Target mass corrections and beyond

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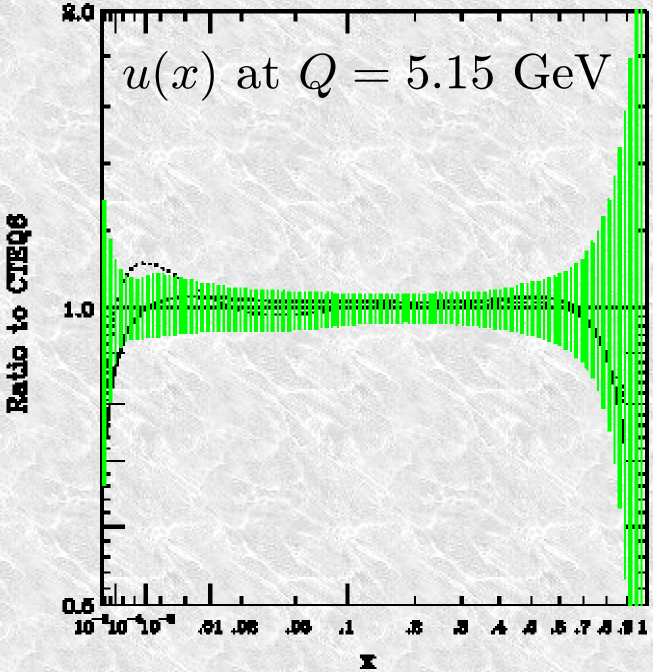
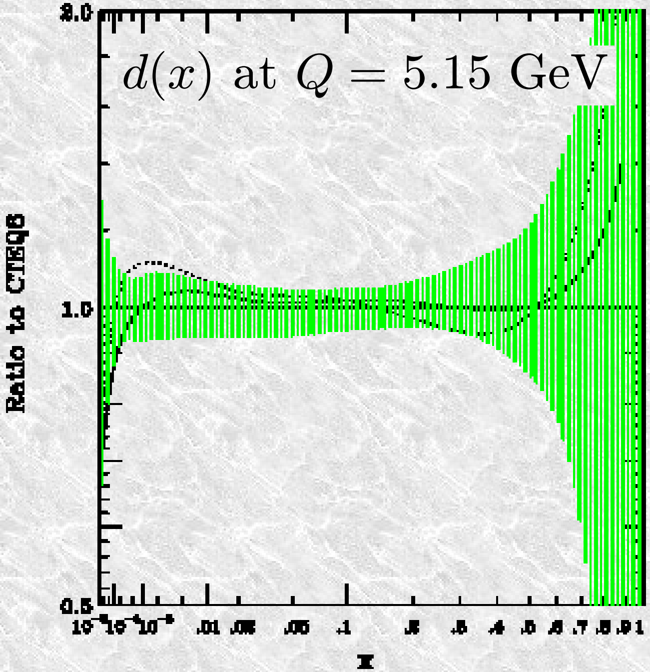
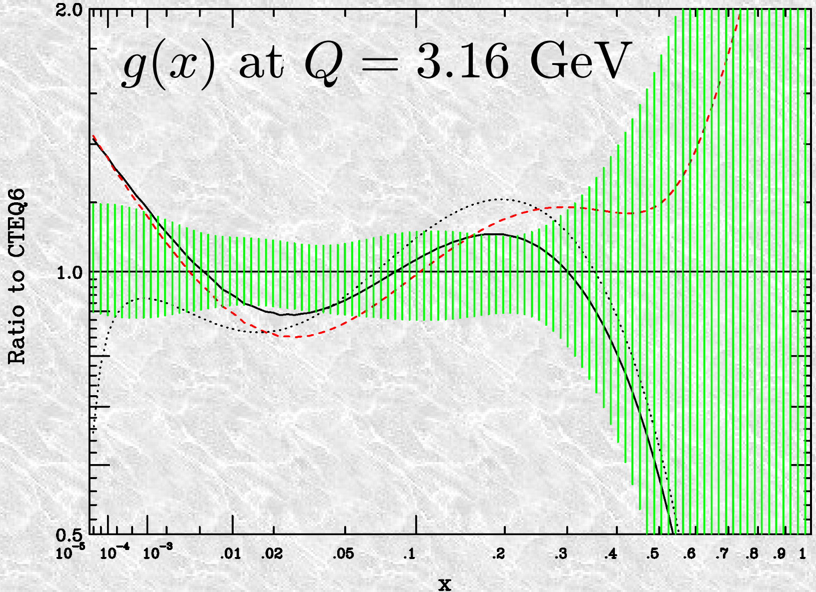
Motivation and outline

Why large x_B and low Q^2 ?

- Large uncertainties in quark and gluon PDF at $x > 0.5$ – e.g., CTEQ6







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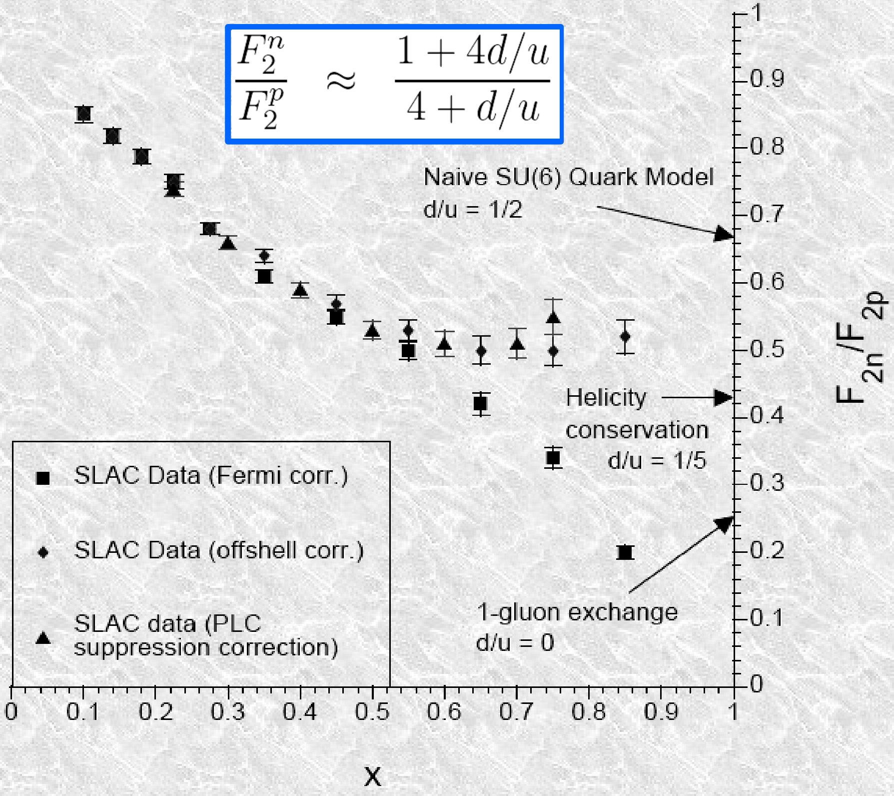
Why large x_B and low Q^2 ?

- Large uncertainties in quark and gluon PDF at $x > 0.5$
- Precise PDF at large x are needed, e.g.,
  - at LHC, Tevatron
    1) New physics as excess in large p_T spectra ⇔ large x PDF
    2) DGLAP evolution feeds large x, low Q^2 into lower x, large Q^2
  - d/u ratio at $x=1$ ⇔ non-perturbative structure of the nucleon



$$\frac{F_2^n}{F_2^p} \approx \frac{1+4d/u}{4+d/u}$$

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- JLab has precision DIS data at large x_B , BUT low Q^2
  - need of theoretical control over
    1) higher twist $\propto \Lambda^2/Q^2$
    2) target mass corrections (TMC) $\propto x_B{}^2\, m_N{}^2/Q^2$
    3) jet mass corrections (JMC) $\propto m_j{}^2/Q^2$
    4) large-x resummation, ...

$\left.\vphantom{\begin{array}{c}1\\2\\3\end{array}}\right\}$ **this talk**

OPE and Target Mass Corrections

[Georgi, Politzer 1976; see review of Schienbein et al. 2007]

$$\int d^4z\, e^{-iq\cdot z} \langle N|T[j^{\dagger\mu}(z)j^\nu(0)]|N\rangle = \sum_k f^{\mu_1\ldots\mu_{2k}} A_{2k} \langle N|\underbrace{\mathcal{O}_{\mu_1\ldots\mu_{2k}}}_{\text{symmetric, traceless}}(0)|N\rangle$$

$$A_{2k} = \int_0^1 dy\, y^{2k} F(y) \qquad F(y) \sim \frac{1}{y^2}\sum_q e_q^2 q(y) \text{ (at LO)} \;\; = \text{“quark function”}$$

- Mellin transform, sum, transform back:

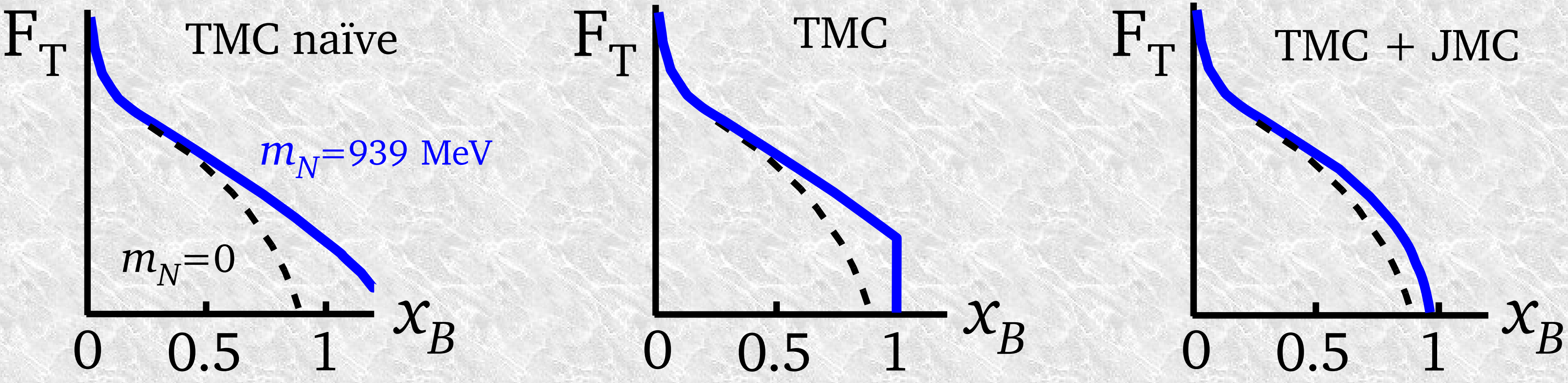
$$F_2^{GP}(x_B,Q^2) = \frac{x_B^2}{\rho_B^3}F(\xi) + 6\frac{m_N^2}{Q^2}\frac{x_B^3}{\rho_B^4}\int_\xi^1 d\xi' F(\xi') + 12\frac{m_N^4}{Q^4}\frac{x^4}{\rho_B^5}\int_\xi^1 d\xi' \int_{\xi'}^1 d\xi'' F(\xi'')$$

$$\xi = \frac{2x_B}{1+\sqrt{1+4x_B^2m_N^2/Q^2}} = \frac{2x_B}{1+\rho_B^2} \quad \text{Nachtmann variable}$$

- Threshold problem: $x_B \leq 1$ implies $0 \leq \xi \leq \xi_{th} \overset{\text{def}}{=} \xi(x_B=1)$
  - Inverse Mellin transform does not give back F(y) !! [Johnson, Tung 1979]
- Unphysical region: $F(y) \sim F_2(y)$ has support over $0<y<1$
  - $F_2^{GP}(x_B) > 0$ also for $x_B>1$!!

Collinear factorization - outline

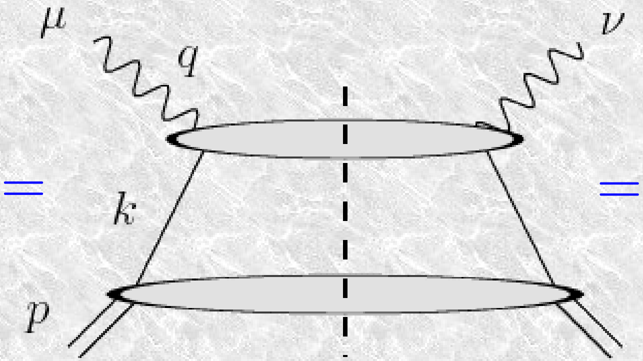
- **Target Mass Corrections – *$O(x_B^2\, m_N^2/Q^2)$***
  - momentum space, no need of Mellin transf.
  - kinematics of handbag diagram
    ⇒ no “unphysical region” at $x_B > 1$ (!!)
  - any order in α_s at leading twist
- **Global PDF fits** – preliminary results
- **Jet Mass Corrections – *$O(m_j^2/Q^2)$***
  - leading order in α_s, leading twist
  - phenomenology of the “jet function”
- **Conclusions**



Target mass corrections

Accardi, Qiu, JHEP '08
Accardi, Melnitchouk, PLB '08

Kinematics with $m_N \neq 0$



$$W^{\mu\nu}(p,q) = \qquad = \frac{1}{8\pi}\int d^4z\, e^{-iq\cdot z}\langle p|j^{\dagger\mu}(z)j^{\nu}(0)|p\rangle$$

- Collinear frames: [Aivazis et al 94]

$$p^\mu = p^+\overline{n}^\mu + \frac{m_N^2}{2p^+}n^\mu$$

$$q^\mu = -\xi p^+\overline{n}^\mu + \frac{Q^2}{2\xi p^+}n^\mu$$

$$k^\mu = xp^+\overline{n}^\mu + \frac{k^2+k_T^2}{2xp^+}n^\mu + k_T^\mu$$

where:

$$x = \frac{k^+}{p^+} \qquad \xi = \frac{2x_B}{1+\sqrt{1+4x_B^2m_N^2/Q^2}}$$

Lorentz invariants:

$$x_B = \frac{-q^2}{2p\cdot q} \qquad Q^2 = -q^2$$

$$x_f = \frac{-q^2}{2k\cdot q} \qquad m_N^2 = p^2$$

Light cone vectors:

$$\boldsymbol{\overline{n}} = (1/\sqrt{2}, \vec{0}_\perp, 1/\sqrt{2})$$

$$n = (1/\sqrt{2}, \vec{0}_\perp, -1/\sqrt{2})$$

$$a^\pm = (a_0 \pm a_3)/\sqrt{2}$$

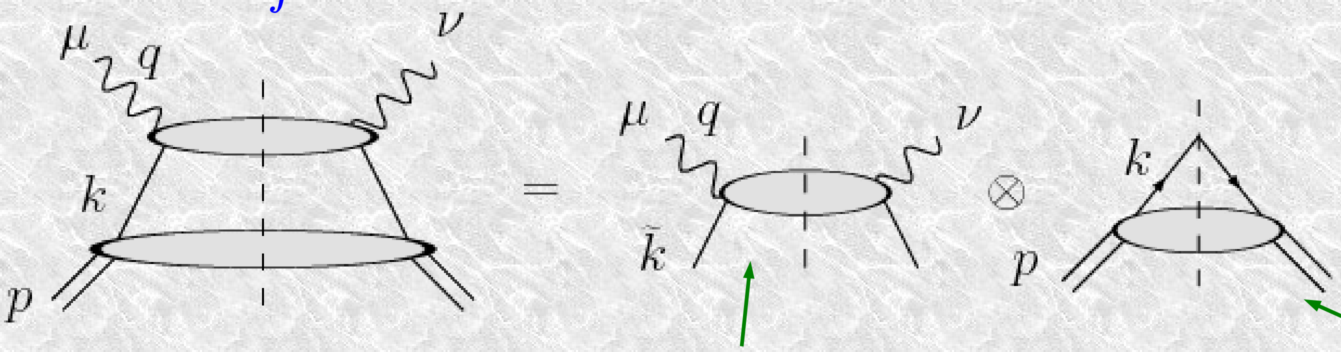
- Bjorken limit: $\xi \to x_B$ recovers the massless ($m_N^2 = 0$) kinematics

Factorization theorem with $m_N \neq 0$

[see also J.W.Qiu's talk at CTEQ meeting 2005]

- Expand around $\tilde{k}^\mu = x p^+ \overline{n}^\mu$ $\tilde{k}^2 = 0$ $\tilde{x}_f = \frac{-q^2}{2\tilde{k}\cdot q} = \frac{\xi}{x}$

$$W_N^{\mu\nu}(p,q) = \sum_f \int \frac{dx}{x}\, \mathcal{H}_f^{\mu\nu}(\tilde{k},q)\, \varphi_{f/N}(x,Q^2) + O(\Lambda^2/Q^2)$$



perturbative: doesn't know about the target's mass

dynamical TMC only from nucleon w.f.

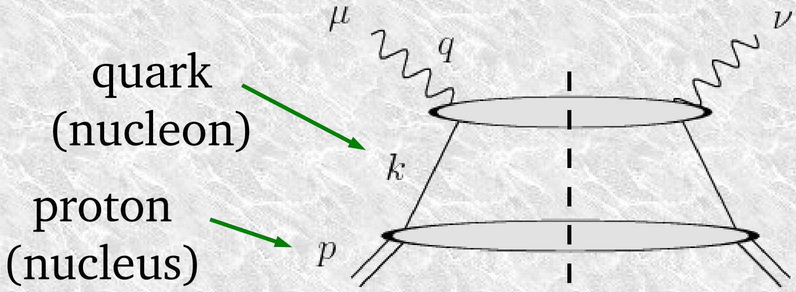
- Helicity structure functions F_T , F_L projected out of $W^{\mu\nu}$: *e.g.*,

$$F_T(x_B,Q^2) = \sum_f \int \frac{dx}{x}\, h_{fT}(\underbrace{\tilde{x}_f}_{= \xi / x}, Q^2)\, \varphi_{f/N}(x,Q^2) + O(\Lambda^2/Q^2)$$

no kinematic prefactors [Aivazis, Olness, Tung 1994]

Kinematic constraints

- General handbag diagram – on shell gluons and light quarks ($\widetilde{k}^2 = 0$):

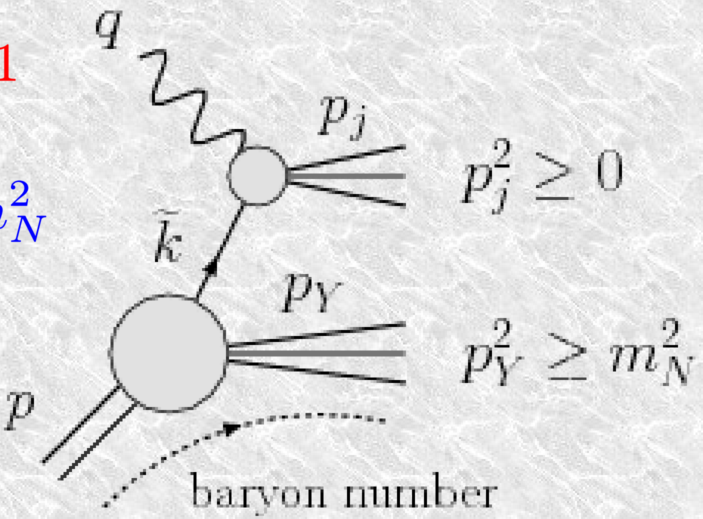


$$x_B \le \widetilde{x}_f \le 1$$

i.e., $\boxed{\xi \le x \le \xi/x_B}$

- Proof (can be generalized to heavy and off-shell quarks – and nuclei)
  - $0 \le p_j^2 = (\widetilde{k} + q)^2 = Q^2\left(\frac{1}{\widetilde{x}_f} - 1\right) \implies \widetilde{x}_f \le 1$
  - $s = (p+q)^2 = (p_j + p_Y)^2 \ge p_j^2 + p_Y^2 \ge p_j^2 + m_N^2$

$$\left.\begin{aligned} p_j^2 &= \left(\frac{1}{\widetilde{x}_f} - 1\right)Q^2 \\ s - m_N^2 &= \left(\frac{1}{x_B} - 1\right)Q^2 \end{aligned}\right\} \implies \widetilde{x}_f \ge x_B$$



- If net baryon number appears in the upper blob (not for pQCD quarks)

$$\frac{x_B}{1 + x_B m_N^2/Q^2} \le \widetilde{x}_f \le \frac{1}{1 + m_N^2/Q^2}$$

No unphysical region!

- **TMC in collinear factorization:**

$$F_T(x_B,Q^2) = \sum_f \int_\xi^{\frac{\xi}{x_B}} \frac{dx}{x} h_{fT}\Big(\frac{\xi}{x},Q^2\Big)\,\varphi_f(x,Q^2)$$

$$\boxed{F_T(x_B,Q^2) = 0 \quad \text{at } x_B > 1}$$

- Bjorken limit $m_N/Q^2 \to 0$ recovers "**massless**" structure functions ($m_N=0$)

$$F_T(x_B,Q^2) \longrightarrow F_T^{(0)}(x_B,Q^2) \equiv \sum_f \int_{x_B}^{1} \frac{dx}{x} h_{fT}\Big(\frac{x_B}{x},Q^2\Big)\,\varphi_f(x,Q^2)$$

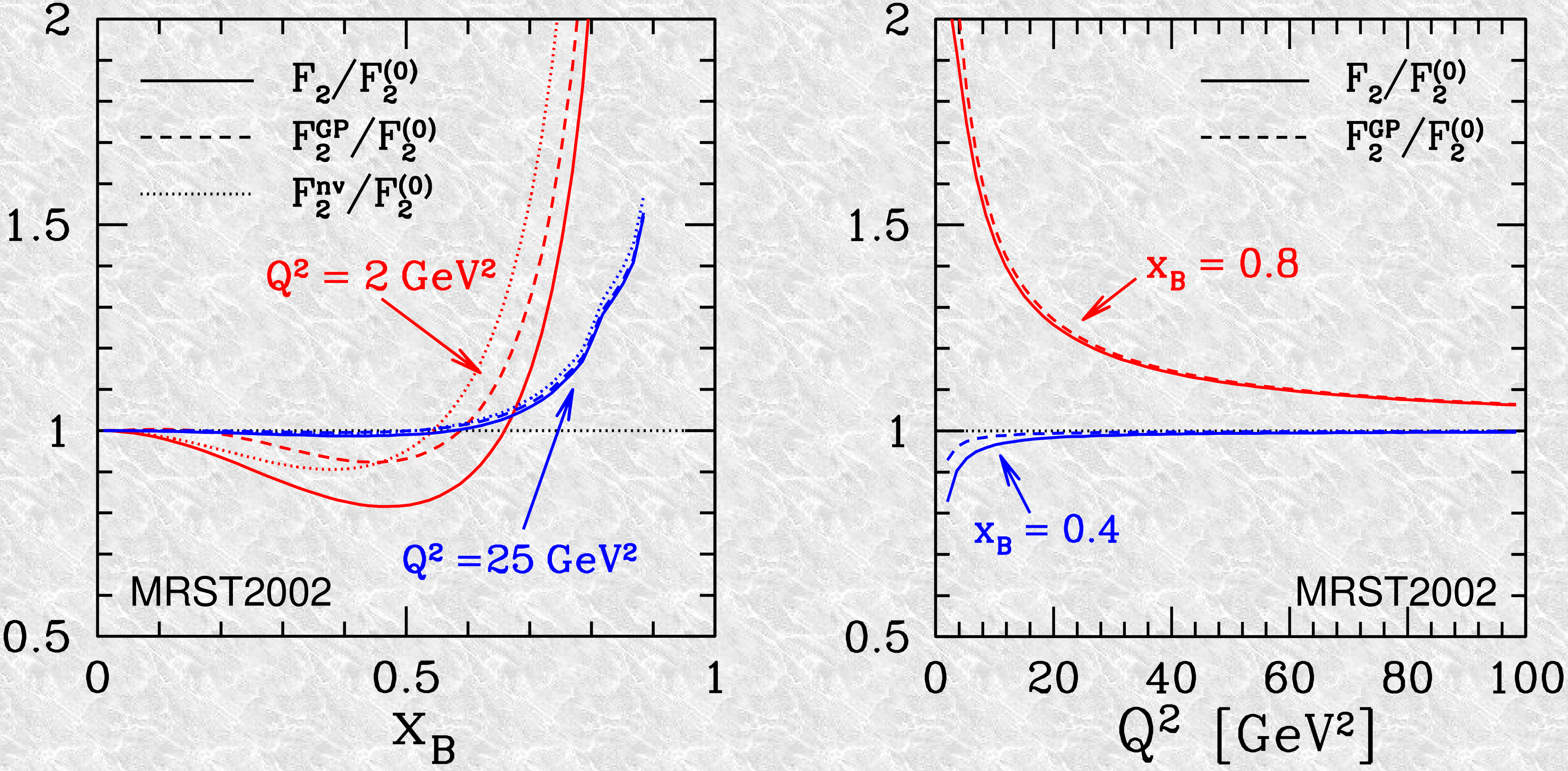
- Different from the "**naive**" collinear factorization TMC [Aivazis et al '94 Kretzer,Reno '02]

$$F_T^{nv}(x_B,Q^2) \equiv F_T^{(0)}(\xi,Q^2) = \sum_f \int_\xi^{1} \frac{dx}{x} h_{fT}\Big(\frac{\xi}{x},Q^2\Big)\,\varphi_{f/N}(x,Q^2)$$

which does not vanish at $x_B > 1$

Target mass corrections – F_2 at NLO

Accardi, Qiu '08   Accardi, Qiu '08



$$F_2^{nv}(x_B) = \frac{1}{1 + 4x_B^2 \frac{m_N^2}{Q^2}} \frac{x_B}{\xi} F_2^{(0)}(\xi)$$

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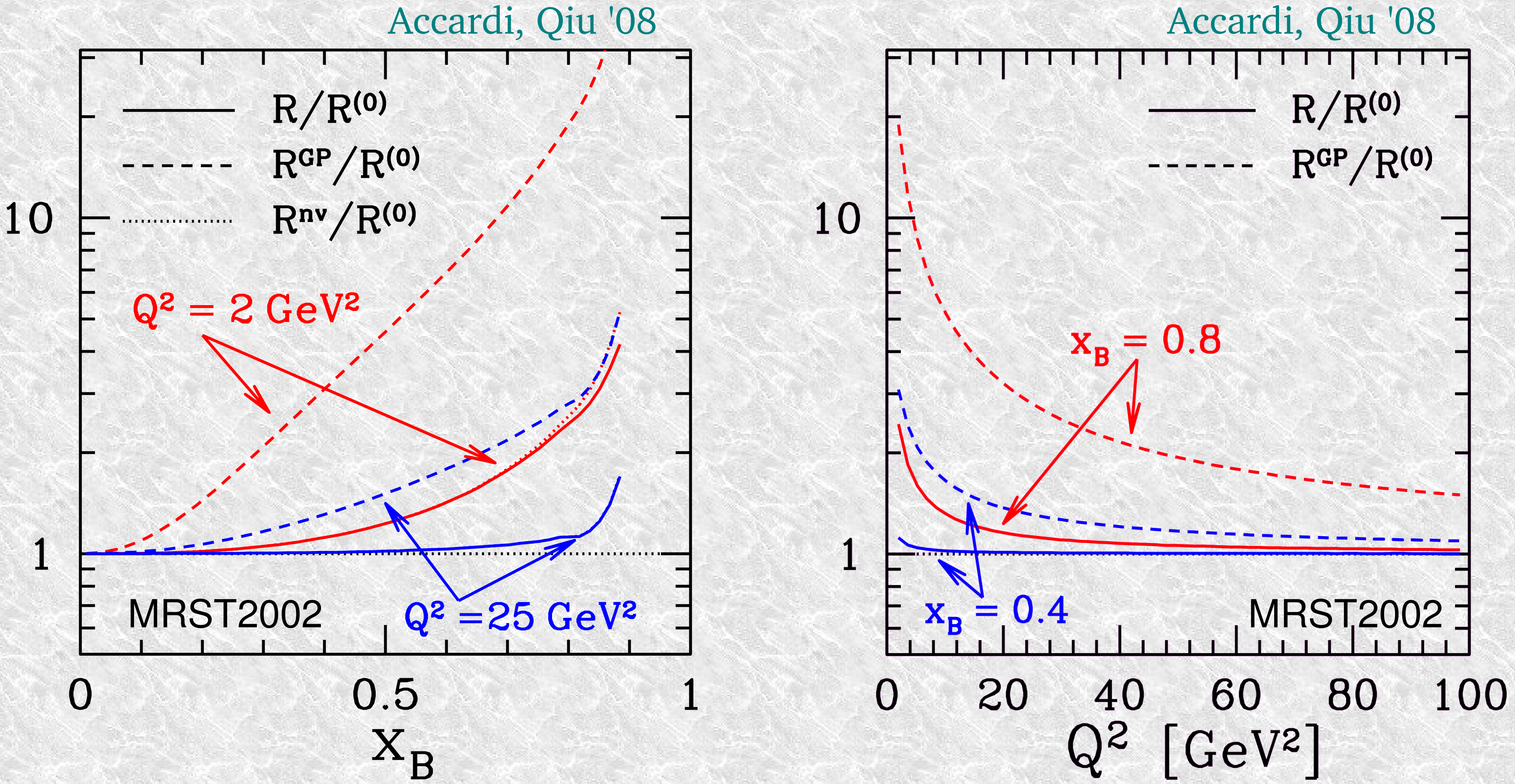
Target mass corrections – σ_L/σ_T at NLO



$$R = \frac{\sigma_L}{\sigma_T} = \frac{F_L}{F_1}$$

$$F^{nv}_{1,L}(x_B) = F^{(0)}_{1,L}(\xi)$$

Polarized DIS

- **TMC for virtual photon asymmetries (leading twist):**

$$\begin{cases} g_1(x_B) - \gamma^2 g_2(x_B) = \sum_f g_{1,f}^{(0)} \otimes \Delta\varphi(\xi) \ + \ \mathrm{HT} \\ \\ g_1(x_B) + g_2(x_B) = 0 \ + \ \mathrm{HT} \end{cases}$$

where

$$\Delta\varphi_f(x) = \varphi_f^+(x) - \varphi_f^-(x) \qquad \gamma^2 = 4x_B^2 \frac{m_N^2}{Q^2} = \rho_B^2 - 1$$

$$g_{1,f}^{(0)} \otimes \Delta\varphi_f(\xi) \equiv \int_\xi^{\frac{\xi}{x_B}} \frac{dx}{x} g_{1,f}^{(0)}\left(\frac{\xi}{x}, Q^2\right) \Delta\varphi_f(x, Q^2)$$

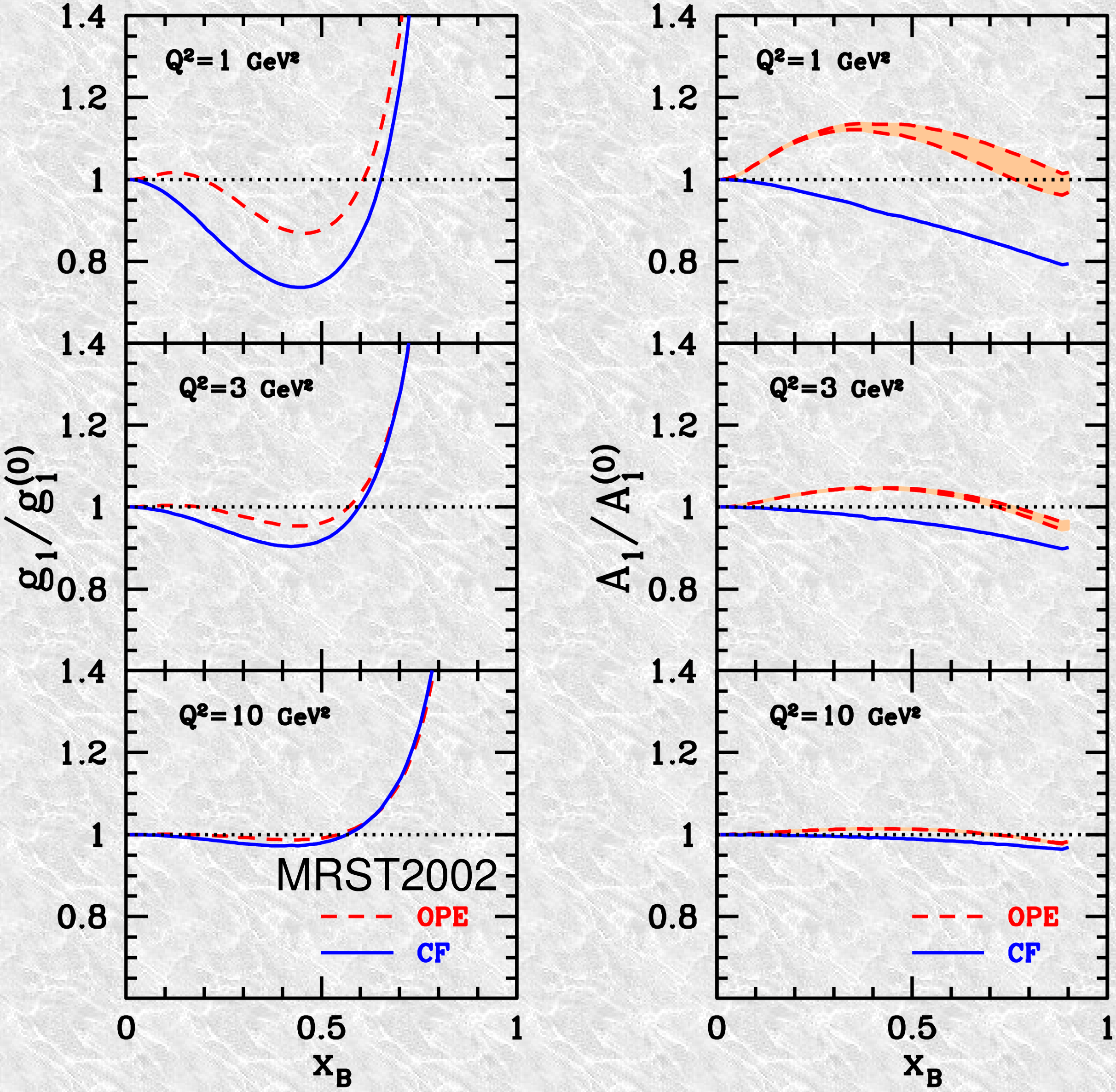
- **TMC for g_1, A_1 at leading twist:**

$$g_1(x_B) = \frac{1}{1+\gamma^2} \sum_f g_{1,f} \otimes \Delta\varphi(\xi)$$

$$A_1(x_B) = \frac{1}{F_1(x_B)} \sum_f g_{1,f} \otimes \Delta\varphi(\xi)$$

Polarized DIS at LO

Accardi, Melnitchouk '08



- g1 similar to F2
- A1 has smaller corrections
- The approximation

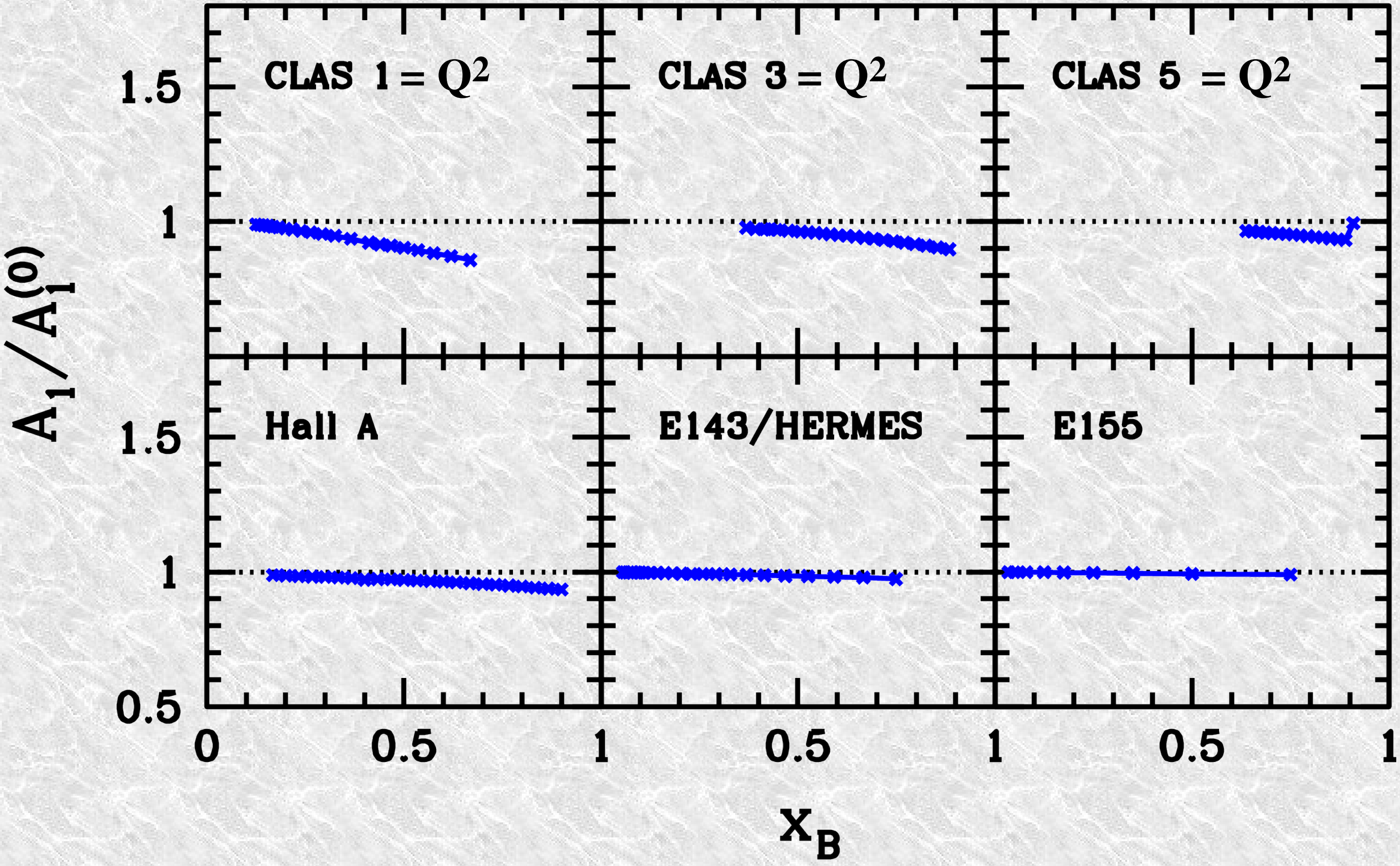
$$A_1 = (1+\gamma^2)\frac{g_1}{F_1} \approx \frac{A_\parallel}{D}$$

which is equivalent to

$$A_1 \approx A_1^{(0)}$$

is **NOT** suitable for precision measurements at Jlab: needs both $A_\parallel$ and $A_\perp$

Polarized DIS at LO

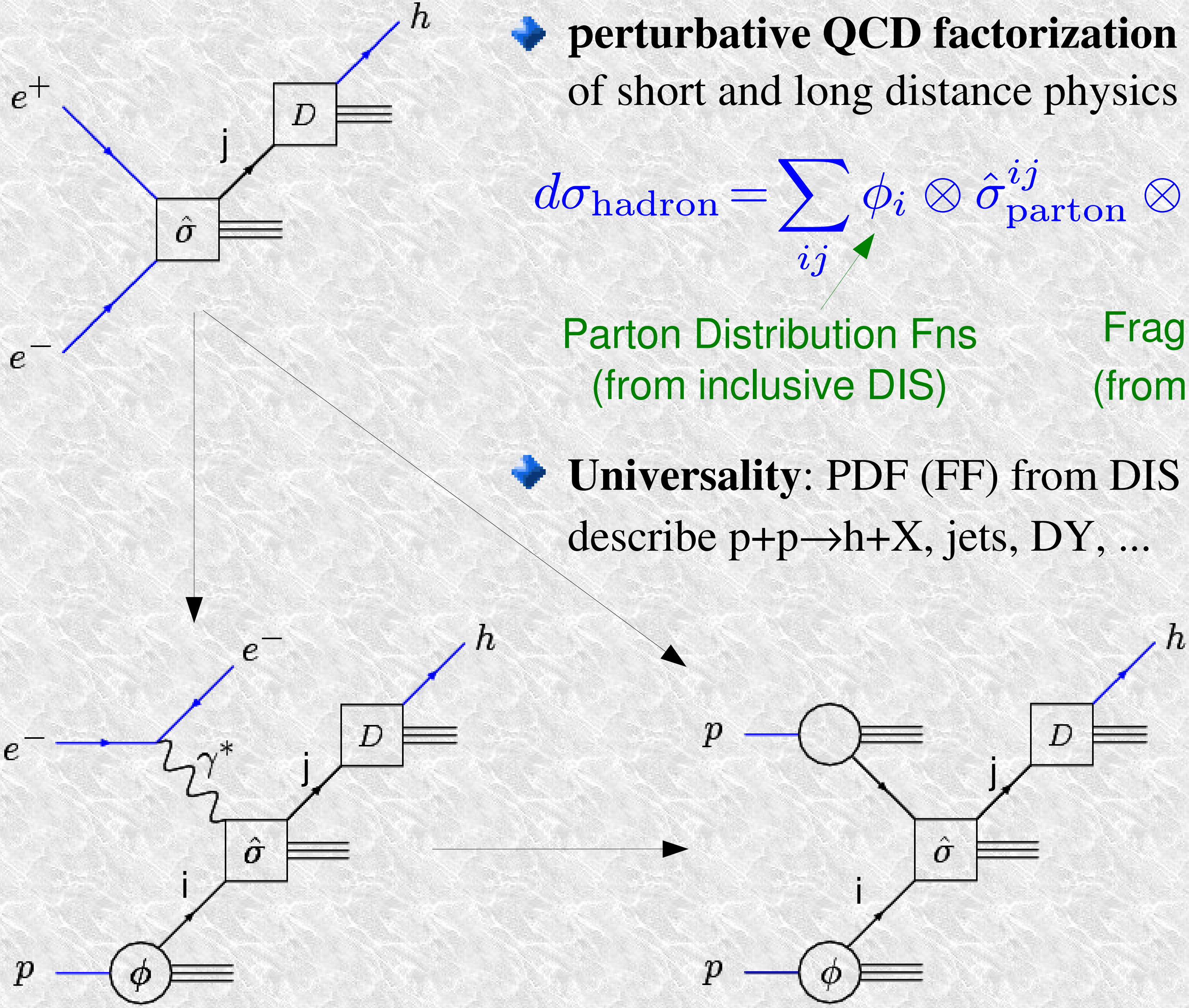


- Precision measurements of A_1 at JLAB requires both $A_{||}$ and $A_\perp$

Global PDF fits

Work in progress with:
E.Christy, C.Keppel, W.Melnitchouk, P.Monaghan, J.Morfín, J.Owens

Factorization of hard scattering processes



- **perturbative QCD factorization** of short and long distance physics

$$d\sigma_{\text{hadron}} = \sum_{ij} \phi_i \otimes \hat{\sigma}^{ij}_{\text{parton}} \otimes D_{j|h}$$

Parton Distribution Fns
(from inclusive DIS)

Fragmentation Fns
(from $e^+ + e^- \rightarrow h + X$)

- **Universality**: PDF (FF) from DIS ($e^+ + e^-$) describe $p+p \rightarrow h+X$, jets, DY, ...

Global PDF fits

- **Problem:** we need a set of PDFs in order to calculate a particular hard-scattering process

- **Solution:**
  - generate a set of PDFs using a parametrized functional form at a given initial scale Q_0 and evolving it at any Q.
  - Choose a data set for a choice of hard scattering processes of different kinds.
  - Repeatedly vary the parameters and evolve the PDFs again, to obtain an optimal fit to a set of data.

- Examples: CTEQ6.1, MRST2002 for unpolarized protons
  DSSV, LSS for polarized protons

- For details, see J. Owens' lectures at the 2007 CTEQ summer school

Collaboration and goals

- Jefferson Lab/Florida State U./Fermilab collaboration:
  - Alberto Accardi, Eric Christy, Thia Keppel, Wally Melnitchouk, Peter Monaghan, Jorge Morfin, Jeff Owens

- Initial Goals:
  - Extend PDF global fits to larger values of x_B and lower values of Q
  - Wealth of data from older SLAC experiments and newer JLab
  - Study effects of different target mass correction methods
  - Explore role of higher twist contributions

- Eventually,
  - see if PDF errors can be reduced using new JLAB data
  - determine an optimized set of PDFs at large x_B

Global fit details

- We are using Jeff Owens' NLO DGLAP fitting package
  - use CTEQ6.1 parametrization of PDFs at $Q^2 = 1.69$ GeV^2
  - option for finite d/u at $x \to 1$ is being considered
  - Can fit DIS, Drell-Yan, W lepton asymmetry, jets (and γ+jet)
  - Multiple TMC and HT terms added
  - Higher-twist contributions by a multiplicative factor
  - Nuclear corrections for deuteron targets added
  - PDF errors computed by the Hessian method

Higher-Twists parametrization

- Parametrize the higher-twist contributions by a multiplicative factor:

$$F_2(data) = F_2(TMC) \times \left(1 + \frac{C(x_B)}{Q^2}\right)$$

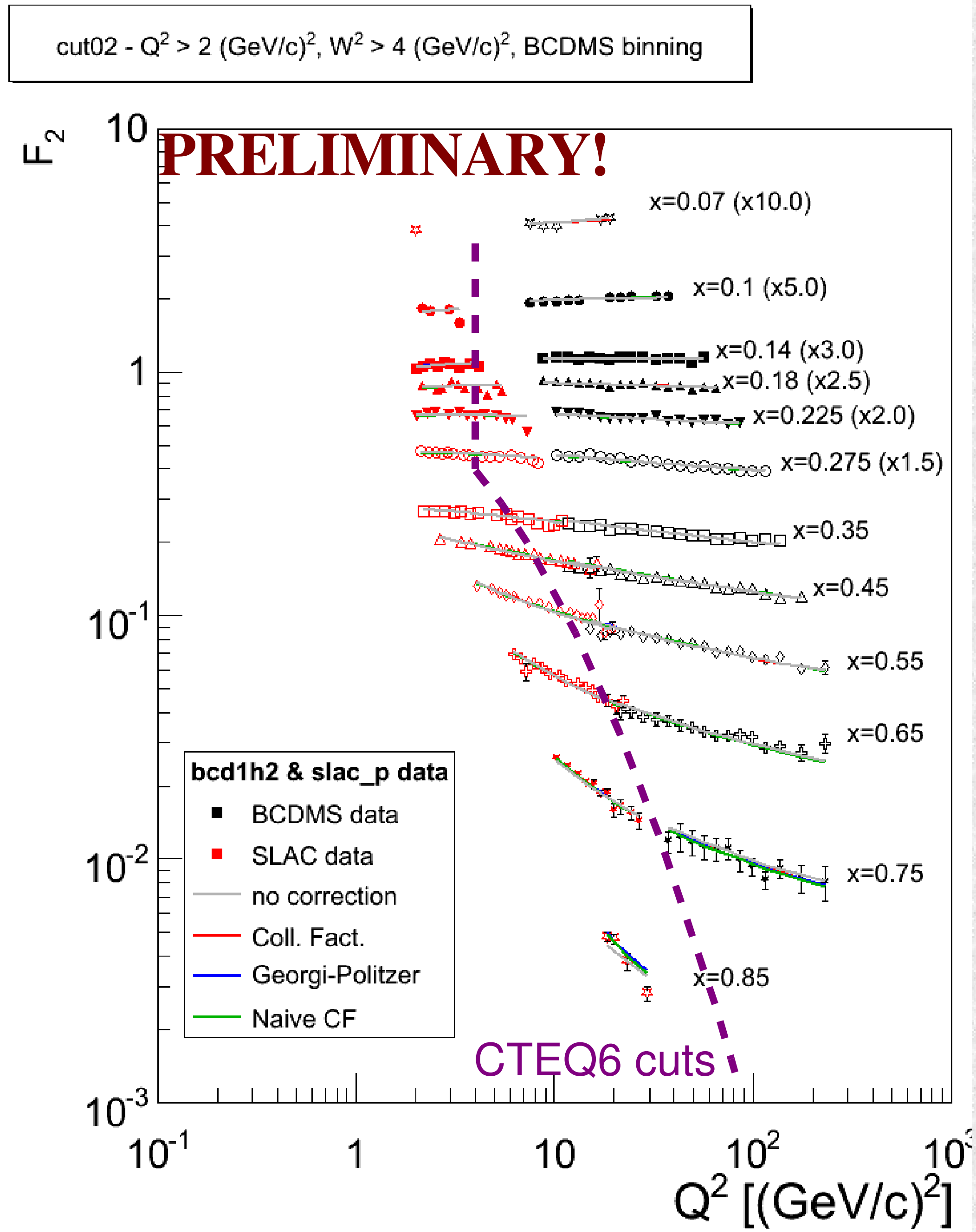
with

$$C(x_B) = a\, x^b\, (1 + c\, x + d\, x^2)$$

- Comments
    - parametrization is sufficiently flexible to give good fits to data
    - typically, parameter d is not needed since at x_B near 1 there is not a lot of difference between x and x^2

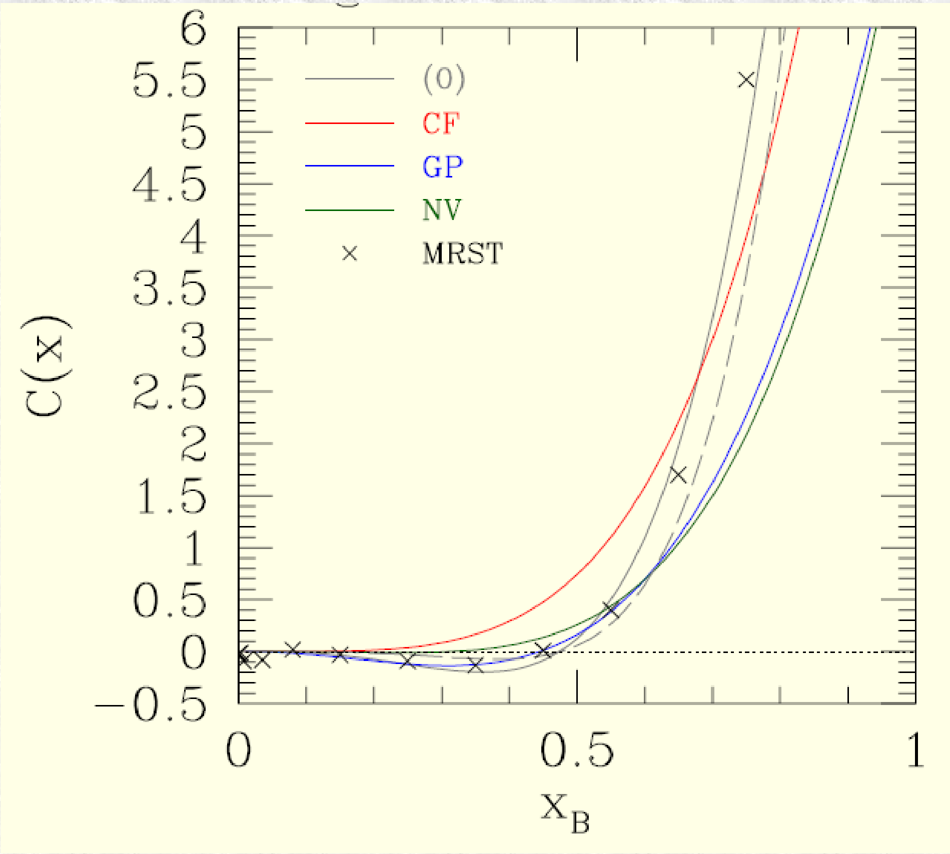
Preliminary results

- Lower the CTEQ6.1 cuts
  - $Q^2>2\ GeV^2$ (was 4 GeV^2)
  - $W^2>4\ GeV^2$ (was 12.25 GeV^2)
  - called “cut02” henceforth
- Include TMC:
  - CF, Gorgi-Politzer, naïve CF
- Use HT parametrization



Preliminary results

- Extracted higher-twist term depends on the type of TMC used



  - $Q^2 > 2$ GeV^2 and $W^2 > 4$ GeV^2 [cut02]
  - Solid curves have $d=0$ and small errors on a, b, and c
  - Dashed curve has $d\neq 0$, but large errors on all four parameters

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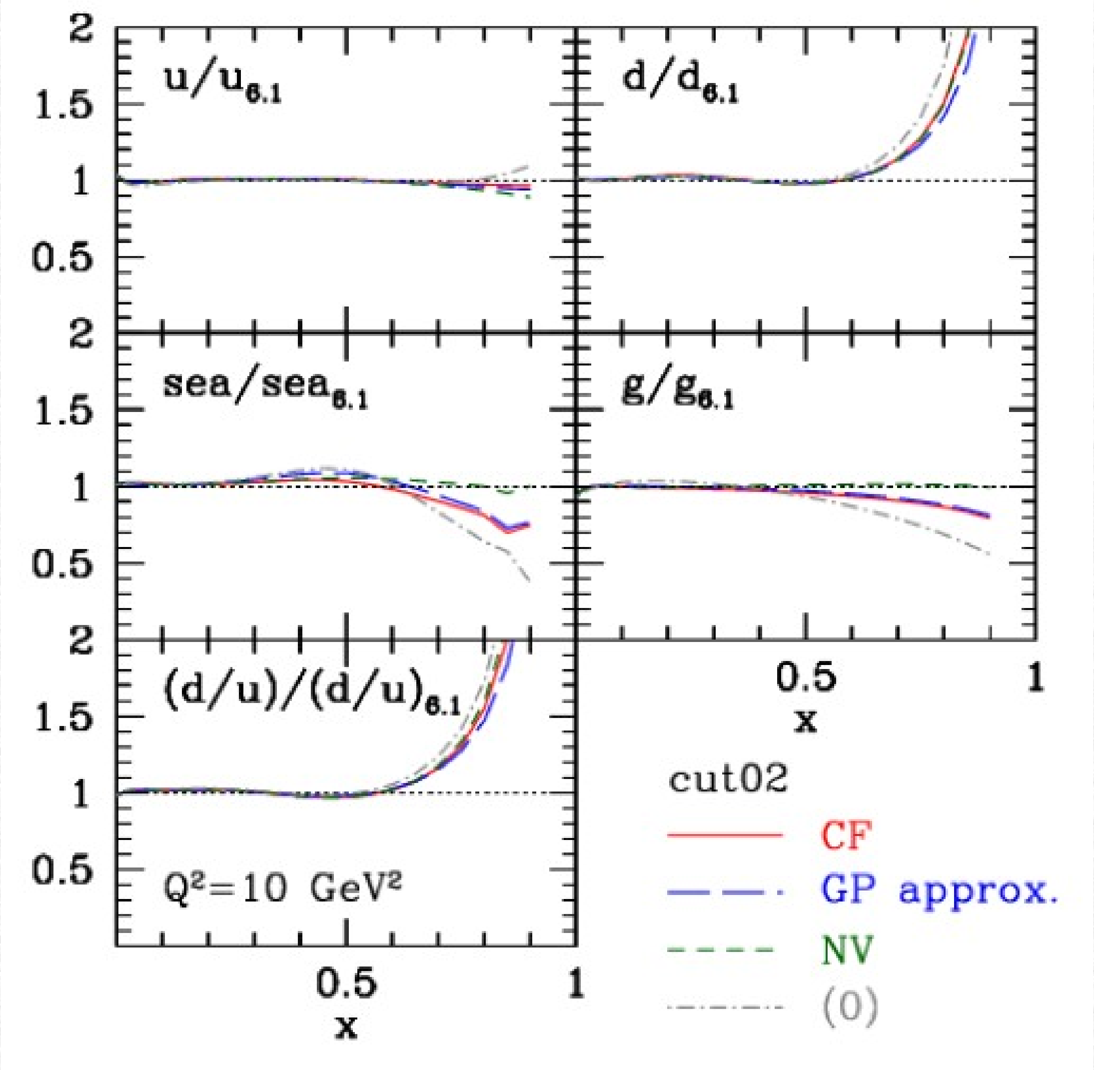
Preliminary results

- Extracted twist-2 PDF much less sensitive to choice of TMC
  - fitted HT function compensates the TMC ⇒ PDFs are rather stable
- Largest effect on *d*-quark distribution (plots relative to CTEQ6.1)



Comments

- Results depend somewhat on the nuclear corrections used for deuteron
  - at large-x_B, mostly Fermi smearing and binding energy
  - important to go beyond Bjorken limit: finite-Q^2 corrections
  - topic is under study

- A similar d-quark enhancement from other studies
  - Global fit including E-866 lepton pair data and NuTeV, CHORUS neutrino data show enhanced d/u ratios
  - DØ W electron asymmetry lie below predictions of current PDFs suggesting an enhanced d/u ratio for x near 0.4-0.5

- Finally, we will quantify the PDF uncertainties using the extended kinematic range and data versus using the previous cuts and data sets
  - effect of JLab data on PDFs and errors

Jet mass corrections

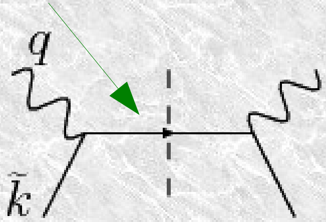
Accardi, Qiu, JHEP '08

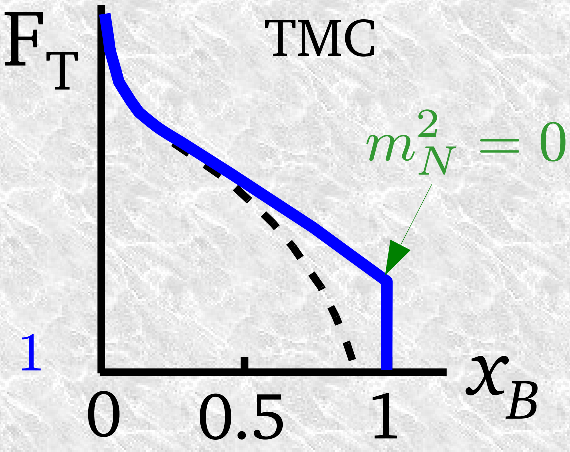
Jet smearing at LO – 1

- At leading order for F_T,

$m_f^2 = 0$

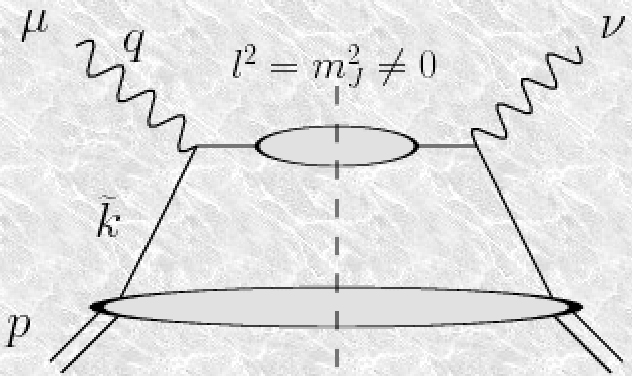
$$h_{fT}\Big(\frac{\xi}{x}, Q^2\Big) = \frac{1}{2} e_f^2 \delta\Big(\frac{\xi}{x} - 1\Big) \quad =$$





$$F_T(x_B, Q^2) = \frac{1}{2} \sum_f e_f^2 \varphi_f(\xi, Q^2) = F_T^{(0)}(\xi, Q^2) \quad \text{at } x_B < 1$$

- Ansatz: jet with a non zero mass, smoohtly distributed in m_j^2



$$(k+q)^2 = m_j^2 \;\longrightarrow\; \delta\Big[x - \xi\Big(1 + \frac{m_j^2}{Q^2}\Big)\Big]$$

jet mass distribution

$$F_T(x_B, Q^2) = \int_0^\infty dm_j^2 \, J_m(m_j^2) \int_\xi^{\frac{\xi}{x_B}} dx \, \frac{1}{2} e_q^2 \delta\Big[x - \xi\Big(1 + \frac{m_j^2}{Q^2}\Big)\Big] \varphi_f(x, Q^2)$$

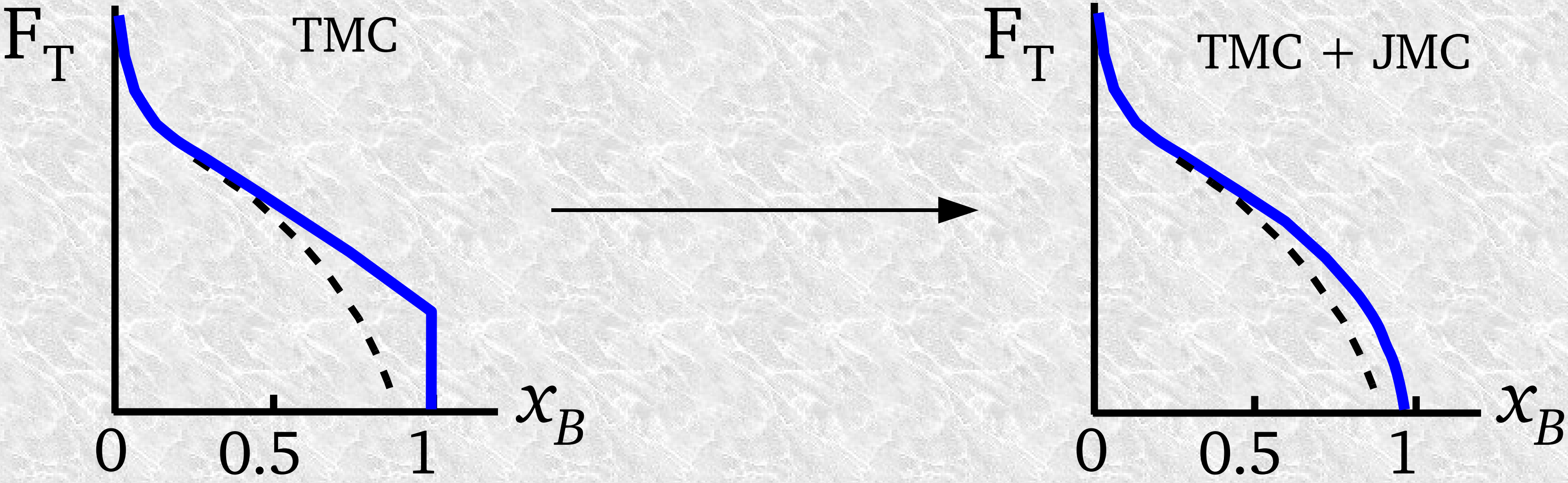
note the limits

$$= \int_0^{\frac{1-x_B}{x_B} Q^2} dm_j^2 \, J_m(m_j^2) \, F_T^{(0)}\Big(\xi\Big(1 + \frac{m_j^2}{Q^2}\Big), Q^2\Big)$$

Jet smearing at LO – 2

$$F_T(x_B,Q^2) = \int_0^\infty dm_j^2\, J_m(m_j^2) \int_\xi^{\frac{\xi}{x_B}} dx\, \frac{1}{2} e_q^2 \delta[x-\xi(1+\frac{m_j^2}{Q^2})]\, \varphi_f(x,Q^2)$$

$$= \int_0^{\frac{1-x_B}{x_B}Q^2} dm_j^2\, J_m(m_j^2)\, F_T^{(0)}(\xi(1+\frac{m_j^2}{Q^2}),Q^2)$$

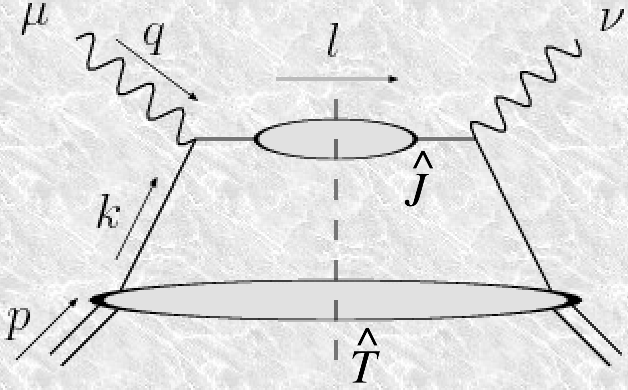


- Rigorously – after some toil:
  - $J(m_j^2)$ is the spectral function of a vacuum quark propagator, smeared by soft momentum exchanges with the target jet

Collinear factorization with a jet function

[see also Collins, Rogers, Stasto, PRD '07]

- Handbag diagram with a quark jet

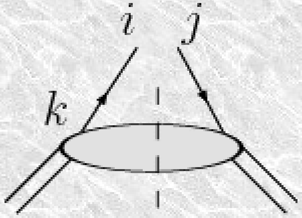


$$l = k + q$$

$$W^{\mu\nu}(p,q) = \frac{e_q^2}{8\pi} \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\big[\hat{T}(k)\gamma^\nu \hat{J}(l)\gamma^\nu\big]$$

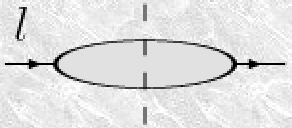
- A hat denotes a Dirac matrix:

$\hat{T}(k) =$



$$= \int d^4z\, e^{iz\cdot k} \langle p|\overline{\psi}_j(z)_i\psi(0)|p\rangle$$

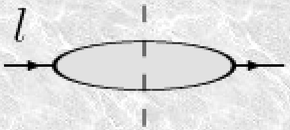
$\hat{J}(l) =$



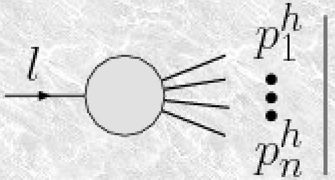
$$= \int d^4z\, e^{iz\cdot l} \langle 0|\overline{\psi}_j(z)_i\psi(0)|0\rangle$$

(color factors are included in T̂)

Jet spectral representation - 1

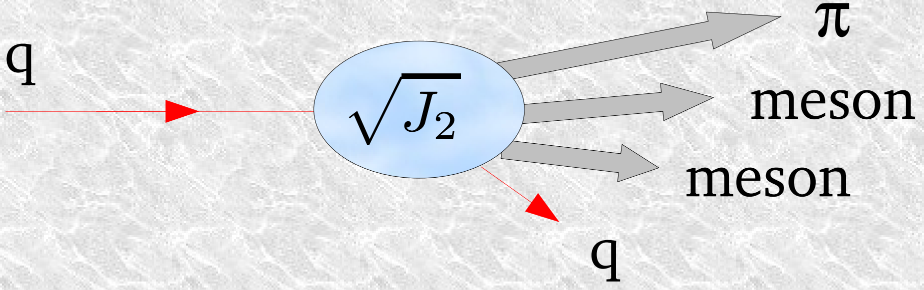


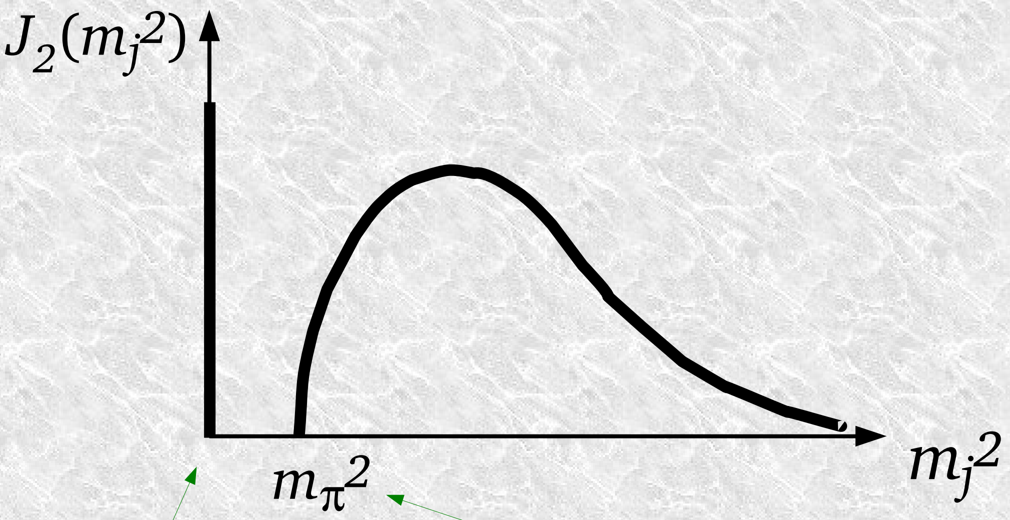
$$= \sum_n (2\pi)^4 \delta^{(4)}(l - \textstyle\sum_1^n p_i^h) \left| \quad \right|^2$$



$$= \int_0^\infty dm_j^2 \Big[J_1(m_j^2)\hat{1} + J_2(m_j^2) \not{l} \Big] 2\pi\delta(l^2 - m_j^2)\theta(l^0)$$

$$\boxed{j_2(l) = \int_0^\infty dm_j^2 J_2(m_j^2) 2\pi\delta(l^2 - m_j^2)\theta(l^0) \quad \text{with} \quad \int_0^\infty dm_j^2 J_2(m_j^2) = 1}$$





- But color must be neutralized...

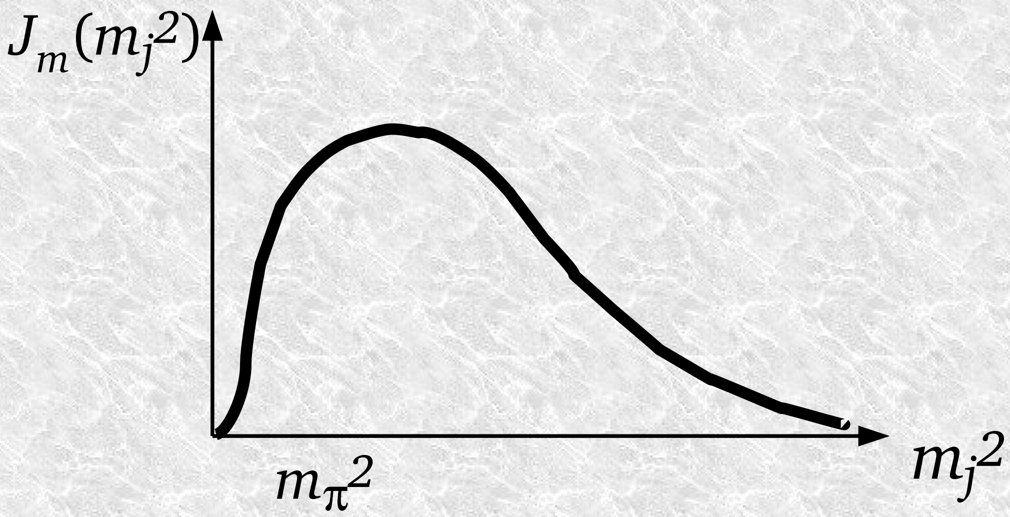
single quark state:
$(Z\times \longrightarrow)$

Continuum:
at least 1 π meson
and 1 parton

Jet spectral representation - 2

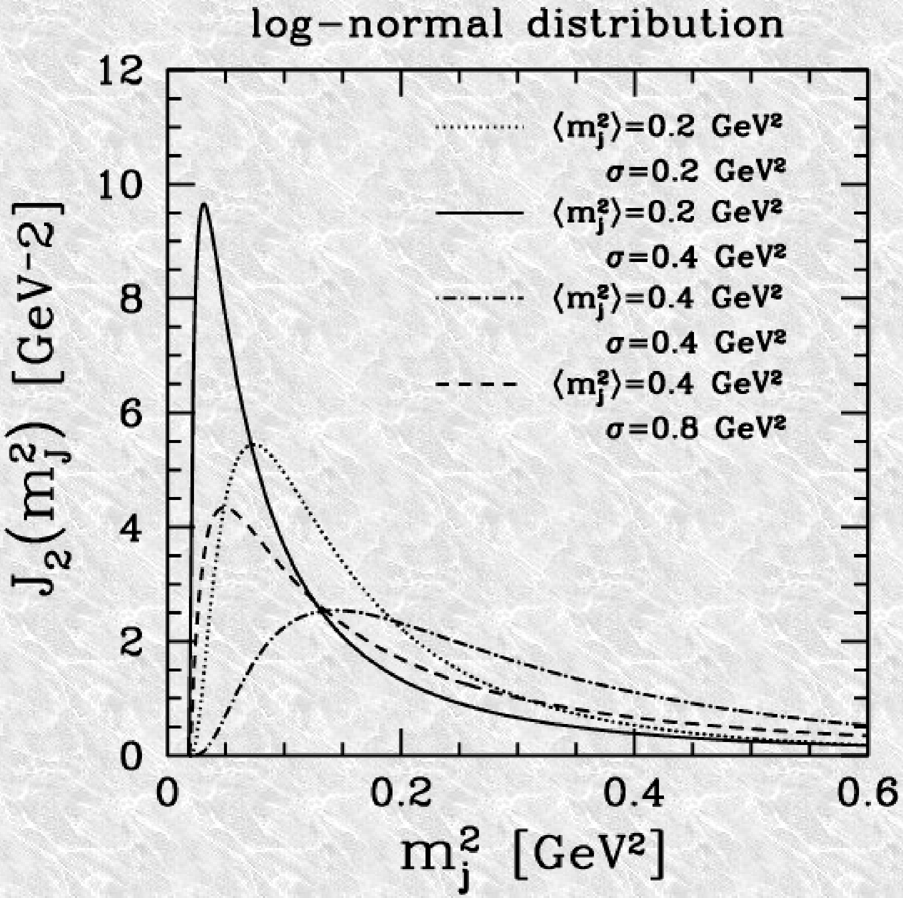
- Assume color neutralization through a soft exchange with the target jet
  - goes beyond the handbag diagram considered
  - would need generalization to fully unintegrated PDFs
    [Collins, Rogers, Stasto PRD '07]

- Phenomenologically:
  - A soft momentum exchange is going to smear out the jet function J_2
  - The smeared jet function J_m is smooth in m_j^2:



Estimate of Jet Mass Corrections

- **Toy jet function**
  - log-normal distribution
  - $<m_j^2> = 0.2 – 0.4\ \mathrm{GeV}^2$
  - $\sigma = 1\text{-}2 <m_j^2>$



Estimate of Jet Mass Corrections

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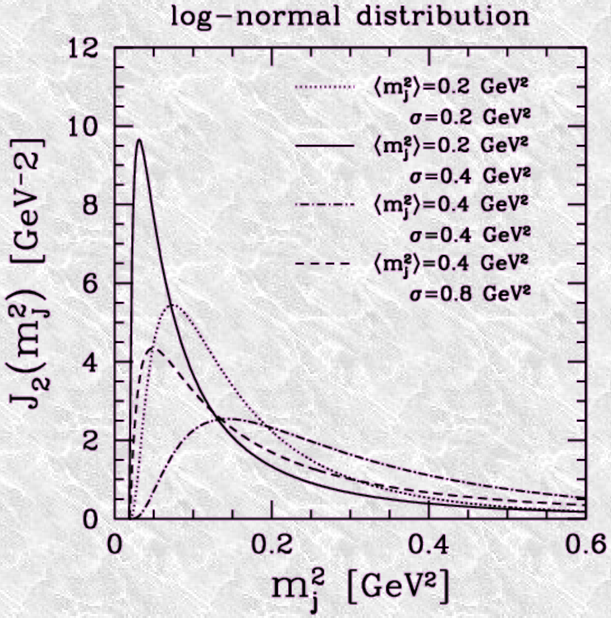

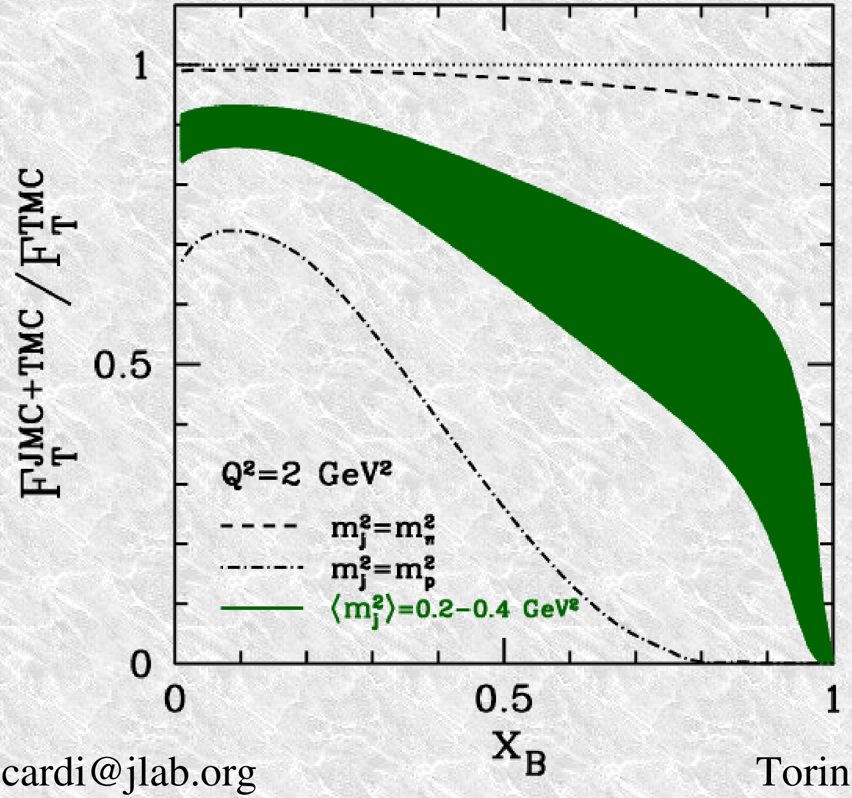

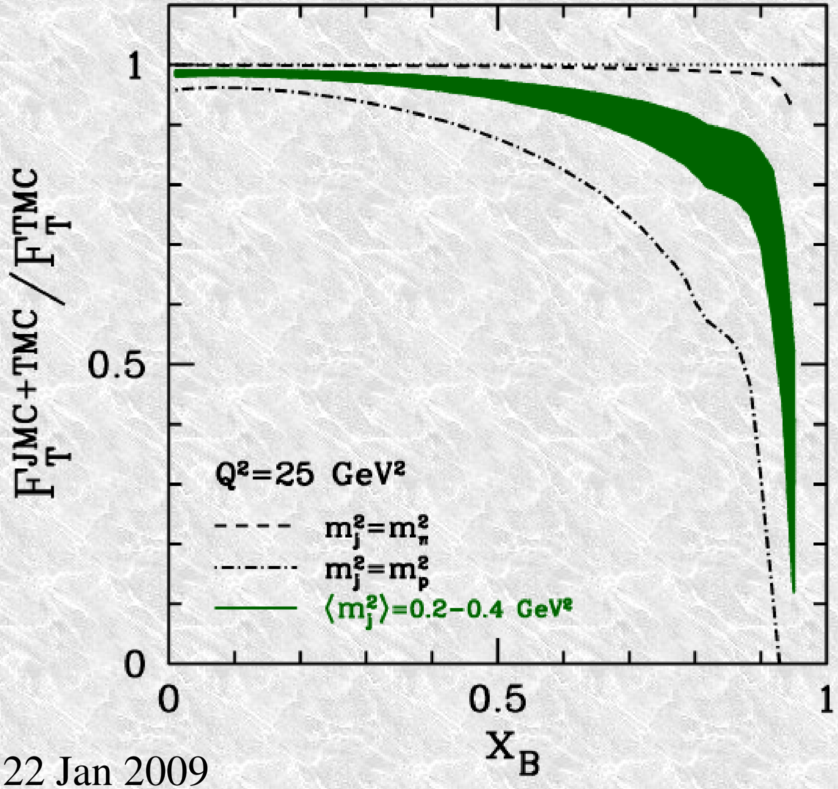

Jet function phenomenology

- We need to develop a “phenomenology” of the jet function:
  - from lattice QCD?
  - from Dyson-Schwinger equations?
  - from $e^+e^- \to$ jets?
  - from Monte Carlo simulations?
  - …

- Should we ultimately regard it only as a phenomenological tool?
  - fit it to DIS data, in the spirit of “global QCD fits”

- Can we compare the fitted $J_m \approx J_2$ to lattice QCD computations ??

$$\int_0^\infty dm_j^2\, J_2(m_j^2)\, 2\pi\delta(l^2 - m_j^2)\,\theta(l^0) = \frac{1}{4l^-}\int d^4z e^{iz\cdot l}\mathrm{Tr}\left[\gamma^-\langle 0|\overline{\psi}(z)\psi(0)|0\rangle\right]$$

  - Landau gauge vs. light-cone gauge
  - Euclidean vs. Minkowski space

Conclusions - 1

- **Collinearly factorized DIS with Target and Jet Mass Corrections**
  - respects $x_B \leq 1$, goes smoothly to 0
  - avoids threshold problem present in OPE formalism (Georgi-Politzer)
  - generalizable to other processes, and nuclear targets
  - fully consistent with CTEQ / MRST global analysis

- **TMC derived at all orders**
  - polarized & unpolarized structure functions, asymmetries
  - numerical differences from OPE corrections, **large for F_L!**

- **JMC rigorously derived only at LO**
  - Clear physical picture
  - For unpolarized str. functions, as yet
  - Need to develop jet fn. phenomenology

Conclusions - 2

- **A new series of global PDF fits is underway ~ CTEQ6.1 + TMC + HT**
  - Expanded kinematic range and enlarged data set
  - Preliminary indications suggest increased *d/u* ratio at large *x*
  - Other analyses and data sets also suggest the need for increased *d/u*

- **Eventual goal:** see if the PDF errors can be reduced using new JLab data

- **Outlook:**
  - TMC (and hadron mass) for SIDIS [w/ Hobbs, Melnitchouck], DY and p+p
  - Large-*x* resummation [w/ Liuti]
  - Effect of Jet Mass Corrections [Accardi,Qiu '08]
    ⇒ new theory, phenomenology, connections to lattice QCD (?), ...
  - Parton-hadron duality – further reduce kinematic cuts

  - Polarized QCD fits?
  - TMDs?

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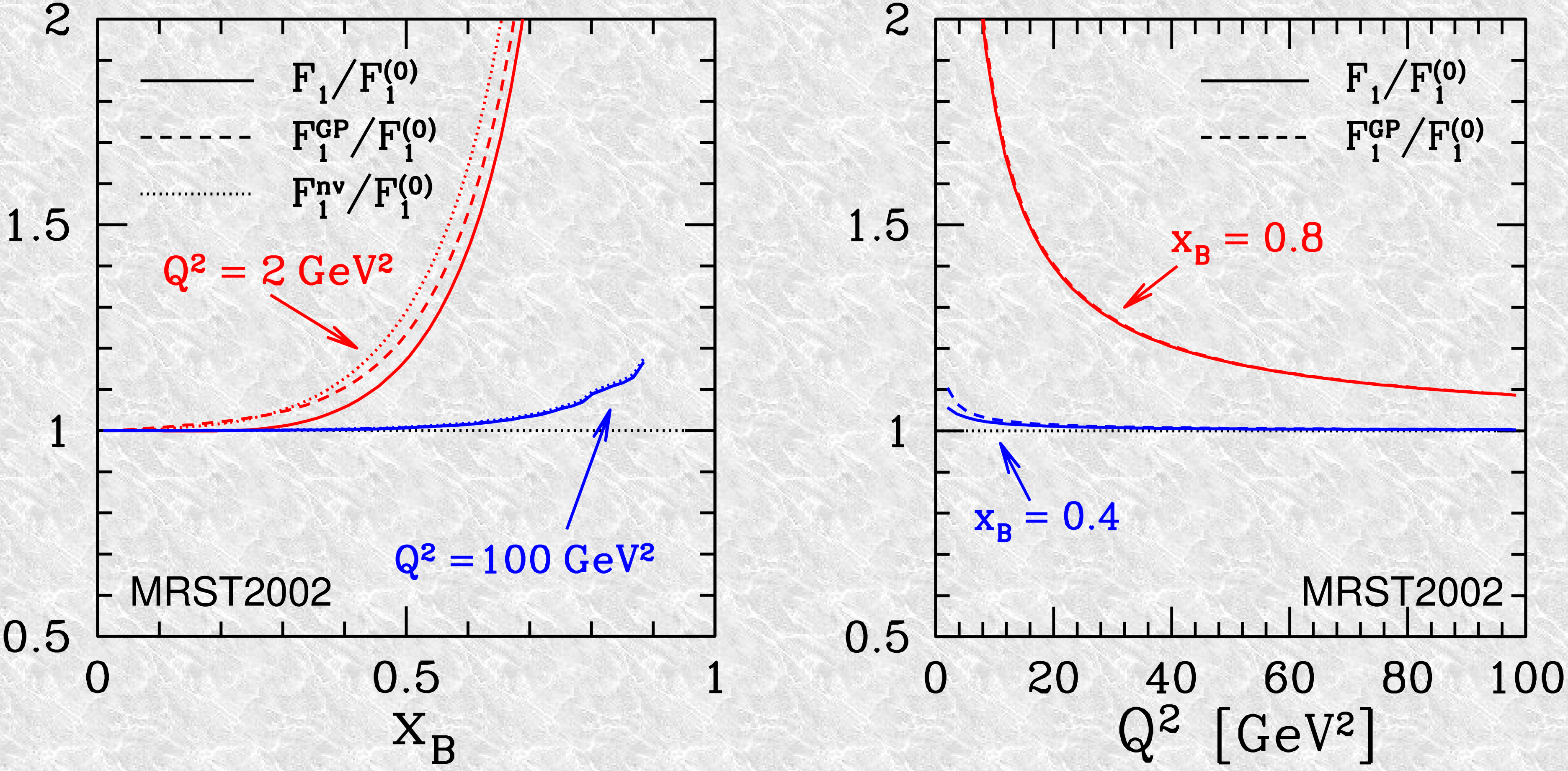
The end

App. A – F1 and GP

Target mass corrections – F_1 at NLO



$$F_1^{nv}(x_B) = F_1^{(0)}(\xi)$$

Target Mass Corrections in OPE formalism

- For unpolarized structure functions,
  [Georgi, Politzer 1976; see review of Schienbein et al. 2007]

$$F_1^{GP}(x_B,Q^2) = \frac{x_B}{\rho_B}\Big[\frac{F_1^{(0)}(\xi,Q^2)}{\xi} + \frac{m_N^2 x_B}{Q^2\rho_B}\Delta_2(x_B,Q^2)\Big]$$

$$F_2^{GP}(x_B,Q^2) = \frac{x_B^2}{\rho_B^3}\Big[\frac{F_2^{(0)}(\xi,Q^2)}{\xi^2} + 6\frac{m_N^2 x_B}{Q^2\rho_B}\Delta_2(x_B,Q^2)\Big]$$

$$F_L^{GP}(x_B,Q^2) = \frac{x_B}{\rho_B}\Big[\frac{F_L^{(0)}(\xi,Q^2)}{\xi} + 2\frac{m_N^2 x_B}{Q^2\rho_B}\Delta_2(x_B,Q^2)\Big]$$

where

$$\xi = \frac{2x_B}{\rho_B^2} \qquad \rho_B^2 = 1 + 4x_B^2 m_N^2/Q^2$$

$$\Delta_2(x_B,Q^2) = \int_\xi^1 dv\Big[1 + 2\frac{m_N^2 x_B}{Q^2\rho_B}(v-\xi)\Big]\frac{F_2^{(0)}(v,Q^2)}{v^2}$$

and, in my conventions,

$$F_L(x_B,Q^2) = \frac{\rho_B^2}{2x_B}F_2(x_B,Q^2) - F_1(x_B,Q^2)$$

Target Mass Corrections in OPE formalism

- For polarized structure functions, [Bluemlein, Tvabkladze, . 2007]

$$g_1^{\mathrm{OPE}}(x_B) = \frac{1}{(1+\gamma^2)^{3/2}} \frac{x_B}{\xi} g_1^{(0)}(\xi)$$

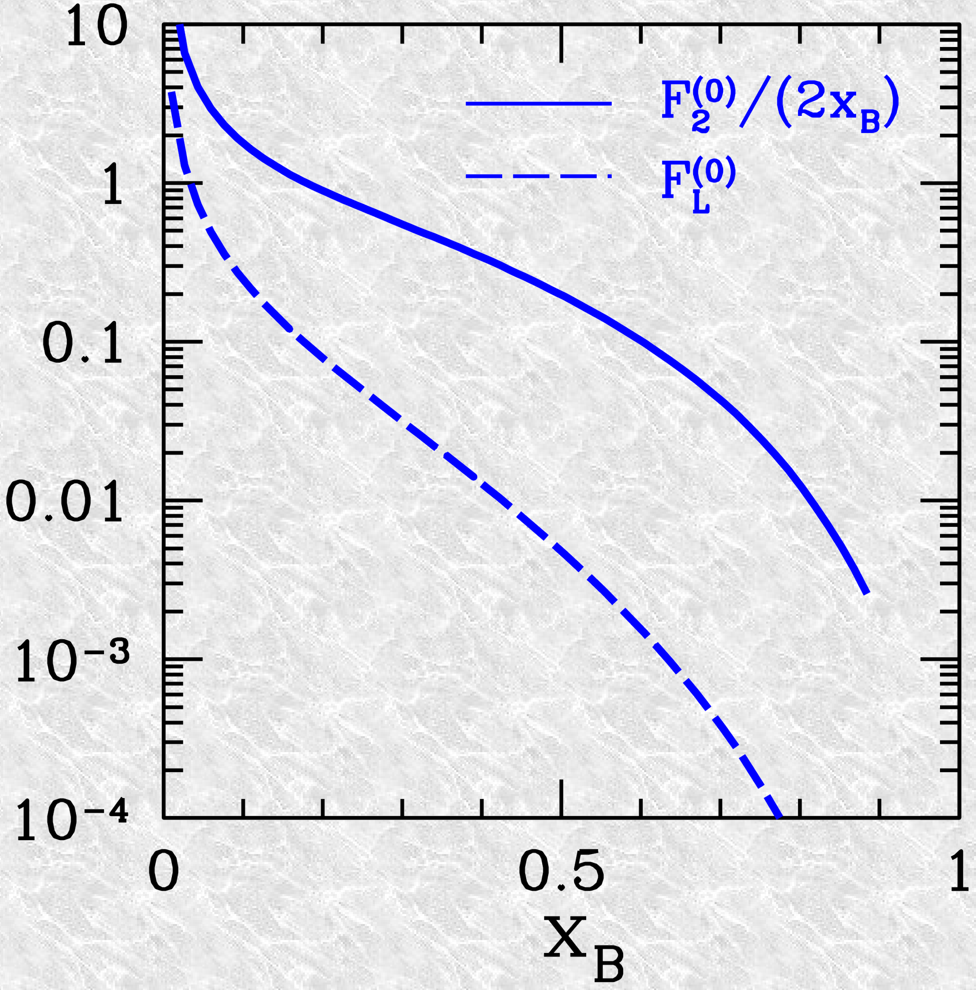
$$+ \frac{\gamma^2}{(1+\gamma^2)^2} \int_\xi^1 \frac{dv}{v} \Big[\frac{x_B+\xi}{\xi} + \frac{\gamma^2-2}{2\sqrt{1+\gamma^2}} \log\left(\frac{v}{\xi}\right) \Big] g_1^{(0)}(v)$$

$$g_2^{\mathrm{OPE}}(x_B) = -g_1^{\mathrm{OPE}}(x_B) \; + \int_{x_B}^1 \frac{dy}{y} g_1^{\mathrm{OPE}}(y)$$

$$A_1^{\mathrm{OPE}}(x_B) = \frac{(1+\gamma^2)}{F_1^{\mathrm{OPE}}(x_B)} \left[g_1^{\mathrm{OPE}}(x_B) - \gamma^2 \int_{x_B}^1 \frac{dy}{y} g_1^{\mathrm{OPE}}(y) \right]$$

Target Mass Corrections in OPE formalism

- Why is the GP corrected FL so large??



$$F_L^{GP}(x_B)$$

$$= \frac{x_B}{\rho_B}\Big[\frac{F_L^{(0)}(\xi)}{\xi} + 2\frac{m_N^2 x_B}{Q^2\rho_B}\Delta_2(x_B)\Big]$$

$$\Delta_2(x_B)$$

$$= \int_\xi^1 dv\big[1 + 2\tfrac{m_N^2 x_B}{Q^2\rho_B}(v-\xi)\big]\frac{F_2^{(0)}(v)}{v^2}$$

App. B

collinear fact. and Jet function

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Factorization procedure

[see Ellis, Furmanski, Petronzio, 1983]

- Expand on a basis of Dirac matrices

$$\hat{T}(k) = \tau_1(k)\hat{1} + \tau_2(k)\not{k} + \tau_3(k)\gamma_5 + \tau_4(k)\not{k}\gamma_5$$

contributes to higher twists

= 0 (T-odd)

cancels for unpolarized targets

$$\hat{J}(l) = j_1(k)\hat{1} + j_2(l)\not{l} + j_3(l)\gamma_5 + j_4(l)\not{l}\gamma_5$$

enters traces with
odd no. of γ 's

= 0 (T-odd)

cancels (quark spin is unobserved)

- Dominance of k^+, l^- in Breit frame suggests to define

$$\tau_2(k) = \frac{1}{4k^+}\mathrm{Tr}\left[\not{n}\hat{T}(k)\right] = \frac{1}{4k^+}\int d^4z\, e^{iz\cdot k}\langle p|\overline{\psi}_j(z)\gamma^+\psi(0)|p\rangle$$

$$j_2(l) = \frac{1}{4l^-}\mathrm{Tr}\left[\not{\bar{n}}\hat{J}(l)\right] = \frac{1}{4l^-}\int d^4z\, e^{iz\cdot l}\langle 0|\overline{\psi}_j(z)\gamma^-\psi(0)|0\rangle$$

Collinear expansion - 1

$$W^{\mu\nu}(p,q) = \int \frac{d^4k}{(2\pi)^4} \underbrace{\frac{e_q^2}{8\pi}\mathrm{Tr}\left[\not{k}\gamma^\nu \not{l}\gamma^\mu\right]}_{=\frac{1}{\pi}H_*^{\mu\nu}(k,l)} j_2(l)\, \tau_2(k)\, \mathbb{K}(k,p,q)$$

kinematic constraints

$$k^\mu = xp^+\overline{n}^\mu + \frac{k^2+k_T^2}{2xp^+}n^\mu + k_T^\mu$$

$$l^\mu = (x-\xi)p^+\overline{n}^\mu + \left(\frac{k^2+k_T^2}{2xp^+} + \frac{Q^2}{2\xi p^+}\right)n^\mu + k_T^\mu$$

1) **Expand *H_*(k,l)*** around $\widetilde{k} \equiv xp^+\overline{n}^\mu$ $\quad [\widetilde{l} \equiv \widetilde{k} + q]$

$$H_*^{\mu\nu}(k,l) = H_*^{\mu\nu}(\widetilde{k},\widetilde{l}) + \frac{\partial H_*^{\mu\nu}}{\partial k^\alpha}(k^\alpha - \widetilde{k}^\alpha) + \ldots$$

leading twist

contributes to Higher Twists [Qiu '90]

NOTE:

- up to now no approximations
- especially, I did not approximate the final state kinematic

Collinear expansion - 2

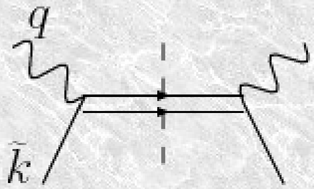
2) Use spectral representation

3) Assume k^-, $k_T \ll (x/\xi)Q^2$ ⇒ $j_2(l) \approx \int_0^\infty dm_j^2\, J_2(m_J^2)\, 2\pi\delta(\tilde{l}^2 - m_J^2)\, \theta(l^0)$

$$W^{\mu\nu}(p,q) = \int_0^\infty dm_j^2\, J_2(m_J^2) \int \frac{d^4k}{(2\pi)^4} H_*^{\mu\nu}(\tilde{k},\tilde{l})\, \delta(\tilde{l}^2 - m_J^2)\, 2\tau_2(k)\, \mathbb{K}(k,p,q)$$

unapproximated!

"fat quark" line:



NOTE:

- Involves a shift in the final state momentum l – **evil !! see [CRS]** but $J_2(m_J^2)$ is unapproximated (improvement over $m_J^2=0$ case)
- OK if $\int d^4l$ dominated by l such that $j_2(l)$ has small slope.

  In terms of the spectral representation we need,

$$\frac{1-x_B}{x_B}Q^2 \gtrsim m_J^2|_{\text{peak}}$$

Collinear expansion - 3

4) Ignore kinematic limits on k^-, k_T : $\mathbb{K}(k,p,q) \approx \mathbb{K}(\tilde{k},p,q)$

$$W^{\mu\nu}(p,q) = \int_0^\infty dm_j^2\, J_2(m_J^2) \int \frac{dx}{x}\, H_*^{\mu\nu}(\tilde{k},\tilde{l})\, \delta(\tilde{l}^2 - m_J^2)\, \varphi_q(x)\, \mathbb{K}(\tilde{k},p,q)$$

where $\varphi_q(x) = \int \frac{dz^-}{2\pi} e^{iz^-k^+} \langle p|\bar{\psi}(z^-n)\frac{\gamma\cdot\bar{n}}{2}\psi(0)|p\rangle$

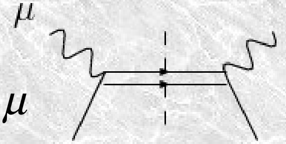
- needed to define collinear PDF
- does not respect 4-momentum conservation – **evil !!** – e.g.,

$$s = (p_J + p_Y)^2 \geq 4k_T^2 \qquad \Longrightarrow \qquad 4k_T^2 \leq \frac{1-\xi}{\xi} Q^2 \left(1 + \xi \frac{m_N^2}{Q^2}\right)$$

5) Set m_J^2=0 inside $H_*(\tilde{k},\tilde{l})$ [CRS]

$$H_*^{\mu\nu}(\tilde{k},\tilde{l}) \approx H_*^{\mu\nu}(\tilde{k},\hat{l}) \quad \text{with } \hat{l}^\mu = \frac{Q^2}{2\xi p^+} n^\mu$$

Needed to:

- respect gauge invariance (otherwise q_μ  $\neq 0$)
- use Ward ids in proof of factorization
- not so evil: does not touch the final state kinematic

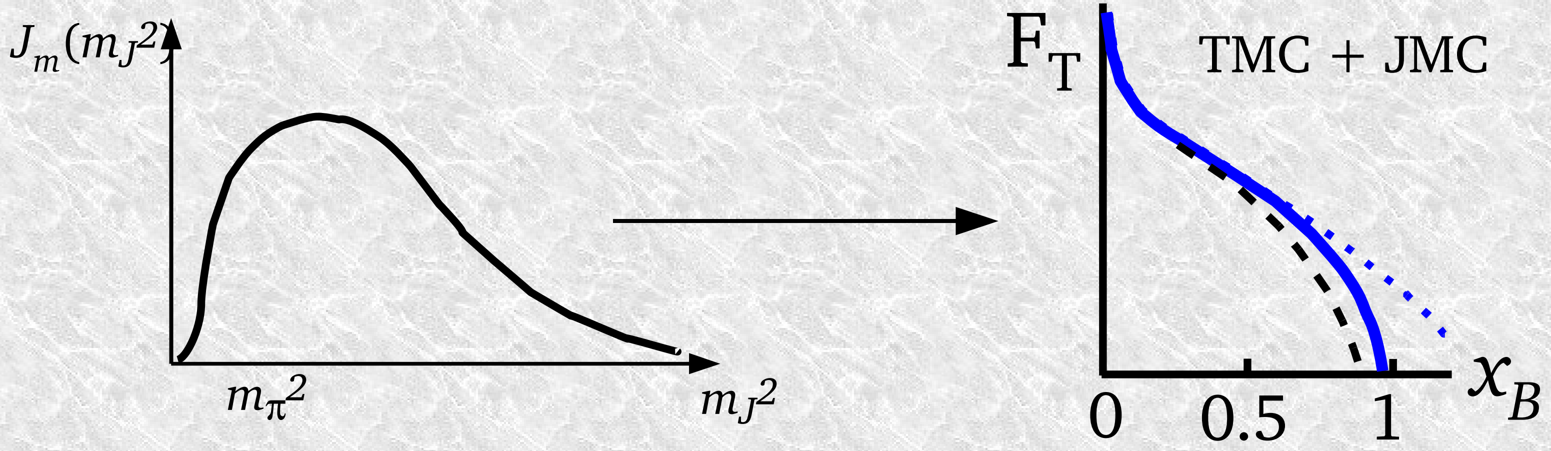
Finally, as promised...

- Collinearly factorized DIS at LO with Target and Jet Mass Corrections
  - respects $x_B \leq 1$, goes smoothly to 0:

$$= \frac{\xi}{Q^2}\delta\left(x - \xi\left(1 + \frac{m_J^2}{Q^2}\right)\right)$$

$$W^{\mu\nu}(p,q) = \int_0^\infty dm_J^2\, J_m(m_J^2) \int_\xi^{\frac{\xi}{x_B}} \frac{dx}{x} \underbrace{\frac{1}{8\pi}\frac{e_q^2}{2}\mathrm{Tr}(\tilde{\not k}\gamma^\nu \hat{\not l}\gamma^\mu) 2\pi\, \delta(\tilde{l}^2 - m_J^2)}_{\mathcal{H}^{\mu\nu}}\, \varphi_q(x)$$

$$F_T(x_B, Q^2) = \int_0^{\frac{1-x_B}{x_B}Q^2} dm_J^2\, J_m(m_j^2)\, F_T^{(0)}\left(\xi\left(1 + \frac{m_J^2}{Q^2}\right), Q^2\right)$$

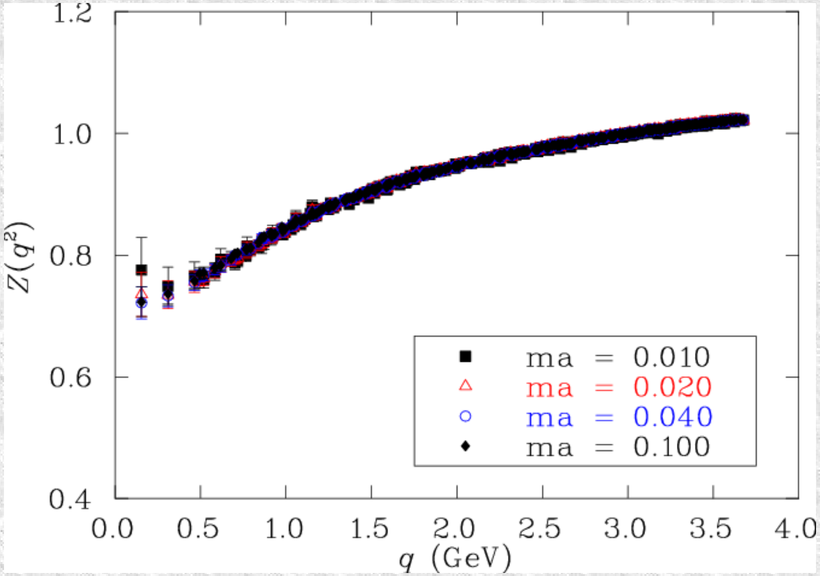


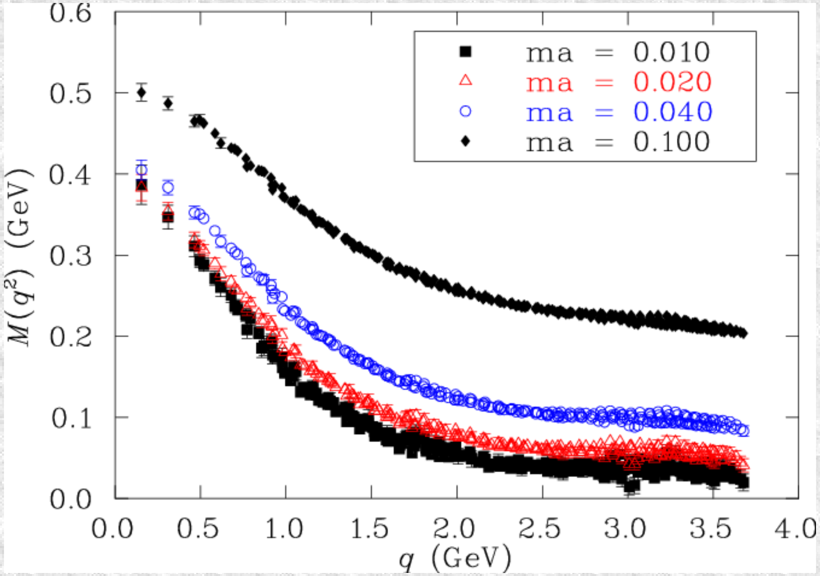
Jet function and lattice QCD

$$\int_0^\infty dm_j^2\, J_2(m_j^2)\, 2\pi\delta(l^2 - m_j^2)\, \theta(l^0) = \frac{1}{4l^-} \int d^4z e^{iz\cdot l} \mathrm{Tr}\left[\gamma^- \langle 0|\overline{\psi}(z)\psi(0)|0\rangle\right]$$

- Quark propagator in lattice QCD [e.g., Bowman et al. '05]

$$\int d^4z e^{iz\cdot q} \langle 0|\overline{\psi}(z)\psi(0)|0\rangle = \frac{Z(q^2)}{i\gamma\cdot q + M(q^2)}$$





- but:
  1) Landau gauge vs. light-cone gauge
  2) Euclidean vs. Minkowski space

Where can we trust the approximations?

- Neglect of integration limits on k$_T$ is OK if

$$\langle k_T^2 \rangle \ll \frac{1-\xi}{4\xi} Q^2 (1 + \xi \frac{m_N^2}{Q^2}) \equiv k_T^2|_{\max}$$

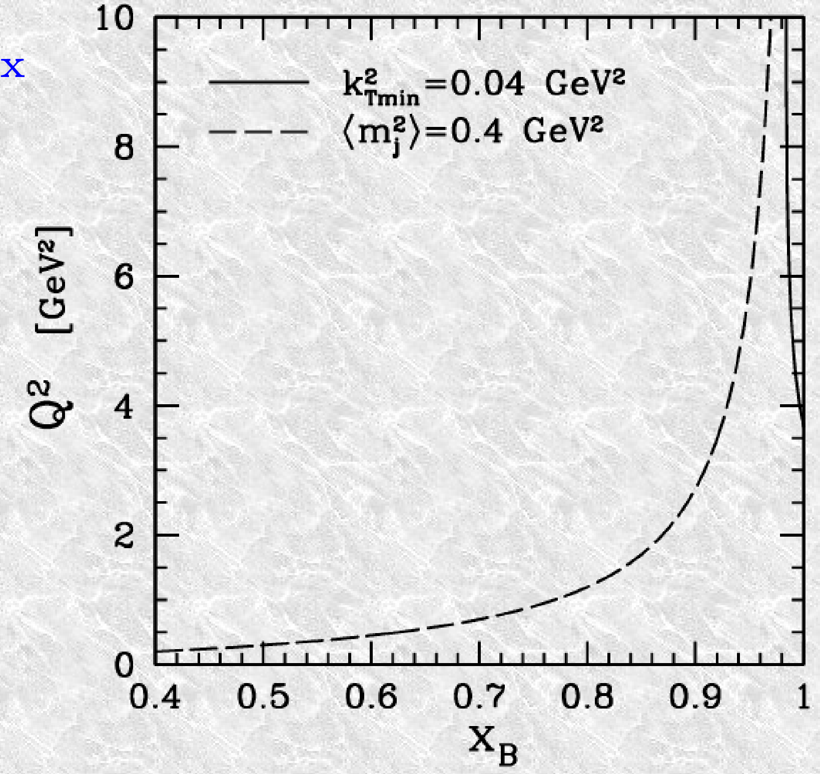
where

$$\langle k_T^2 \rangle \approx k_T^2|_{\rm intr.} \left[1 + \alpha_s \log \left(\frac{Q^2}{k_T^2|_{\rm intr.}} \right) \right]$$

⇒ solid line

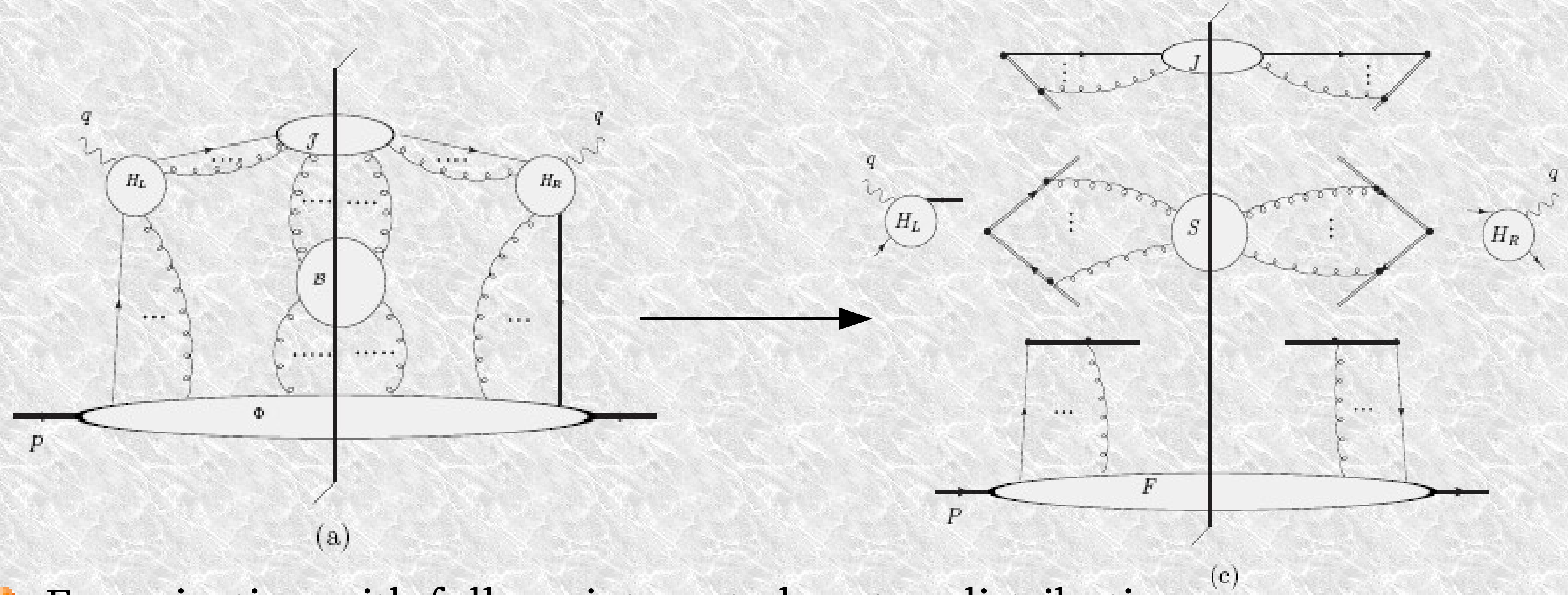
- Replacement $l^\mu \longrightarrow \hat{l}^\mu$ is OK if

$$Q^2_{\max} = (\frac{1}{x_B} - 1) Q^2 \gtrsim \langle m_J^2 \rangle$$



“Proof” of collinear factorization - 1

- Generalized handbag diagram with a quark jet [Collins, Rogers, Stasto, 2007]



- Factorization with fully unintegrated parton distributions (for an abelian theory of massive gluons – QCD to come soon) [CRS]

$$P_{\mu\nu}W^{\mu\nu} = \int \frac{d^4k_{\rm T}}{(2\pi)^4}\frac{d^4k_{\rm J}}{(2\pi)^4}\frac{d^4k_S}{(2\pi)^4}(2\pi)^4\delta^{(4)}(q+P-k_{\rm T}-k_{\rm J}-k_S)\times$$

$$\times\,|H(Q,\mu)|^2\; S_2(k_S,y_s,\mu)\; F(k_{\rm T},y_p,y_s,\mu)\; J(k_{\rm J},y_s,\mu)$$

soft PCF target PCF jet PCF

"Proof" of collinear factorization - 2

- Start from

$$P_{\mu\nu}W^{\mu\nu} = \int \frac{d^4k_\mathrm{T}}{(2\pi)^4}\frac{d^4k_\mathrm{J}}{(2\pi)^4}\frac{d^4k_S}{(2\pi)^4}(2\pi)^4\delta^{(4)}(q+P-k_\mathrm{T}-k_\mathrm{J}-k_S)\times$$

$$\times\,|H(Q,\mu)|^2\; S_2(k_S,y_s,\mu)\; F(k_\mathrm{T},y_p,y_s,\mu)\; J(k_\mathrm{J},y_s,\mu).$$

$$\tilde F(w,y_p,y_s,\mu) \;=\; \langle p|\bar\psi(w)V_w^\dagger(n_s)I_{n_s;w,0}\frac{\gamma^+}{2}V_0(n_s)\psi(0)|p\rangle.$$

$$J(k_J,y_s\,m) = \langle 0|\bar\psi(w)V_w^\dagger(-n_s)I_{-n_s;w,0}\gamma^-V_0(-n_s)\psi(0)|0\rangle$$

$$V_w(n) = P\exp\left(-ig\int_0^\infty d\lambda\, n\cdot A(w+\lambda n)\right)$$

- neglect soft jet-target interactions, use $P - k_T = k$, $k_J = l$
- the hard function H is the same as our $h_{T,L,\ldots}$
- integrate out k_J , use spectral represantation for $J(k_J)$
- expand H , repeat approximations 3, 4
- use $n_s{\cdot}A=0$ gauge