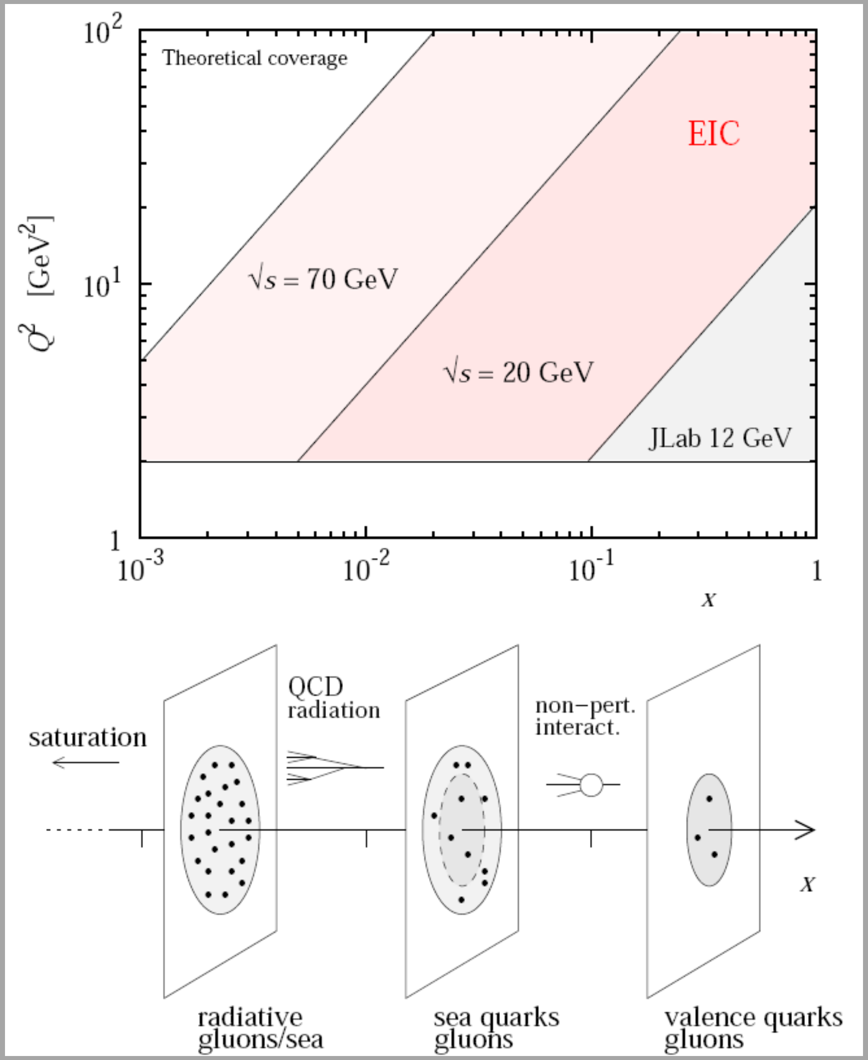
Drell-Yan workshop 21-25 May 2012, ECT* Trento

Alexei Prokudin

# Evolution of TMDs and the process dependence of the Sivers function

Jefferson Lab
EXPLORING THE NATURE OF MATTER

# Nucleon landscape




Plot courtesy of Christian Weiss

Nucleon is a many body dynamical system of quarks and gluons

Changing x we probe different aspects of nucleon wave function

How partons move and how they are distributed in space is one of the future directions of development of nuclear physics

Technically such information is encoded into Generalised Parton Distributions

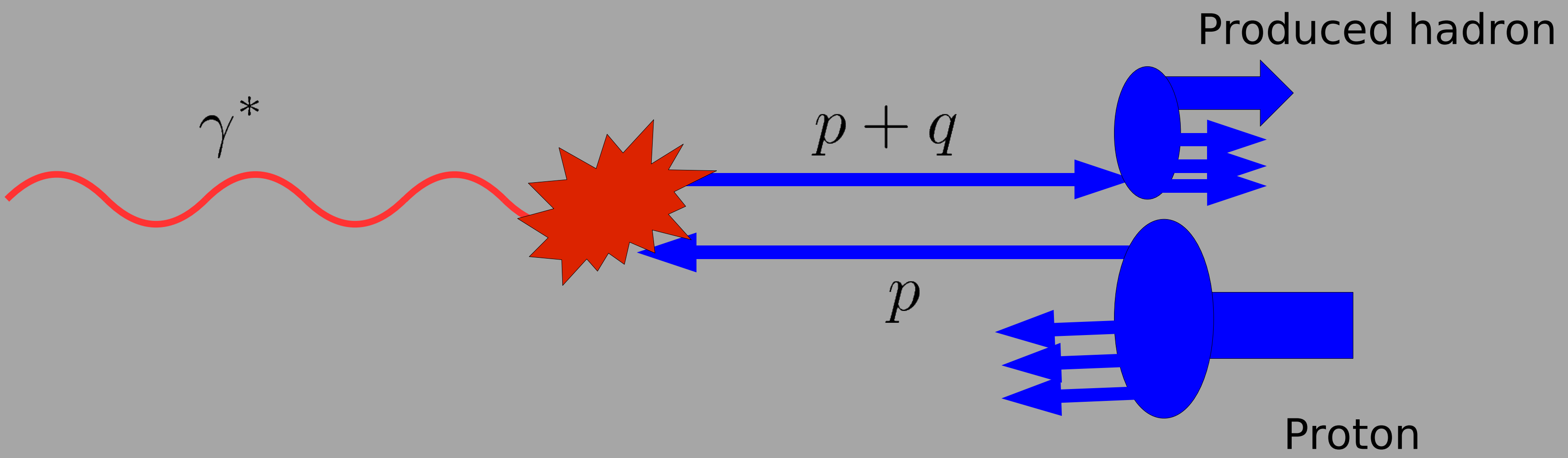
Markus Diehl (2003)
Matthias Burkardt (2003)

and Transverse Momentum Dependent distributions

EIC report, Boer, Diehl, Milner, Venugopalan, Vogelsang et al , 2011

# QCD and parton model

Let us calculate SIDIS cross section in parton model:




We work in Infinite Momentum Frame and all partons are collinear to the proton, thus

$$\frac{d\sigma}{dP_T^2} \sim \delta(P_T^2)$$

# QCD and parton model

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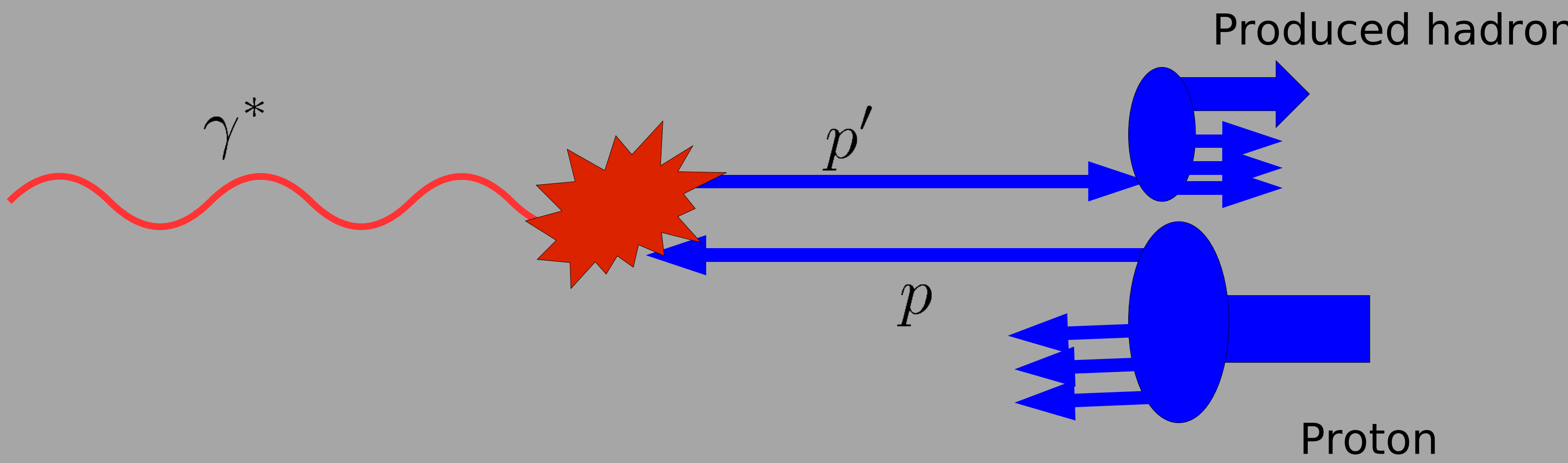
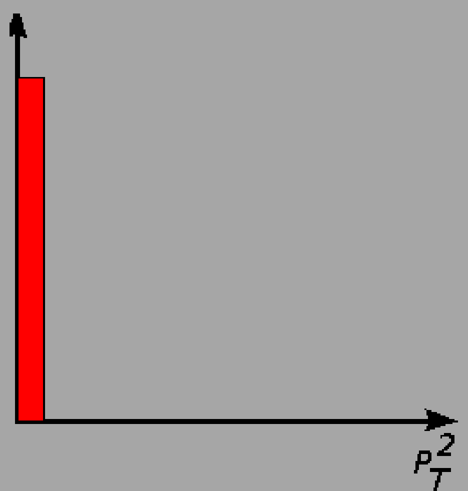



“Theory”



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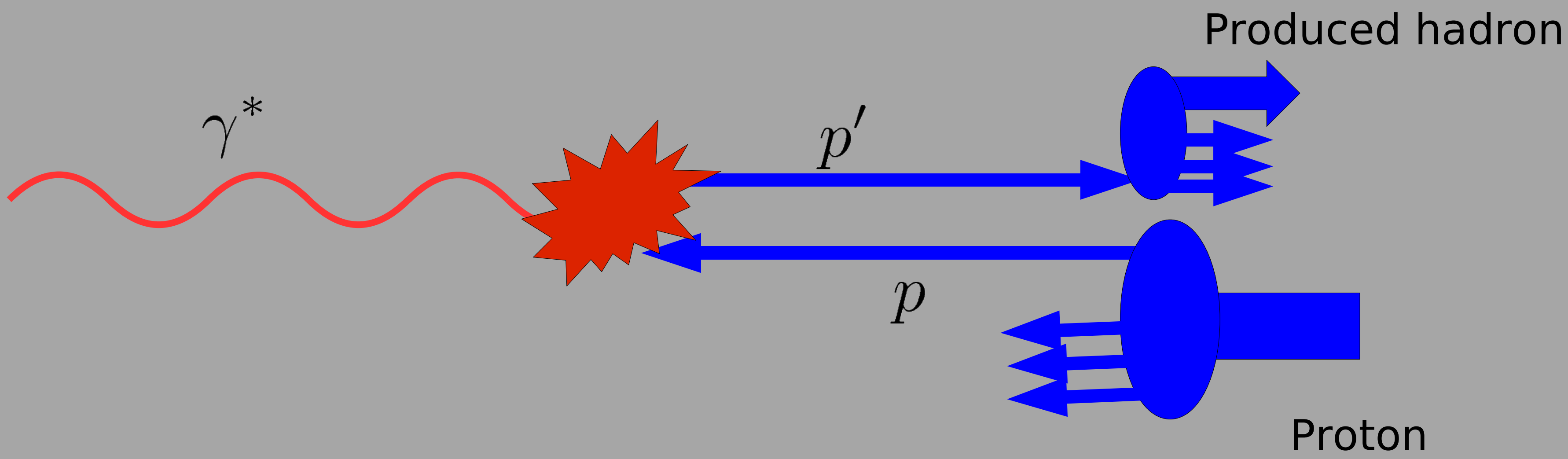
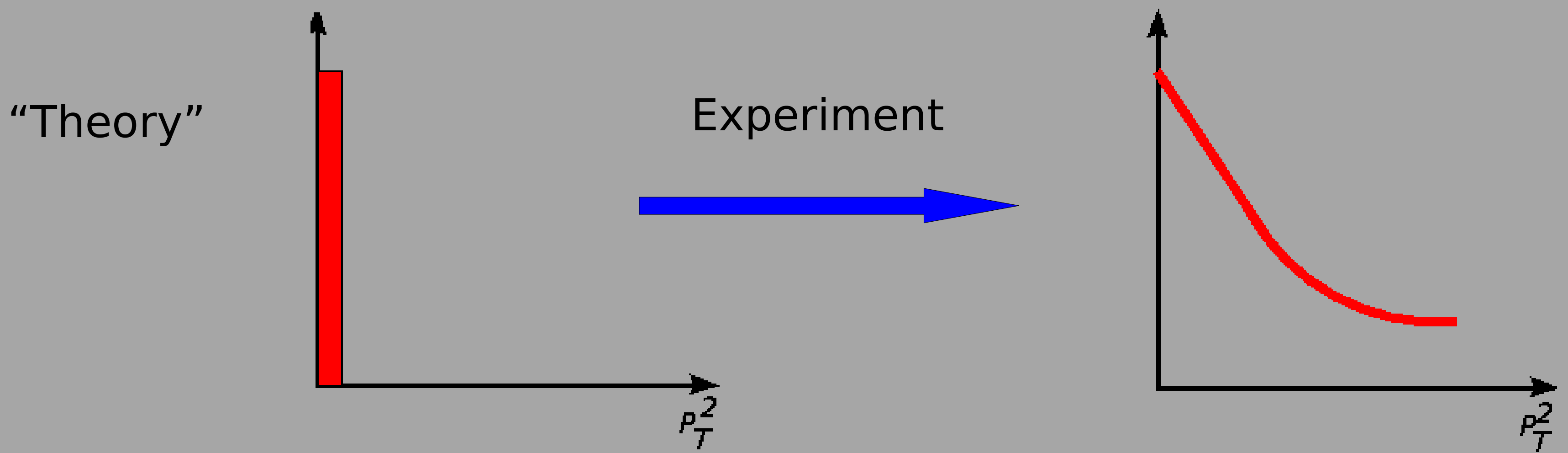
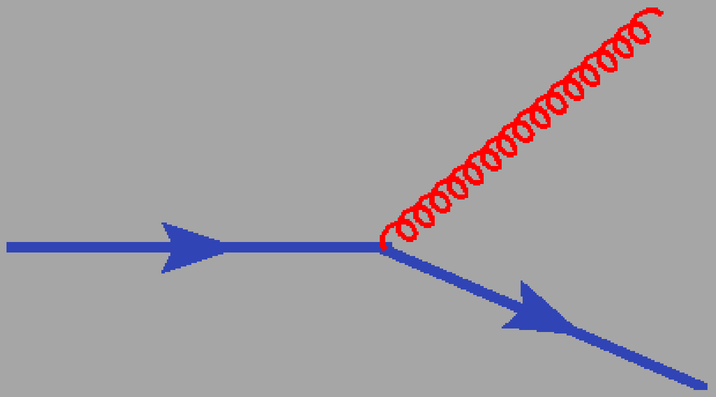


# SIDIS and parton model

"QCD improved" parton model:



Radiation of gluons create transverse momenta

Terms like this appear

$$(\alpha_s)^n \left( \ln \frac{Q^2}{P_T^2} \right)^m$$

Result at $P_T \to 0$ needs to be resummed

Dokshitzer, Dyakonov, Troyan 1980
Parizi, Petronzio 1979
Collins, Soper 1982
Collins, Soper, Sterman 1985

} Implementation of resummation In QCD

# Resummation

Dokshitzer, Dyakonov, Troyan 1980
Parizi, Petronzio 1979
Collins, Soper 1982
Collins, Soper, Sterman 1985

Resummation (CSS)is in configuration space
Fourier transform is needed for observables

For Drell-Yan

$$\frac{d\sigma}{dq_T} \sim \int d^2 b_T e^{i q_T \cdot b_T} \hat{W}(x_1, x_2, b_T) e^{-S(b_T, Q)} + Y(q_T, Q)$$

Collinear distributions
are contained here

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Corrections for large $q_T$

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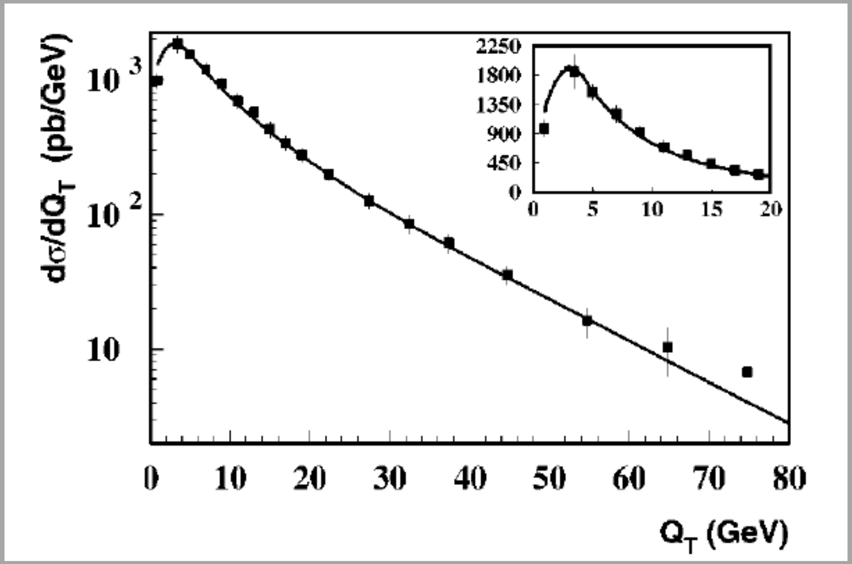
A lot of phenomenology done. Energies from 20 GeV to 2 TeV.

Brock, Landry, Nadolsky, Yuan 2003
Qiu, Zhang 2001



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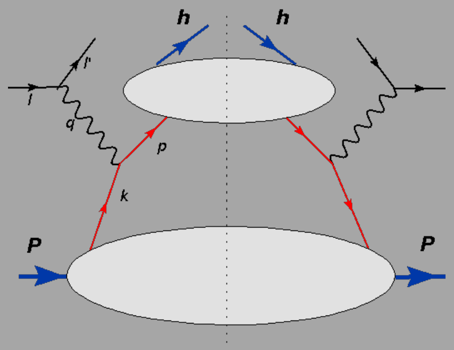
Drawbacks:

- Process dependent fits
- No direct connection to TMDs
- Designed for large energies

# Transverse Momentum Dependent distributions

**SIDIS**




$$\mathbf{l} + \mathbf{P} \to \mathbf{l}' + \mathbf{h} + \mathbf{X}$$

Designed for low transverse momenta

If produced hadron has low transverse momentum

$$P_{hT} \sim \Lambda_{QCD} << Q$$

it will be sensitive to quark transverse momentum $k_\perp$

TMD factorization

Parton model: Field, Feynman (1977)
Polarised case: Kotzinian (1995)
Mulders, Tangerman (1995)

QCD: Ji, Ma, Yuan (2002)
Collins(2011)


$$\Phi_{ij}(x, \mathbf{k}_\perp) = \int \frac{d\xi^-}{(2\pi)} \frac{d^2\xi_\perp}{(2\pi)^2} e^{ixP^+\xi^- - i\mathbf{k}_\perp\xi_\perp} \langle P, S_P | \bar{\psi}_j(0)\mathcal{U}(\mathbf{0}, \xi)\psi_i(\xi) | P, S_P \rangle$$

# Transverse Momentum Dependent distributions

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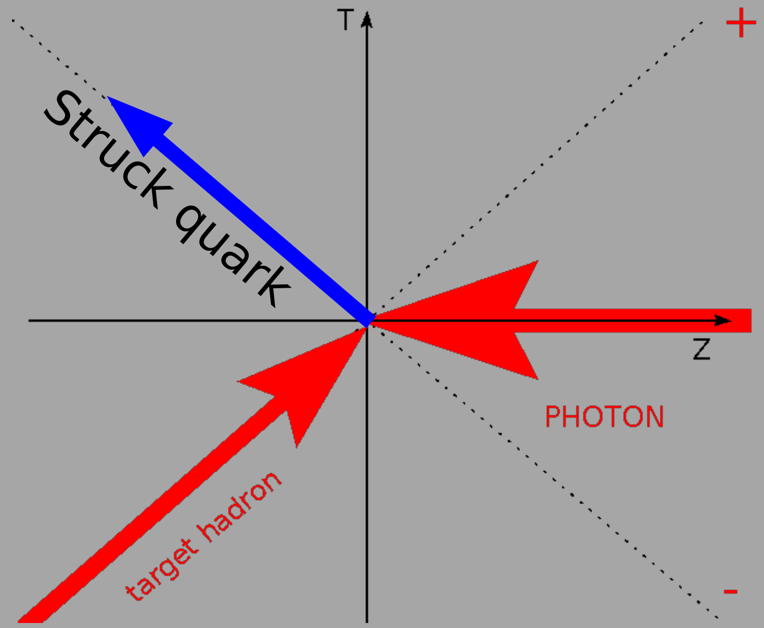
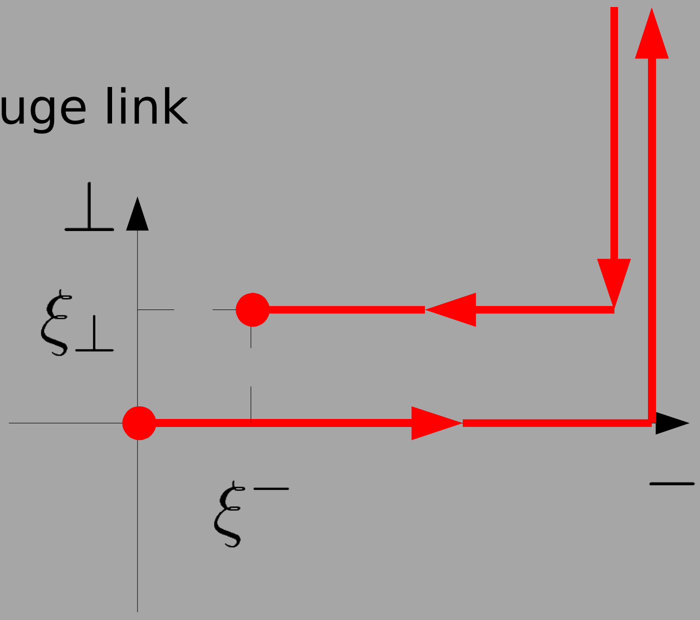
SIDIS in Infinite Momentum Frame:




Gauge link

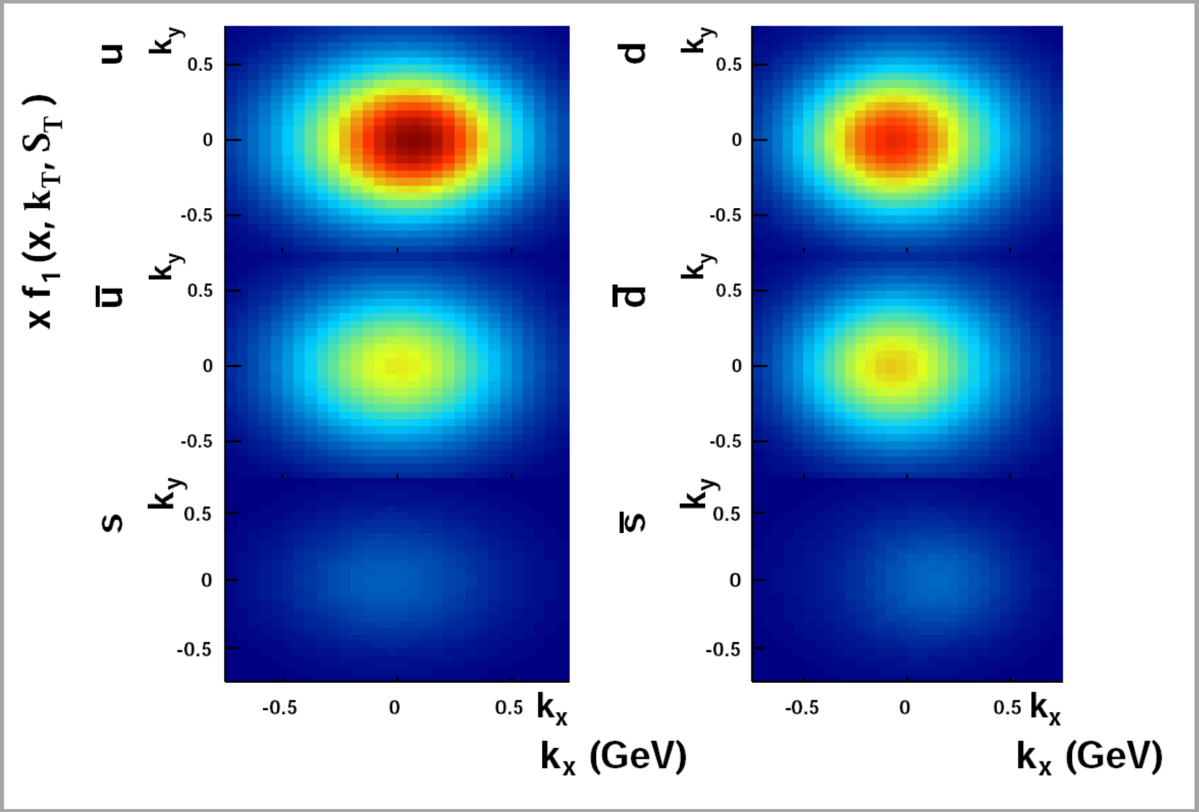



Transverse separation is due to presence of transverse parton momentum

Struck quark propagates in the gauge field of the remnant and forms gauge link

# TMDs give us 3D distributions

$$f(x, \mathbf{k_T}, \mathbf{S_T}) = f_1(x, \mathbf{k_T^2}) - f_{1T}^{\perp}(x, \mathbf{k_T^2}) \frac{\mathbf{k_{T1}}}{M}$$




The slice is at:

$$x = 0.1$$

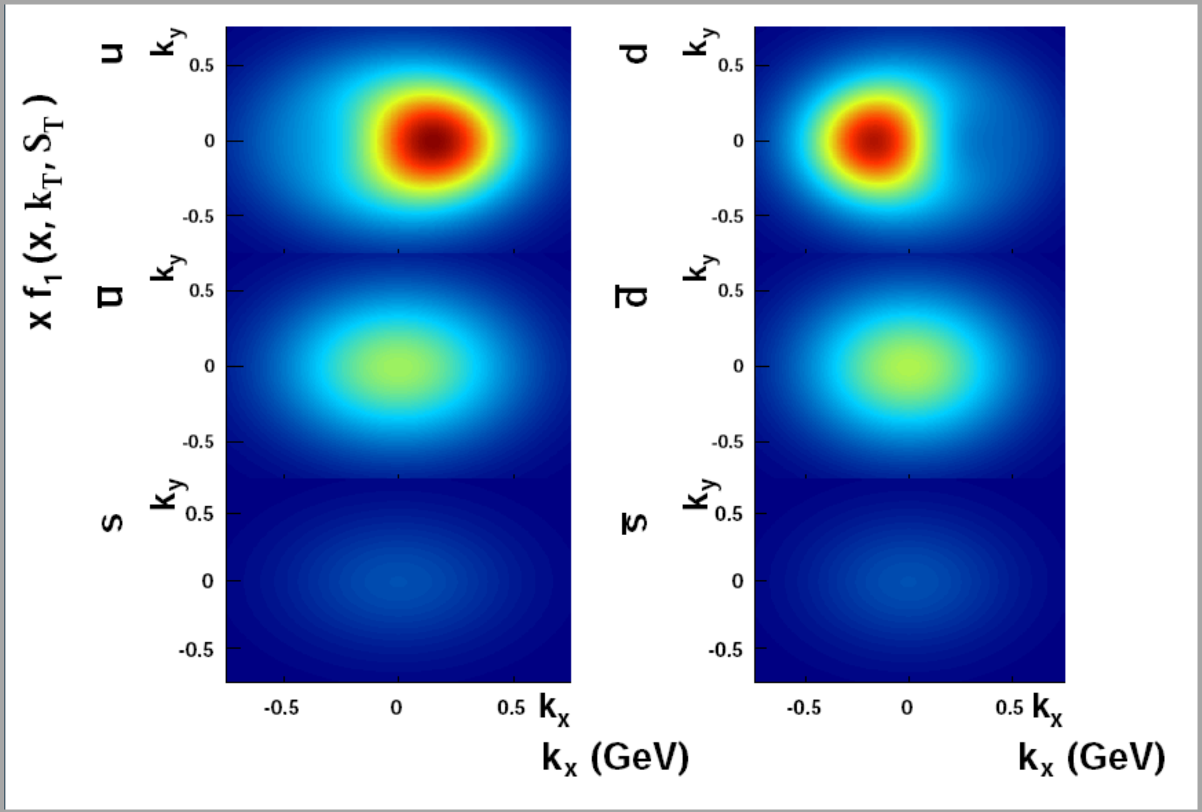
Low-x and high-x region is uncertain
JLab 12 and EIC will contribute

No information on sea quarks

Picture is still quite uncertain

# TMDs give us 3D distributions

$$f(x,\mathbf{k_T},\mathbf{S_T}) = f_1(x,\mathbf{k_T^2}) - f_{1T}^{\perp}(x,\mathbf{k_T^2})\frac{\mathbf{k_{T1}}}{M}$$




The slice is at:

$x = 0.1$

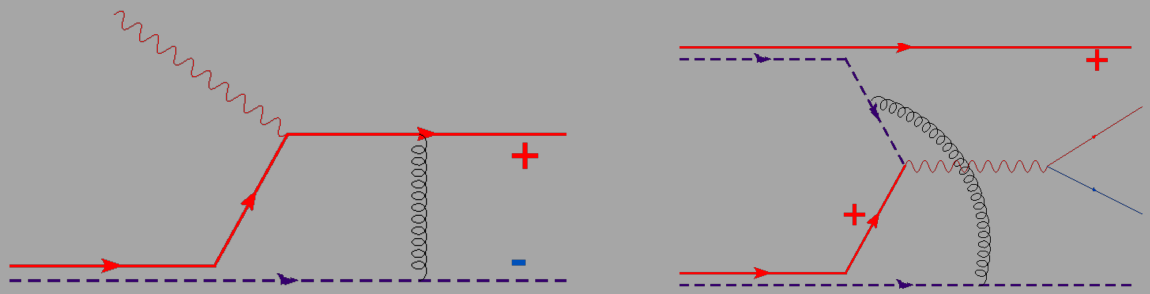
Low-x and high-x region is uncertain
JLab 12 and EIC will contribute

No information on sea quarks

In future we will obtain much clearer picture

# Physics of gauge links

Colored objects are surrounded by gluons, profound consequence of gauge invariance.
Sivers function has opposite sign when gluon couple after quark scatters (SIDIS) or before quark annihilates (Drell-Yan)




Brodsky,Hwang, Schmidt
Belitsky,Ji,Yuan
Collins
Boer,Mulders,Pijlman,
etc

$$f_{1T}^{\perp SIDIS} = -f_{1T}^{\perp DY}$$

One of the main goals is to verify this relation.
It goes beyond "just" check of TMD factorization.
Motivates Drell-Yan experiments

AnDY, COMPASS, JPARC, PAX etc
Barone et al., Anselmino et al., Yuan,Vogelsang, Schlegel et al., Kang,Qiu, Metz,Zhou

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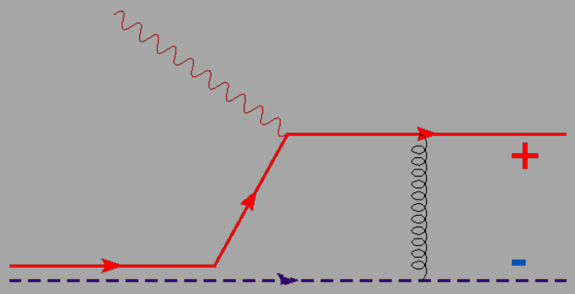


man,

$$f_{1T}^{\perp SIDIS} = -$$

Drell-Yan is at much different resolution scale Q.
EIC will operate at higher Q.
What do we know about evolution of TMDs?

One of the main goals is to verify ution.
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Motivates Drell-Yan experiments

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# Evolution of TMDs

One needs a unique definition of TMDs

Foundations of perturbative QCD
Collins 2011

$$W^{\mu\nu} = \sum_f |H_f(Q^2, \mu|^{\mu\nu}$$
$$\times \int d^2\mathbf{k}_{1T} d^2\mathbf{k}_{2T} F_{f/P_1}(x_1, \mathbf{k}_{1T}; \mu, \zeta_F) F_{\bar{f}/P_1}(x_2, \mathbf{k}_{2T}; \mu, \zeta_F)$$
$$\times \delta^{(2)}(\mathbf{k}_{1T} + \mathbf{k}_{2T} - \mathbf{q}_T) + Y(\mathbf{q}_T, Q)$$

$F_{f/P_1}(x_1, \mathbf{k}_{1T}; \mu, \zeta_F)$ **TMD distribution of partons in hadron**

Renorm group (RG) renormalization ($\mu$)

Rapidity divergence regulator ($\zeta_F$)

# Evolution of TMDs

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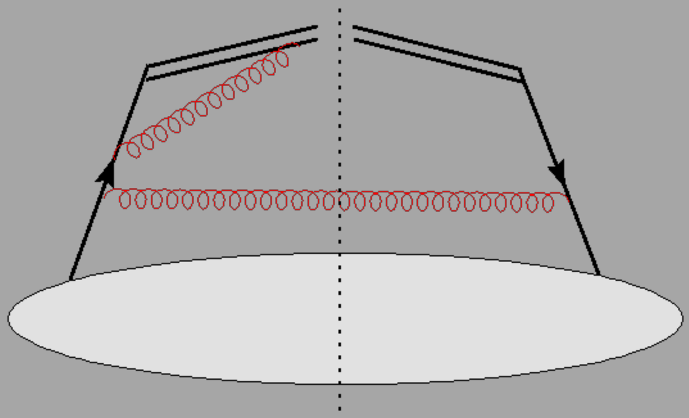
Foundations of perturbative QCD
Collins 2011



Infinite rapidity of the gluon creates so called rapidity divergence

In collinear PDFs this divergence is cancelled between virtual and real gluon diagrams

It is not the case for TMDs
Thus new regulator $\zeta_F$ is needed

$$F_{f/P_1}(x_1, \mathbf{k}_{1T}; \mu, \zeta_F)$$

Rapidity divergence regulator

Renorm group (RG) renormalization

# Evolution of TMDs

Evolution of TMDs is done in coordinate space $\mathbf{b}_T$

$$F_{f/P}(x, \mathbf{k}_T; \mu, \zeta_F) = \frac{1}{(2\pi)^2} \int d^2\mathbf{b}_T e^{i\mathbf{k}_T\cdot\mathbf{b}_T} \tilde{F}_{f/P}(x, \mathbf{b}_T; \mu, \zeta_F)$$

Colins Soper 1982
Foundations of perturbative QCD Collins 2011

Why coordinate space?

Convolutions become simple products:

Collins, Soper 1982
Collins, Soper, Sterman 1985
Idilbi, Ji, Ma, Yuan 2004
Boer, Gamberg, Musch, AP 2011

$$W^{\mu\nu} = \sum_f |H_f(Q^2, \mu|^{\mu\nu}$$
$$\times \int d^2\mathbf{b}_T e^{i\mathbf{b}_T\mathbf{q}_T} \tilde{F}_{f/P_1}(x_1, \mathbf{b}_T; \mu, \zeta_F) \tilde{F}_{\bar{f}/P_1}(x_2, \mathbf{b}_T; \mu, \zeta_F)$$

In principle experimental study of functions in coordinate space
Is possible

Boer, Gamberg, Musch, AP 2011

# Evolution of TMDs

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$$F_{f/P}(x, \mathbf{k}_T; \mu, \zeta_F) = \frac{1}{(2\pi)^2} \int d^2\mathbf{b}_T e^{i\mathbf{k}_T \cdot \mathbf{b}_T} \tilde{F}_{f/P}(x, \mathbf{b}_T; \mu, \zeta_F)$$

Colins Soper 1982
Foundations of perturbative QCD Collins 2011

Complicated in case of Sivers function

Aybat, Collins, Qiu, Rogers 2012

$$F_{f/P^\uparrow}(x, \mathbf{k}_T, \mathbf{S}_T; \mu, \zeta_F) = F_{f/P}(x, \mathbf{k}_T; \mu, \zeta_F) - F_{1T}^{\perp f}(x, \mathbf{k}_T; \mu, \zeta_F) \frac{\epsilon_{ij} k_T^i S^j}{M_p}$$

Unpolarised part:

$$\tilde{F}_{f/P}(x, b_T; \mu, \zeta_F) = (2\pi) \int_0^\infty dk_T k_T J_0(k_T b_T) F_{f/P}(x, k_T; \mu, \zeta_F)$$

Sivers function:

$$\tilde{F}_{1T}^{\prime \perp f}(x, b_T; \mu, \zeta_F) = -(2\pi) \int_0^\infty dk_T k_T^2 J_1(k_T b_T) F_{1T}^{\perp f}(x, k_T; \mu, \zeta_F)$$

# TMD evolution

Energy evolution

$$\frac{\partial \ln \tilde{F}(x, b_T, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T, \mu)$$

→ Collins-Soper kernel in coordinate space

Renormalization group equations

TMD:
Collins 2011
Rogers, Aybat 2011
Aybat, Collins, Qiu, Rogers 2011

$$\frac{d\tilde{K}(b_T, \mu)}{d \ln \mu} = -\gamma_K(g(\mu))$$

$$\frac{d \ln \tilde{F}(x, b_T, \mu, \zeta)}{d \ln \mu} = -\gamma_F(g(\mu), \zeta)$$

# TMD evolution

Energy evolution

$$\frac{\partial \ln \tilde{F}(x, b_T, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T, \mu)$$

→ Collins-Soper kernel in coordinate space

At small $\mathbf{b}_T$ perturbative treatment is possible

TMD:
Collins 2011
Rogers, Aybat 2011
Aybat, Collins, Qiu, Rogers 2011

$$\tilde{K}(b_T, \mu) = -\frac{\alpha_s C_F}{\pi}\Big(\ln(\mu^2 b_T^2) - \ln 4 + 2\gamma_E\Big) + \mathcal{O}(\alpha_s^2)$$

Large $\mathbf{b}_T$ nonperturbative – matching via $\mathbf{b}_*$ Collins Soper 1982

$$b_*(b_T) = \frac{b_T}{\sqrt{1 + b_T^2/b_{max}^2}}$$

# TMD evolution

Energy evolution

$$\frac{\partial \ln \tilde{F}(x, b_\perp, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_\perp, \mu)$$

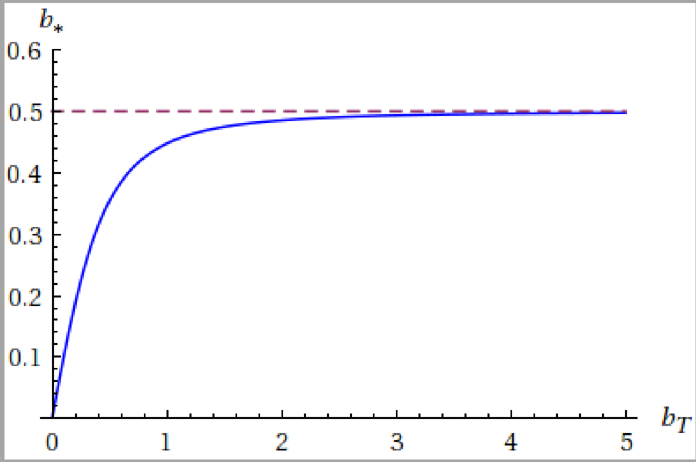
Collins-Soper kernel in coordinate space

Large $\mathbf{b}_T$ nonperturbative – matching via $\mathbf{b}_*$ Collins Soper 1982

$$b_*(b_T) = \frac{b_T}{\sqrt{1 + b_T^2/b_{max}^2}}$$

$$b_{max} = 0.5\ (\mathrm{GeV}^{-1})$$

Brock, Landry, Nadolsky, Yuan 2003



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Collins-Soper kernel in coordinate space

Large $\mathbf{b}_T$ nonperturbative – matching via $\mathbf{b}_*$ Collins Soper 1982

$$\tilde{K}(b_T, \mu) = \tilde{K}(b_*, \mu) - g_K(b_T)$$

Always perturbative

Non perturbative

$$g_K(b_T) = \frac{1}{2} g_2 b_T^2$$

$$g_2 \simeq 0.68 \; (GeV^2)$$

This function is universal for different partons!

Brock, Landry, Nadolsky, Yuan 2003

# TMD evolution

Relation to collinear treatment:

$$\tilde{F}_f(x, b_T, \mu, \zeta) = \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{j/f}\left(\frac{x}{\hat{x}}, b_T, \mu, \zeta\right) f_j(\hat{x}, \mu) + \mathcal{O}(\Lambda_{QCD} b_T)$$

Collins Soper 1982

Valid at small $\mathbf{b}_T$ , lowest order:

$$\tilde{C}_{j/f}(\frac{x}{\hat{x}}, b_T, \mu, \zeta) = \delta_{jf} \delta\left(\frac{x}{\hat{x}} - 1\right) + \mathcal{O}(\alpha_s)$$

Higher order for TMD PDFs

Aybat Rogers 2011

Higher order for Sivers function

Kang, Xiao, Yuan 2011

# TMD evolution

Solution

Rogers, Aybat 2011
Aybat, Collins, Qiu, Rogers 2011

$$\tilde{F}_{f/P}(x,b_T;Q,\zeta_F) = \tilde{F}_{f/P}(x,b_T;Q_0,Q_0^2)$$
$$\times \exp\Big[-g_K(b_T)\ln\frac{Q}{Q_0}\Big]$$

Non perturbative

$$\times \exp\Big[\ln\frac{Q}{Q_0}\tilde{K}(b_*;\mu_b) + \int_{Q_0}^{Q}\frac{d\mu'}{\mu'}\Big[\gamma_F(g(\mu');1) - \ln\frac{Q}{\mu'}\gamma_K(g(\mu'))\Big]$$
$$+ \int_{Q_0}^{\mu_b}\frac{d\mu'}{\mu'}\ln\frac{Q}{Q_0}\gamma_K(g(\mu'))\Big]$$

Perturbative

Typically for TMDs:

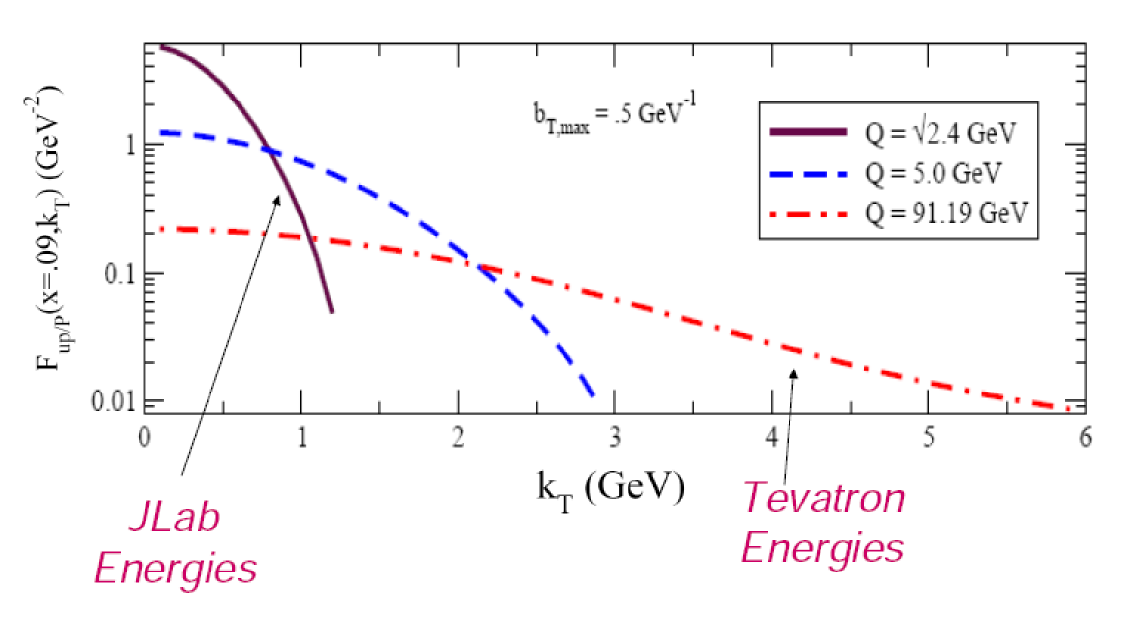
$$\tilde{F}_{f/P}(x,b_T;Q_0,Q_0^2) = F_{f/P}(x;Q_0)\exp\Big(-\frac{\langle k_T^2\rangle}{4}b_T^2\Big)$$

# TMD evolution

Solution  Rogers, Aybat 2011
Aybat, Collins, Qiu, Rogers 2011

$$\tilde{F}_{f/P}(x, b_T; Q, \zeta_F) = F_{f/P}(x; Q_0) \exp\Big( - \Big[\frac{\langle k_T^2\rangle}{4} + \frac{g_2}{2}\ln\frac{Q}{Q_0}\Big] b_T^2 \Big)$$

Non perturbative

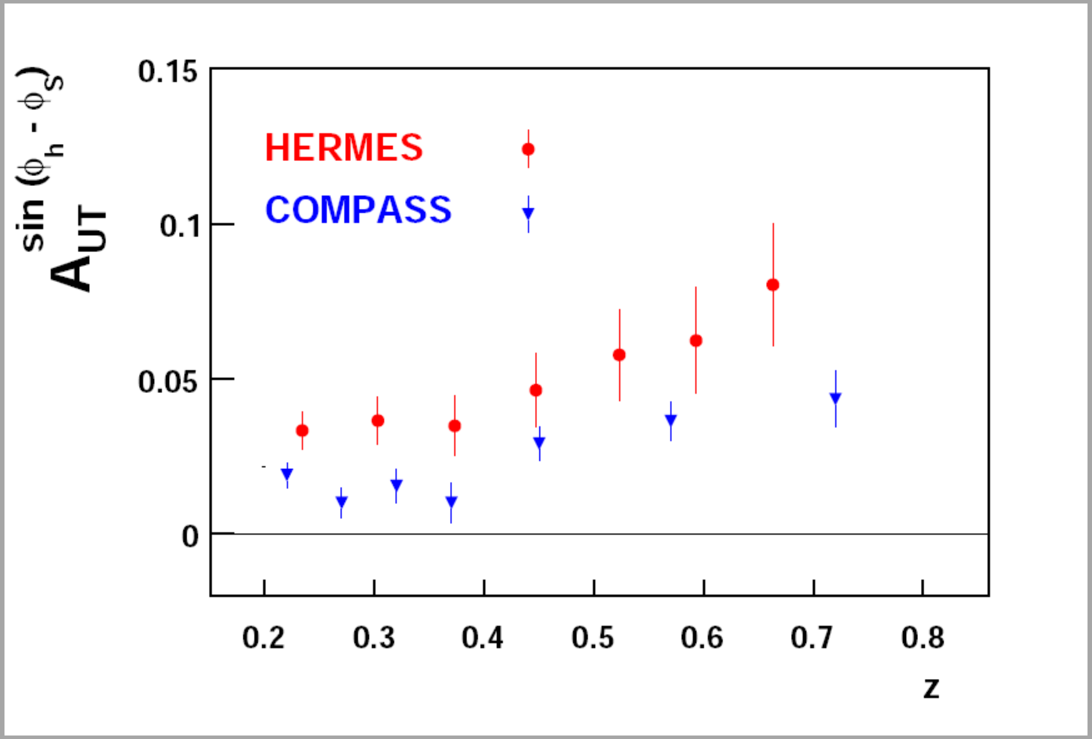



Gaussian behaviour is appropriate only in a limited range

TMDs change with energy and resolution scale

# TMD evolution

Can we see signs of evolution in the experimental data?




Aybat, AP, Rogers  2011

COMPASS data is at

$$\langle Q^2 \rangle \simeq 3.6 \ (GeV^2)$$

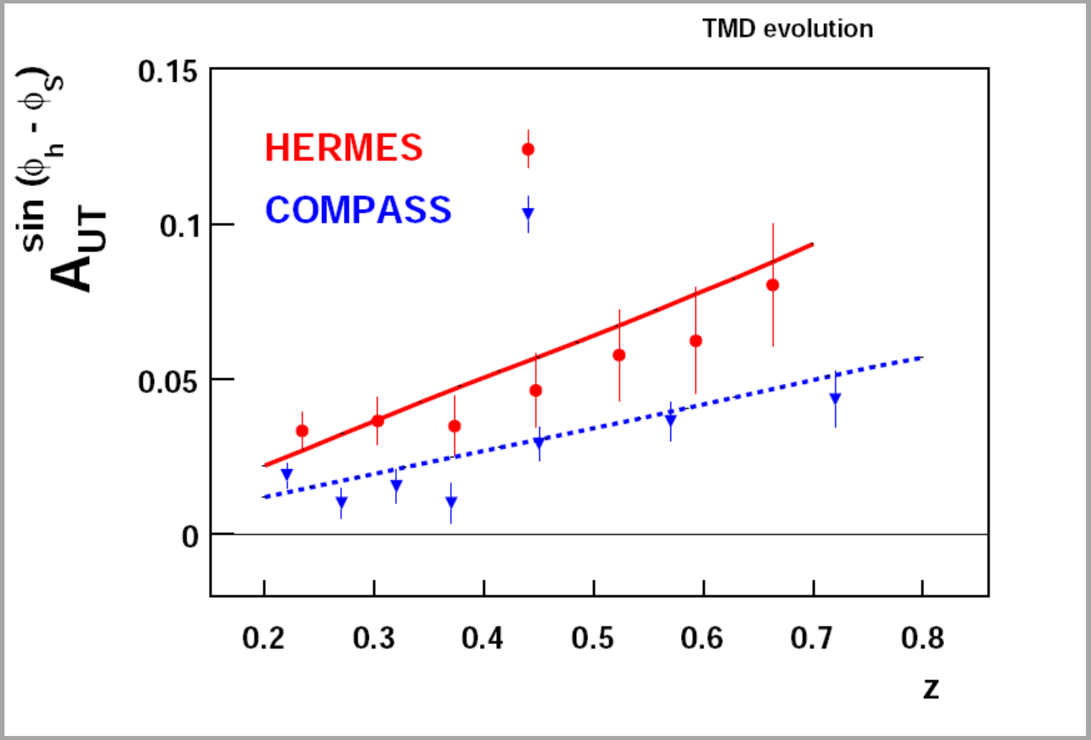
HERMES data is at

$$\langle Q^2 \rangle \simeq 2.4 \ (GeV^2)$$

# TMD evolution

Can we explain the experimental data?

Full TMD evolution is needed!




Aybat, AP, Rogers 2011

COMPASS dashed line

$$\langle Q^2 \rangle \simeq 3.6 \; (GeV^2)$$

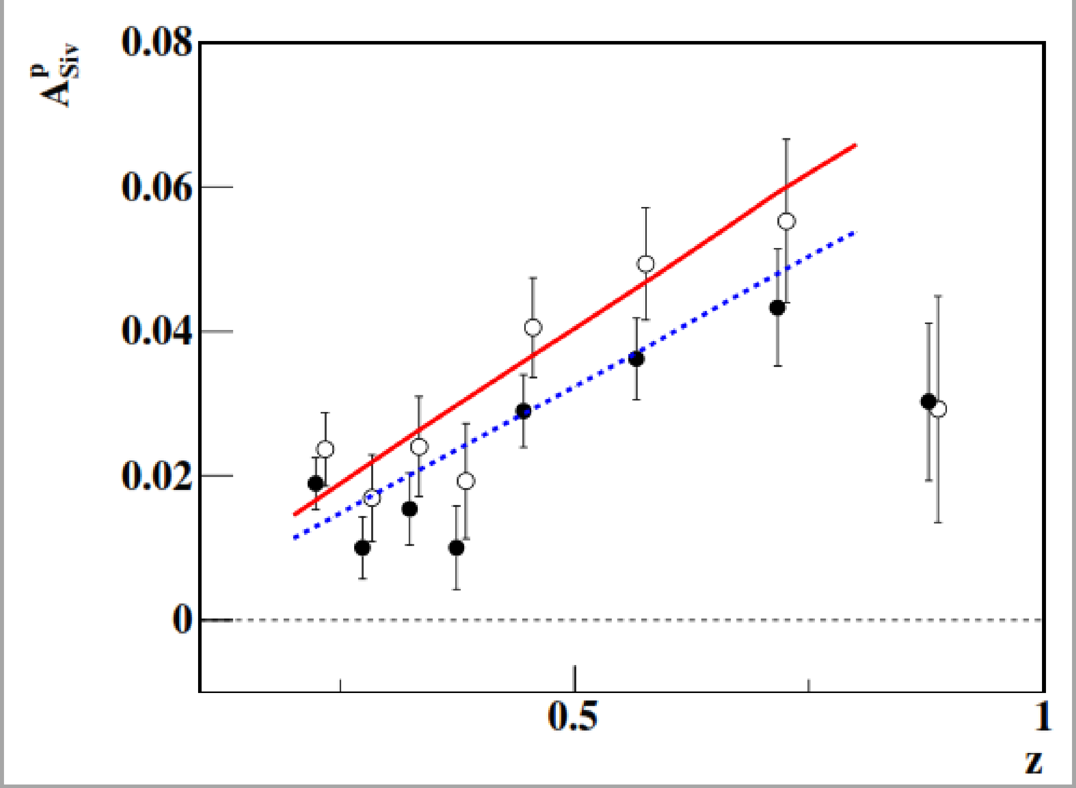
HERMES solid line

$$\langle Q^2 \rangle \simeq 2.4 \; (GeV^2)$$

# Blindfolded comparison

Can we predict the experimental data that we have not seen?

Aybat, AP, Rogers 2011




COMPASS red line

$0.032 < x < 0.7$

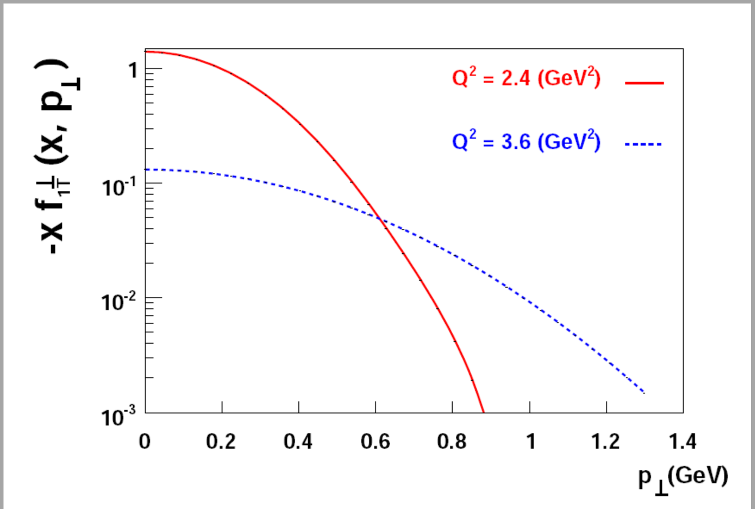
COMPASS dashed line

full x range of COMPASS

See talk of Anna Martin at
QCD EVOLUTION 2012
JLAB 14-17 May, 2012

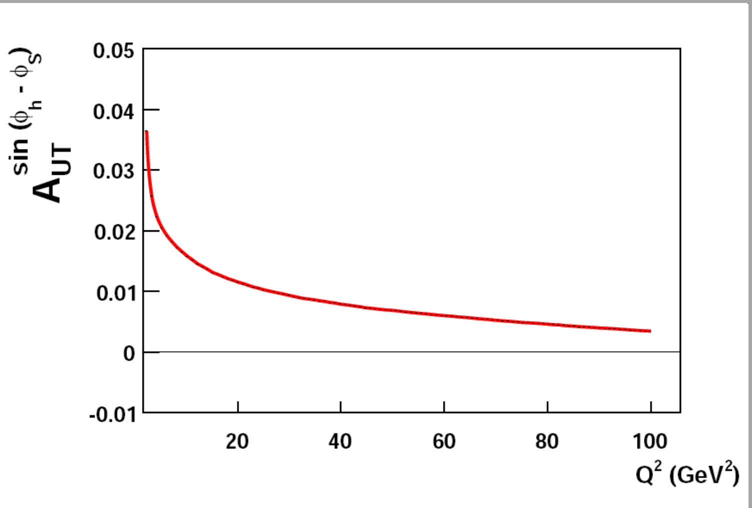
# TMD evolution

This is the first implementation of TMD evolution for observables




Aybat, AP, Rogers 2011

Asymmetry changes with $Q^2$

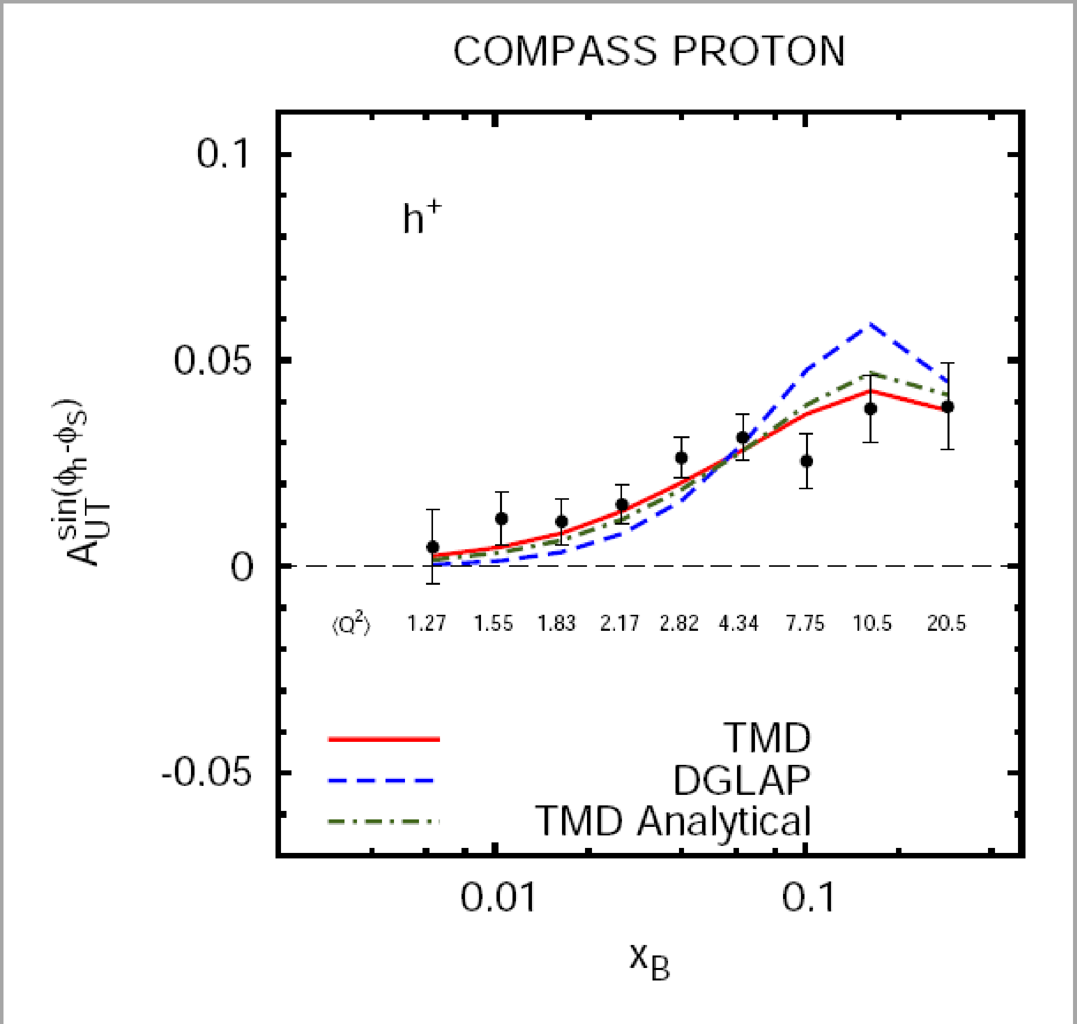



Functions change with energy

Phenomenological analysis with evolution is now possible

# TMD evolution

The same conclusions in




Anselmino, Boglione, Melis 2012

Solid line – TMD evolution fit
Dashed line – DGLAP fit

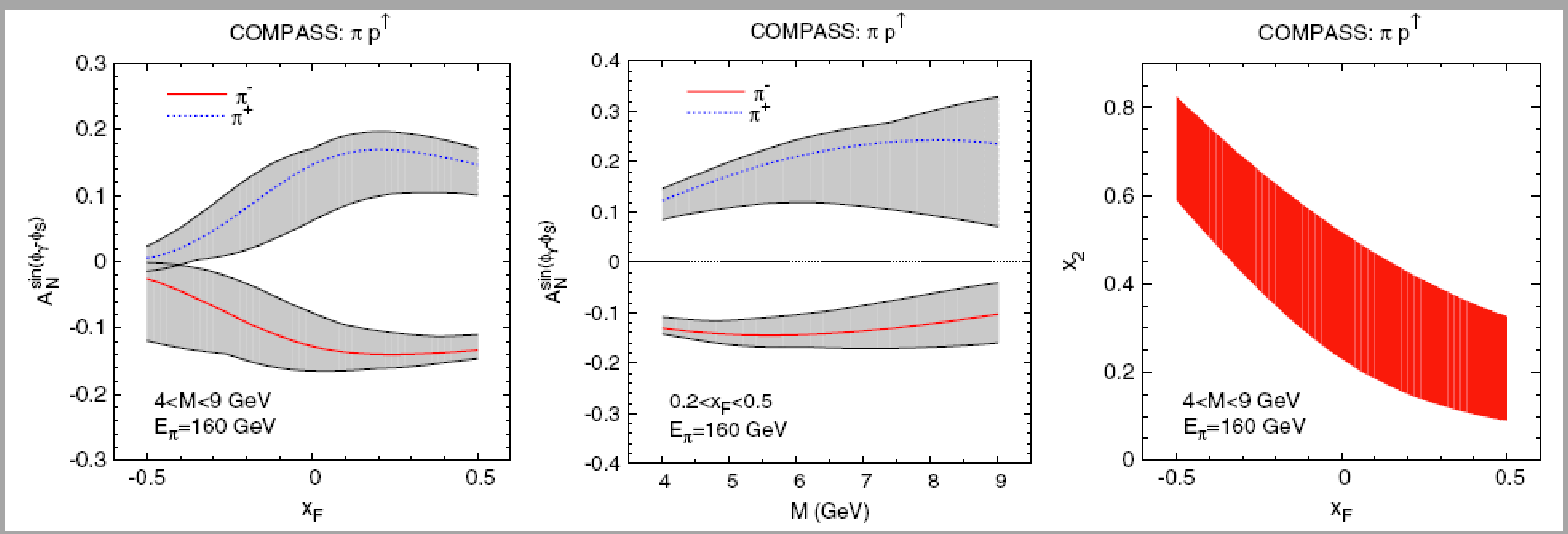
# Drell Yan

$$\mathbf{A_N} = \frac{\sum_q \mathbf{f_{1T}^{\perp q}(x_2, p_T) \otimes f_1^{\bar{q}}(x_1, p_T)}\sigma_{\mathbf{q\bar{q}}}}{\sum_q \mathbf{f_1^{q}(x_2, p_T) \otimes f_1^{\bar{q}}(x_1, p_T)}\sigma_{\mathbf{q\bar{q}}}}$$

Analysis at LO in hadronic cm frame

Anselmino et al (2009)

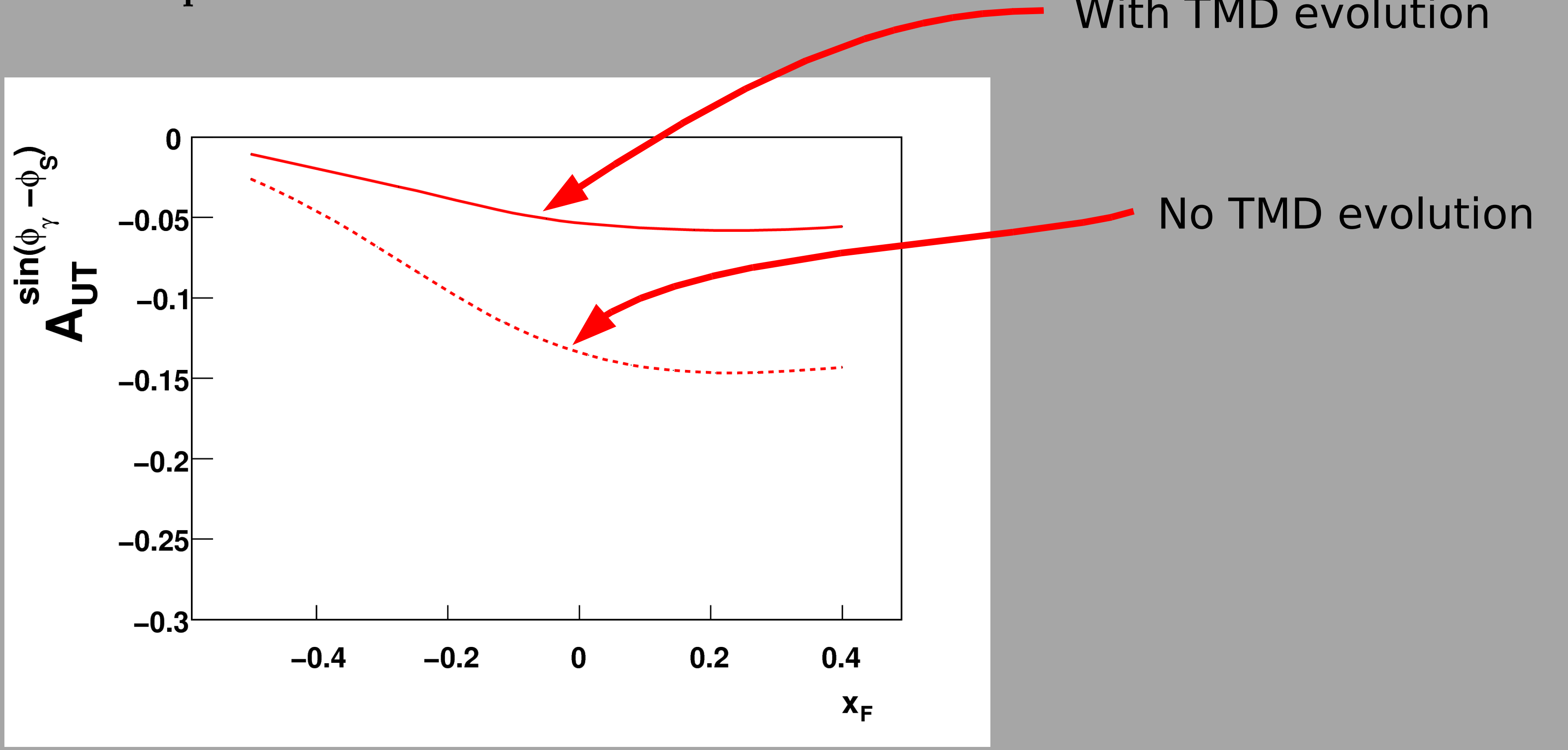
$$\sqrt{s} = 17.4 \ \ (\mathrm{GeV})$$



# Drell Yan

$$\mathbf{A_N = \frac{\sum_q f_{1T}^{\perp q}(x_2, p_T) \otimes f_1^{\bar{q}}(x_1, p_T)\sigma_{q\bar{q}}}{\sum_q f_1^{q}(x_2, p_T) \otimes f_1^{\bar{q}}(x_1, p_T)\sigma_{q\bar{q}}}}$$

**Analysis in hadronic cm frame**



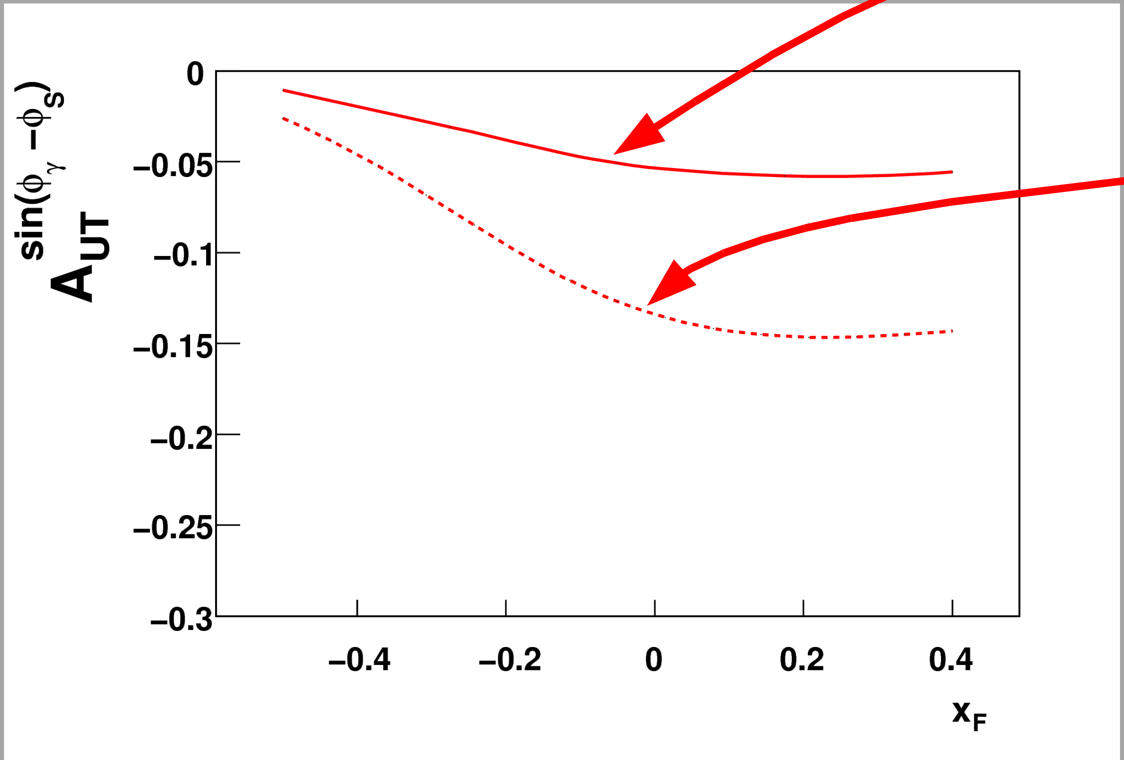

Asymmetry is suppesed with respect to LO analysis

# Drell Yan

$$A_N = \frac{\sum_q f_{1T}^{\perp q}(x_2, p_T) \otimes f_1^{\bar{q}}(x_1, p_T)\sigma_{q\bar{q}}}{\sum_q f_1^{q}(x_2, p_T) \otimes f_1^{\bar{q}}(x_1, p_T)\sigma_{q\bar{q}}}$$

**Analysis in hadronic cm frame**

With TMD evolution




No TMD evolution


Result with TMD Evolution depends on the choice of $g_K(b_T)$

Unpolarised x-sections should be reanalyzed

Asymmetry is suppesed with respect to LO analysis

# What is needed?

In order to fix TMD evolution fits one would need to have

- Unpolarised cross-sections as a function of $P_T$ for SIDIS, Drell-Yan and $e^+e^-$
- Unpolarised cross-sections at different energies and different values of $Q^2$
- Asymmetries at different energies and different values of $Q^2$

**COMPASS will be source of a lot of information!**

# Theoretical uncertainties

Relation to collinear treatment:

$$\tilde{F}'^{\perp}_{1T}(x, b_T, \mu, \zeta) = \sum_j \frac{M_p b_T}{2} \int_x^1 \frac{d\hat{x}_1}{\hat{x}_1} \frac{d\hat{x}_2}{\hat{x}_2} \tilde{C}^{Sivers}_{j/f}\Big(\hat{x}_1, \hat{x}_2, b_T, \mu, \zeta\Big) T_{Fj}(\hat{x}_1, \hat{x}_2, \mu)$$

Aybat, Collins, Qiu, Rogers 2011

Valid at small $\mathbf{b}_T$ , lowest order:

$$\tilde{C}_{j/f}(\hat{x}_1, \hat{x}_2, b_T, \mu, \zeta) = \delta_{jf}\delta\Big(\frac{x}{\hat{x}_1} - 1\Big)\delta\Big(\frac{x}{\hat{x}_2} - 1\Big) + \mathcal{O}(\alpha_s)$$

Higher order for Sivers function Kang, Xiao, Yuan 2011

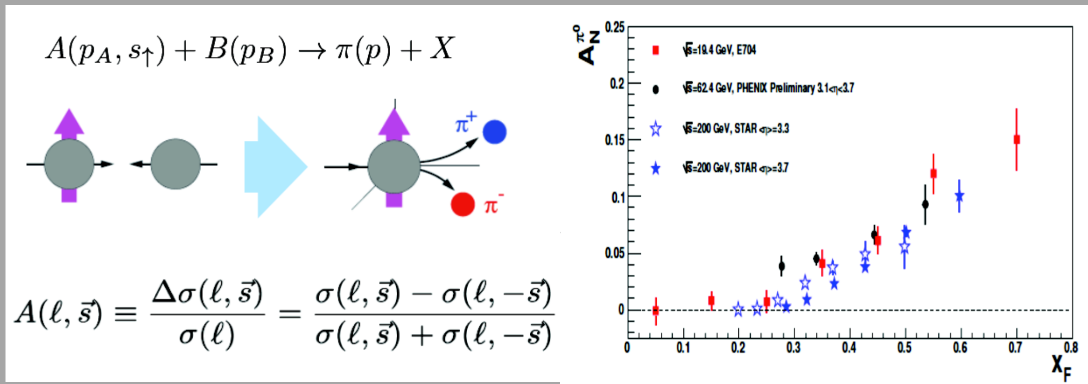
At lowest order we have:

$$\int \mathbf{d^2 p_T} \frac{\mathbf{p_T^2}}{\mathbf{M}} \mathbf{f^{\perp}_{1T}}(\mathbf{x}, \mathbf{p_T^2}) + \mathbf{UVCT}(\mu^2) = \mathbf{T_F}(\mathbf{x}, \mathbf{x}, \mu^2) \qquad \mathbf{f^{\perp(1)}_{1T}} \equiv \int \mathbf{d^2 p_T} \frac{\mathbf{p_T^2}}{\mathbf{2M^2}} \mathbf{f^{\perp}_{1T}}(\mathbf{x}, \mathbf{p_T^2})$$

Sivers function is related to TF, but counterterm matters!

# Data analysis

## **Proton Proton** Left -Right asymmetry




Only one scale $P_T$

Collinear analysis:

Kouvaris, Qiu,

Vogelsang, Yuan (2006)

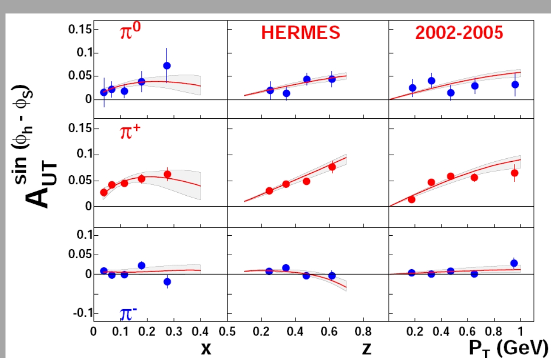
Kanazava, Koike (2010)

TMD analysis:

Anselmino et al (2006)

## SIDIS

$$A_{UT} = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} \qquad d\sigma^\uparrow - d\sigma^\downarrow \propto \underbrace{f_{1T}^\perp \otimes D_1 \sin(\phi_h - \phi_S)}_{\text{Sivers effect}}$$




Two scales $P_T, Q$

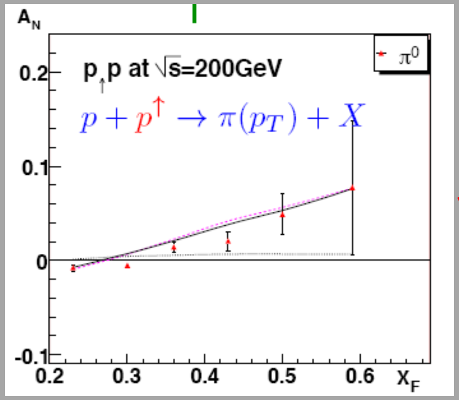
$$\Lambda^2_{QCD} < P^2_{h\perp} \ll Q^2$$

TMD analysis: Anselmino et al (2008); Collins et al (2007) ; Vogelsang, Yuan (2006)

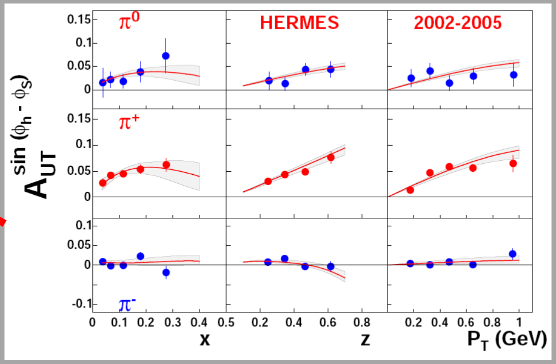
# Comparison of results

Kang, Qiu, Vogelsang, Yuan (2011)

$$g_s T_F(x,x) = -2M f_{1T}^{\perp(1)}(x)$$




Collinear analysis: Kouvaris, Qiu, Vogelsang, Yuan (2006)



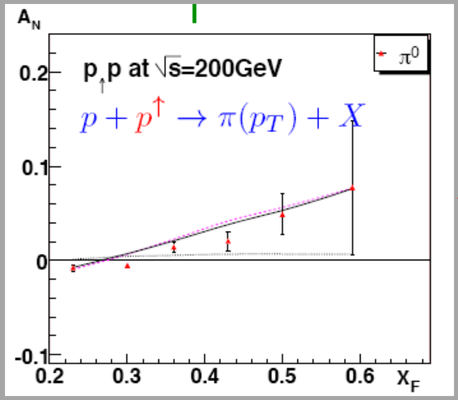

TMD analysis:

Anselmino et al (2008)

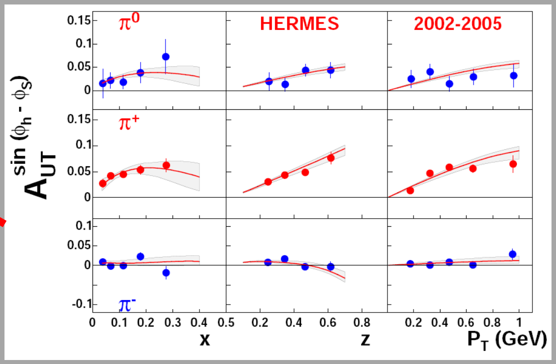
# Comparison of results

Kang, Qiu, Vogelsang, Yuan (2011)

$$g_s T_F(x,x) = -2M f_{1T}^{\perp(1)}(x)$$

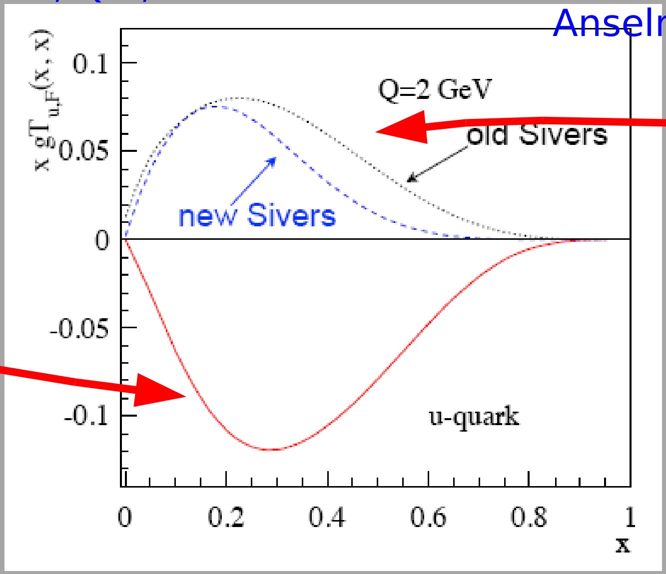



Compare




Collinear analysis: Kouvaris, Qiu, Vogelsang, Yuan (2006)

TMD analysis: Anselmino et al (2008)




Alexei Prokudin

# Comparison of results

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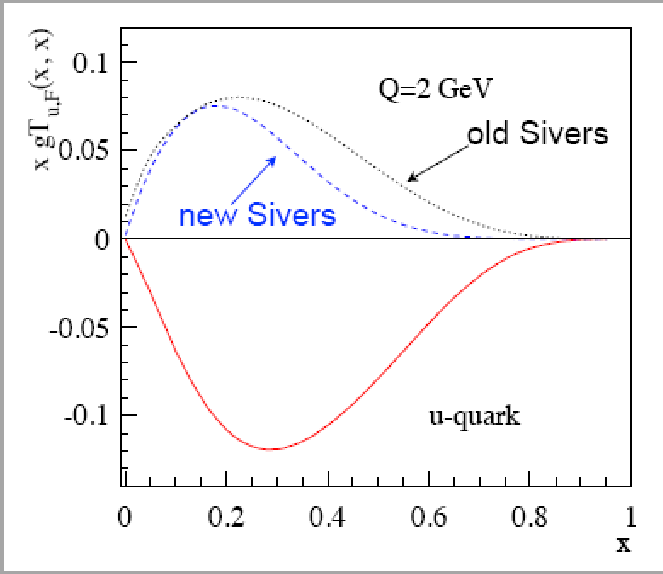
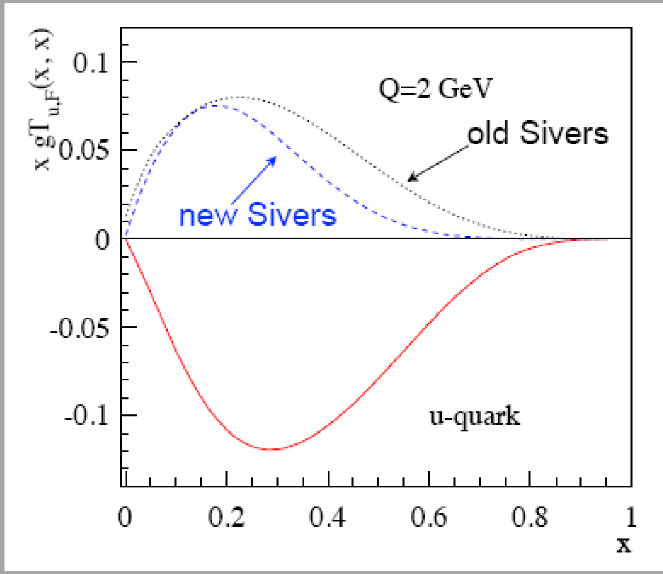



Kang, Qiu, Vogelsang, Yuan (2011)

- Magnitudes are similar
- Sign is opposite

# Comparison of results

$$g_s T_F(x,x) = -2M f_{1T}^{\perp(1)}(x)$$




Kang, Qiu, Vogelsang, Yuan (2011)

- Magnitudes are similar
- Sign is opposite

**It is a puzzle!**

# Possible explanations

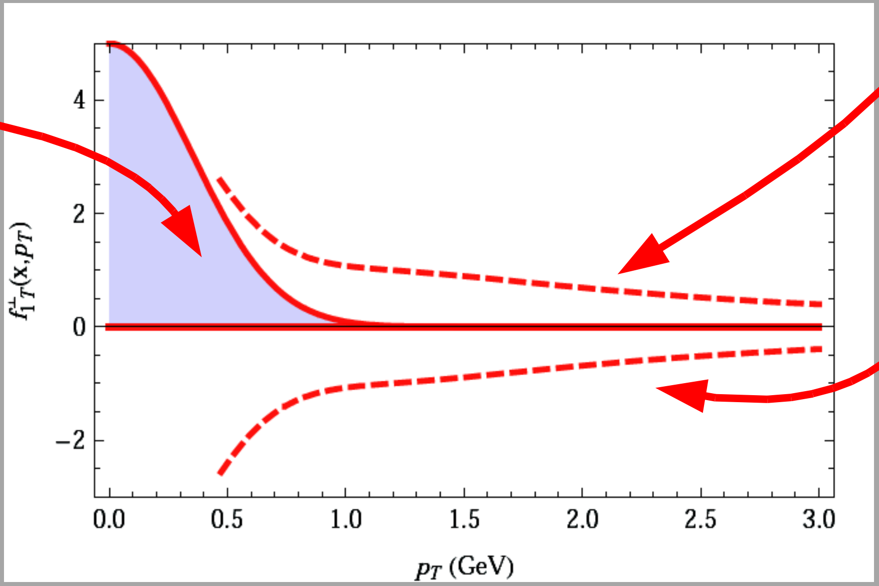
Sivers function can have nodes in $\mathbf{k_T}$.

Kang, AP (2012)

SIDIS $\mathbf{T_F > 0}$

Perturbarive tail $\propto \frac{\mathbf{M^2}}{\mathbf{k_T^4}}\alpha_\mathbf{s}$

Bacchetta, Boer, Deihl, Mulders 2008



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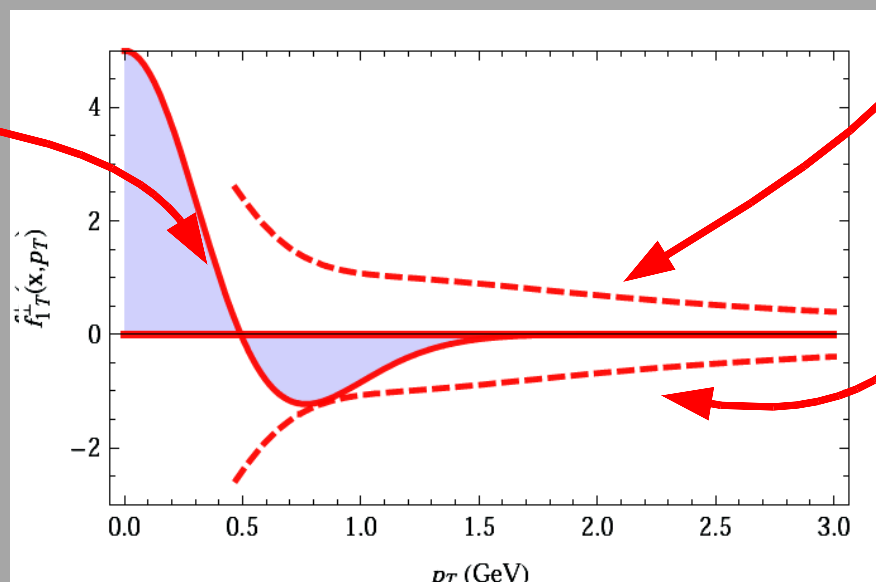
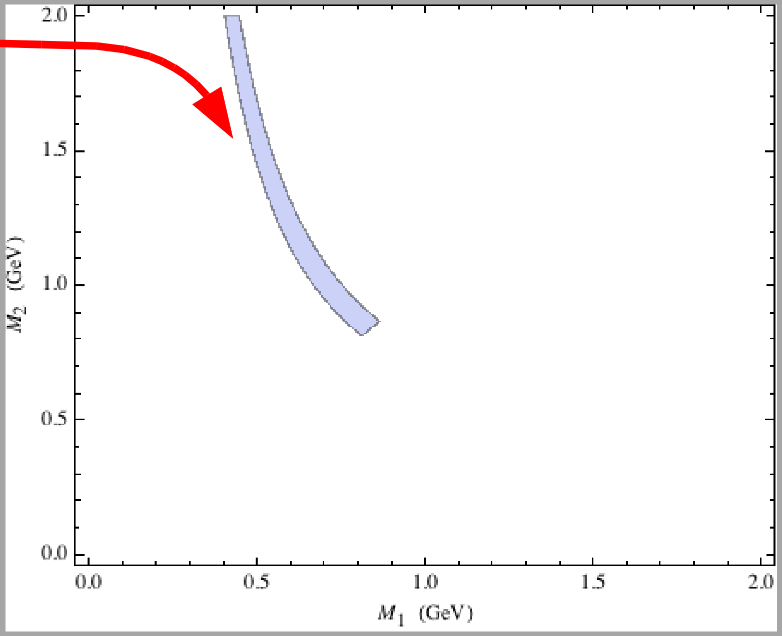



Bacchetta, Boer, Deihl, Mulders 2008

# Possible explanations

Sivers function can have nodes in $\mathbf{k_T}$.
Kang, AP (2012)

Allowed region in parameter space




Appears to be not a natural solution!

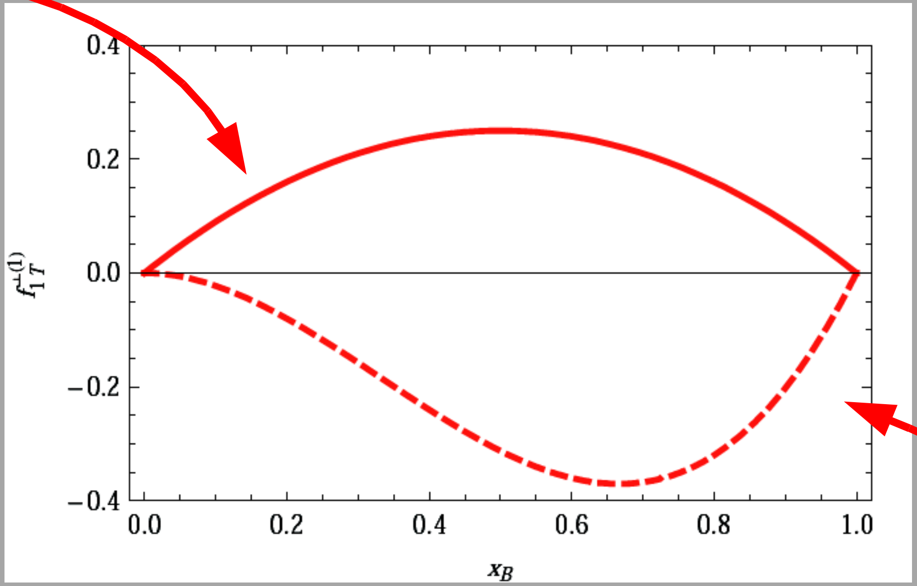
# Possible explanations

Sivers function can have nodes in x.

Boer (2011)

Bacchetta et al, model calculation (2010), Kang, AP (2012)

SIDIS



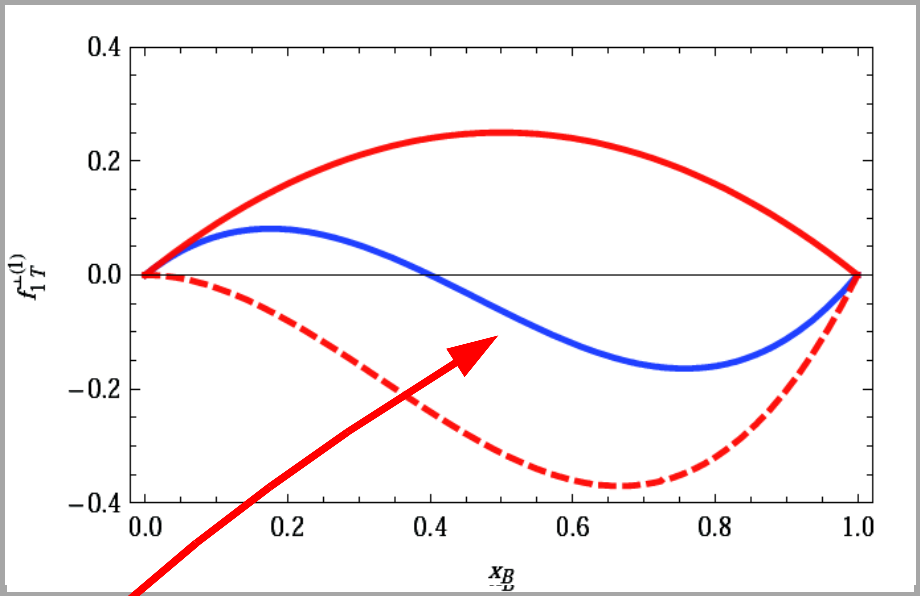

PP

# Possible explanations

Sivers function can have nodes in x.

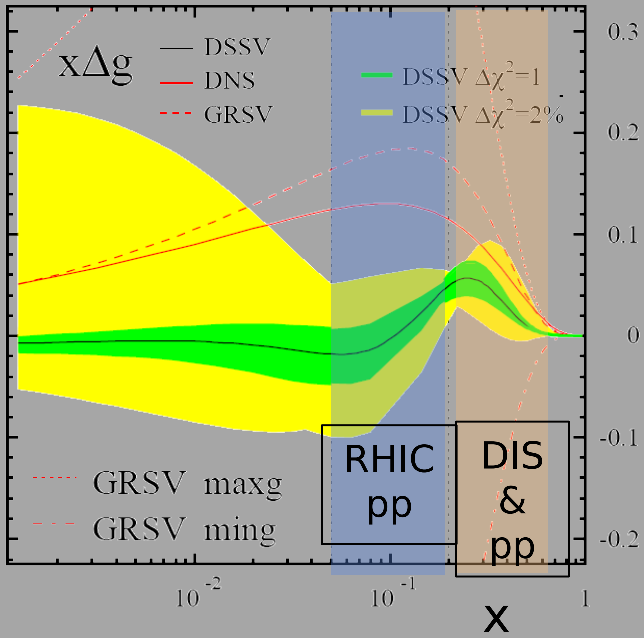
Boer (2011)

Bacchetta et al, model calculation (2010)




If PP and SIDIS probe different regions of x

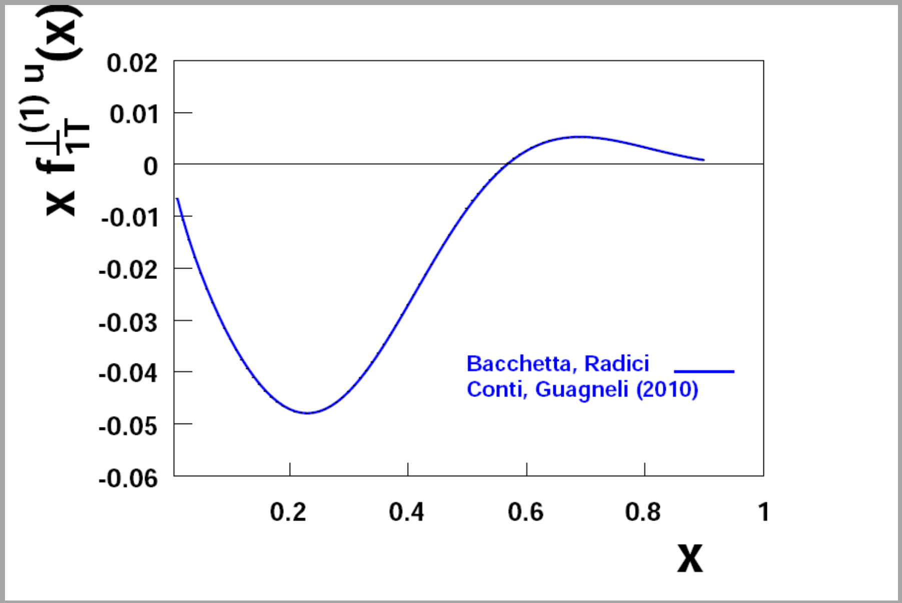
# Are nodes so strange?




Node in $\mathbf{\Delta g(x)}$ from DSSV global fit De Florian, Sassot, Stratmann, Vogelsang

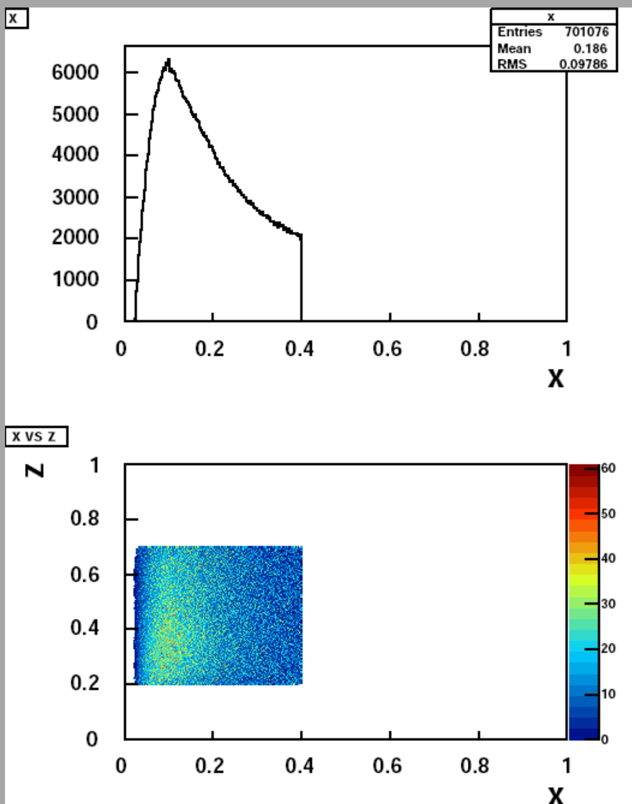
$$\mathbf{\Delta f \propto f(S) - f(-S)}$$

Node in Sivers function
Bacchetta, Radici,
Conti, Guagneli (2010)



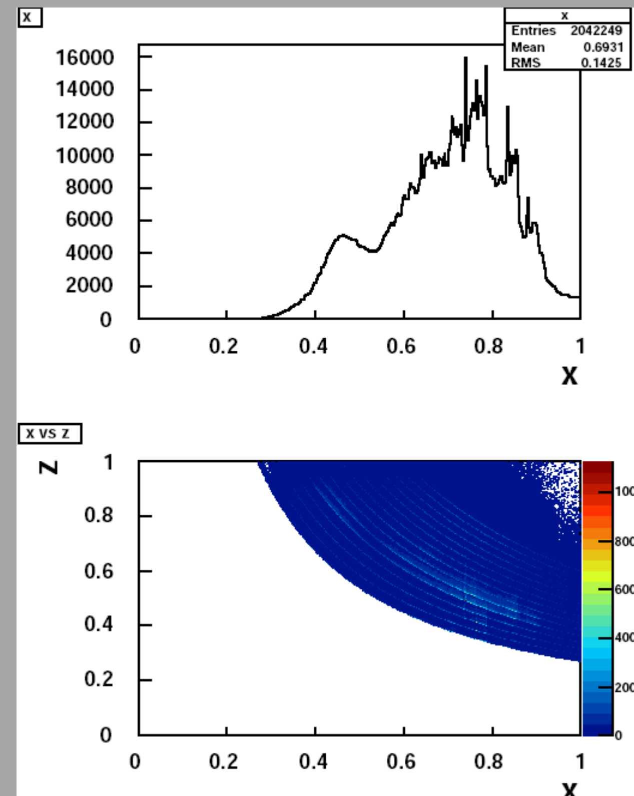
# SIDIS vs PP kinematics

**SIDIS HERMES**




$$\mathbf{x < 0.4}$$

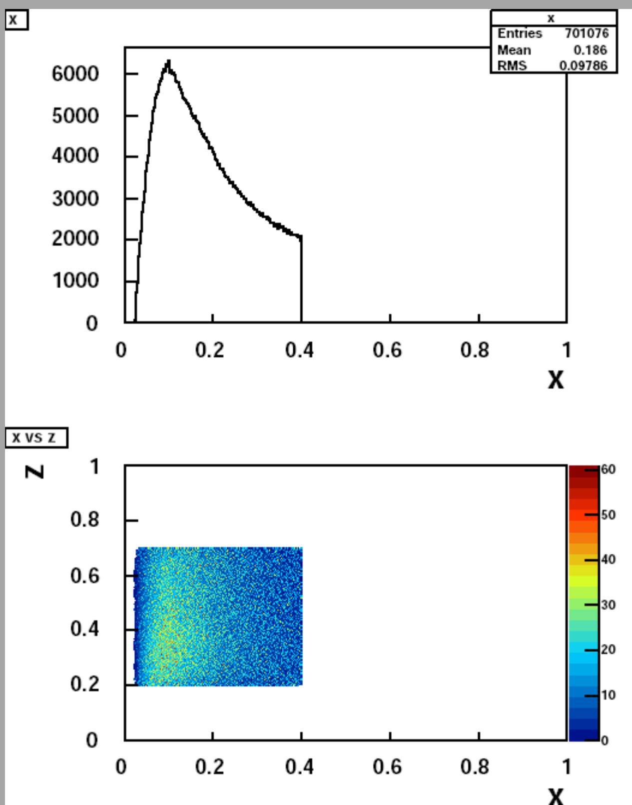
**PP STAR**




$$\mathbf{x > 0.4}$$

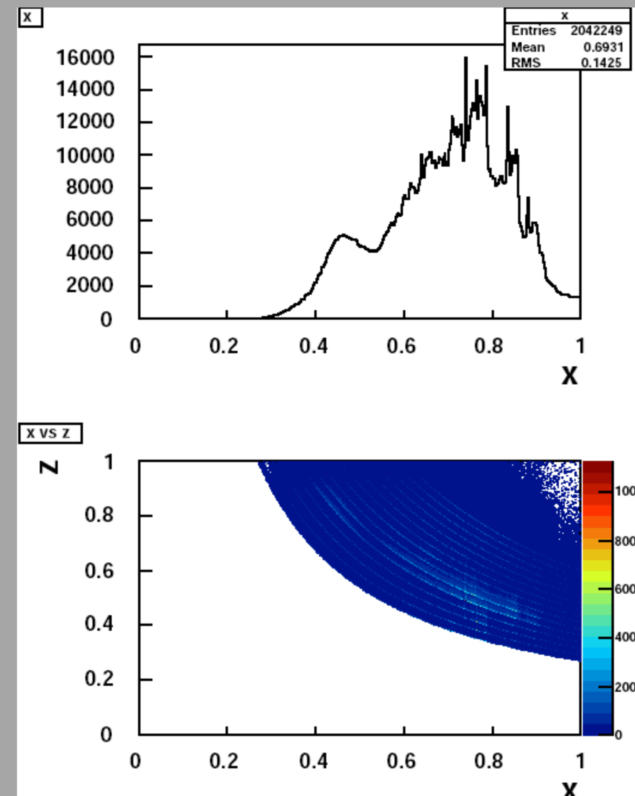
# SIDIS vs PP kinematics

**SIDIS HERMES**




$$\mathbf{x < 0.4}$$

**PP STAR**




$$\mathbf{x > 0.4}$$

SIDIS and PP probe different regions in x !

# Parametrization

$$\mathbf{f_{1T}^{\perp q}} \propto \mathbf{x}^{\alpha_\mathbf{q}}(\mathbf{1}-\mathbf{x})^{\beta_\mathbf{q}}(\mathbf{1}-\eta_\mathbf{q}\mathbf{x})$$

as in De Florian, Sassot, Stratmann, Vogelsang (2009)

$\mathbf{1}-\eta_\mathbf{q}\mathbf{x}$ has a node if $\eta_\mathbf{q} > \mathbf{0}$

**SIDIS: HERMES, COMPASS data** $\pi^\pm$

TMD

$$\mathbf{A_{UT}^{\sin(\Phi_h-\Phi_S)}} \sim \mathbf{f_{1T}^{\perp}} \otimes \sigma \otimes \mathbf{D_1}$$

**PP: STAR data** $\pi^\mathbf{0}$ **BRAHMS data** $\pi^\pm$

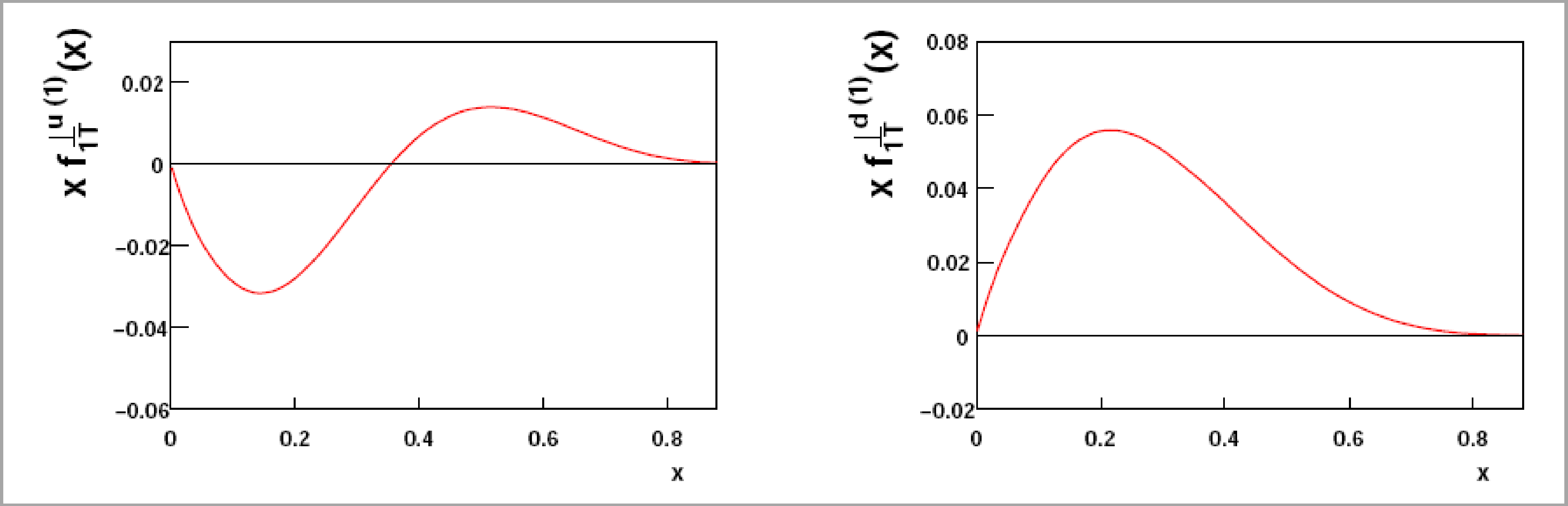
Twist-3

$$\mathbf{A_N} \sim \mathbf{T_F} \otimes \sigma \otimes \mathbf{D_1}$$

•using PDF GRV98 and FF DSSV

# Results: Sivers function

Kang, AP (2012)

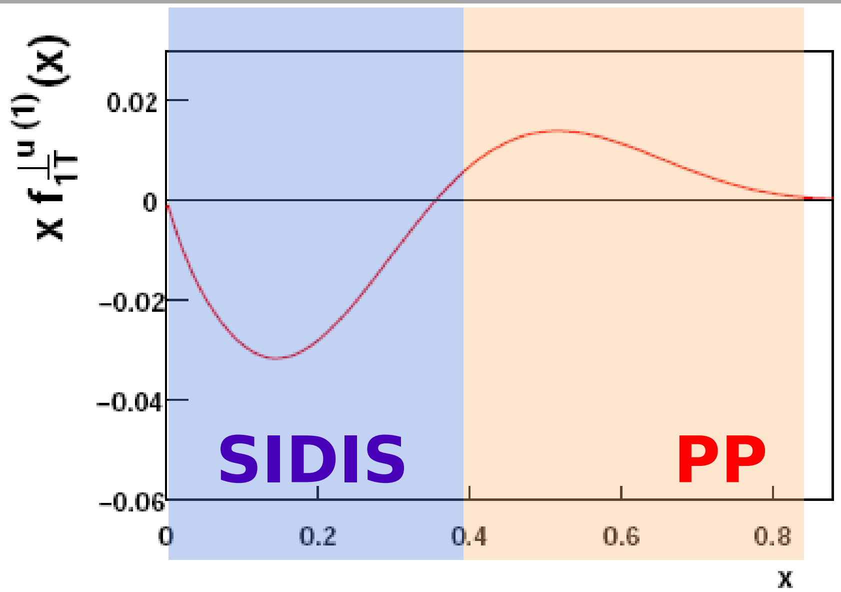


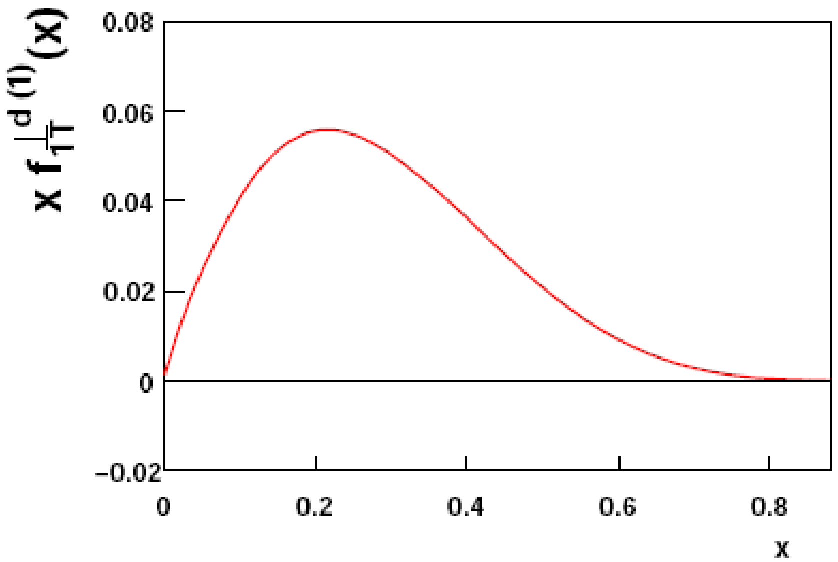

**Sivers function can have a node!**

$$\mathbf{x_{node} \sim 0.35}$$

# Results: Sivers function

Kang, AP (2012)

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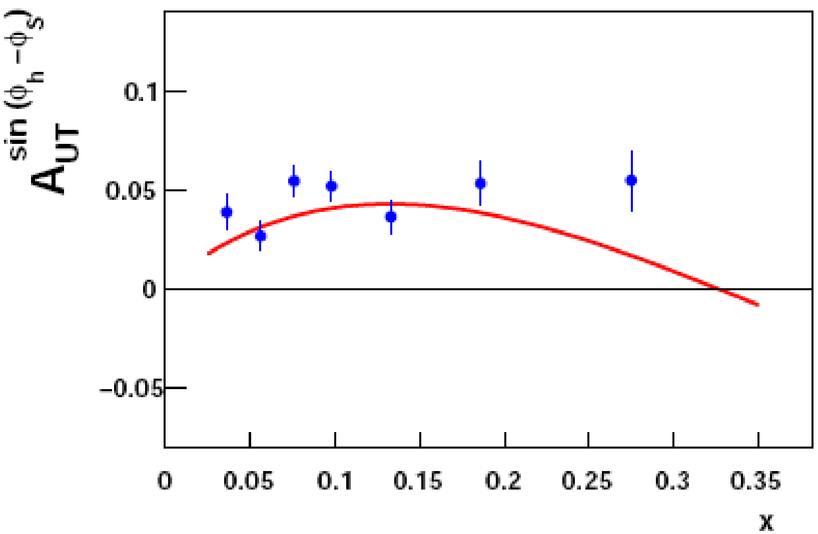
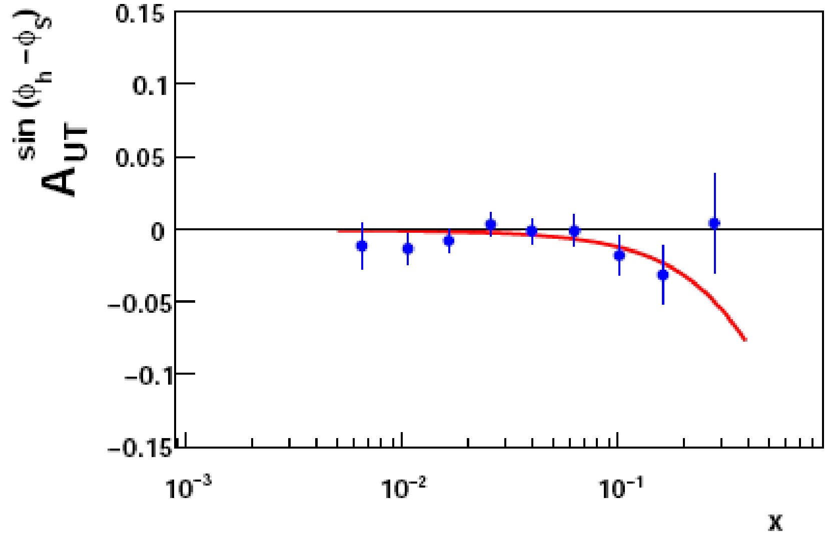
# Results: SIDIS

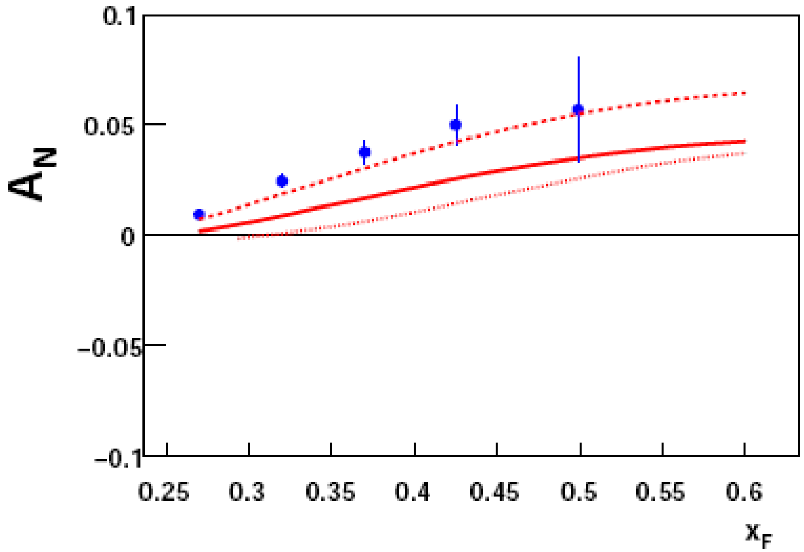


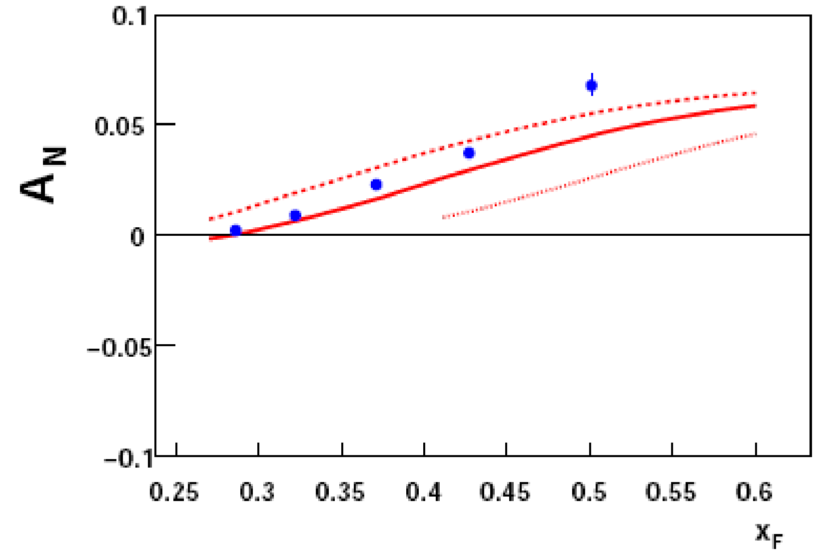




**HERMES data**

**COMPASS data**

# Results: PP



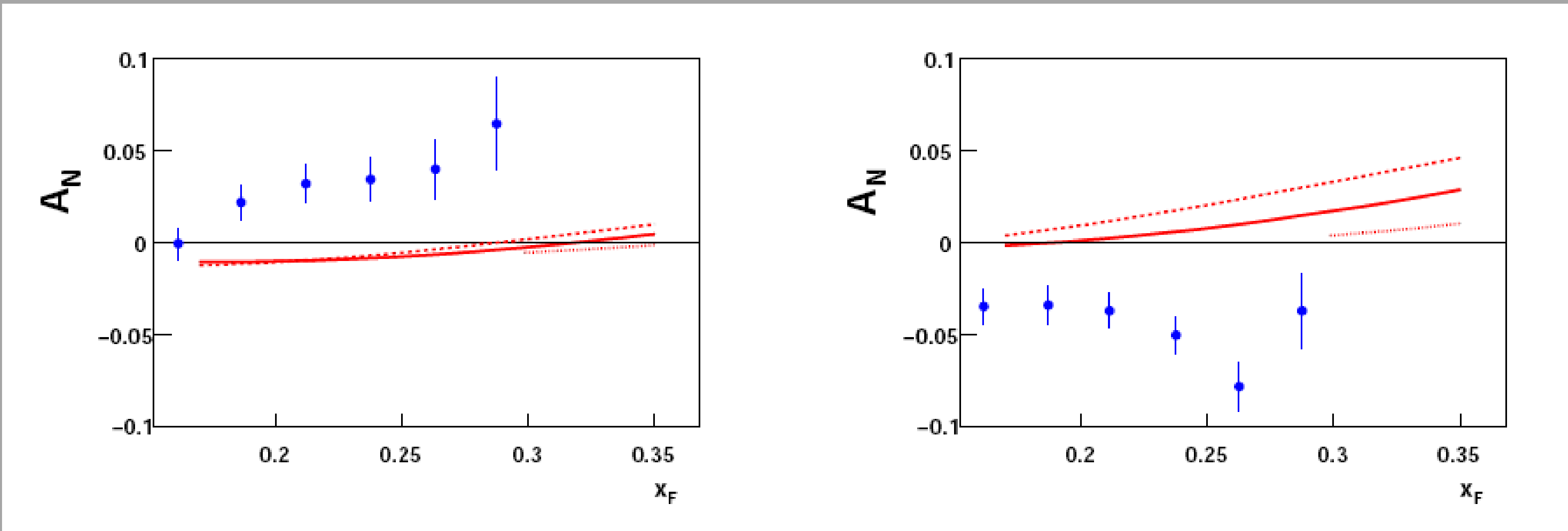




**STAR data** $\pi^0, \ \mathbf{y = 3.7}$**, reasonable description**

$$\mathbf{Q = P_T/2...2PT}$$

# Results: PP

$\pi^+$ $\pi^-$




**BRAHMS data** $\theta = 4^o$ **,** **wrong sign**

**SIGN PUZZLE IS STILL UNRESOLVED!**

# What is missing?

Twist-3 formalism:

$$\mathbf{A_N} \sim \mathbf{T_F} \otimes \sigma \otimes \mathbf{D_1} + \mathbf{h_1} \otimes \sigma \otimes \mathbf{H_F} + \ldots$$

We considered only Sivers effect, Soft Gluon Pole. Other parts should be added: sea-quarks, Soft Fermionic Pole contribution. Fragmentation part:Collins effect in particular.

For global analysis one should combine SIDIS, PP and $e^+e^-$ data

TMD Collins effect in PP: Anselmino et al in preparation

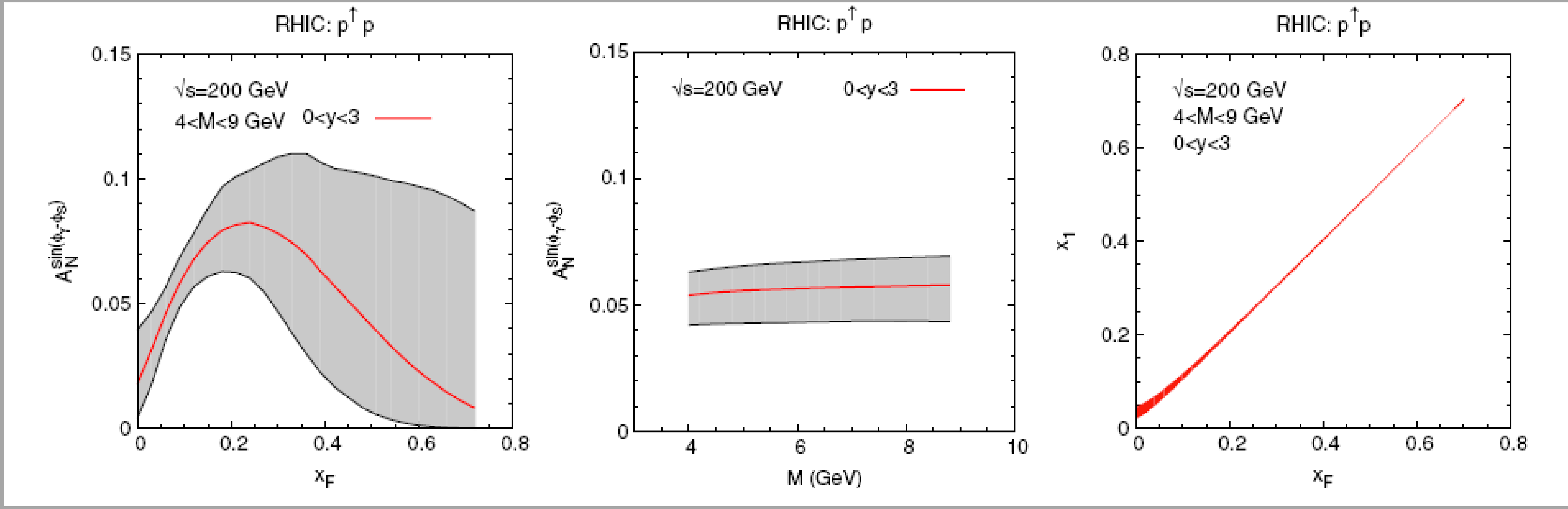
# Drell Yan

$$\mathbf{A_N} = \frac{\sum_q \mathbf{f_{1T}^{\perp q}(x_1, p_T) \otimes f_1^{\bar{q}}(x_1, p_T)}\sigma_{q\bar{q}}}{\sum_q \mathbf{f_1^{q}(x_1, p_T) \otimes f_1^{\bar{q}}(x_1, p_T)}\sigma_{q\bar{q}}}$$

Analysis at LO in hadronic cm frame
Anselmino et al (2009)

$$\mathbf{x_1} = \frac{\mathbf{x_F} + \sqrt{\mathbf{x_F^2} + 4\mathbf{M^2}/\mathbf{s}}}{2} \approx \mathbf{x_F}$$

In DY we probe Sivers function at $\mathbf{x_F}$
Anselmino et al (2009)

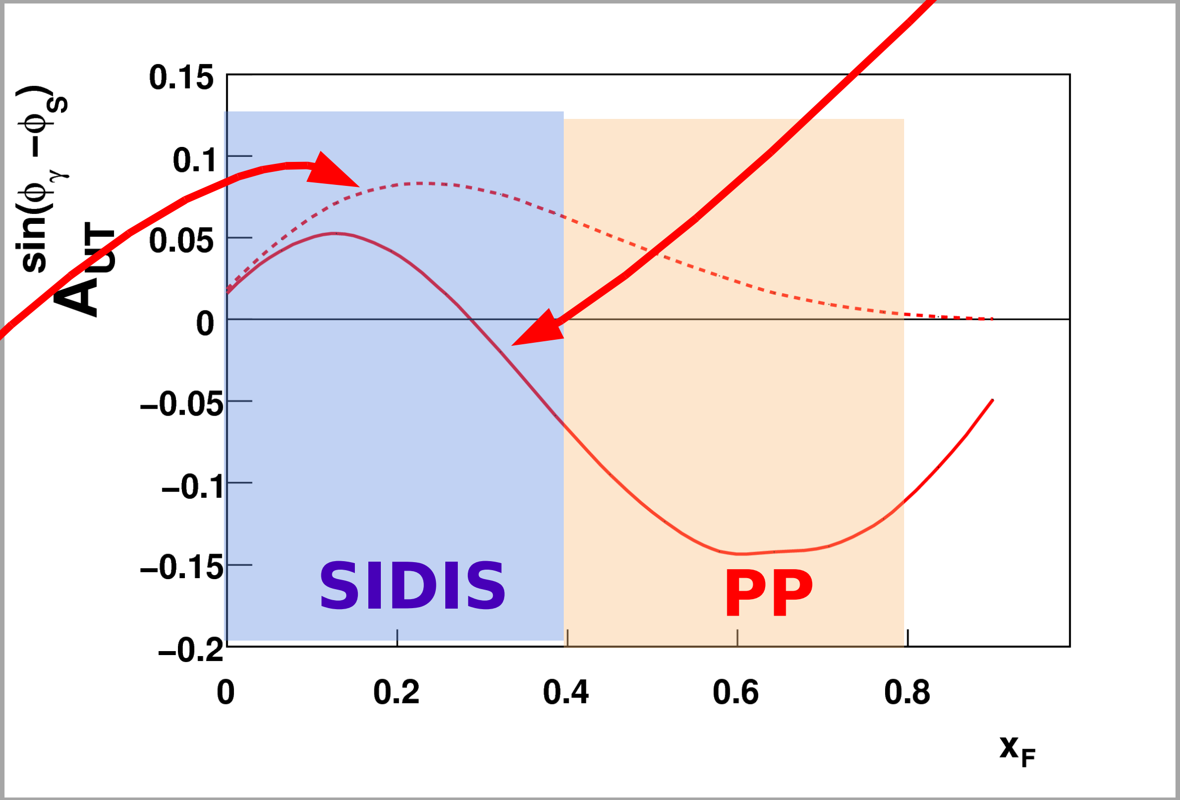


# Drell Yan

$$A_N = \frac{\sum_q f_{1T}^{\perp q}(x_1, p_T) \otimes f_1^{\bar{q}}(x_1, p_T)\sigma_{q\bar{q}}}{\sum_q f_1^{q}(x_1, p_T) \otimes f_1^{\bar{q}}(x_1, p_T)\sigma_{q\bar{q}}}$$

**Analysis at LO in hadronic cm frame**

Kang, AP (2011)

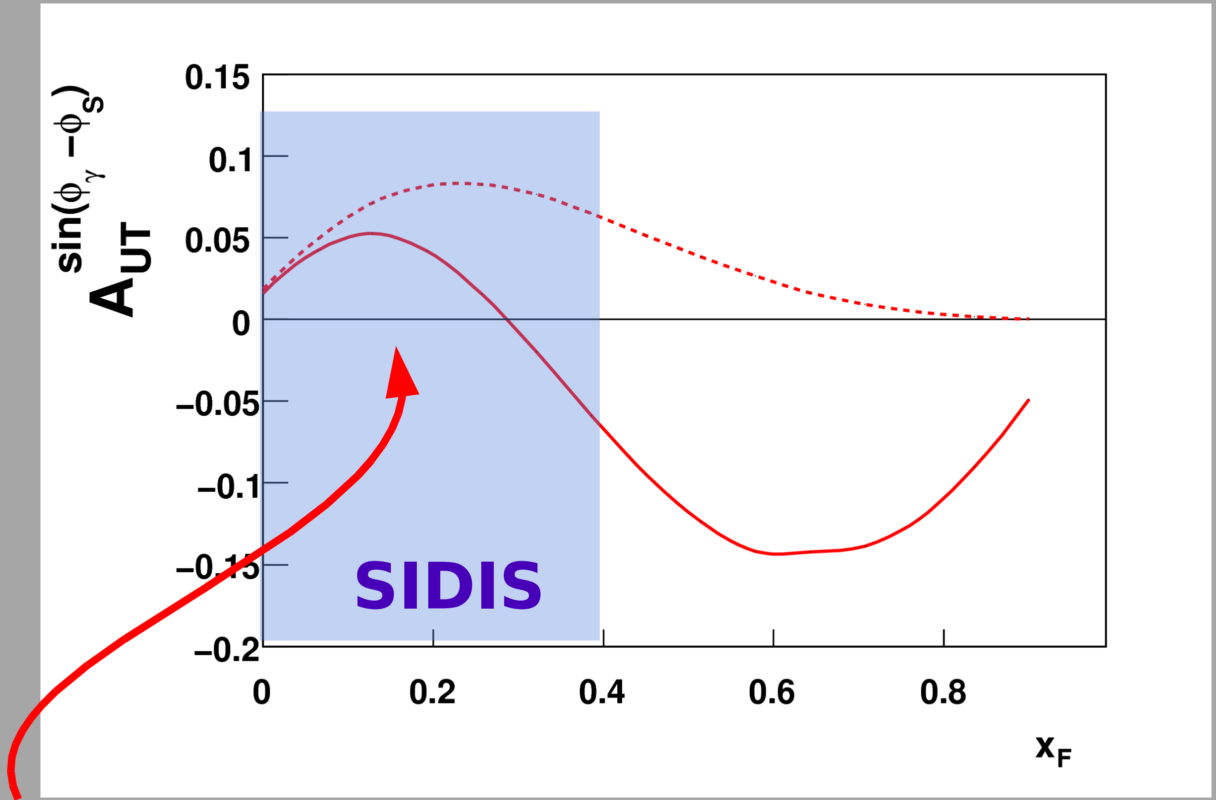



Anselmino et al (2009) no node

# Drell Yan

$$A_N = \frac{\sum_q f_{1T}^{\perp q}(x_1, p_T) \otimes f_1^{\bar{q}}(x_1, p_T)\sigma_{q\bar{q}}}{\sum_q f_1^q(x_1, p_T) \otimes f_1^{\bar{q}}(x_1, p_T)\sigma_{q\bar{q}}}$$

**Analysis at LO in hadronic cm frame**
Kang, AP (2011)



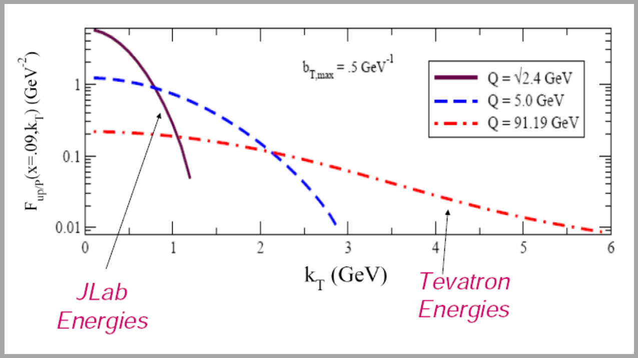

**To measure in order to check**

$$-f_{1T}^{\perp}|_{DY} = f_{1T}^{\perp}|_{SIDIS}$$

# TMD&Twist-3 phenomenology

Global analysis of SIDIS, PP and $e^+e^-$ data using TMD and twist-3 formalisms.
Kang, AP (2012), ...

TMD phenomenology:
NLO accuracy
Collins (2011), Aybat, Rogers (2011), ...




Twist-3 phenomenology:
NLO accuracy of hard functions
Vogelsang, Yuan (2009), ...

$$A_N \propto \Delta\sigma(Q, S_\perp) \propto T_f^{(3)}(x,x) \otimes \hat{H}_f \otimes \ldots$$

**Beyond LO!**

# CONCLUSIONS

- TMD phenomenology is possible with evolution
- HERMES and COMPASS data are compatible with TMD evolution
- Future measurements at Electron Ion Collider and Drell-Yan experiments at COMPASS are important for both confirmation of sign change of Sivers function and TMD evolution effects.