# Collinear factorization for DIS at large $x_B$ and low $Q^2$

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CTEQ meeting at Michigan State U.
9-10 November 2007

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# Motivation and outline

# Why large $x_B$ and low $Q^2$ ?

- Large uncertainties in quark and gluon PDF at $x > 0.5$

- Precise PDF at large $x$ are needed, e.g.,
  - at LHC, Tevatron
    1) New physics as excess in large $p_T$ spectra $\Leftrightarrow$ large $x$ PDF
    2) DGLAP evolution feeds large $x$, low $Q^2$ into lower $x$, large $Q^2$
  - d/u ratio at $x=1$ $\Leftrightarrow$ non-perturbative structure of the nucleon

- JLAB has precision DIS data at large $x_B$ , BUT low $Q^2$
  - need of theoretical control over
    1) higher twist $\propto \Lambda^2/Q^2$
    2) target mass corrections (TMC) $\propto x_B{}^2\, m_N{}^2/Q^2$
    3) jet mass corrections (JMC) $\propto m_J{}^2/Q^2$

    } this talk (items 2 and 3)

# OPE and Target Mass Corrections

[Georgi, Politzer 1976; see review of Schienbein et al. 2007]

$$\int d^4z\, e^{-iq\cdot z} \langle N|T[j^{\dagger\mu}(z) j^\nu(0)]|N\rangle = \sum_k f^{\mu_1\ldots\mu_{2k}} A_{2k} \langle N|\underbrace{\mathcal{O}_{\mu_1\ldots\mu_{2k}}(0)}_{\text{symmetric, traceless}}|N\rangle$$

$$A_{2k} = \int_0^1 dy\, y^{2k} F(y)$$

$F(y) \sim \Sigma\, e_q^2\, q(y)$ (at LO) = "quark function"

- Mellin transform, resum, transform back:

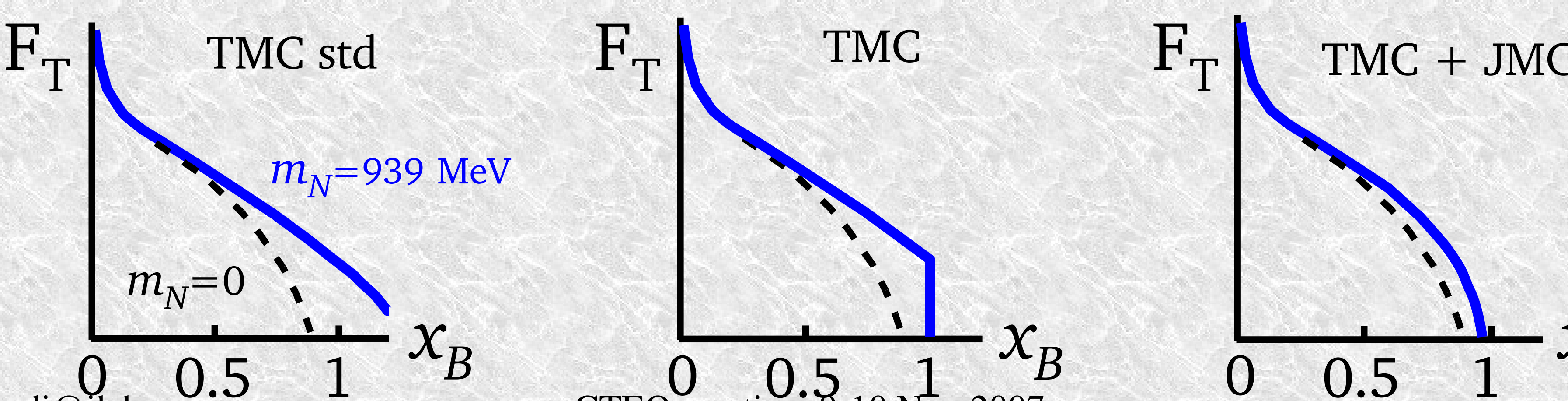
$$F_2^{GP}(x, Q^2) = \frac{x^2}{r^3} F(\xi) + 6\frac{M^2}{Q^2}\frac{x^3}{r^4}\int_\xi^1 d\xi'\, F(\xi') + 12\frac{M^4}{Q^4}\frac{x^4}{r^5}\int_\xi^1 d\xi' \int_{\xi'}^1 d\xi''\, F(\xi'')$$

$$\xi = \frac{2x_B}{1+\sqrt{1+4x_B^2 m^2/Q^2}}$$ = Nachtmann variable

- Threshold problem: $x_B \le 1$ implies $0 \le \xi \le \xi_{th} \stackrel{\text{def}}{=} \xi(x_B{=}1)$
  - Inverse Mellin transform does not give back F(y) !! [Johnson, Tung 1979]
- Unphysical region: q(y) has support over $0<y<1$
  - $F_2(x_B) > 0$ also for $x_B>1$ !!

# Collinear factorization - outline

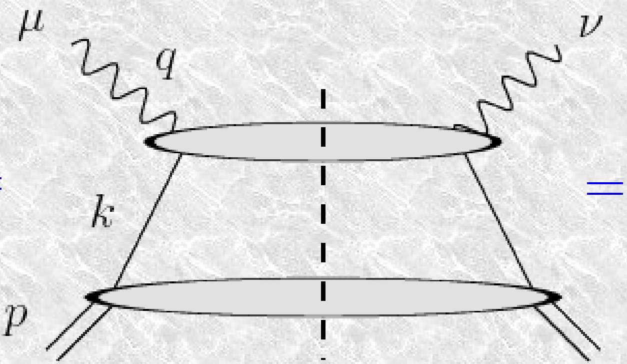
- **Target Mass Corrections – $O(x_B^2\, m_N^2/Q^2)$**
  - momentum space, no need of Mellin transf.
  - analiticity of handbag diagram
    $\Rightarrow$ no "unphysical region" at $x_B > 1$ (!!)
  - any order in $\alpha_s$ at leading twist
- **Jet Mass Corrections – $O(m_J^2/Q^2)$**
  - leading order in $\alpha_s$, leading twist
- **Conclusion:**
  - **factorized formula with TMC + JMC**
  - remarks, open issues



# Target mass corrections

# Kinematics with $m_N \neq 0$

$$W^{\mu\nu}(p,q) =$$




$$= \frac{1}{8\pi} \int d^4z \, e^{-iq\cdot z} \langle p | j^{\dagger\mu}(z) j^{\nu}(0) | p \rangle$$

- Collinear frames: [Aivazis et al 94]

$$p^\mu = p^+ \overline{n}^\mu + \frac{m_N^2}{2p^+} n^\mu$$

$$q^\mu = -\xi p^+ \overline{n}^\mu + \frac{Q^2}{2\xi p^+} n^\mu$$

$$k^\mu = x p^+ \overline{n}^\mu + \frac{k^2 + k_T^2}{2xp^+} n^\mu + k_T^\mu$$

where:

$$x = \frac{k^+}{p^+} \qquad \xi = \frac{2x_B}{1 + \sqrt{1 + 4x_B^2 m_N^2 / Q^2}}$$

Lorentz invariants:

$$x_B = \frac{-q^2}{2p\cdot q} \qquad Q^2 = -q^2$$

$$x_q = \frac{-q^2}{2k\cdot q} \qquad m_N^2 = p^2$$

Light cone vectors:

$$\overline{n} = (1/\sqrt{2}, \vec{0}_\perp, 1/\sqrt{2})$$

$$n = (1/\sqrt{2}, \vec{0}_\perp, -1/\sqrt{2})$$

$$a^\pm = (a_0 \pm a_3)/\sqrt{2}$$

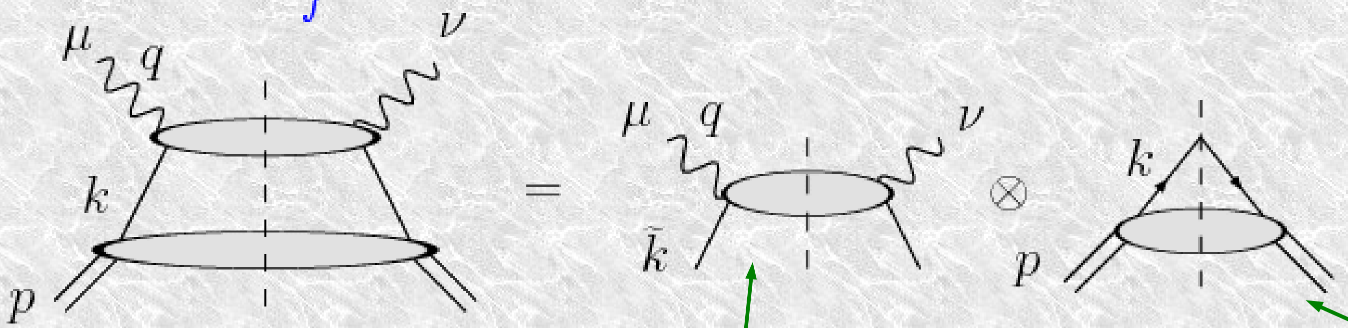
- Bjorken limit: $\xi \to x_B \qquad x_q \to x_B / x$

# Factorization theorem with $m_N \neq 0$

[see also Qiu's talk at CTEQ meeting 2005]

- Expand around $\tilde{k}^\mu = x p^+ \overline{n}^\mu$ $\quad \tilde{k}^2 = 0 \quad \tilde{x}_q = \dfrac{-q^2}{2\tilde{k}\cdot q} = \dfrac{\xi}{x}$

$$W_N^{\mu\nu}(p,q) = \sum_f \int \frac{dx}{x} \mathcal{H}_f^{\mu\nu}(\tilde{k},q)\, \varphi_{f/N}(x,Q^2) + O(\Lambda^2/Q^2)$$




perturbative: doesn't know about the target's mass

dynamical TMC only from nucleon w.f.

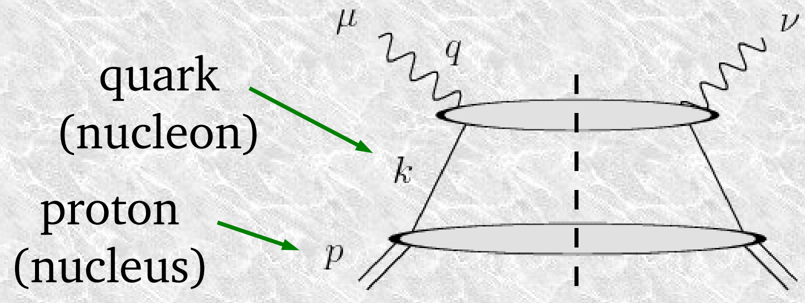
- Helicity structure functions $F_T$, $F_L$ projected out of $W^{\mu\nu}$: *e.g.*,

$$F_T(x_B,Q^2) = \sum_f \int \frac{dx}{x} h_{fT}(\underbrace{\tilde{x}_q}_{=\xi/x},Q^2)\, \varphi_{f/N}(x,Q^2) + O(\Lambda^2/Q^2)$$

no kinematic prefactors [Aivazis, Olness, Tung 1994]

# Kinematic constraints

- General handbag diagram:

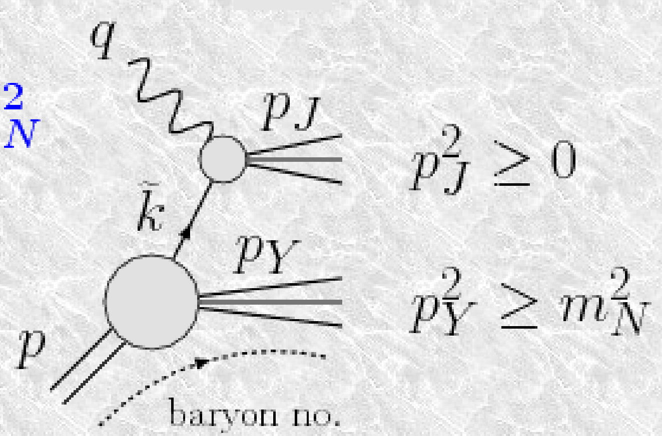



$$\alpha = \frac{k \cdot q}{p \cdot q} = \frac{x_B}{x_q}$$

- Analiticity, ∞ momentum frame ⇒ $x_B \leq \alpha \leq 1$
  - for us, $k = \tilde{k}$, $\tilde{\alpha} = \xi / x$ ⇒ $\xi \leq x \leq \xi / x_B$

- Simple proof for on-shell $\tilde{k}^2 = 0$, $\tilde{\alpha} = \tilde{k} \cdot q / p \cdot q$ (works also for heavy quarks)
  - $0 \leq (\tilde{k} + q)^2 = Q^2 \left( \frac{1}{\tilde{x}_q} - 1 \right) \implies \tilde{x}_q \leq 1 \implies \tilde{\alpha} \geq x_B$
  - $s = (p+q)^2 = (p_J + p_Y)^2 \geq p_J^2 + p_Y^2 \geq p_J^2 + m_N^2$

$$\left. \begin{aligned} p_J^2 &= \left( \frac{\tilde{\alpha}}{\tilde{x}_B} - 1 \right) Q^2 \\ s - m_N^2 &= \left( \frac{1}{\tilde{x}_B} - 1 \right) Q^2 \end{aligned} \right\} \implies \tilde{\alpha} \leq 1$$



# No unphysical region!

$$F_T(x_B,Q^2) = \sum_f \int_\xi^{\frac{\xi}{x_B}} \frac{dx}{x}\, h_{fT}\Big(\frac{\xi}{x},Q^2\Big)\, \varphi_{f/N}(x,Q^2)$$

$$\boxed{F_T(x_B,Q^2) = 0 \quad \text{at } x_B \geq 0}$$

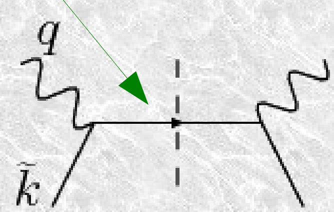
- Bjorken limit recovers "massless" structure functions (*$m_N$=0*)

$$F_T(x_B,Q^2) \longrightarrow \sum_f \int_{x_B}^{1} \frac{dx}{x}\, h_{fT}\Big(\frac{x_B}{x},Q^2\Big)\, \varphi_{f/N}(x,Q^2) \equiv F_T^{(0)}(x_B,Q^2)$$

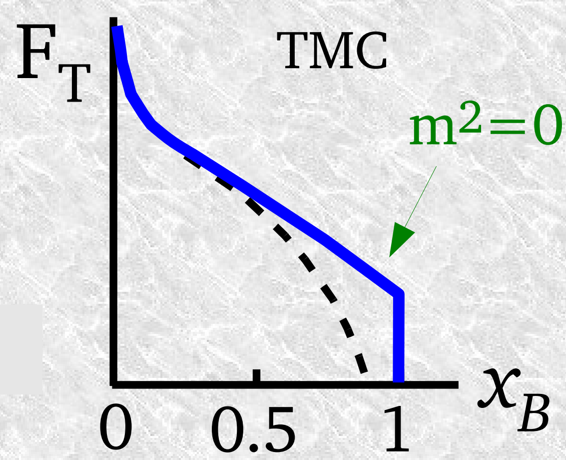
- But... at leading order,

m²=0

$$h_{fT}\big(\frac{\xi}{x},Q^2\big) = x\frac{1}{2}e_f^2\delta\Big(\frac{\xi}{x}-1\Big) \quad =$$

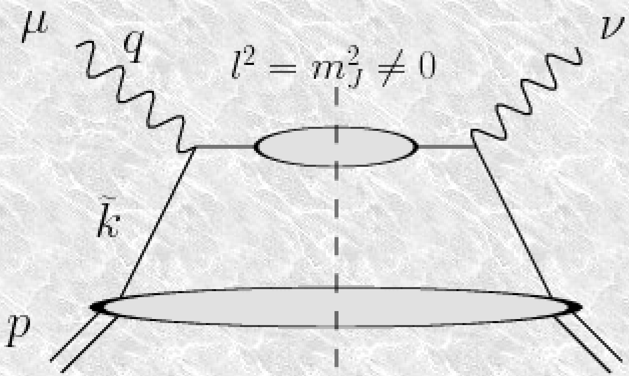



$$F_T(x_B,Q^2) = \frac{1}{2}\sum_f e_f^2\phi(\xi) = F_T^{(0)}(\xi,Q^2) \quad \text{at } x_B < 1$$



# Jet smearing, heuristically

- Ansatz: jet with a non zero mass, smoohtly distributed in $m_J^2$



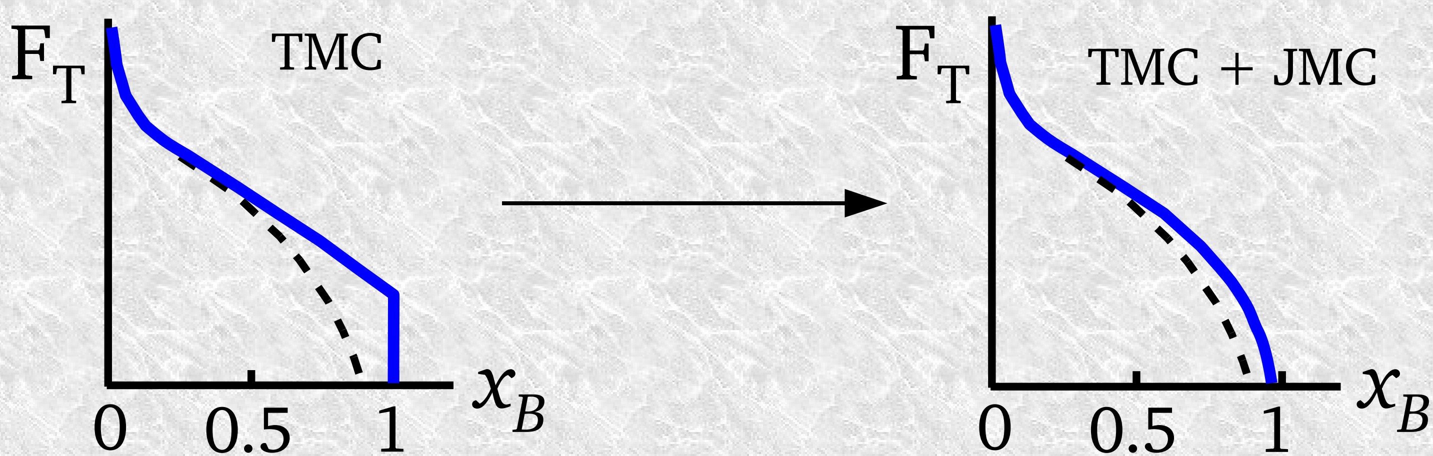

$$(k+q)^2 = m_J^2 \quad \longrightarrow \quad \delta\left[x - \xi\left(1 + \frac{m_J^2}{Q^2}\right)\right]$$

jet mass distribution

$$F_T(x_B, Q^2) = \int_0^\infty dm_J^2 \, J(m_j^2) \int_\xi^{\frac{\xi}{x_B}} dx \, \frac{1}{2} e_q^2 \delta\left[x - \xi\left(1 + \frac{m_J^2}{Q^2}\right)\right] \varphi_f(x, Q^2)$$

note the int. limits

$$= \int_0^{\frac{1-x_B}{x_B}Q^2} dm_J^2 \, J(m_j^2) \, F_T^{(0)}\left(\xi\left(1 + \frac{m_J^2}{Q^2}\right), Q^2\right)$$

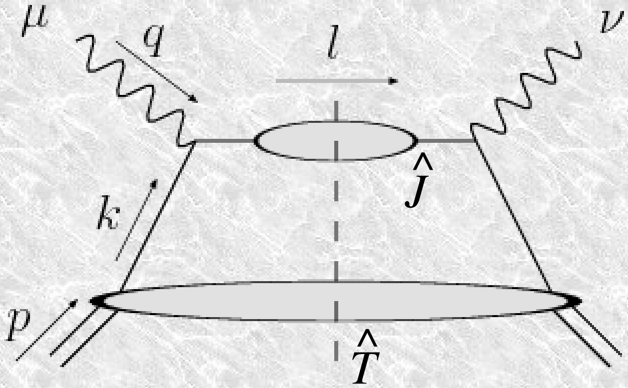


# Jet mass corrections

# Collinear factorization with a jet function

[see also Collins, Rogers, Stasto, 2007]

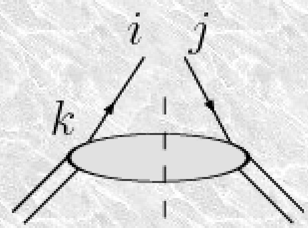
- Handbag diagram with a quark jet

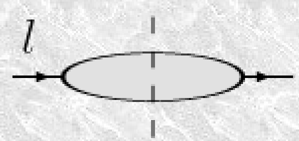



$$l = k + q$$

$$W^{\mu\nu}(p,q) = \frac{e_q^2}{8\pi} \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{T}(k)\gamma^\nu \hat{J}(l)\gamma^\mu\right]$$

- A hat denotes a Dirac matrix:

$\hat{T}(k) =$  $= \int d^4z e^{iz\cdot k} \langle p|\overline{\psi}_j(z)\psi_i(0)|p\rangle$


$\hat{J}(l) =$  $= \int d^4z e^{iz\cdot l} \langle 0|\overline{\psi}_j(z)\psi_i(0)|0\rangle$


(color factors are included in $\hat{\mathrm{T}}$)

# Factorization procedure

[Ellis, Furmanski, Petronzio, 1983]

- Expand on a basis of Dirac matrices

$$\hat{T}(k) = \tau_1(k)\hat{\mathbb{I}} + \tau_2(k)\not{k} + \tau_3(k)\gamma_5 + \tau_4(k)\not{k}\gamma_5$$

=0 for massless quarks (for $\tau_1$); cancel for unpolarized targets (for $\tau_3$, $\tau_4$)

$$\hat{J}(l) = j_1(l)\hat{\mathbb{I}} + j_2(l)\not{l} + j_3(l)\gamma_5 + j_4(l)\not{l}\gamma_5$$

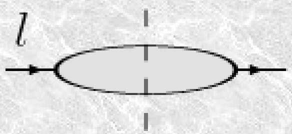
enter traces with odd no. of γ's (for $j_1$); =0 in pure QCD + EM (parity invariance) (for $j_3$, $j_4$)

- Dominance of $k^+$, $l^-$ in Breit frame suggests to define

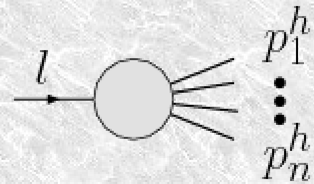
$$\tau_2(k) = \frac{1}{4k^+}\mathrm{Tr}\left[\not{n}\hat{T}(k)\right] = \frac{1}{4k^+}\int d^4z e^{iz\cdot k}\langle p|\overline{\psi}_j(z)\gamma^+\psi(0)|p\rangle$$

$$j_2(k) = \frac{1}{4l^-}\mathrm{Tr}\left[\not{\overline{n}}\hat{J}(l)\right] = \frac{1}{4l^-}\int d^4z e^{iz\cdot l}\langle 0|\overline{\psi}_j(z)\gamma^-\psi(0)|0\rangle$$

# Jet spectral representation




$$= \sum_n (2\pi)^4 \delta^{(4)}(l - \textstyle\sum_1^n p_i^h) \Big| \quad \Big|^2$$



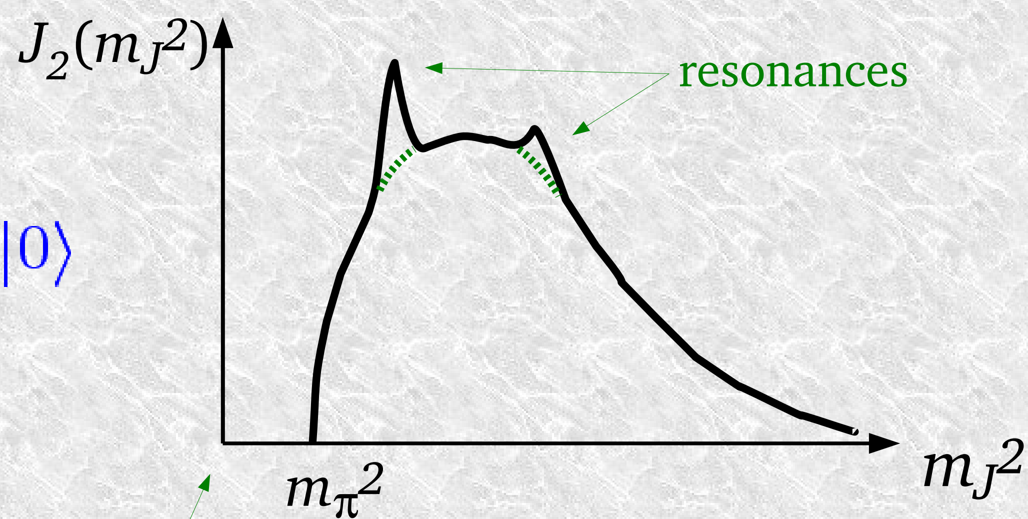

$$= \int_0^\infty dm_j^2 \, [J_1(m_J^2)\hat{\mathbb{I}} + J_2(m_J^2) \not{l}\,] 2\pi\delta(l^2 - m_J^2)\theta(l^0)$$

$$j_2(l) = \int_0^\infty dm_j^2 \, J_2(m_J^2) \, 2\pi\delta(l^2 - m_J^2) \, \theta(l^0) \quad \text{with} \quad \int_0^\infty dm_j^2 \, J_2(m_J^2) = 1$$

- $J_2(m_J{}^2)$ measurable in lattice QCD!

$$J_2(m_J^2) \propto F.T. \langle 0|\overline{\psi}_j(z)\gamma^- \psi_i(0)|0\rangle$$

- I am assuming color neutralization through a (neglected) soft exchange with the target jet




QCD vacuum is confining:
no zero mass hadrons

# Collinear expansion - 1

$$W^{\mu\nu}(p,q) = \int \frac{d^4k}{(2\pi)^4} \underbrace{\frac{e_q^2}{8\pi} \mathrm{Tr}\left[\not k \gamma^\nu \not l \gamma^\mu\right]}_{= \frac{1}{2\pi} H_*^{\mu\nu}(k,l)} j_2(l)\, \tau_2(k)\, \mathbb{K}(k,p,q)$$

kinematic constraints

$$k^\mu = x p^+ \overline{n}^\mu + \frac{k^2 + k_T^2}{2xp^+} n^\mu + k_T^\mu$$

$$l^\mu = (x-\xi) p^+ \overline{n}^\mu + \left(\frac{k^2 + k_T^2}{2xp^+} + \frac{Q^2}{2\xi p^+}\right) n^\mu + k_T^\mu$$

1) **Expand** ***H*** $_*$ ***(k,l)*** around $\widetilde{k} \equiv x p^+ \overline{n}^\mu$ $\quad [\widetilde{l} \equiv \widetilde{k} + q]$

$$H_*^{\mu\nu}(k,l) = H_*^{\mu\nu}(\widetilde{k},\widetilde{l}) + \frac{\partial H_*^{\mu\nu}}{\partial k^\alpha}(k^\alpha - \widetilde{k}^\alpha) + \ldots$$

leading twist — contributes to Higher Twists [Qiu '90]

NOTE:

- up to now no approximations
- especially, I did not approximate the final state kinematic

# Collinear expansion - 2

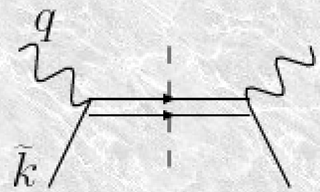
**2) Use spectral representation**

**3) Assume $k^-$, $k_T \ll (x/\xi)Q^2$ $\Rightarrow$** $j_2(l) \approx \int_0^\infty dm_j^2\, J_2(m_J^2)\, 2\pi\delta(\tilde{l}^2 - m_J^2)\, \theta(l^0)$

$$W^{\mu\nu}(p,q) = \int_0^\infty dm_j^2\, J_2(m_J^2) \int \frac{d^4k}{(2\pi)^4} H_*^{\mu\nu}(\tilde{k},\tilde{l})\, \delta(\tilde{l}^2 - m_J^2)\, 2\tau_2(k)\, \mathbb{K}(k,p,q)$$

unapproximated!

"fat quark" line:




NOTE:

- Involves a shift in the final state momentum $l$ – **evil !! see [CRS]** but $J_2(m_J^2)$ is unapproximated (improvement over $m_J^2=0$ case)
- OK if $\int d^4l$ dominated by $l$ such that $j_2(l)$ has small slope.

  In terms of the spectral representation we need,

$$\frac{1-x_B}{x_B} Q^2 \gtrsim m_J^2|_{\rm peak}$$

# Collinear expansion - 3

**4) Ignore kinematic limits on $k^-$, $k_T$ :** $\mathbb{K}(k,p,q) \approx \mathbb{K}(\widetilde{k},p,q)$

$$W^{\mu\nu}(p,q) = \int_0^\infty dm_j^2\, J_2(m_J^2) \int \frac{dx}{x}\, H_*^{\mu\nu}(\widetilde{k},\widetilde{l})\, \delta(\widetilde{l}^2 - m_J^2)\, \varphi_q(x)\, \mathbb{K}(\widetilde{k},p,q)$$

where $\varphi_q(x) = \int \frac{dz^-}{2\pi} e^{iz^-k^+} \langle p|\bar\psi(z^- n)\frac{\gamma\cdot\overline{n}}{2}\psi(0)|p\rangle$

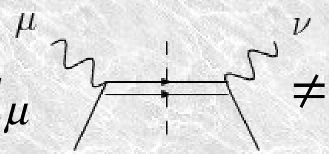
- needed to define collinear PDF
- does not respect 4-momentum conservation – **evil !!** – e.g.,

$$s = (p_J + p_Y)^2 \geq 4k_T^2 \qquad \Longrightarrow \qquad 4k_T^2 \leq \frac{1-\xi}{\xi} Q^2 \big(1 + \xi \frac{m_N^2}{Q^2}\big)$$

**5) Set $m_J^2 = 0$ inside $H_*(\widetilde{k},\widetilde{l})$** [CRS]

$$H_*^{\mu\nu}(\widetilde{k},\widetilde{l}) \approx H_*^{\mu\nu}(\widetilde{k},\hat{l}) \quad \text{with } \hat{l}^\mu = \frac{Q^2}{2\xi p^+} n^\mu$$

Needed to:

- respect gauge invariance (otherwise $q_\mu$ 

$\neq 0$)
- use Ward ids in proof of factorization
- not so evil: does not touch the final state kinematic

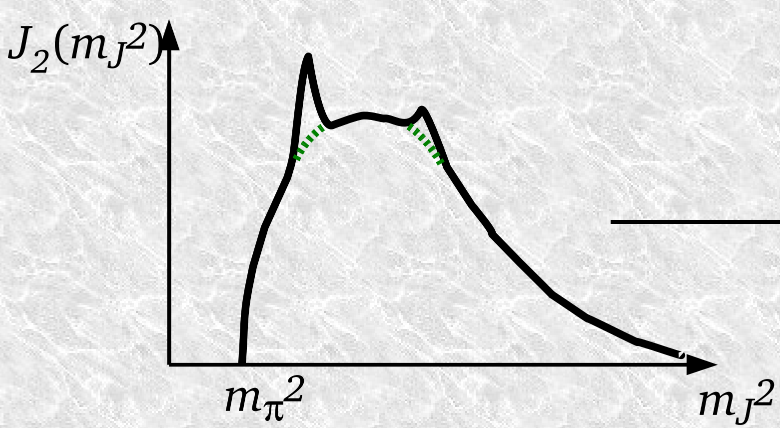
# Conclusion

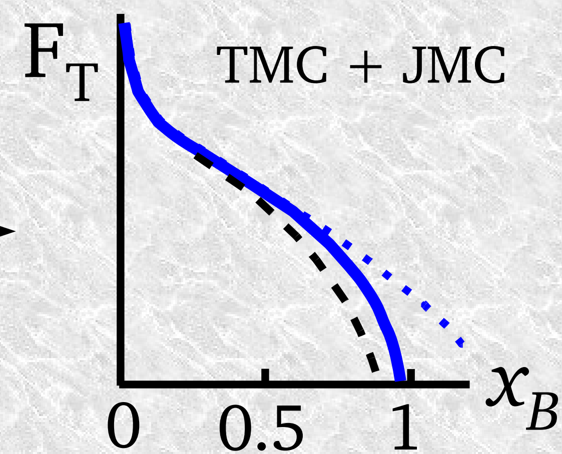
- Collinearly factorized DIS at LO with Target and Jet Mass Corrections
  - respects $x_B \leq 1$, goes smoothly to 0:

$$= \frac{\xi}{Q^2}\delta\left(x - \xi\left(1 + \frac{m_J^2}{Q^2}\right)\right)$$

$$W^{\mu\nu}(p,q) = \int_0^\infty dm_J^2\, J_2(m_J^2) \int_\xi^{\frac{\xi}{x_B}} \frac{dx}{x} \underbrace{\frac{1}{8\pi}\frac{e_q^2}{2}\mathrm{Tr}(\tilde{\not k}\gamma^\nu \hat{\not l}\gamma^\mu) 2\pi\,\delta(\tilde{l}^2 - m_J^2)}_{\mathcal{H}^{\mu\nu}}\, \varphi_q(x)$$

$$F_T(x_B, Q^2) = \int_0^{\frac{1-x_B}{x_B}Q^2} dm_J^2\, J(m_J^2)\, F_T^{(0)}\left(\xi\left(1 + \frac{m_J^2}{Q^2}\right), Q^2\right)$$



# Remarks

- Kinematic approximations are explicit
  - estimates of validity in $(x_B, Q^2)$ plane

- $J_2(m_J^2)$ has explicit operator definition
  - Accessible in lattice QCD
  - Clear physical picture, allows for phenomenology

- Fully consistent with collinear factorization
  - generalizable to other processes
  - fully consistent with CTEQ global analysis

- Collinear factorization is pushed to its limit (or close to it)
  - further improvement requires:
    - ✓ jet-target soft interactions, finite rapidity separation
    - ✓ unapproximated final-state kinematics
  - Fully unintegrated parton distributions of [CRS]

# Open Issues

- **Next-to-leading orders** (see also [CRS])
  - real gluon emission: already in $J_2(l)$?
  - how to compute the hard tensor?

- **Higher twist terms**
  - Just $x_B \to \xi\,(1+m_J^2/Q^2)$ ?
  - Dynamical TM correlations among different twists because of equations of motion? [EFP]
  - Expansion around $l=\hat{l}+\ldots$ ? (in analogy to $k=\tilde{k}+\ldots$)

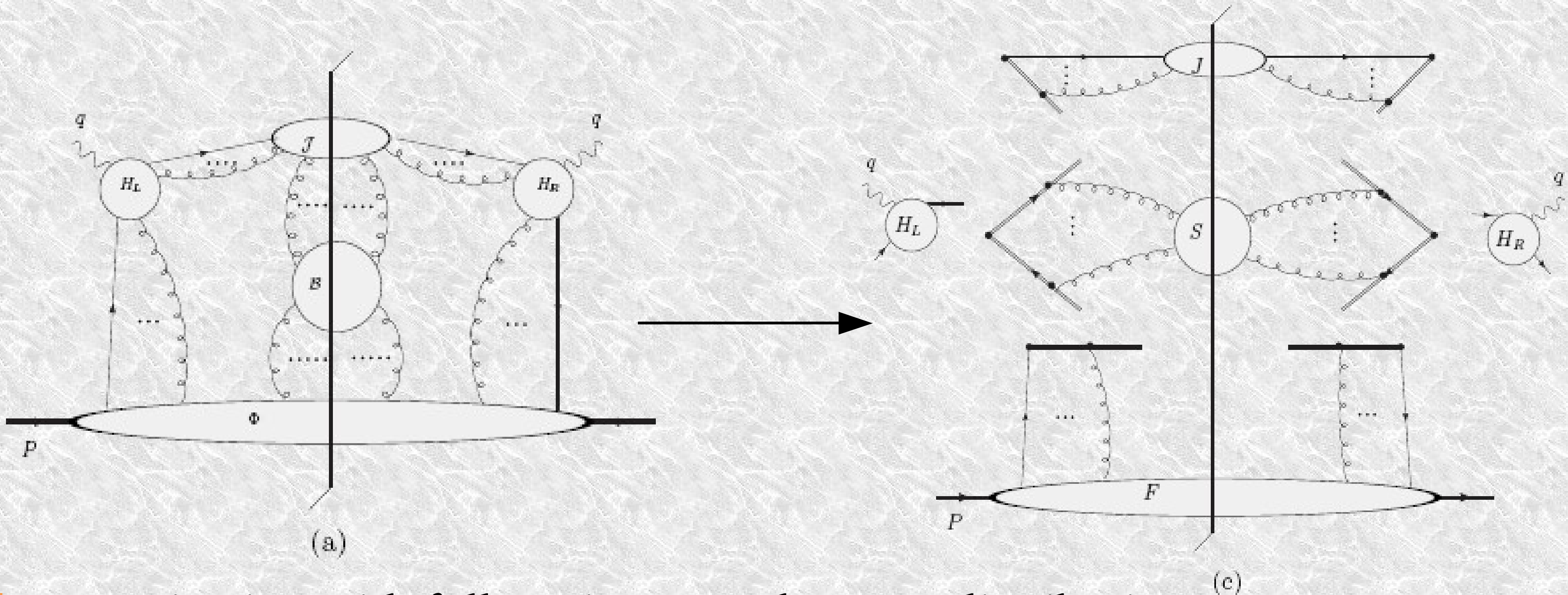
- **Fits to experimental data**
  - $J_2(m_J^2) \Rightarrow$ parametrization / lattice QCD based phenomenology
  - HT $\Rightarrow$ C$[\xi\,(1+m_J^2/Q^2)]\,/\,Q^2$ ?
  - dynamical TMC in HT terms ?

# The end

# Appendices

# "Proof" of collinear factorization - 1

- Generalized handbag diagram with a quark jet [Collins, Rogers, Stasto, 2007]




- Factorization with fully unintegrated parton distributions (for an abelian theory of massive gluons – QCD to come soon) [CRS]

$$P_{\mu\nu} W^{\mu\nu} = \int \frac{d^4 k_{\rm T}}{(2\pi)^4} \frac{d^4 k_{\rm J}}{(2\pi)^4} \frac{d^4 k_S}{(2\pi)^4} (2\pi)^4 \delta^{(4)}(q + P - k_{\rm T} - k_{\rm J} - k_S) \times$$
$$\times \left| H(Q,\mu) \right|^2 S_2(k_S, y_s, \mu)\, F(k_{\rm T}, y_p, y_s, \mu)\, J(k_{\rm J}, y_s, \mu).$$

soft PCF    target PCF    jet PCF

# "Proof" of collinear factorization - 2

- Start from

$$P_{\mu\nu}W^{\mu\nu} = \int \frac{d^4k_{\rm T}}{(2\pi)^4}\frac{d^4k_{\rm J}}{(2\pi)^4}\frac{d^4k_S}{(2\pi)^4}(2\pi)^4\delta^{(4)}(q+P-k_{\rm T}-k_{\rm J}-k_S)\times$$
$$\times\, |H(Q,\mu)|^2\; S_2(k_S,y_s,\mu)\; F(k_{\rm T},y_p,y_s,\mu)\; J(k_{\rm J},y_s,\mu).$$

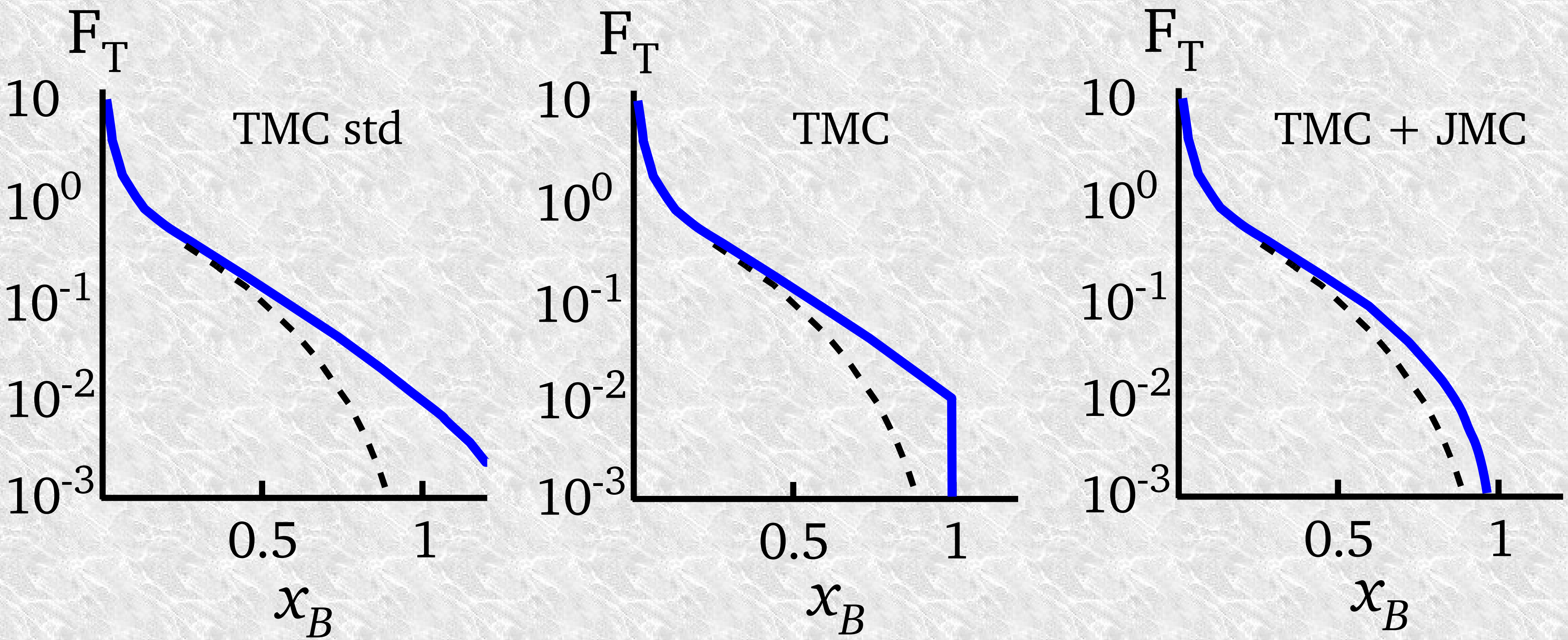
$$\tilde{F}(w,y_p,y_s,\mu) \;= \langle p|\bar{\psi}(w)V_w^\dagger(n_s)I_{n_s;w,0}\frac{\gamma^+}{2}V_0(n_s)\psi(0)|p\rangle.$$

$$J(k_J,y_s\,m) = \langle 0|\bar{\psi}(w)V_w^\dagger(-n_s)I_{-n_s;w,0}\gamma^-V_0(-n_s)\psi(0)|0\rangle$$

$$V_w(n) = P\exp\left(-ig\int_0^\infty d\lambda\, n\cdot A(w+\lambda n)\right)$$

- neglect soft jet-target interactions, use $P - k_T = k$ , $k_J = l$
- the hard function $H$ is the same as our $h_{T,L,\dots}$
- integrate out $k_J$ , use spectral represantation for $J(k_J)$
- expand $H$ , repeat approximations 3, 4
- use $n_s{\cdot}A=0$ gauge

- Transverse structure function at LO in $\alpha_s$ with CTEQ5L parton distributions




(the only cartoon)