A new formalism for target-mass corrections

Alberto Accardi
Jlab & Hampton U.

Hall C meeting
18 January 2008

Work in progress with: J.W. Qiu
W. Melnitchouk, T. Keppel, E. Christy

Why large x_B and low Q^2 ?

- Large uncertainties in quark and gluon PDF at $x > 0.5$

- Precise PDF at large x are needed, e.g.,
  - at LHC, Tevatron
    1) New physics as excess in large p_T spectra $\Leftrightarrow$ large x PDF
    2) DGLAP evolution feeds large x, low Q^2 into lower x, large Q^2
  - d/u ratio at $x=1$ $\Leftrightarrow$ non-perturbative structure of the nucleon

- JLAB has precision DIS data at large x_B , BUT low Q^2
  - need of theoretical control over
    1) higher twist $\propto \Lambda^2/Q^2$
    2) target mass corrections (TMC) $\propto x_B^2\, m_N^2/Q^2$
    3) jet mass corrections (JMC) $\propto m_J^2/Q^2$

$$\left.\begin{array}{l}\text{2) TMC}\\ \text{3) JMC}\end{array}\right\}\ \textbf{this talk}$$

OPE and Target Mass Corrections

[Georgi, Politzer 1976; see review of Schienbein et al. 2007]

$$\int d^4z\, e^{-iq\cdot z} \langle N|T[j^{\dagger\mu}(z) j^\nu(0)]|N\rangle = \sum_k f^{\mu_1\ldots\mu_{2k}} A_{2k} \langle N|\underbrace{\mathcal{O}_{\mu_1\ldots\mu_{2k}}(0)}_{\text{symmetric, traceless}}|N\rangle$$

$$A_{2k} = \int_0^1 dy\, y^{2k} F(y) \qquad F(y) \sim \sum_q e_q^2 q(y) \ \text{(at LO)} \ = \text{“quark function”}$$

- Mellin transform, resum, transform back:

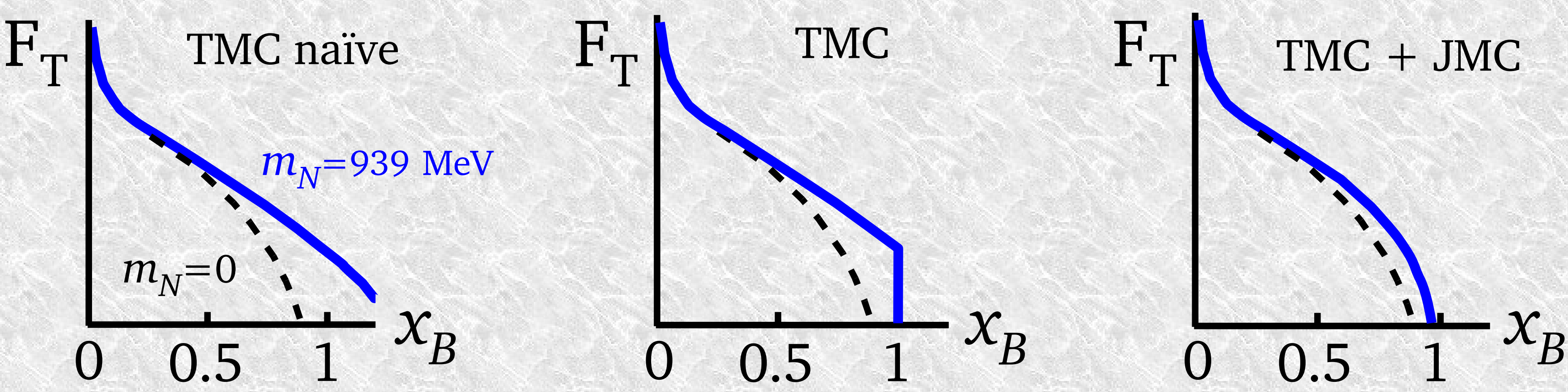
$$F_2^{GP}(x_B, Q^2) = \frac{x_B^2}{\rho_B^3} F(\xi) + 6\frac{m_N^2}{Q^2}\frac{x_B^3}{\rho_B^4}\int_\xi^1 d\xi' F(\xi') + 12\frac{m_N^4}{Q^4}\frac{x^4}{\rho_B^5}\int_\xi^1 d\xi' \int_{\xi'}^1 d\xi'' F(\xi'')$$

$$\xi = \frac{2x_B}{1+\sqrt{1+4x_B^2 m_N^2/Q^2}} = \frac{2x_B}{1+\rho_B} \quad \text{Nachtmann variable}$$

- <u>Threshold problem</u>: $x_B \leq 1$ implies $0 \leq \xi \leq \xi_{\text{th}} \stackrel{\text{def}}{=} \xi(x_B{=}1)$
  - Inverse Mellin transform does not give back F(y) !! [Johnson, Tung 1979]
- <u>Unphysical region</u>: $F(y) \sim F_2(y)$ has support over $0<y<1$
  - $F_2^{GP}(x_B) > 0$ also for $x_B>1$!!

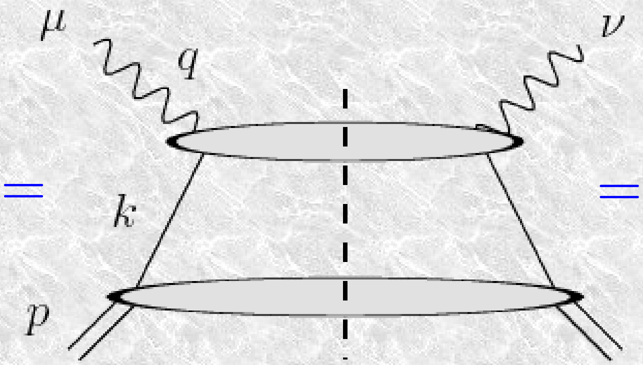
Collinear factorization - outline

- **Target Mass Corrections – $O(x_B^2\, m_N^2/Q^2)$**
  - momentum space, no need of Mellin transf.
  - kinematics of handbag diagram
    $\Rightarrow$ no "unphysical region" at $x_B > 1$ (!!)
  - any order in α_s at leading twist
- **Jet Mass Corrections – $O(m_J^2/Q^2)$**
  - leading order in α_s, leading twist
- **Conclusion:**
  - **factorized formula with TMC + JMC**



Kinematics with $m_N \neq 0$

$$W^{\mu\nu}(p,q) =$$



$$= \frac{1}{8\pi}\int d^4z\, e^{-iq\cdot z}\langle p|j^{\dagger\mu}(z)j^{\nu}(0)|p\rangle$$

- Collinear frames: [Aivazis et al 94]

$$p^\mu = p^+\overline{n}^\mu + \frac{m_N^2}{2p^+}n^\mu$$

$$q^\mu = -\xi p^+\overline{n}^\mu + \frac{Q^2}{2\xi p^+}n^\mu$$

$$k^\mu = xp^+\overline{n}^\mu + \frac{k^2+k_T^2}{2xp^+}n^\mu + k_T^\mu$$

where:

$$x = \frac{k^+}{p^+} \qquad \xi = \frac{2x_B}{1+\sqrt{1+4x_B^2m_N^2/Q^2}}$$

Lorentz invariants:

$$x_B = \frac{-q^2}{2p\cdot q} \qquad Q^2 = -q^2$$

$$x_f = \frac{-q^2}{2k\cdot q} \qquad m_N^2 = p^2$$

Light cone vectors:

$$\overline{\boldsymbol{n}} = (1/\sqrt{2}, \vec{0}_\perp, 1/\sqrt{2})$$

$$n = (1/\sqrt{2}, \vec{0}_\perp, -1/\sqrt{2})$$

$$a^\pm = (a_0 \pm a_3)/\sqrt{2}$$

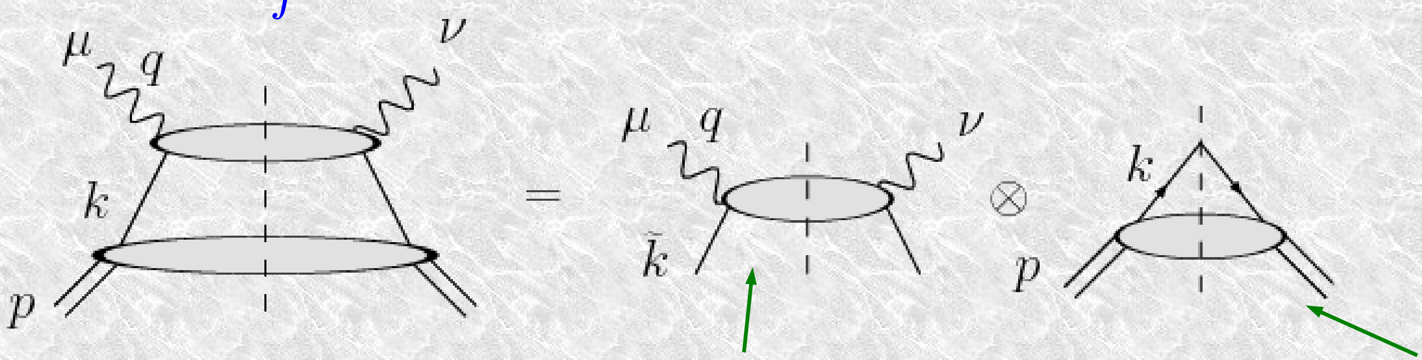
- Bjorken limit: $\xi \to x_B$ recovers the massless ($m_N{}^2=0$) kinematics

Factorization theorem with $m_N \neq 0$

[see also Qiu's talk at CTEQ meeting 2005]

- Expand around $\tilde{k}^\mu = x p^+ \overline{n}^\mu$ $\qquad \tilde{k}^2 = 0$ $\qquad \tilde{x}_f = \dfrac{-q^2}{2\tilde{k}\cdot q} = \dfrac{\xi}{x}$

$$W_N^{\mu\nu}(p,q) = \sum_f \int \frac{dx}{x}\, \mathcal{H}_f^{\mu\nu}(\tilde{k},q)\, \varphi_{f/N}(x,Q^2) + O(\Lambda^2/Q^2)$$



perturbative: doesn't know
about the target's mass

dynamical TMC
only from nucleon w.f.

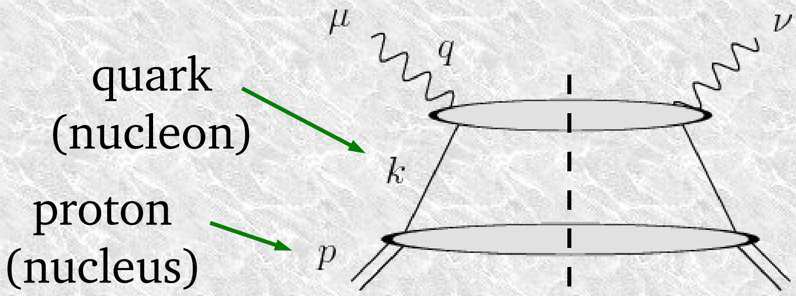
- Helicity structure functions F_T , F_L projected out of $W^{\mu\nu}$: *e.g.*,

$$F_T(x_B,Q^2) = \sum_f \int \frac{dx}{x}\, h_{fT}(\underbrace{\tilde{x}_f}_{=\,\xi\,/\,x},Q^2)\, \varphi_{f/N}(x,Q^2) + O(\Lambda^2/Q^2)$$

no kinematic prefactors [Aivazis, Olness, Tung 1994]

Kinematic constraints

- General handbag diagram – on shell gluons and light quarks ($\widetilde{k}^2 = 0$):



$$x_B \leq \widetilde{x}_f \leq 1$$

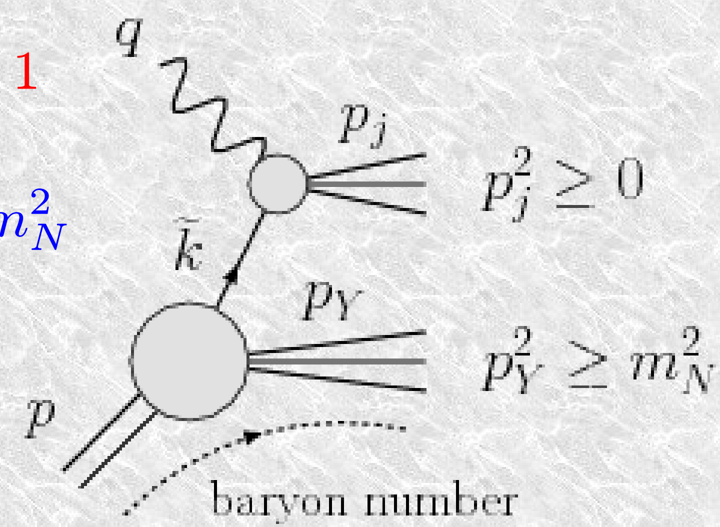
i.e., $\boxed{\xi \leq x \leq \xi/x_B}$

- Proof (can be generalized to heavy and off-shell quarks – and nuclei)

  - $0 \leq p_j^2 = (\widetilde{k} + q)^2 = Q^2\Big(\dfrac{1}{\widetilde{x}_f} - 1\Big) \implies \widetilde{x}_f \leq 1$

  - $s = (p+q)^2 = (p_j + p_Y)^2 \geq p_j^2 + p_Y^2 \geq p_j^2 + m_N^2$

$$\left.\begin{aligned} p_j^2 &= \Big(\frac{1}{\widetilde{x}_f} - 1\Big)Q^2 \\ s - m_N^2 &= \Big(\frac{1}{x_B} - 1\Big)Q^2 \end{aligned}\right\} \implies \widetilde{x}_f \geq x_B$$



- If baryon number flows in the upper blob (not the case for pQCD quarks)

$$\frac{x_B}{1 + x_B m_N^2/Q^2} \leq \widetilde{x}_f \leq \frac{1}{1 + m_N^2/Q^2}$$

No unphysical region!

- **TMC in collinear factorization:**

$$F_T(x_B,Q^2) = \sum_f \int_\xi^{\frac{\xi}{x_B}} \frac{dx}{x}\, h_{fT}\Big(\frac{\xi}{x}, Q^2\Big)\, \varphi_f(x,Q^2)$$

$$\boxed{F_T(x_B,Q^2) = 0 \quad \text{at } x_B \geq 1}$$

- Bjorken limit recovers "**massless**" structure functions (m_N=*0*)

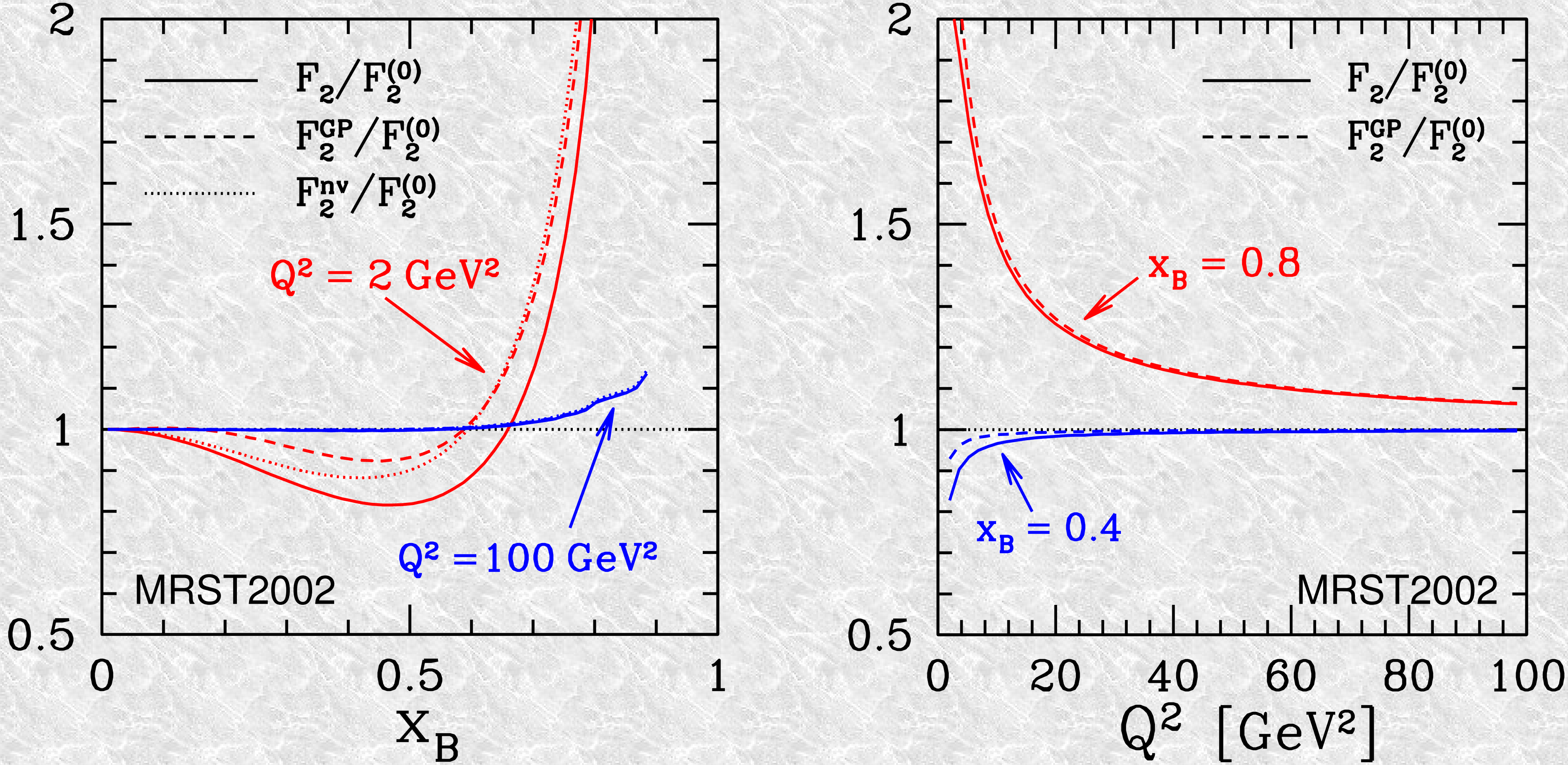
$$F_T(x_B,Q^2) \longrightarrow F_T^{(0)}(x_B,Q^2) \equiv \sum_f \int_{x_B}^{1} \frac{dx}{x}\, h_{fT}\Big(\frac{x_B}{x}, Q^2\Big)\, \varphi_f(x,Q^2)$$

- Different from the "**naive**" collinear factorization TMC [Aivazis et al '94 Kretzer,Reno '02]

$$F_T^{nv}(x_B,Q^2) \equiv F_T^{(0)}(\xi,Q^2) = \sum_f \int_\xi^{1} \frac{dx}{x}\, h_{fT}\Big(\frac{\xi}{x}, Q^2\Big)\, \varphi_{f/N}(x,Q^2)$$

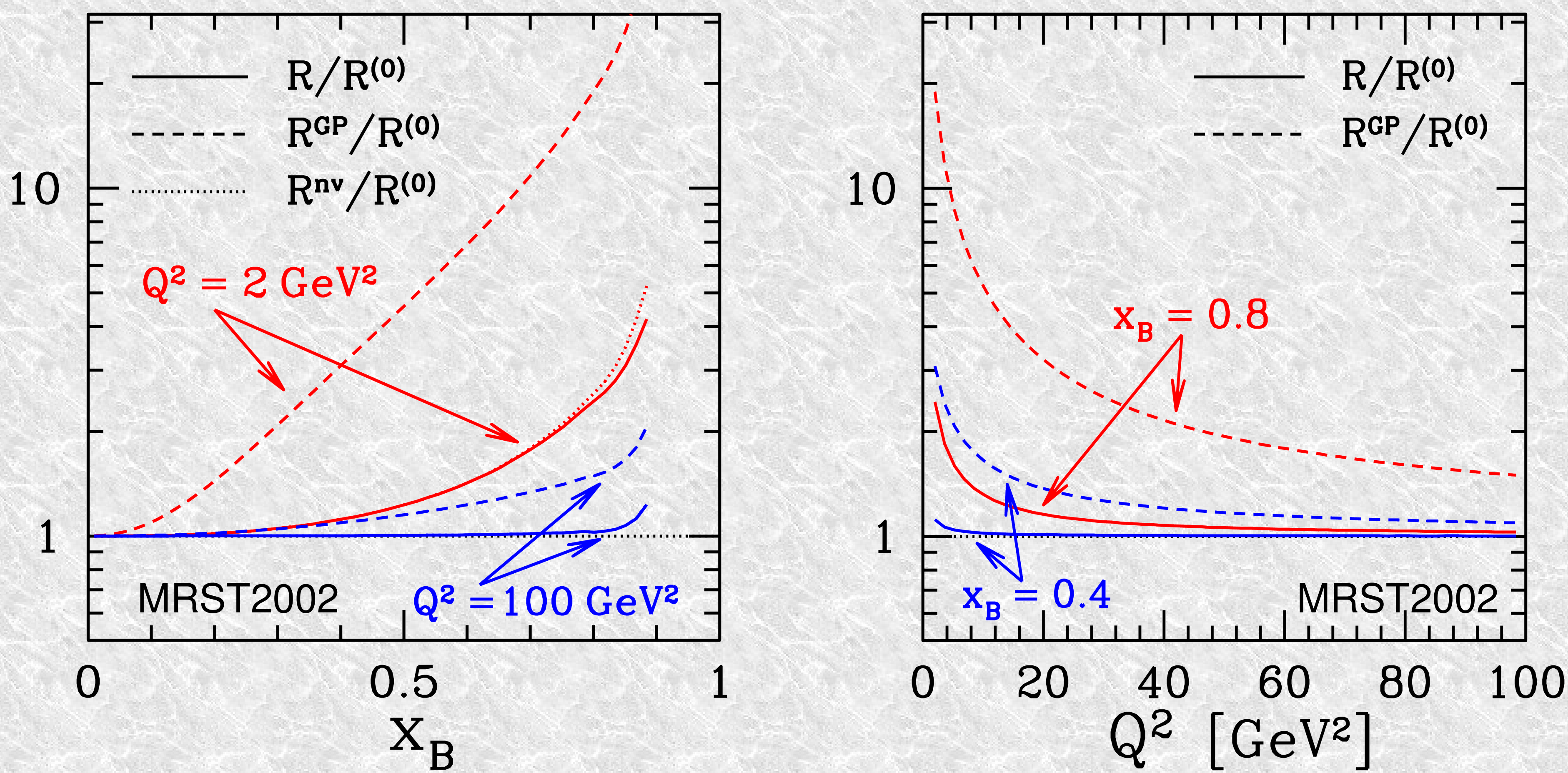
which does not vanish at $x_B > 1$

Target mass corrections – F_2 at NLO



$$F_2^{nv}(x_B) = \frac{1}{1 + 4x_B^2 \frac{m_N^2}{Q^2}} \frac{x_B}{\xi} F_2^{(0)}(\xi)$$

Target mass corrections – σ_L/σ_T at NLO



$$R = \frac{\sigma_L}{\sigma_T} = \frac{F_L}{F_1}$$

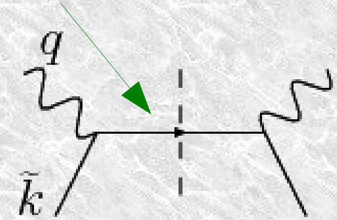
$$F_{1,L}^{nv}(x_B) = F_{1,L}^{(0)}(\xi)$$

Jet smearing at LO – 1

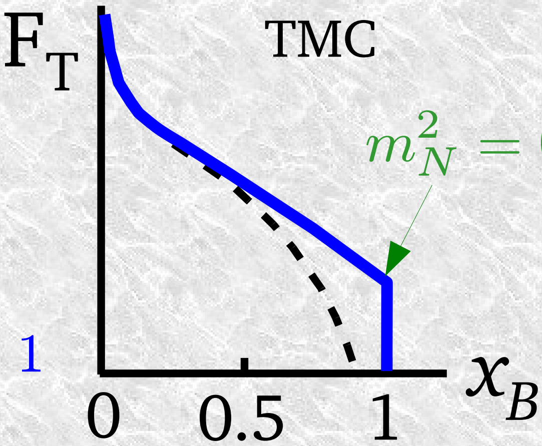
- But… at leading order,

$m_f^2 = 0$

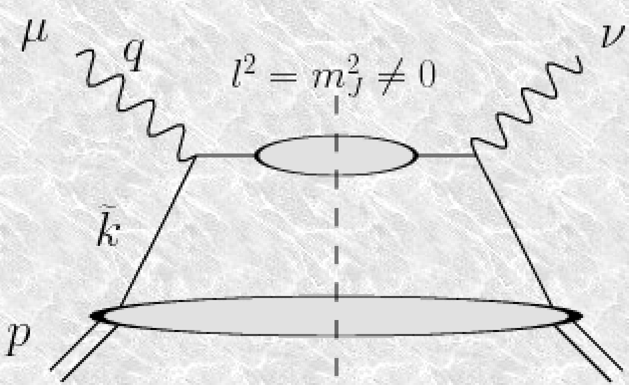
$$h_{fT}\left(\frac{\xi}{x}, Q^2\right) = \frac{1}{2} e_f^2 \delta\left(\frac{\xi}{x} - 1\right) \quad =$$



$$F_T(x_B, Q^2) = \frac{1}{2}\sum_f e_f^2 \varphi_f(\xi, Q^2) = F_T^{(0)}(\xi, Q^2) \quad \text{at } x_B < 1$$



- Ansatz: jet with a non zero mass, smoohtly distributed in m_J^2



$$(k+q)^2 = m_j^2 \longrightarrow \delta\left[x - \xi\left(1 + \frac{m_j^2}{Q^2}\right)\right]$$

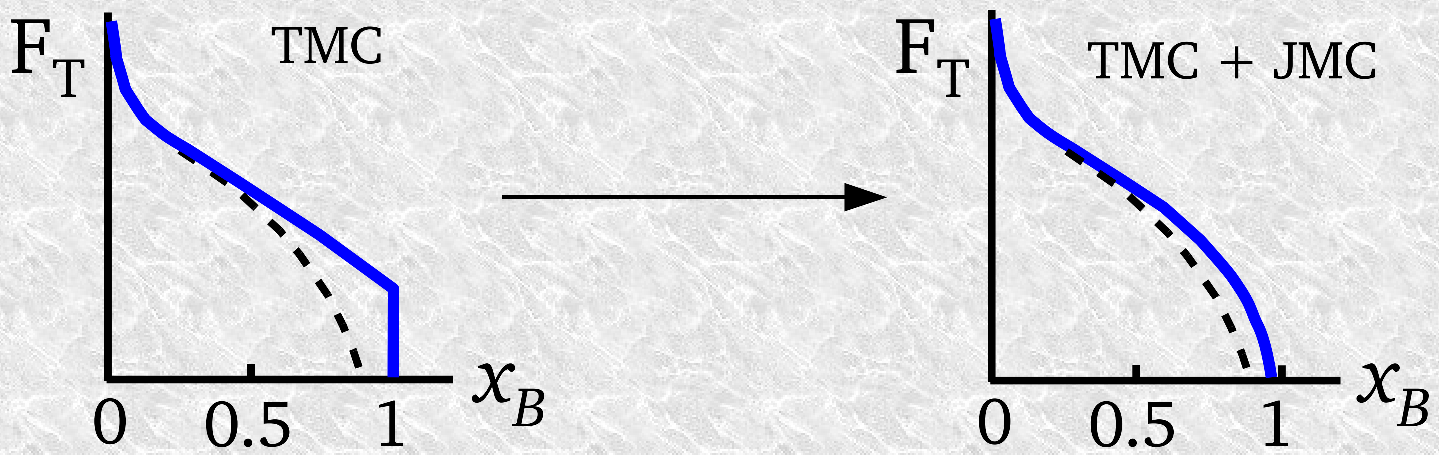
jet mass distribution

$$F_T(x_B, Q^2) = \int_0^\infty dm_j^2\, J_2(m_j^2) \int_\xi^{\frac{\xi}{x_B}} dx\, \frac{1}{2} e_q^2 \delta\left[x - \xi\left(1 + \frac{m_j^2}{Q^2}\right)\right] \varphi_f(x, Q^2)$$

note the limits

$$= \int_0^{\frac{1-x_B}{x_B}Q^2} dm_j^2\, J_2(m_j^2)\, F_T^{(0)}\left(\xi\left(1 + \frac{m_j^2}{Q^2}\right), Q^2\right)$$

Jet smearing at LO – 2



- Rigorously – after some toil:
  - $J(m_J{}^2)$ is the spectral function of a vacuum quark propagator

$$\int_0^\infty dm_j^2\, J_2(m_j^2)\, 2\pi\delta(l^2 - m_j^2)\, \theta(l^0) = \frac{1}{4l^-}\int d^4z e^{iz\cdot l}\mathrm{Tr}\left[\gamma^- \langle 0|\overline{\psi}(z)\psi(0)|0\rangle\right]$$

  - $\langle 0|\overline{\psi}\psi|0\rangle$ computable in lattice QCD – [e.g., Bowman et al. '05] – but

    1) Landau gauge vs. light-cone gauge

    2) Euclidean vs. Minkowski space

Conclusions

- **Collinearly factorized DIS with Target and Jet Mass Corrections**
  - respects $x_B \leq 1$, goes smoothly to 0
  - avoids threshold problem present in OPE formalism (Georgi-Politzer)
  - generalizable to other processes, and nuclear targets
  - fully consistent with CTEQ / MRST global analysis

- **TMC derived at all orders**
  - numerical differences from OPE corrections
    - ✓ 10-20% for F_2
    - ✓ very large for F_L

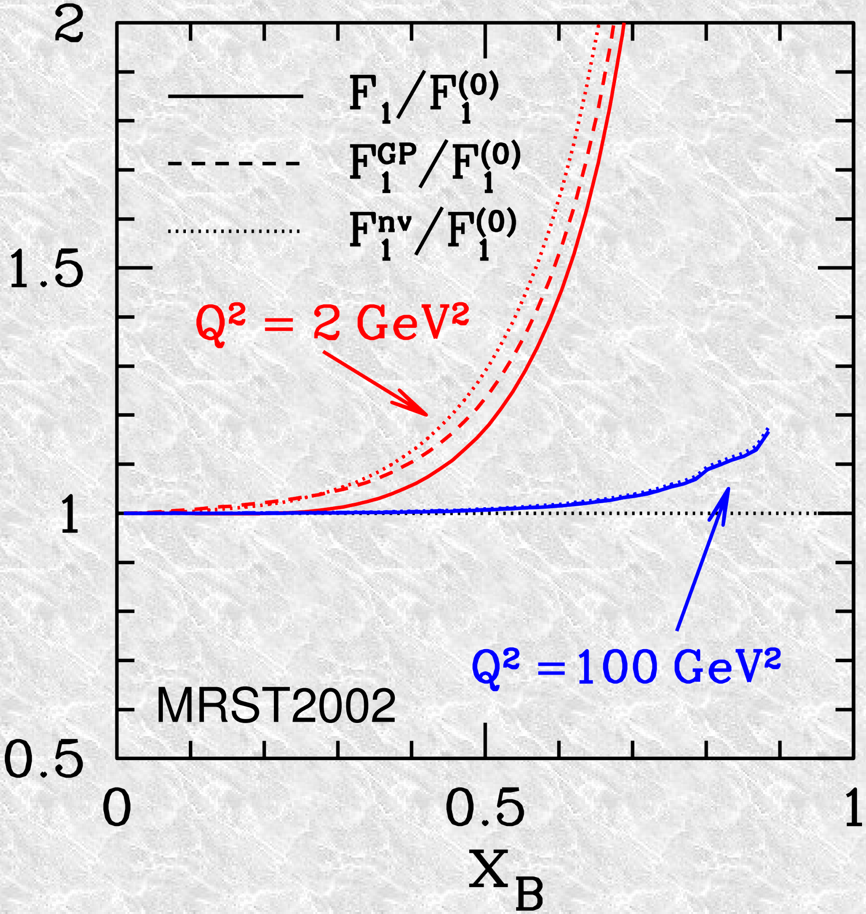
- **JMC rigorously derived only at LO**
  - Clear physical picture – $J_2(m_J^2)$ accessible in lattice QCD

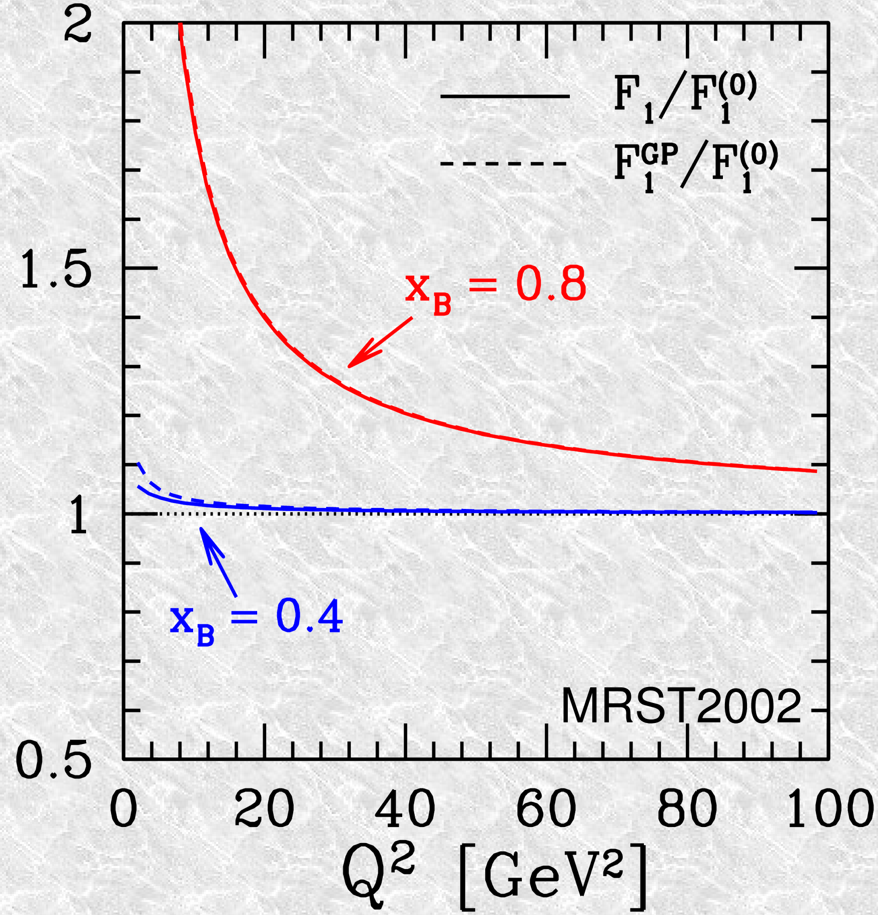
- **Open issues:**
  - higher-twist terms – dynamical TMC
  - JMC at NLO
  - fits to experimental data

The end

Appendices

Target mass corrections – F_1 at NLO





$$F_1^{nv}(x_B) = F_1^{(0)}(\xi)$$

Georgi-Politzer Target Mass Corrections

[Georgi, Politzer 1976; see review of Schienbein et al. 2007]

- In the OPE formalism

$$F_1^{GP}(x_B,Q^2) = \frac{x_B}{\rho_B}\Big[\frac{F_1^{(0)}(\xi,Q^2)}{\xi} + \frac{m_N^2 x_B}{Q^2\rho_B}\Delta_2(x_B,Q^2)\Big]$$

$$F_2^{GP}(x_B,Q^2) = \frac{x_B^2}{\rho_B^3}\Big[\frac{F_2^{(0)}(\xi,Q^2)}{\xi^2} + 6\frac{m_N^2 x_B}{Q^2\rho_B}\Delta_2(x_B,Q^2)\Big]$$

$$F_L^{GP}(x_B,Q^2) = \frac{x_B}{\rho_B}\Big[\frac{F_L^{(0)}(\xi,Q^2)}{\xi} + 2\frac{m_N^2 x_B}{Q^2\rho_B}\Delta_2(x_B,Q^2)\Big]$$

where

$$\xi = \frac{2x_B}{\rho_B^2} \qquad \rho_B^2 = 1 + 4x_B^2 m_N^2/Q^2$$

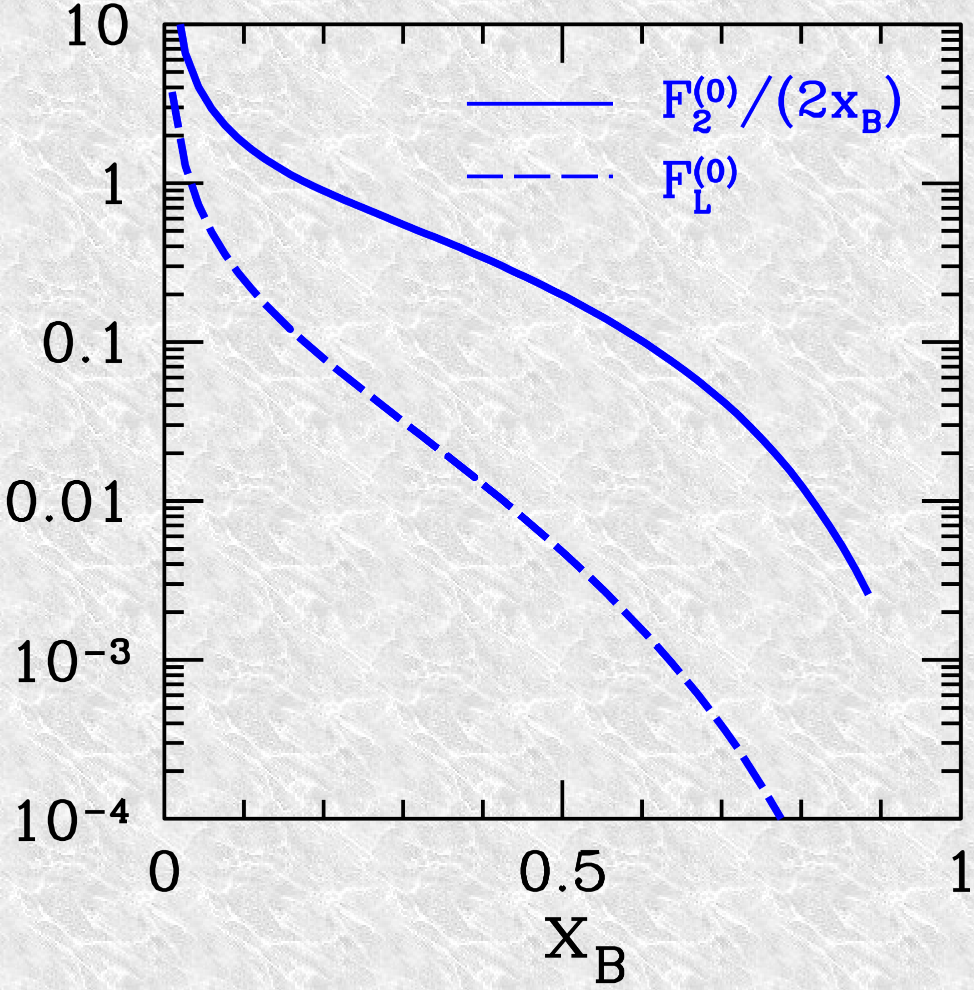
$$\Delta_2(x_B,Q^2) = \int_\xi^1 dv\Big[1 + 2\frac{m_N^2 x_B}{Q^2\rho_B}(v-\xi)\Big]\frac{F_2^{(0)}(v,Q^2)}{v^2}$$

and, in my conventions,

$$F_L(x_B,Q^2) = \frac{\rho_B^2}{2x_B}F_2(x_B,Q^2) - F_1(x_B,Q^2)$$

Georgi-Politzer Target Mass Corrections

- Why is the GP corrected FL so large??



$$F_L^{GP}(x_B)$$

$$= \frac{x_B}{\rho_B}\Big[\frac{F_L^{(0)}(\xi)}{\xi} + 2\frac{m_N^2 x_B}{Q^2\rho_B}\Delta_2(x_B)\Big]$$

$$\Delta_2(x_B)$$

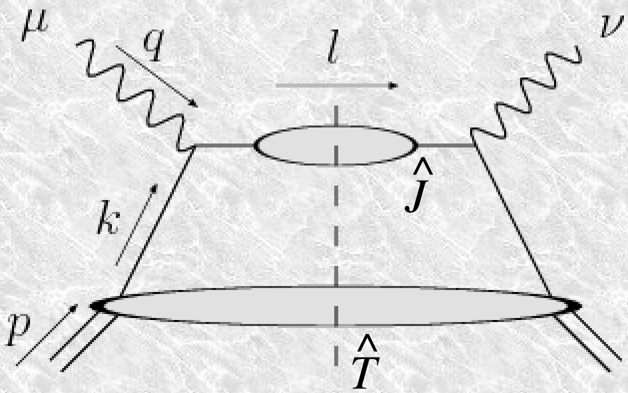
$$= \int_\xi^1 dv\Big[1 + 2\tfrac{m_N^2 x_B}{Q^2\rho_B}(v-\xi)\Big]\frac{F_2^{(0)}(v)}{v^2}$$

Jet mass corrections

Collinear factorization with a jet function

[see also Collins, Rogers, Stasto, 2007]

- Handbag diagram with a quark jet

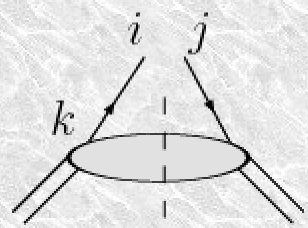


$$l = k + q$$

$$W^{\mu\nu}(p,q) = \frac{e_q^2}{8\pi} \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{T}(k)\gamma^\nu \hat{J}(l)\gamma^\mu\right]$$

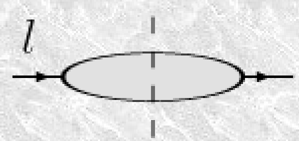
- A hat denotes a Dirac matrix:

$$\hat{T}(k) =$$



$$= \int d^4z e^{iz\cdot k} \langle p|\overline{\psi}_j(z)\psi_i(0)|p\rangle$$

$$\hat{J}(l) =$$



$$= \int d^4z e^{iz\cdot l} \langle 0|\overline{\psi}_j(z)\psi_i(0)|0\rangle$$

(color factors are included in $\hat{\mathrm{T}}$)

Factorization procedure

[Ellis, Furmanski, Petronzio, 1983]

- Expand on a basis of Dirac matrices

$$\hat{T}(k) = \tau_1(k)\hat{\mathbb{I}} + \tau_2(k)\not{k} + \tau_3(k)\gamma_5 + \tau_4(k)\not{k}\gamma_5$$

=0 for massless quarks

cancel for unpolarized targets

$$\hat{J}(l) = j_1(l)\hat{\mathbb{I}} + j_2(l)\not{l} + j_3(l)\gamma_5 + j_4(l)\not{l}\gamma_5$$

enter traces with odd no. of γ's

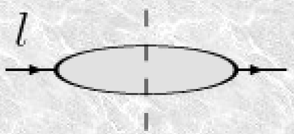
=0 in pure QCD + EM (parity invariance)

- Dominance of k^+, l^- in Breit frame suggests to define

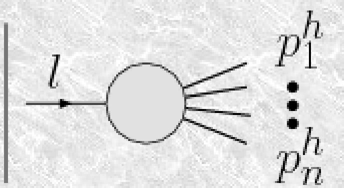
$$\tau_2(k) = \frac{1}{4k^+}\mathrm{Tr}\left[\not{n}\hat{T}(k)\right] = \frac{1}{4k^+}\int d^4z e^{iz\cdot k}\langle p|\overline{\psi}_j(z)\gamma^+\psi(0)|p\rangle$$

$$j_2(l) = \frac{1}{4l^-}\mathrm{Tr}\left[\not{\overline{n}}\hat{J}(l)\right] = \frac{1}{4l^-}\int d^4z e^{iz\cdot l}\langle 0|\overline{\psi}_j(z)\gamma^-\psi(0)|0\rangle$$

Jet spectral representation



$$= \sum_n (2\pi)^4 \delta^{(4)}(l - \textstyle\sum_1^n p_i^h) \left| \quad \right|^2$$



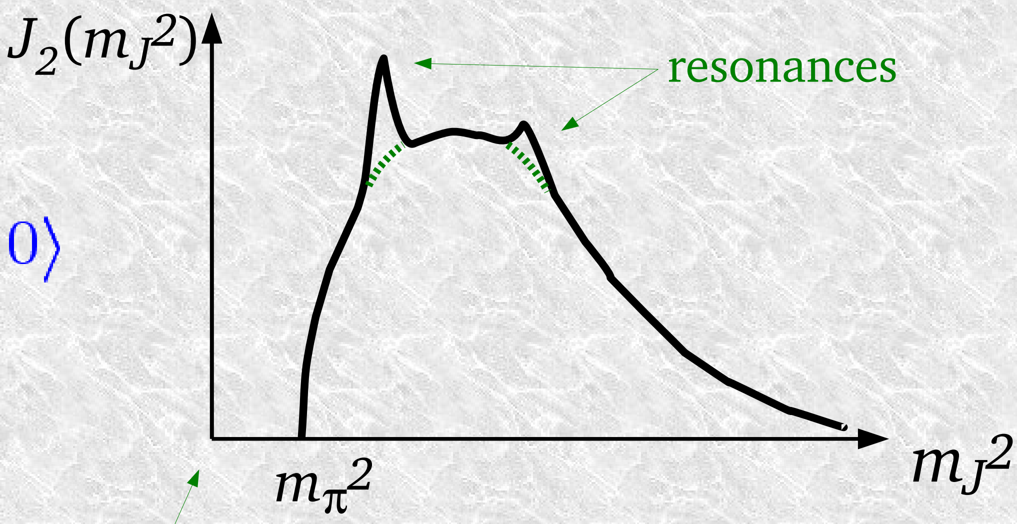
$$= \int_0^\infty dm_j^2 \, [J_1(m_J^2)\hat{\mathbb{I}} + J_2(m_J^2)\not{l}\,] 2\pi\delta(l^2 - m_J^2)\theta(l^0)$$

$$\boxed{j_2(l) = \int_0^\infty dm_j^2 \, J_2(m_J^2) \, 2\pi\delta(l^2 - m_J^2) \, \theta(l^0) \quad \text{with} \quad \int_0^\infty dm_J^2 \, J_2(m_J^2) = 1}$$

- $J_2(m_J{}^2)$ measurable in lattice QCD!

$$J_2(m_J^2) \propto F.T. \langle 0 | \overline{\psi}_j(z) \gamma^- \psi_i(0) | 0 \rangle$$

- I am assuming color neutralization through a (neglected) soft exchange with the target jet



QCD vacuum is confining:
no zero mass hadrons

Collinear expansion - 1

$$W^{\mu\nu}(p,q) = \int \frac{d^4k}{(2\pi)^4} \underbrace{\frac{e_q^2}{8\pi} \mathrm{Tr}\left[\not k \gamma^\nu \not l \gamma^\mu\right]}_{= \frac{1}{\pi} H_*^{\mu\nu}(k,l)} j_2(l)\, \tau_2(k)\, \mathbb{K}(k,p,q)$$

kinematic constraints

$$k^\mu = xp^+ \overline{n}^\mu + \frac{k^2 + k_T^2}{2xp^+} n^\mu + k_T^\mu$$

$$l^\mu = (x-\xi) p^+ \overline{n}^\mu + \left(\frac{k^2 + k_T^2}{2xp^+} + \frac{Q^2}{2\xi p^+}\right) n^\mu + k_T^\mu$$

1) **Expand *H$_*$(k,l)*** around $\widetilde{k} \equiv xp^+ \overline{n}^\mu \quad [\widetilde{l} \equiv \widetilde{k} + q]$

$$H_*^{\mu\nu}(k,l) = H_*^{\mu\nu}(\widetilde{k},\widetilde{l}) + \frac{\partial H_*^{\mu\nu}}{\partial k^\alpha}(k^\alpha - \widetilde{k}^\alpha) + \ldots$$

leading twist

contributes to Higher Twists [Qiu '90]

NOTE:

- up to now no approximations
- especially, I did not approximate the final state kinematic

Collinear expansion - 2

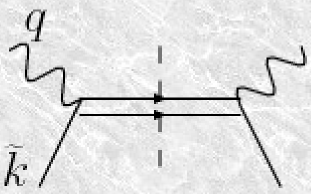
2) **Use spectral representation**

3) **Assume k^-, $k_T \ll (x/\xi)Q^2 \Rightarrow$** $j_2(l) \approx \int_0^\infty dm_J^2\, J_2(m_J^2)\, 2\pi\delta(\tilde{l}^2 - m_J^2)\,\theta(l^0)$

$$W^{\mu\nu}(p,q) = \int_0^\infty dm_J^2\, J_2(m_J^2) \int \frac{d^4k}{(2\pi)^4} H_*^{\mu\nu}(\tilde{k},\tilde{l})\,\delta(\tilde{l}^2 - m_J^2)\, 2\tau_2(k)\,\mathbb{K}(k,p,q)$$

unapproximated!

"fat quark" line:



NOTE:

- Involves a shift in the final state momentum l – **evil !! see [CRS]** but $J_2(m_J^2)$ is unapproximated (improvement over $m_J^2=0$ case)
- OK if $\int d^4l$ dominated by l such that $j_2(l)$ has small slope.

  In terms of the spectral representation we need,

$$\frac{1-x_B}{x_B} Q^2 \gtrsim m_J^2|_{\text{peak}}$$

Collinear expansion - 3

4) **Ignore kinematic limits on k^-, k_T :** $\mathbb{K}(k,p,q) \approx \mathbb{K}(\tilde{k},p,q)$

$$W^{\mu\nu}(p,q) = \int_0^\infty dm_j^2\, J_2(m_J^2) \int \frac{dx}{x}\, H_*^{\mu\nu}(\tilde{k},\tilde{l})\, \delta(\tilde{l}^2 - m_J^2)\, \varphi_q(x)\, \mathbb{K}(\tilde{k},p,q)$$

where $\varphi_q(x) = \int \frac{dz^-}{2\pi} e^{iz^-k^+} \langle p|\bar{\psi}(z^-n)\frac{\gamma\cdot\overline{n}}{2}\psi(0)|p\rangle$

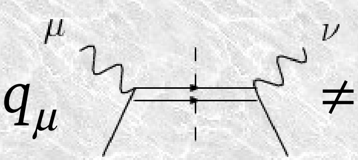
- needed to define collinear PDF
- does not respect 4-momentum conservation – **evil !!** – e.g.,

$$s = (p_J + p_Y)^2 \geq 4k_T^2 \qquad \Longrightarrow \qquad 4k_T^2 \leq \frac{1-\xi}{\xi} Q^2 \Big(1 + \xi \frac{m_N^2}{Q^2}\Big)$$

5) **Set $m_J^2 = 0$ inside $H_*(\tilde{k},\tilde{l})$** [CRS]

$$H_*^{\mu\nu}(\tilde{k},\tilde{l}) \approx H_*^{\mu\nu}(\tilde{k},\hat{l}) \quad \text{with } \hat{l}^\mu = \frac{Q^2}{2\xi p^+} n^\mu$$

Needed to:

- respect gauge invariance (otherwise q_μ  $\neq 0$)
- use Ward ids in proof of factorization
- not so evil: does not touch the final state kinematic

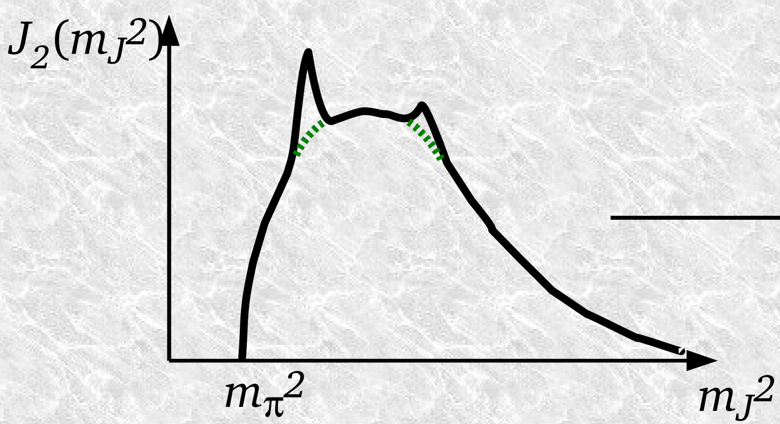
Conclusion

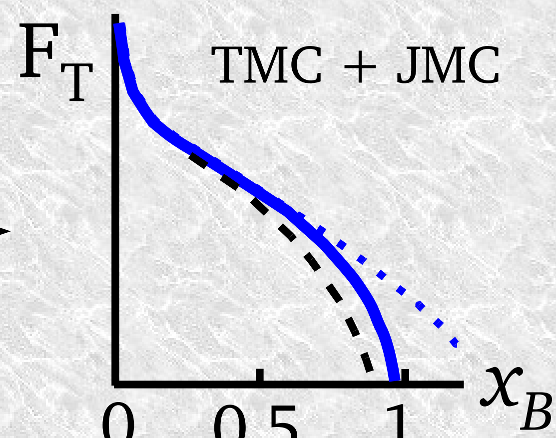
- Collinearly factorized DIS at LO with Target and Jet Mass Corrections
  - respects $x_B \leq 1$, goes smoothly to 0:

$$= \frac{\xi}{Q^2}\delta\left(x - \xi\left(1 + \frac{m_J^2}{Q^2}\right)\right)$$

$$W^{\mu\nu}(p,q) = \int_0^\infty dm_J^2\, J_2(m_J^2) \int_\xi^{\frac{\xi}{x_B}} \frac{dx}{x} \underbrace{\frac{1}{8\pi}\frac{e_q^2}{2}\mathrm{Tr}(\tilde{\not{k}}\gamma^\nu \hat{\not{l}}\gamma^\mu)2\pi\,\delta(\tilde{l}^2 - m_J^2)}_{\mathcal{H}^{\mu\nu}}\,\varphi_q(x)$$

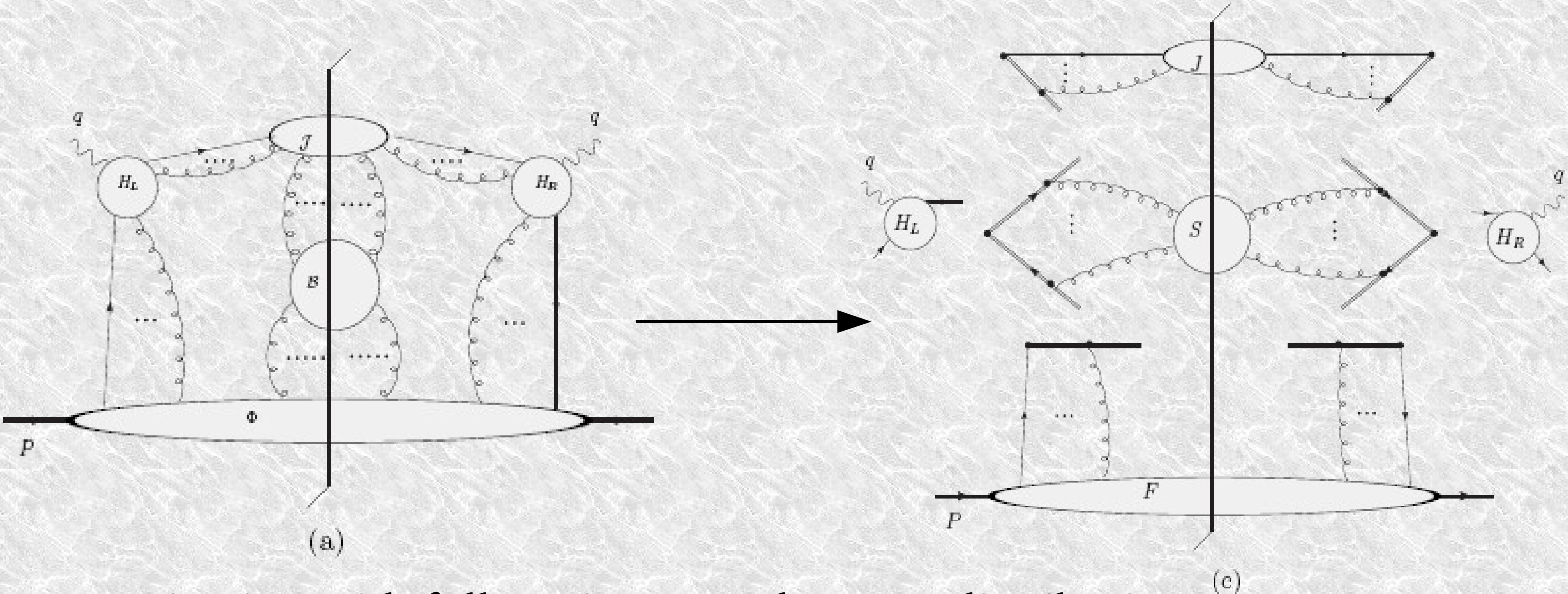
$$F_T(x_B, Q^2) = \int_0^{\frac{1-x_B}{x_B}Q^2} dm_J^2\, J(m_J^2)\, F_T^{(0)}\left(\xi\left(1 + \frac{m_J^2}{Q^2}\right), Q^2\right)$$





"Proof" of collinear factorization - 1

- Generalized handbag diagram with a quark jet [Collins, Rogers, Stasto, 2007]



- Factorization with fully unintegrated parton distributions
  (for an abelian theory of massive gluons – QCD to come soon) [CRS]

$$P_{\mu\nu}W^{\mu\nu} = \int \frac{d^4 k_{\mathrm{T}}}{(2\pi)^4}\frac{d^4 k_{\mathrm{J}}}{(2\pi)^4}\frac{d^4 k_S}{(2\pi)^4}(2\pi)^4\delta^{(4)}(q+P-k_{\mathrm{T}}-k_{\mathrm{J}}-k_S)\times$$

$$\times\, |H(Q,\mu)|^2\ S_2(k_S,y_s,\mu)\ F(k_{\mathrm{T}},y_p,y_s,\mu)\ J(k_{\mathrm{J}},y_s,\mu)$$

soft PCF target PCF jet PCF

“Proof” of collinear factorization - 2

- Start from

$$P_{\mu\nu}W^{\mu\nu} = \int \frac{d^4k_{\rm T}}{(2\pi)^4}\frac{d^4k_{\rm J}}{(2\pi)^4}\frac{d^4k_S}{(2\pi)^4}(2\pi)^4\delta^{(4)}(q+P-k_{\rm T}-k_{\rm J}-k_S)\times$$

$$\times\, |H(Q,\mu)|^2\; S_2(k_S,y_s,\mu)\; F(k_{\rm T},y_p,y_s,\mu)\; J(k_{\rm J},y_s,\mu).$$

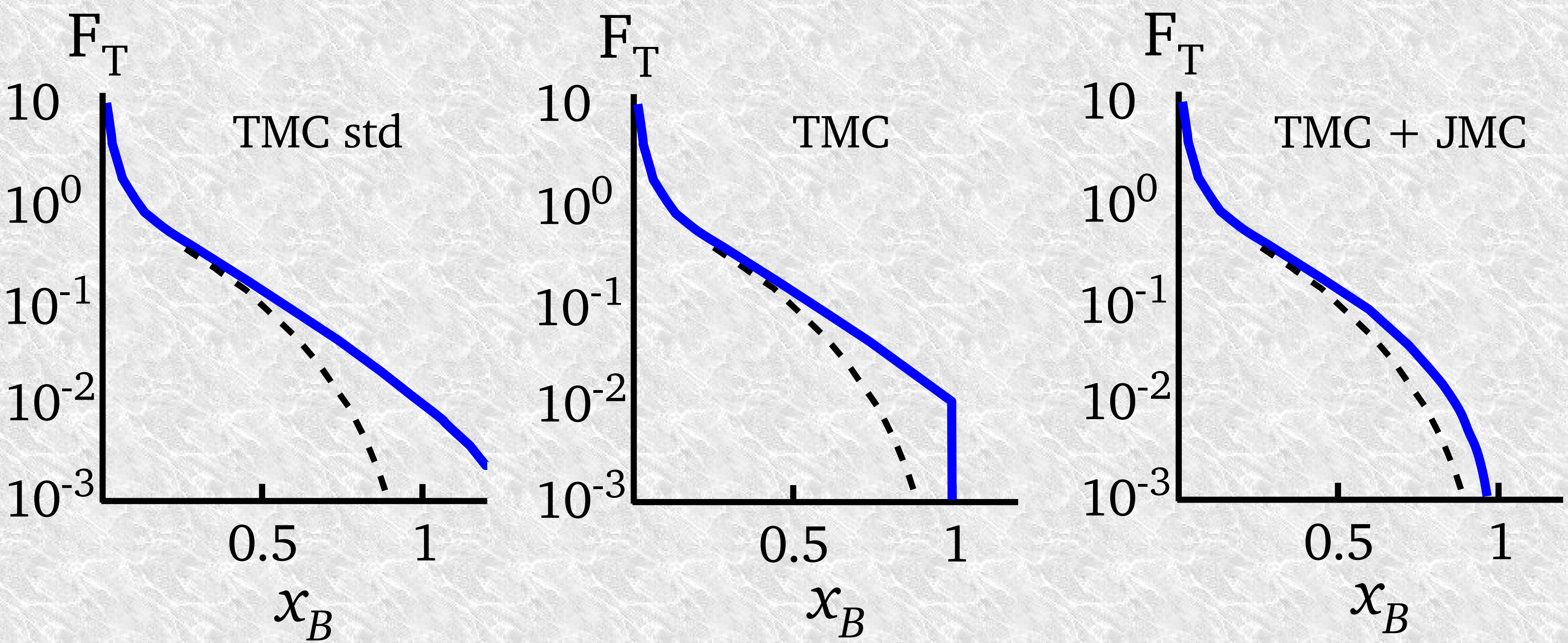
$$\tilde F(w,y_p,y_s,\mu) \;=\langle p|\bar\psi(w)V_w^\dagger(n_s)I_{n_s;w,0}\frac{\gamma^+}{2}V_0(n_s)\psi(0)|p\rangle.$$

$$J(k_J,y_s\,m) = \langle 0|\bar\psi(w)V_w^\dagger(-n_s)I_{-n_s;w,0}\gamma^-V_0(-n_s)\psi(0)|0\rangle$$

$$V_w(n) = P\exp\left(-ig\int_0^\infty d\lambda\, n\cdot A(w+\lambda n)\right)$$

  - neglect soft jet-target interactions, use $P - k_T = k$, $k_J = l$
  - the hard function H is the same as our $h_{T,L,...}$
  - integrate out k_J , use spectral represantation for $J(k_J)$
  - expand H , repeat approximations 3, 4
  - use $n_s{\cdot}A=0$ gauge

- Transverse structure function at LO in α_s with CTEQ5L parton distributions



(the only cartoon in this talk)