Progresses with
Transverse Momentum Distributions

Alessandro Bacchetta

Jefferson Lab
N. Isgur Fellowship

Outline

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- Some words about the relevance of TMDs

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- Some theory

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TMDs physics touches all this points

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X. Ji, PRL 91 (03), see talk by M. Schlegel

for even more dim. (8), see Collins, Rogers, Stasto, PRD77 (08)

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  - 3 dim. in coordinate space (GPDs)
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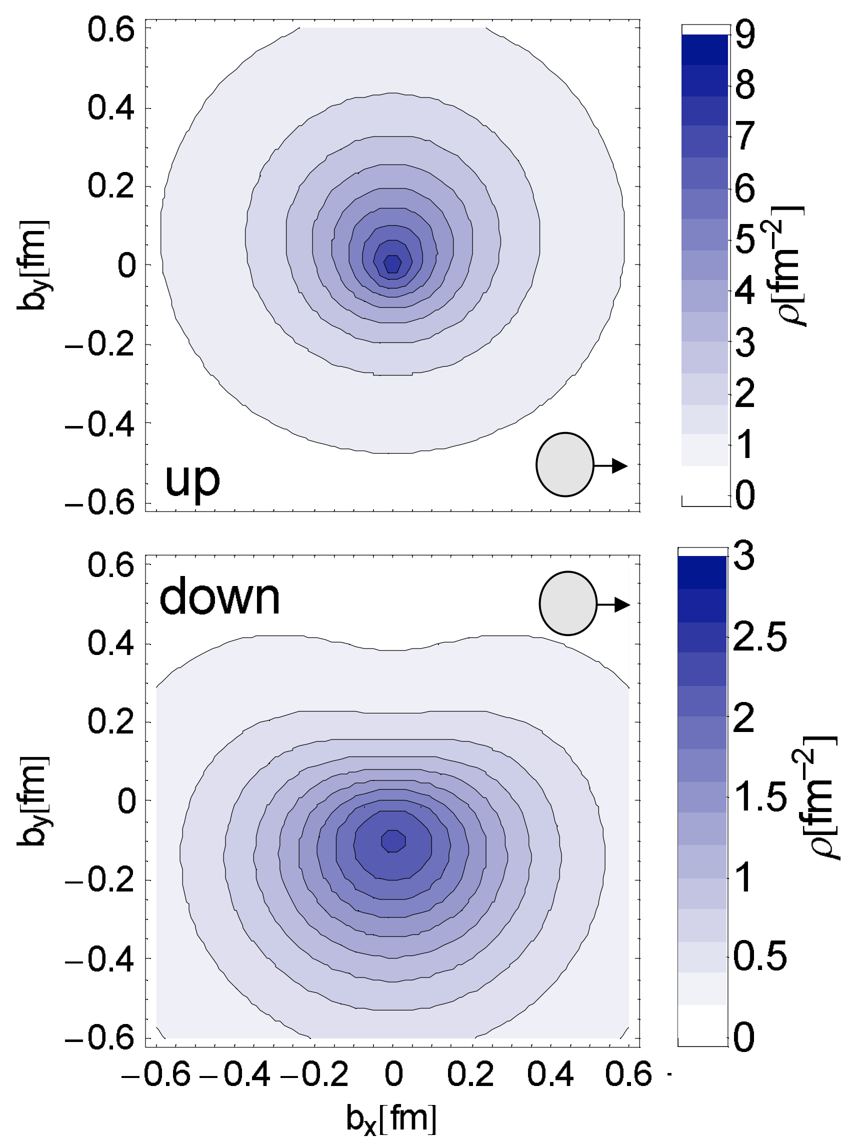
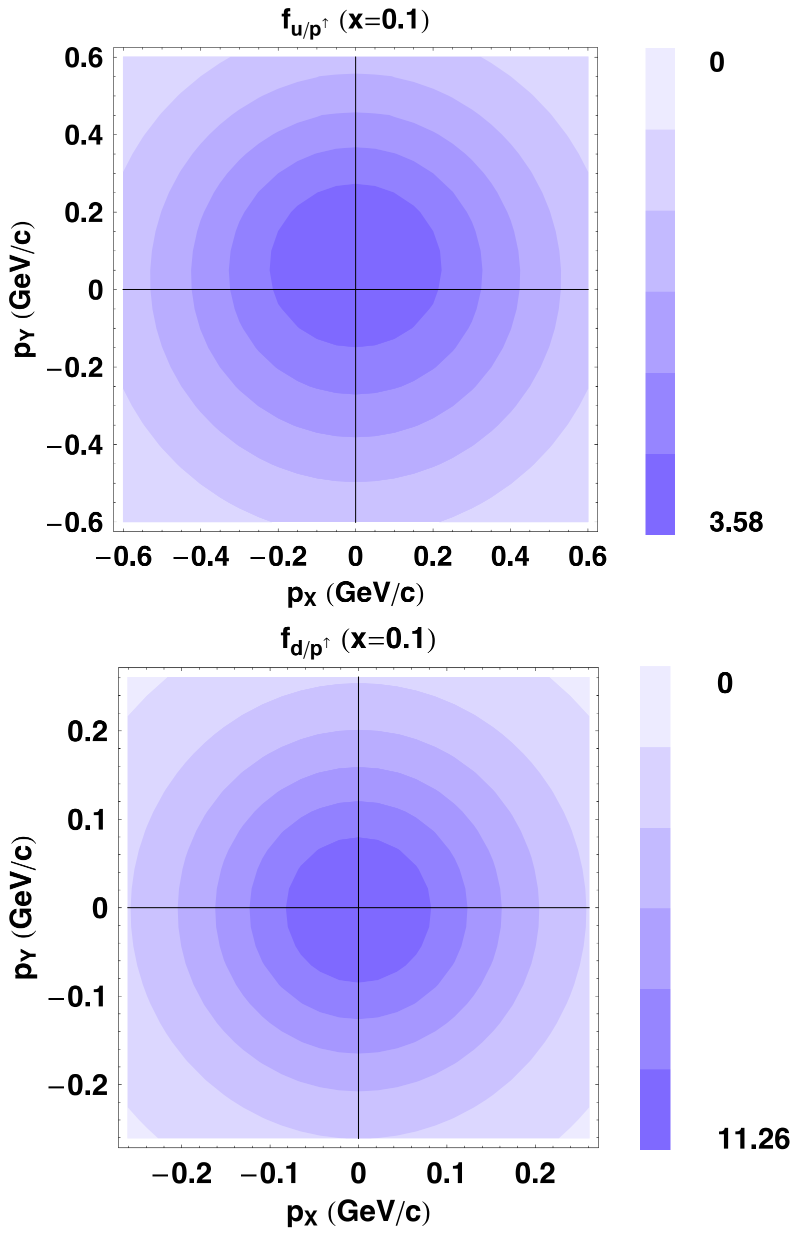
New dimensions

Transverse position   Transverse momentum





QCDSF/UKQCD, PRL 98 (07)

A.B., F. Conti, M. Radici, in preparation

Momentum distributions

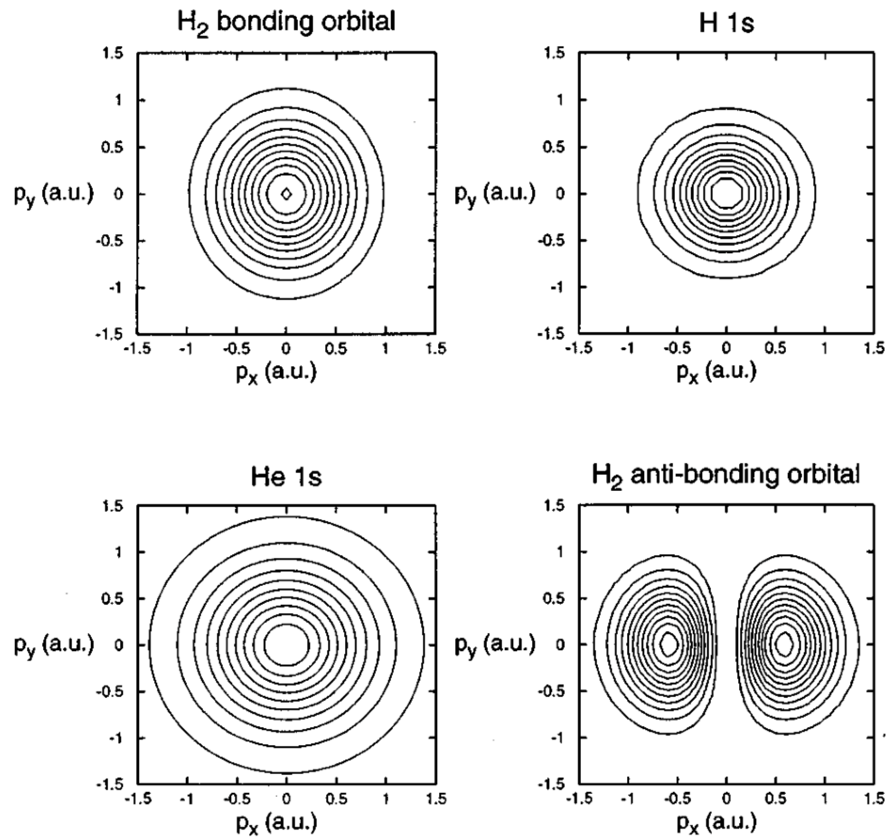


Fig. 6. Chemical bonding in momentum space. In the top panel we show the momentum density distribution of the bonding orbital for a hydrogen molecule oriented along the x axis. As the electrons become more delocalized along the x axis the distribution becomes narrower along the p_x axis. At large distances the electrons probe the attractive potential of two protons screened by one electron. The resulting momentum distribution for the bonding orbital is then between those of the $1s$ orbital of the hydrogen atom and the $1s$ orbital of helium. The antibonding orbital peaks at larger momentum values and thus has more kinetic energy.

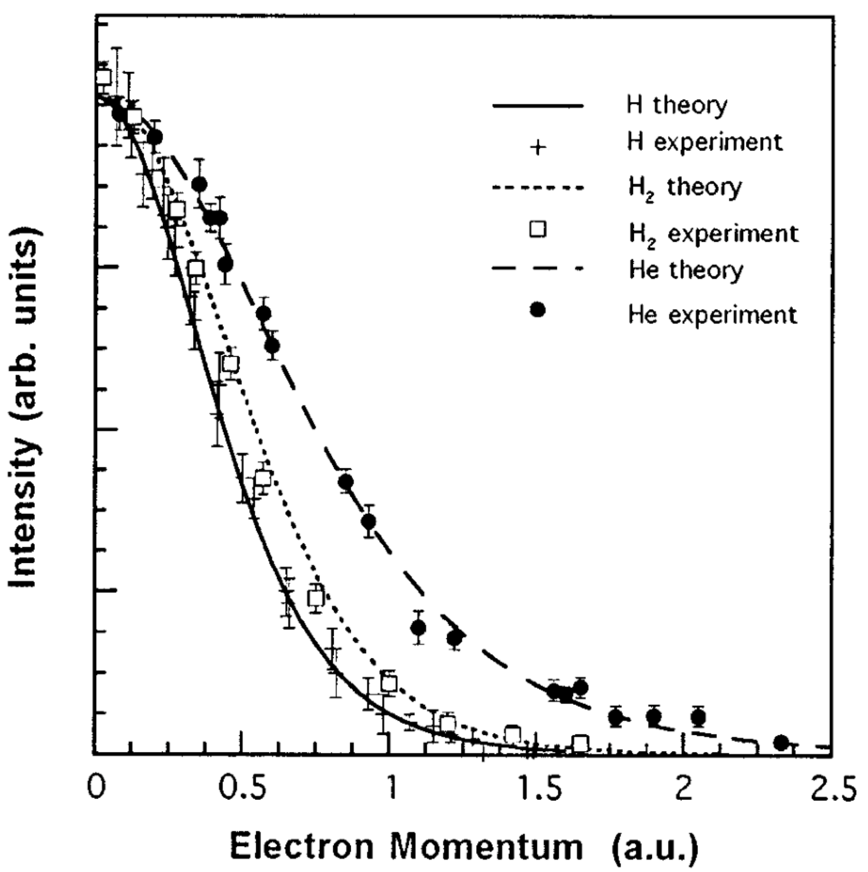
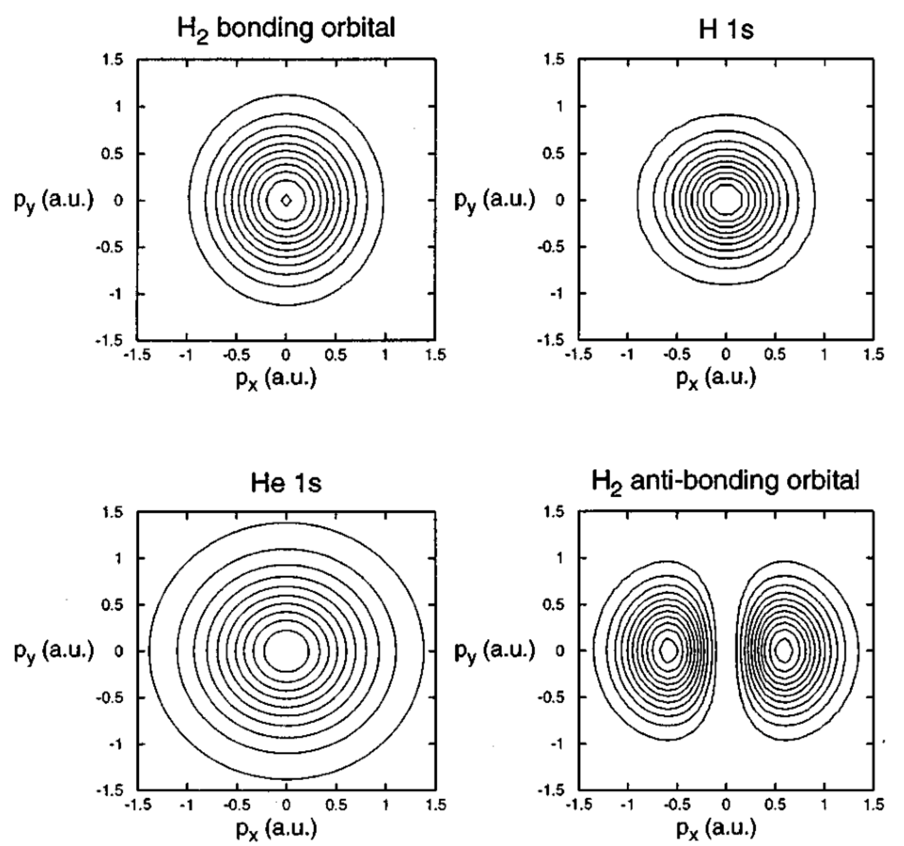


Fig. 7. Experimental momentum-density profiles for the hydrogen atom, the hydrogen molecule and the helium atom. The curves are calculated from the exact solution of the Schrödinger equation for the hydrogen atom and SCF approximations for the two-electron cases. The data are arbitrarily normalized to the same zero-momentum value.

Vos, McCarthy, Am. J. Phys. 65 (97)

Momentum distributions



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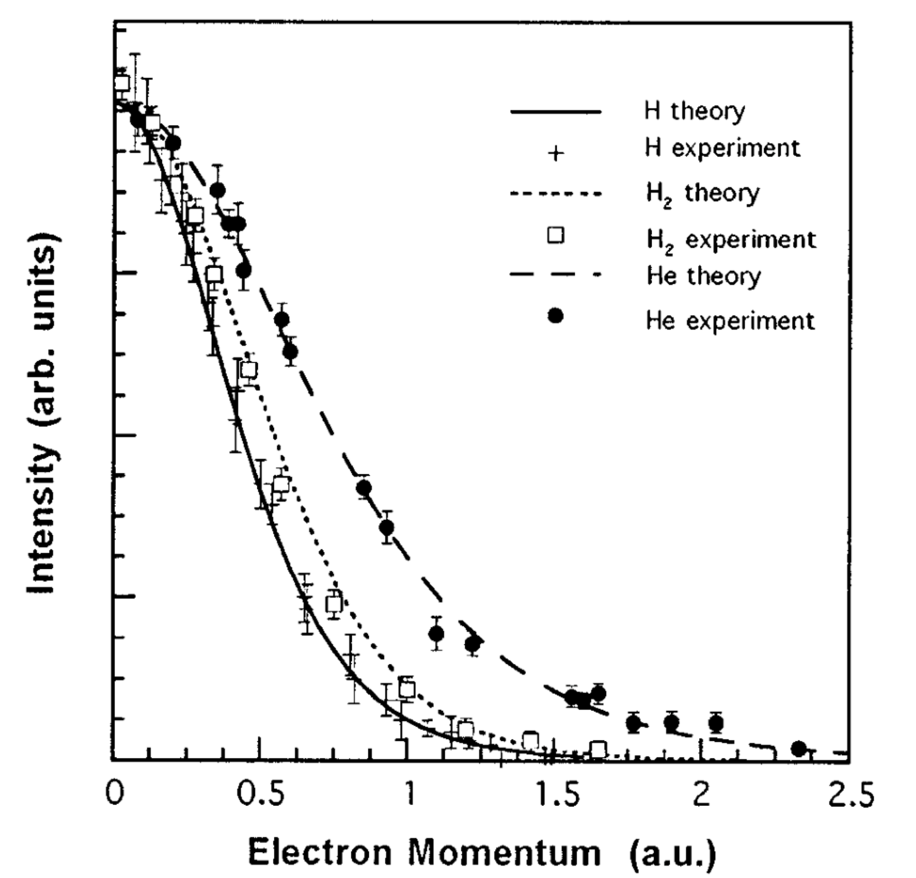

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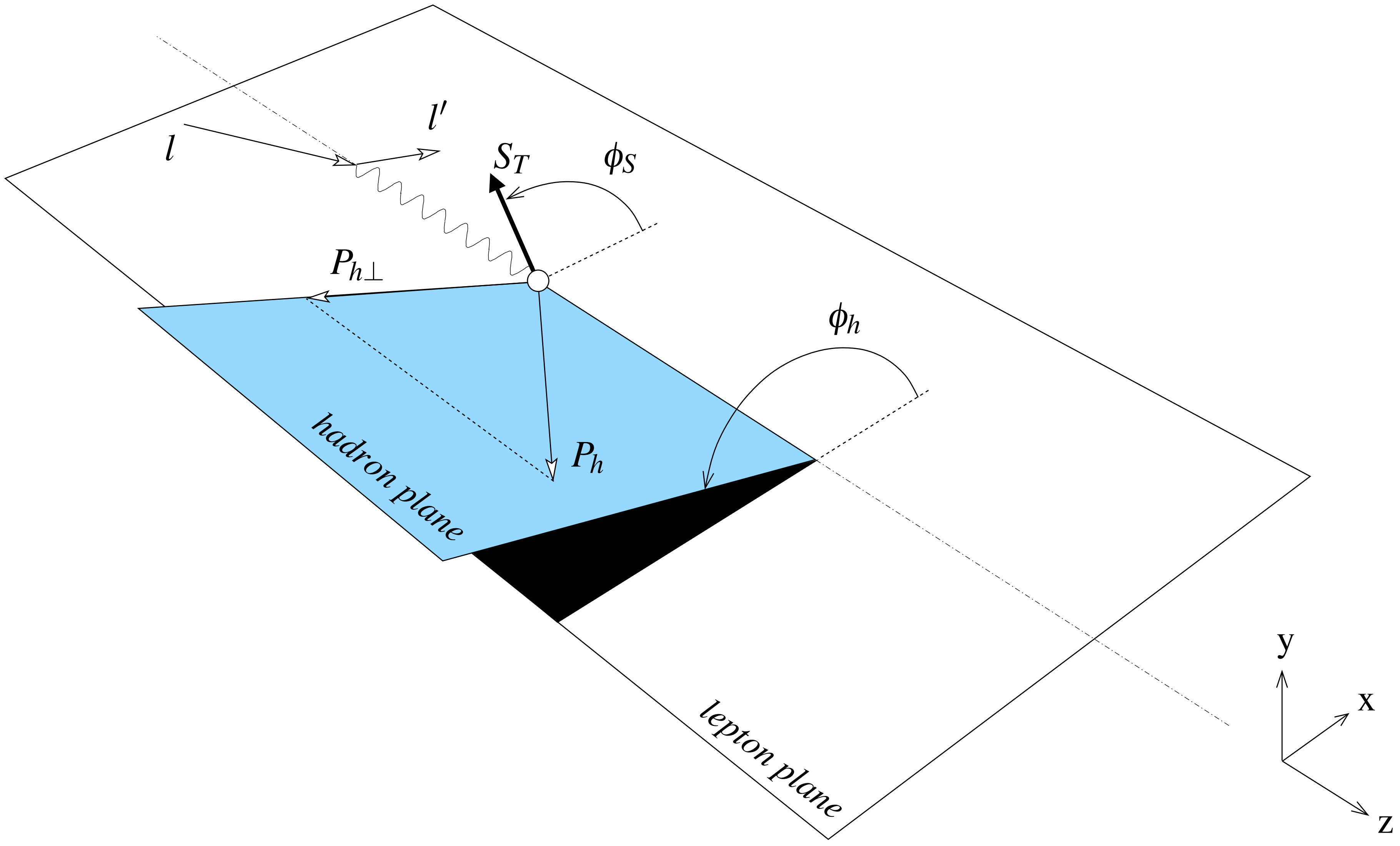
In scattering experiments, measuring momentum distributions is the closest we get to "imaging" a quantum object

Theory background

SIDIS



SIDIS cross section

$$\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2}$$

$$= \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left\{F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}\right.$$

$$+ \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} + S_L\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right]$$

$$+ S_L\,\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right]$$

$$+ S_T\left[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}\right.$$

$$+ \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}$$

$$\left. + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\right] + S_T\lambda_e\left[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}\right.$$

$$\left.\left. + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\right]\right\}$$

SIDIS cross section

$$F_{UU,T}(x, z, P_{h\perp}^2, Q^2)$$

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$$+ S_L\, \lambda_e \left[\sqrt{1-\varepsilon^2}\, F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos\phi_h\, F_{LL}^{\cos\phi_h} \right]$$

$$+ S_T \Bigg[\sin(\phi_h - \phi_S) \left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\, F_{UT,L}^{\sin(\phi_h-\phi_S)} \right) + \varepsilon\, \sin(\phi_h + \phi_S)\, F_{UT}^{\sin(\phi_h+\phi_S)}$$

$$+ \varepsilon\, \sin(3\phi_h - \phi_S)\, F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin\phi_S\, F_{UT}^{\sin\phi_S}$$

$$+ \sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin(2\phi_h - \phi_S)\, F_{UT}^{\sin(2\phi_h-\phi_S)} \Bigg] + S_T \lambda_e \Bigg[\sqrt{1-\varepsilon^2}\, \cos(\phi_h - \phi_S)\, F_{LT}^{\cos(\phi_h-\phi_S)}$$

$$+ \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos\phi_S\, F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos(2\phi_h - \phi_S)\, F_{LT}^{\cos(2\phi_h-\phi_S)} \Bigg] \Bigg\}$$

High and low transverse momentum

Q = photon virtuality

M = hadron mass

$P_{h\perp}$ = hadron transverse momentum

$q_T^2 \approx P_{h\perp}^2 / z$

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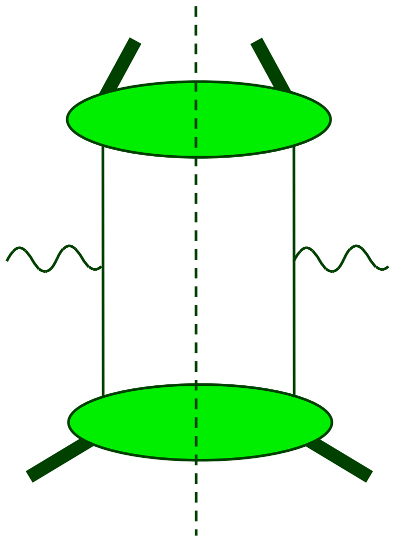

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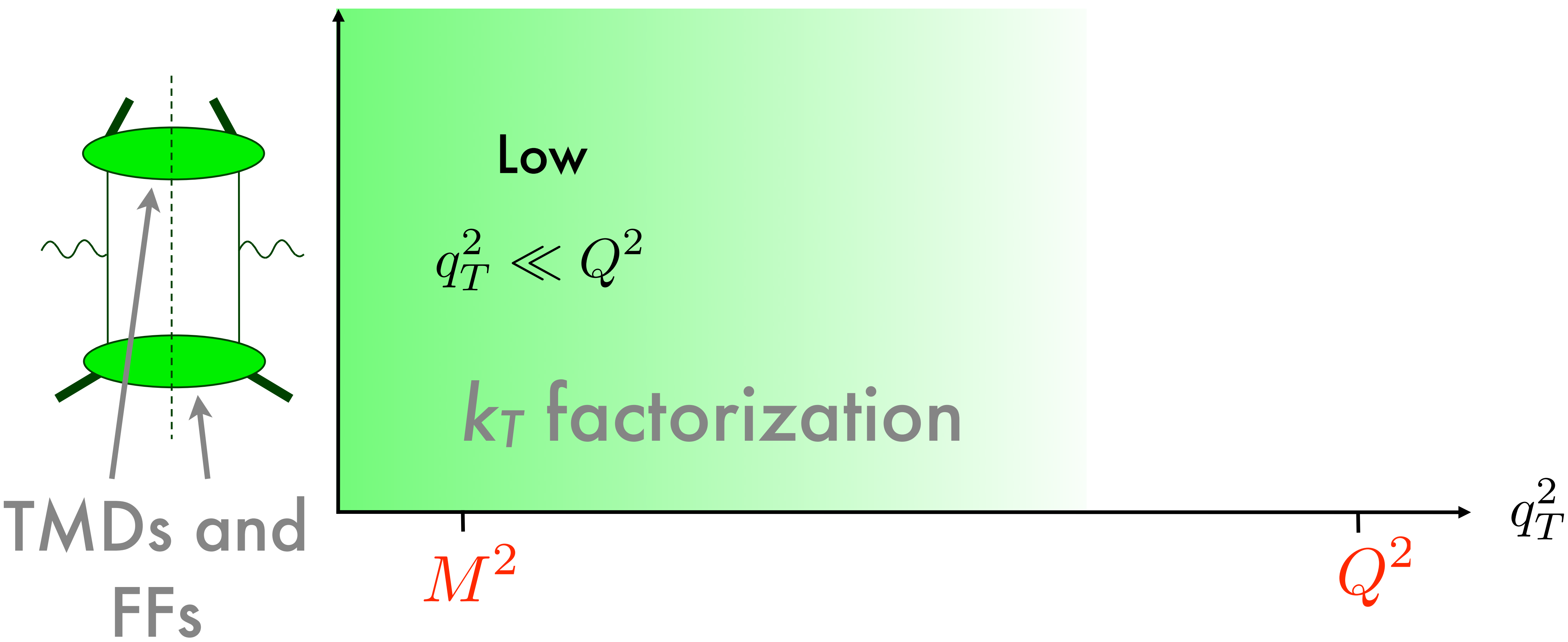

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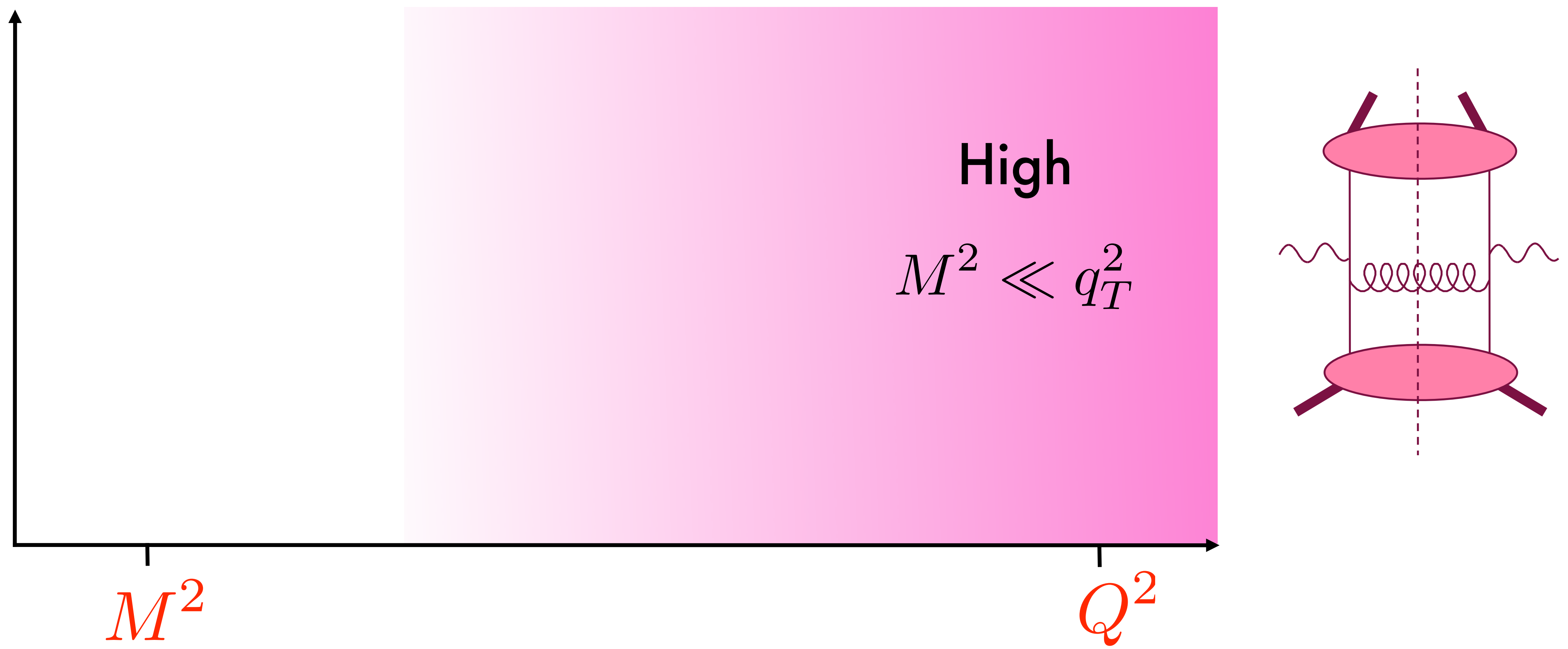

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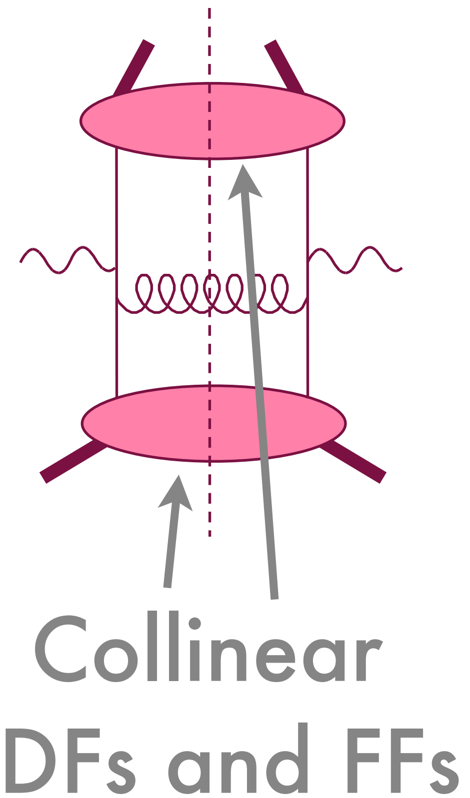

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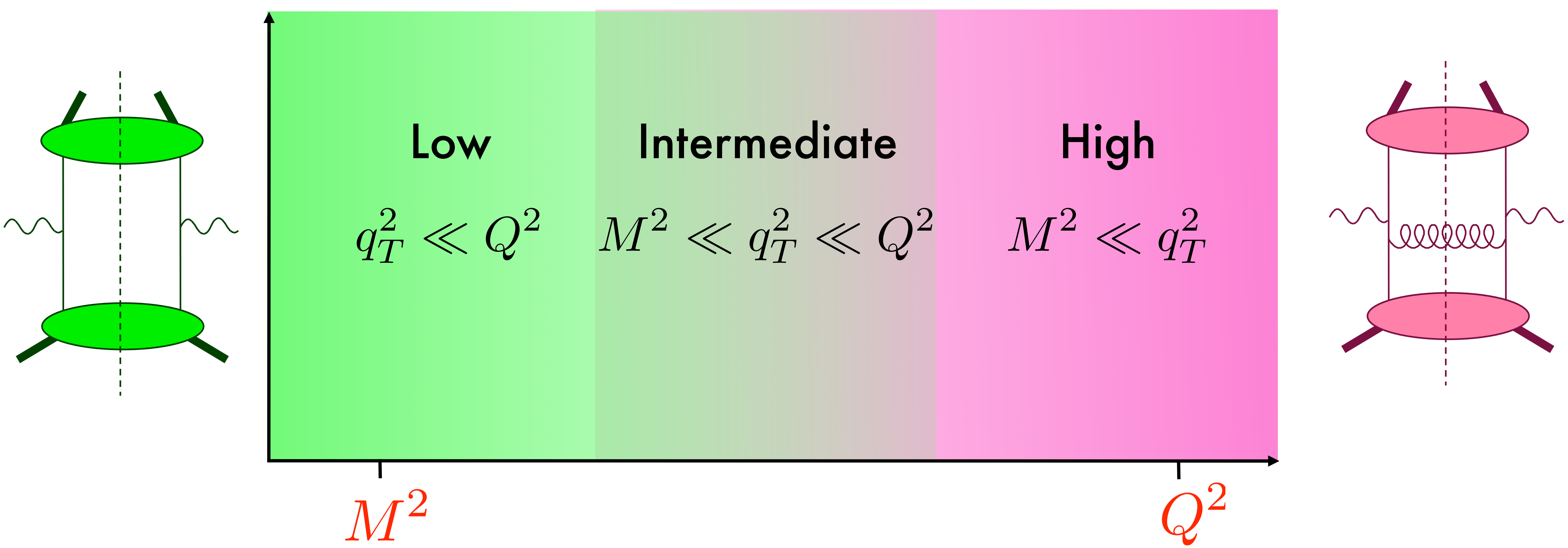

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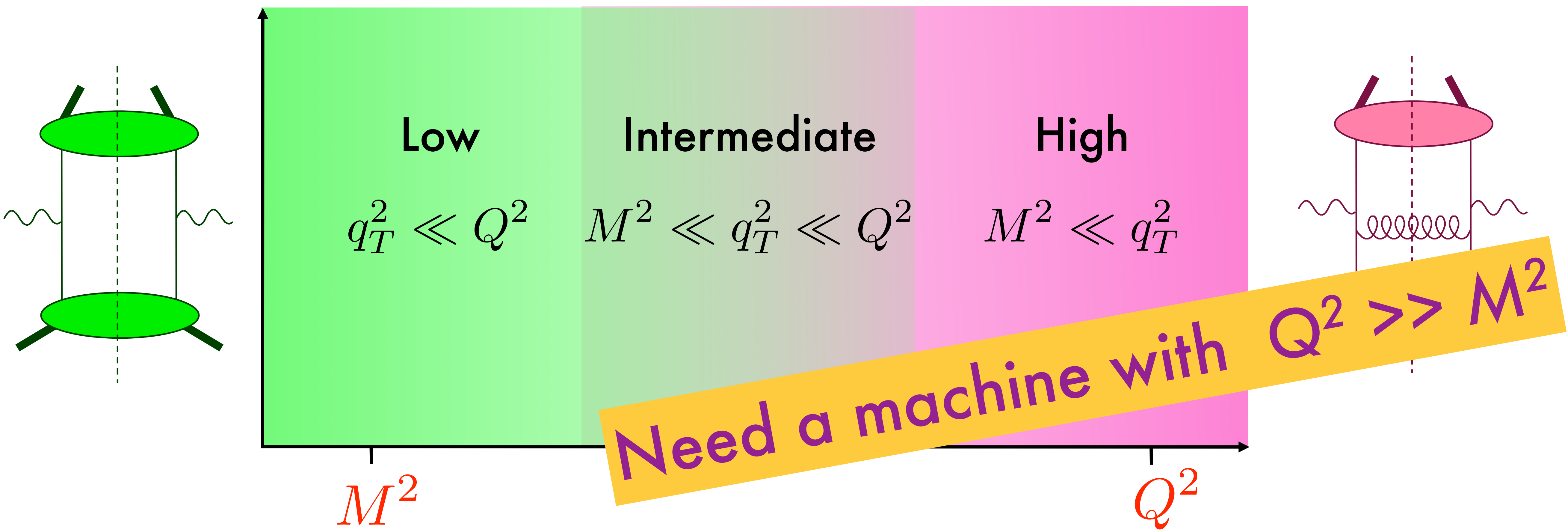
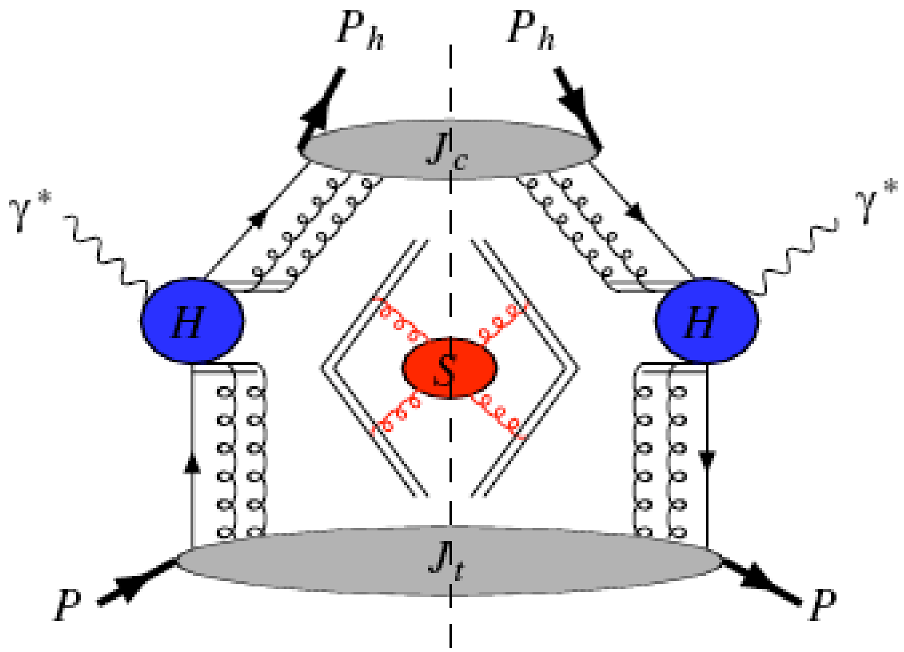


Predictions in intermediate region

	low-q_T calculation			high-q_T calculation			
observable	twist	order	power	twist	order	power	powers match
$F_{UU,T}$	2	α_s	$1/q_T^2$	2	α_s	$1/q_T^2$	yes
$F_{UU,L}$	4			2	α_s	$1/Q^2$	?
$F_{UU}^{\cos\phi_h}$	3	α_s	$1/(Q\,q_T)$	2	α_s	$1/(Q\,q_T)$	yes
$F_{UU}^{\cos 2\phi_h}$	2	α_s	$1/q_T^4$	2	α_s	$1/Q^2$	no
$F_{LU}^{\sin\phi_h}$	3	α_s^2	$1/(Q\,q_T)$	2	α_s^2	$1/(Q\,q_T)$	yes
$F_{UL}^{\sin\phi_h}$	3	α_s^2	$1/(Q\,q_T)$				?
$F_{UL}^{\sin 2\phi_h}$	2	α_s	$1/q_T^4$				?
F_{LL}	2	α_s	$1/q_T^2$	2	α_s	$1/q_T^2$	yes
$F_{LL}^{\cos\phi_h}$	3	α_s	$1/(Q\,q_T)$	2	α_s	$1/(Q\,q_T)$	yes
$F_{UT,T}^{\sin(\phi_h-\phi_S)}$	2	α_s	$1/q_T^3$	3	α_s	$1/q_T^3$	yes
$F_{UT,L}^{\sin(\phi_h-\phi_S)}$	4			3	α_s	$1/(Q^2\,q_T)$	?
$F_{UT}^{\sin(\phi_h+\phi_S)}$	2	α_s	$1/q_T^3$	3	α_s	$1/q_T^3$	yes
$F_{UT}^{\sin(3\phi_h-\phi_S)}$	2	α_s^2	$1/q_T^3$	3	α_s	$1/(Q^2\,q_T)$	no
$F_{UT}^{\sin\phi_S}$	3	α_s	$1/(Q\,q_T^2)$	3	α_s	$1/(Q\,q_T^2)$	yes
$F_{UT}^{\sin(2\phi_h-\phi_S)}$	3	α_s	$1/(Q\,q_T^2)$	3	α_s	$1/(Q\,q_T^2)$	yes
$F_{LT}^{\cos(\phi_h-\phi_S)}$	2	α_s	$1/q_T^3$				?
$F_{LT}^{\cos\phi_S}$	3	α_s	$1/(Q\,q_T^2)$				?
$F_{LT}^{\cos(2\phi_h-\phi_S)}$	3	α_s	$1/(Q\,q_T^2)$				?

A.B., Boer, Diehl, Mulders, 0803.0227

k_T factorization



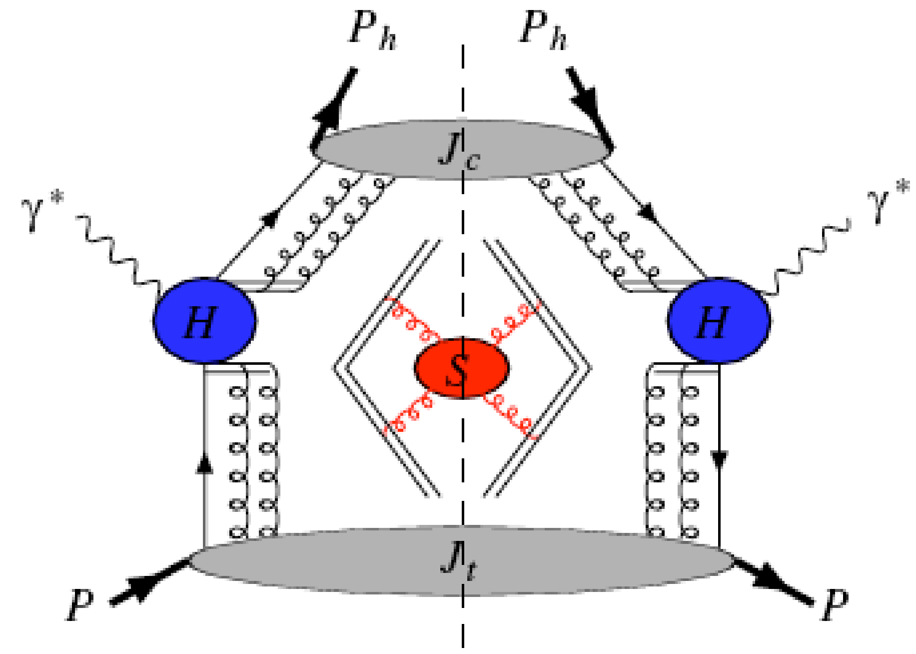
$$F_{UU,T}(x,z,P_{h\perp}^2,Q^2) = \mathcal{C}\big[f_1 D_1\big]$$

$$= \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, d^2\boldsymbol{l}_T\, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T + \boldsymbol{l}_T - \boldsymbol{P}_{h\perp}/z\big)$$

$$x \sum_a e_a^2\, f_1^a(x, p_T^2, \mu^2)\, D_1^a(z, k_T^2, \mu^2)\, U(l_T^2, \mu^2) H(Q^2, \mu^2)$$

Collins, Soper, NPB 193 (81)
Ji, Ma, Yuan, PRD 71 (05)

k_T factorization



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TMD PDF — TMD FF — Soft factor — Hard part

Collins, Soper, NPB 193 (81)
Ji, Ma, Yuan, PRD 71 (05)

Collins–Soper evolution equations

$$\zeta \frac{\partial}{\partial \zeta} Q(x, k_\perp, x\zeta) = \int [K + G] \otimes Q(x, k_\perp, x\zeta)$$

k_T space

$$\zeta \frac{\partial}{\partial \zeta} Q(x, b, x\zeta) = [K(b, \mu, \rho) + G(x\zeta, \mu, \rho)] \times Q(x, b, x\zeta)$$

b space

Collins, Soper, NPB 193 (81)
Ji, Ma, Yuan, PRD 70 (04)

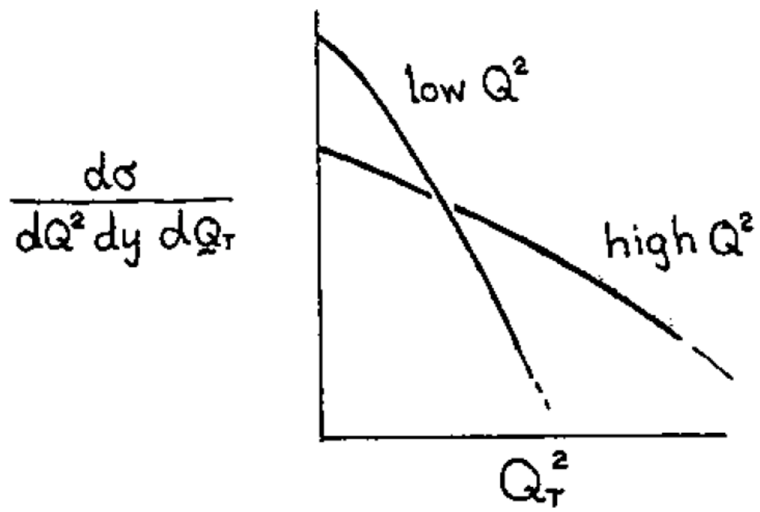

Fig 3 Broadening of the Q_T distribution

Collins, Soper, Sterman, talk at Fermilab Workshop on Drell-Yan Process, Batavia, Ill., Oct 7-8, 1982

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$$\zeta \frac{\partial}{\partial \zeta} Q(x, b, x\zeta) = [K(b, \mu, \rho) + G(x\zeta, \mu, \rho)] \times Q(x, b, x\zeta)$$ b space

Collins, Soper, NPB 193
Ji, Ma, Y

Evolution equations for TMDs are NOT standard DGLAP

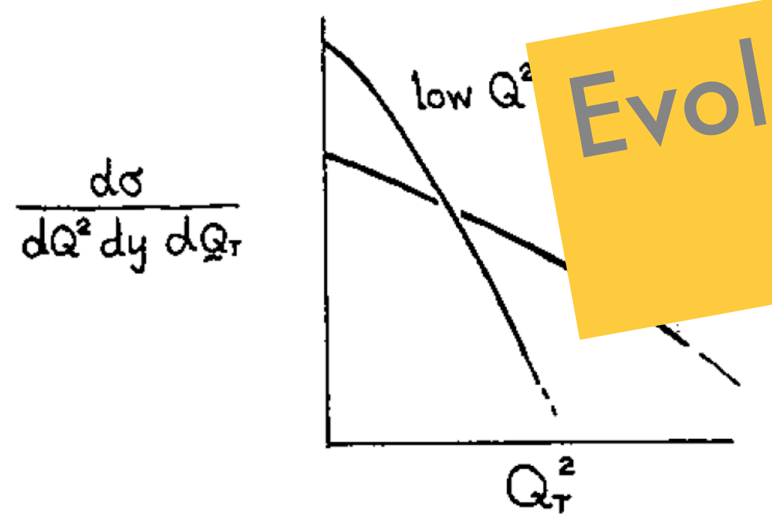

Fig 3 Broadening of the Q_T distribution

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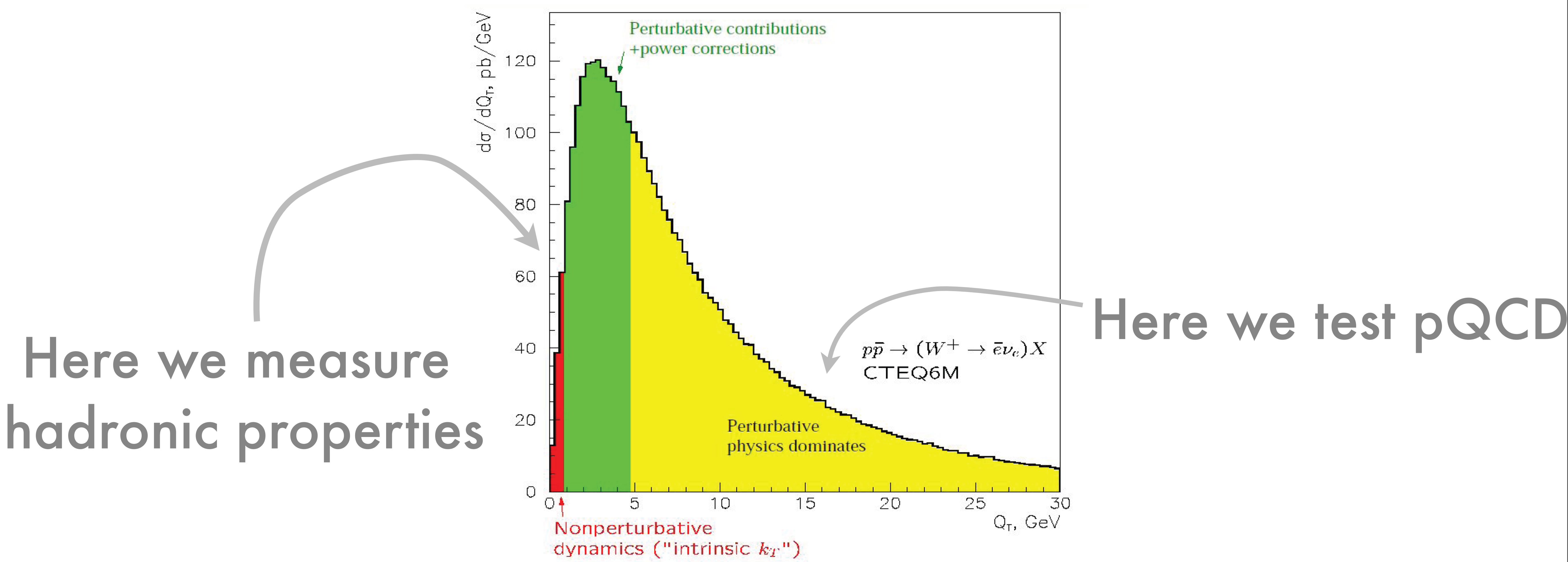
Resummation

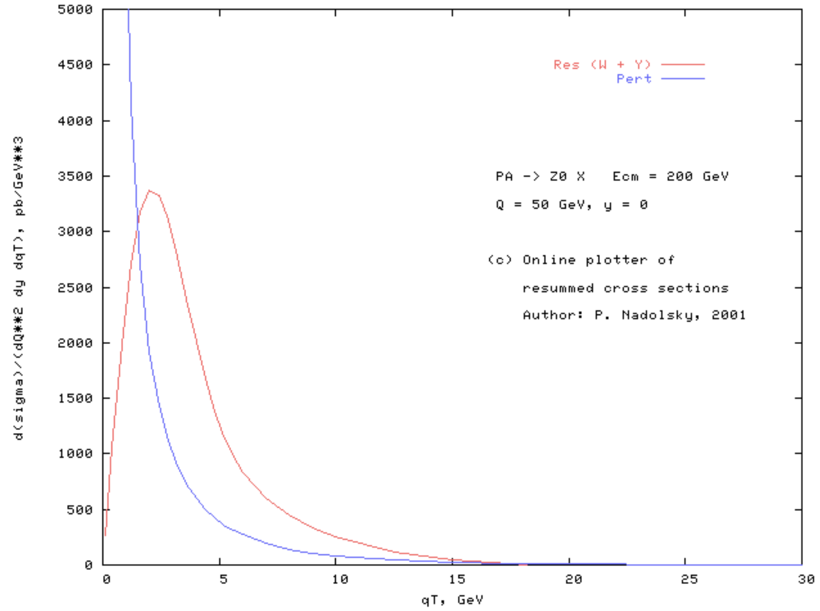
$$F_{UU,T}(x,z,b,Q^2) = x\sum_a e_a^2\,(f_1^i \otimes \mathcal{C}_{ia})\,(\mathcal{C}_{aj}\otimes D_1^j)\,e^{-S_{\rm nonpert.}}\,e^{-S_{\rm pert.}}$$

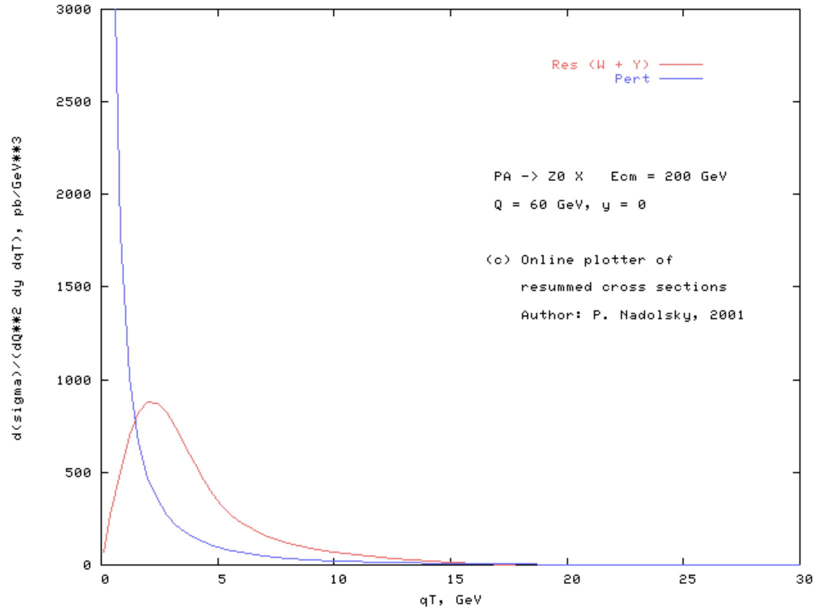
The calculation automatically fulfills Collins-Soper evolution and requires:

- collinear PDFs
- calculable pQCD contributions
- nonperturbative part of TMDs



Resummation results





http://hep.pa.msu.edu/resum/index.html

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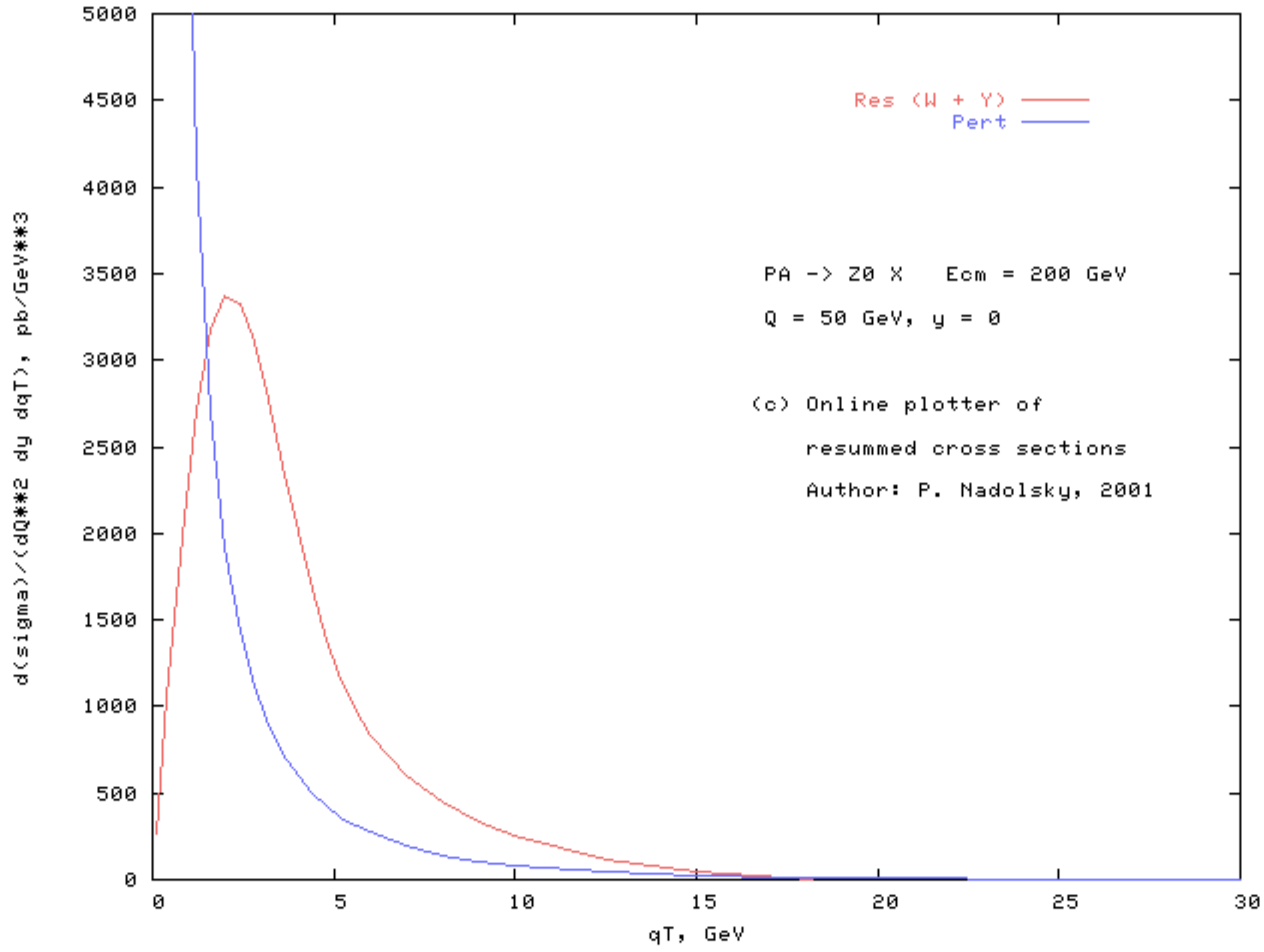

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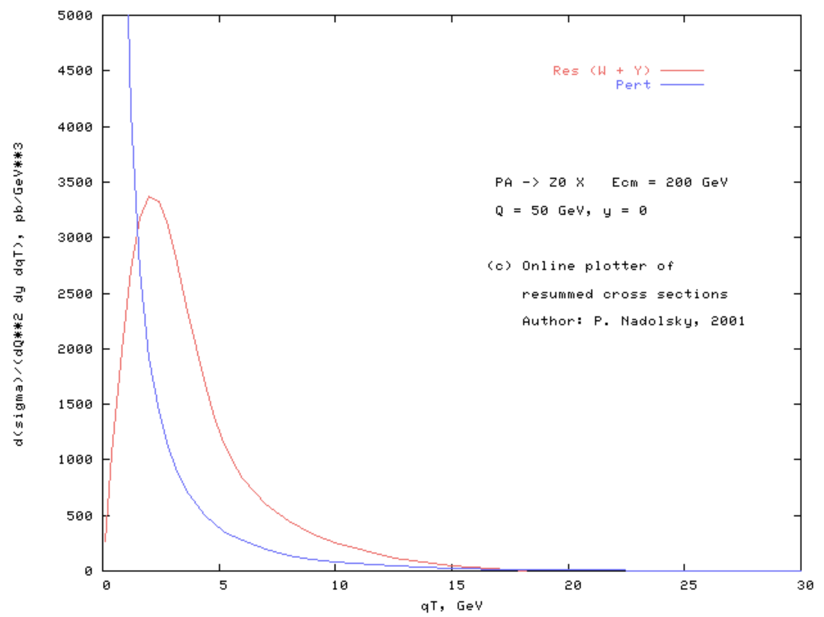

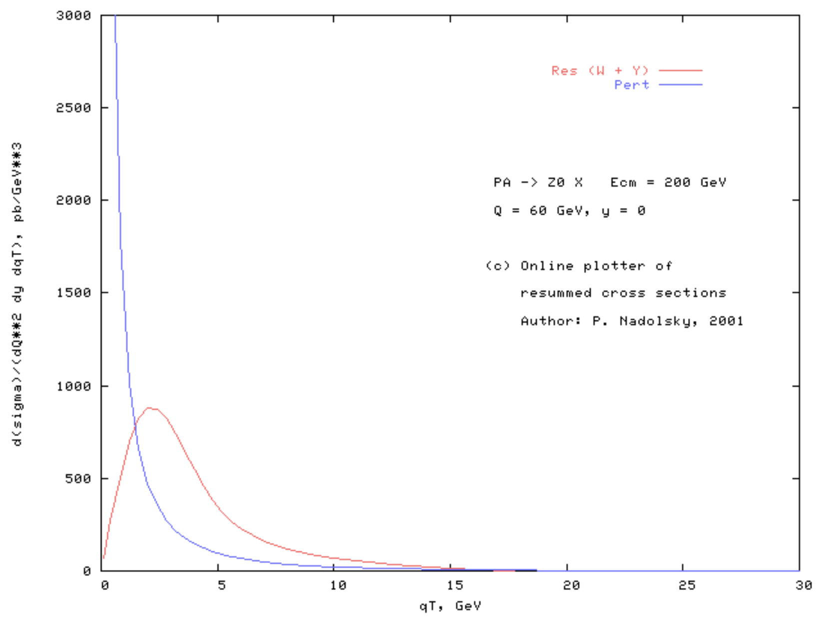

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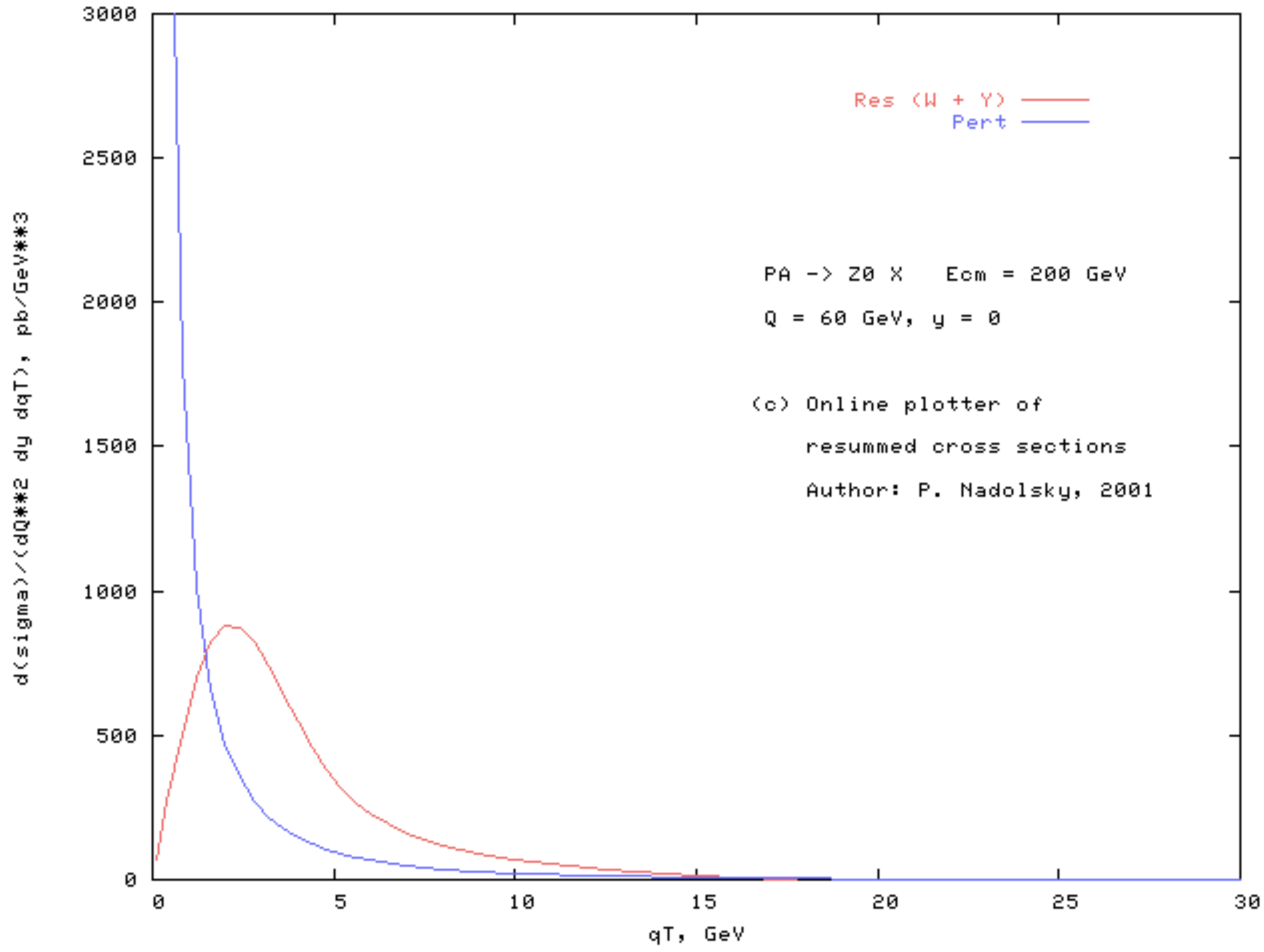

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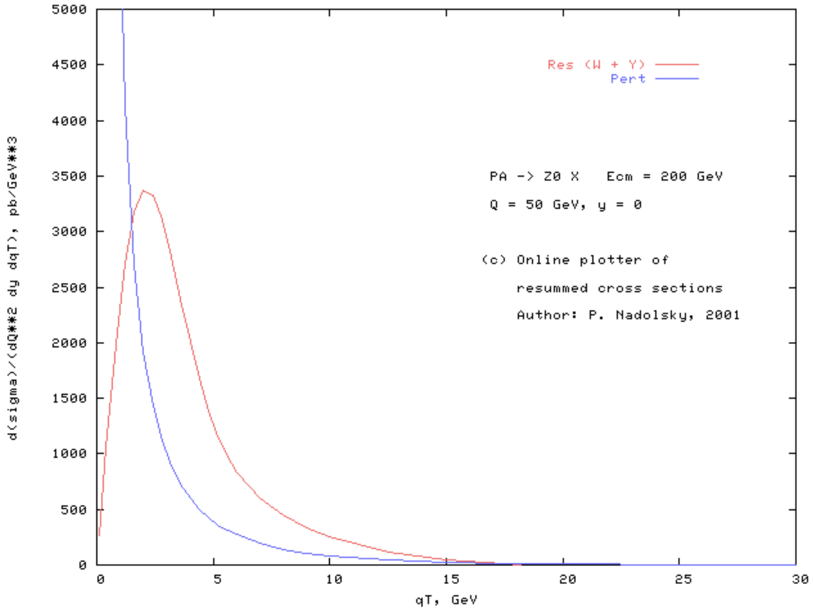

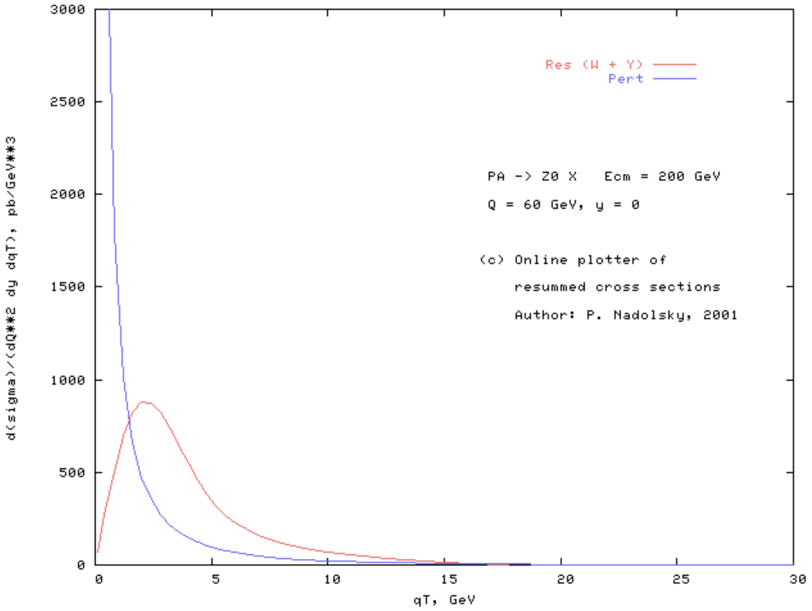

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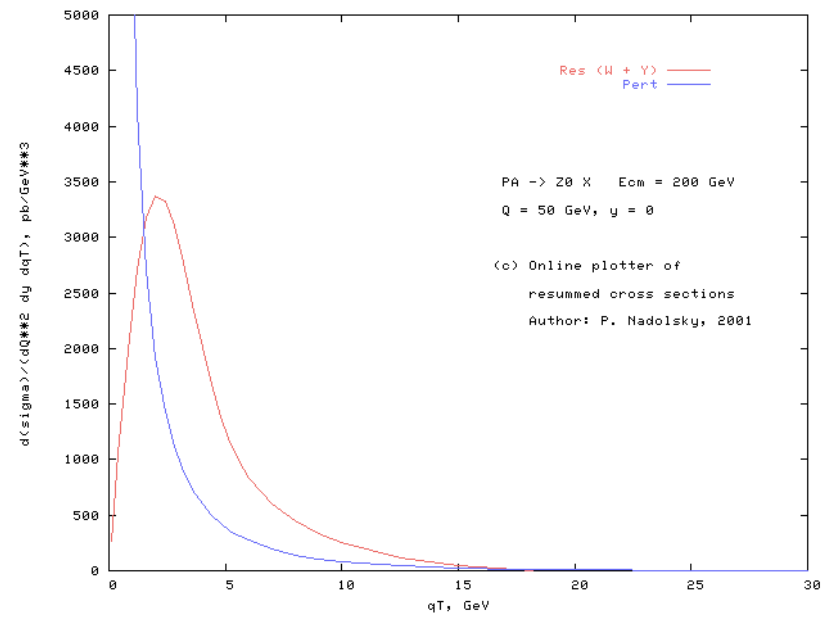

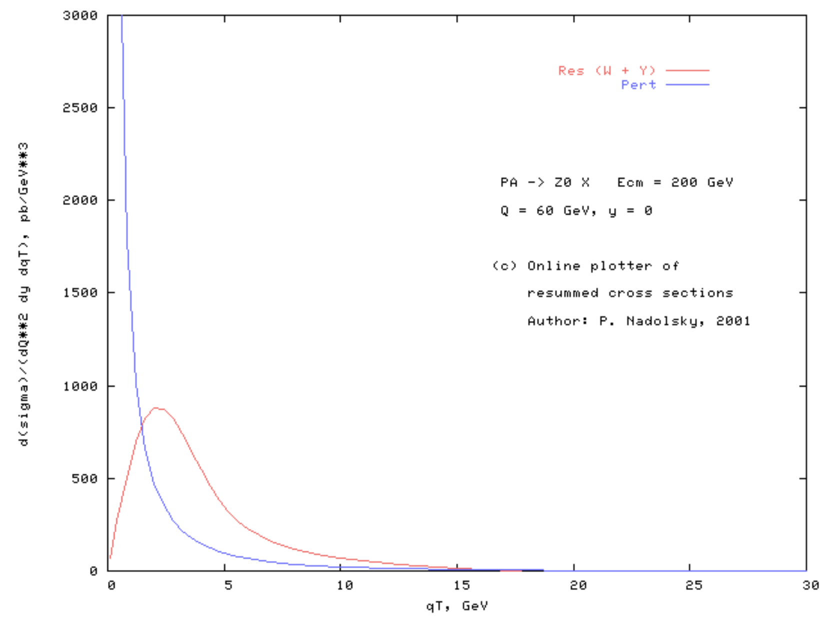

Most studies done for Drell–Yan

http://hep.pa.msu.edu/resum/index.html

Theoretical activity

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- Issues related to k_T-factorization in pp collisions

Collins, Qiu, PRD75 (07)
Qiu,Vogelsang, Yuan, PRD76 (07)
Bomhof, Mulders, Pijlman, A.B., PRD72 (05)

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- Issues related to resummation in other structure functions

Boer, Vogelsang, PRD74 (06)
Berger, Qiu, Rodriguez-Pedraza, PRD76 (07)
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Collins, Qiu, PRD75 (07)
Qiu,Vogelsang, Yuan, PRD76 (07)
Bomhof, Mulders, Pijlman, A.B., PRD72 (05)

- Issues related to resummation in other structure functions

Boer, Vogelsang, PRD74 (06)
Berger, Qiu, Rodriguez-Pedraza, PRD76 (07)
A.B., Boer, Diehl, Mulders, 0803.0227

Can have impact on LHC physics

Unpolarized TMDs

Nonperturbative transverse momentm

$$F_{UU,T}(x,z,b,Q^2) = x \sum_a e_a^2 \left(f_1^i \otimes \mathcal{C}_{ia}\right)\left(\mathcal{C}_{aj} \otimes D_1^j\right) e^{-S_{\text{nonpert.}}} e^{-S_{\text{pert.}}}$$

The nonperturbative part has to be fitted to data
Usually assumed to be Gaussian

Available extractions

- In b space

$$\exp\left[-g_2 b^2 \ln\left(\frac{Q}{2Q_0}\right) - g_1 b^2 + g_1 g_3 b^2 \ln(100 x_A x_B)\right]$$

$$g_1 = 0.21 \pm 0.01 \text{ GeV}^2,$$

$$g_2 = 0.68 \pm 0.02 \text{ GeV}^2,$$

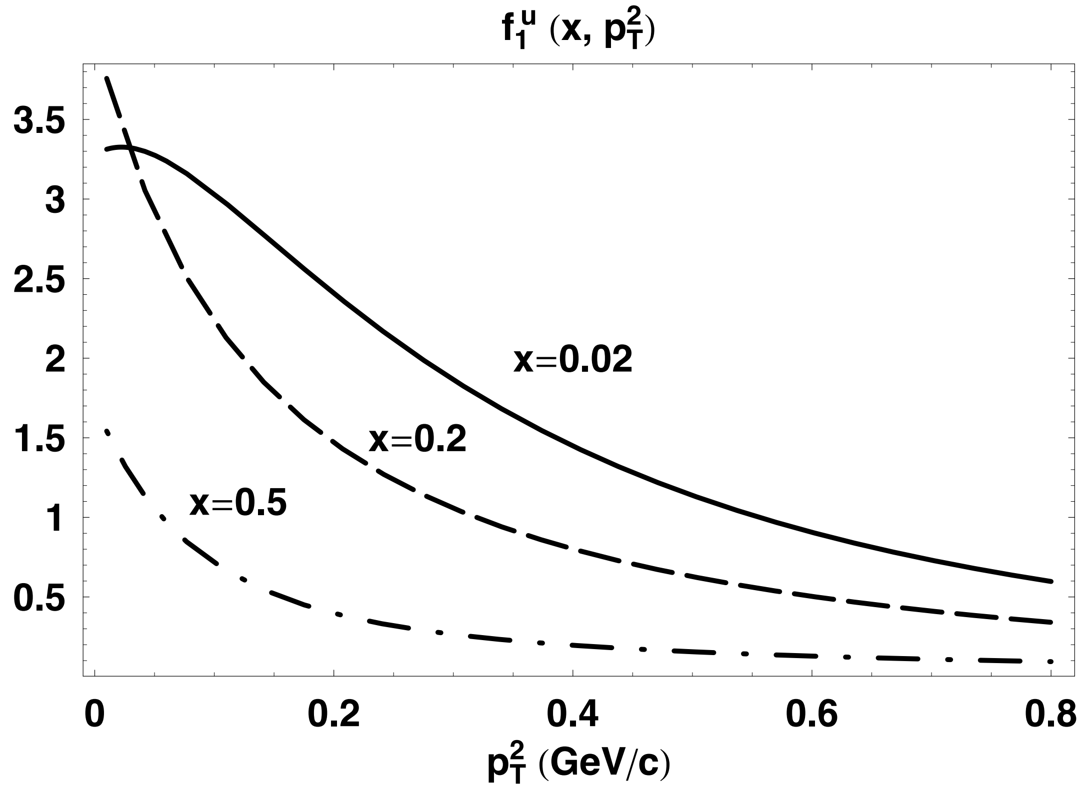
$$g_3 = -0.60^{+0.05}_{-0.04} \text{ GeV}^2.$$

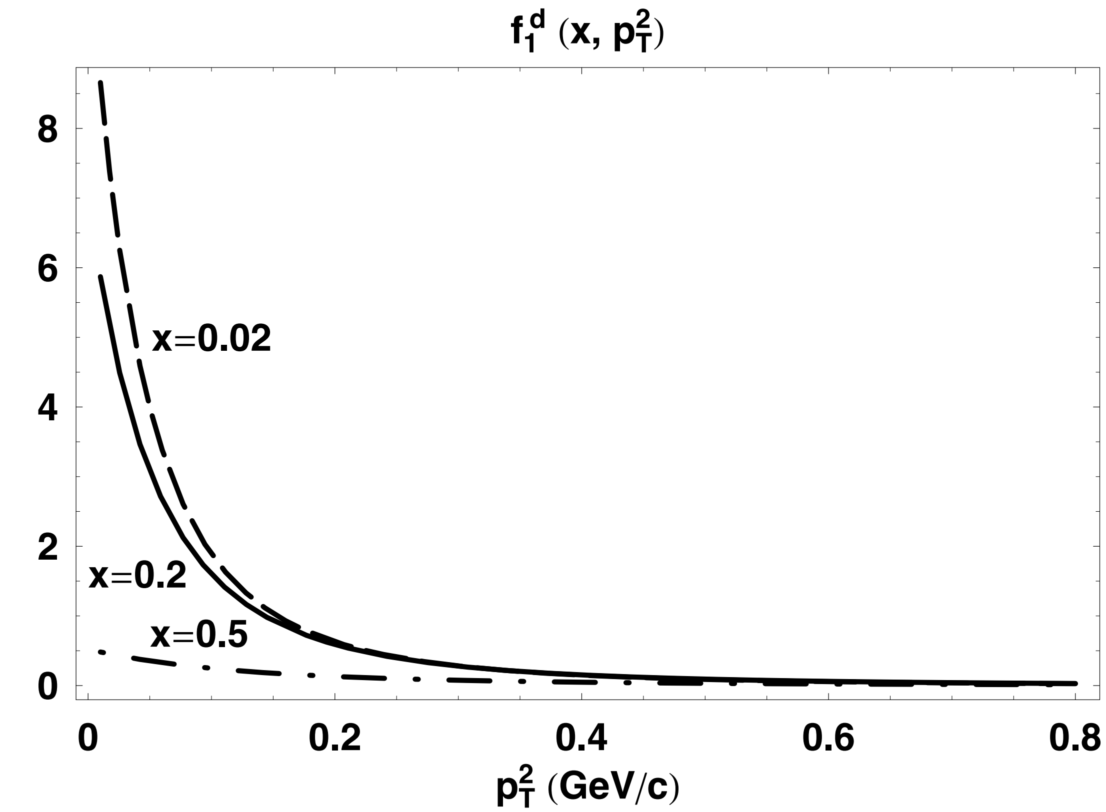
$$Q_0 = 1.6 \text{ GeV}$$

111 data points (Drell-Yan)

Brock, Landry, Nadolsky, Yuan, PRD67 (03)

Is it sufficient?





Simple model calculation suggests

- flavor dependence
- deviation from a simple Gaussian (required also by orbital angular momentum)

EIC tasks

EIC tasks

- Gain better knowledge of the nonperturbative part (also for fragmentation functions)

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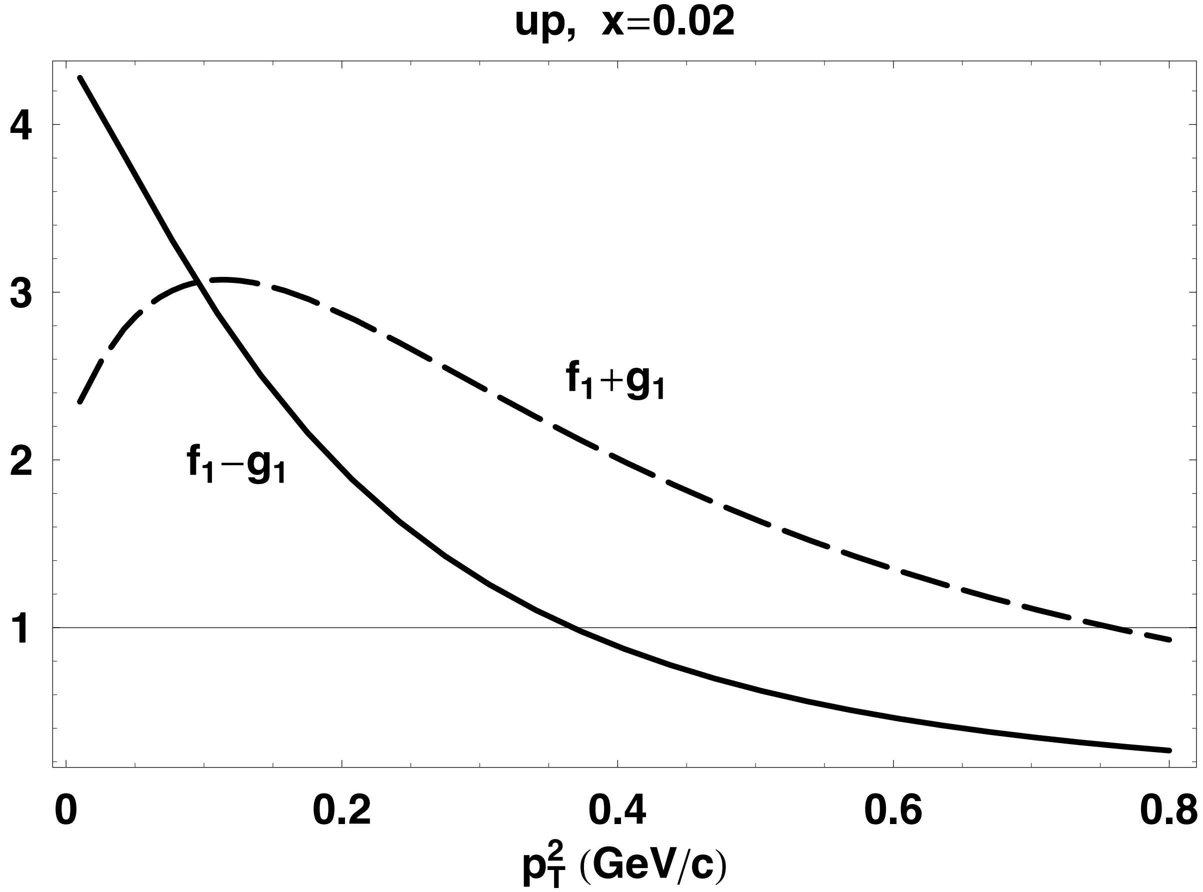
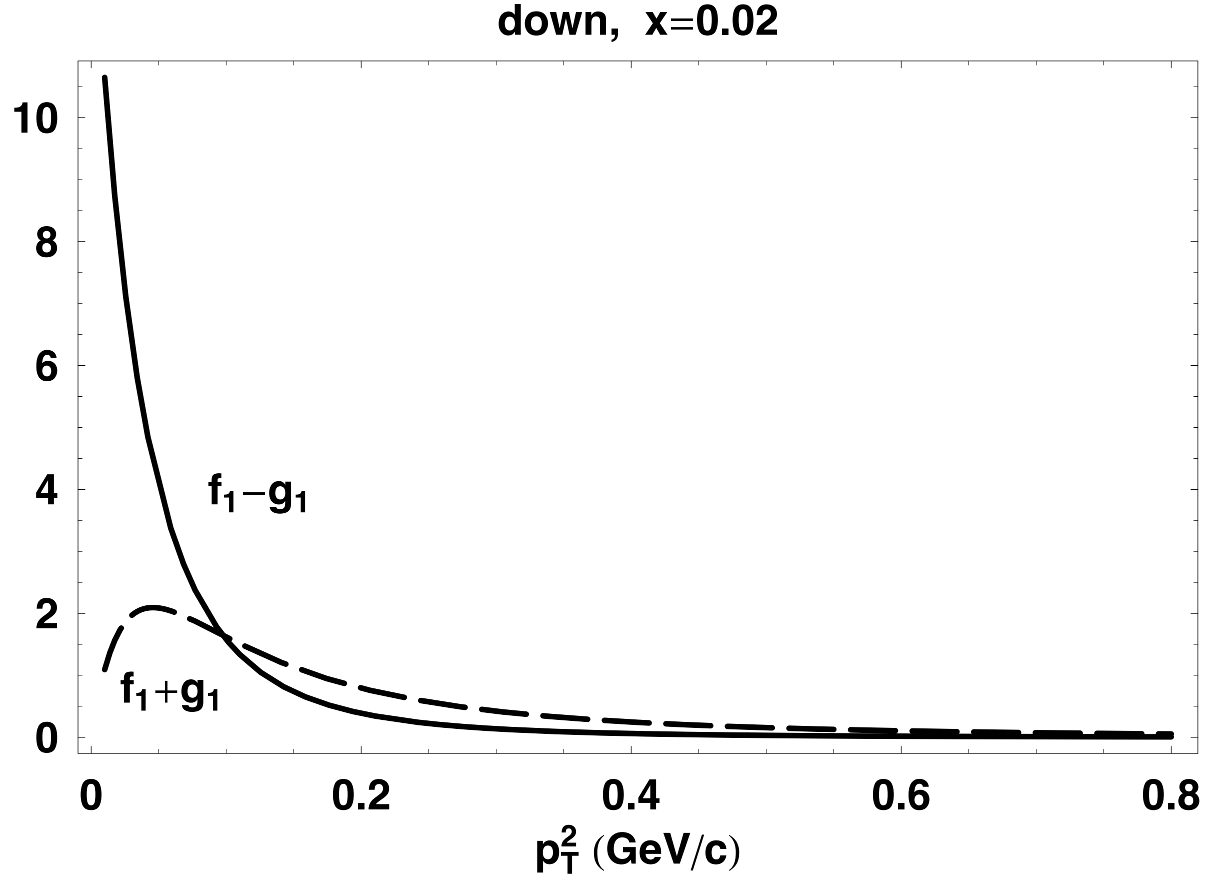
- Flavor separation (also for fragmentation functions)
- Unintegrated gluon distribution function
- Global fits with unintegrated PDFs
- Cleaner information from jet-SIDIS?

Longitudinal spin

Everything can be done also with helicity TMDs





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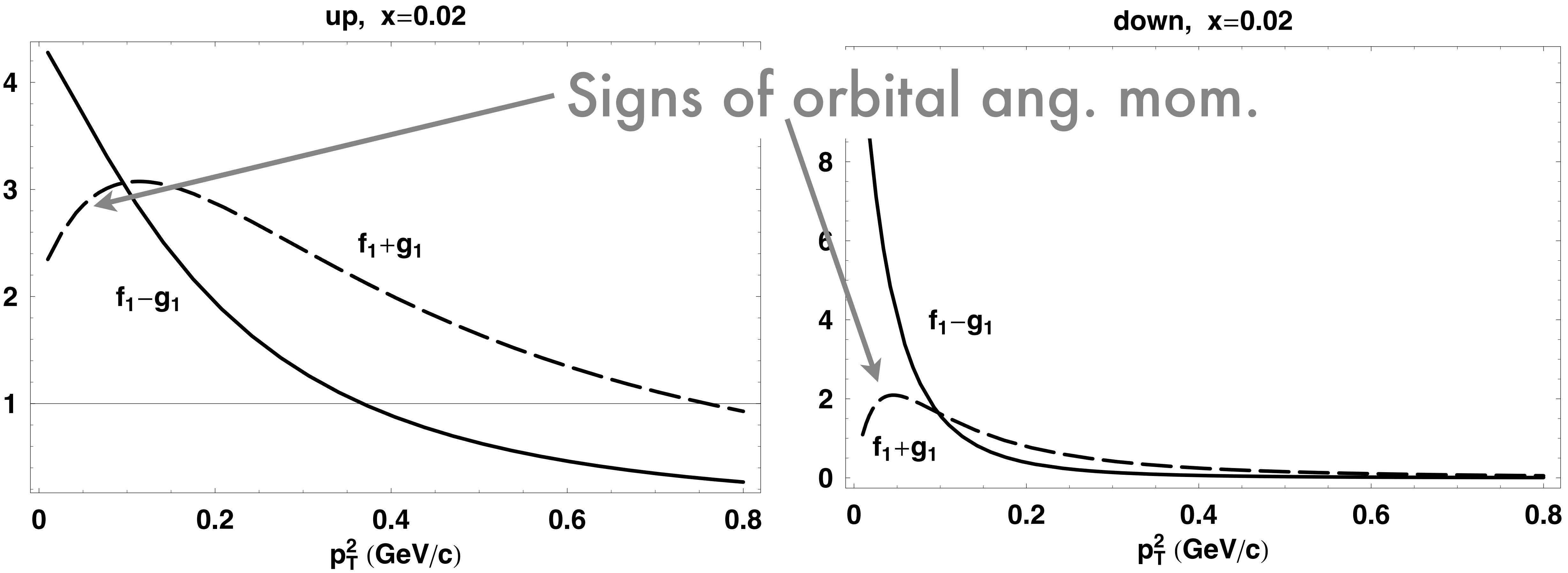

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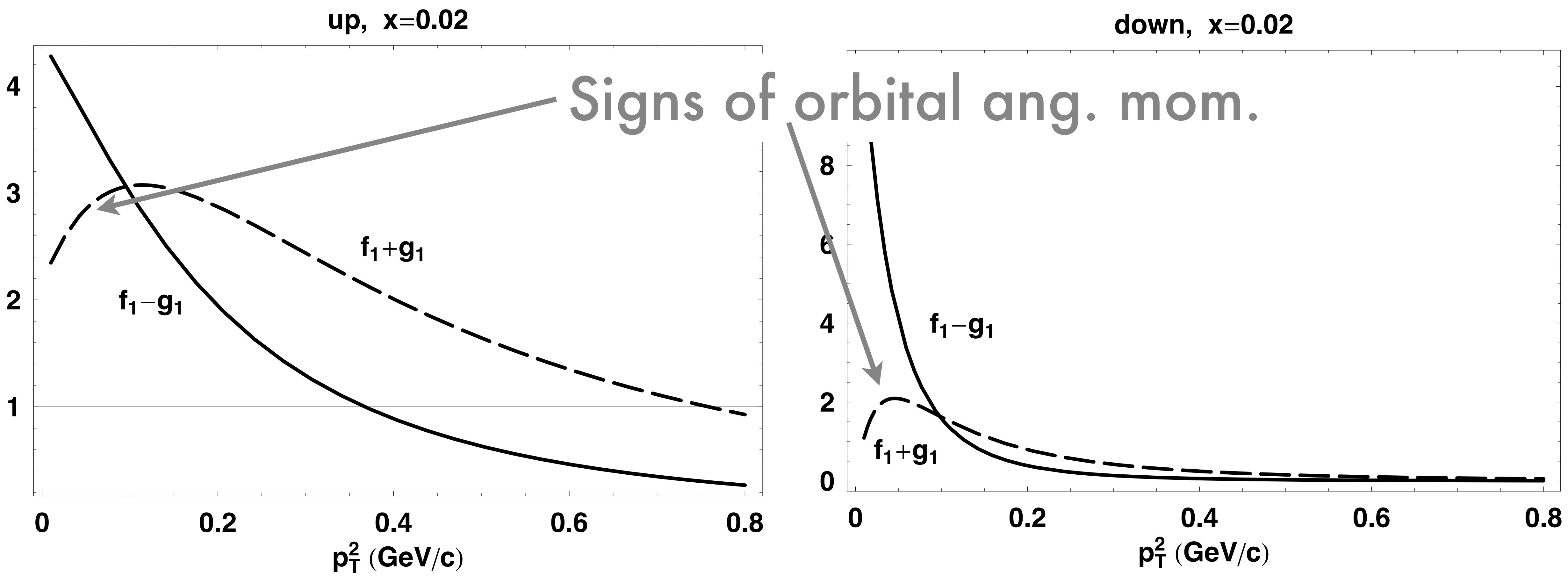

Impossible to reproduce using simple Gaussians

Sivers function

Formalism

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\boldsymbol{p}_T\cdot\hat{h}}{M}\, f_{1T}^{\perp}(x,p_T^2,\mu^2)D_1(z,k_T^2,\mu^2)\right]$$

Formalism

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- Collins–Soper evolution always required. Not only for the Sivers function, but also for D_1

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- Collins–Soper evolution always required. Not only for the Sivers function, but also for D_1
- We are far from the sofistication reached for F_{UU}

Weighted asymmetries

$$\left\langle \frac{P_{h\perp}}{z} \sin(\phi_h - \phi_S) \right\rangle = -x \sum_a e_a^2 \, f_{1T}^{\perp(1)}(x,\mu^2) D_1(z,\mu^2)$$

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- Not clear if it's possible to recover DGLAP-like evolution equations for Sivers, but certainly for D_1

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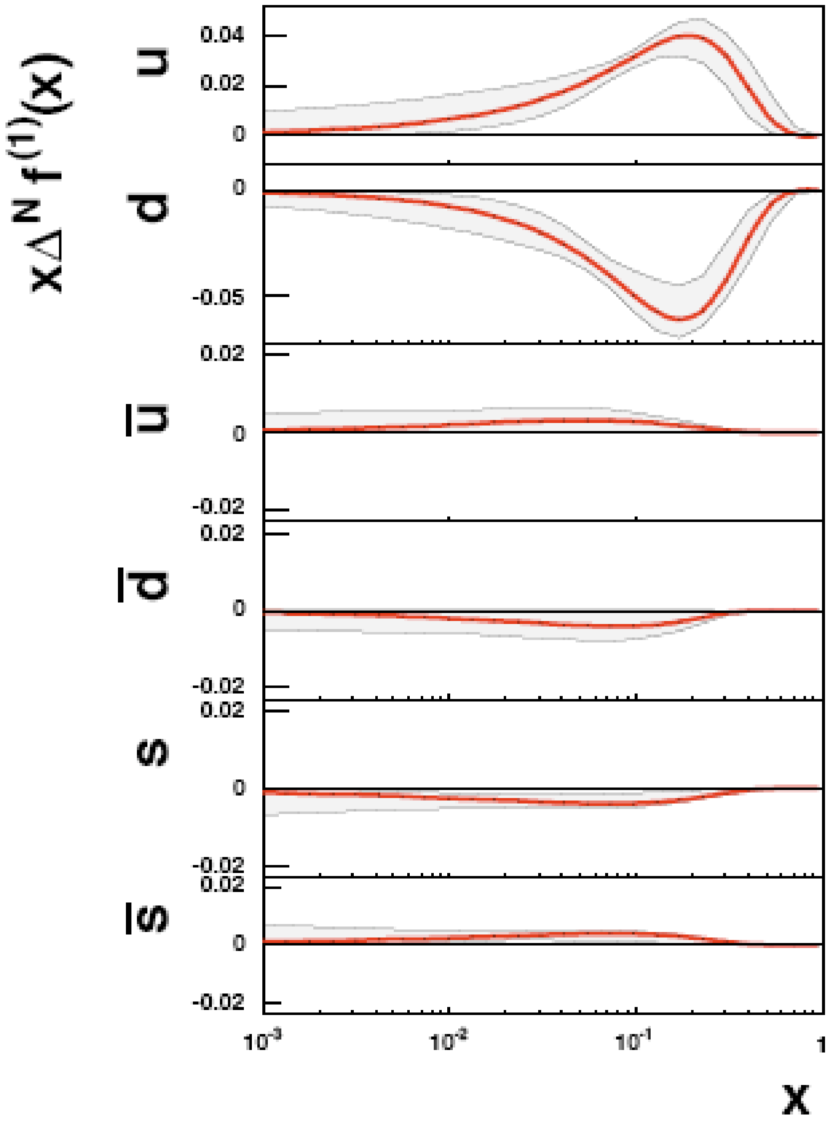
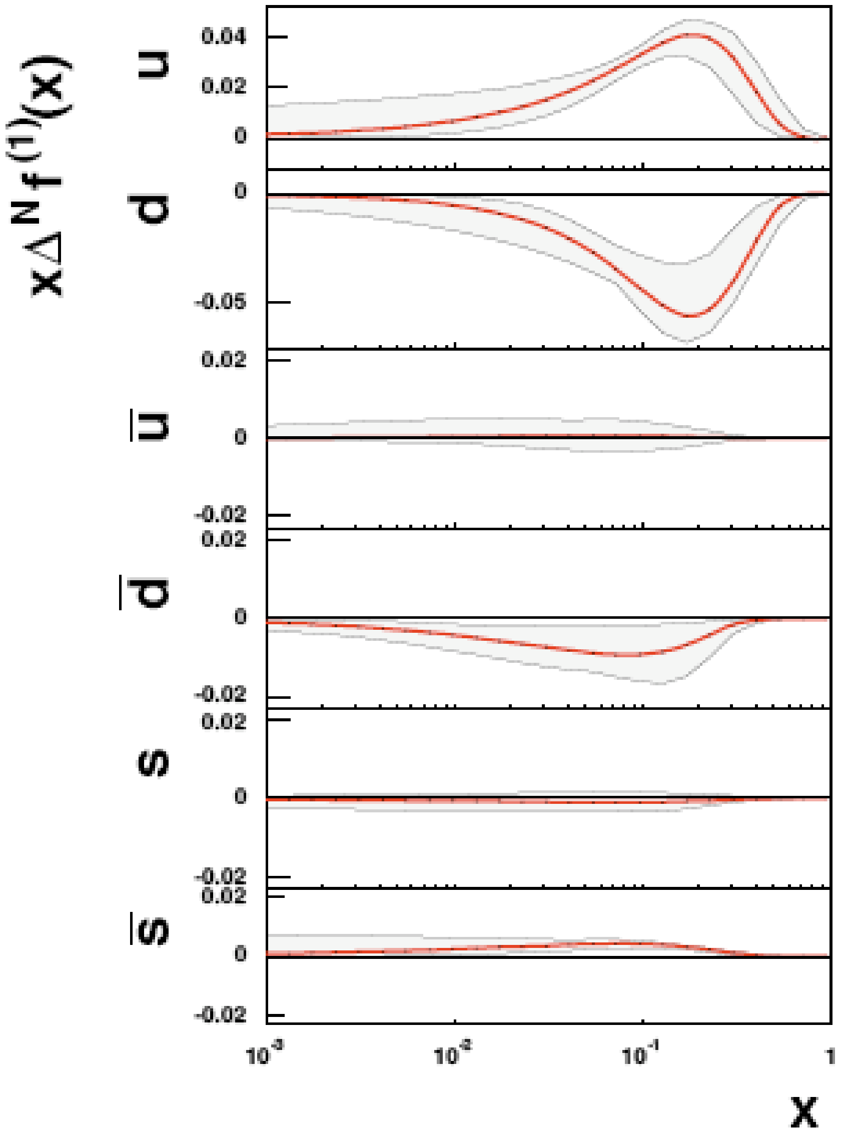
- Not clear if it's possible to recover DGLAP-like evolution equations for Sivers, but certainly for D_1
- It's important that the structure function falls fast enough with transv. mom.

Sivers function - Torino

"Symmetric sea"



Free fit



Anselmino et al., 0805.2677

Sivers function - Bochum

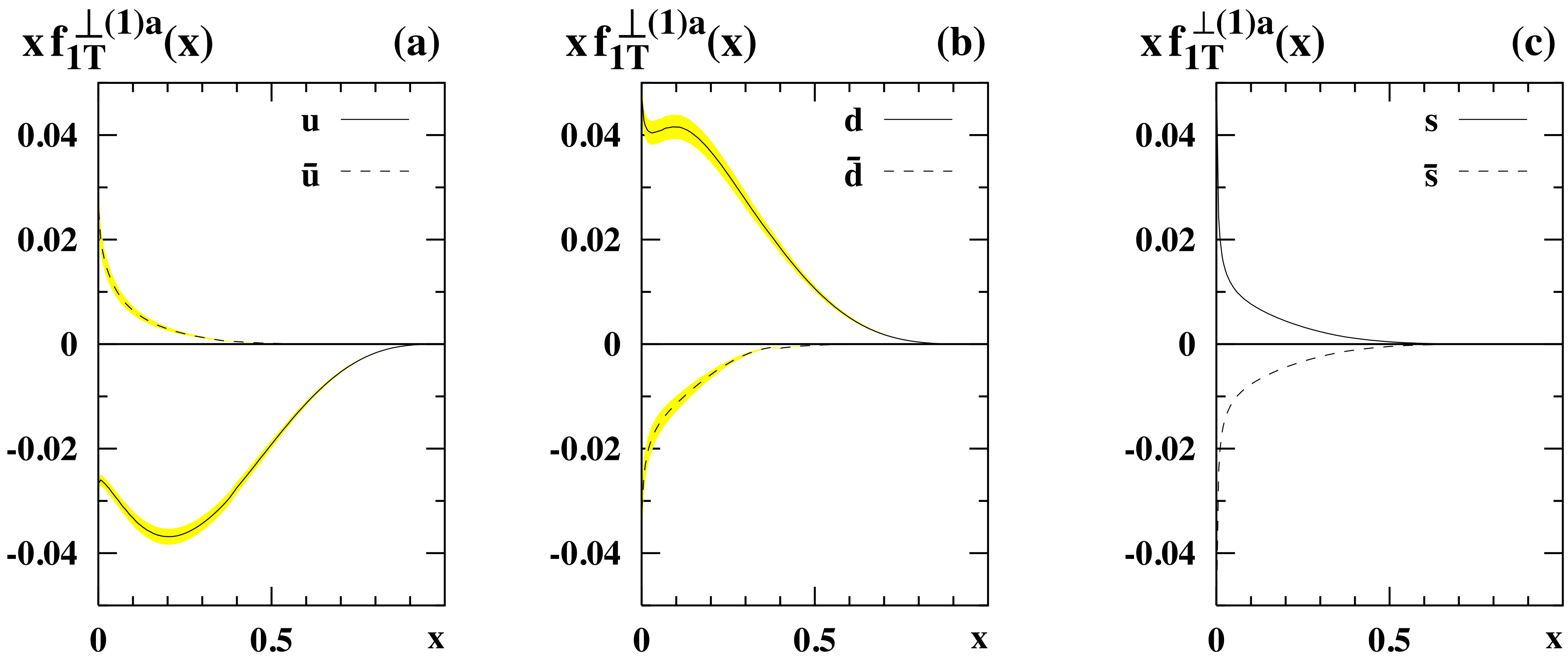


FIGURE 7. The $x f_{1T}^{\perp(1)a}(x)$ vs. x as extracted from preliminary HERMES and COMPASS data [10, 11]. (a) The flavours u and $\overline{u}$. (b) The flavours d and $\overline{d}$. (c) The flavours s and $\overline{s}$ that were fixed to $\pm$ positivity bounds (17) for reasons explained in Sec. 7, see also Eqs. (18, 19). The shaded areas in (a) and (b) show the respective 1-σ-uncertainties.

Arnold et al. , 0805.2137

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- Evolution equations neglected (will be very relevant at EIC)

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- Limited x range (EIC can improve on both sides)
- 173 data points (cf. 467 points in Δq fits)

Gluon Sivers function

$$\sum_{a=q,\bar{q},g} \int_0^1 dx\, f_{1T}^{\perp(1)a}(x) = 0\,, \quad \text{with} \quad f_{1T}^{\perp(1)a}(x) = \int d^2\vec{k}_T \frac{\vec{k}_T^2}{2M^2} f_{1T}^{\perp a}(x, \vec{k}_T^2)$$

M. Burkardt, PRD69 (04)

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M. Burkardt, PRD69 (04)

- Based on the Burkardt sum rule, the gluon Sivers function is claimed to be small
- Limited *x* range
- There could be nodes in the function
- Connection with orbital angular momentum is not straightforward

EIC tasks

EIC tasks

- Provide more data

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- Measure weighted asymmetries

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- Extend x, Q range

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EIC tasks

- Provide more data
- Measure weighted asymmetries
- Extend x, Q range
- Demand theory improvements
- Explore gluon Sivers function

Connection with orbital angular momentum

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- Several TMD observables can help constrain models with OAM

see also next talk by H. Avakian

Connection with orbital angular momentum

- There is no direct connection between Sivers function and OAM
- Several TMD observables can help constrain models with OAM

see also next talk by H. Avakian

- Global effort to put together GPDs and TMDs information is required

Other observables

Leading-twist functions

Leading-twist functions

- There are 8 leading-twist TMDs

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Leading-twist functions

- There are 8 leading-twist TMDs
- There are 8 leading-twist structure functions where the PDFs are connected with either D_1 or Collins function
- 4 can be studies also through jet-SIDIS without fragmentation functions

EIC tasks

EIC tasks

- It's difficult to choose the most relevant measurements at the moment

EIC tasks

- It's difficult to choose the most relevant measurements at the moment
- Shall EIC measure them all?

see COMPASS Coll. 0705.2402

see also next talks

Post Scriptum

- I did not mention the issue of AdS/QCD correspondence

see work of Brodsky, de Teramond

- EIC can map a new dimension: the distribution of parton in transverse momentum space

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- EIC will be a precision machine for TMDs