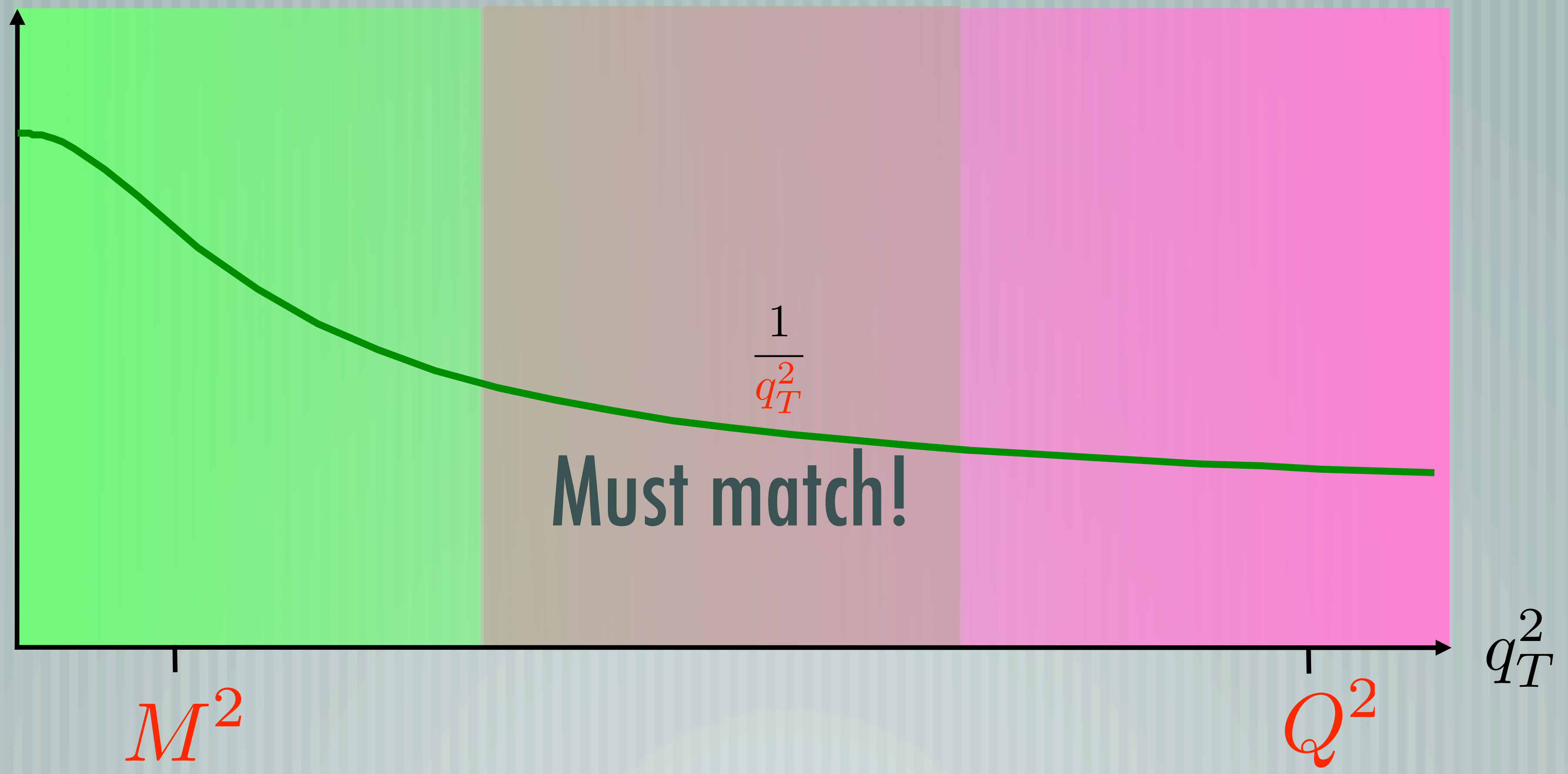
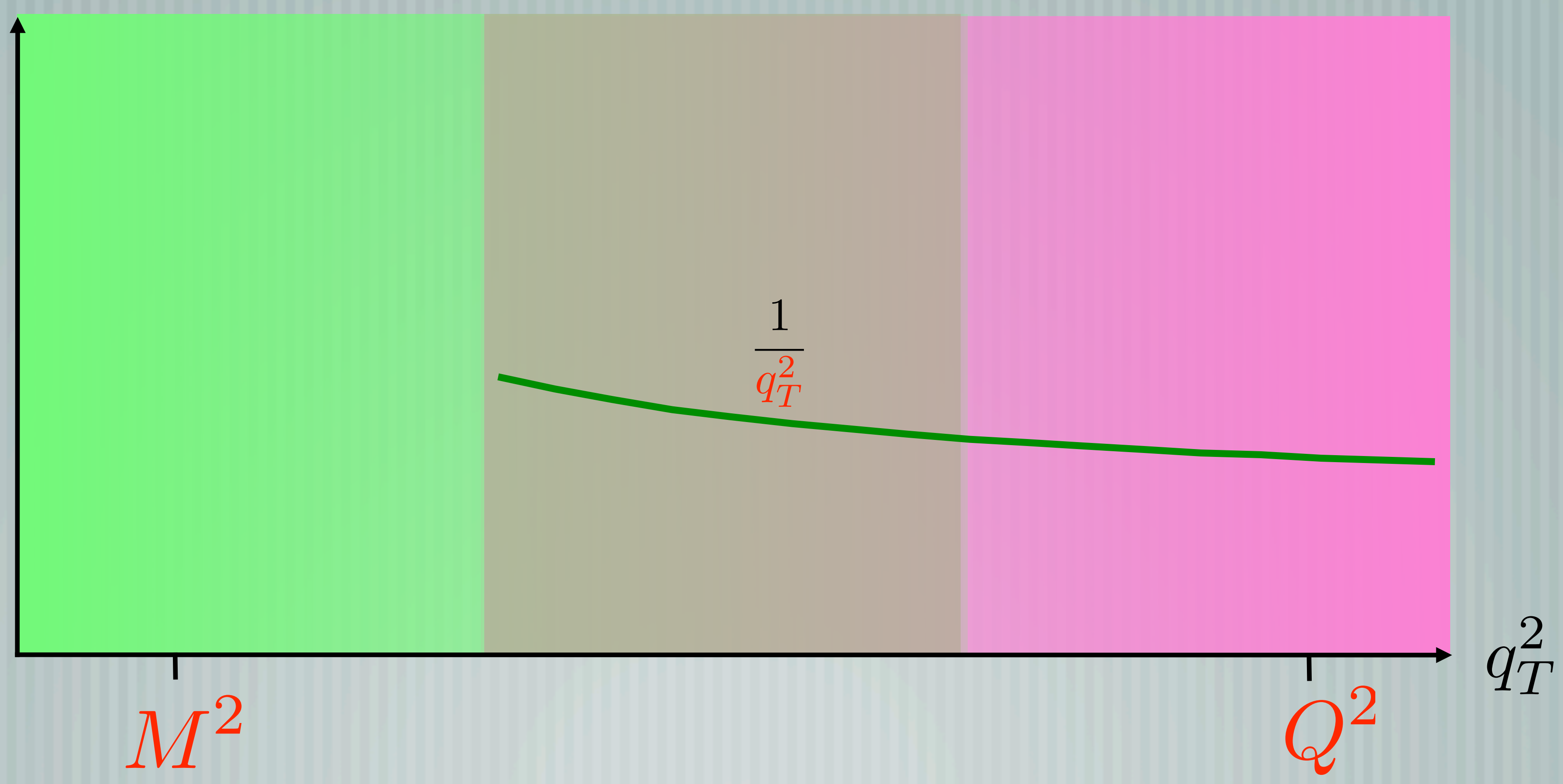
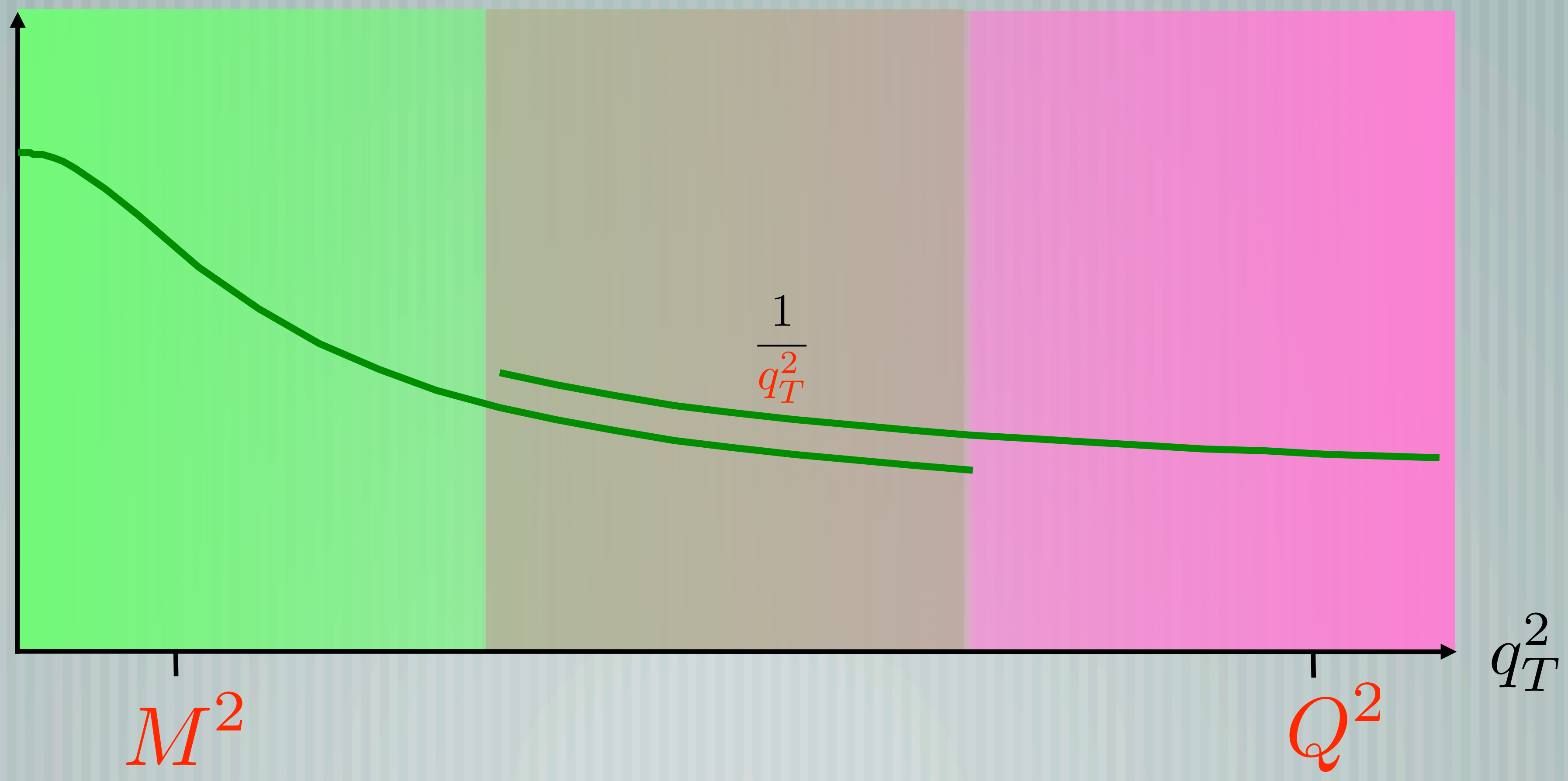


The perturbative part is the "tail" of the nonperturbative part

Unexpected mismatch

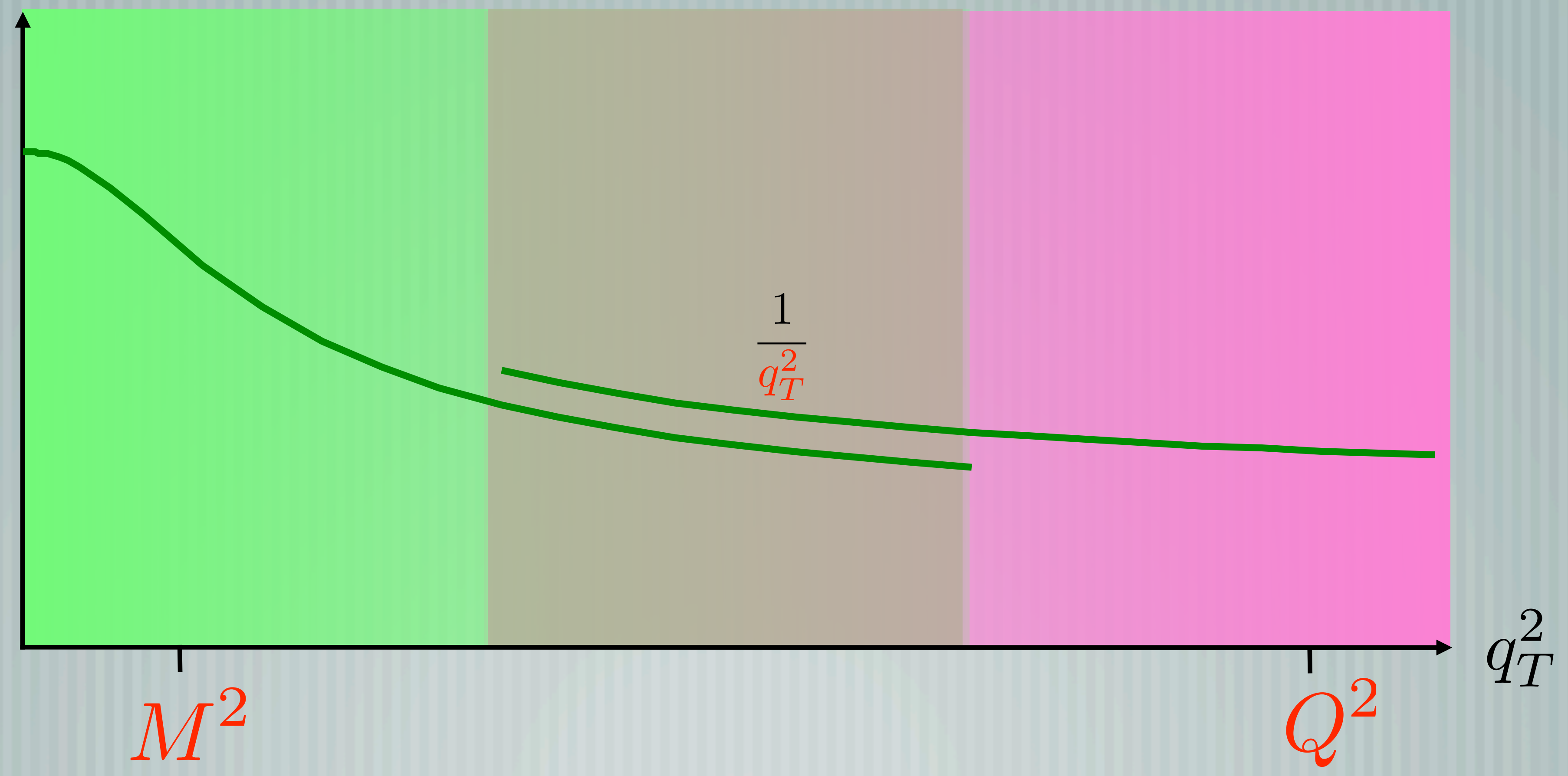


Unexpected mismatch

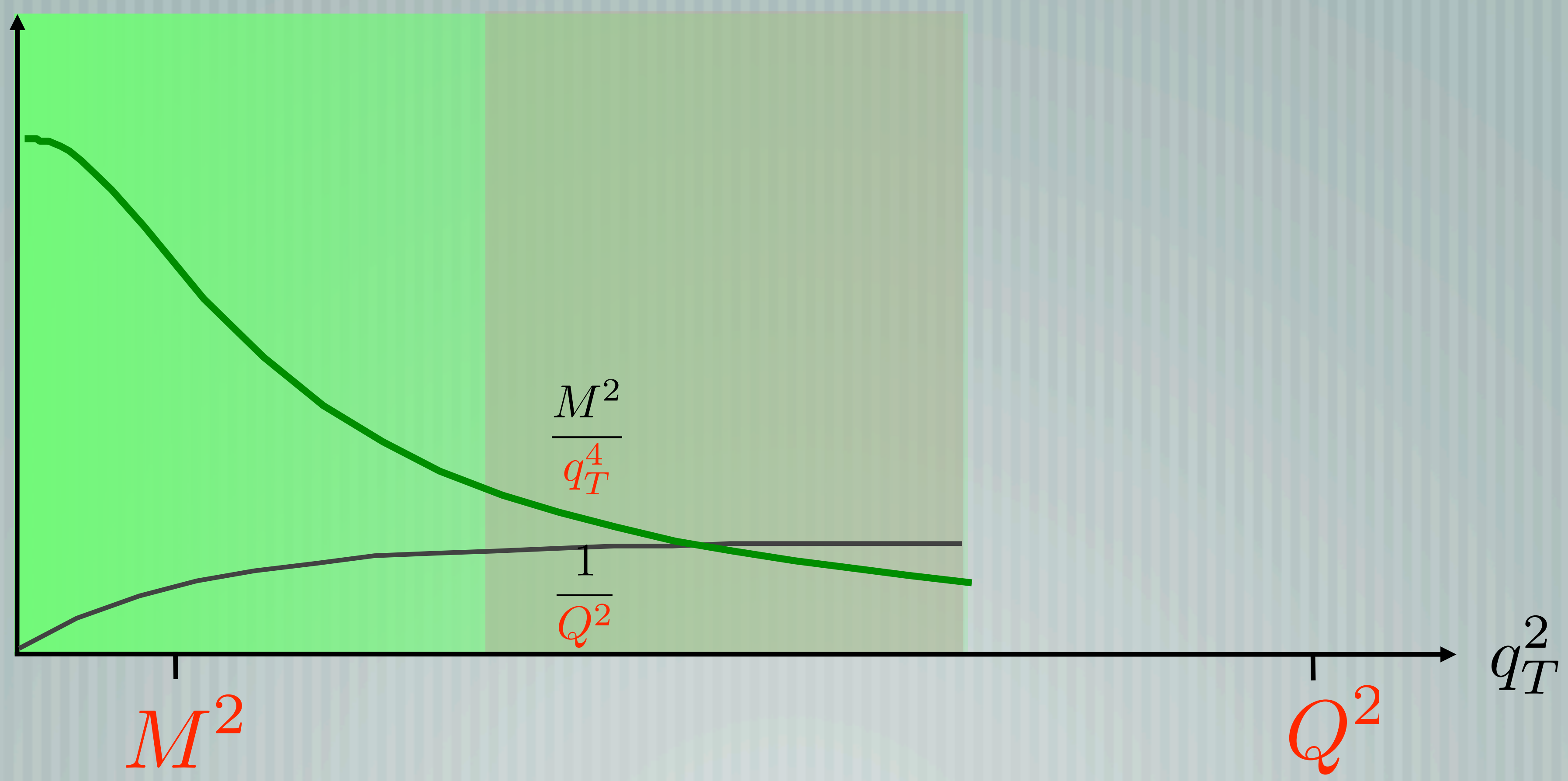


Unexpected mismatch

We are neglecting something that cannot be neglected...



Expansion at low trans. momentum



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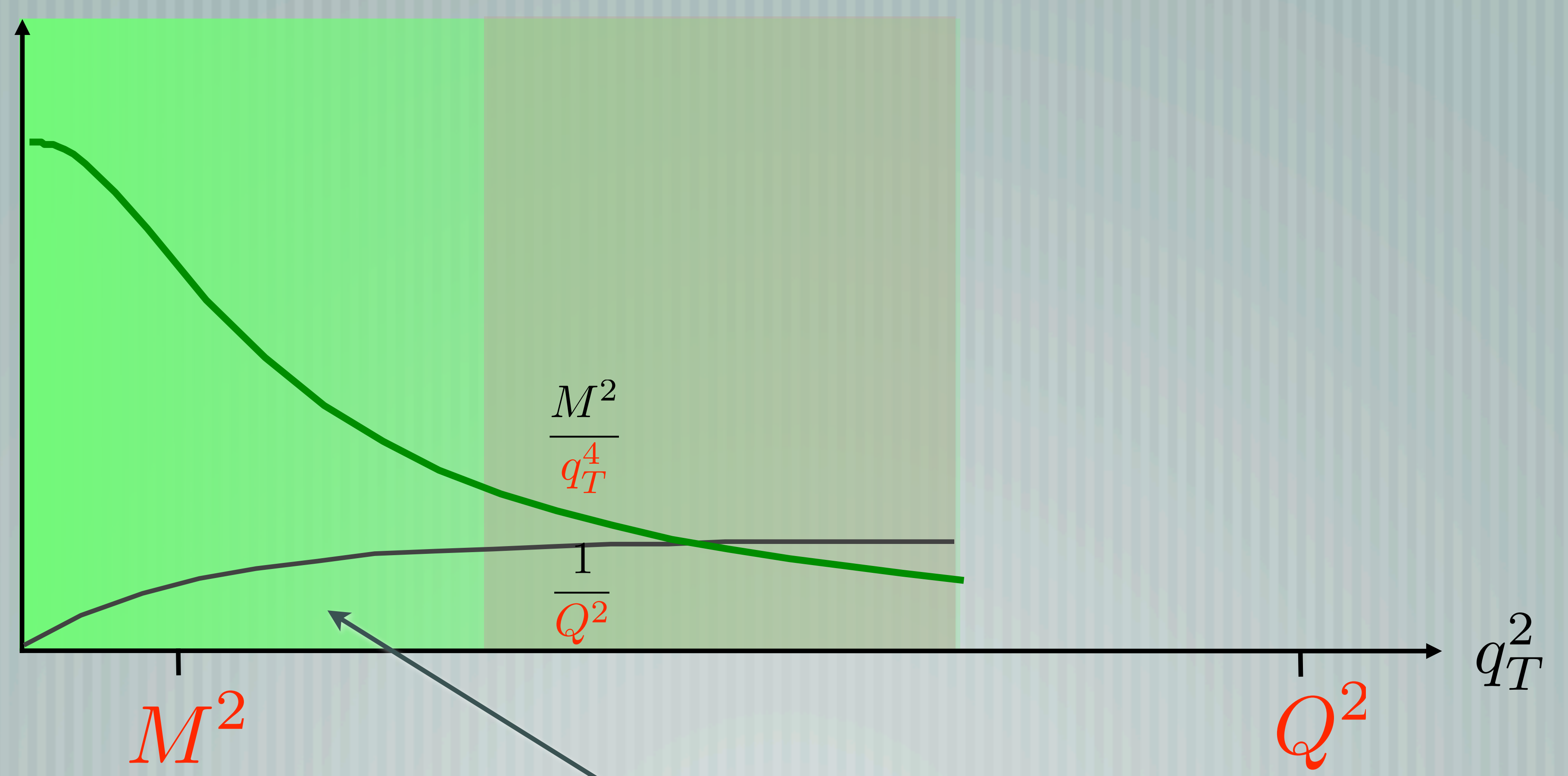

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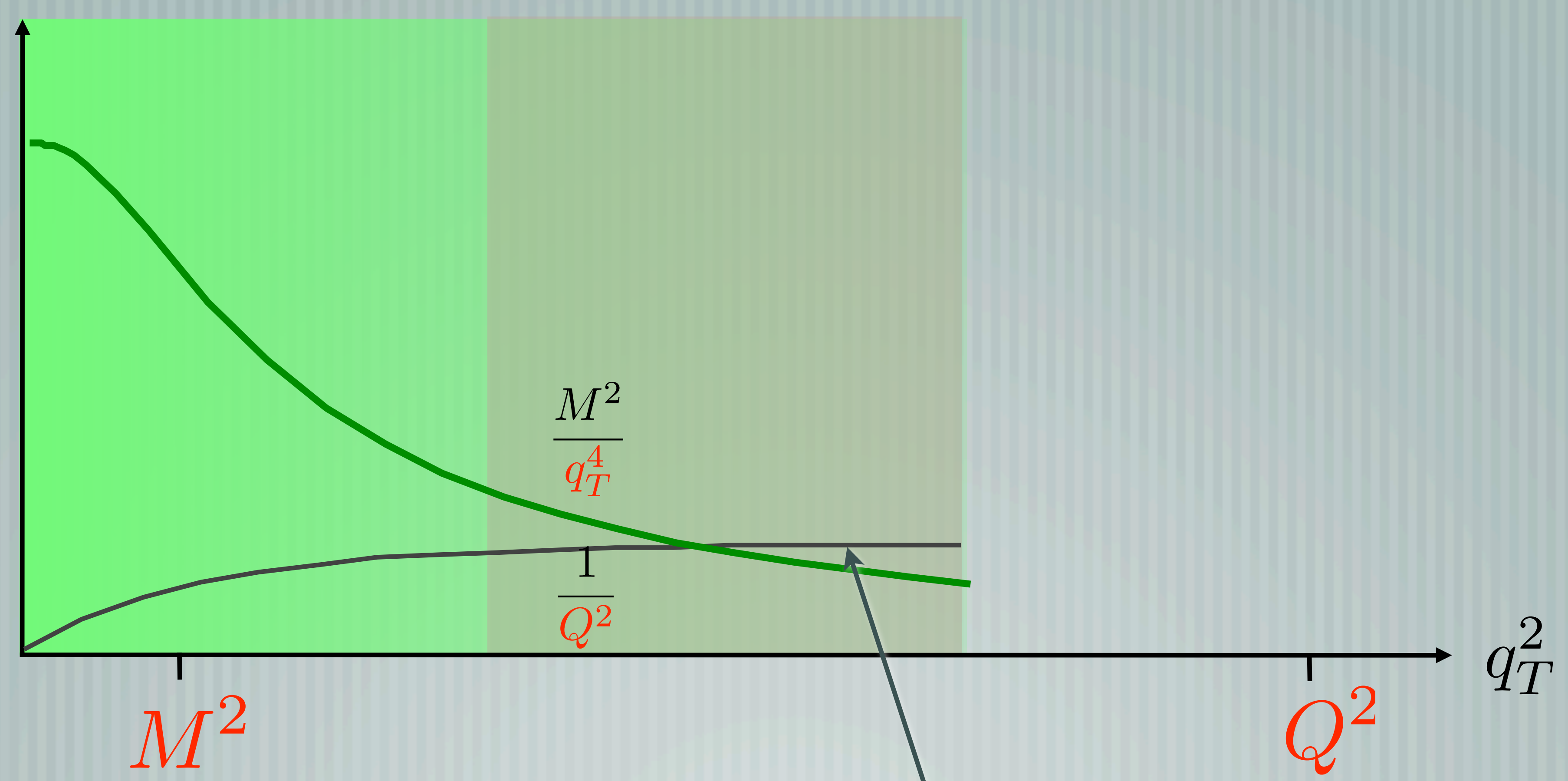
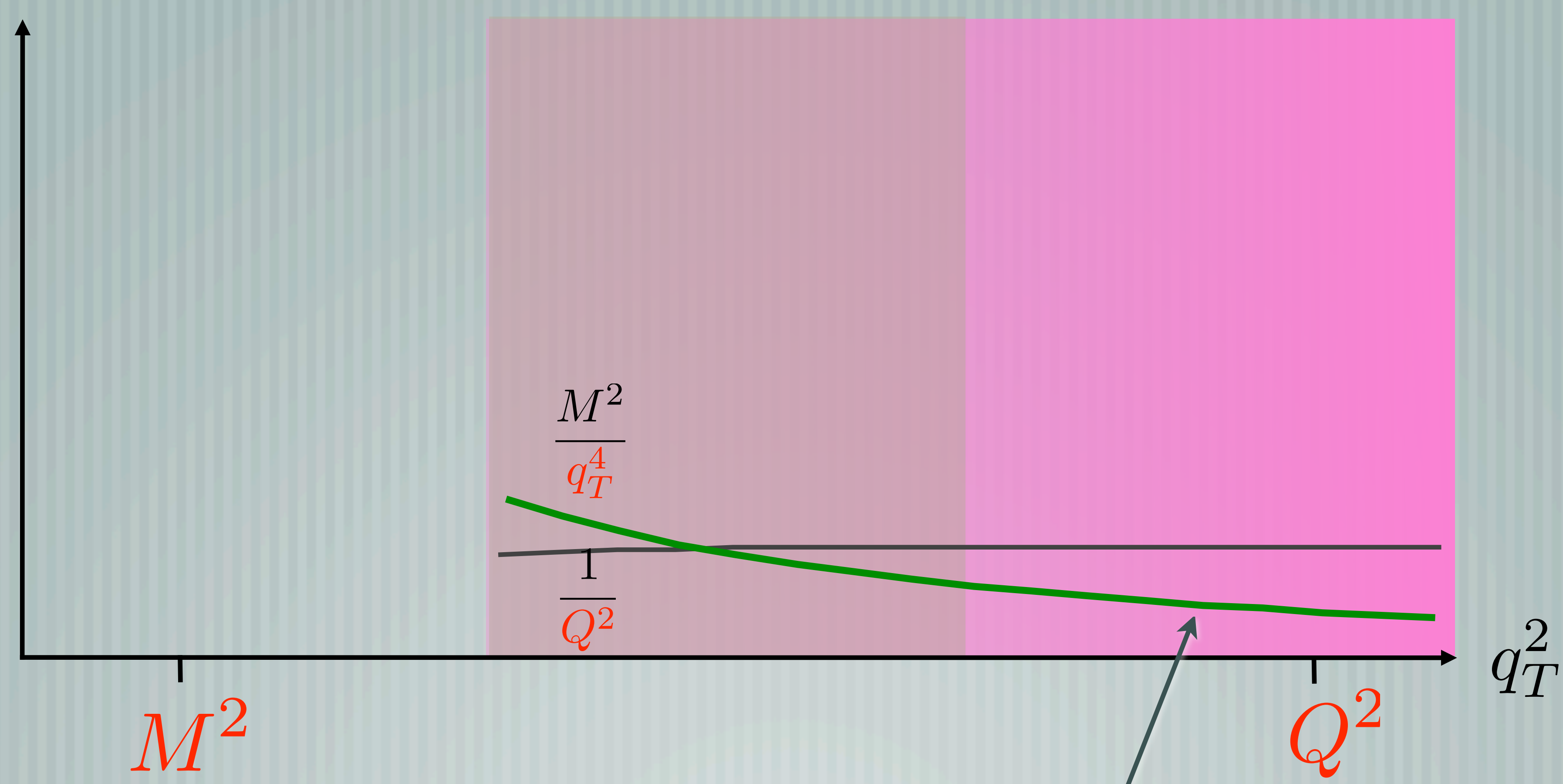
Expansion at low trans. momentum



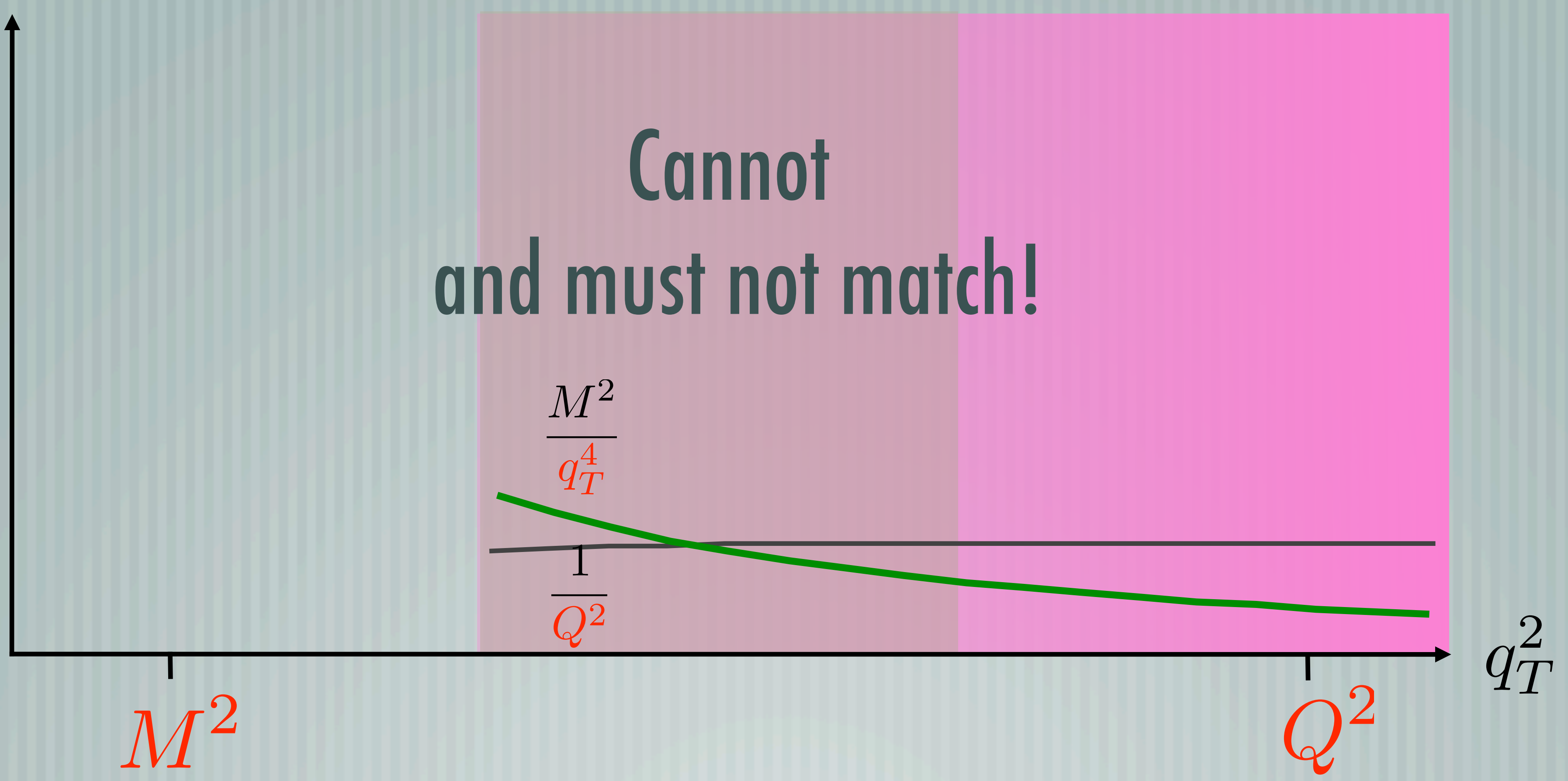
At some point this term becomes dominant

Expansion at high trans. momentum

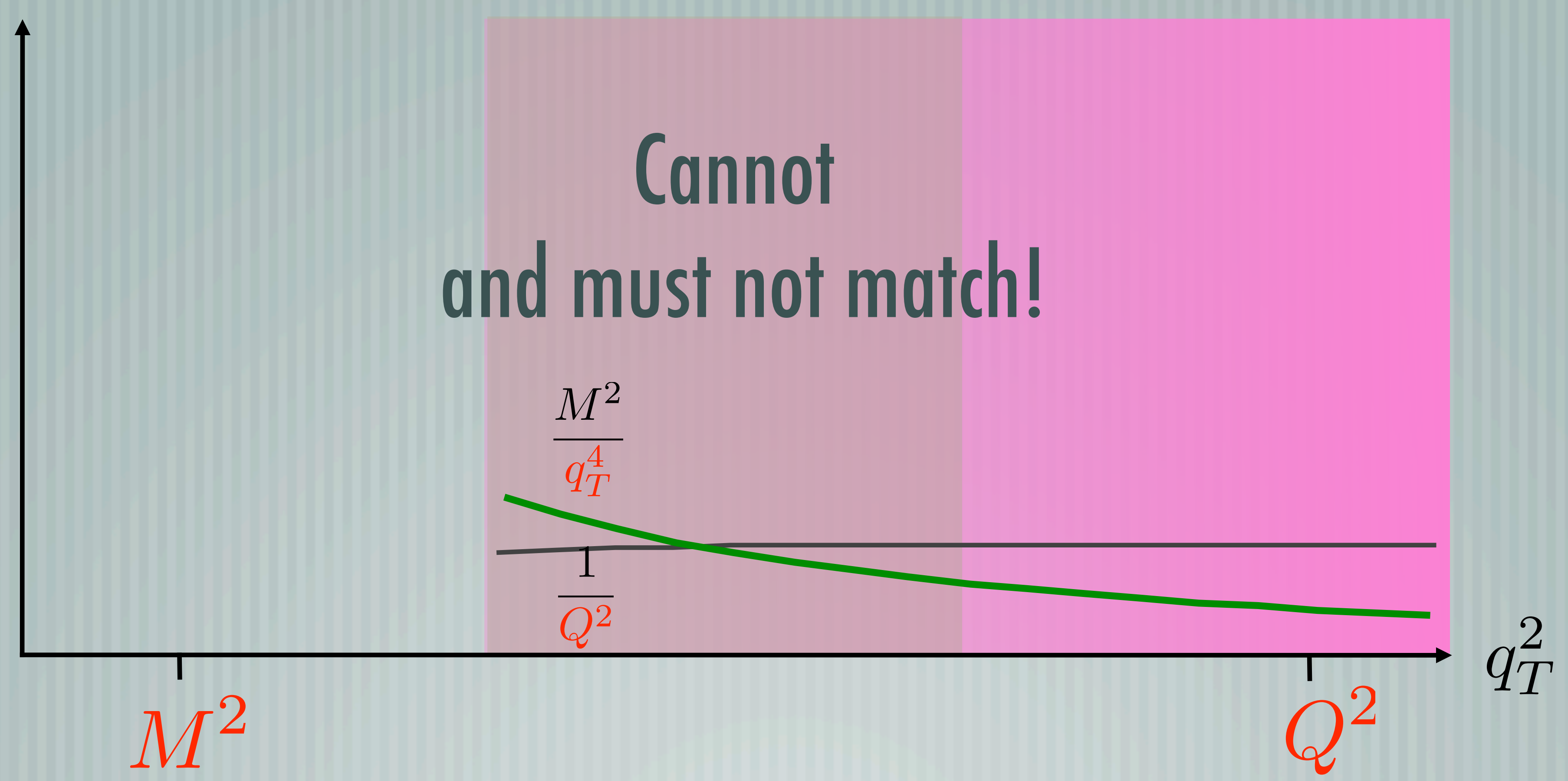


At high q_T this term is suppressed

Expected mismatch



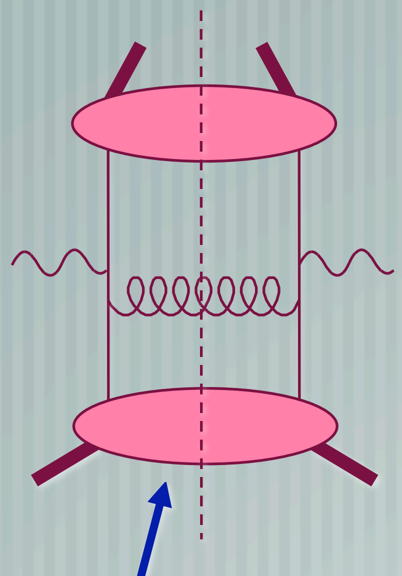
Expected mismatch

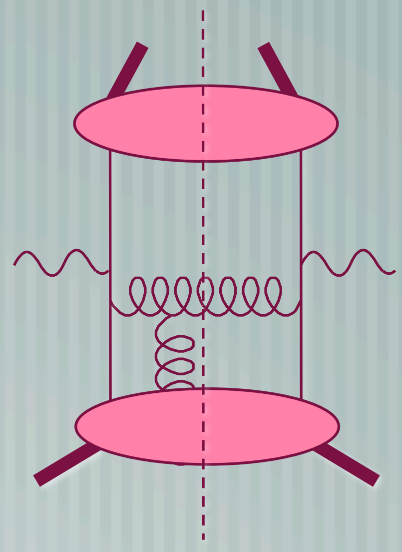


Two distinct mechanisms are involved

Calculation at high q_T

Collinear factorization





Twist-2 integrated PDFs

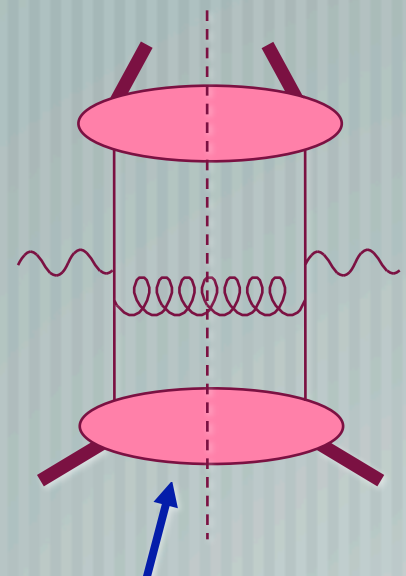
e.g. $f_1(x)$

e.g. $G_F(x_1, x_2)$

see e.g. Koike, Nagashima, Vogelsang, NPB744 (06)

Calculation at high q_T

Collinear factorization



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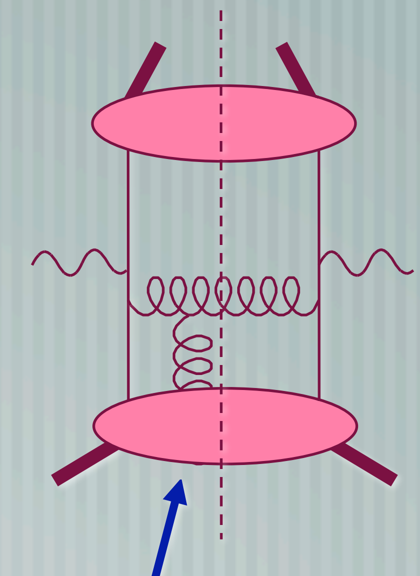

Twist-3 integrated PDFs

e.g. $G_F(x_1, x_2)$

Eguchi, Koike, Tanaka, NPB752 (06) & NPB763 (07)

Example of analytic formula

$$F_{UU,T} = \frac{1}{Q^2} \frac{\alpha_s}{(2\pi z)^2} \sum_a x e_a^2 \int_x^1 \frac{\hat{x}}{\hat{x}} \int_z^1 \frac{\hat{z}}{\hat{z}} \, \delta\Big(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}} \Big)$$

$$\times \Big[f_1^a\Big(\frac{x}{\hat{x}}\Big) D_1^a\Big(\frac{z}{\hat{z}}\Big) C_{UU,T}^{(\gamma^* q \to q g)} + f_1^a\Big(\frac{x}{\hat{x}}\Big) D_1^g\Big(\frac{z}{\hat{z}}\Big) C_{UU,T}^{(\gamma^* q \to g q)} + f_1^g\Big(\frac{x}{\hat{x}}\Big) D_1^a\Big(\frac{z}{\hat{z}}\Big) C_{UU,T}^{(\gamma^* g \to q \bar{q})} \Big]$$

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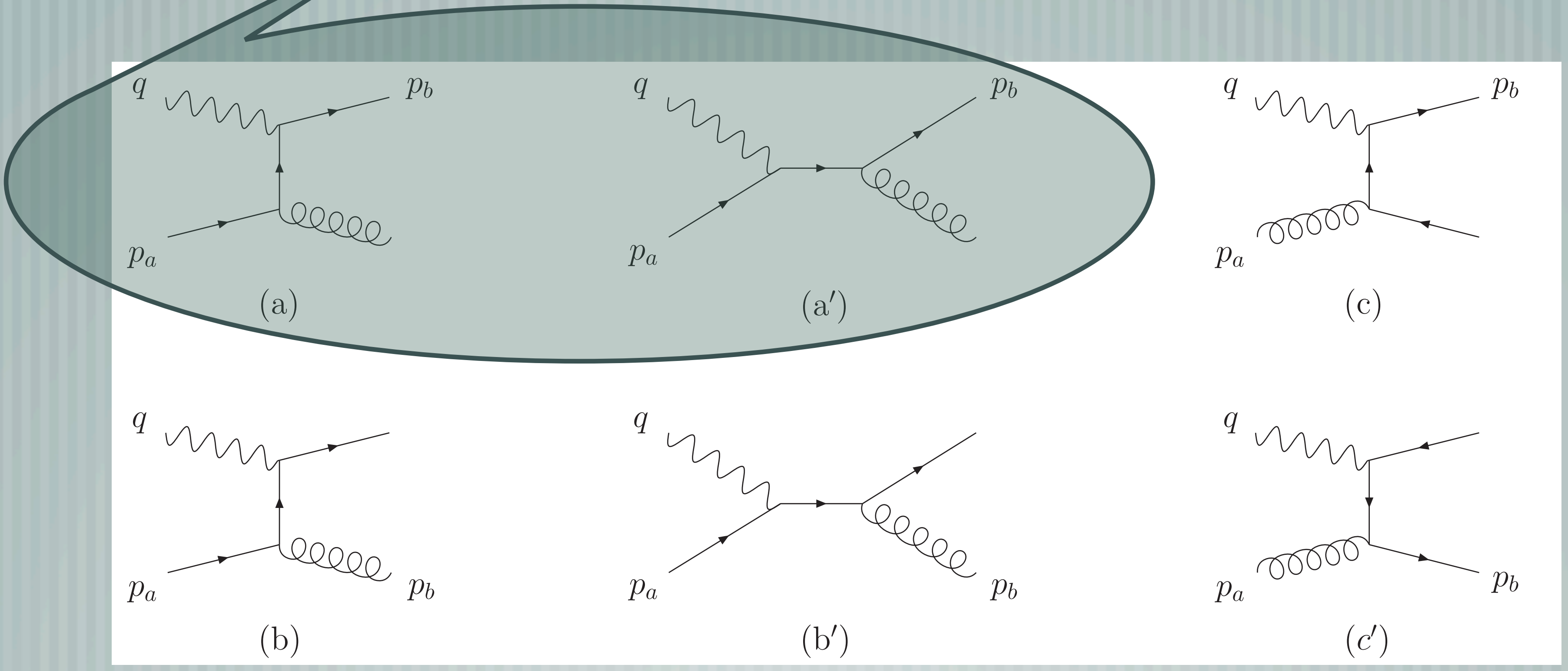

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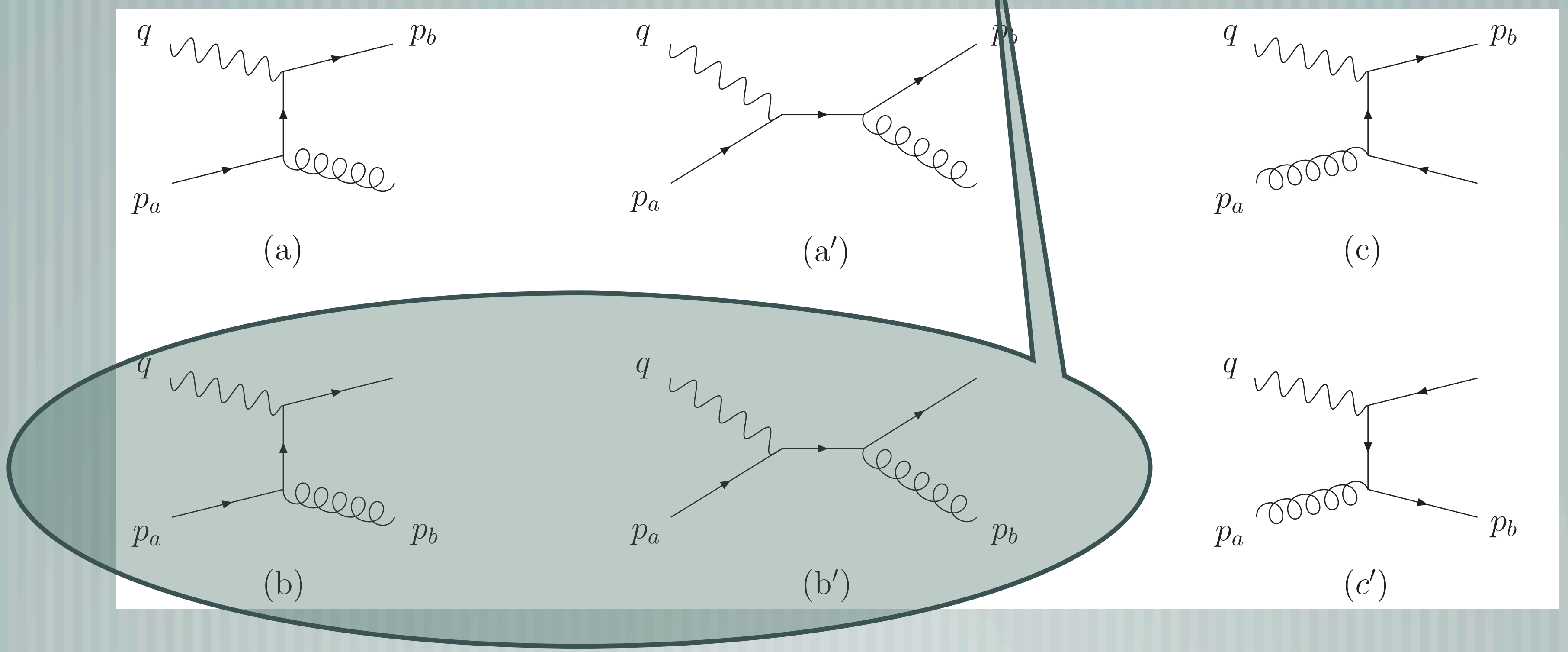

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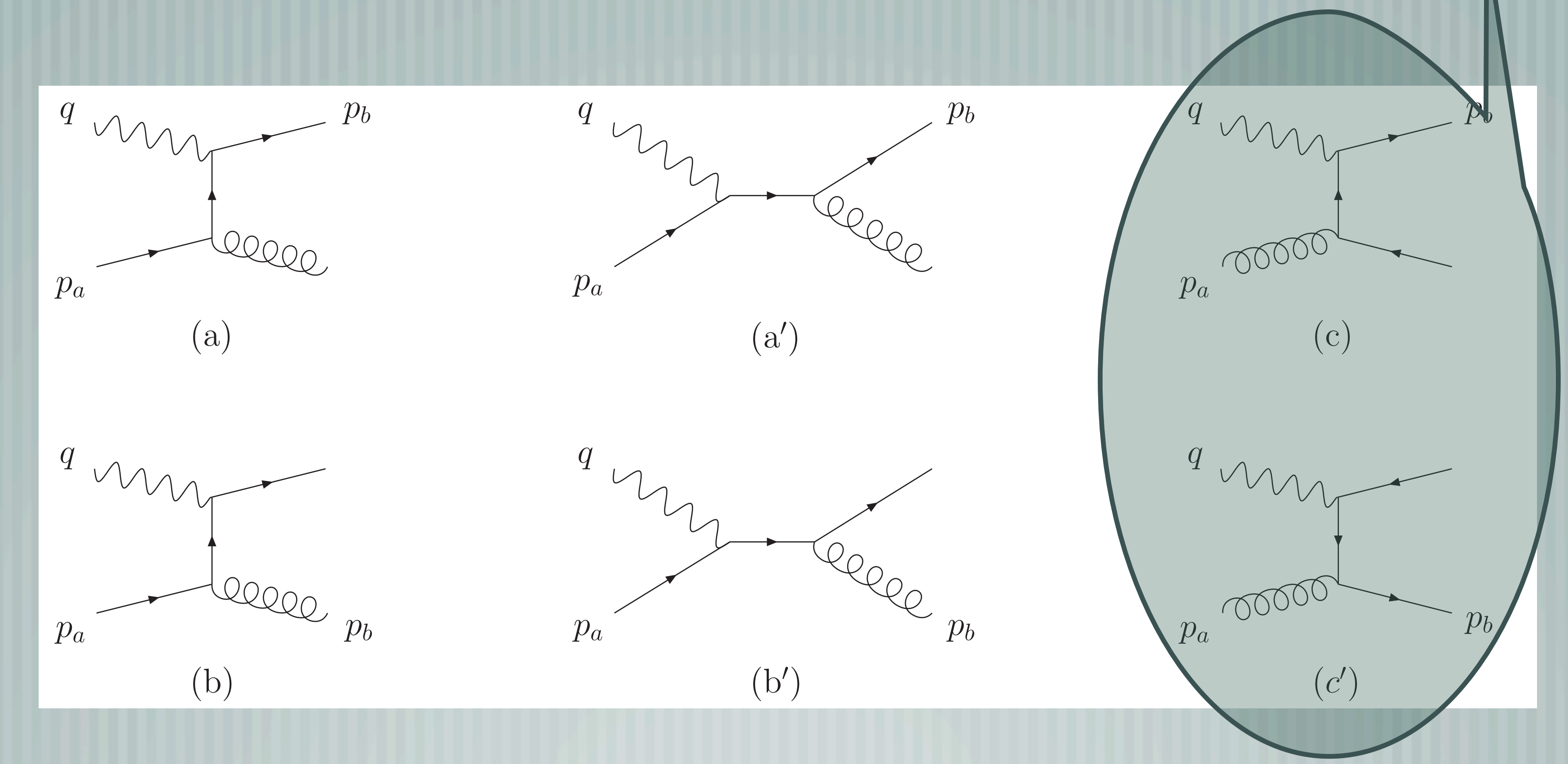

From high to intermediate

High q_T

$$F_{UU,T} = \frac{1}{Q^2}\,\frac{\alpha_s}{(2\pi z)^2}\sum_a x e_a^2 \int_x^1 \frac{\hat{x}}{\hat{x}} \int_z^1 \frac{\hat{z}}{\hat{z}}\,\delta\Big(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}}\Big)$$
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use

$$\delta\Big(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}}\Big) = \delta(1-\hat{x})\,\delta(1-\hat{z})\,\ln\frac{Q^2}{q_T^2} + \frac{\hat{x}}{(1-\hat{x})_+}\,\delta(1-\hat{z}) + \frac{\hat{z}}{(1-\hat{z})_+}\,\delta(1-\hat{x})$$

From high to intermediate

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Intermediate q_T

$$F_{UU,T} = \frac{1}{q_T^2}\frac{\alpha_s}{2\pi^2 z^2}\sum_a x e_a^2\Big[f_1^a(x)\,D_1^a(z)\,L\Big(\frac{Q^2}{q_T^2}\Big) + f_1^a(x)\,\big(D_1^a\otimes P_{qq} + D_1^g\otimes P_{gq}\big)(z) + \big(P_{qq}\otimes f_1^a + P_{qg}\otimes f_1^g\big)(x)\,D_1^a(z)\Big]$$

where

$$L\Big(\frac{Q^2}{q_T^2}\Big) = 2C_F\ln\frac{Q^2}{q_T^2} - 3C_F$$

From high to intermediate

High q_T

$$F_{UU,T} = \frac{1}{Q^2}\,\frac{\alpha_s}{(2\pi z)^2}\sum_a x e_a^2 \int_x^1 \frac{\hat{x}}{\hat{x}} \int_z^1 \frac{\hat{z}}{\hat{z}}\,\delta\Big(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}}\Big)$$

$$\times \Big[f_1^a\Big(\frac{x}{\hat{x}}\Big)\, D_1^a\Big(\frac{z}{\hat{z}}\Big)\, C_{UU,T}^{(\gamma^* q\to qg)} + f_1^a\Big(\frac{x}{\hat{x}}\Big)\, D_1^g\Big(\frac{z}{\hat{z}}\Big)\, C_{UU,T}^{(\gamma^* q\to gq)} + f_1^g\Big(\frac{x}{\hat{x}}\Big)\, D_1^a\Big(\frac{z}{\hat{z}}\Big)\, C_{UU,T}^{(\gamma^* g\to q\bar{q})} \Big]$$

use

$$\delta\Big(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}}\Big) = \delta(1-\hat{x})\,\delta(1-\hat{z})\,\ln\frac{Q^2}{q_T^2} + \frac{\hat{x}}{(1-\hat{x})_+}\,\delta(1-\hat{z}) + \frac{\hat{z}}{(1-\hat{z})_+}\,\delta(1-\hat{x})$$

Intermediate q_T

$$F_{UU,T} = \frac{1}{q_T^2}\,\frac{\alpha_s}{2\pi^2 z^2}\sum_a x e_a^2 \Big[f_1^a(x)\, D_1^a(z)\, L\Big(\frac{Q^2}{q_T^2}\Big) + f_1^a(x)\,\big(D_1^a \otimes P_{qq} + D_1^g \otimes P_{gq}\big)(z)$$

$$+ \big(P_{qq}\otimes f_1^a + P_{qg}\otimes f_1^g\big)(x)\, D_1^a(z) \Big]$$

DGLAP splitting functions

where $$L\Big(\frac{Q^2}{q_T^2}\Big) = 2C_F \ln\frac{Q^2}{q_T^2} - 3C_F$$

From high to intermediate

High q_T

$$F_{UU,T} = \frac{1}{Q^2}\,\frac{\alpha_s}{(2\pi z)^2}\sum_a x e_a^2 \int_x^1 \frac{\hat{x}}{\hat{x}} \int_z^1 \frac{\hat{z}}{\hat{z}}\, \delta\Big(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}}\Big)$$

$$\times \Big[f_1^a\Big(\frac{x}{\hat{x}}\Big)\, D_1^a\Big(\frac{z}{\hat{z}}\Big)\, C_{UU,T}^{(\gamma^* q\to qg)} + f_1^a\Big(\frac{x}{\hat{x}}\Big)\, D_1^g\Big(\frac{z}{\hat{z}}\Big)\, C_{UU,T}^{(\gamma^* q\to gq)} + f_1^g\Big(\frac{x}{\hat{x}}\Big)\, D_1^a\Big(\frac{z}{\hat{z}}\Big)\, C_{UU,T}^{(\gamma^* g\to q\bar{q})}\Big]$$

use

$$\delta\Big(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}}\Big) = \delta(1-\hat{x})\,\delta(1-\hat{z})\,\ln\frac{Q^2}{q_T^2} + \frac{\hat{x}}{(1-\hat{x})_+}\,\delta(1-\hat{z}) + \frac{\hat{z}}{(1-\hat{z})_+}\,\delta(1-\hat{x})$$

Intermediate q_T

$$F_{UU,T} = \frac{1}{q_T^2}\,\frac{\alpha_s}{2\pi^2 z^2}\sum_a x e_a^2 \Big[f_1^a(x)\, D_1^a(z)\, L\Big(\frac{Q^2}{q_T^2}\Big) + f_1^a(x)\,\big(D_1^a \otimes P_{qq} + D_1^g \otimes P_{gq}\big)(z) + \big(P_{qq}\otimes f_1^a + P_{qg}\otimes f_1^g\big)(x)\, D_1^a(z)\Big]$$

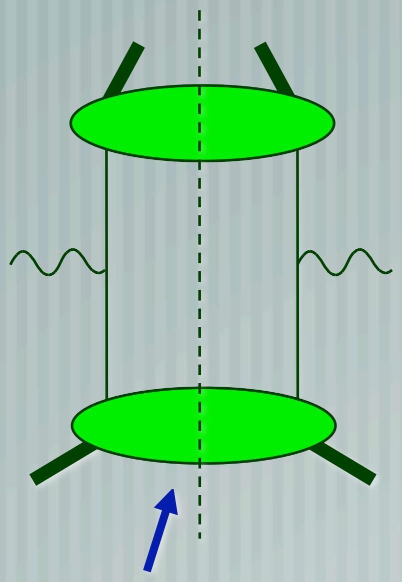
Large log, needs resummation

DGLAP splitting functions

where $$L\Big(\frac{Q^2}{q_T^2}\Big) = 2C_F \ln\frac{Q^2}{q_T^2} - 3C_F$$

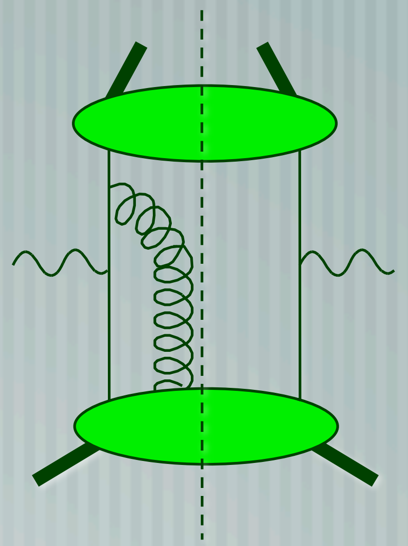
Calculation at low q_T

k_T-factorization



Twist-2 TMDs

e.g. $f_1(x, p_T^2)$



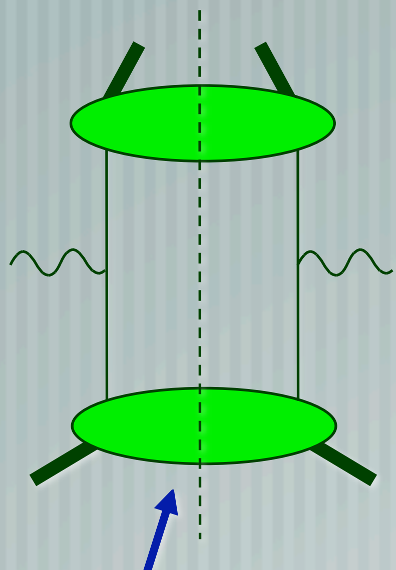
Twist-3 TMDs

e.g. $f^{\perp}(x, p_T^2)$

see e.g. AB, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP093 (07)

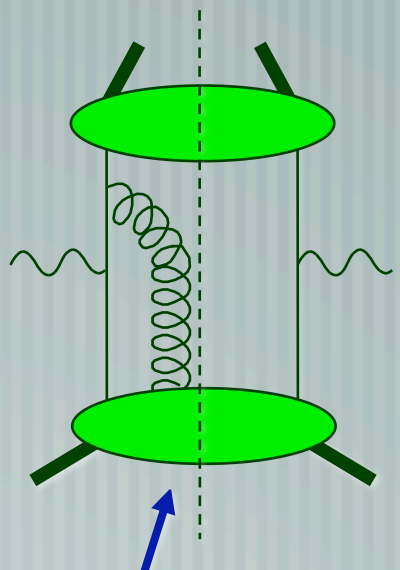
Calculation at low q_T

k_T-factorization



Twist-2 TMDs

e.g. $f_1(x, p_T^2)$



Twist-3 TMDs

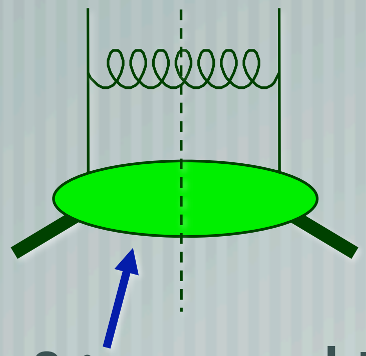
e.g. $f^{\perp}(x, p_T^2)$

see e.g. AB, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP093 (07)

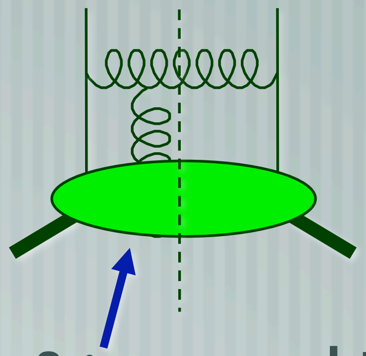
From low to intermediate

see e.g. Ji, Qiu, Vogelsang, Yuan, PLB638 (06)

- Compute the high-transverse-momentum behavior of the TMD PDFs by considering diagrams such as



Twist-2 integrated PDFs

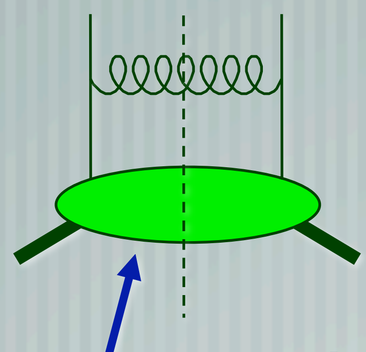


Twist-3 integrated PDFs

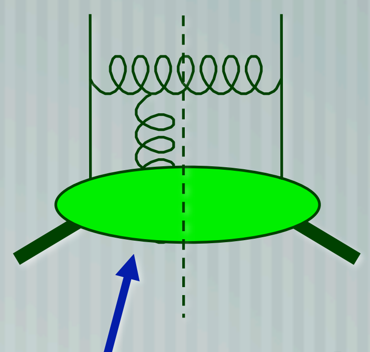
From low to intermediate

see e.g. Ji, Qiu, Vogelsang, Yuan, PLB638 (06)

- Compute the high-transverse-momentum behavior of the TMD PDFs by considering diagrams such as



Twist-2 integrated PDFs



Twist-3 integrated PDFs

- Consider also the high-transverse-momentum contribution of the soft factor

Collins, Soper, NPB193 (81)

From low to intermediate

Low q_T $$F_{UU,T} = \sum_a x e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, d^2\boldsymbol{l}_T\, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T + \boldsymbol{l}_T + \boldsymbol{q}_T\big)\, f_1^a(x, p_T^2)\, D_1^a(z, k_T^2)\, U(l_T^2)$$

From low to intermediate

Low q_T

$$F_{UU,T} = \sum_a x e_a^2 \int d^2\boldsymbol{p}_T \, d^2\boldsymbol{k}_T \, d^2\boldsymbol{l}_T \, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T + \boldsymbol{l}_T + \boldsymbol{q}_T\big) \, f_1^a(x, p_T^2) \, D_1^a(z, k_T^2) \, U(l_T^2)$$

From low to intermediate

Low q_T

$$F_{UU,T} = \sum_a x e_a^2 \int d^2\boldsymbol{p}_T \, d^2\boldsymbol{k}_T \, d^2\boldsymbol{l}_T \, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T + \boldsymbol{l}_T + \boldsymbol{q}_T\big) f_1^a(x, p_T^2) \, D_1^a(z, k_T^2) \, U(l_T^2)$$

$$F_{UU,T} = \sum_a x e_a^2 \left[f_1^a(x, q_T^2) \, \frac{D_1^a(z)}{z^2} + f_1^a(x) \, D_1^a(z, q_T^2) + f_1^a(x) \, \frac{D_1^a(z)}{z^2} \, U(q_T^2) \right]$$

From low to intermediate

Low q_T

$$F_{UU,T} = \sum_a x e_a^2 \int d^2\boldsymbol{p}_T \, d^2\boldsymbol{k}_T \, d^2\boldsymbol{l}_T \, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T + \boldsymbol{l}_T + \boldsymbol{q}_T\big) \, f_1^a(x, p_T^2) \, D_1^a(z, k_T^2) \, U(l_T^2)$$

⇩

$$F_{UU,T} = \sum_a x e_a^2 \left[f_1^a(x, q_T^2) \, \frac{D_1^a(z)}{z^2} + f_1^a(x) \, D_1^a(z, q_T^2) + f_1^a(x) \, \frac{D_1^a(z)}{z^2} \, U(q_T^2) \right]$$

use

$$f_1^q(x, p_T^2) = \frac{\alpha_s}{2\pi^2} \frac{1}{\boldsymbol{p}_T^2} \left[\frac{L(\eta^{-1})}{2} \, f_1^q(x) - C_F f_1^q(x) + \big(P_{qq} \otimes f_1^q + P_{qg} \otimes f_1^g\big)(x) \right],$$

From low to intermediate

Low q_T $$F_{UU,T} = \sum_a x e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, d^2\boldsymbol{l}_T\, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T + \boldsymbol{l}_T + \boldsymbol{q}_T\big)\, f_1^a(x, p_T^2)\, D_1^a(z, k_T^2)\, U(l_T^2)$$

$$F_{UU,T} = \sum_a x e_a^2 \left[f_1^a(x, q_T^2)\, \frac{D_1^a(z)}{z^2} + f_1^a(x)\, D_1^a(z, q_T^2) + f_1^a(x)\, \frac{D_1^a(z)}{z^2}\, U(q_T^2) \right]$$

use $$f_1^q(x, p_T^2) = \frac{\alpha_s}{2\pi^2} \frac{1}{\boldsymbol{p}_T^2} \left[\frac{L(\eta^{-1})}{2} f_1^q(x) - C_F f_1^q(x) + \big(P_{qq} \otimes f_1^q + P_{qg} \otimes f_1^g\big)(x) \right],$$

$$D_1^q(z, k_T^2) = \frac{\alpha_s}{2\pi^2} \frac{1}{z^2 \boldsymbol{k}_T^2} \left[\frac{L(\eta_h^{-1})}{2} D_1^q(z) - C_F D_1^q(z) + \big(D_1^q \otimes P_{qq} + D_1^g \otimes P_{gq}\big)(z) \right],$$

From low to intermediate

Low q_T $$F_{UU,T} = \sum_a x e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, d^2\boldsymbol{l}_T\, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T + \boldsymbol{l}_T + \boldsymbol{q}_T\big) f_1^a(x, p_T^2)\, D_1^a(z, k_T^2)\, U(l_T^2)$$

$$F_{UU,T} = \sum_a x e_a^2 \left[f_1^a(x, q_T^2)\, \frac{D_1^a(z)}{z^2} + f_1^a(x)\, D_1^a(z, q_T^2) + f_1^a(x)\, \frac{D_1^a(z)}{z^2}\, U(q_T^2) \right]$$

use

$$f_1^q(x, p_T^2) = \frac{\alpha_s}{2\pi^2} \frac{1}{\boldsymbol{p}_T^2} \left[\frac{L(\eta^{-1})}{2} f_1^q(x) - C_F f_1^q(x) + \big(P_{qq} \otimes f_1^q + P_{qg} \otimes f_1^g\big)(x) \right],$$

$$D_1^q(z, k_T^2) = \frac{\alpha_s}{2\pi^2} \frac{1}{z^2 \boldsymbol{k}_T^2} \left[\frac{L(\eta_h^{-1})}{2} D_1^q(z) - C_F D_1^q(z) + \big(D_1^q \otimes P_{qq} + D_1^g \otimes P_{gq}\big)(z) \right],$$

$$U(q_T^2) = \frac{\alpha_s C_F}{\pi^2} \frac{1}{q_T^2}$$

From low to intermediate

Low q_T

$$F_{UU,T} = \sum_a x e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, d^2\boldsymbol{l}_T\, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T + \boldsymbol{l}_T + \boldsymbol{q}_T\big) f_1^a(x, p_T^2)\, D_1^a(z, k_T^2)\, U(l_T^2)$$

Intermediate q_T

$$F_{UU,T} = \sum_a x e_a^2 \left[f_1^a(x, q_T^2)\, \frac{D_1^a(z)}{z^2} + f_1^a(x)\, D_1^a(z, q_T^2) + f_1^a(x)\, \frac{D_1^a(z)}{z^2}\, U(q_T^2) \right]$$

use

$$f_1^q(x, p_T^2) = \frac{\alpha_s}{2\pi^2} \frac{1}{\boldsymbol{p}_T^2} \left[\frac{L(\eta^{-1})}{2}\, f_1^q(x) - C_F f_1^q(x) + \big(P_{qq} \otimes f_1^q + P_{qg} \otimes f_1^g\big)(x) \right],$$

$$D_1^q(z, k_T^2) = \frac{\alpha_s}{2\pi^2} \frac{1}{z^2 \boldsymbol{k}_T^2} \left[\frac{L(\eta_h^{-1})}{2}\, D_1^q(z) - C_F D_1^q(z) + \big(D_1^q \otimes P_{qq} + D_1^g \otimes P_{gq}\big)(z) \right],$$

$$U(q_T^2) = \frac{\alpha_s C_F}{\pi^2} \frac{1}{q_T^2}$$

$$F_{UU,T} = \frac{1}{q_T^2} \frac{\alpha_s}{2\pi^2 z^2} \sum_a x e_a^2 \Bigg[f_1^a(x)\, D_1^a(z)\, L\left(\frac{Q^2}{q_T^2}\right) + f_1^a(x)\, \big(D_1^a \otimes P_{qq} + D_1^g \otimes P_{gq}\big)(z)$$
$$+ \big(P_{qq} \otimes f_1^a + P_{qg} \otimes f_1^g\big)(x)\, D_1^a(z) \Bigg]$$

Selected examples

$$
\begin{aligned}
&\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2} \\
&= \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\,\Bigg\{ F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} \\
&\quad + \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} + S_L\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
&\quad + S_L\,\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
&\quad + S_T\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)} \\
&\qquad + \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S} \\
&\qquad + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] + S_T\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)} \\
&\qquad + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\}
\end{aligned}
$$

Selected examples

$$
\begin{aligned}
&\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2} \\
&= \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\Bigg\{ F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} \\
&\quad + \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} + S_L\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
&\quad + S_L\,\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
&\quad + S_T\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)} \\
&\qquad + \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S} \\
&\qquad + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] + S_T\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)} \\
&\qquad + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\}
\end{aligned}
$$

Selected examples

talks of Kafer and Giordano

$$
\begin{aligned}
&\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2} \\
&= \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\Bigg\{ F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} \\
&\quad + \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} + S_L\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
&\quad + S_L\,\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
&\quad + S_T\bigg[\sin(\phi_h-\phi_S)\Big(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\Big) + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)} \\
&\qquad + \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S} \\
&\qquad + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\bigg] + S_T\lambda_e\bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)} \\
&\qquad + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\bigg]\Bigg\}
\end{aligned}
$$

Selected examples

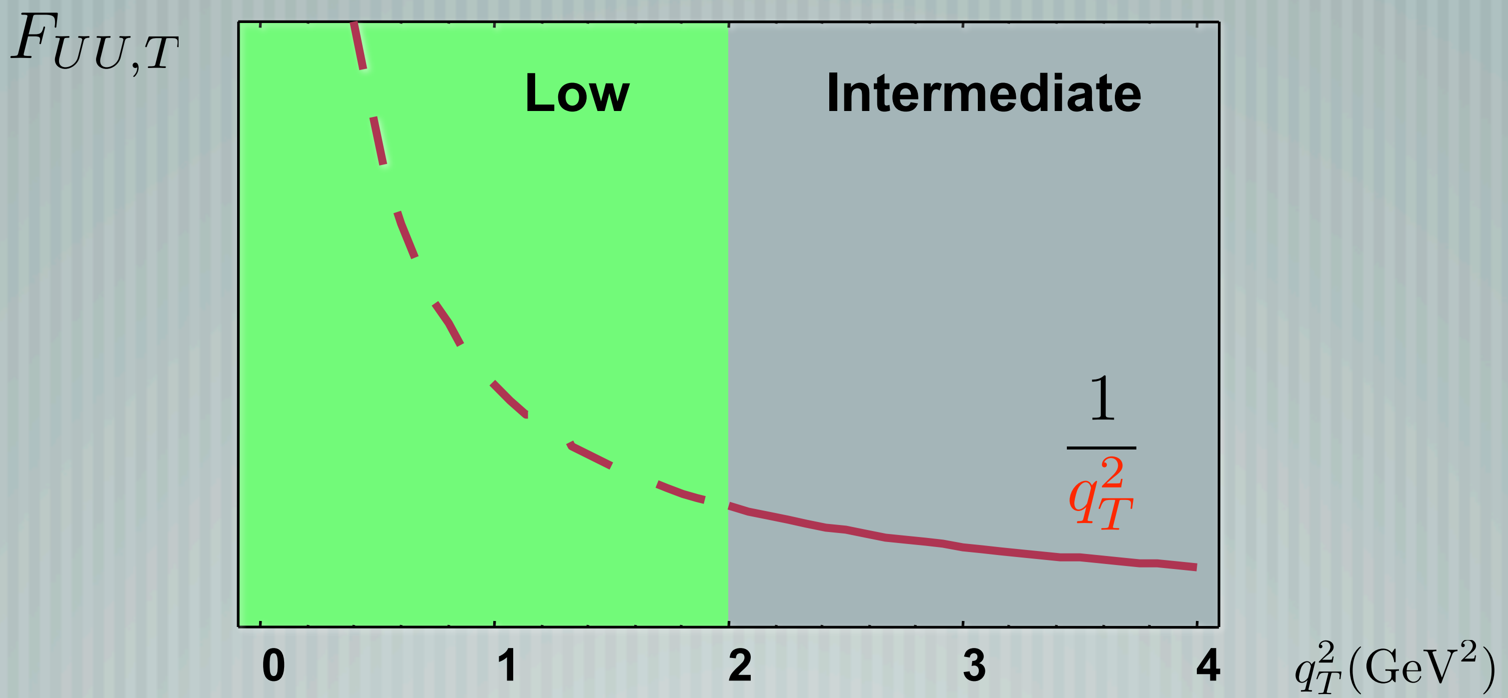
talks of Kafer and Giordano

$$\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2}$$
$$= \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\bigg\{F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}$$
$$+ \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} + S_L\bigg[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\bigg]$$
$$+ S_L\,\lambda_e\bigg[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\bigg]$$

talks of wednesday

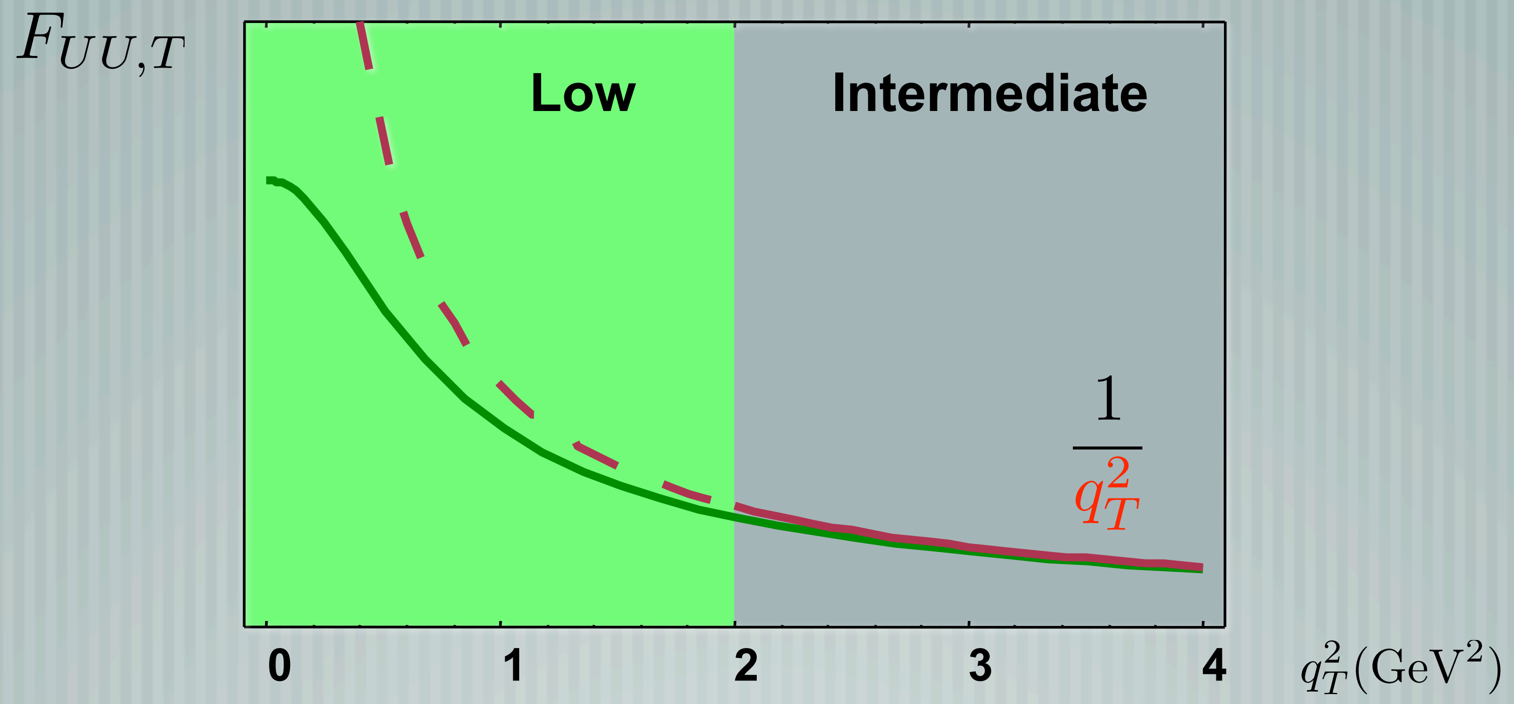
$$+ S_T\bigg[\sin(\phi_h-\phi_S)\Big(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\Big) + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}$$
$$+ \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}$$
$$+ \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\bigg] + S_T\lambda_e\bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}$$
$$+ \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\bigg]\bigg\}$$

$F_{UU,T}$ structure function



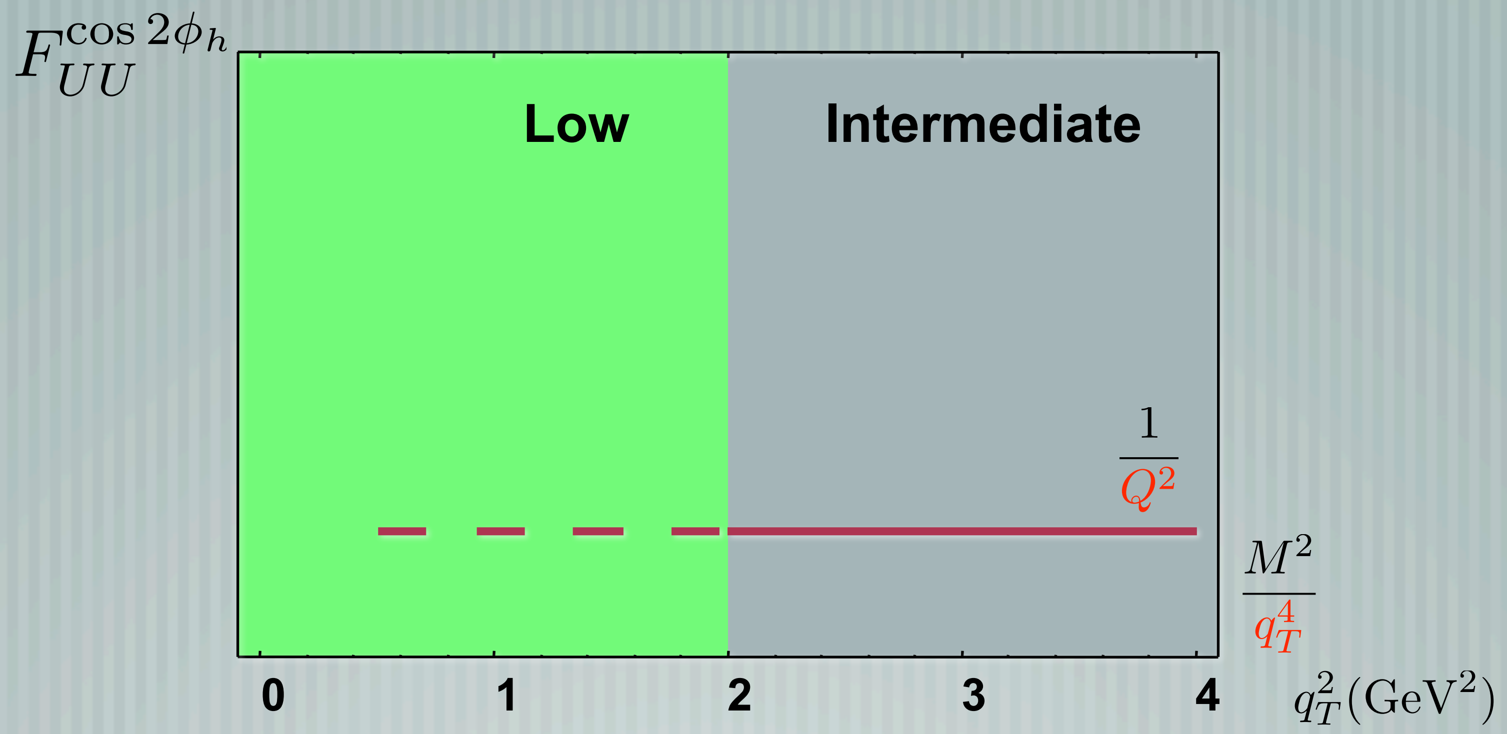
Collins, Soper, Sterman, NPB250 (85)

$F_{UU,T}$ structure function

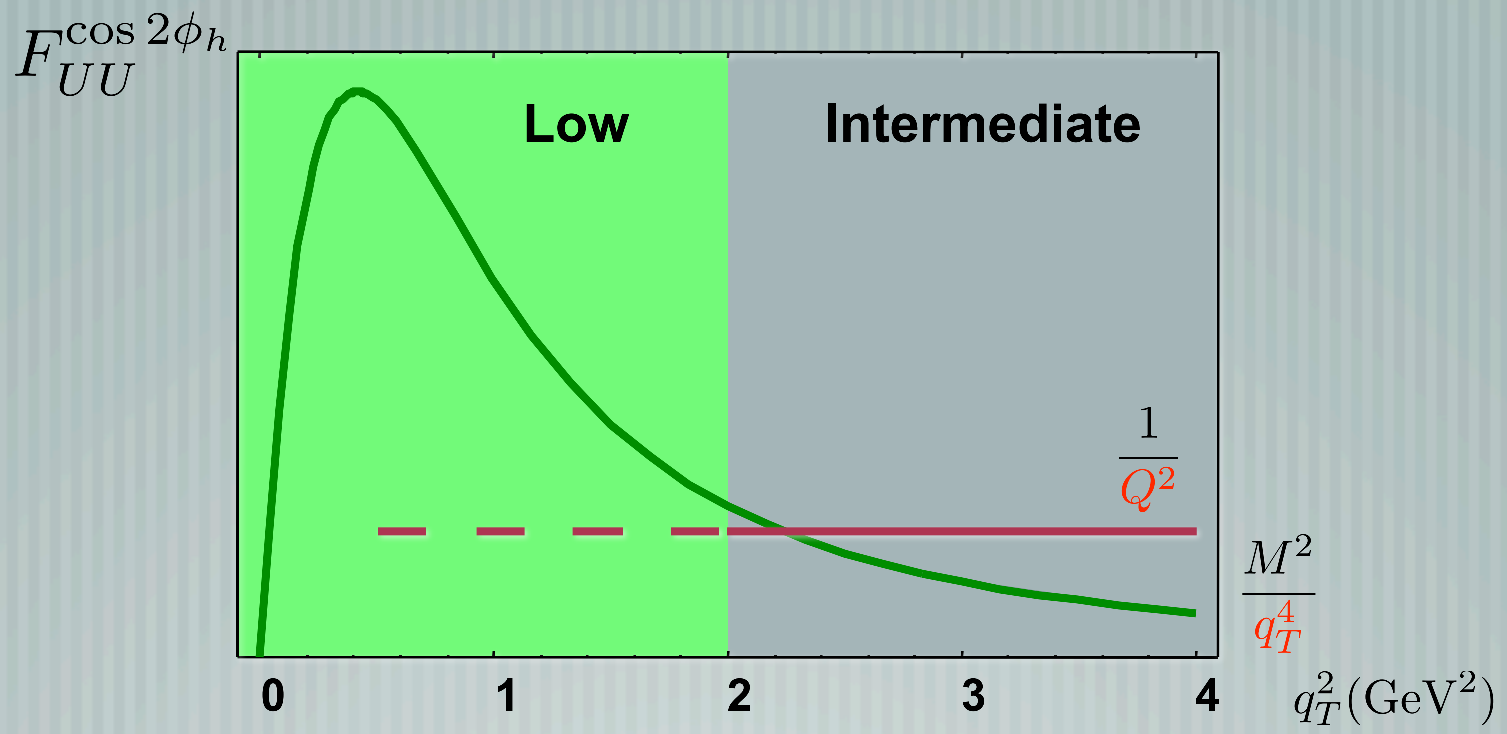


Collins, Soper, Sterman, NPB250 (85)

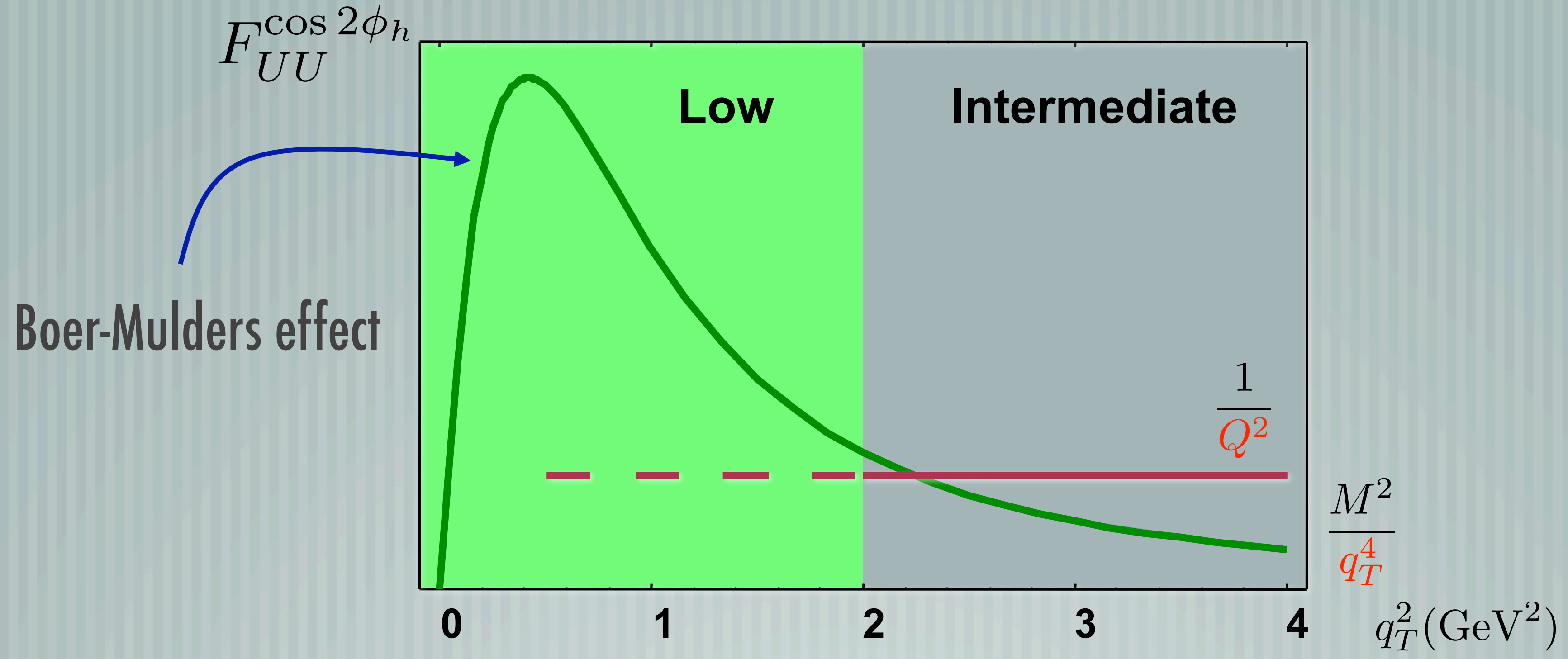
$F_{UU}^{\cos 2\phi_h}$ structure function



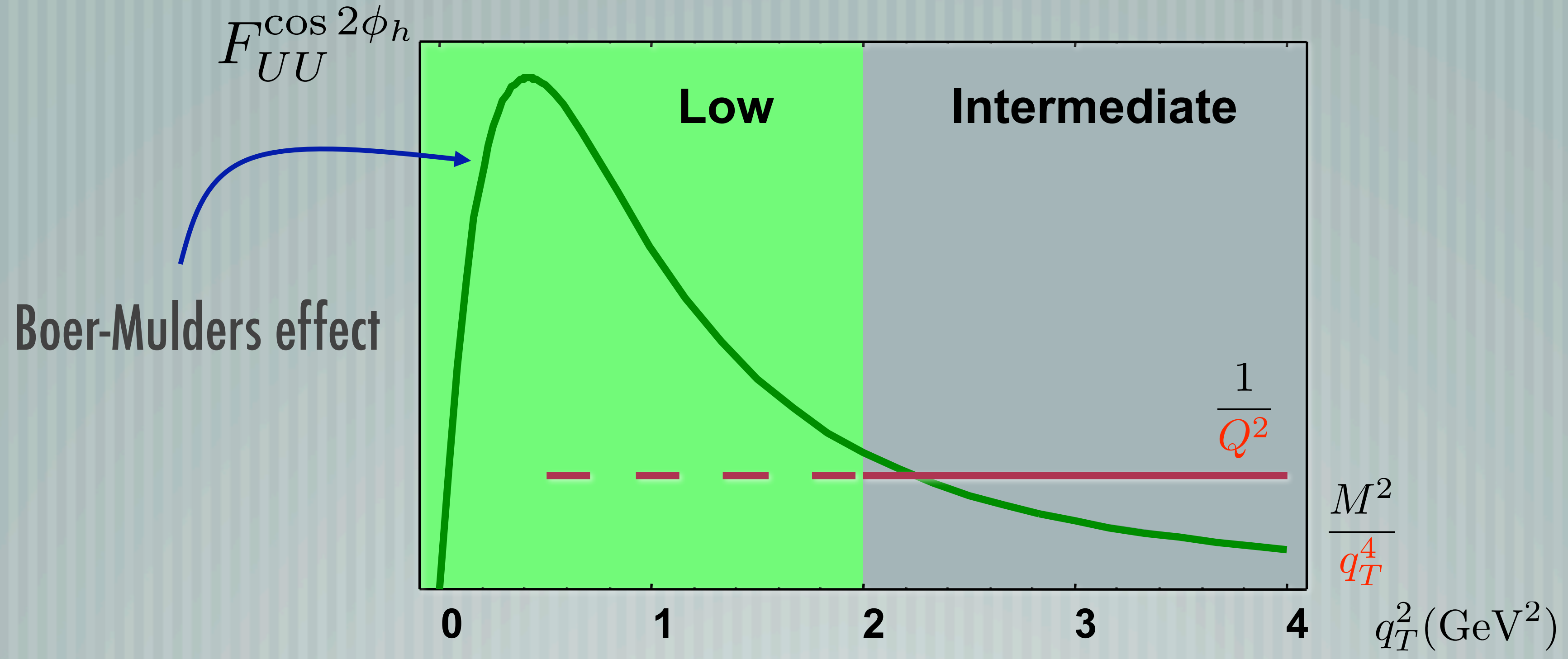
$F_{UU}^{\cos 2\phi_h}$ structure function



$F_{UU}^{\cos 2\phi_h}$ structure function



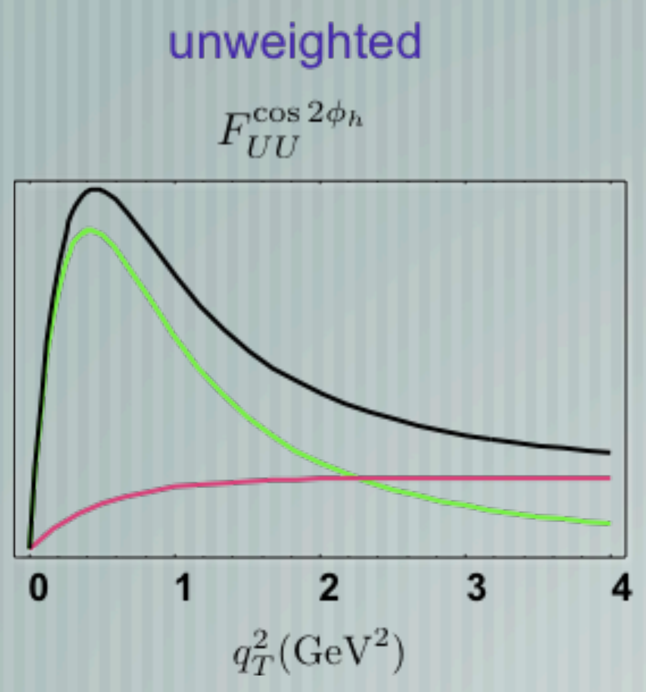
$F_{UU}^{\cos 2\phi_h}$ structure function

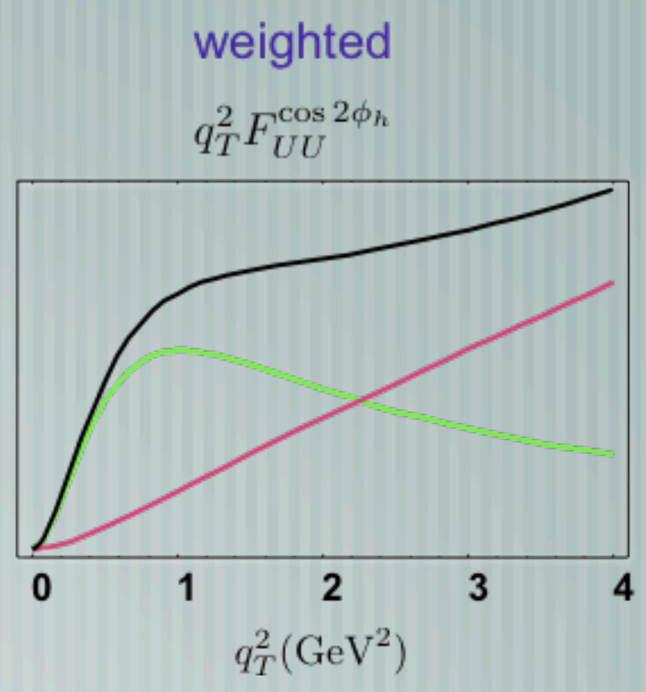


Expected mismatch: high and low calculations represent two distinct mechanisms

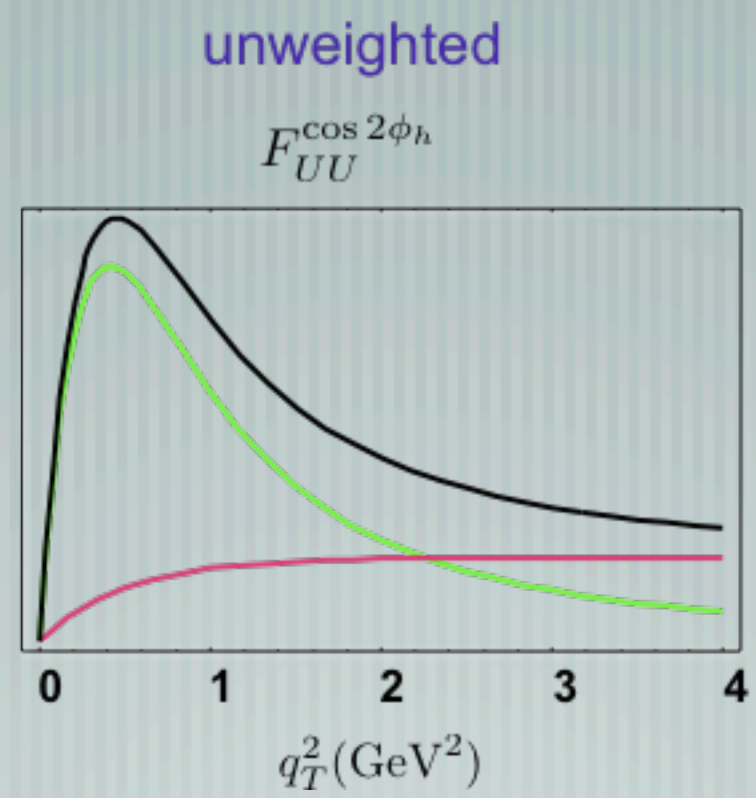
NOTE: it's a twist-2 calculation

$F_{UU}^{\cos 2\phi_h}$ and weighting

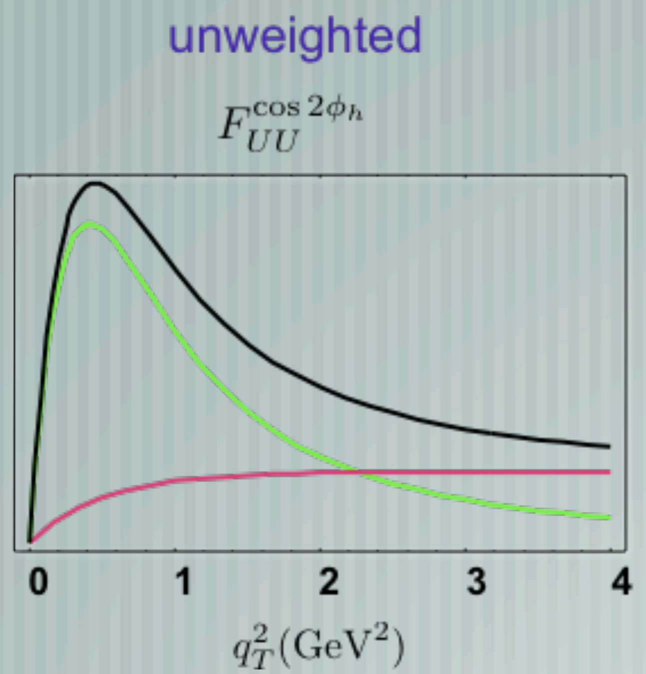


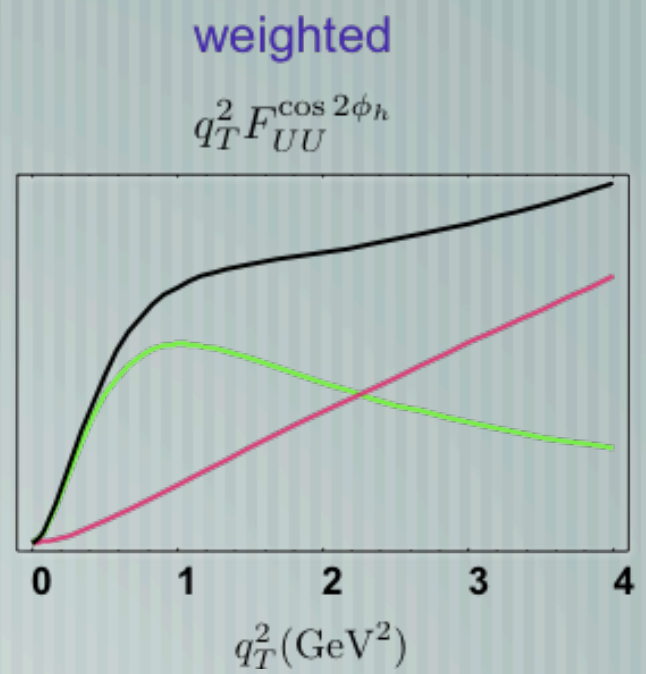


$F_{UU}^{\cos 2\phi_h}$ and weighting

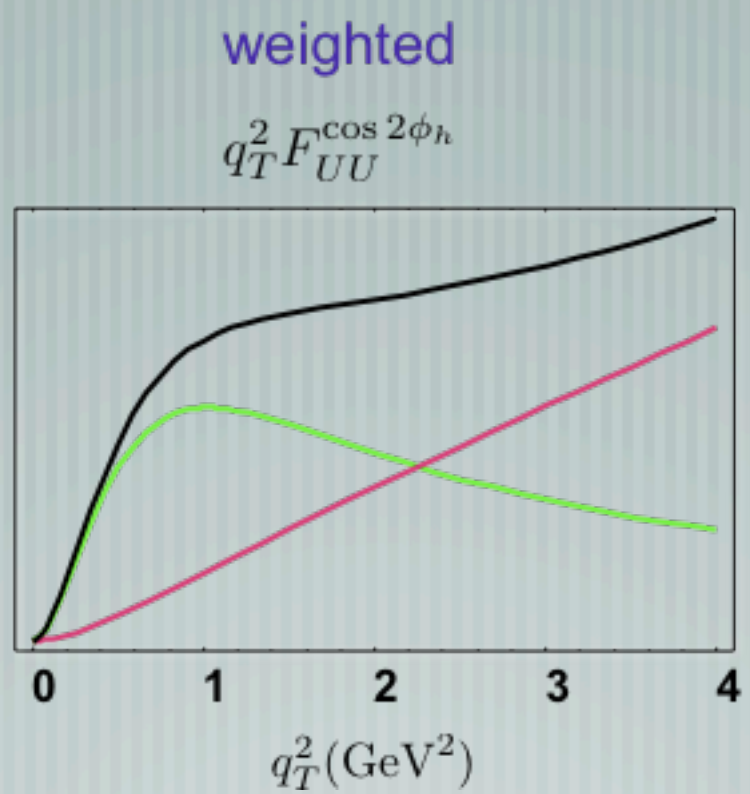


$F_{UU}^{\cos 2\phi_h}$ and weighting

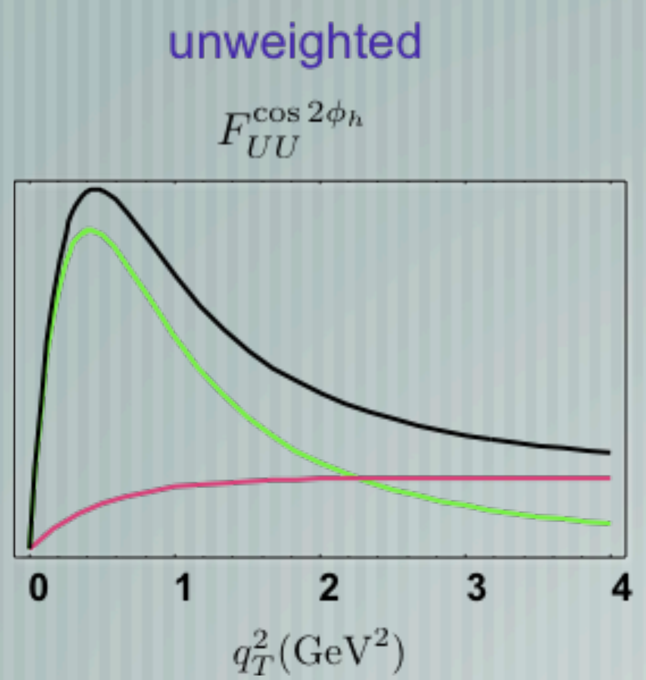


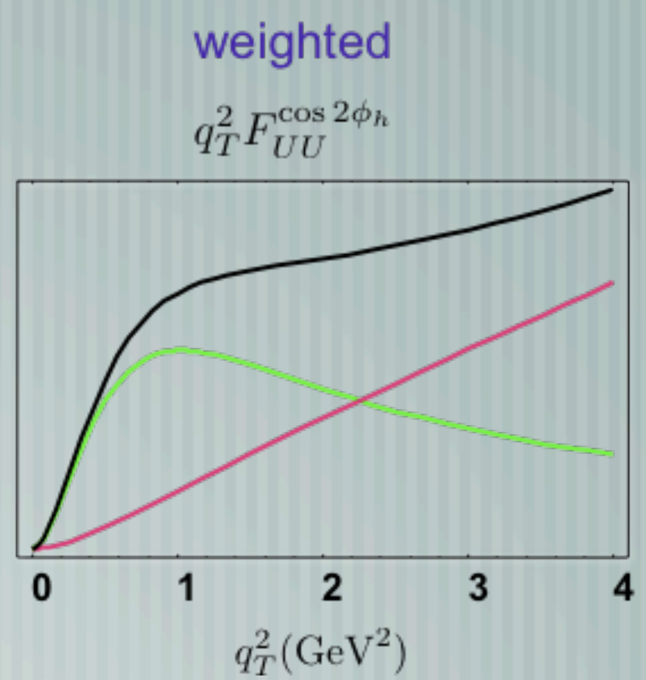


$F_{UU}^{\cos 2\phi_h}$ and weighting

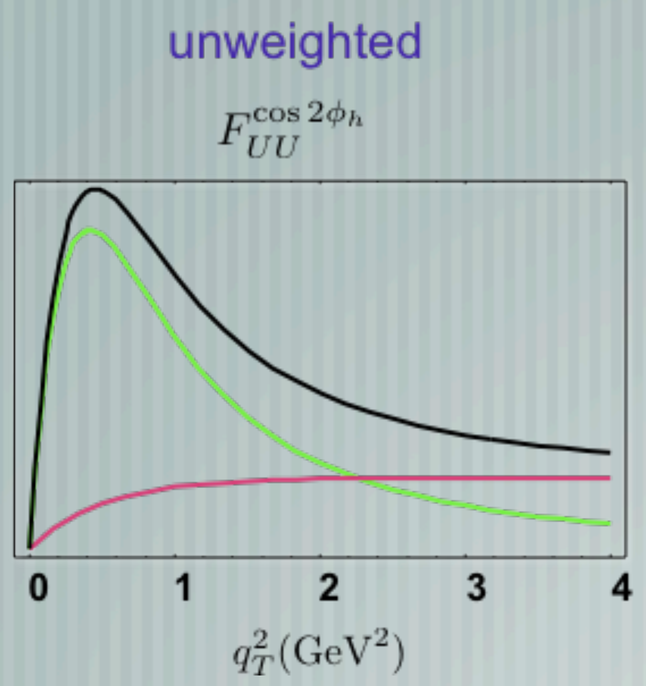


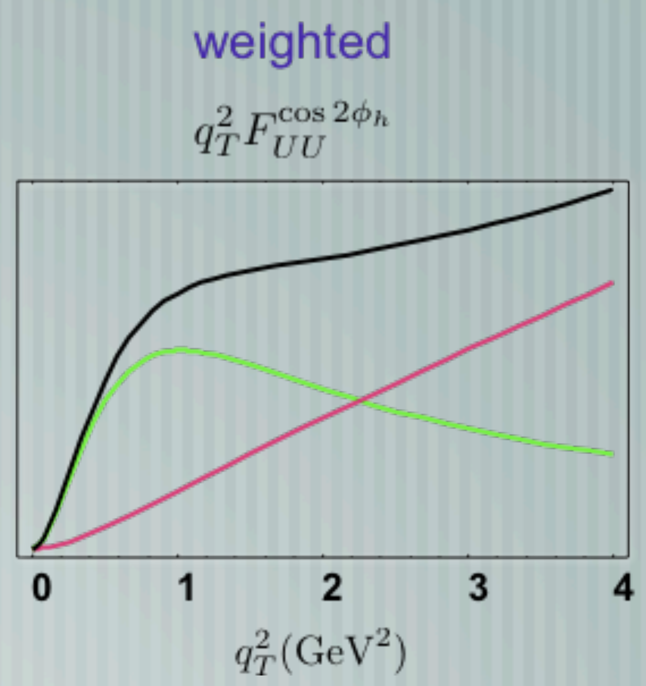
$F_{UU}^{\cos 2\phi_h}$ and weighting





$F_{UU}^{\cos 2\phi_h}$ and weighting

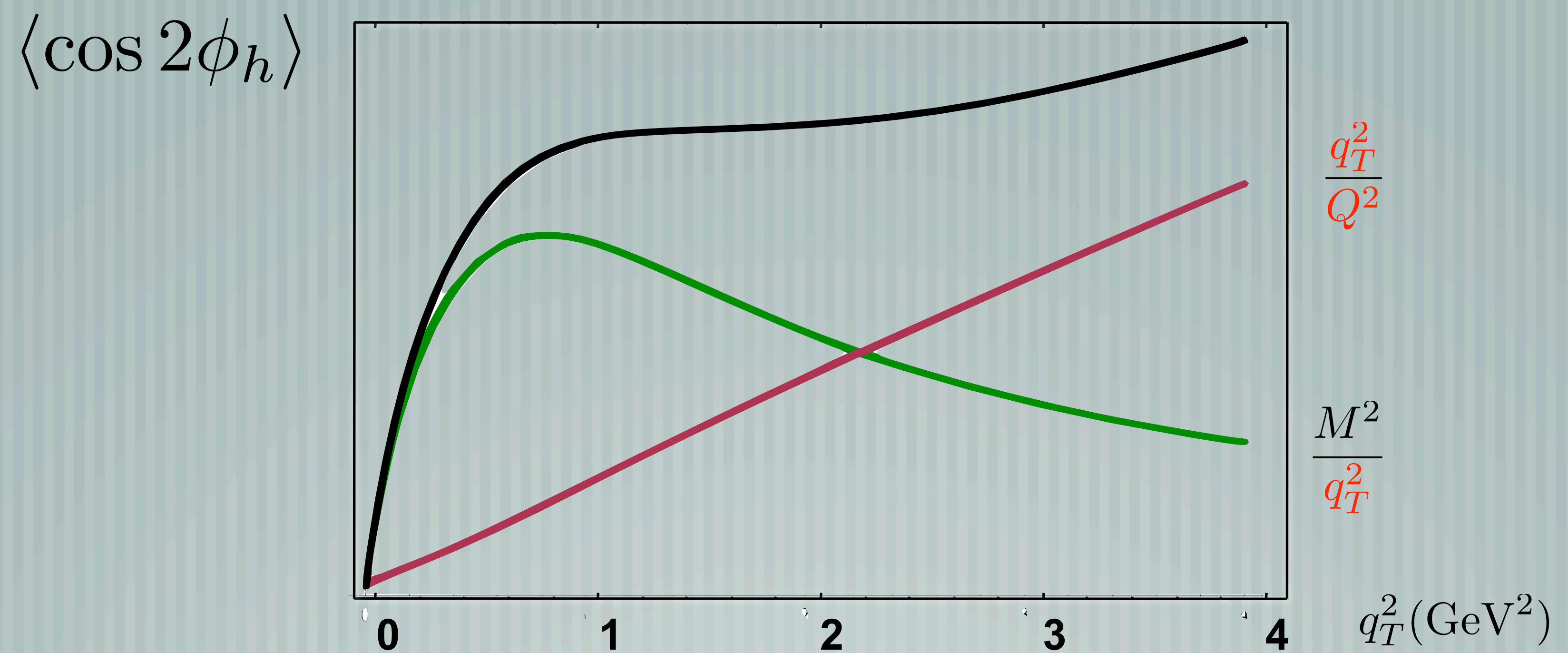




Weighting is not a good idea to access Boer-Mulders function

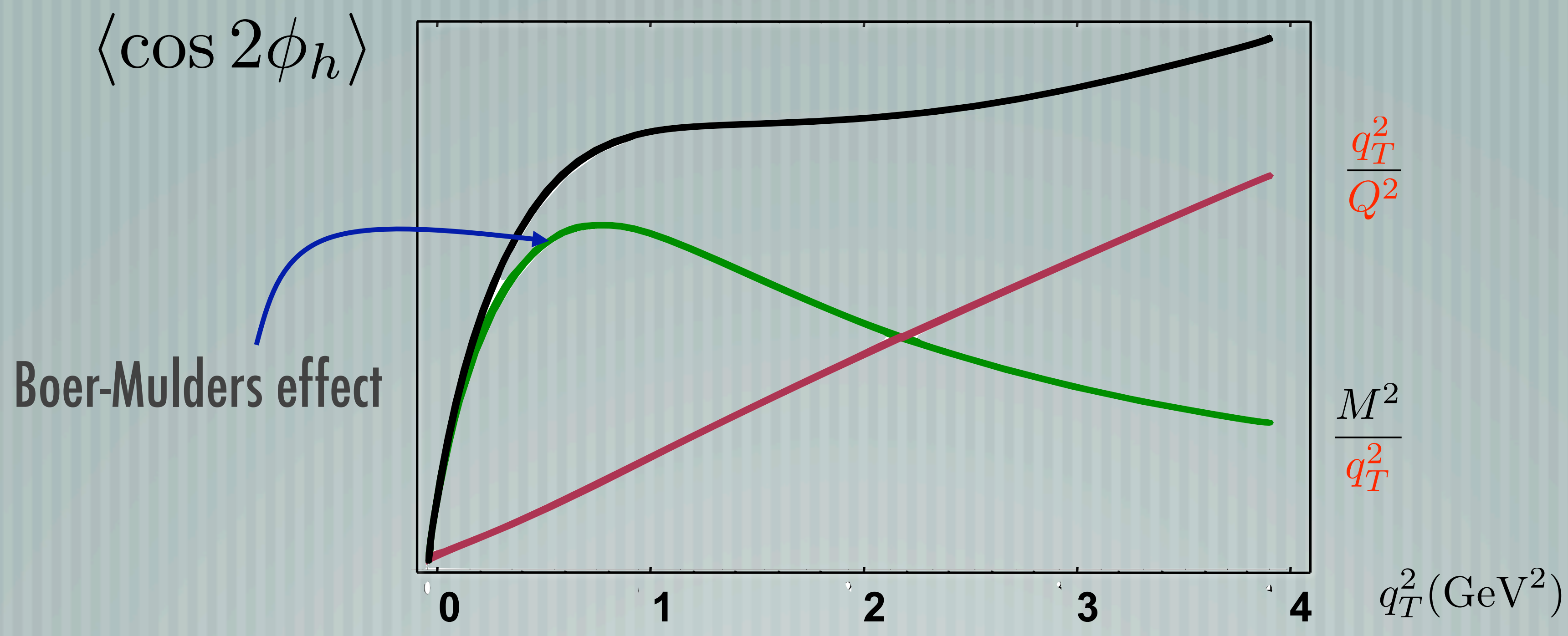
$\cos 2\phi_h$ asymmetry

see also Barone, Prokudin, Ma 0804.3024



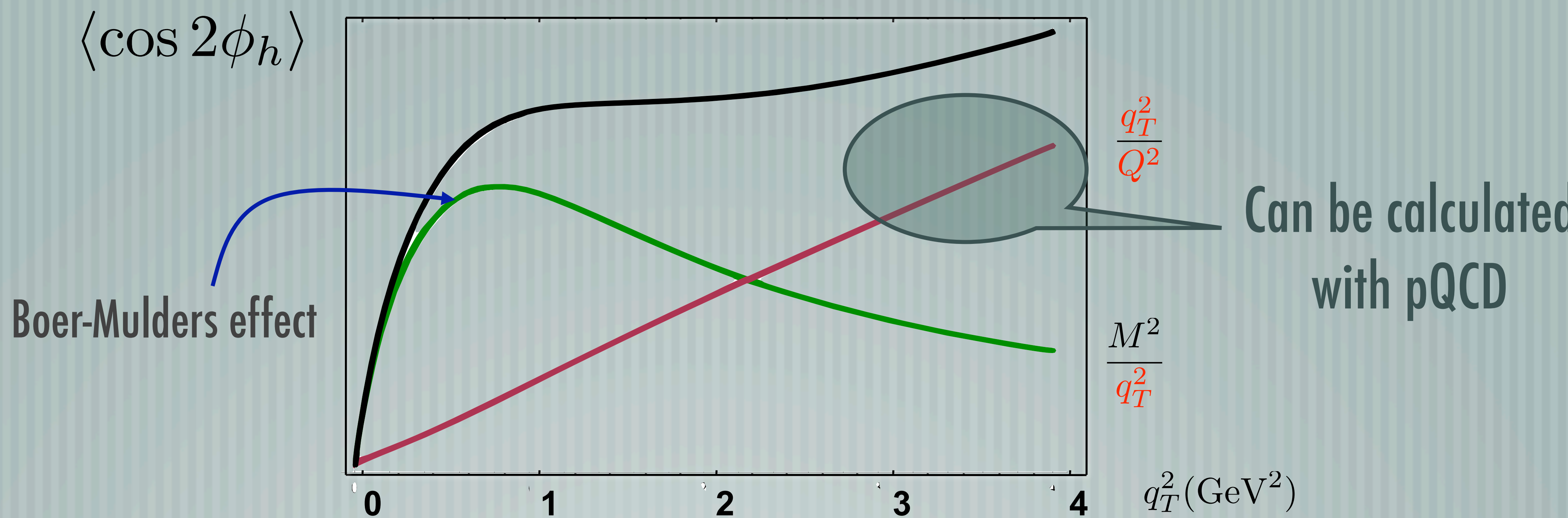
$\cos 2\phi_h$ asymmetry

see also Barone, Prokudin, Ma 0804.3024



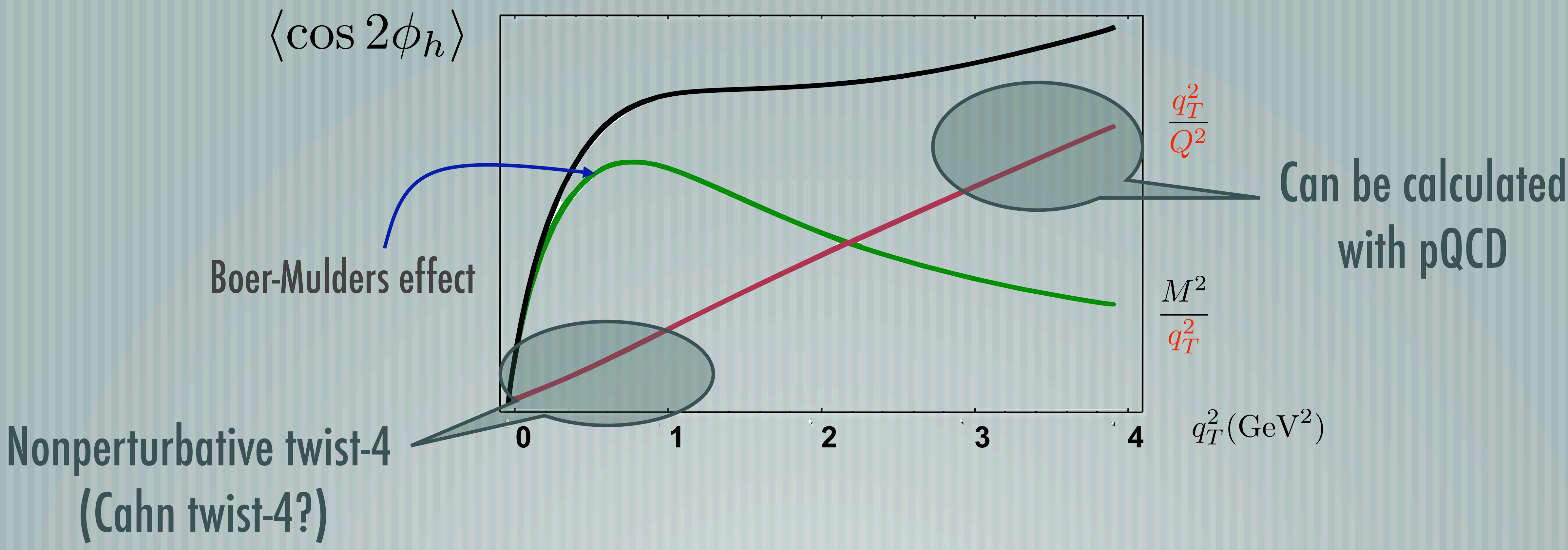
$\cos 2\phi_h$ asymmetry

see also Barone, Prokudin, Ma 0804.3024



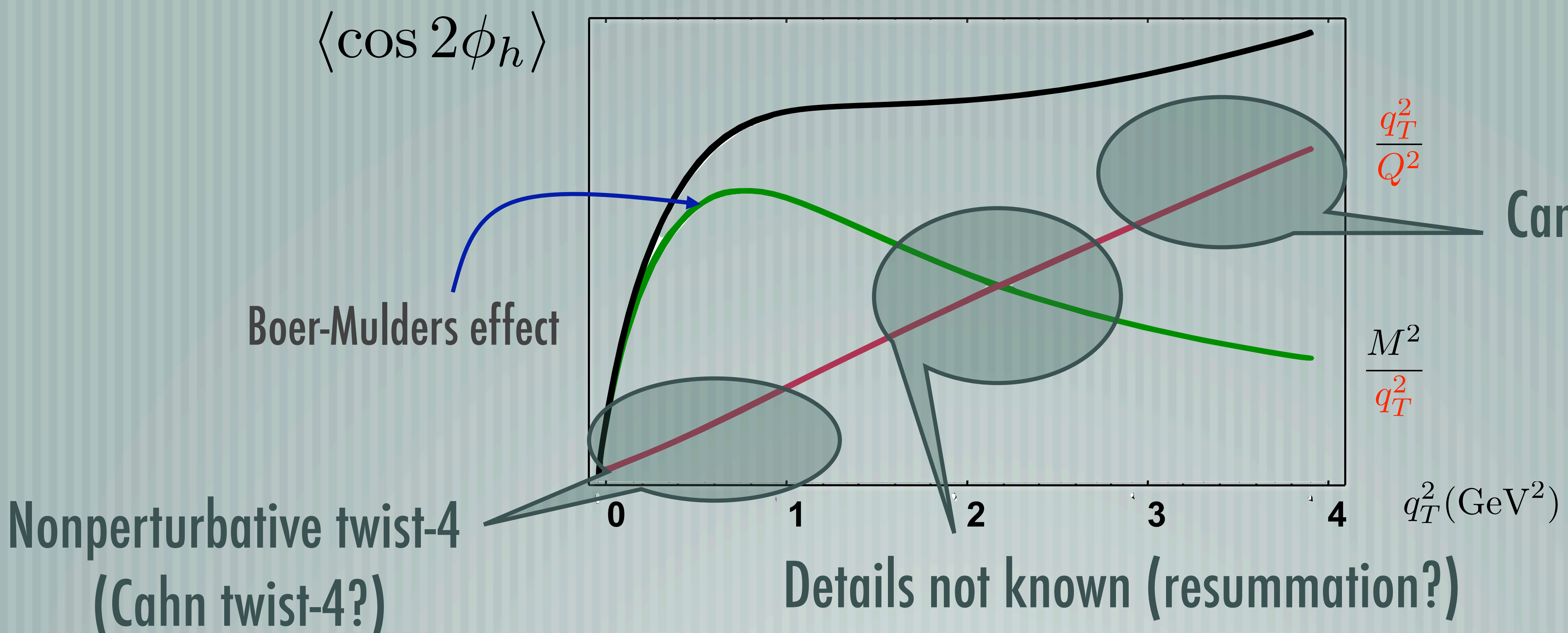
$\cos 2\phi_h$ asymmetry

see also Barone, Prokudin, Ma 0804.3024



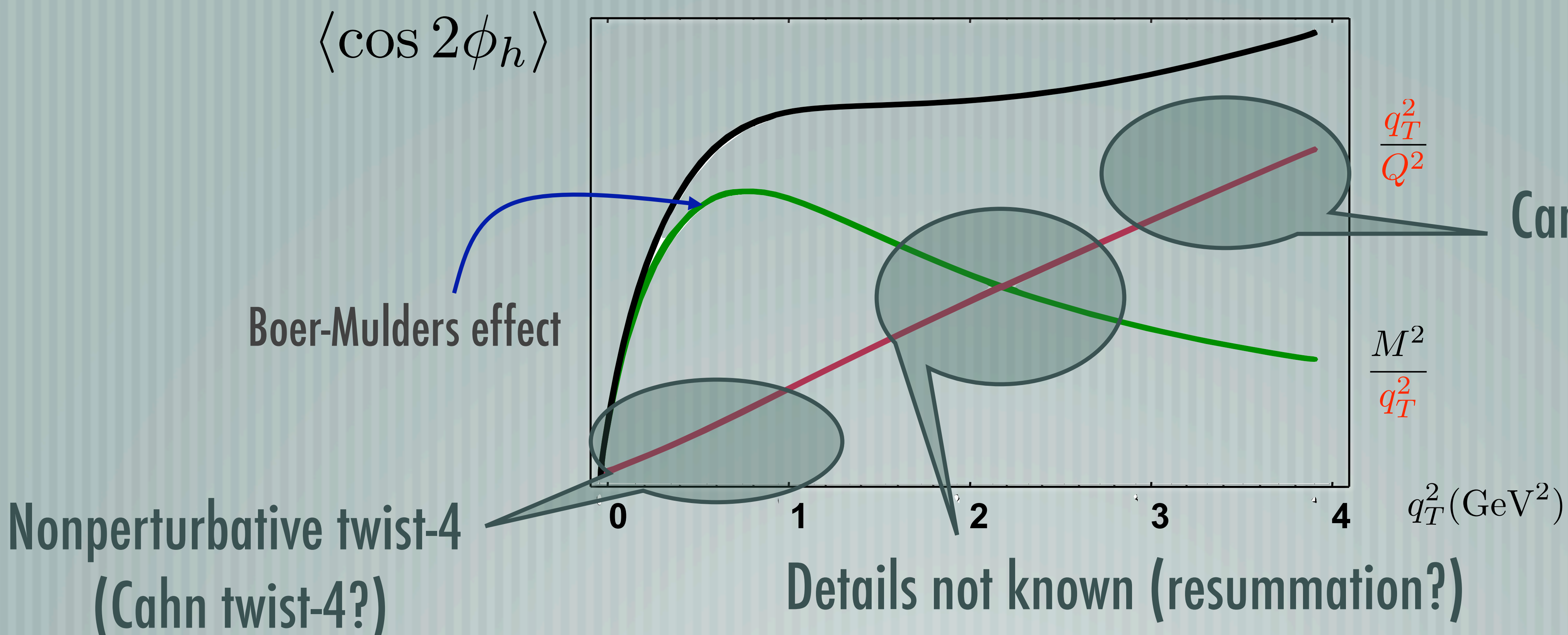
$\cos 2\phi_h$ asymmetry

see also Barone, Prokudin, Ma 0804.3024



$\cos 2\phi_h$ asymmetry

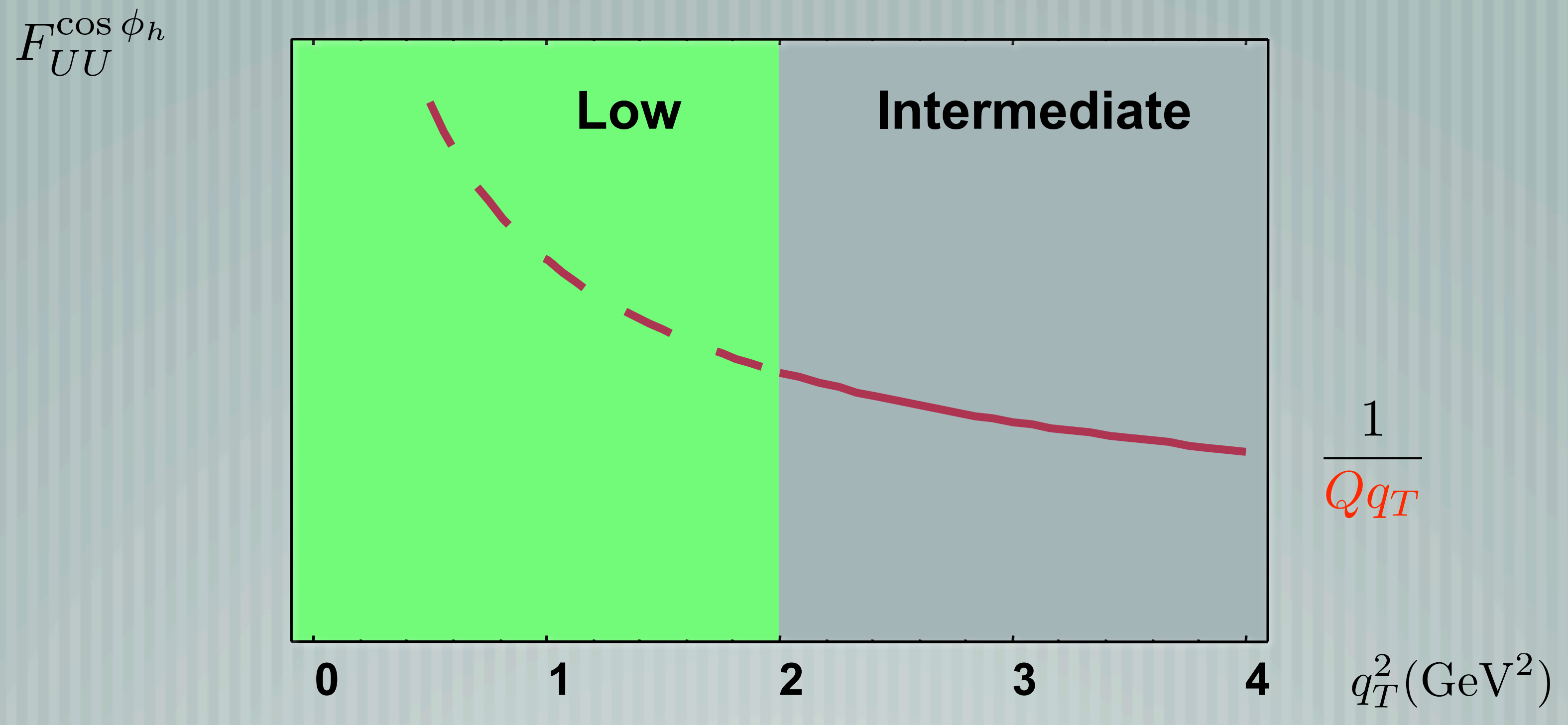
see also Barone, Prokudin, Ma 0804.3024



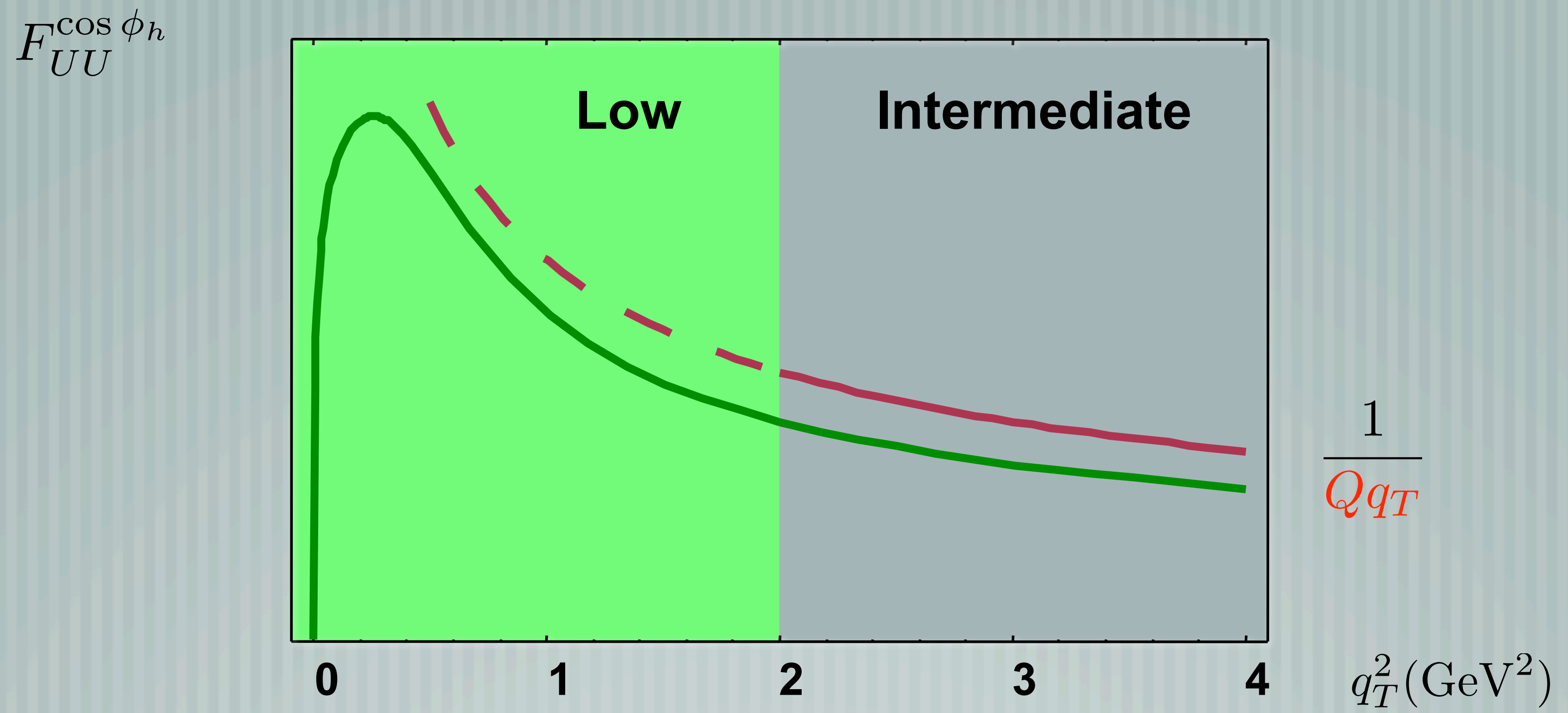
Similarly for Drell-Yan Boer-Mulders measurement and Belle Collins measurement

talks of M. Grosse-Perdekamp and J.-C. Peng

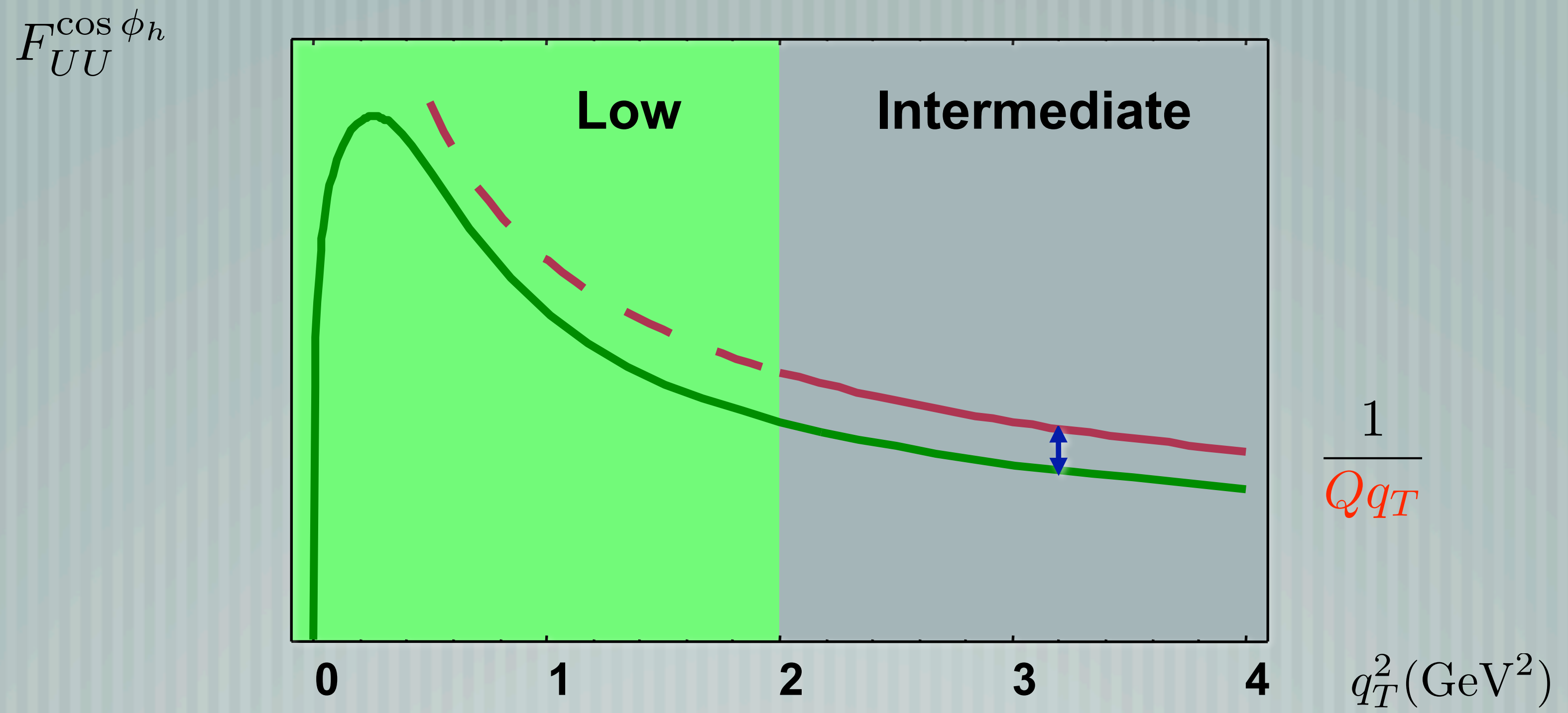
$F_{UU}^{\cos\phi_h}$ structure function



$F_{UU}^{\cos\phi_h}$ structure function



$F_{UU}^{\cos\phi_h}$ structure function



Unexpected mismatch: same power behavior, but they don't match

Problems with the formalism at low transverse momentum!

From low to intermediate

Low q_T $$F_{UU}^{\cos\phi_h} = \frac{2M}{Q}\,\mathcal{C}\Big[-\frac{\hat{h}k_T}{M_h}\Big(xh\,H_1^{\perp} + \frac{M_h}{M}\,f_1\frac{\tilde{D}^{\perp}}{z}\Big) - \frac{\hat{h}p_T}{M}\Big(xf^{\perp}D_1 + \frac{M_h}{M}\,h_1^{\perp}\frac{\tilde{H}}{z}\Big)\Big]$$

From low to intermediate

Low q_T $$F_{UU}^{\cos\phi_h} = \frac{2M}{Q}\,\mathcal{C}\Big[-\frac{\hat{h}k_T}{M_h}\Big(xh\,H_1^{\perp} + \frac{M_h}{M}\,f_1\frac{\tilde{D}^{\perp}}{z}\Big) - \frac{\hat{h}p_T}{M}\Big(xf^{\perp}D_1 + \frac{M_h}{M}\,h_1^{\perp}\frac{\tilde{H}}{z}\Big)\Big]$$

⇩

Intermediate q_T

From low to intermediate

Low q_T

$$F_{UU}^{\cos\phi_h} = \frac{2M}{Q}\,\mathcal{C}\Big[-\frac{\hat{h}k_T}{M_h}\Big(xh\,H_1^{\perp} + \frac{M_h}{M}\,f_1\frac{\tilde{D}^{\perp}}{z}\Big) - \frac{\hat{h}p_T}{M}\Big(xf^{\perp}D_1 + \frac{M_h}{M}\,h_1^{\perp}\frac{\tilde{H}}{z}\Big)\Big]$$

⇩

Intermediate q_T

$$F_{UU}^{\cos\phi_h} = -\frac{2q_T}{Q}\sum_a xe_a^2\left[xf^{\perp a}(x,q_T^2)\,\frac{D_1^a(z)}{z^2} - f_1^a(x)\,\frac{\tilde{D}^{\perp a}(z,q_T^2)}{z}\right]$$

From low to intermediate

Low q_T $F_{UU}^{\cos\phi_h} = \frac{2M}{Q}\,\mathcal{C}\Big[-\frac{\hat{h}k_T}{M_h}\Big(xh\,H_1^{\perp} + \frac{M_h}{M}\,f_1\frac{\tilde{D}^{\perp}}{z}\Big) - \frac{\hat{h}p_T}{M}\Big(xf^{\perp}D_1 + \frac{M_h}{M}\,h_1^{\perp}\frac{\tilde{H}}{z}\Big)\Big]$

Intermediate q_T $F_{UU}^{\cos\phi_h} = -\frac{2q_T}{Q}\sum_a xe_a^2\left[xf^{\perp a}(x,q_T^2)\,\frac{D_1^a(z)}{z^2} - f_1^a(x)\,\frac{\tilde{D}^{\perp a}(z,q_T^2)}{z}\right]$

use $xf^{\perp q}(x,p_T^2) = \frac{\alpha_s}{2\pi^2}\,\frac{1}{2p_T^2}\left[\frac{L(\eta^{-1})}{2}\,f_1^q(x) + \big(P'_{qq}\otimes f_1^q + P'_{qg}\otimes f_1^g\big)(x)\right]$

From low to intermediate

Low q_T $$F_{UU}^{\cos\phi_h} = \frac{2M}{Q}\,\mathcal{C}\Big[-\frac{\hat{h}k_T}{M_h}\Big(xh\,H_1^{\perp} + \frac{M_h}{M}\,f_1\frac{\tilde{D}^{\perp}}{z}\Big) - \frac{\hat{h}p_T}{M}\Big(xf^{\perp}D_1 + \frac{M_h}{M}\,h_1^{\perp}\frac{\tilde{H}}{z}\Big)\Big]$$

absent in "Cahn effect"

Intermediate q_T $$F_{UU}^{\cos\phi_h} = -\frac{2q_T}{Q}\sum_a xe_a^2\Big[xf^{\perp a}(x,q_T^2)\,\frac{D_1^a(z)}{z^2} - f_1^a(x)\,\frac{\tilde{D}^{\perp a}(z,q_T^2)}{z}\Big]$$

use $$xf^{\perp q}(x,p_T^2) = \frac{\alpha_s}{2\pi^2}\,\frac{1}{2p_T^2}\Big[\frac{L(\eta^{-1})}{2}\,f_1^q(x) + \big(P'_{qq}\otimes f_1^q + P'_{qg}\otimes f_1^g\big)(x)\Big]$$

not consistent with "Cahn effect"

Cf. "Cahn effect calculations" of Anselmino, Boglione, Prokudin, Turk, EPJA31

From low to intermediate

Low q_T

$$F_{UU}^{\cos\phi_h} = \frac{2M}{Q}\,\mathcal{C}\Big[-\frac{\hat{h}k_T}{M_h}\Big(xh\,H_1^{\perp} + \frac{M_h}{M}\,f_1\frac{\tilde{D}^{\perp}}{z}\Big) - \frac{\hat{h}p_T}{M}\Big(xf^{\perp}D_1 + \frac{M_h}{M}\,h_1^{\perp}\frac{\tilde{H}}{z}\Big)\Big]$$

⇩

Intermediate q_T

$$F_{UU}^{\cos\phi_h} = -\frac{2q_T}{Q}\sum_a xe_a^2\left[xf^{\perp a}(x,q_T^2)\,\frac{D_1^a(z)}{z^2} - f_1^a(x)\,\frac{\tilde{D}^{\perp a}(z,q_T^2)}{z}\right]$$

use

$$xf^{\perp q}(x,p_T^2) = \frac{\alpha_s}{2\pi^2}\,\frac{1}{2p_T^2}\left[\frac{L(\eta^{-1})}{2}\,f_1^q(x) + \big(P'_{qq}\otimes f_1^q + P'_{qg}\otimes f_1^g\big)(x)\right]$$

From low to intermediate

Low q_T $$F_{UU}^{\cos\phi_h} = \frac{2M}{Q}\,\mathcal{C}\Big[-\frac{\hat{h}k_T}{M_h}\Big(xh\,H_1^{\perp} + \frac{M_h}{M}\,f_1\frac{\tilde{D}^{\perp}}{z}\Big) - \frac{\hat{h}p_T}{M}\Big(xf^{\perp}D_1 + \frac{M_h}{M}\,h_1^{\perp}\frac{\tilde{H}}{z}\Big)\Big]$$

Intermediate q_T $$F_{UU}^{\cos\phi_h} = -\frac{2q_T}{Q}\sum_a xe_a^2\left[xf^{\perp a}(x,q_T^2)\,\frac{D_1^a(z)}{z^2} - f_1^a(x)\,\frac{\tilde{D}^{\perp a}(z,q_T^2)}{z}\right]$$

use

$$xf^{\perp q}(x,p_T^2) = \frac{\alpha_s}{2\pi^2}\,\frac{1}{2p_T^2}\left[\frac{L(\eta^{-1})}{2}\,f_1^q(x) + \big(P'_{qq}\otimes f_1^q + P'_{qg}\otimes f_1^g\big)(x)\right]$$

$$\frac{\tilde{D}^{\perp q}(z,k_T^2)}{z} = -\frac{\alpha_s}{2\pi^2}\,\frac{1}{2z^2k_T^2}\left[\frac{L(\eta_h^{-1})}{2}\,D_1^q(z) - 2C_F D_1^q(z) + \big(D_1^q\otimes P'_{qq} + D_1^g\otimes P'_{gq}\big)(z)\right]$$

From low to intermediate

Low q_T

$$F_{UU}^{\cos\phi_h} = \frac{2M}{Q}\,\mathcal{C}\Big[-\frac{\hat{h}k_T}{M_h}\Big(xh\,H_1^{\perp} + \frac{M_h}{M}\,f_1\frac{\tilde{D}^{\perp}}{z}\Big) - \frac{\hat{h}p_T}{M}\Big(xf^{\perp}D_1 + \frac{M_h}{M}\,h_1^{\perp}\frac{\tilde{H}}{z}\Big)\Big]$$

Intermediate q_T

$$F_{UU}^{\cos\phi_h} = -\frac{2q_T}{Q}\sum_a xe_a^2\Big[xf^{\perp a}(x,q_T^2)\,\frac{D_1^a(z)}{z^2} - f_1^a(x)\,\frac{\tilde{D}^{\perp a}(z,q_T^2)}{z}\Big]$$

use

$$xf^{\perp q}(x,p_T^2) = \frac{\alpha_s}{2\pi^2}\,\frac{1}{2p_T^2}\Big[\frac{L(\eta^{-1})}{2}\,f_1^q(x) + \big(P'_{qq}\otimes f_1^q + P'_{qg}\otimes f_1^g\big)(x)\Big]$$

$$\frac{\tilde{D}^{\perp q}(z,k_T^2)}{z} = -\frac{\alpha_s}{2\pi^2}\,\frac{1}{2z^2k_T^2}\Big[\frac{L(\eta_h^{-1})}{2}\,D_1^q(z) - 2C_F D_1^q(z) + \big(D_1^q\otimes P'_{qq} + D_1^g\otimes P'_{gq}\big)(z)\Big]$$

$$F_{UU}^{\cos\phi_h} = -\frac{1}{Qq_T}\,\frac{\alpha_s}{2\pi^2z^2}\sum_a xe_a^2\Big[f_1^a(x)\,D_1^a(z)\,L\Big(\frac{Q^2}{q_T^2}\Big) + f_1^a(x)\,\big(D_1^a\otimes P'_{qq} + D_1^a\otimes P'_{gq}\big)(z)$$
$$+ \big(P'_{qq}\otimes f_1^a + P'_{qg}\otimes f_1^g\big)(x)\,D_1^a(z) - 2C_F\,f_1^a(x)\,D_1^a(z)\Big]$$

From low to intermediate

Low q_T

$$F_{UU}^{\cos\phi_h} = \frac{2M}{Q}\,\mathcal{C}\Big[-\frac{\hat{h}k_T}{M_h}\Big(xh\,H_1^{\perp} + \frac{M_h}{M}\,f_1\frac{\tilde{D}^{\perp}}{z}\Big) - \frac{\hat{h}p_T}{M}\Big(xf^{\perp}D_1 + \frac{M_h}{M}\,h_1^{\perp}\frac{\tilde{H}}{z}\Big)\Big]$$

Intermediate q_T

$$F_{UU}^{\cos\phi_h} = -\frac{2q_T}{Q}\sum_a xe_a^2\left[xf^{\perp a}(x,q_T^2)\,\frac{D_1^a(z)}{z^2} - f_1^a(x)\,\frac{\tilde{D}^{\perp a}(z,q_T^2)}{z}\right]$$

use

$$xf^{\perp q}(x,p_T^2) = \frac{\alpha_s}{2\pi^2}\,\frac{1}{2p_T^2}\left[\frac{L(\eta^{-1})}{2}\,f_1^q(x) + \big(P'_{qq}\otimes f_1^q + P'_{qg}\otimes f_1^g\big)(x)\right]$$

$$\frac{\tilde{D}^{\perp q}(z,k_T^2)}{z} = -\frac{\alpha_s}{2\pi^2}\,\frac{1}{2z^2k_T^2}\left[\frac{L(\eta_h^{-1})}{2}\,D_1^q(z) - 2C_F D_1^q(z) + \big(D_1^q\otimes P'_{qq} + D_1^g\otimes P'_{gq}\big)(z)\right]$$

$$F_{UU}^{\cos\phi_h} = -\frac{1}{Qq_T}\,\frac{\alpha_s}{2\pi^2z^2}\sum_a xe_a^2\Big[f_1^a(x)\,D_1^a(z)\,L\Big(\frac{Q^2}{q_T^2}\Big) + f_1^a(x)\,\big(D_1^a\otimes P'_{qq} + D_1^a\otimes P'_{gq}\big)(z)$$

$$+\big(P'_{qq}\otimes f_1^a + P'_{qg}\otimes f_1^g\big)(x)\,D_1^a(z) - 2C_F\,f_1^a(x)\,D_1^a(z)\Big]$$

Not the same as high trans. mom. calculation!

From low to intermediate

Low q_T

$$F_{UU}^{\cos\phi_h} = \frac{2M}{Q}\,\mathcal{C}\Big[-\frac{\hat{h}k_T}{M_h}\Big(xh\,H_1^{\perp} + \frac{M_h}{M}\,f_1\frac{\tilde{D}^{\perp}}{z}\Big) - \frac{\hat{h}p_T}{M}\Big(xf^{\perp}D_1 + \frac{M_h}{M}\,h_1^{\perp}\frac{\tilde{H}}{z}\Big)\Big]$$

It seems so easy to correct it...

Intermediate q_T

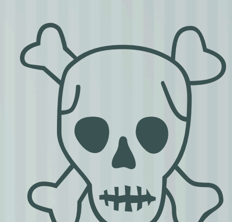
$$F_{UU}^{\cos\phi_h} = -\frac{2q_T}{Q}\sum_a xe_a^2\left[xf^{\perp a}(x,q_T^2)\,\frac{D_1^a(z)}{z^2} - f_1^a(x)\,\frac{\tilde{D}^{\perp a}(z,q_T^2)}{z}\right]$$

use

$$xf^{\perp q}(x,p_T^2) = \frac{\alpha_s}{2\pi^2}\,\frac{1}{2p_T^2}\left[\frac{L(\eta^{-1})}{2}\,f_1^q(x) + \big(P'_{qq}\otimes f_1^q + P'_{qg}\otimes f_1^g\big)(x)\right]$$

$$\frac{\tilde{D}^{\perp q}(z,k_T^2)}{z} = -\frac{\alpha_s}{2\pi^2}\,\frac{1}{2z^2k_T^2}\left[\frac{L(\eta_h^{-1})}{2}\,D_1^q(z) - 2C_F D_1^q(z) + \big(D_1^q\otimes P'_{qq} + D_1^g\otimes P'_{gq}\big)(z)\right]$$

$$F_{UU}^{\cos\phi_h} = -\frac{1}{Qq_T}\,\frac{\alpha_s}{2\pi^2z^2}\sum_a xe_a^2\left[f_1^a(x)\,D_1^a(z)\,L\Big(\frac{Q^2}{q_T^2}\Big) + f_1^a(x)\,\big(D_1^a\otimes P'_{qq} + D_1^a\otimes P'_{gq}\big)(z)\right.$$
$$\left. + \big(P'_{qq}\otimes f_1^a + P'_{qg}\otimes f_1^g\big)(x)\,D_1^a(z) - 2C_F\,f_1^a(x)\,D_1^a(z)\right]$$



Not the same as high trans. mom. calculation!

From low to intermediate

Low q_T

$$F_{UU}^{\cos\phi_h} = \frac{2M}{Q}\,\mathcal{C}\Big[-\frac{\hat{h}k_T}{M_h}\Big(xh\,H_1^{\perp} + \frac{M_h}{M}\,f_1\frac{\tilde{D}^{\perp}}{z}\Big) - \frac{\hat{h}p_T}{M}\Big(xf^{\perp}D_1 + \frac{M_h}{M}\,h_1^{\perp}\frac{\tilde{H}}{z}\Big)\Big]$$

It seems so easy to correct it...

Intermediate q_T

$$F_{UU}^{\cos\phi_h} = -\frac{2q_T}{Q}\sum_a xe_a^2\left[xf^{\perp a}(x,q_T^2)\,\frac{D_1^a(z)}{z^2} - f_1^a(x)\,\frac{\tilde{D}^{\perp a}(z,q_T^2)}{z}\right] + f_1^a(x)\,\frac{D_1^a(z)}{z^2}\,\frac{U(q_T^2)}{2}$$

use

$$xf^{\perp q}(x,p_T^2) = \frac{\alpha_s}{2\pi^2}\,\frac{1}{2p_T^2}\left[\frac{L(\eta^{-1})}{2}\,f_1^q(x) + \big(P'_{qq}\otimes f_1^q + P'_{qg}\otimes f_1^g\big)(x)\right]$$

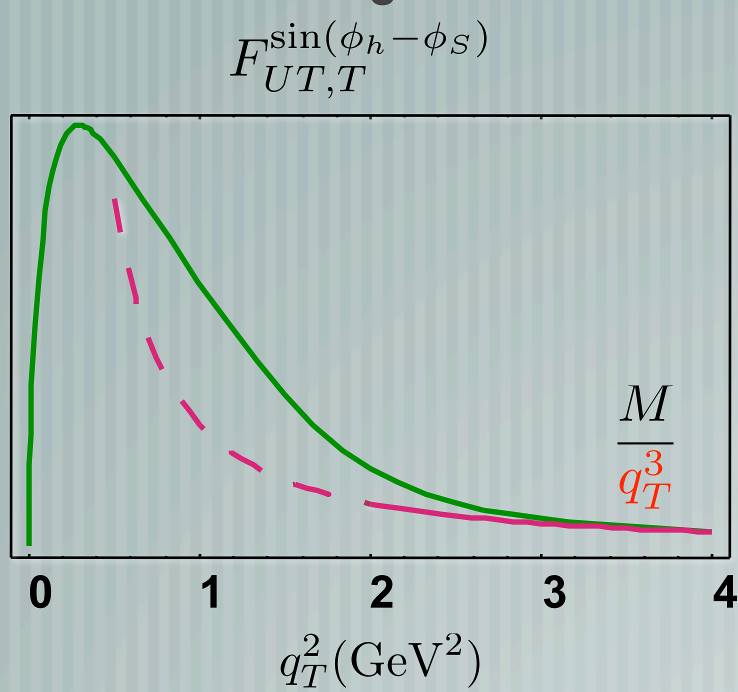
$$\frac{\tilde{D}^{\perp q}(z,k_T^2)}{z} = -\frac{\alpha_s}{2\pi^2}\,\frac{1}{2z^2k_T^2}\left[\frac{L(\eta_h^{-1})}{2}\,D_1^q(z) - 2C_F D_1^q(z) + \big(D_1^q\otimes P'_{qq} + D_1^g\otimes P'_{gq}\big)(z)\right]$$

$$F_{UU}^{\cos\phi_h} = -\frac{1}{Qq_T}\,\frac{\alpha_s}{2\pi^2z^2}\sum_a xe_a^2\Big[f_1^a(x)\,D_1^a(z)\,L\Big(\frac{Q^2}{q_T^2}\Big) + f_1^a(x)\,\big(D_1^a\otimes P'_{qq} + D_1^a\otimes P'_{gq}\big)(z)$$
$$+ \big(P'_{qq}\otimes f_1^a + P'_{qg}\otimes f_1^g\big)(x)\,D_1^a(z) - 2C_F f_1^a(x)\,D_1^a(z)\Big]$$

☠ Not the same as high trans. mom. calculation!

$F_{UT,T}^{\sin(\phi_h-\phi_S)}$ (Sivers) structure funct.

unweighted

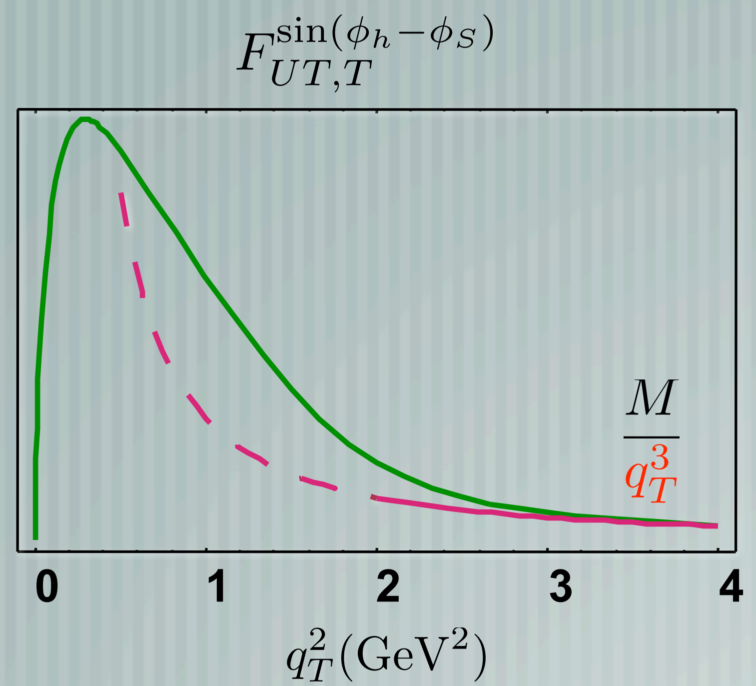


Ji, Qiu, Vogelsang, Yuan, PLB638 (06)

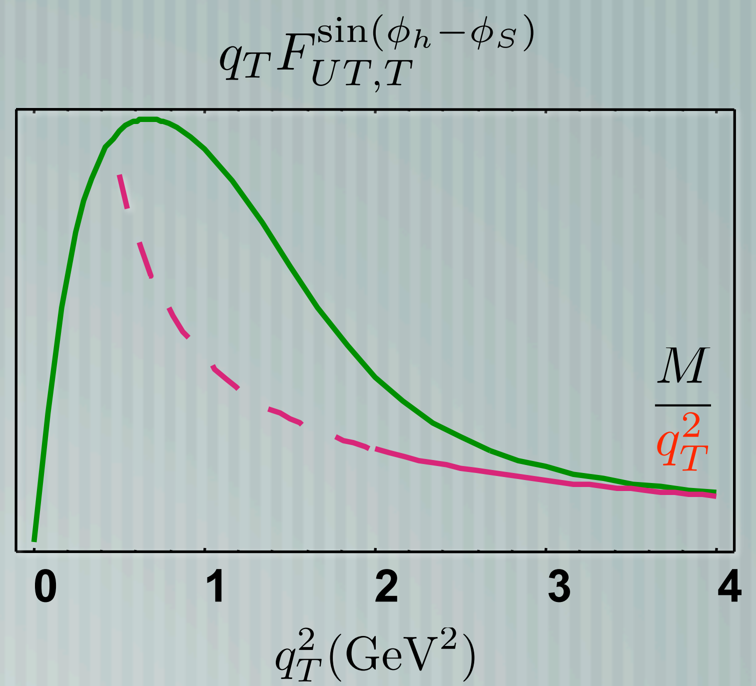
Koike, Vogelsang, Yuan, arXiv:0711.0636

$F_{UT,T}^{\sin(\phi_h-\phi_S)}$ (Sivers) structure funct.

unweighted



weighted



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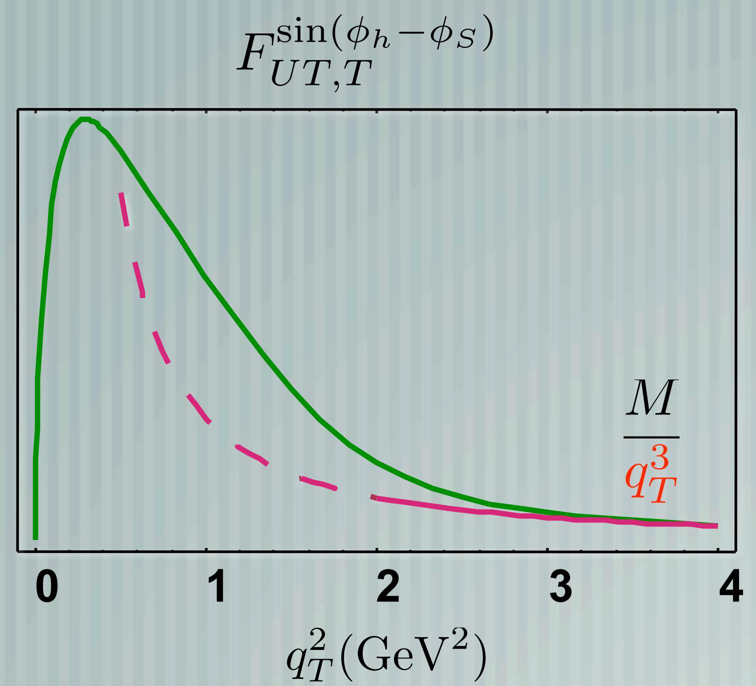
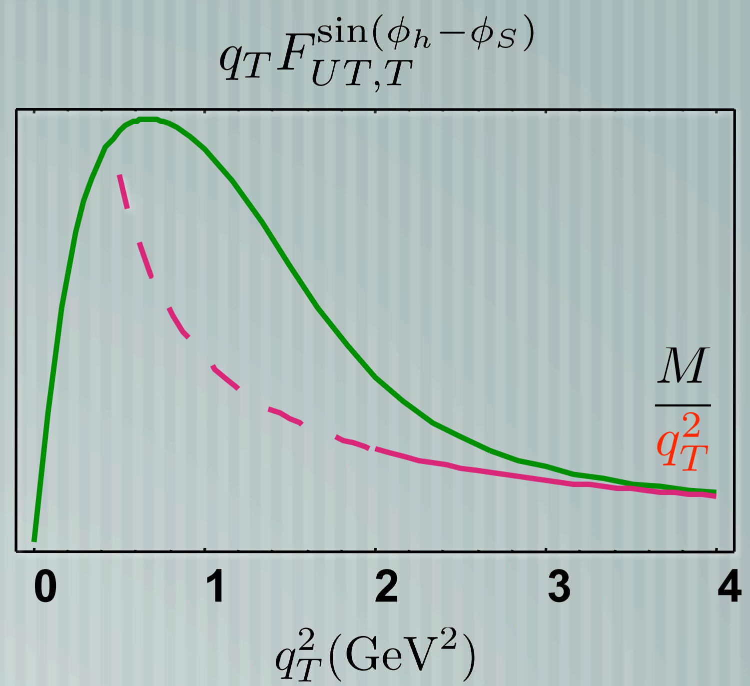
Koike, Vogelsang, Yuan, arXiv:0711.0636

$F_{UT,T}^{\sin(\phi_h-\phi_S)}$ (Sivers) structure funct.

unweighted



weighted



Ji, Qiu, Vogelsang, Yuan, PLB638 (06)

Koike, Vogelsang, Yuan, arXiv:0711.0636

Weighting is good!

Power behaviors and expected mismatches

	low-q_T calculation			high-q_T calculation			
observable	twist	order	power	twist	order	power	powers match
$F_{UU,T}$	2	α_s	$1/q_T^2$	2	α_s	$1/q_T^2$	yes
$F_{UU,L}$	4			2	α_s	$1/Q^2$	?
$F_{UU}^{\cos\phi_h}$	3	α_s	$1/(Q\,q_T)$	2	α_s	$1/(Q\,q_T)$	yes
$F_{UU}^{\cos 2\phi_h}$	2	α_s	$1/q_T^4$	2	α_s	$1/Q^2$	no
$F_{LU}^{\sin\phi_h}$	3	α_s^2	$1/(Q\,q_T)$	2	α_s^2	$1/(Q\,q_T)$	yes
$F_{UL}^{\sin\phi_h}$	3	α_s^2	$1/(Q\,q_T)$				?
$F_{UL}^{\sin 2\phi_h}$	2	α_s	$1/q_T^4$				?
F_{LL}	2	α_s	$1/q_T^2$	2	α_s	$1/q_T^2$	yes
$F_{LL}^{\cos\phi_h}$	3	α_s	$1/(Q\,q_T)$	2	α_s	$1/(Q\,q_T)$	yes
$F_{UT,T}^{\sin(\phi_h-\phi_S)}$	2	α_s	$1/q_T^3$	3	α_s	$1/q_T^3$	yes
$F_{UT,L}^{\sin(\phi_h-\phi_S)}$	4			3	α_s	$1/(Q^2\,q_T)$	?
$F_{UT}^{\sin(\phi_h+\phi_S)}$	2	α_s	$1/q_T^3$	3	α_s	$1/q_T^3$	yes
$F_{UT}^{\sin(3\phi_h-\phi_S)}$	2	α_s^2	$1/q_T^3$	3	α_s	$1/(Q^2\,q_T)$	no
$F_{UT}^{\sin\phi_S}$	3	α_s	$1/(Q\,q_T^2)$	3	α_s	$1/(Q\,q_T^2)$	yes
$F_{UT}^{\sin(2\phi_h-\phi_S)}$	3	α_s	$1/(Q\,q_T^2)$	3	α_s	$1/(Q\,q_T^2)$	yes
$F_{LT}^{\cos(\phi_h-\phi_S)}$	2	α_s	$1/q_T^3$				?
$F_{LT}^{\cos\phi_S}$	3	α_s	$1/(Q\,q_T^2)$				?
$F_{LT}^{\cos(2\phi_h-\phi_S)}$	3	α_s	$1/(Q\,q_T^2)$				?

Power behaviors and expected mismatches

observable	low-q_T calculation twist	order	power	high-q_T calculation twist	order	power	powers match
$F_{UU,T}$	2	α_s	$1/q_T^2$	2	α_s	$1/q_T^2$	yes
$F_{UU,L}$	4			2	α_s	$1/Q^2$	?
$F_{UU}^{\cos\phi_h}$	3	α_s	$1/(Q\,q_T)$	2	α_s	$1/(Q\,q_T)$	yes
$F_{UU}^{\cos 2\phi_h}$	2	α_s	$1/q_T^4$	2	α_s	$1/Q^2$	no
$F_{LU}^{\sin\phi_h}$	3	α_s^2	$1/(Q\,q_T)$	2	α_s^2	$1/(Q\,q_T)$	yes
$F_{UL}^{\sin\phi_h}$	3	α_s^2	$1/(Q\,q_T)$				
$F_{UL}^{\sin 2\phi_h}$	2	α_s	$1/q_T^4$				
F_{LL}	2	α_s	$1/q_T^2$	2	α_s	$1/q_T^2$	yes
$F_{LL}^{\cos\phi_h}$	3	α_s	$1/(Q\,q_T)$	2	α_s	$1/(Q\,q_T)$	yes
$F_{UT,T}^{\sin(\phi_h-\phi_S)}$	2	α_s	$1/q_T^3$	3	α_s	$1/q_T^3$	yes
$F_{UT,L}^{\sin(\phi_h-\phi_S)}$	4			3	α_s	$1/(Q^2\,q_T)$	?
$F_{UT}^{\sin(\phi_h+\phi_S)}$	2	α_s	$1/q_T^3$	3	α_s	$1/q_T^3$	yes
$F_{UT}^{\sin(3\phi_h-\phi_S)}$	2	α_s^2	$1/q_T^3$	3	α_s	$1/(Q^2\,q_T)$	no
$F_{UT}^{\sin\phi_S}$	3	α_s	$1/(Q\,q_T^2)$	3	α_s	$1/(Q\,q_T^2)$	yes
$F_{UT}^{\sin(2\phi_h-\phi_S)}$	3	α_s	$1/(Q\,q_T^2)$	3	α_s	$1/(Q\,q_T^2)$	yes
$F_{LT}^{\cos(\phi_h-\phi_S)}$	2	α_s	$1/q_T^3$				
$F_{LT}^{\cos\phi_S}$	3	α_s	$1/(Q\,q_T^2)$				
$F_{LT}^{\cos(2\phi_h-\phi_S)}$	3	α_s	$1/(Q\,q_T^2)$				

conjectures!

Table of unexpected mismatches

observable	low-q_T calculation twist	order	power	high-q_T calculation twist	order	power	powers match	exact match
$F_{UU,T}$	2	α_s	$1/q_T^2$	2	α_s	$1/q_T^2$	yes	✓
$F_{UU,L}$	4			2	α_s	$1/Q^2$		
$F_{UU}^{\cos\phi_h}$	3	α_s	$1/(Q\,q_T)$	2	α_s	$1/(Q\,q_T)$	yes	✗
$F_{UU}^{\cos 2\phi_h}$	2	α_s	$1/q_T^4$	2	α_s	$1/Q^2$	no	
$F_{LU}^{\sin\phi_h}$	3	α_s^2	$1/(Q\,q_T)$	2	α_s^2	$1/(Q\,q_T)$	yes	?
$F_{UL}^{\sin\phi_h}$	3	α_s^2	$1/(Q\,q_T)$					
$F_{UL}^{\sin 2\phi_h}$	2	α_s	$1/q_T^4$					
F_{LL}	2	α_s	$1/q_T^2$	2	α_s	$1/q_T^2$	yes	✓
$F_{LL}^{\cos\phi_h}$	3	α_s	$1/(Q\,q_T)$	2	α_s	$1/(Q\,q_T)$	yes	✗
$F_{UT,T}^{\sin(\phi_h-\phi_S)}$	2	α_s	$1/q_T^3$	3	α_s	$1/q_T^3$	yes	✓
$F_{UT,L}^{\sin(\phi_h-\phi_S)}$	4			3	α_s	$1/(Q^2\,q_T)$		
$F_{UT}^{\sin(\phi_h+\phi_S)}$	2	α_s	$1/q_T^3$	3	α_s	$1/q_T^3$	yes	?
$F_{UT}^{\sin(3\phi_h-\phi_S)}$	2	α_s^2	$1/q_T^3$	3	α_s	$1/(Q^2\,q_T)$	no	
$F_{UT}^{\sin\phi_S}$	3	α_s	$1/(Q\,q_T^2)$	3	α_s	$1/(Q\,q_T^2)$	yes	?
$F_{UT}^{\sin(2\phi_h-\phi_S)}$	3	α_s	$1/(Q\,q_T^2)$	3	α_s	$1/(Q\,q_T^2)$	yes	?
$F_{LT}^{\cos(\phi_h-\phi_S)}$	2	α_s	$1/q_T^3$					
$F_{LT}^{\cos\phi_S}$	3	α_s	$1/(Q\,q_T^2)$					
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$F_{UU}^{\cos\phi_h}$	3	α_s	$1/(Q\,q_T)$	2	α_s	$1/(Q\,q_T)$	yes	✘
$F_{UU}^{\cos 2\phi_h}$	2	α_s	$1/q_T^4$	2	α_s	$1/Q^2$	no	
$F_{LU}^{\sin\phi_h}$	3	α_s^2	$1/(Q\,q_T)$	2	α_s^2	$1/(Q\,q_T)$	yes	✘
$F_{UL}^{\sin\phi_h}$	3	α_s^2	$1/(Q\,q_T)$					
$F_{UL}^{\sin 2\phi_h}$	2	α_s	$1/q_T^4$					
F_{LL}	2	α_s	$1/q_T^2$	2	α_s	$1/q_T^2$	yes	✓
$F_{LL}^{\cos\phi_h}$	3	α_s	$1/(Q\,q_T)$	2	α_s	$1/(Q\,q_T)$	yes	✘
$F_{UT,T}^{\sin(\phi_h-\phi_S)}$	2	α_s	$1/q_T^3$	3	α_s	$1/q_T^3$	yes	✓
$F_{UT,L}^{\sin(\phi_h-\phi_S)}$	4			3	α_s	$1/(Q^2\,q_T)$		
$F_{UT}^{\sin(\phi_h+\phi_S)}$	2	α_s	$1/q_T^3$	3	α_s	$1/q_T^3$	yes	✓
$F_{UT}^{\sin(3\phi_h-\phi_S)}$	2	α_s^2	$1/q_T^3$	3	α_s	$1/(Q^2\,q_T)$	no	
$F_{UT}^{\sin\phi_S}$	3	α_s	$1/(Q\,q_T^2)$	3	α_s	$1/(Q\,q_T^2)$	yes	✘
$F_{UT}^{\sin(2\phi_h-\phi_S)}$	3	α_s	$1/(Q\,q_T^2)$	3	α_s	$1/(Q\,q_T^2)$	yes	✘
$F_{LT}^{\cos(\phi_h-\phi_S)}$	2	α_s	$1/q_T^3$					
$F_{LT}^{\cos\phi_S}$	3	α_s	$1/(Q\,q_T^2)$					
$F_{LT}^{\cos(2\phi_h-\phi_S)}$	3	α_s	$1/(Q\,q_T^2)$					

conjectures!

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- The last case indicates a violation of factorization with twist-3 TMD PDFs
- The study has several phenomenological consequences